Flow through Granular Bed & Packed Columns

Introduction

- **The presence of solid particles in a bed or a column will cause a pressure drop when the fluid passes through it.**
- **Examples: Dryers used solid material, Gas absorption columns using different types of packing, Sand filters.**
- **Formulating the pressure drop in terms of the controlling properties and parameters will lead to an important design formula.**

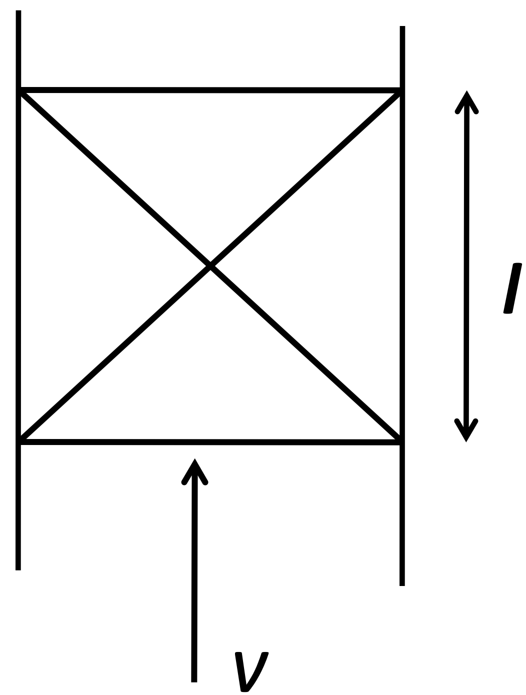
Introduction

Finding the pressure drop through a bed is also a one method used to measure the surface area and the particle size of a powder material as mentioned in the beginning of the course.

This is called Permeability cell technique for measuring the particle size of a powder material

Flow of a single fluid through a granular bed

Packed bed



Darcy's Law and Permeability

- Darcy observed that the flow of water through a packed bed of sand was governed by the relationship:

$$\begin{bmatrix} pressure \\ gradient \end{bmatrix} \alpha \begin{bmatrix} liquid \\ velocity \end{bmatrix} \text{ or } \frac{(-\Delta p)}{l} \alpha \ u$$

Where

$-\Delta p$: pressure drop across the bed

l : bed depth

u : superficial velocity= fluid volumetric flow rate/cross sectional area, ***(1/A) (dv/dt)***

A : cross sectional area of the bed.

$$\therefore \quad u = -K\left(\frac{\Delta P}{l}\right) = -\left(\frac{B}{\mu}\right)\left(\frac{\Delta P}{l}\right)$$

where

K : proportionality constant that depends on the properties of bed and fluid = B/μ.

B : permeability coeff. of the bed ‘depends on the properties of solid particles’.

μ : viscosity of the fluid.

Specific surface and voidage

(i) specific surface area of the bed, $S_B \Rightarrow$ 'surface area of the bed which is available to hold the fluid per unit volume of the bed' m^2/m^3 or $[length]^{-1}$.

(ii) fractional voidage of the bed, e $\Rightarrow$'fraction of volume of bed that not occupied by solid material'. (1-e) represents the fraction of bed volume that occupied by solid material.

Notes

- **S : sp. Surface area of particles. For a sphere; S=6/d**
- **$S \neq S_B$ 'due to voidage, e'**
- **$S_B = S(1-e)$**
- *As* **e** *is increased, flow through the bed becomes easier and so the permeability coefficient* ***B*** *increases.*
- If the particles are **randomly packed**, then **e** *should be approximately constant* throughout the bed and the resistance to flow is the same in all directions.

Table 4.1 shows the properties of beds (S, e, B) of some common regular-shaped materials.

See also the text

Table 4.1. Properties of beds of some regular-shaped materials[2]

Solid constituents		Porous mass		
No.	Description	Specific surface area S (m^2/m^3)	Fractional voidage, e (–)	Permeability coefficient B (m^2)
	Spheres			
1	0.794 mm diam. ($\frac{1}{32}$ in.)	7600	0.393	6.2×10^{-10}
2	1.588 mm diam. ($\frac{1}{16}$ in.)	3759	0.405	2.8×10^{-9}
3	3.175 mm diam. ($\frac{1}{8}$ in.)	1895	0.393	9.4×10^{-9}
4	6.35 mm diam. ($\frac{1}{4}$ in.)	948	0.405	4.9×10^{-8}
5	7.94 mm diam. ($\frac{5}{16}$ in.)	756	0.416	9.4×10^{-8}
	Cubes			
6	3.175 mm ($\frac{1}{8}$ in.)	1860	0.190	4.6×10^{-10}
7	3.175 mm ($\frac{1}{8}$ in.)	1860	0.425	1.5×10^{-8}
8	6.35 mm ($\frac{1}{4}$ in.)	1078	0.318	1.4×10^{-8}
9	6.35 mm ($\frac{1}{4}$ in.)	1078	0.455	6.9×10^{-8}

Table 4.3. Design data for various packings

	Size (in.)	Size (mm)	Wall thickness (in.)	Wall thickness (mm)	Number (/ft^3)	Number (/m^3)	Bed density (lb/ft^3)	Bed density (kg/m^3)	Contact surface S_B (ft^2/ft^3)	Contact surface S_B (m^2/m^3)	Free space % (100 e)	Packing factor F (ft^2/ft^3)	Packing factor F (m^2/m^3)
Ceramic Raschig Rings	0.25	6	0.03	0.8	85,600	3,020,000	60	960	242	794	62	1600	5250
	0.38	9	0.05	1.3	24,700	872,000	61	970	157	575	67	1000	3280
	0.50	12	0.07	1.8	10,700	377,000	55	880	112	368	64	640	2100
	0.75	19	0.09	2.3	3090	109,000	50	800	73	240	72	255	840
	1.0	25	0.14	3.6	1350	47,600	42	670	58	190	71	160	525
	1.25	31			670	23,600	46	730			71	125	410
	1.5	38			387	13,600	43	680			73	95	310
	2.0	50	0.25	6.4	164	5790	41	650	29	95	74	65	210
	3.0	76			50	1765	35	560			78	36	120
Metal Raschig Rings	0.25	6	0.03	0.8	88,000	3,100,000	133	2130			72	700	2300
	0.38	9	0.03	0.8	27,000	953,000	94	1500			81	390	1280
	0.50	12	0.03	0.8	11,400	402,000	75	1200	127	417	85	300	980
	0.75	19	0.03	0.8	3340	117,000	52	830	84	276	89	185	605

Raschig rings

Raschig rings are pieces of tube (approximately equal in length and diameter) used in large numbers as a packed bed within columns for distillations and other chemical engineering processes

Structured packing

Structured packing refers to a range of specially designed materials for use in absorption, distillation columns , chemical reactors and cooling towers.

Pall rings

Saddles

General expressions for flow through Beds

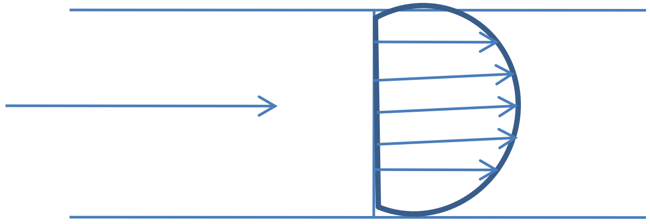
- **The flow of fluid through a bed can be analyzed in terms of the fluid flow through tubes.**

- ***Streamline flow***

  Start with the Hagen-Poiseuille equation for laminar flow through a circular tube

$$\frac{(-\Delta P)}{l} = \frac{32\mu\, u_t}{d_t^2} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

OR



$$u_t = \frac{d_t^2}{32\mu} \frac{(-\Delta P)}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

- **But in case of a packed bed, the free space can be assumed to form of a series of tortuous (twist) channels. Therefore, the previous eq. can be modified as follows:**

$$u_1 = \frac{d_{m'}^2}{K'\mu}\frac{(-\Delta P)}{l'} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where

$d_{m'}$:equivalent diam of the pore channels

K':dimensionless cons. depends on the bed structure.

l' :average length of Tortuous path of Capillaries.

u_1:average velocity through the pore channels.

Since the equivalent diameter = flow area/wetted perimeter

For a packed bed; flow area $= e A$

Wetted perimeter = surface area of bed/l

$$= S_B A l / l = S_B A$$

$$\therefore dm' = \frac{\text{flow area}}{\text{wetted perimeter}} = \frac{e}{S_B} = \frac{e}{S(1-e)}$$

average velocity, $u_1 = u / e$

and, $l' \alpha l$

Equation (3) becomes

$$u = \frac{1}{K''} \frac{e^3}{S^2 (1-e)^2} \frac{1}{\mu} \frac{(-\Delta P)}{l} \quad \text{................}(4)$$

This equation is called Carman-Kozeny equation.

The constant K'' depends on porosity, particle shape and other factors.

In general, K'' = 5.0 .

Note 1

Compare Carman-Kozeny equation with Darcy equation, you can deduce that

$$B = \frac{1}{K''} \frac{e^3}{S^2(1-e)^2}$$

Note 2

For spherical particle $S = 6/d$

$$u = \frac{1}{180} \frac{e^3 d^2}{(1-e)^2} \frac{1}{\mu} \frac{(-\Delta P)}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Note3

For non-spherical particle, the surface mean diameter, d_s , can be substituted instead of d

Streamline and Turbulent Flow

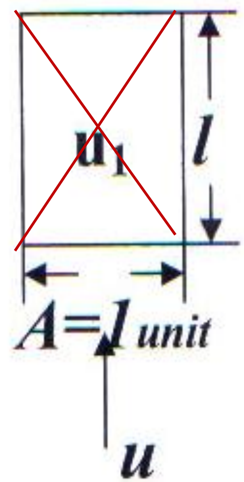
Modified Reynolds number, Re_1 :

$$Re_1 = \left(\frac{u}{e}\right) \frac{e}{S(1-e)} \frac{\rho}{\mu} = \frac{u\rho}{\mu(1-e)S} = B$$

Vol of particles in bed = *$l\,A\,(1-e) = l\,(1-e)$*

total surface = *$S\,A\,l\,(1-e) = S\,l\,(1-e)$*

the friction factor= $\frac{R_1}{\rho u_1^2}$



where R_1 is the drag force per unit wetted projected surface area.

The resistance force or drag force = $R_1 S l (1-e)$

This force equals to the force resulting from a pressure difference ΔP across the bed.

$$(-\Delta P)(A)(e) = R_1 S l (1-e)$$

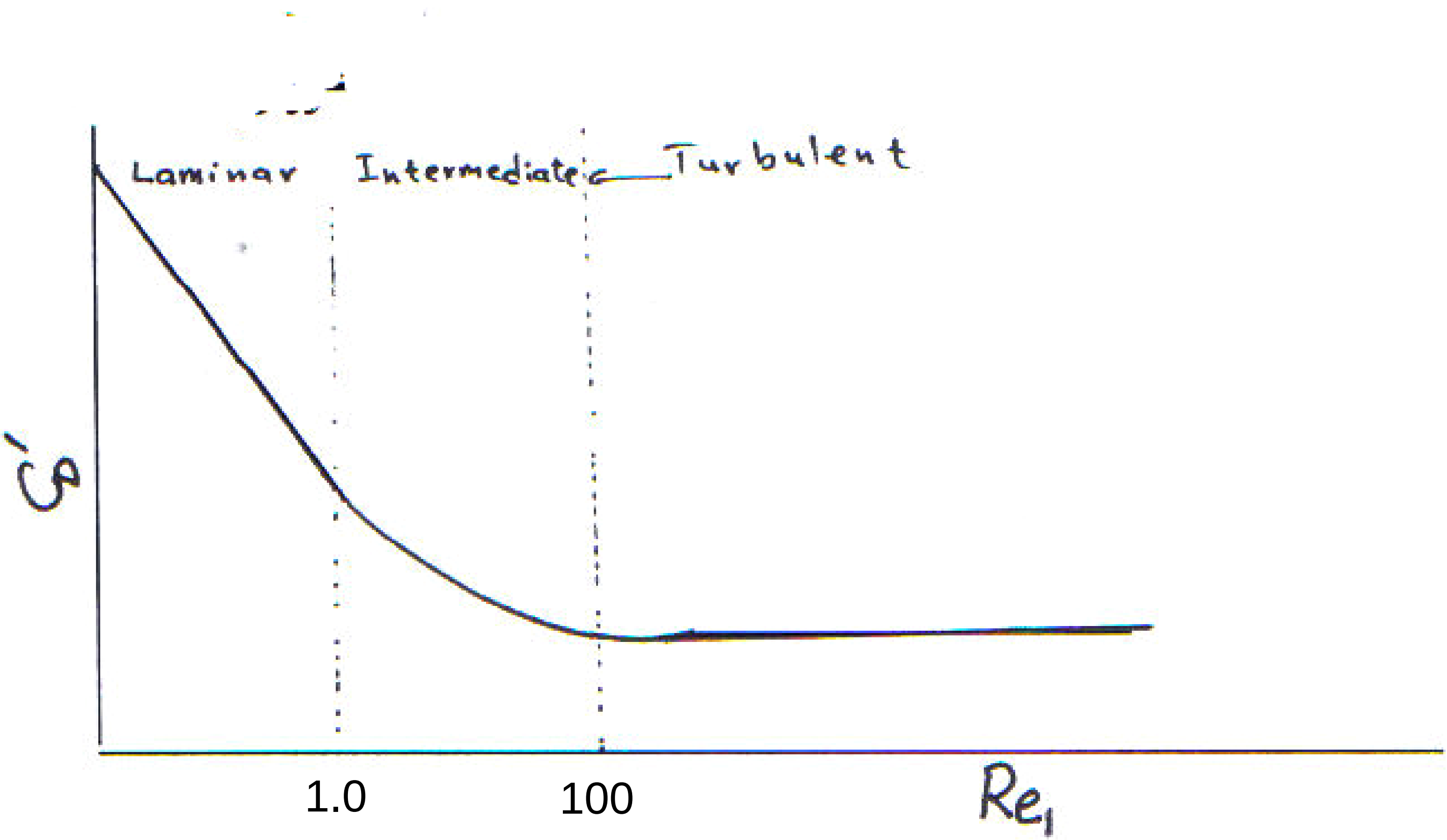
$$(-\Delta P)(e) = R_1 S l (1-e)$$

$$R_1 = e(-\Delta P)/ S(1-e)l$$

$$C_D{}' = \frac{R_1}{\rho u_1^2} = \frac{e}{S(1-e)} \cdot \frac{(-\Delta P)}{l} \cdot \frac{1}{\rho (\frac{u}{e})^2}$$

$$= \frac{e^3}{S(1-e)} \cdot \frac{(-\Delta P)}{l} \cdot \frac{1}{\rho u^2}$$

C_D against Re_l



Carman curve ***can be approximated by this eq.:***

$$\frac{R_1}{\rho u_1^2} = 5\, Re_1^{-1} + 0.4\, Re_1^{-0.1}$$

The 1st term $5/Re_1$ represents friction coefficient For streamline flow $Re_1<1.0$.

For intermediate flow through a packed bed, the friction coefficient is the sum of the two terms $5/Re_1+0.4/Re_1^{0.1}$.

For turbulent regime $Re_1>100$, the friction coefficient is
$0.4/Re_1^{0.1}$

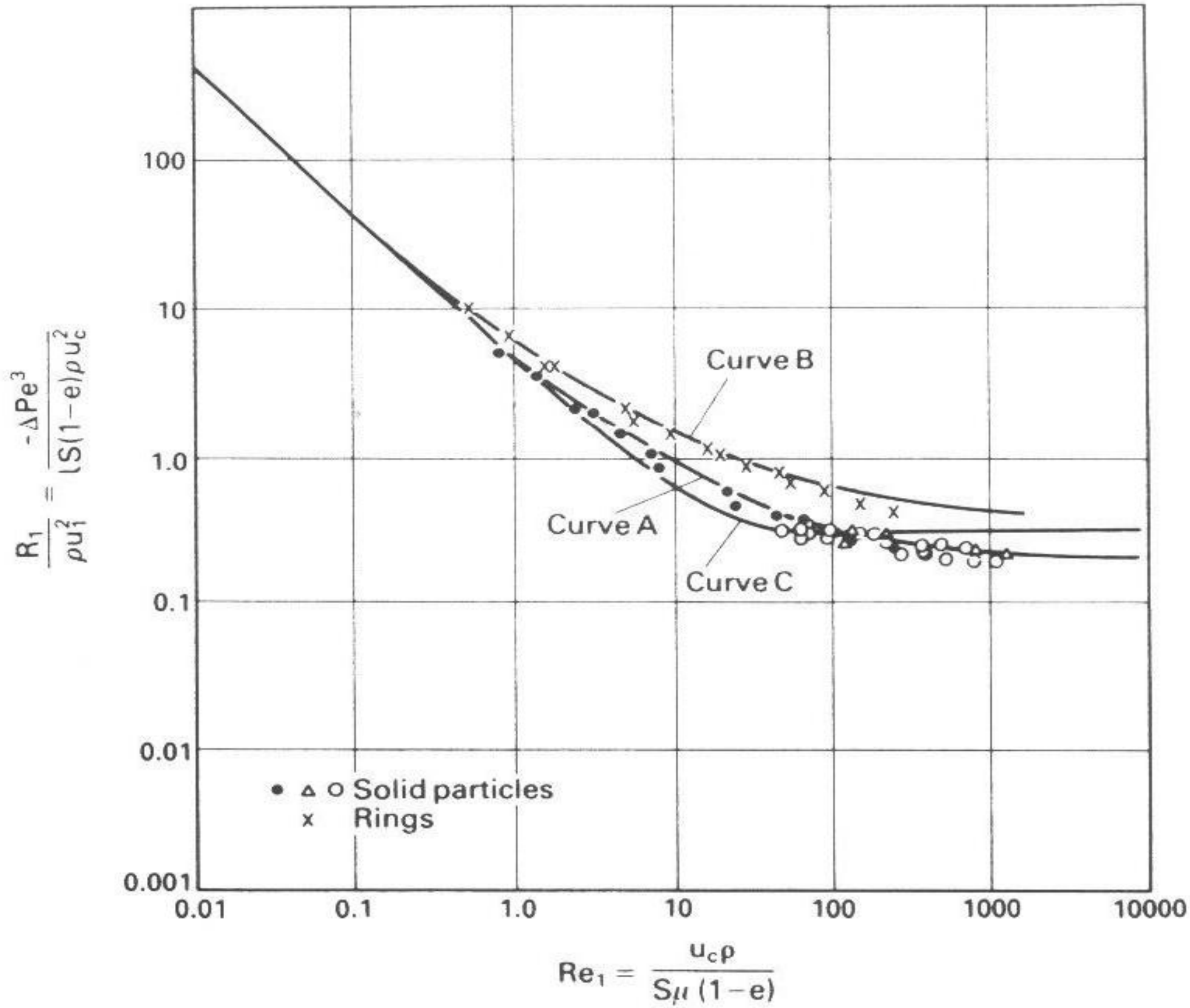


FIG. 4.1. Carman's graph of $R_1/\rho u_1^2$ against Re_1

Note:

- **The constant K'' is usually dependent on the structure of the bed, the shape of the cross-section of a channel, solid size and voidage. In general, $K'' = (l'/l)^2 K_o$ 'see the text'**
- **Wall effect ' f_w'**

  $f_w = (1+0.5S_c/S)^2$

  where S_c is the surface of the container per unit volume of bed.

  This correction factor **' f_w' must be multiplied by Kozeny equation.**

where (l'/l) is the tortuosity and is a measure of the fluid path length through the bed compared with the actual depth of the bed,

K''_o is a factor which depends on the shape of the cross-section of a channel **through which fluid is passing.**

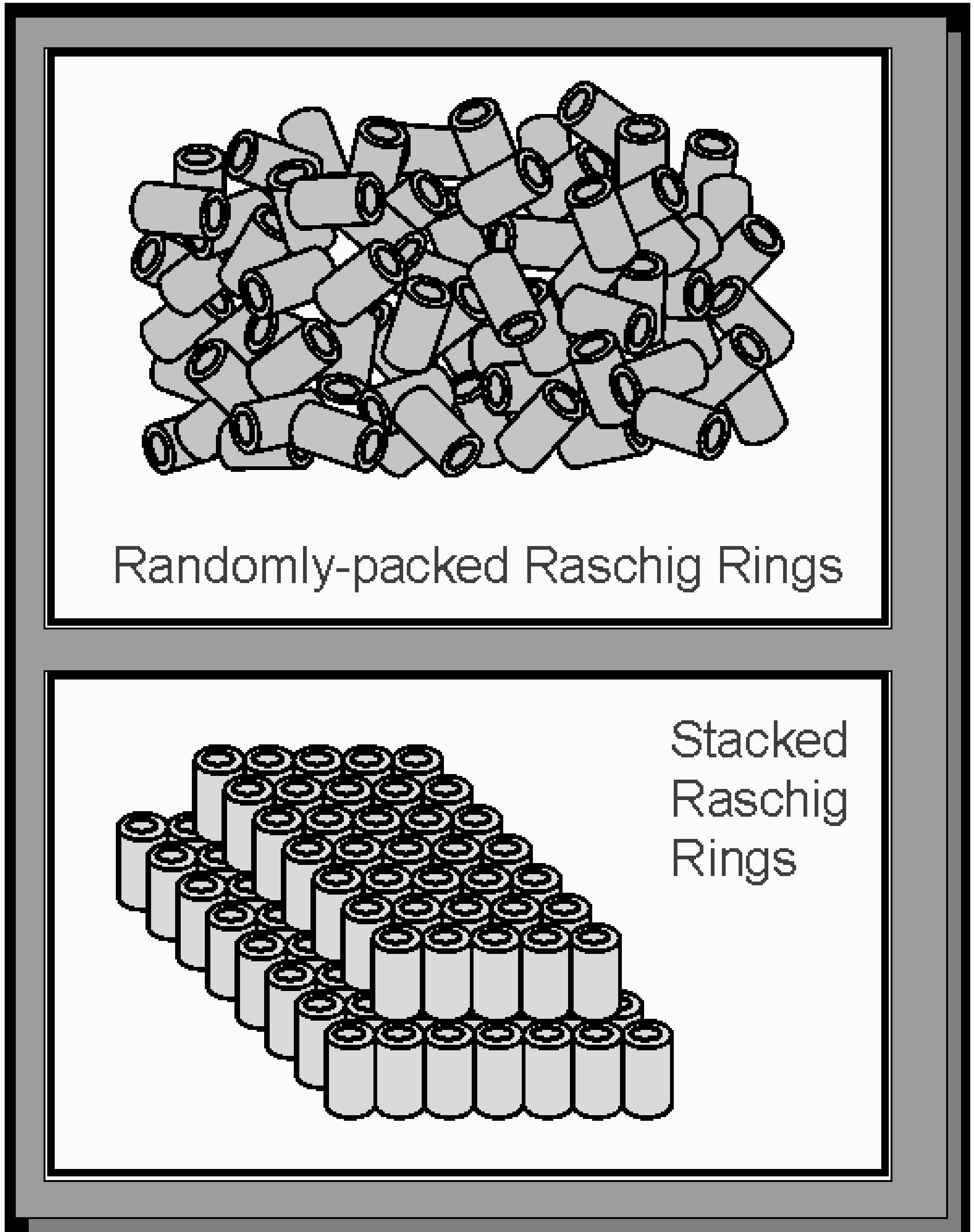
Table A Comparison between packed bed & packed column

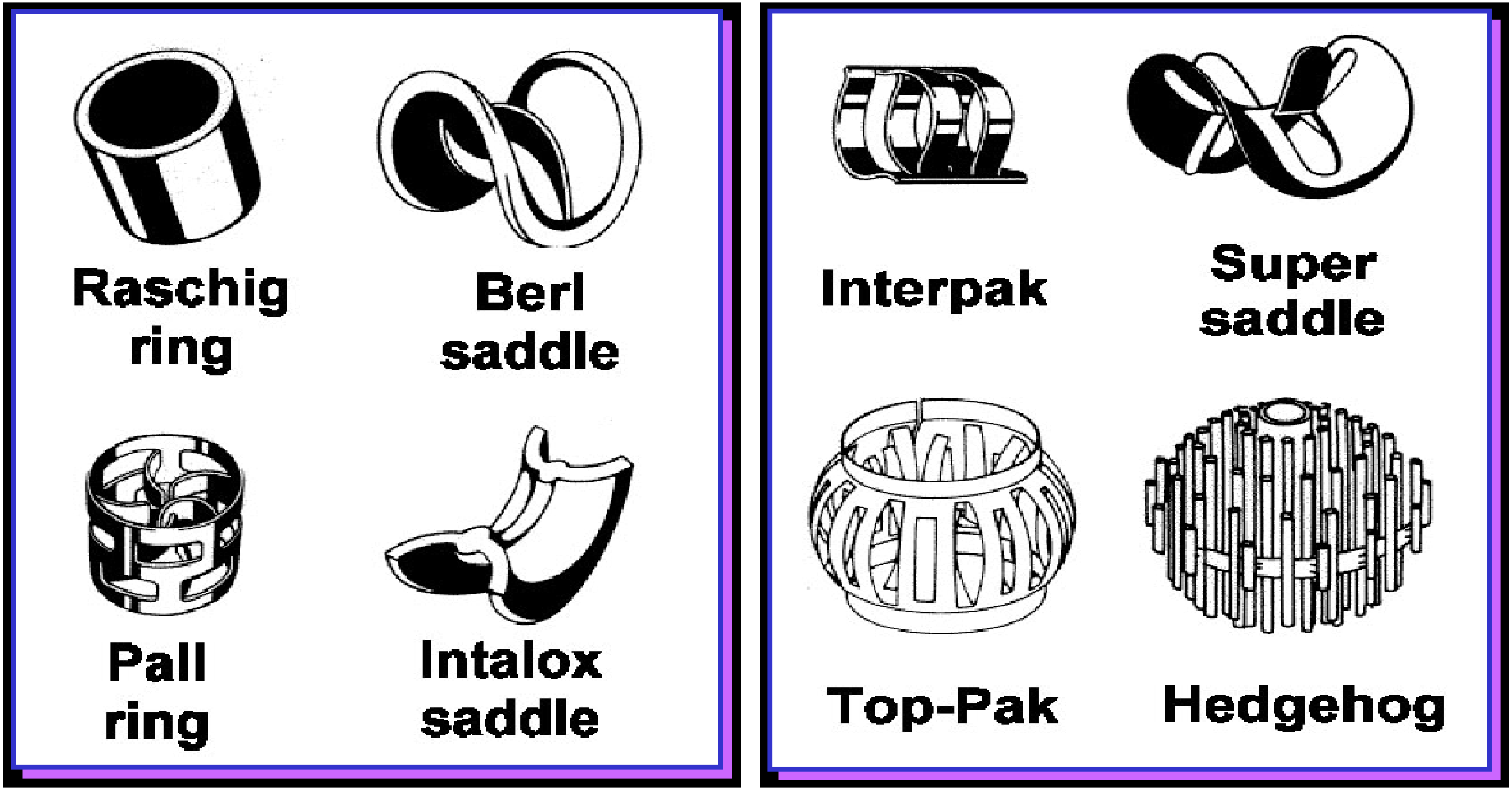
Packed bed	Packed column
Single flow through bed	Usually, two flows(liquid +gas)
Size of packing: small	Size of packing: large
Packing element: usually solid	Packing element: usually hollow with large internal surface area and small pressure gradient.
Regime of flow: Laminar	Regime of flow: Turbulent

Types of Packing

1. **Particulate: dumped or stacked Raschig Rings, Lessing rings, Berl Saddles, Stoneware, Porcelain, Carbon, Metal.**
2. **Grid Packings: wood, metal, carbon, plastic.**
3. **Wire-Mesh & Knit-mesh packing**

‘ See the Text for details’


Randomly-packed Raschig Rings
Stacked
Raschig
Rings


Raschig ring
Berl saddle
Pall ring
Intalox saddle
Interpak
Super saddle
Top-Pak
Hedgehog

Multi knit mesh columns packing

Stainless steel knitted wire mesh packing

TABLE 4.1. *Properties of Beds of Some Regular-shaped Materials*[2]

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No.	Description	Specific surface area S (m²/m³)	Fractional voidage, e	Permeability coefficient B (m²)
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1	$\frac{1}{32}$ in. diam. (0.794 mm)	7600	0.393	6.2×10^{-10}
2	$\frac{1}{16}$ in. diam. (1.588 mm)	3759	0.405	2.8×10^{-9}
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5	$\frac{5}{16}$ in. diam. (7.94 mm)	756	0.416	9.4×10^{-8}
	Cubes			
6	$\frac{1}{8}$ in. (3.175 mm)	1860	0.190	4.6×10^{-10}
7	$\frac{1}{8}$ in. (3.175 mm)	1860	0.425	1.5×10^{-8}
8	$\frac{1}{4}$ in. (6.35 mm)	1078	0.318	1.4×10^{-8}
9	$\frac{1}{4}$ in. (6.35 mm)	1078	0.455	6.9×10^{-8}
	Hexagonal prisms			
10	$\frac{3}{16}$ in. $\times$ $\frac{3}{16}$ in. thick (4.76 mm $\times$ 4.76 mm)	1262	0.355	1.3×10^{-8}
11	$\frac{3}{16}$ in. $\times$ $\frac{3}{16}$ in. thick (4.76 mm $\times$ 4.76 mm)	1262	0.472	5.9×10^{-8}
	Triangular pyramids			
12	$\frac{1}{4}$ in. length $\times$ 0.113 in. ht. (6.35 mm $\times$ 2.87 mm)	2410	0.361	6.0×10^{-9}
13	$\frac{1}{4}$ in. length $\times$ 0.113 in. ht. (6.35 mm $\times$ 2.87 mm)	2410	0.518	1.9×10^{-8}
	Cylinders			
14	$\frac{1}{8}$ in. diam. $\times$ $\frac{1}{8}$ in. (3.175 mm $\times$ 3.175 mm)	1840	0.401	1.1×10^{-8}
15	$\frac{1}{8}$ in. diam. $\times$ $\frac{1}{4}$ in. (3.175 mm $\times$ 6.35 mm)	1585	0.397	1.2×10^{-8}
16	$\frac{1}{4}$ in. diam. $\times$ $\frac{1}{4}$ in. (6.35 mm $\times$ 6.35 mm)	945	0.410	4.6×10^{-8}
	Plates			
17	$\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. $\times$ $\frac{1}{32}$ in. (6.35 mm $\times$ 6.35 mm $\times$ 0.794 mm)	3033	0.410	5.0×10^{-9}
18	$\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. $\times$ $\frac{1}{16}$ in. (6.35 mm $\times$ 6.35 mm $\times$ 1.59 mm)	1984	0.409	1.1×10^{-8}
	Discs			
19	$\frac{1}{8}$ in. diam. $\times$ $\frac{1}{16}$ in. (3.175 mm $\times$ 1.59 mm)	2540	0.398	6.3×10^{-9}
	Porcelain Berl saddles			
20	0.236 in. (6 mm)	2450	0.685	9.8×10^{-8}
21	0.236 in. (6 mm)	2450	0.750	1.73×10^{-7}
22	0.236 in. (6 mm)	2450	0.790	2.94×10^{-7}
23	0.236 in. (6 mm)	2450	0.832	3.94×10^{-7}
24	Lessing rings (6 mm)	5950	0.870	1.71×10^{-7}
25	Lessing rings (6 mm)	5950	0.889	2.79×10^{-7}

Metal, Ceramic and carbon Raschig and Lessing rings

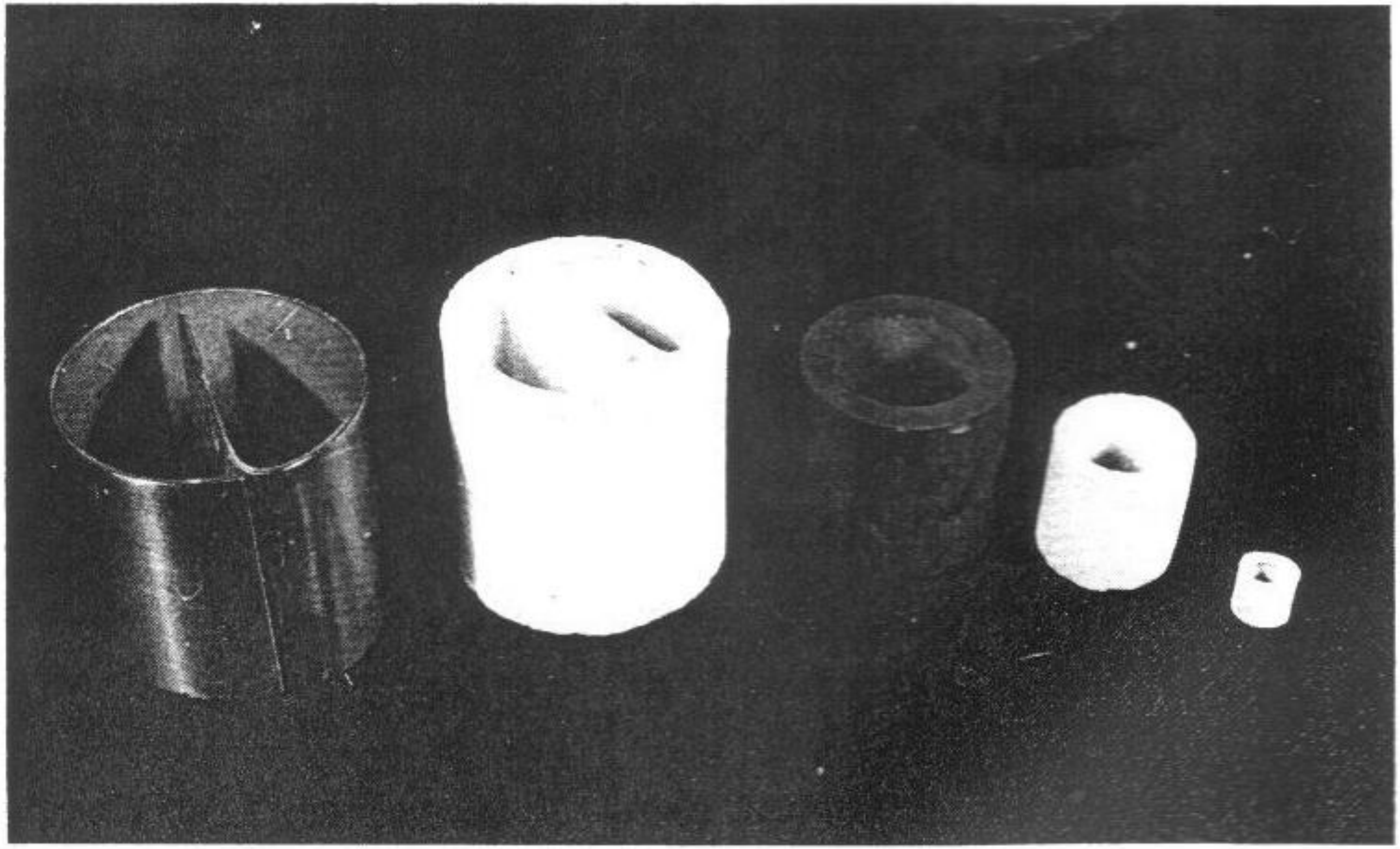


FIG. 4.13. Metal, ceramic and carbon Raschig and Lessing rings

Plastic, Ceramic and metal Pall rings

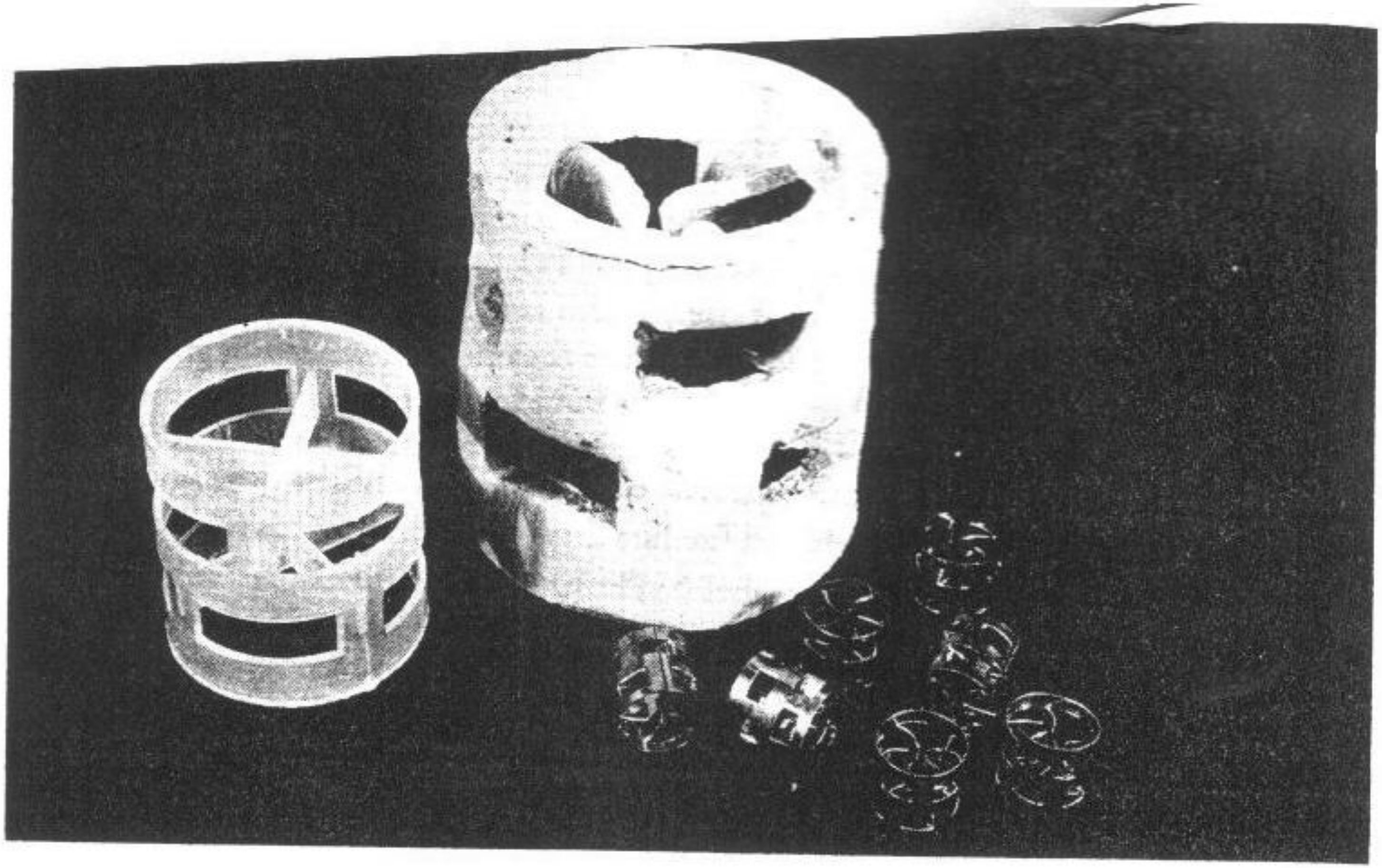


FIG. 4.14. Plastics, ceramic and metal Pall rings

Ceramic Intalox Berl Saddles and Intalox Metal

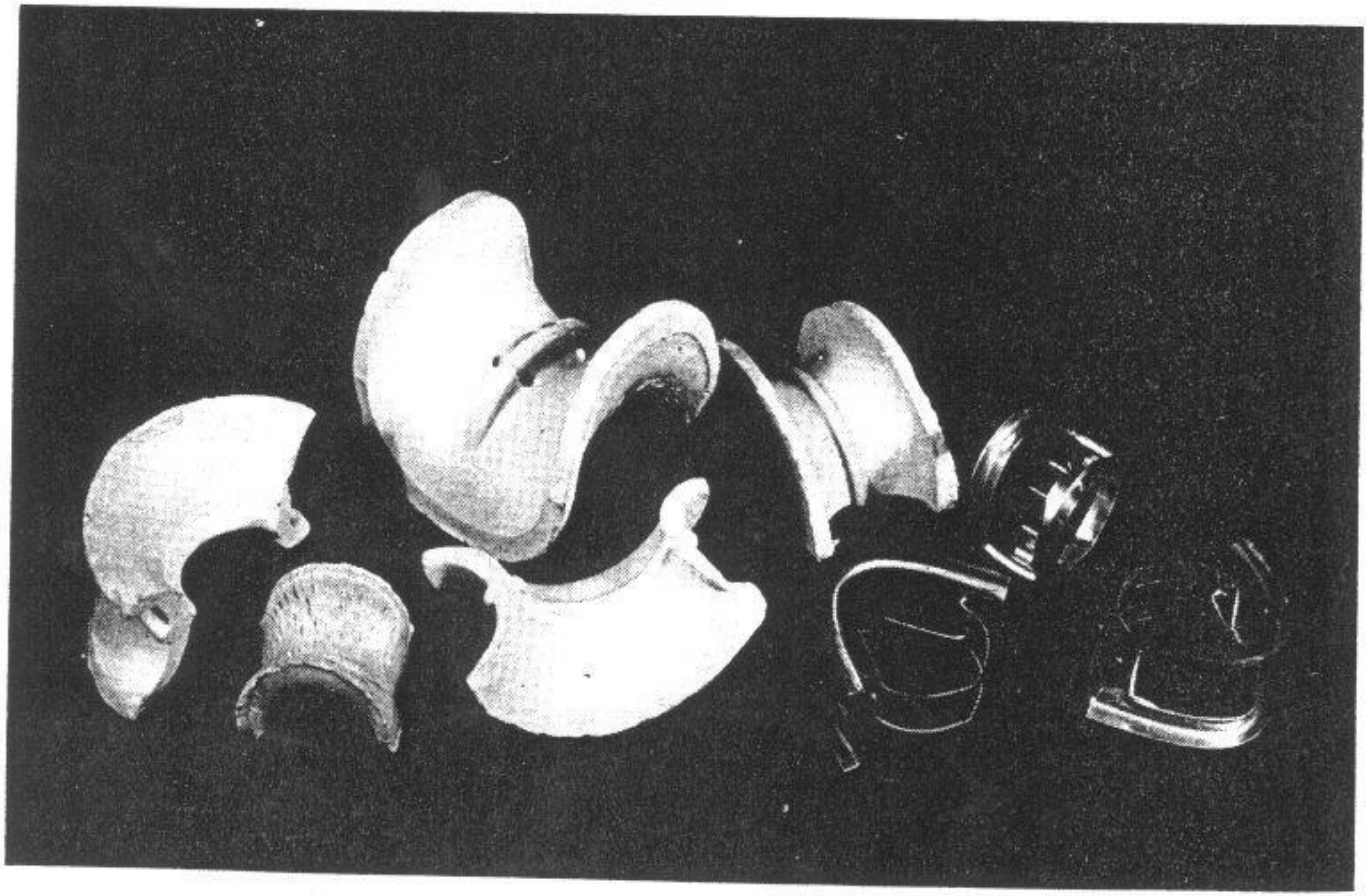


FIG. 4.15. Ceramic Intalox and Berl saddles and Intalox Metal

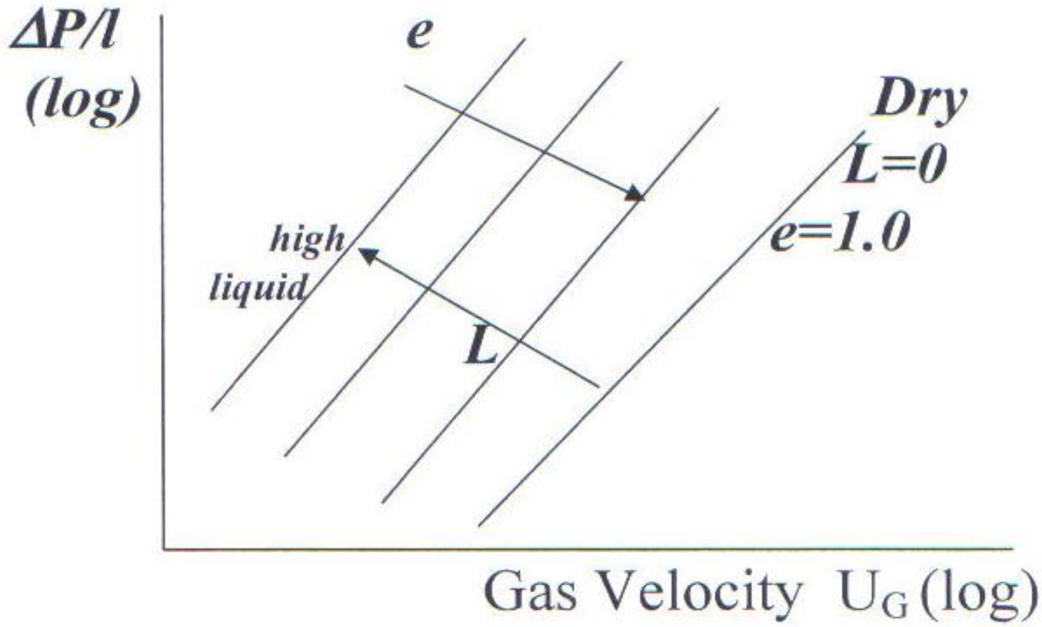
Design Data For Various Packings

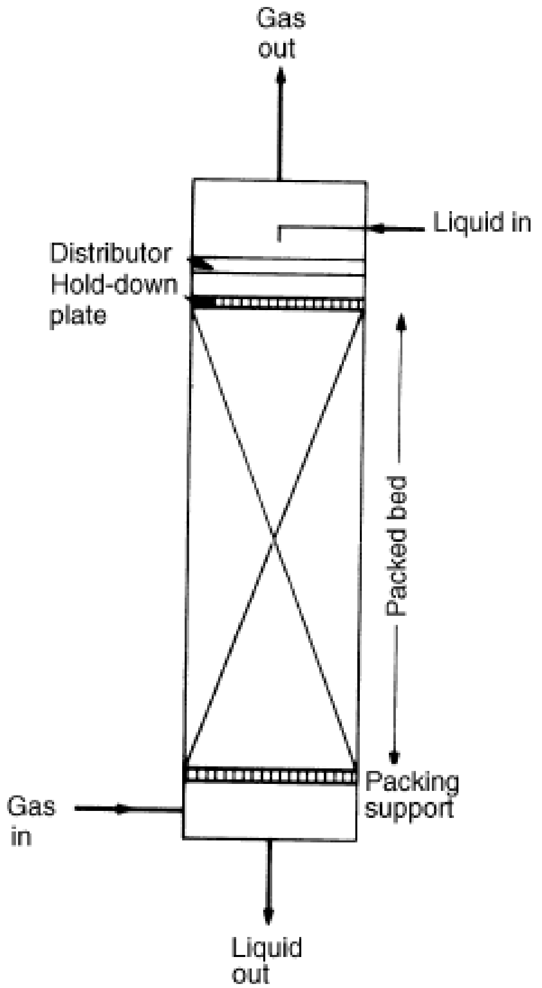
	Size in.	Size mm	Wall thickness in.	Wall thickness mm	Number /ft^3	Number /m^3
Ceramic Raschig Rings	0.25	6	0.03	0.8	85,600	3,020,000
	0.38	9	0.05	1.3	24,700	872,000
	0.50	12	0.07	1.8	10,700	377,000
	0.75	19	0.09	2.3	3090	109,000
	1.0	25	0.14	3.6	1350	47,600
	1.25	31			670	23,600
	1.5	38			387	13,600
	2.0	50	0.25	6.4	164	5790
	3.0	76			50	1765
Metal Raschig Rings	0.25	6	0.03	0.8	88,000	3,100,000
	0.38	9	0.03	0.8	27,000	953,000
	0.50	12	0.03	0.8	11,400	402,000
	0.75	19	0.03	0.8	3340	117,000
(N.B. Bed densities	0.75	19	0.06	1.6	3140	110,000
are for mild	1.0	25	0.03	0.8	1430	50,000
steel; multiply	1.0	25	0.06	1.6	1310	46,200
by 1.105, 1.12, 1.37,	1.25	31	0.06	1.6	725	25,600
1.115 for stainless	1.5	38	0.06	1.6	400	14,100
steel, copper, aluminium,	2.0	50	0.06	1.6	168	5930
and monel respectively)	3.0	76	0.06	1.6	51	1800

	Bed density lb/ft³	Bed density kg/m³	Contact surface S_B ft²/ft³	Contact surface S_B m²/m³	Free space % (100 e)	Packing factor F ft²/ft³	Packing factor F m²/m³
Ceramic Raschig Rings	60	960	242	794	62	1600	5250
	61	970	157	575	67	1000	3280
	55	880	112	368	64	640	2100
	50	800	73	240	72	255	840
	42	670	58	190	71	160	525
	46	730			71	125	410
	43	680			73	95	310
	41	650	29	95	74	65	210
	35	560			78	36	120
Metal Raschig Rings	133	2130			72	700	2300
	94	1500			81	390	1280
	75	1200	127	417	85	300	980
	52	830	84	276	89	185	605
(N.B. Bed densities	94	1500			80	230	750
are for mild	39	620	63	207	92	115	375
steel; multiply	71	1130			86	137	450
by 1.105, 1.12, 1.37,	62	990			87	110	360
1.115 for stainless	49	780			90	83	270
steel, copper, aluminium,	37	590	31	102	92	57	190
and monel respectively)	25	400	22	72	95	32	105

Flow Through Packed Towers

Gas flow through packed tower in presence of liquid flow





$(\Delta P/l)_{gas} \alpha \; U_G^n$

U_G superficial gas velocity < 1m/s
l bed height
L mass flow rate of liquid 'L' liquid mass velocity'
In general, L>1 kg/s.m^2 tower

G mass flow rate of gas 'G′ Gas mass velocity'

e voidage

n index ; $1.6 < n > 2.0 \Rightarrow \therefore n \approx 1.8$

$\therefore$ *In general,*

$$\frac{\Delta P}{l} \alpha U_G^{1.8}$$

Types of towers

- **Dry tower $\Rightarrow$ Single gas flow ($-\Delta P/l$); *use* modified Reynolds number, Re_1 . '$Re_1>30$'**
- **Wet Drained Tower L=0.0**

$$(-\Delta P_w) = (1 + k_1/d_n)\,\Delta P_d$$

Where

ΔP_w : pressure drop across the wet drained tower

ΔP_d : Pressure drop across the dry tower

d_n : size of solids 'normal size of element in mm'

k_1: constant, for broken solids: $k_1 = 5.5$, $d_n = 100mm$

For Raschig rings : $k_1 = 3.3$

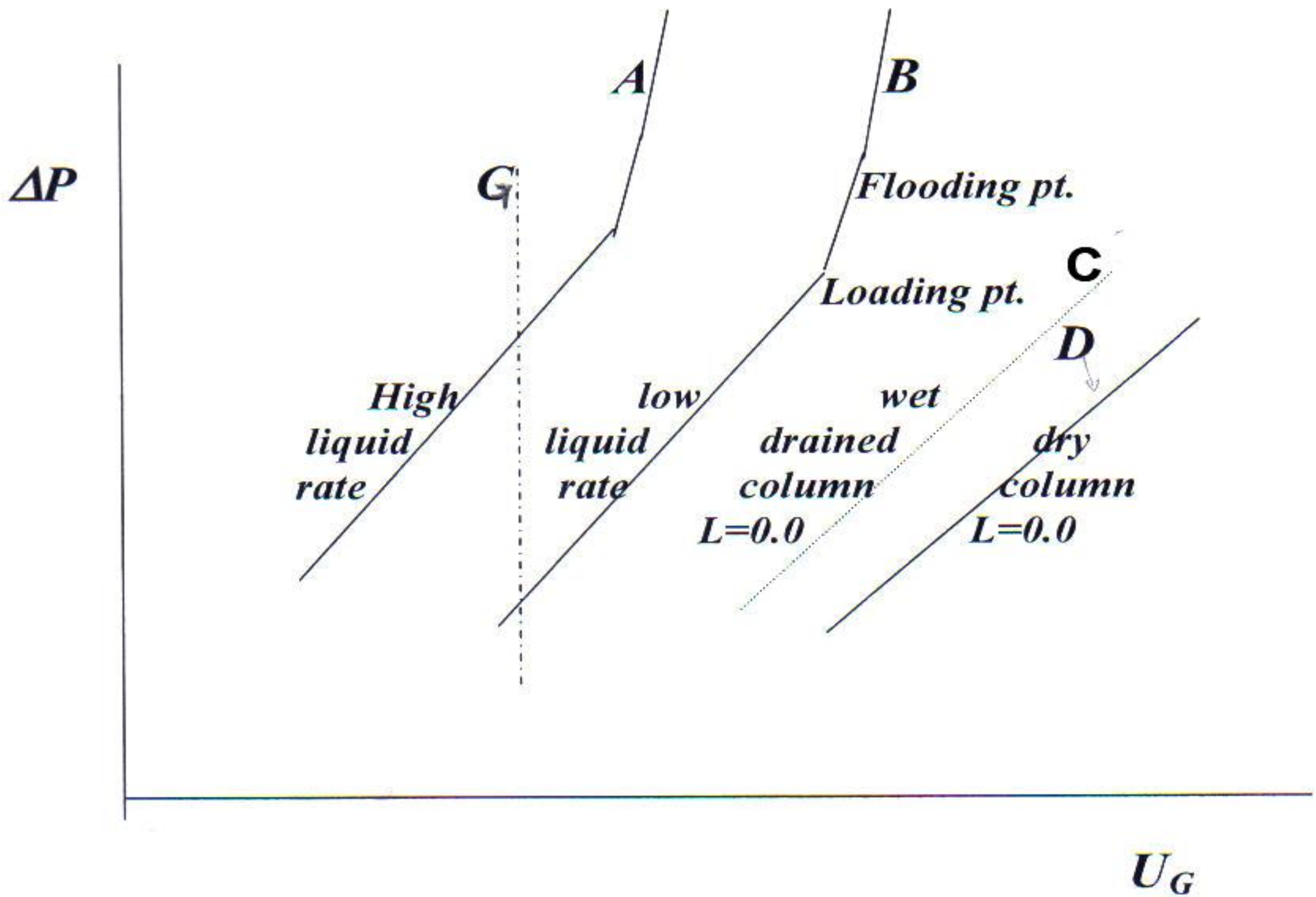
- Irrigated tower $L>0.0$

  To obtain pressure drop use the following Eq.

$$(-\Delta P_i) = (1 + k\, L/d_n)\, \Delta P_d$$

For details see the text and the Perry

Loading & Flooding Points



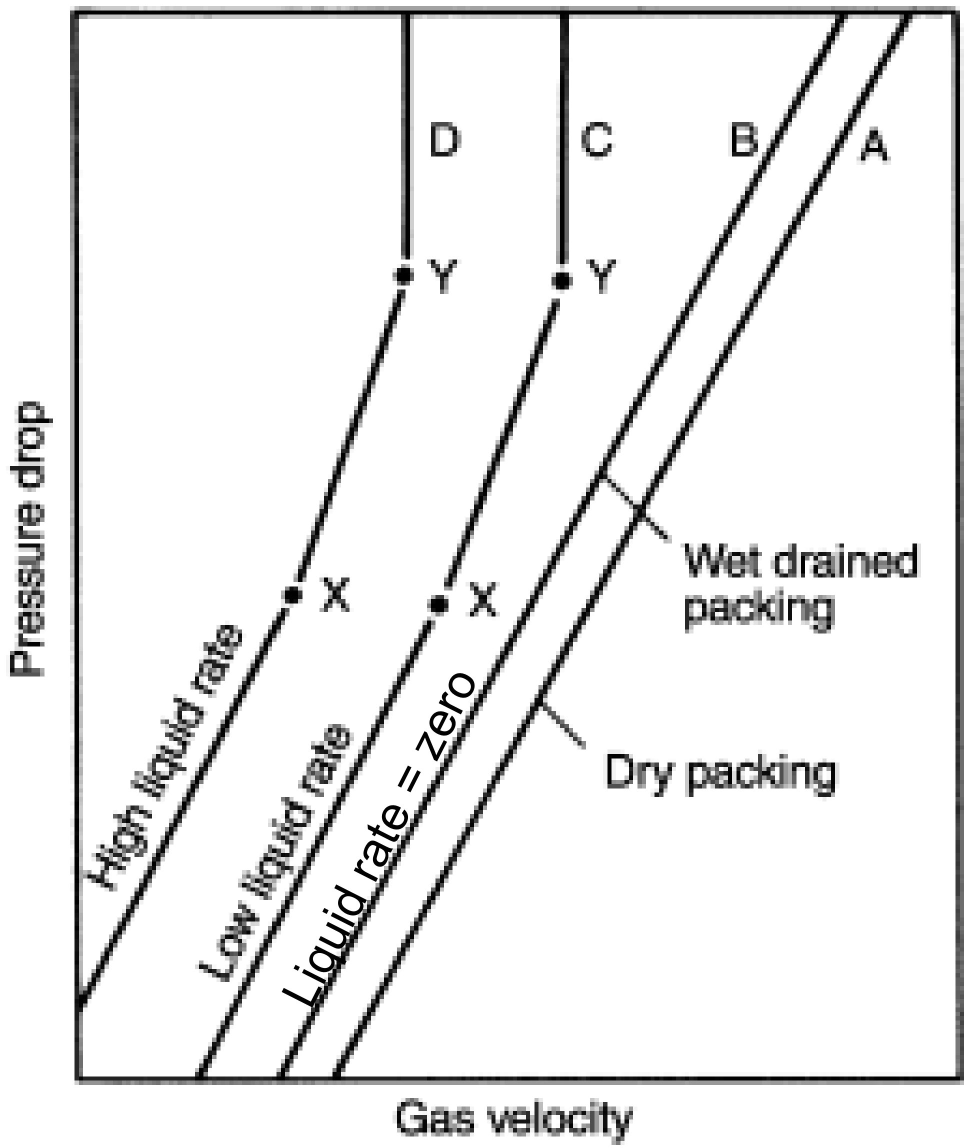
- ***In general, the pressure drop of a packed column is influenced by both gas and liquid flow rates as shown above.***

- ***For a constant gas velocity $\Delta P \uparrow$ as L $\uparrow$ line G.***

- ***Each type of packing has a certain void fraction available for liquid passage; as L $\uparrow$ the voids or e $\downarrow$ since part of it will be filled by liquid, hence reducing the cross-sectional area available for gas flow.***

- ***Consider transparent column and line A or B.***

Pressure drops in wet packing (logarithmic axes)



See the text for details

X means loading point
Y means flooding Point

Flooding Rates in Packed columns

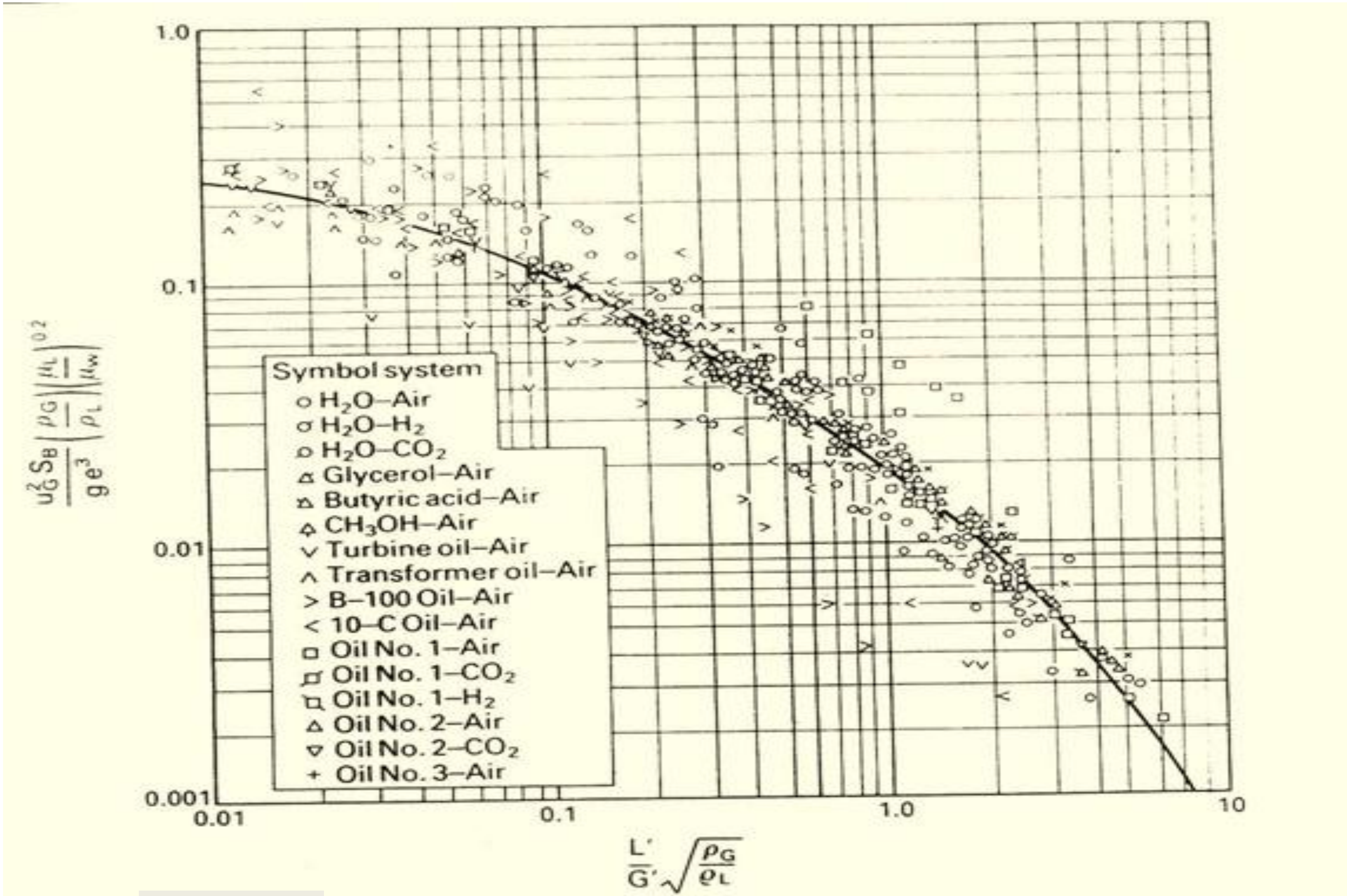
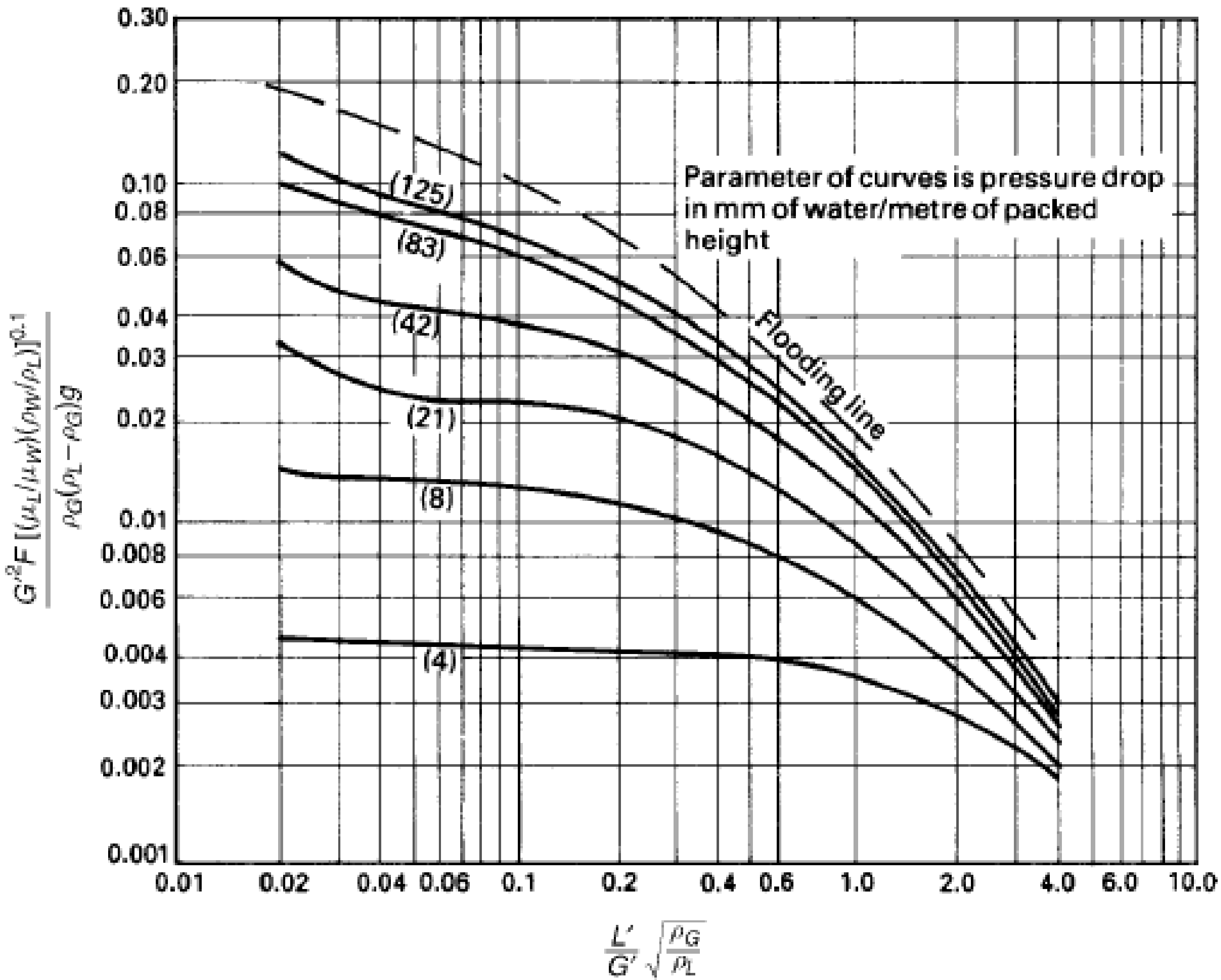


Fig 4.17 . Generalised correlation for flooding rates in packed towers

Generalized pressure drop correlation Fig 4.18



Notes over fig 4.18

- Most of the data on which it is based are obtained for cases where the liquid is water and the correction factor $[(\mu_L/\mu_w)/(\rho_w/\rho_L)]^{0.1}$*, in which* μ_w *and* ρ_w *refer to* water at 293 K, is introduced to enable it to be used for other liquids.
- The packing factor *F which is employed in the correlation is a modification of the specific surface of the* packing which is used in Figure 4.17.
- In practice, a pressure drop is selected for a given duty and use is made of the correlation to determine the gas flow rate per unit area *G'* from which the tower diameter may be calculated for the required flows.