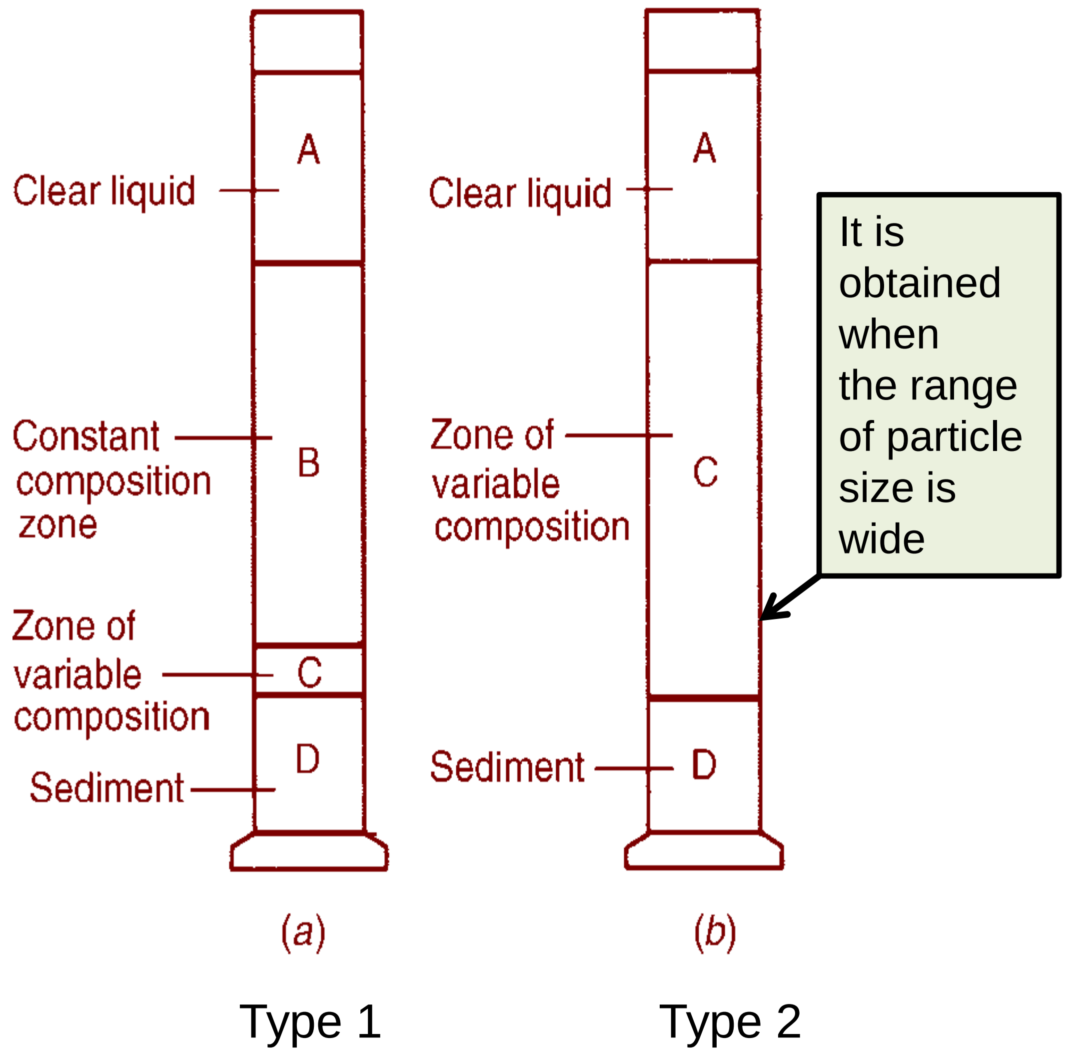
Sedimentation

- Gravitational sedimentation.
- Meaning
- Batch settling test .
- 4 zones can be observed: see fig.
- Critical settling point $\Rightarrow$ only zones A and D.

Comparison between Type 1 and Type 2

Settling of fine particles (<100μm) can be divided into two types according to the zones:

- ➢Type 1 contains zones A, B,C and D;
- ➢ Type 2 contains zones A, C and D.



Gravity sedimentation

After critical settling pt., compression and consolidation of solid layer ‘D’ will take place with liquid being forced upwards into the clear liquid zone ‘A’. Hence the height of layer D will be decreased.

Stokes' law

For a suspension of particles in a fluid, Stokes' law is assumed to apply but an effective viscosity and effective average density are used.

- **The modified settling or sedimentation velocity in suspension is:**

$$u_c = K'' \frac{d^2(\rho_s - \rho_c)g}{\mu_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where

K'' : *cons.*

$$(\rho_s - \rho_c) = \rho_s - \{\rho_s(1-e) + \rho e\} = e(\rho_s - \rho) \ \ldots\ldots\ldots (2)$$

$$\mu_c = \mu(1 + k''c) \quad \leftrightarrow \quad c \text{ up to } 0.02 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where

k'' : *cons. for a given shape of particles(2.5 for spheres)*

c : *volumetric conc. of particles.*

μ : *fluid viscosity*

But

$$\mu_c = \mu\, e^{k''c/(1+a'c)} \quad \leftrightarrow \quad c \succ 0.02 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where a' : *cons.* $= 0.609$ *for spheres*

- It has been shown for a suspension of uniform particles, the velocity of particle relative to the fluid, u_p, is given by:

$$u_p = \frac{d^2(\rho_s - \rho_c)g}{18\mu} \mathrm{f}(e) \quad \text{......................(5)}$$

or

$$u_p = \frac{u_c}{e} \quad \text{.......................................(6)}$$

- f(e) could be as

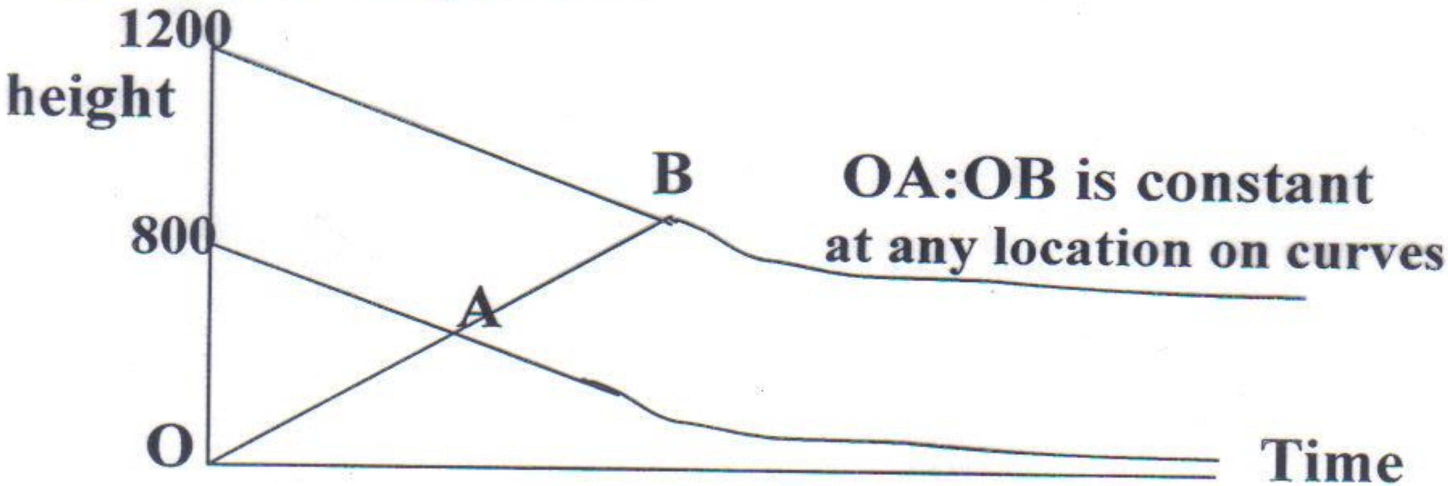
$$f(e) = 10^{-1.82(1-e)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Substituting 6,7,2, into 5

$$u_C = \frac{e^2 d^2 (\rho_S - \rho) g}{18\mu} 10^{-1.82(1-e)} \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Main factors which affect sedimentation

1. **Height of suspension**



Suspension height has no effect on rate of sedimentation

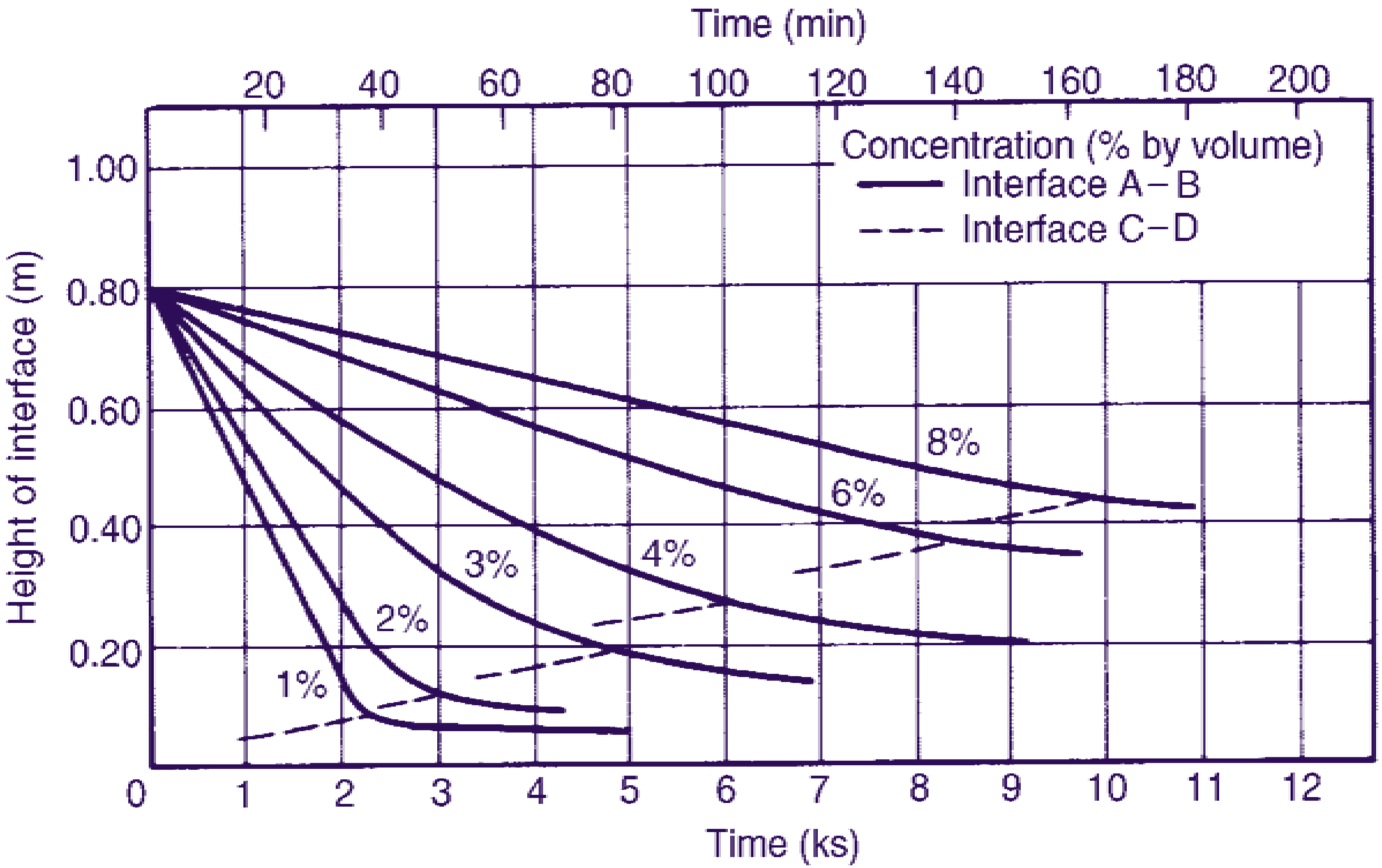
2. **Diameter of Vessel**

If $D_c/D_p > 100$, the walls have no effect on sedimentation rate.

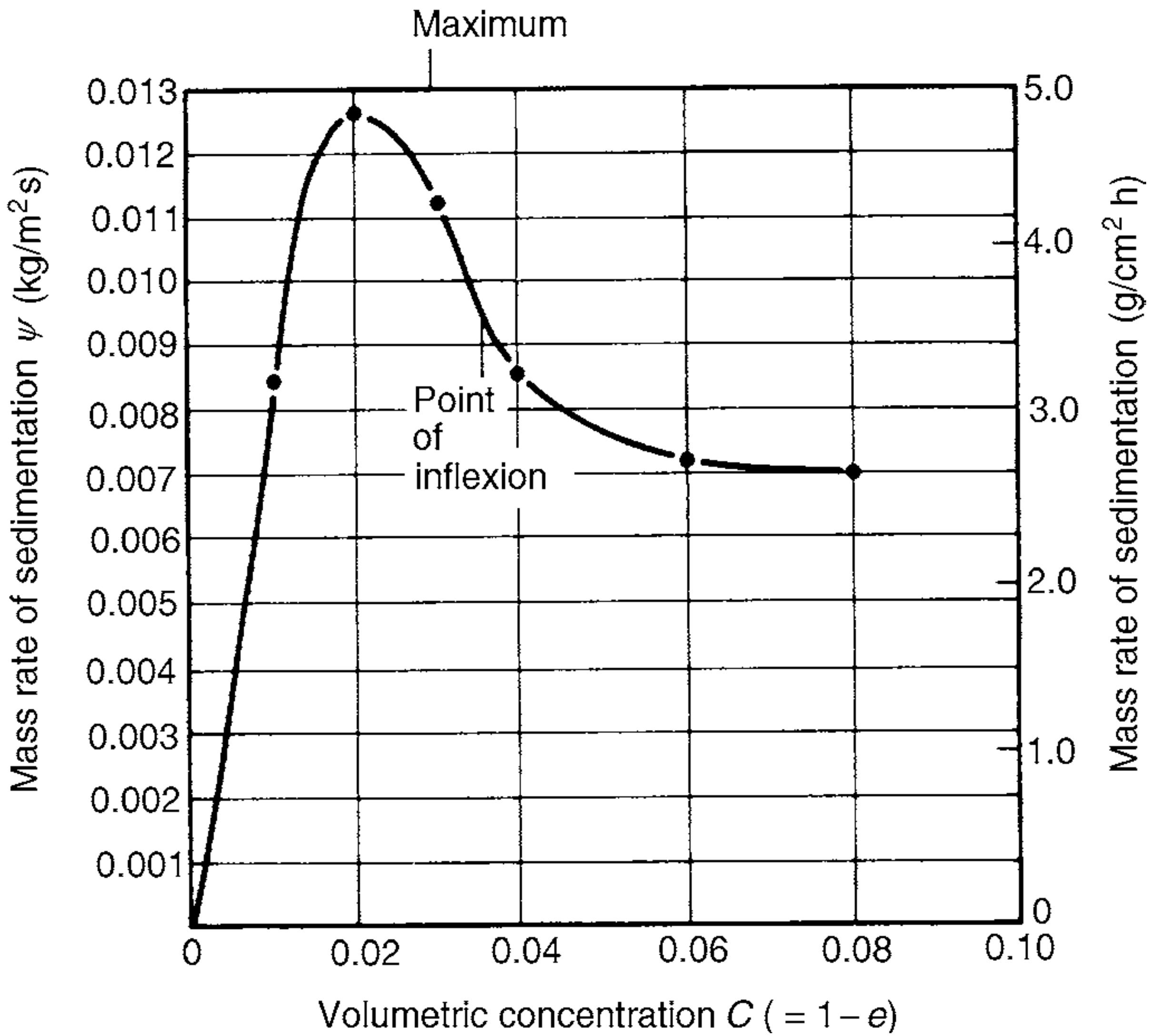
If $D_c/D_p < 100$, walls reduce the sedimentation rate

3. **Concentration of suspension**

Effect of concentration on the sedimentation of calcium carbonate suspensions



Effect of concentration on mass rate of sedimentation of calcium carbonate



Note

- **Rate of sedimentation during consolidation or solid-compression period is**

$$-\frac{dH}{dt} = b(H - H_{\infty})$$

$$On\ integration\ with\ B.C^{S} : t = o, H = H_{C} ; t = t,\ H = H$$

$$ln(H - H_{\infty}) - ln(H_{C} - H_{\infty}) = -bt$$

Where H_C : the height of sluge at critical settling pt.

H_{∞}: the final height of the sediment

b : cons. For a given suspension

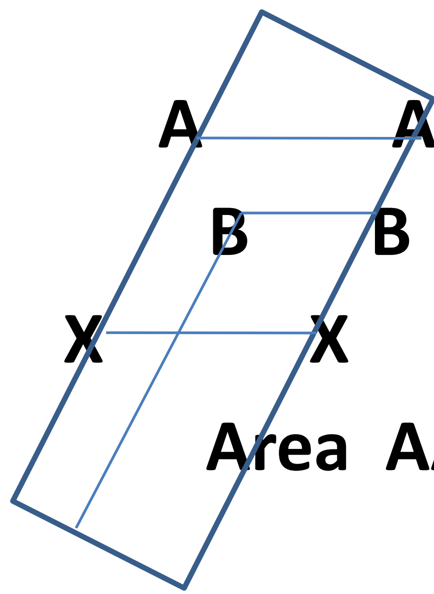
- If $\ln(H-H_\infty)$ is plotted against t, hence b can be found from the slope of the given line.

4. Shape of vessel

- It has little effect
- If the container

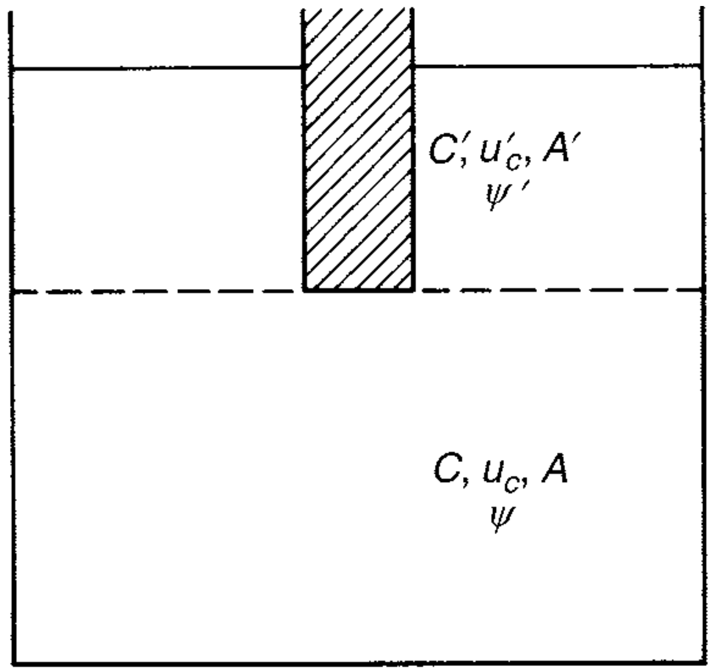
  has a slope

  with horizontal



Area AAXX = shaded area

- **Obstructed vessel**



Sedimentation in partially obstructed vessel

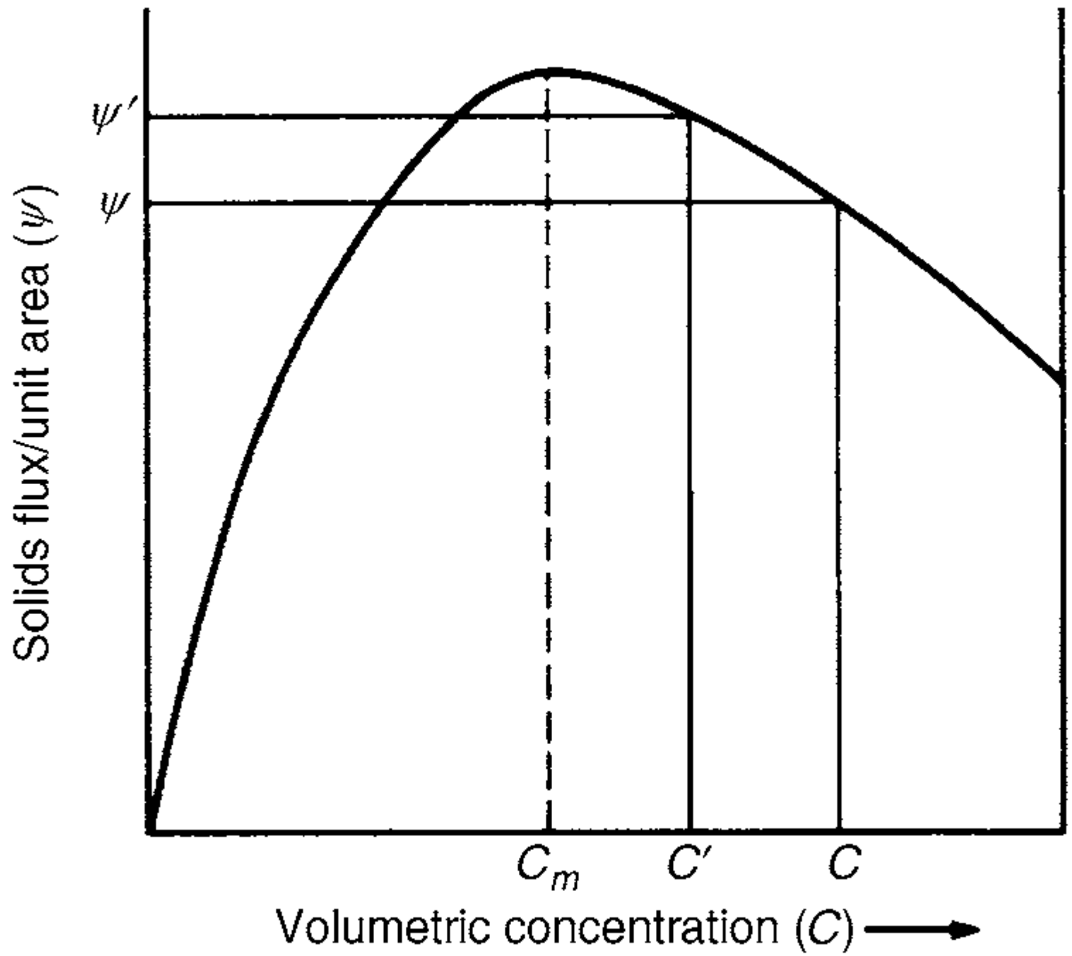
$$A\psi = Acu_C$$

$$A'\psi' = A'c'u'_C$$

For contiuity at the bottom of construction

$$\psi' = \frac{A}{A'}\psi \quad \therefore \psi' \succ \psi \quad \textit{since } A' \prec A$$

Solids flux per unit area as a function of volumetric concentration



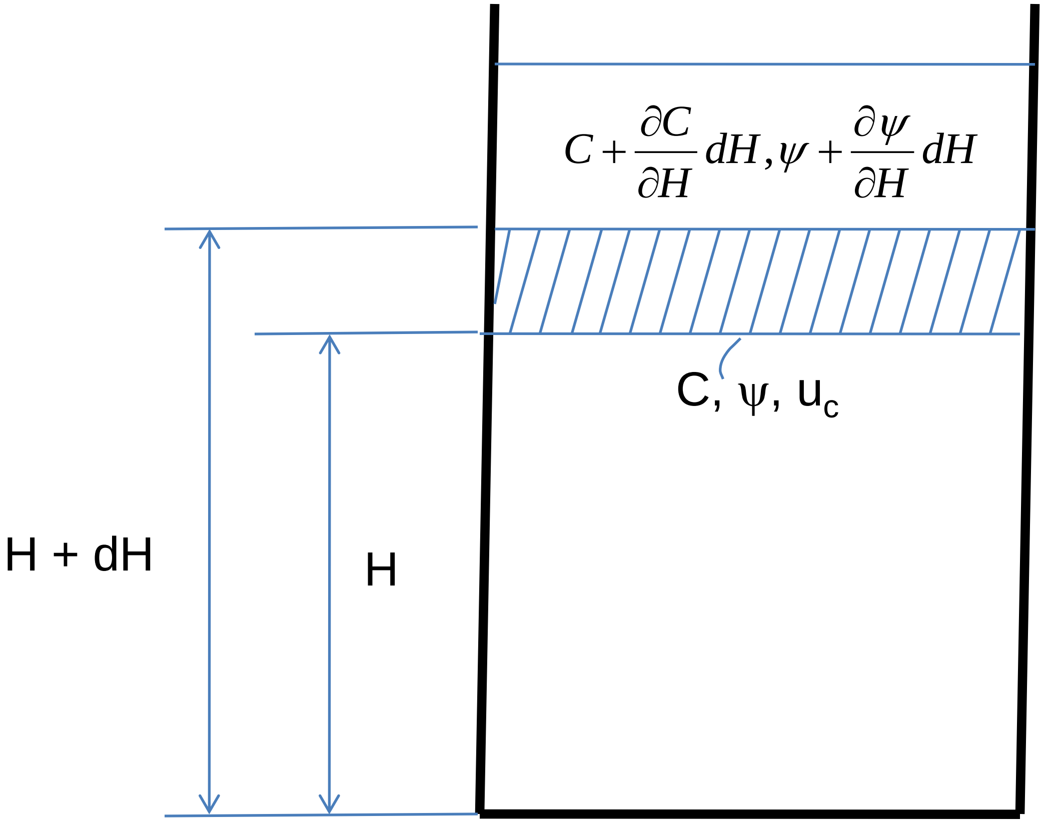
Kynch Theory

The mechanism of sedimentation of concentrated suspension can be studied via using the concept of continuity.

<u>Some basic assumption</u>:

- **Uniform conc. at any level.**
- **No wall effects**
- **Velocity of fall of particles depends on the local conc. of particles.**
- **There is no differential settling**
- **Sedimentation velocity = 0.0 at the sediment layer at the bottom.**

Kynch Theory



- **In general, ψ = c u_c**
  Making a material balance between the height h and the height H+dH

$$\left\{ \left(\psi + \frac{\partial \psi}{\partial H} dH\right) - \psi \right\} dt = \frac{\partial}{\partial t}(c dH) dt$$

$$\frac{\partial \psi}{\partial H} = \frac{\partial c}{\partial t} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$note: \;\; Right\ term \quad \frac{\partial dH}{\partial t} c + \frac{\partial c}{\partial t} dH$$

$$\frac{\partial \psi}{\partial H} = \frac{\partial \psi}{\partial c} \cdot \frac{\partial c}{\partial H} = \frac{d\psi}{dc} \cdot \frac{\partial c}{\partial H} \; since\ \psi\ depends\ only\ on\ c$$

Substituting in (1)

$$\frac{\partial c}{\partial t} - \frac{d\psi}{dc} \cdot \frac{\partial c}{\partial H} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$But\ c = f(H, t)$$

$$dc = \frac{\partial c}{\partial H} dH + \frac{\partial c}{\partial t} dt$$

$$Assume\ constant\ conc.$$

$$\frac{\partial c}{\partial H} dH + \frac{\partial c}{\partial t} dt = 0$$

$$\frac{\partial c}{\partial H} = -\frac{\partial c}{\partial t} \Big/ \frac{dH}{dt}$$

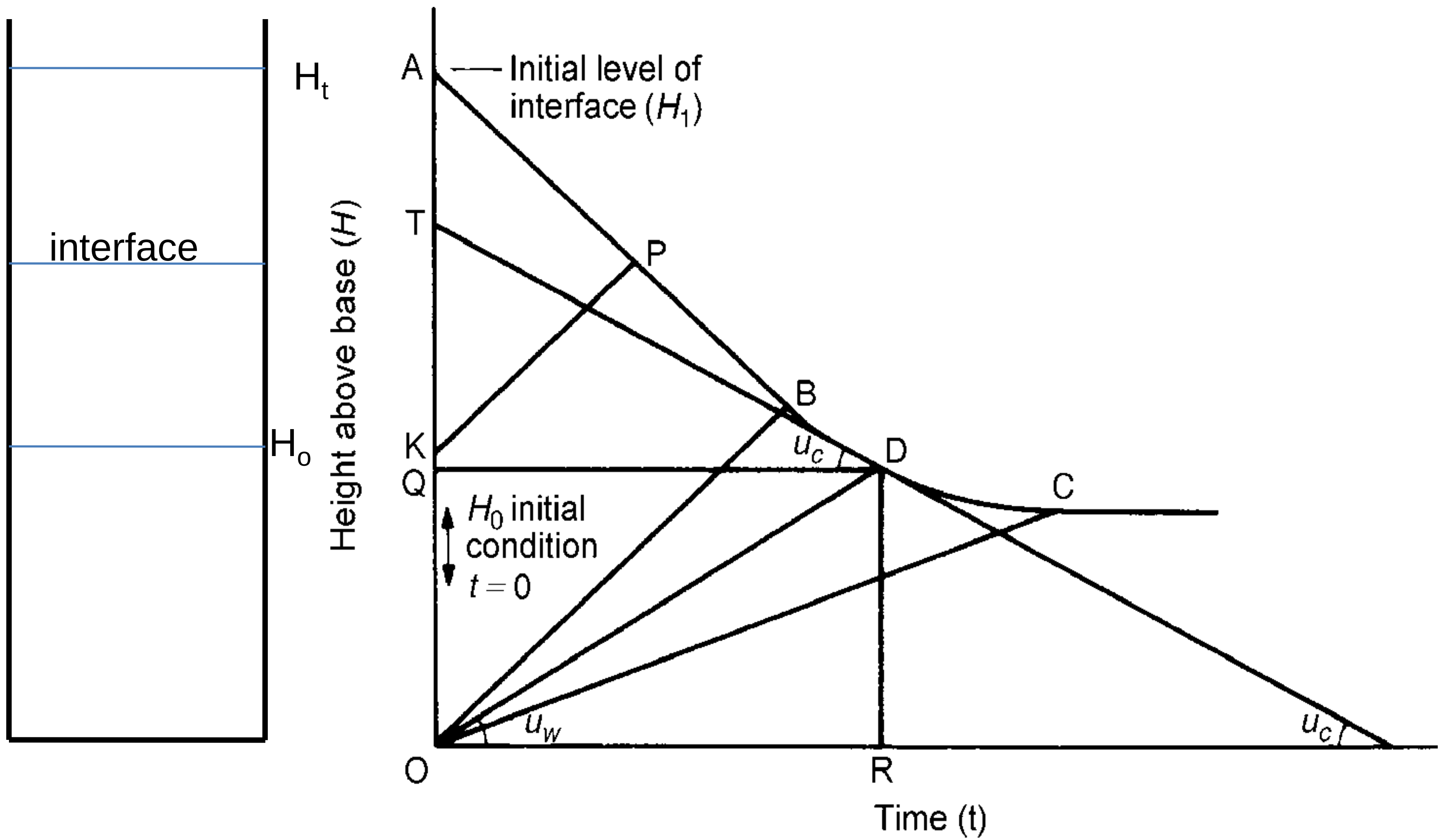
$$sub.\ in\ (2)$$

$$\frac{\partial c}{\partial t} - \frac{d\psi}{dc}\left\{-\frac{\partial c}{\partial t} \Big/ \frac{dH}{dt}\right\} = 0$$

$$\therefore \quad -\frac{d\psi}{dc} = \frac{dH}{dt} = u_w \quad \text{.......................}(3)$$

Where u_w velocity of propagation of a zone of constant concentration ‘velocity of propagation of concentration wave’. Note u_w is constant for any concentration.

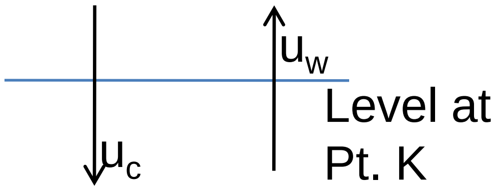
The relationship between flux ‘Ψ’ and concentration ‘c’



- lines such as KP and OB represent constant concentrations with slopes = dH/dt = u_w
- line AB represents the fall down of interface 'between clear liquid and suspension' with fall down velocity 'sedimentation velocity' ,u_c= slope of this line = - dH/dt
- the total volume of particles that pass through a certain plane in time

  't' per unit area is:

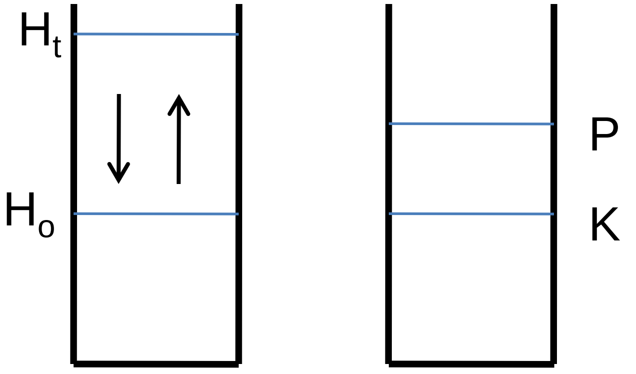
  $V = C_o (u_c + u_w) t$



- Total volume, V, must equal to the total volume of particles was originally above the K, for example.

$$C_o\ (u_c + u_w)\ t = C_o\ (H_t - H_o)$$

$$(u_c + u_w)\ t = (H_t - H_o)$$



- For intermediate concentration curve BC $C_o < C < C_{max}$
- Consider pt. D and apply the same previous approach:

See text for details

- Here the line OD represents a wave starts from pt. O 'the base of the container' and terminates at pt. D ' the interface between clear liquid and suspension' on the sedimentation curve.

$C (u_c + u_w) t = C_o H_t$ ....(i)

But $H_t = OA$ 'see the fig.'

$u_c = QT/t$

or $u_c t = QT$ .... (ii)

and $u_w t = RD = OQ$ ....(iii)

add (iii) to (ii)

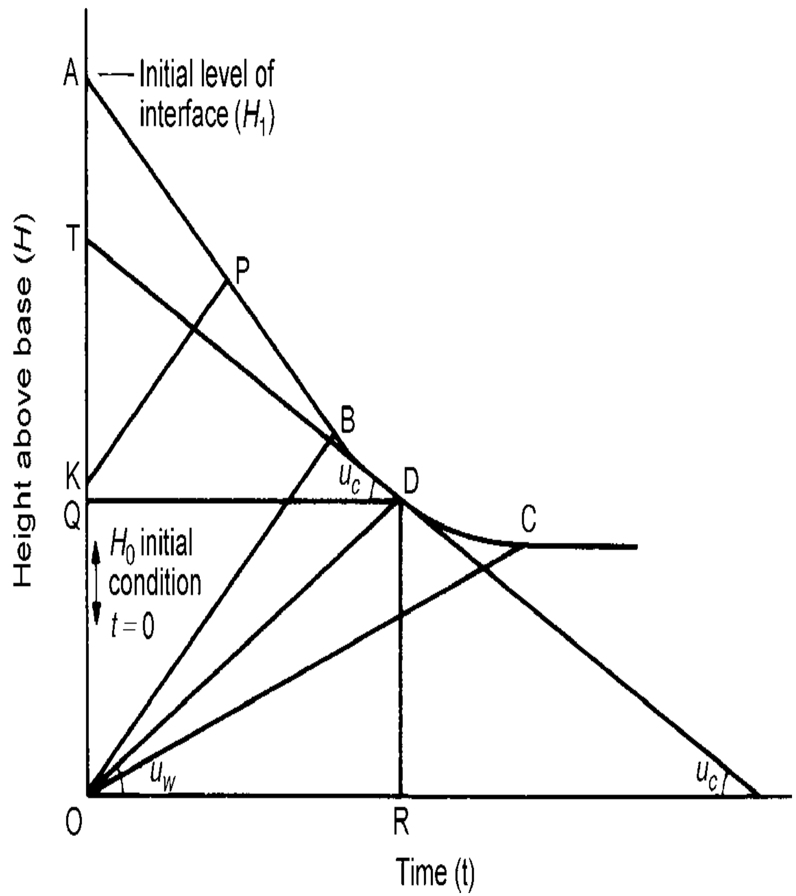
$(u_c + u_w) t = OT$ .... (iv)

divide (i) by (iv)

$C = C_o (OA/OT)$ ###

The corresponding Flux is

$\Psi = C u_c = C_o (OA/OT) u_c$ ###



Thickener design

Thickening zone

Thickening zone

- **<u>Good design requires</u>**:

  1. adequate or suitable diameter to obtain reasonable clarification.

  2.Enough depth to obtain the required degree of thickening.

Required condition for good design

- In a continuous thickener, the area required for thickening must be such that the total solids flux at any depth does not exceed the rate at which the solids can be transmitted downwards.
- If this condition is not met, solids will build up and unsteady-state operation will take place.

Required Area

- To calculate the required area , it is very important to obtain the concentration at which the rate of sedimentation or the flux is a minimum.
- Total flux, ψ_t, consists of two componets:

  a. Due to the sedimentation of solid in the liquid (as measured by batch sedimentation experiment, $\psi = u_c$ C).

  b. Arising from the removal of the underflow at the base of the thickener, $\psi_u = u_u$ C.

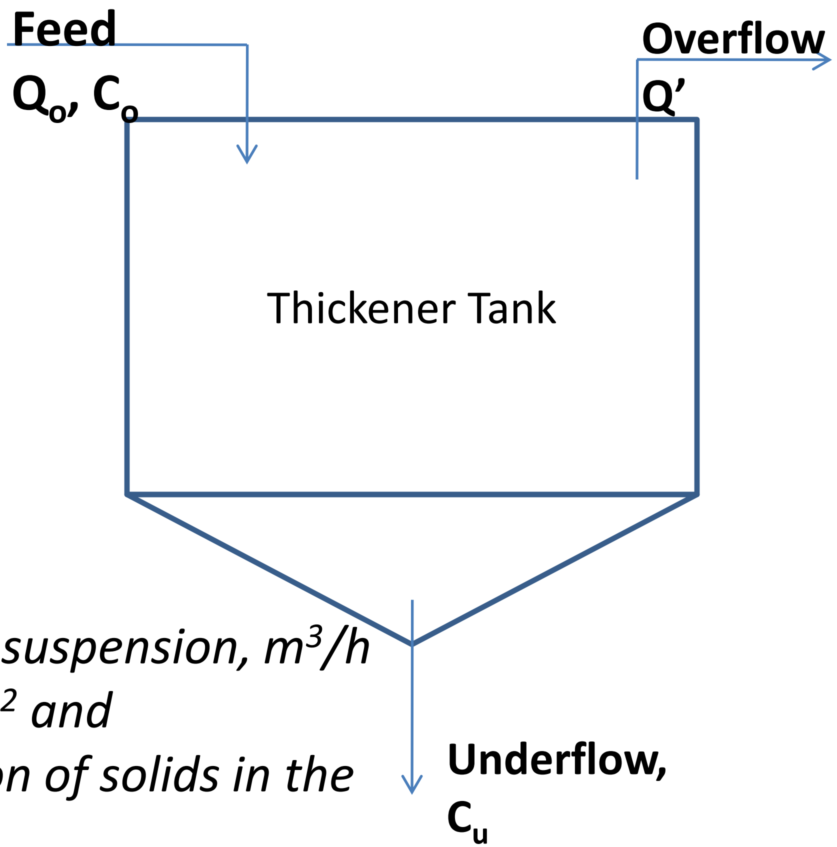
$$\therefore \text{Total flux is,}$$

$$\Psi_T = \Psi + \Psi_u$$

$$= \Psi + u_u C$$

This flux equals the volumetric flow rate per unit area at which the solids are fed to the thickener

$$\Psi_T = \left(\frac{Q_0}{A}\right) C_o$$



Where;

Q_0 is the volumetric feed rate of suspension, m^3/h

A is the area of the thickener, m^2 and

C_0 is the volumetric concentration of solids in the feed.

$$\frac{\partial \psi_T}{\partial C} = \frac{\partial \psi}{\partial C} + u_u$$

The minimum value of ψ_T $(=\psi_{TL})$ occurs when $\frac{\partial \psi_T}{\partial C} = 0$;

i.e. when:

$$\frac{\partial \psi}{\partial C} = -u_u$$

See the text for details

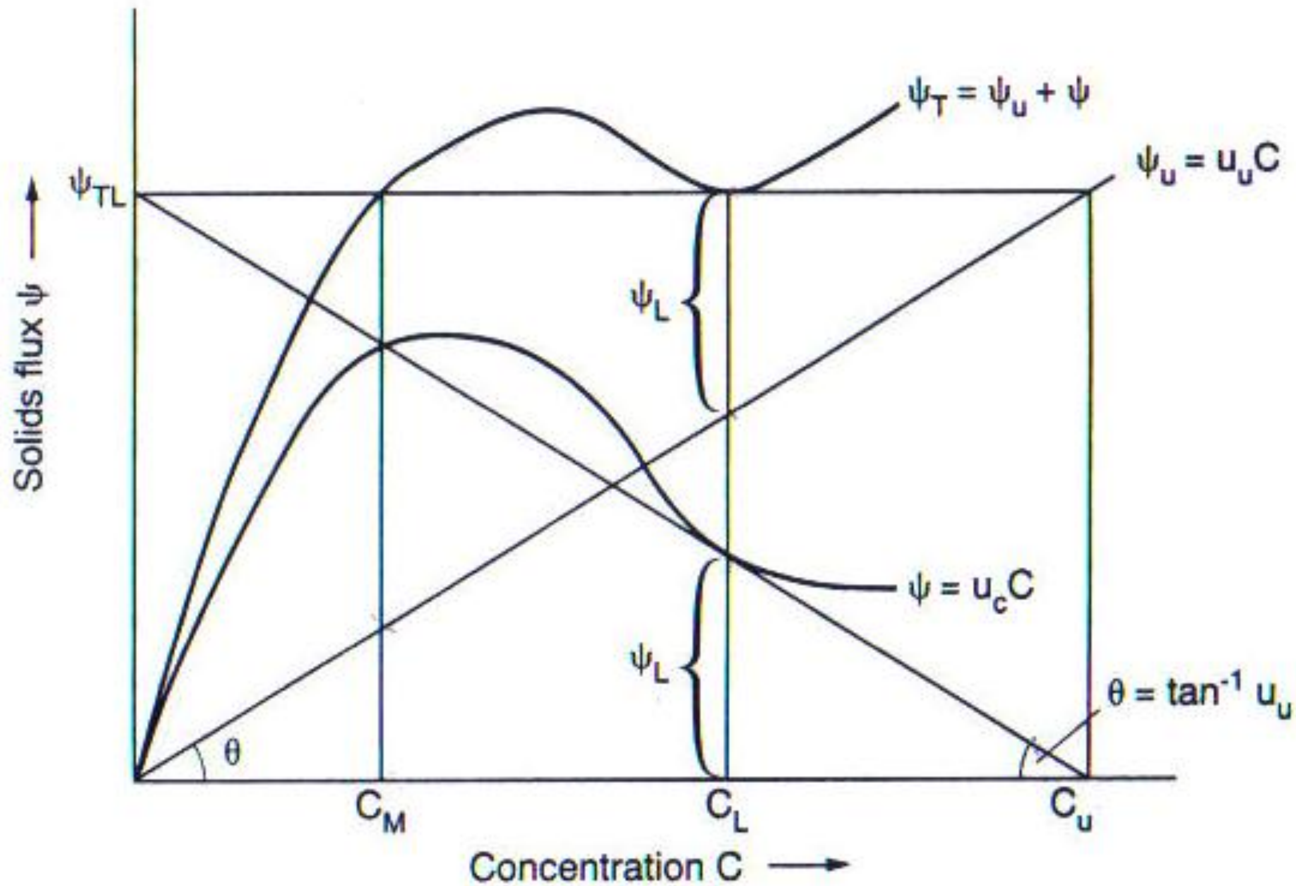


FIG. 5.13. Solids fluxes as functions of concentration and Yoshioka construction.

$$\frac{\psi_{TL}}{C_u} = \frac{\psi_L}{C_u - C_L}$$

where ψ_L is the value of ψ at the concentration C_L.

Since $\psi_L = u_{cL}\, C_L$

$$A = \frac{Q_0\, C_0}{\psi_{TL}}$$

$$= Q_0\, C_0 \left[\frac{\dfrac{1}{C_L} - \dfrac{1}{C_u}}{u_{cL}}\right]$$

where u_{cL} is the value of u_c at the concentration C_L. Thus, the minimum necessary area of the thickener may be obtained from the maximum value of

$$\left[\frac{\dfrac{1}{C} - \dfrac{1}{C_u}}{u_c}\right] \text{ which will be designated } \left[\frac{\dfrac{1}{C} - \dfrac{1}{C_u}}{u_c}\right]_{\text{max}}.$$

Concentrations can also be expressed as mass per unit volume (using c in place of C) to give:

$$A = Q_0\, c_0 \left[\frac{[(1/c) - (1/c_u)]}{u_c}\right]_{\text{max}}$$

Overflow

The liquid flowrate in the overflow (Q') is the difference between that in the feed and in the underflow.

Thus:

$$Q' = Q_0 (1 - C_0) - (Q_0 - Q') (1 - C_u)$$

or:

$$\frac{Q'}{Q_0} = 1 - \frac{C_0}{C_u} \tag{5.52}$$

At any depth below the feed point, the upward liquid velocity must not exceed the settling velocity of the particles. (u_c) Where the concentration is C, the required area is therefore given by:

$$A = Q_0 \frac{1}{u_c} \left[1 - \frac{C}{C_u} \right] \tag{5.53}$$

It is therefore necessary to calculate the *maximum* value of A for all the values of C which may be encountered.

Equation 5.53 can usefully be rearranged in terms of the mass ratio of liquid to solid in the feed (Y) and the corresponding value (U) in the underflow:

$$Y = \frac{1 - C}{C} \frac{\rho}{\rho_s} \quad \text{and} \quad U = \frac{1 - C_u}{C_u} \frac{\rho}{\rho_s}$$

Then:

$$C = \frac{1}{1 + Y(\rho_s/\rho)} \qquad C_u = \frac{1}{1 + U(\rho_s/\rho)}$$

$$A = \frac{Q_0}{u_c}\left\{1 - \frac{1 + U(\rho_s/\rho)}{1 + Y(\rho_s/\rho)}\right\}$$

$$= \frac{Q_0(Y - U)C\rho_s}{u_c\rho}$$

The values of *A* should be calculated for the whole range of concentrations present in the thickener, and the design should then be based on the maximum value so obtained.

Depth of the thickener Zone, H_t

The required depth of the thickening region is

$$H_t = \left\{ \frac{W t_R}{A \rho_s} + W \frac{t_R}{A \rho} X \right\} = \frac{W t_R}{A \rho_s} \left(1 + \frac{\rho_s}{\rho} X \right)$$

where:

t_R is the required time of retention of the solids, as determined experimentally,

W is the mass rate of feed of solids to the thickener,

X is the average value of the mass ratio of liquid to solids in the thickening portion, and

ρ and ρ_s are the densities of the liquid and solid respectively.

See the text for details