Mean Particle Size

Example 1

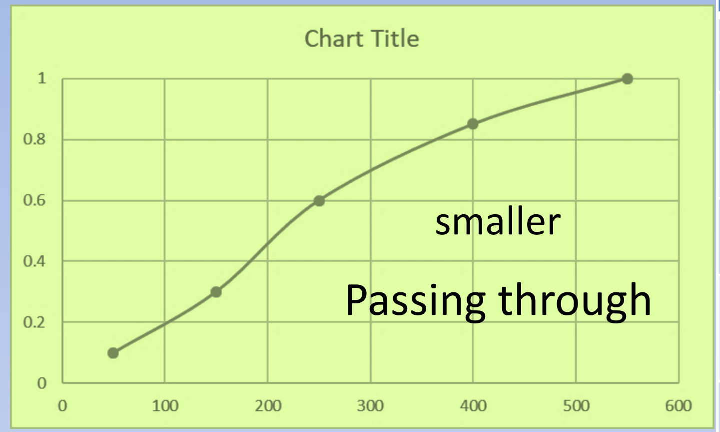
Calculate the volume diameter, surface volume diameter for a cuboid of side length of 1, 2, 4 mm. compare the results with the surface equivalent diameter.

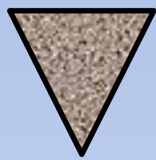
Cumulative screen analysis curve

Plot Cumulative mass fraction vs Average particle size

Example for Discussion

Size range	x	d	x_{cum}
0 - 100	0.1	50	0.1
100 – 200	0.2	150	0.3
200 – 300	0.3	250	0.6
300 – 500	0.25	400	0.85
500 - 600	0.15	550	1.00
total	1.00		

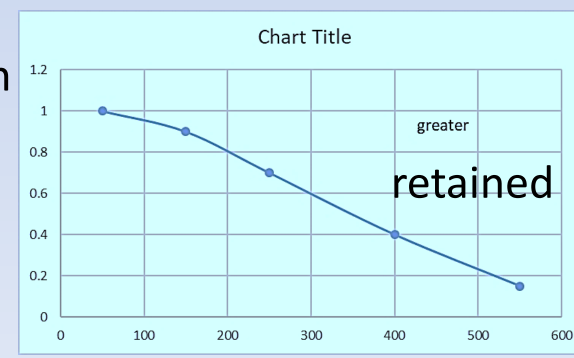


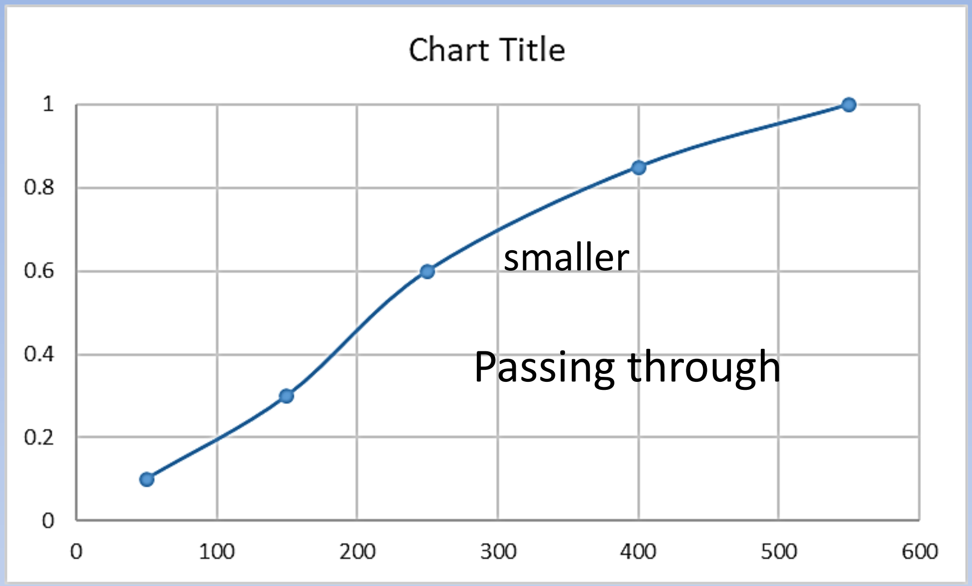


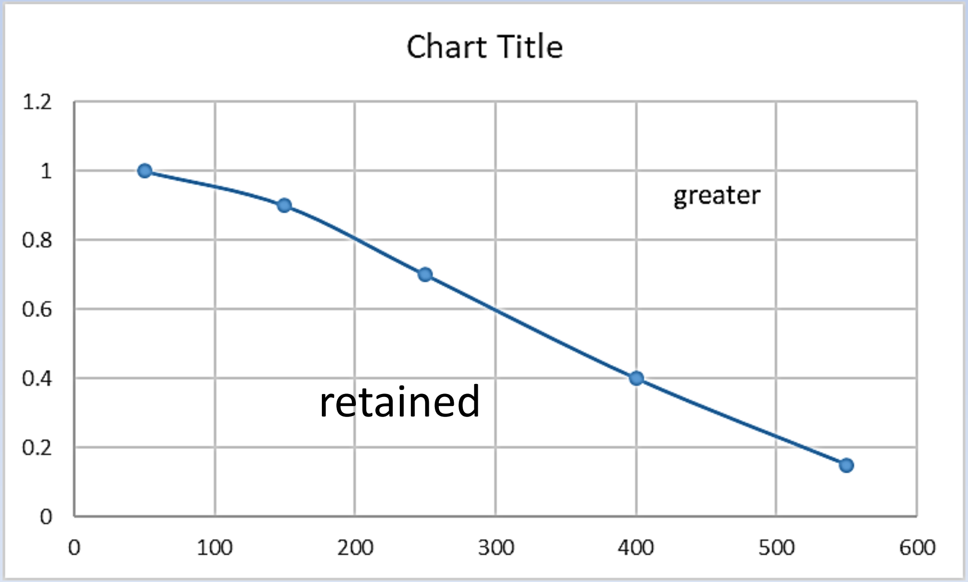
Sample
100 g

Arbitrary values

	Wt on sieve	Fraction, x	x cum	davg
600	0 g	0.00 0r 0%		
500	15 g	0.15 or 15%	15%	550 µm
300	25 g	0.25 or 25%	40%	400 µm
200	30 g	0.30 or 30%	70%	250 µm
100	20 g	0.20 0r 20%	90%	150 µm
Pan	10 g	0.10 0r 10%	100%	50 µm
Total	100 g	Total 1.00 100%		







Plot Cumulative mass fraction vs Average particle size

Particle size distribution

cumulative mass fraction curve, in which the proportion of particles *(x) smaller* than a certain size *(d) is plotted against that size (d)*

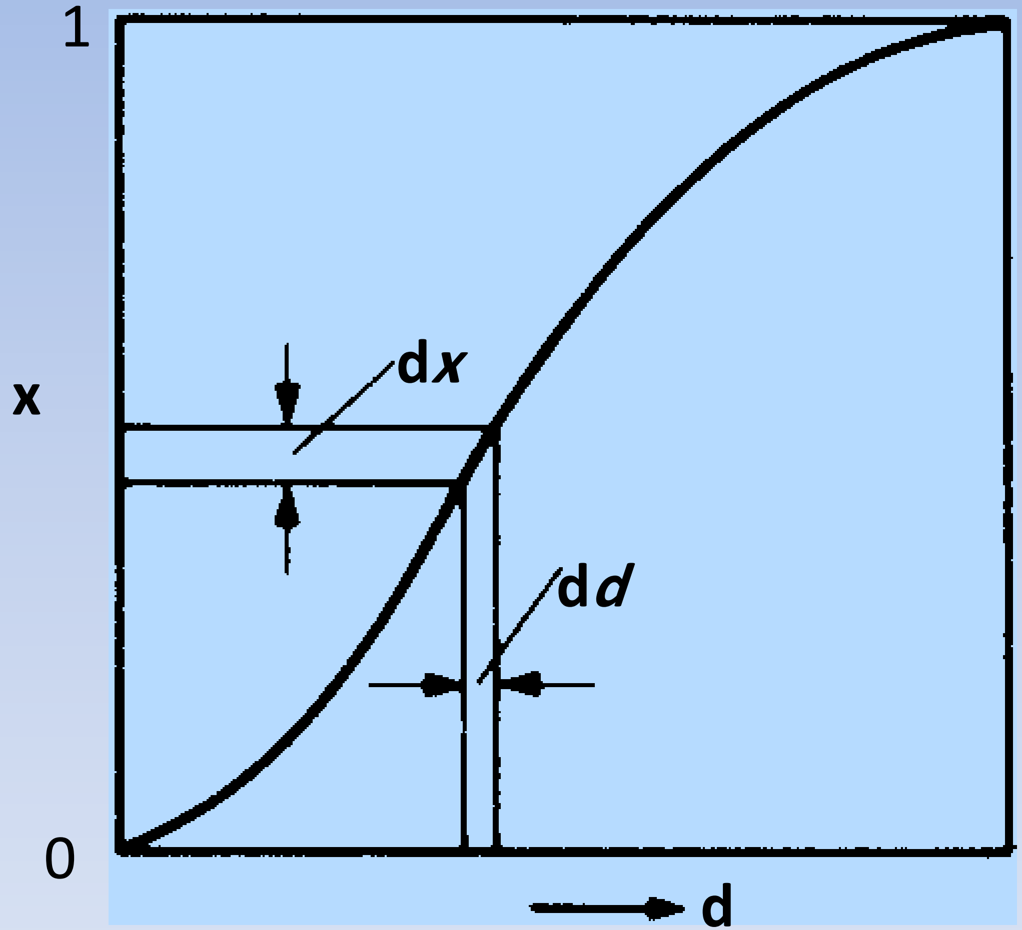


Figure 1.5. Size distribution curve—cumulative basis

Particle size distribution

the slope *(dx/dd) of the cumulative* curve (Figure 1.5) is plotted against particle size *(d).*

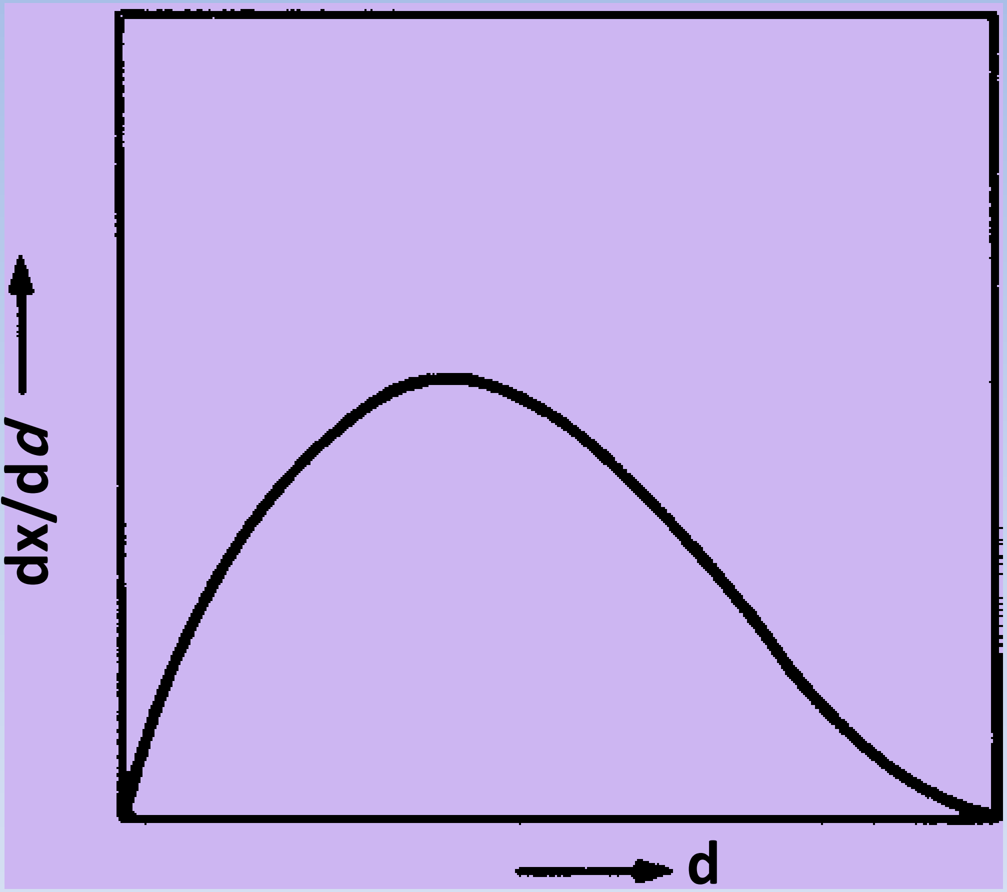


Figure 1.6. Size distribution curve—frequency basis

Three values are very important

I. Mode
II. Median
III. Mean

These values can be found directly from the frequency and cumulative curves.

Mean values can be found algebraically based on size intervals.

Consider a unit mass of n_1 particles of characteristic length d_1 with mass fraction x_1 and so on

$$\left\{ \begin{matrix} n_1 & d_1 & x_1 \\ n_2 & d_2 & x_2 \\ . & . & . \\ . & . & . \\ n_x & d_x & x_x \end{matrix} \right\} \textbf{unit mass}$$

Note:

Aggregate length $n_i d_i$

Aggregate surface $n_i d_i^2$

Aggregate volume $n_i d_i^3$

Mean particle size

- Considering unit mass of particles consisting of n_1 *particles of characteristic dimension* d_1, *constituting a mass fraction* x_1, n_2 *particles of size* d_2, *and so on, then:*

$$x_i = n_i k_i d_i^3 \rho_s \qquad (1.4)$$

and: $$\Sigma x_i = 1 = \rho_s k \Sigma (n_i d_i^3) \qquad (1.5)$$

Thus: $$n_i = (1 / \rho_s k_i)(x_i / d_i^3) \qquad (1.6)$$

- If the size distribution can be represented by a continuous function, then:

$dx = \rho_s k_i d^3 dn$

or:

$$\frac{dx}{dn} = \rho_s k_1 d^3 \qquad (1.7)$$

And:

$$\int_0^1 dx = 1 = \rho_s k_1 \int d^3 dn \qquad (1.8)$$

where ρ_s *is the density of the particles, and*

k_1 *is a constant whose value depends on the shape of the particle.*

Summary
Means based on volume

- Volume mean diameter, dv

$$d_v = \frac{\sum (n_i d_i) v_i}{\sum n_i v_i} = \frac{\sum n_i d_i^4}{\sum n_i d_i^3}$$

in terms of x_i :

$$d_v = \frac{\sum d_i x_i}{\sum x_i} = \sum d_i x_i$$

- Mean volume diameter dv'

$$d_{v'}^3 \sum n_i = \sum n_i d_i^3$$

$$d_{v'} = \sqrt[3]{\frac{\sum n_i d_i^3}{\sum n_i}} \quad \text{, since } n_i = x_i \big/ \rho_s k_i d_i^3$$

$$\therefore \quad d_{v'} = \sqrt[3]{\frac{\sum x_i}{\sum (x_i / d_i^3)}}$$

Means based on surface

- Surface mean diam, ds

$$d_s = \frac{\sum (n_i d_i) s_i}{\sum n_i s_i} = \frac{\sum n_i d_i^3}{\sum n_i d_i^2}$$

in terms of x_i :

$$d_s = \frac{\sum x_i}{\sum (x_i / d_i)} = \frac{1}{\sum (x_i / d_i)}$$

Sauter mean diameter

- Mean surface diam, ds'

$$d_{s'}^2 \sum n_i = \sum n_i s_i = \sum n_i d_i^2$$

$$d_{s'} = \sqrt{\frac{\sum n_i d_i^2}{\sum n_i}} \quad \text{, since} \quad n_i = x_i \Big/ \rho_s k_i d_i^3$$

$$\therefore \qquad d_{s'} = \sqrt{\frac{\sum (x_i / d_i)}{\sum (x_i / d_i^3)}}$$

Means based on length

- Length mean diam, d_l

$$d_l = \frac{\sum (n_i d_i) d_i}{\sum n_i d_i} = \frac{\sum n_i d_i^2}{\sum n_i d_i}$$

in terms of x_i :

$$d_l = \frac{\sum (x_i / d_i)}{\sum (x_i / d_i^2)}$$

- Mean length diam, d_l'

$$d_{l'} \sum n_i = \sum n_i d_i$$

$$d_{l'} = \frac{\sum n_i d_i}{\sum n_i}$$

in terms x_i

$$\therefore \qquad d_{l'} = \frac{\sum (x_i / d_i^2)}{\sum (x_i / d_i^3)}$$

Exercise 1

Sl. No.	Mesh No.	Screen Opening D_{pi} (cm)	Mass retained on a screen m_i (gm)
	4		0
	6		25
	8		125
	10		325
	14		250
	20		160
	28		50
	35		20
	48		10
	65		8
	100		6
	150		4
	200		3
	pan		2

Plot the cumulative curve, and calculate the Sauter mean diameter. Use mat lab or excel programs