Motion of Particles in a Fluid

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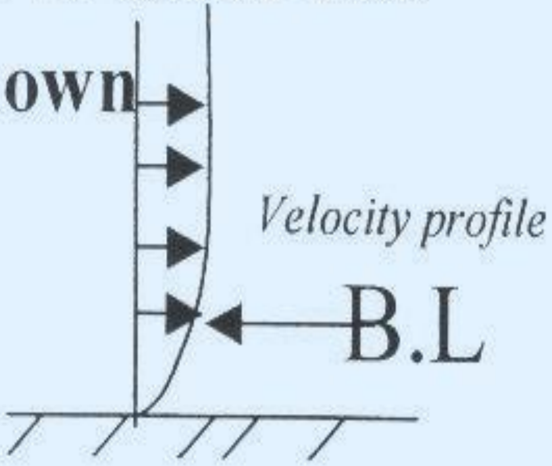
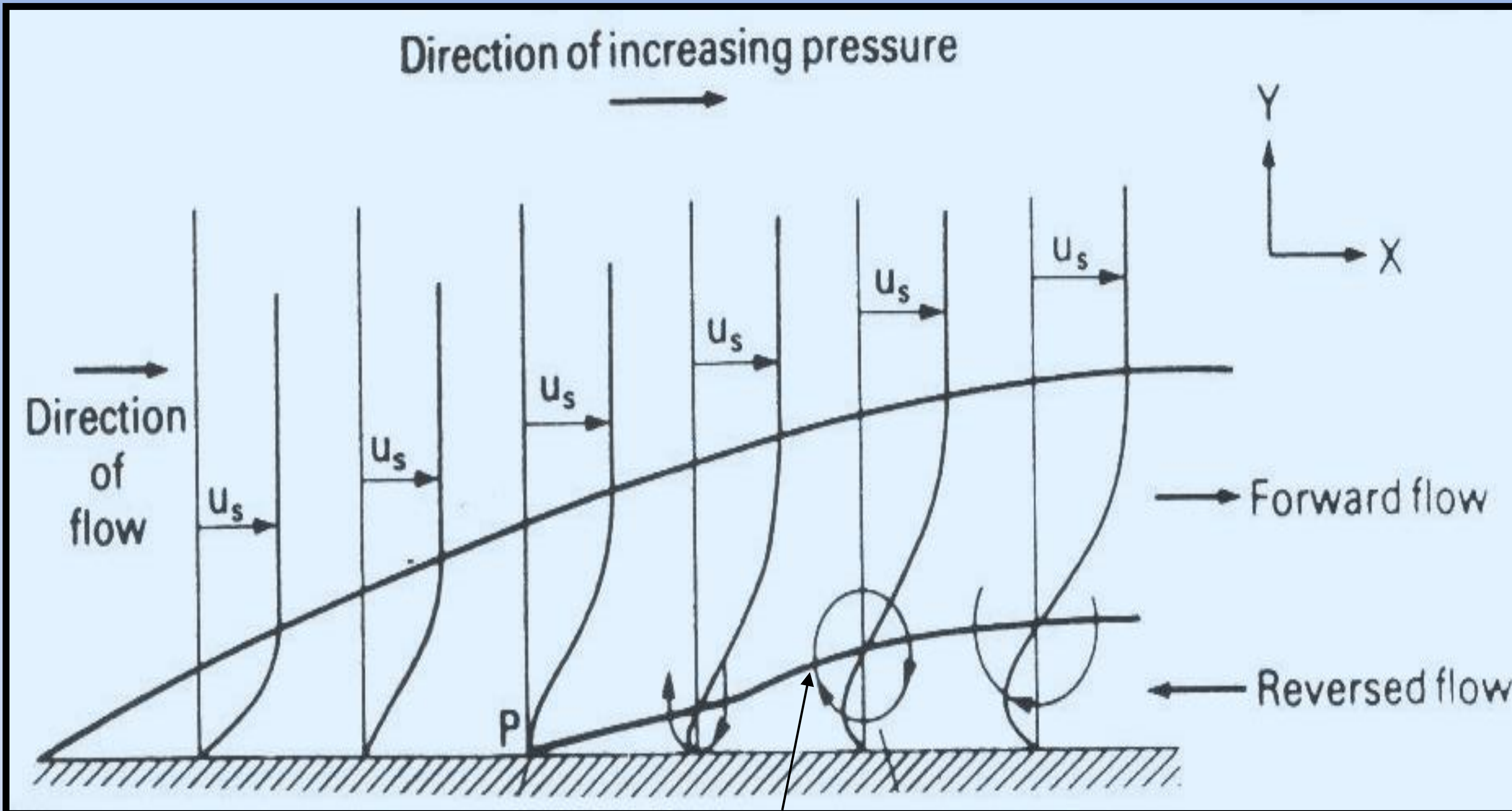
INTRODUCTION

1. Flow over a flat surface
   a. Drag force ~ fluid flows over a solid flat surface. Velocity gradient as shown Drag force arises due to the effect Of retardation of fluid at the Surface



b. Boundary layer (B.L) ~ a specific region where the velocity profile is changed with distance.

Thickness of B.L = f (distance from the leading edge)



Eddy formation

NOTE: The force acting on the fluid at some point in the boundary layer may then be sufficient to bring it to rest or to cause flow in the reverse direction with the result that an eddy current is set up. A region of reverse flow then exists near the surface where the boundary layer has separated as shown in the above Figure.

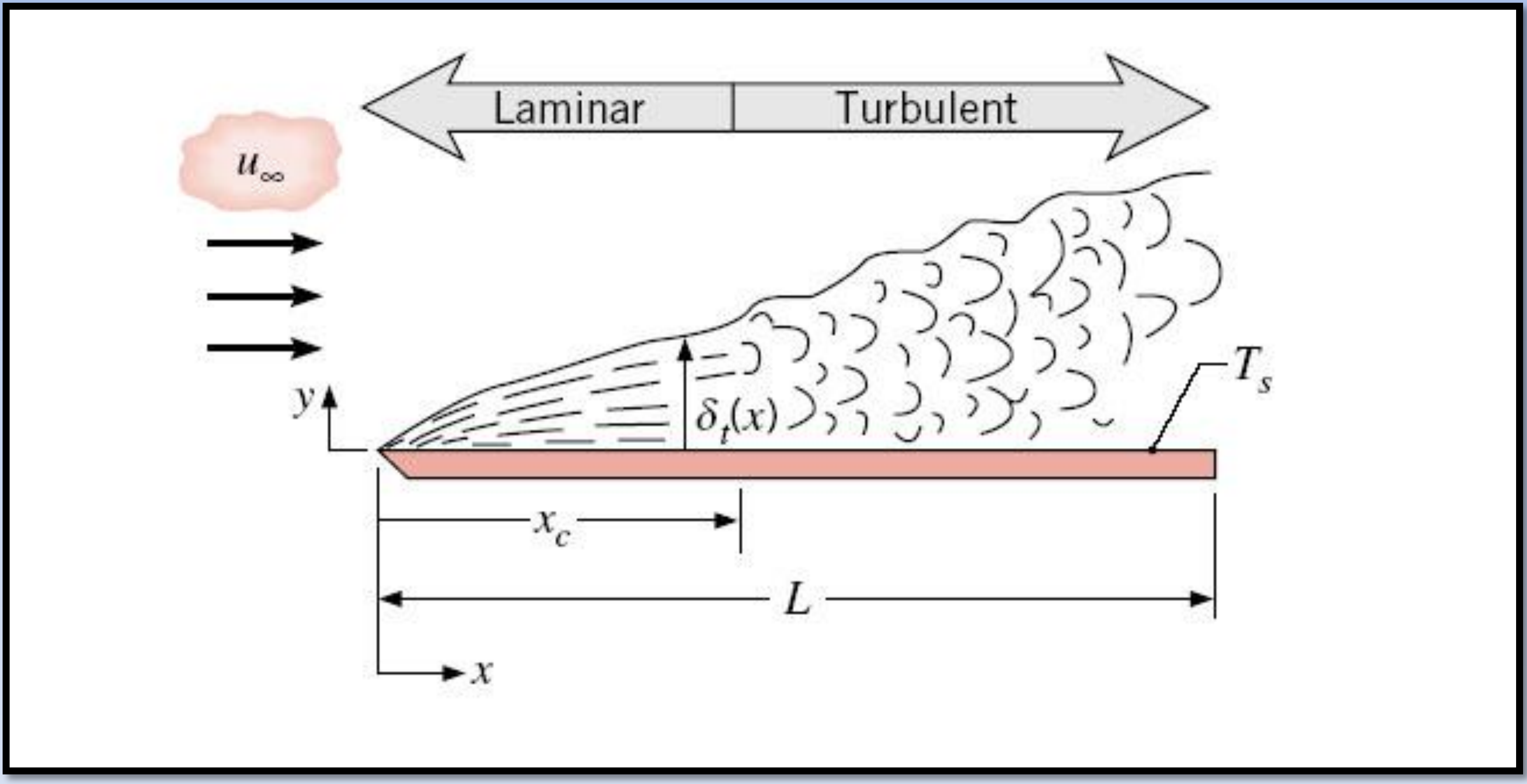
Notes

- The velocity at any pt. in the B.L varies from 0 at surface to the velocity of undisturbed stream, U_s.
- For a short distance on the surface, the flow is streamline.
- At a certain critical distance, x_c, the flow is changed for streamline to turbulent, except thin

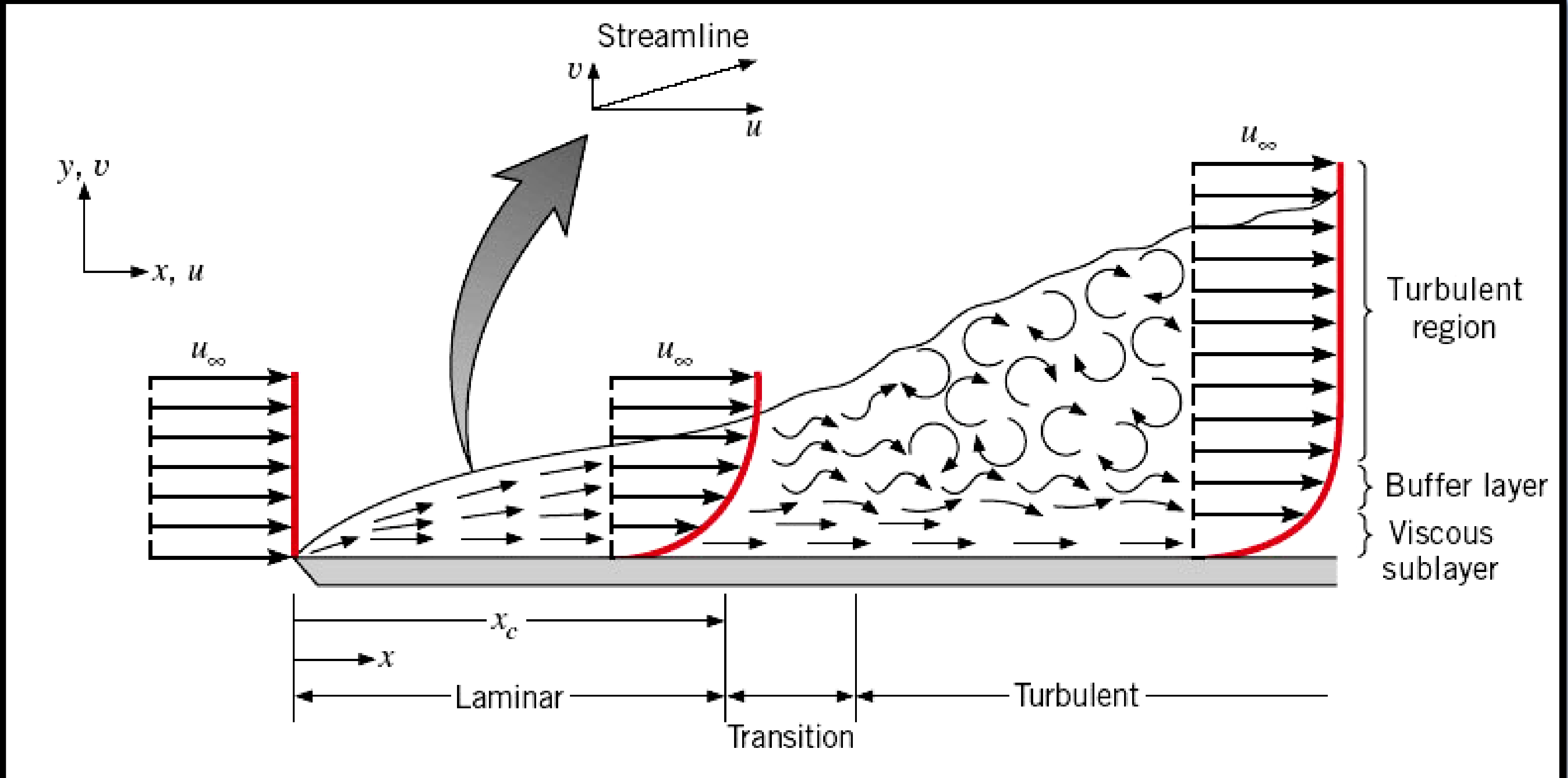
layer near the surface where it remains streamline 'called laminar sub-layer'.

- There is a thin layer between the sub-layer and the turbulent , the regime is transient and the layer is called 'buffer layer'!
- X_c = F (shape of leading edge, roughness of solid surface, properties of fluid, fluid velocity)
- Transition in regime can be examined via Re.

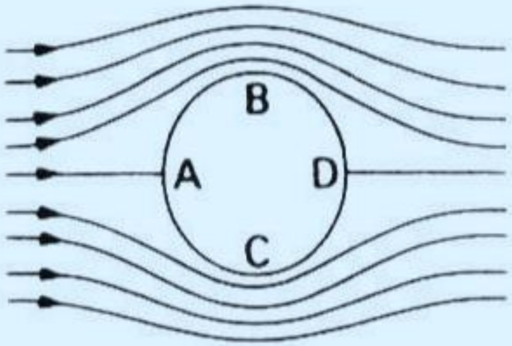
Laminar and turbulent flow over a flat surface



Velocity B.L developed on a flat surface



2. Flow round (past) a Cylinder



Flow round a·cylinder

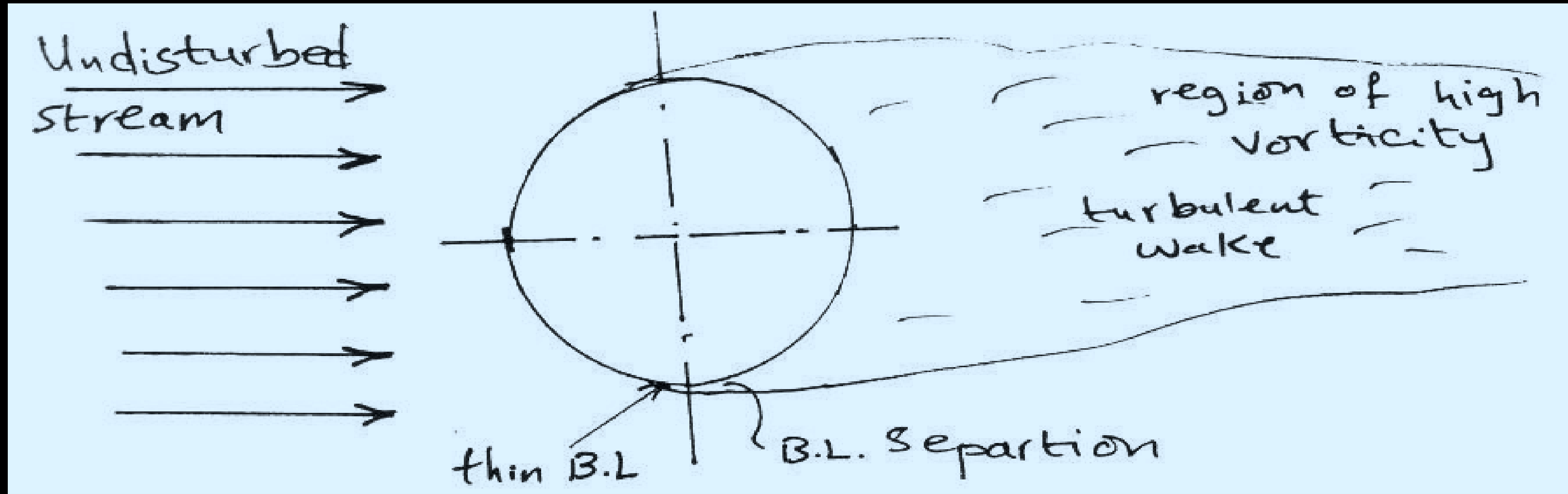
- For viscous/non-viscous fluid, the velocity, u, varies round the wall of cylinder $\Rightarrow$ at A & D the fluid is stagnant i.e $u = 0$ $\Rightarrow$ whilst at B & C u is max. { K.E is max at B & C and zero at A & D

- Also the Pressure falls from A to B and rises from B to D
- $\sum$ {K.E + Pressure Energy} is constant at all the points on the surface.

Note 1: when the pressure falls in the direction of flow, the retardation of the fluid will be less and the B.L will be thinner and visa versa.

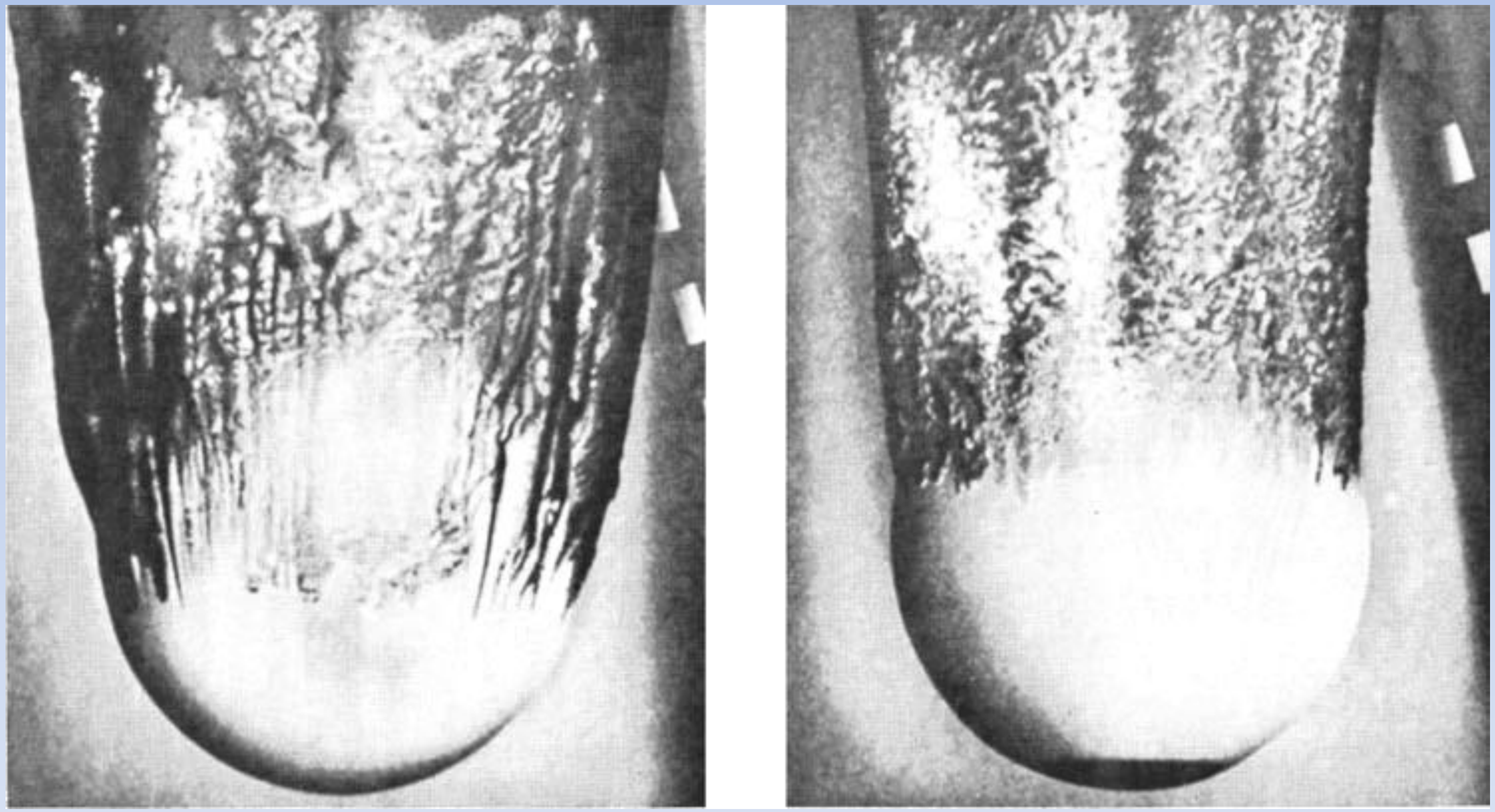
Note 2: Thin B.L is noted in the front of the cylinder, whilst a thick B.L takes place at the back or in the wake of the cylinder that tends to .separate from the surface. If separation occurs, the eddy currents are builds up in the wake and drag force (form drag) is made up.

Flow past a sphere



- Similar observations as flow past a cylinder.
- At large values of Re number, we can see two main zones: (i) a region of high vorticity comprising a thin B.L over the front half of the sphere and a turbulent wake on the downstream side of the sphere, and (ii) an external irrotational flow region outside the B.L and wake.

Effect of roughening front face of a sphere *(a) 216 mm diameter ball entering water at 298 K (b) As before, except for 100 mm diameter patch of sand on nose*



(a) (b)

Note

- ✓ Turbulence may arise either from an **increased fluid velocity** or from **artificial roughening** of the forward face of the immersed body. Prandtl roughened the forward face of a sphere by fixing a hoop to it, with the result that the **drag was considerably reduced**.
- ✓ Further experiments have been carried out in which sand particles have been stuck to the front face, as shown in previous Figure. The tendency for separation, and hence the magnitude of the form drag, are also dependent on the shape of the body.

Drag force on a spherical particle

The drag force, F, on a sphere is usually given in terms of a drag coefficient, C_D.

$$C_D = \frac{F}{\frac{1}{2}\rho\, u^2 A}$$

Note: C_D is analogous to friction factor, f, for pipe flow 'funning friction factor'.

Stokes showed a formula for the drag force for a sphere moving at a low velocity (creeping motion) in a continuos fluid.

$$F = 3\pi\mu\, d\, u$$

Where

F: total drag-resisting motion

ρ, μ: density and viscosity of a fluid, respectively.

d: diameter of a spherical particle.

u: velocity of the fluid relative to the particle.

A: projected area

Note 1: F consists of two components; a pressure drag force, F_p, and a shear stress force, F_s.

$$F_p = 2\pi\mu\, d\, u \qquad\qquad F_S = \pi\mu\, d\, u$$

Note 2: Experimentally, Stoke's Law is found to hold almost exactly for single particle Reynolds number, $Re \le 0.1$.

$$\text{Particle}\quad Re = ud\rho/\mu \quad \text{and} \quad C_D = f(Re)$$

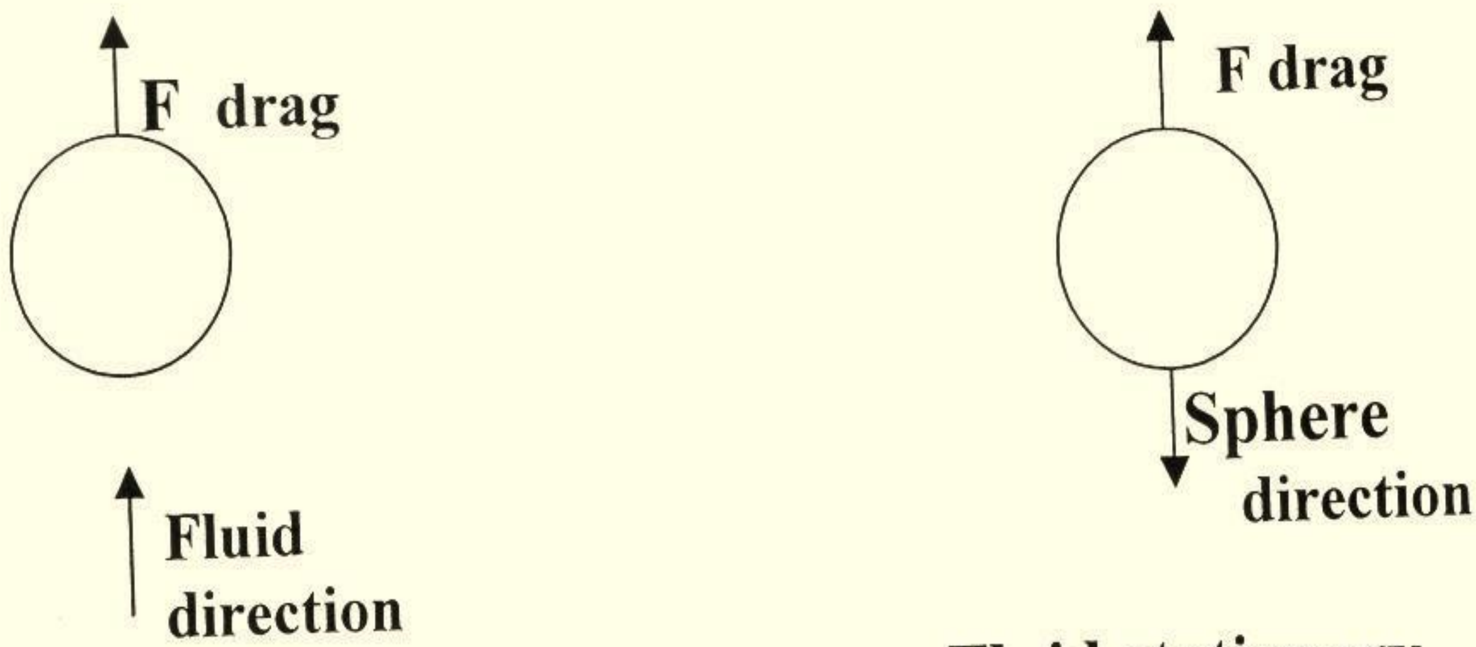
Where ρ is the density of the fluid, μ is the viscosity **of the fluid,** ***d is the diameter of the sphere, and u is the velocity of the fluid relative to*** **the particle.**

For spherical particle $A=(\pi/4)d^2$

$$\therefore C_D = \frac{F}{\frac{1}{2}\rho\, u^2 \left(\frac{\pi}{4}\right) d^2}$$

$$\therefore\ F = \frac{\pi}{8} C_D \rho\, u^2\, d^2$$

Direction of Drag:



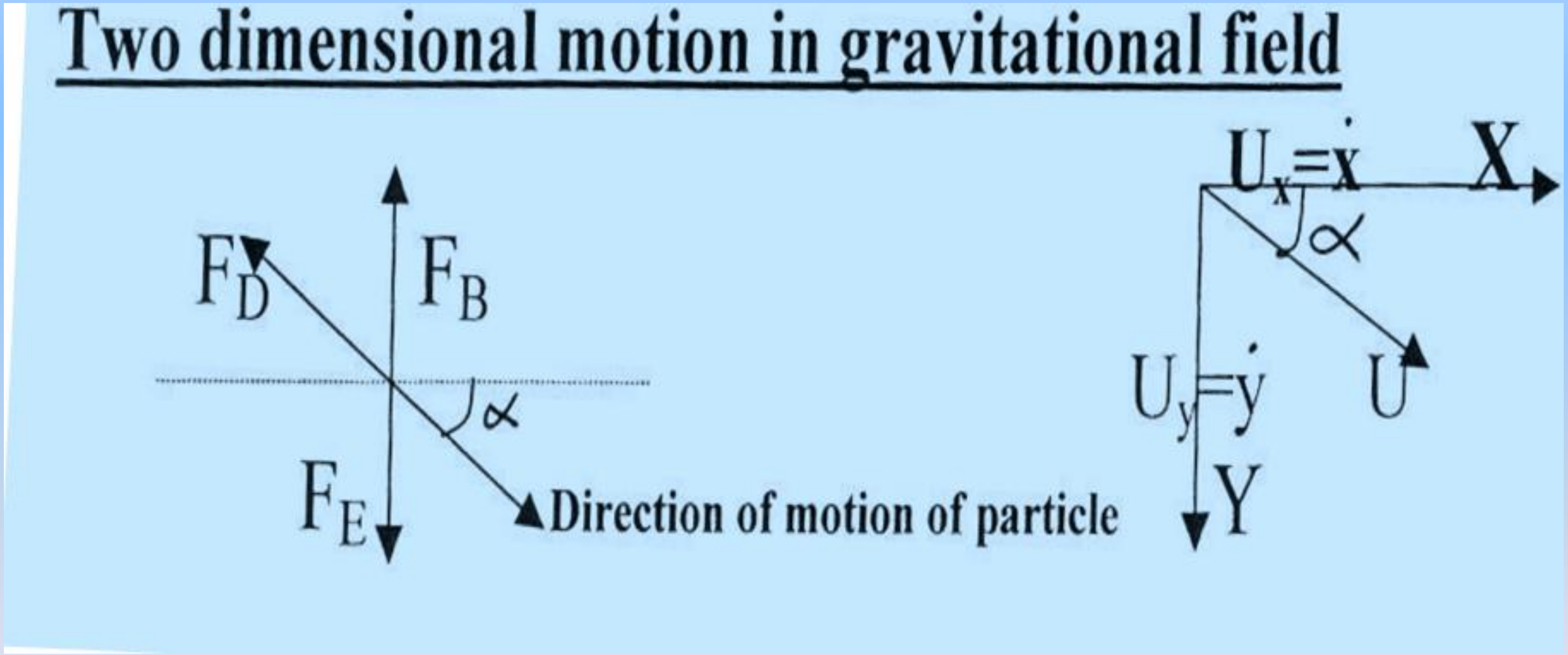
Body stationary **Fluid stationary**

Equations of motion of particles:

Forces acting on particle

- *External force* $F_E \Rightarrow$ *gravity, centrifugal etc.*
- *Buoyancy force* $F_B \Rightarrow$ *if* $\rho_f \neq \rho_p$ *acts vertically upwards.*
- *Drag force* $F_D \Rightarrow$ *acts parallel to relative velocity in opposite direction.*

Motion of a particle in gravitational field



$$\dot{x} = U\cos\alpha$$

$$\dot{y} = U\sin\alpha$$

$$U = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$F_E = m\ g$$

$$F_B = m\ \frac{\rho_f}{\rho_p}\ g$$

$$F_D = C_D \frac{1}{2} \rho_f\ U^2\ A$$

Force balance in X direction

Gravity – Buoyancy – drag = acceleration force

$$-F_D \cos\alpha = m\ddot{x}$$

$$-\frac{1}{2}\rho_f C_D A(\dot{x}^2+\dot{y}^2)\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} = m\ddot{x}$$

$$-\frac{1}{2}\rho_f C_D A\dot{x}\sqrt{(\dot{x}^2+\dot{y}^2)} = m\ddot{x}$$

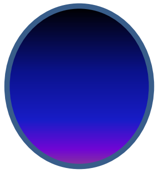
Force balance in Y direction

Gravity – Buoyancy – drag = acceleration force

$$mg\left(1-\frac{\rho_f}{\rho_p}\right)-\frac{1}{2}\rho_f C_D A\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}=m\ddot{y}$$

For spherical particle

For Sphere $\Rightarrow A=(\pi/4)\, d^2$, $m=(\pi/6)\, d^3\, \rho_p$



$$\ddot{x} = -\frac{3}{4}\frac{C_D\, \rho_f}{d\, \rho_p}\dot{x}\sqrt{\dot{x}^2+\dot{y}^2} \qquad (1)$$

$$\ddot{y} = -\frac{3}{4}\frac{C_D\, \rho_f}{d\, \rho_p}\dot{y}\sqrt{\dot{x}^2+\dot{y}^2} + g\,(1-\frac{\rho_f}{\rho_p}) \qquad (2)$$

Note

<u>**Special case**</u>

Spherical particle in stokes' regime Re<0.1⇒C_D=24/Re

$$\ddot{x} = -\frac{3}{4}\left(\frac{24\mu}{\rho_f d u}\right)\left(\frac{\rho_f}{\rho_p d}\right)\dot{x}u$$

$$\ddot{x} = -\frac{18\mu}{d^2\rho_p}\dot{x} = -a\dot{x}$$

Note

Similarly

$$\ddot{y} = -a\dot{y} + g(1 - \frac{\rho_f}{\rho_p}) = -a\dot{y} + b$$

where a and b are constants.

In order to solve the previous equations, use the following B.C $\Rightarrow$at t=0 $\rightarrow$ x=0, y=0

$$\dot{x} = u_0, \quad \dot{y} = v_o$$

Equilibrium or terminal velocity

- Assume a particle that falls from rest in a fluid. The particle will initially accelerate as the shear stress drag (which increases with velocity) will be small. As the particle accelerates the drag force increases, causing the acceleration to reduce. Later on a force balance will be achieved where the acceleration is zero and a maximum or terminal velocity is reached.

Single Particle terminal velocity

- This is known as the single particle terminal velocity.

- For a spherical particle, the following equation becomes:

gravity –buoyancy-drag = acceleration force

$$F_E - F_B - F_D = 0$$

Or

Or

$$F_D = F_E - F_B = mg\,(1 - \rho_f/\rho_p)$$

$$F_D = (\pi/6)\, d^3\, \rho_p\, g\,(1 - \rho_f/\rho_p)$$

$$\therefore\ F_D = (\pi/6)\, d^3\, g\,(\rho_p - \rho_f)$$

for stokes' regime $F_D = 3\pi\mu u_o d$, where u_o is terminal velocity.

$$\therefore\ 3\pi\mu u_o d = (\pi/6)\, d^3\, g\,(\rho_p - \rho_f)$$

$$u_o = \frac{d^2 g}{18\mu}\left(\rho_p - \rho_f\right) = \frac{b}{a}$$

Note that in the stokes' law region the terminal velocity is proportional to the square of the particle diameter.

For the form drag regime (Newton's Law), C_D=0.44

$$F_D = (1/2)\,\rho_f\, u^2\, A\, C_D$$

At force balance

$$\frac{0.44}{8}\pi d^2 \rho_f u_o^2 = \frac{d^3}{6}\pi g(\rho_p - \rho_f)$$

$$u_o = 1.75\left(d\,g\frac{(\rho_p - \rho_f)}{\rho_f}\right)^{1/2}$$

<u>Note</u> that in this region the terminal velocity is independent of the fluid viscosity and proportional to the square root of the particle size.

General Method to obtain Terminal Velocity

$$\because \frac{C_D}{8}\pi d^2 \rho_f u_o = \frac{d^3\pi}{6} g\,(\rho_p - \rho_f)$$

$$\therefore C_D = \frac{4}{3}\,\frac{d\,g}{u_o^2}\left(\frac{\rho_p}{\rho_f} - 1\right) \quad \text{..............................} (a)$$

$$Re = \frac{u_o\,d\,\rho_f}{\mu} \quad \text{..} (b)$$

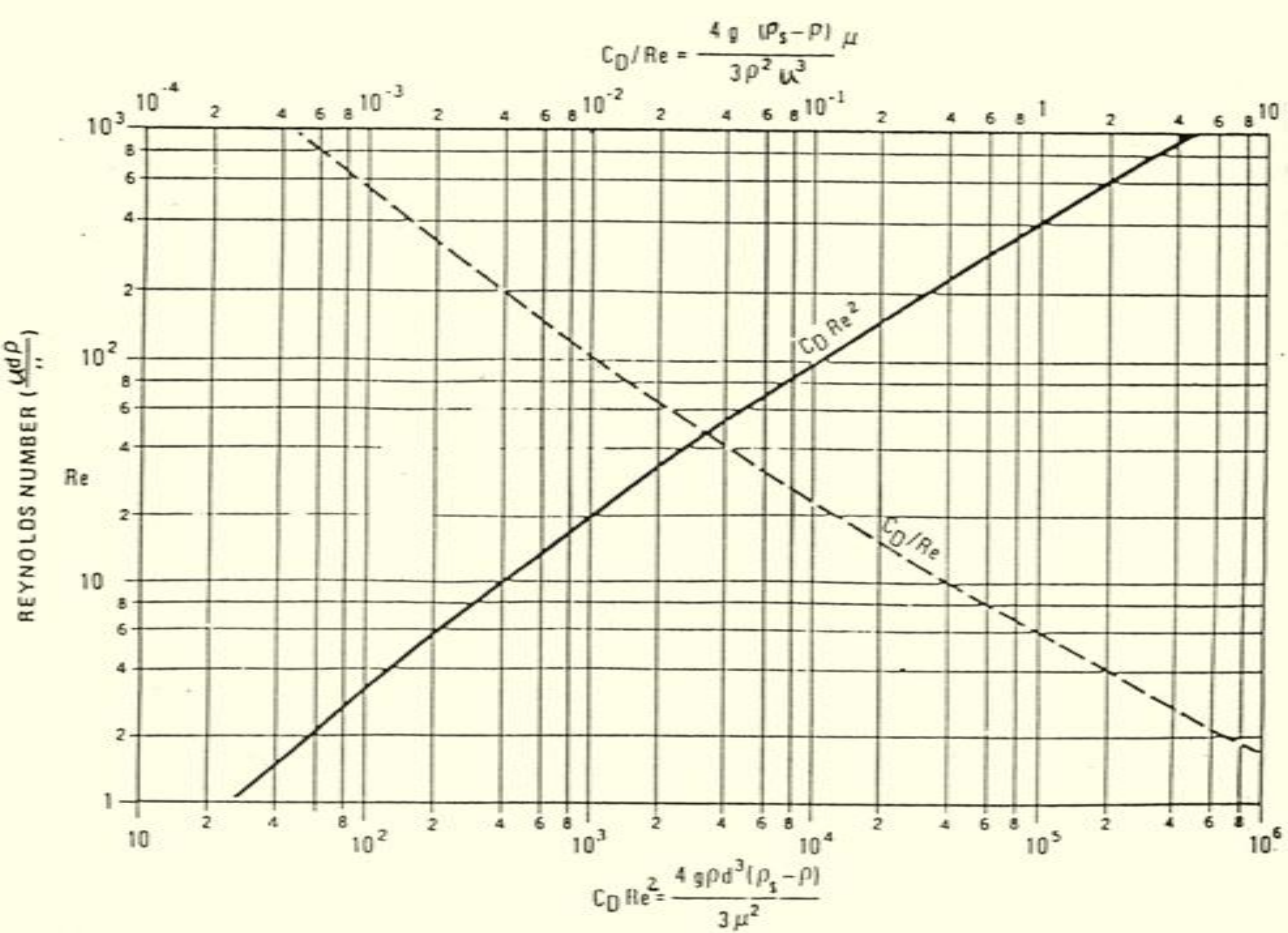
Using equations a and b, form the following dimensionless equations:

$$C_D Re^2 = \frac{4}{3}\frac{d^3 g \rho_f}{\mu^2}(\rho_p - \rho_f) = \frac{4}{3} Ga \text{ ....(c)}$$

$$\frac{C_D}{Re} = \frac{4}{3}\frac{\mu g}{u_0^3}\left(\frac{\rho_p - \rho_f}{\rho_f^2}\right) \text{ ....(d)}$$

Procedure to find u_0

- Eq. (c) is free of u_o, which can therefore be calculated, if d specified.
- Eq. (d) is free of d, which can therefore be calculated, if u_o specified.
- In order to obtain u_o or d, use tables or charts given in the text.
- The method covers all the regimes; Stokes', Transition, and Form drag "Newton's".



$C_D/Re = \frac{4g(\rho_s-\rho)\mu}{3\rho^2 u^3}$
REYNOLDS NUMBER ($\frac{u d \rho}{\mu}$)
Re
$C_D Re^2$
C_D/Re
$C_D Re^2 = \frac{4 g \rho d^3(\rho_s-\rho)}{3\mu^2}$

Generally speaking, for free settling, the terminal velocity of a particle in a given fluid tends to be higher as both its particle size and density are increased.

Suppose we have two particles: particle A and particle B with densities and sizes: ρ_A, ρ_B , d_A , and d_B, respectively.

In case of Stokes' regime $\Rightarrow$ Terminal velocities are

$$u_{oA} = \frac{d_A^2 g}{18\mu}(\rho_A - \rho_f)$$

$$u_{oB} = \frac{d_B^2 g}{18\mu}(\rho_B - \rho_f)$$

If $u_{oA} = u_{oB}$

$$\frac{d_B}{d_A} = \left[\frac{\rho_A - \rho_f}{\rho_B - \rho_f}\right]^{1/2}$$

In case Newton's regime:

$$u_{oA}^2 = \frac{d_A g}{0.33}\left(\frac{\rho_A - \rho_f}{\rho_f}\right)$$

$$u_{oB}^2 = \frac{d_B g}{0.33}\left(\frac{\rho_B - \rho_f}{\rho_f}\right)$$

For equal settling velocities

$$\frac{d_B}{d_A} = \left[\frac{\rho_A - \rho_f}{\rho_B - \rho_f}\right]$$

In general, for equal settling velocities

$$\frac{d_B}{d_A} = \left[\frac{\rho_A - \rho_f}{\rho_B - \rho_f}\right]^S$$

Where the index S= 1/2 for stokes' regime

Fine particle

S = 1 for Newton's regime

Coarse particle

1> S>1/2 for transient regime

Note

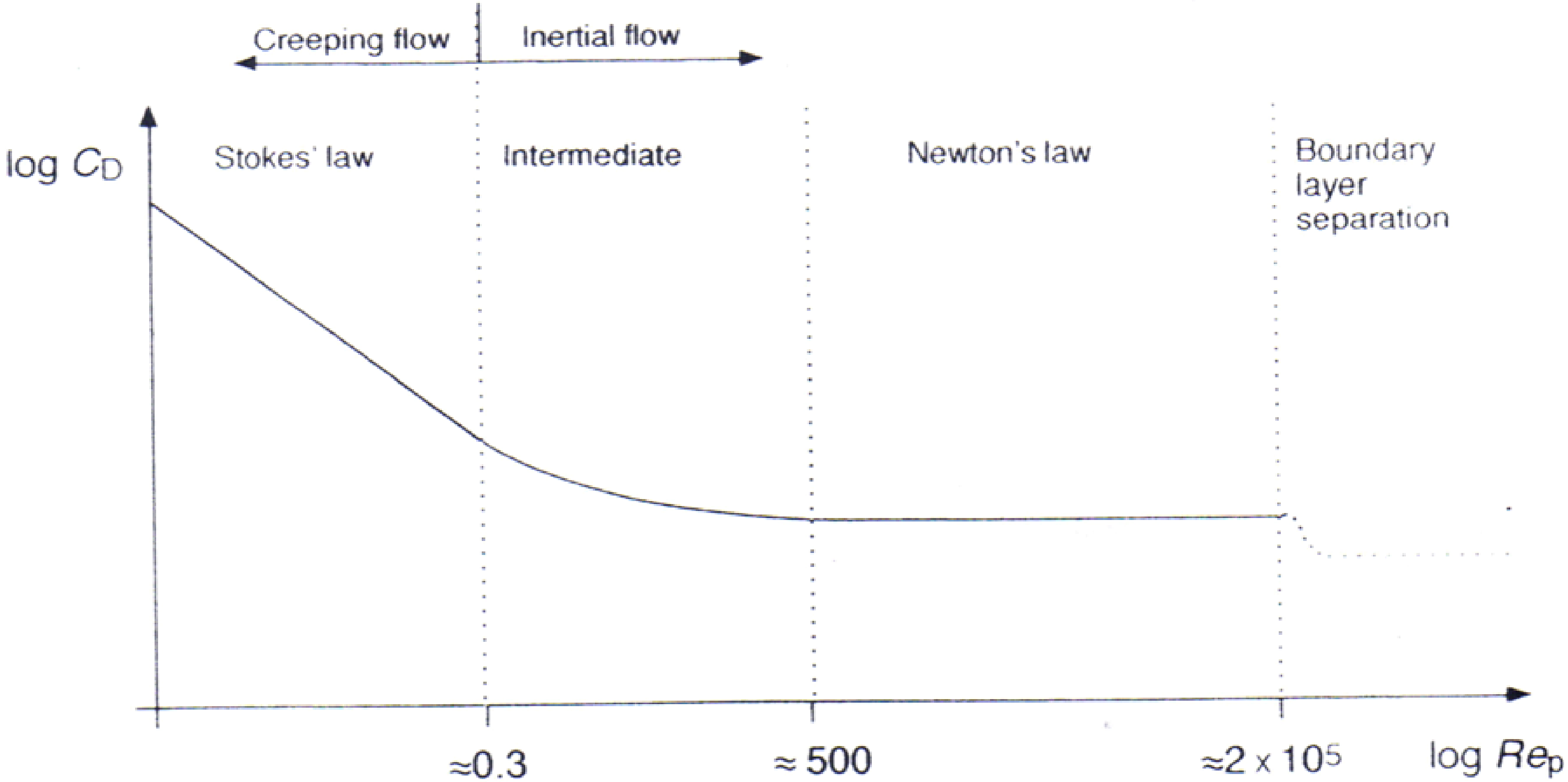
- If, for example, it is desired to separate particles of a relatively dense material **A of** density ρ_A *from particles of a less dense material* ***B and the size range is large, the terminal*** falling velocities of the largest particles of **B of density *ρ_B may be greater than those of*** the smallest particles of **A, and therefore a complete separation will not be possible. The** maximum range of sizes that can be separated is calculated from the ratio of the sizes of the particles of the two materials which have the same terminal falling velocities as given by the previous equation.

Example 2

A finely ground mixture of galena and limestone in the proportion of 1 to 4 by mass is subjected to elutriation by an upward-flowing stream of water flowing at a velocity of 5 mm/s. Assuming that the size distribution for each material is the same, and is as shown in the following table, estimate the percentage of galena in the material carried away and in the material left behind. The viscosity of water is 1 mN s/m^2 and Stokes' equation (3.1) may be used. The densities of galena and limestone are 7500 and 2700 kg/m^3, respectively.

Diameter (μm)	20	30	40	50	60	70	80	100
Undersize (per cent by mass)	15	28	48	54	64	72	78	88

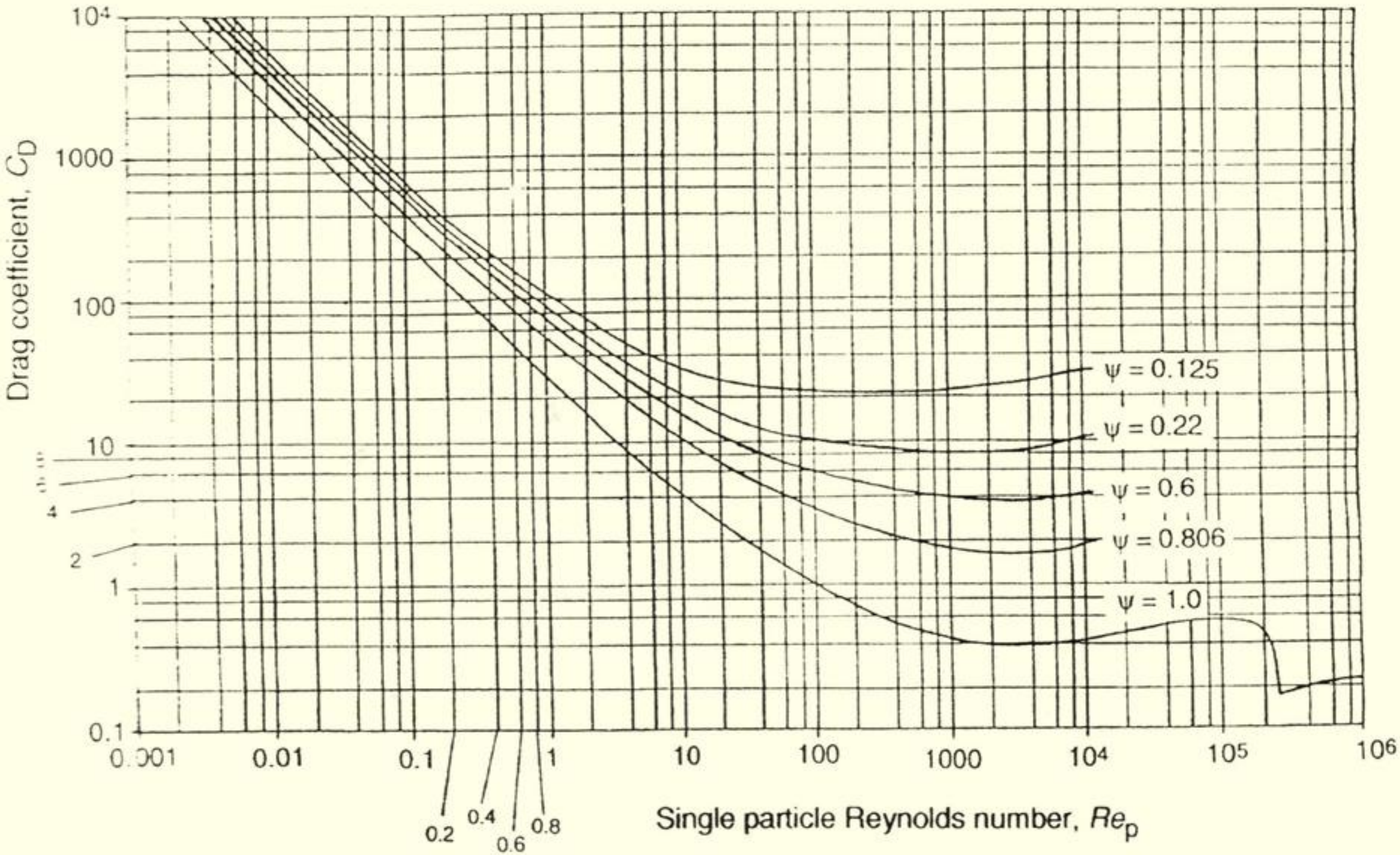
Solution



Standard drag curve for motion of a sphere in a fluid

Reynolds number ranges for single particle drag coefficient correlations

Region	Stokes	Intermediate	Newton's Law
Re_p range	< 0.3	$0.3 < Re_p < 500$	$500 < Re_p > 2 \times 10^5$
C_D	$24/Re_p$	$\approx 24/Re_p + 0.44$	≈ 0.44



Drag coefficient C_D versus Reynolds number Re_p for particles of sphericity ψ ranging form 0.125 to 1.0 (note Re_p uses the equivalent volume diameter)

Notes

$$\therefore \quad C_D = \frac{F_D}{\frac{1}{2}\rho u^2 A_p}$$

$$= \frac{F_D}{\frac{1}{2}\rho u^2 \frac{\pi}{4} d^2}$$

$$= \frac{4 F_D}{\frac{1}{2}\rho u^2 \pi d^2}$$

R'

Let $R' = \dfrac{4 F_D}{\pi d^2}$ force per projected area

C_D'

$$\therefore \quad C_D = \frac{2 R'}{\rho u^2}$$

Note

$\therefore \; C_D = 2\, C_D'$

analogous to Fanning friction factor f

analogous to friction factor ϕ for pipe flow

For Stokes' Law $C_D = 24\, Re^{-1}$

Or $\dfrac{R'}{\rho u^2} = 12\, Re^{-1} = 12\, \dfrac{\mu}{u d \rho}$

C_D'

Figure 3.4. $R'/\rho u^2$ versus Re' for spherical particles

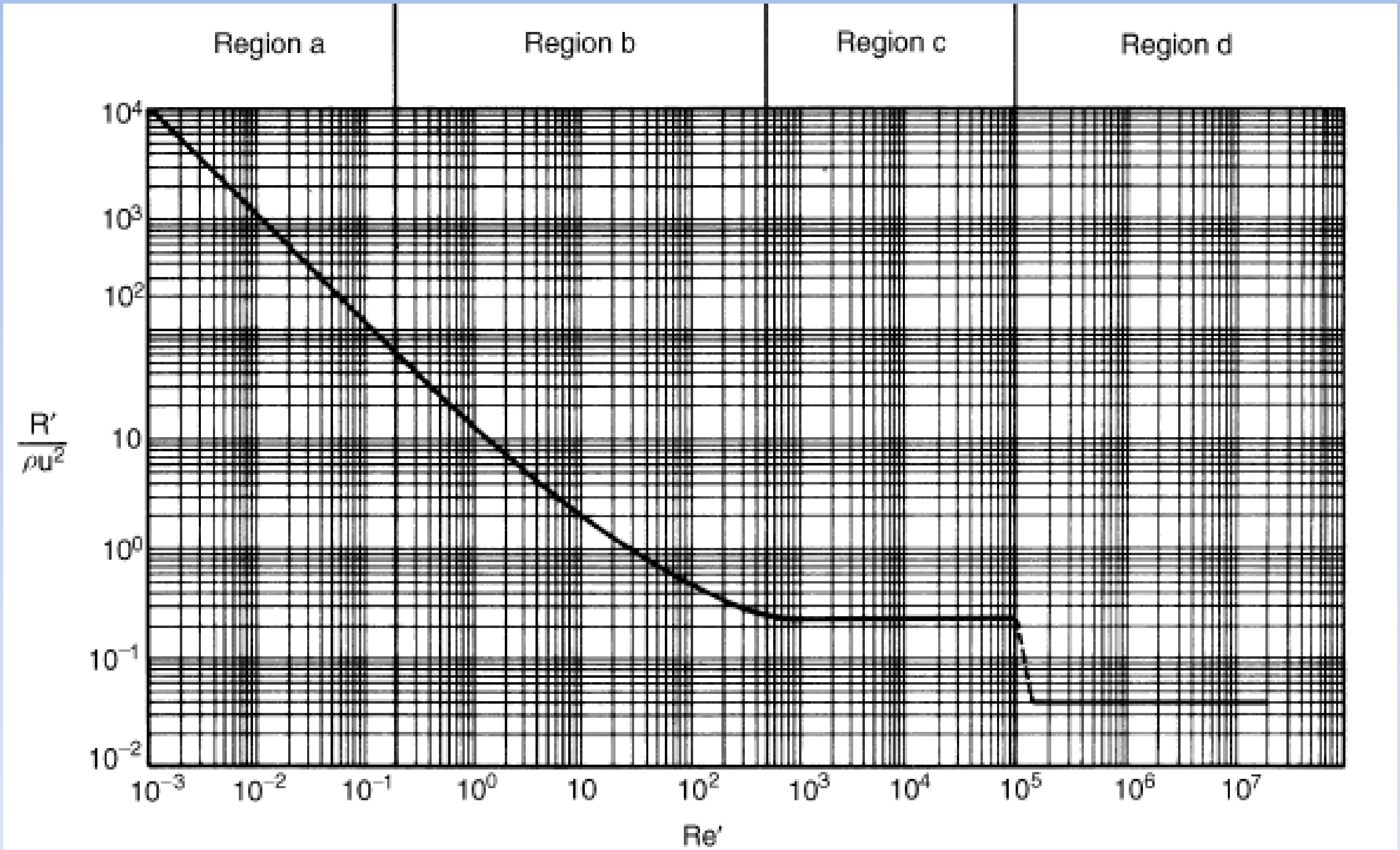


Table 3.2. $R'/\rho u^2$, $(R'/\rho u^2)Re'^2$ and $(R'/\rho u^2)Re'^{-1}$ as a function of Re'

Re'	$R'/\rho u^2$	$(R'/\rho u^2)Re'^2$	$(R'/\rho u^2)Re'^{-1}$
10^{-3}	12,000		
2×10^{-3}	6000		
5×10^{-3}	2400		
10^{-2}	1200	1.20×10^{-1}	1.20×10^{5}
2×10^{-2}	600	2.40×10^{-1}	3.00×10^{4}
5×10^{-2}	240	6.00×10^{-1}	4.80×10^{3}
10^{-1}	124	1.24	1.24×10^{3}
2×10^{-1}	63	2.52	3.15×10^{2}
5×10^{-1}	26.3	6.4	5.26×10
10^{0}	13.8	1.38×10	1.38×10
2×10^{0}	7.45	2.98×10	3.73
5×10^{0}	3.49	8.73×10	7.00×10^{-1}
10	2.08	2.08×10^{2}	2.08×10^{-1}
2×10	1.30	5.20×10^{2}	6.50×10^{-2}
5×10	0.768	1.92×10^{3}	1.54×10^{-2}
10^{2}	0.547	5.47×10^{3}	5.47×10^{-3}
2×10^{2}	0.404	1.62×10^{4}	2.02×10^{-3}
5×10^{2}	0.283	7.08×10^{4}	5.70×10^{-4}
10^{3}	0.221	2.21×10^{5}	2.21×10^{-4}
2×10^{3}	0.22	8.8×10^{5}	1.1×10^{-4}
5×10^{3}	0.22	5.5×10^{6}	4.4×10^{-5}
10^{4}	0.22	2.2×10^{7}	2.2×10^{-5}
2×10^{4}	0.22		
5×10^{4}	0.22		
10^{5}	0.22		
2×10^{5}	0.05		
5×10^{5}	0.05		
10^{6}	0.05		
2×10^{6}	0.05		
5×10^{6}	0.05		
10^{7}	0.05		

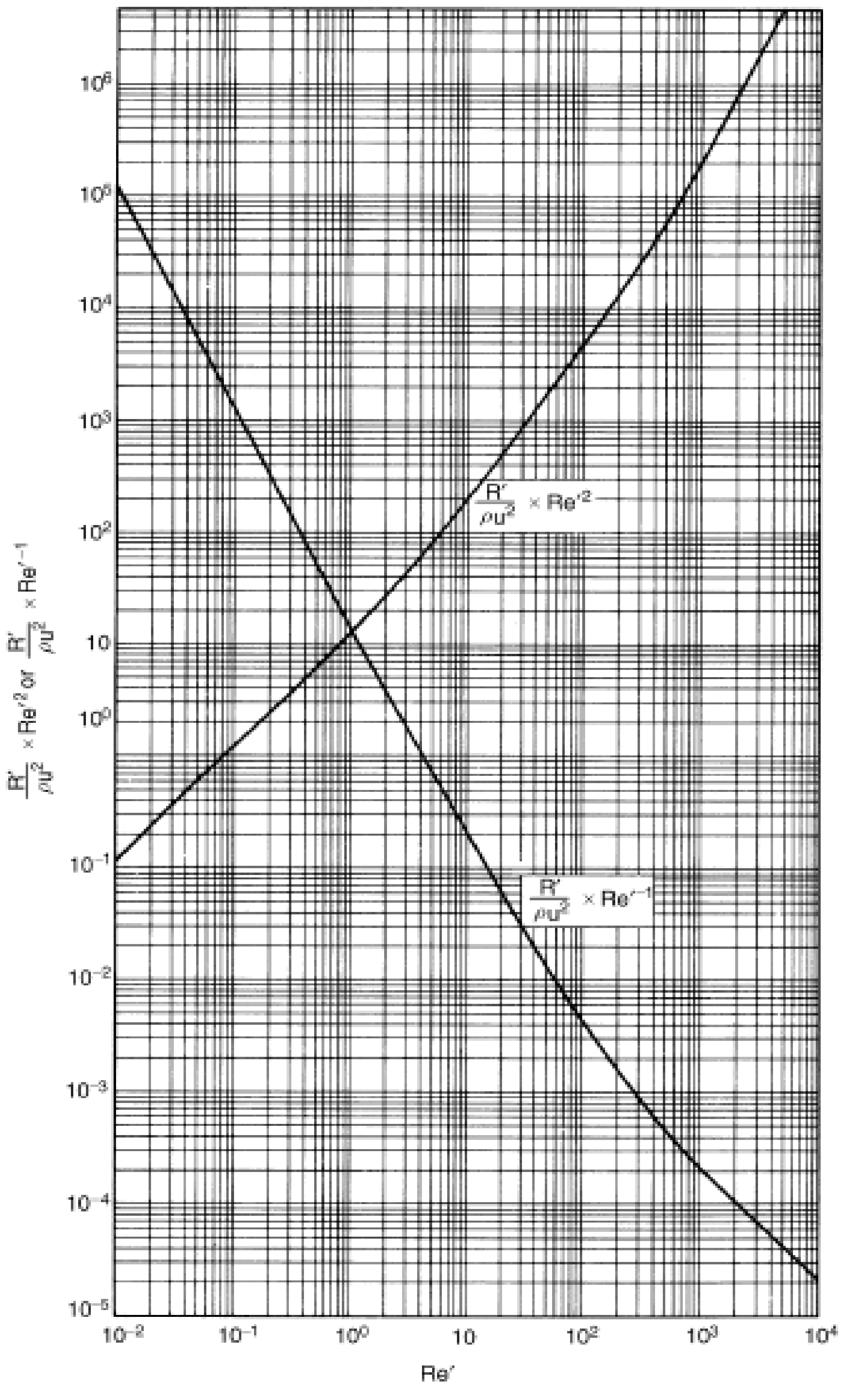


Figure 3.6. $(R'/\rho u^2)Re'^2$ and $(R'/\rho u^2)Re'^{-1}$ versus Re' for spherical particles

Table 3.4. Values of $\log Re'$ as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{2}$								$\bar{3}.620$	$\bar{3}.720$	$\bar{3}.819$
$\bar{1}$	$\bar{3}.919$	$\bar{2}.018$	$\bar{2}.117$	$\bar{2}.216$	$\bar{2}.315$	$\bar{2}.414$	$\bar{2}.513$	$\bar{2}.612$	$\bar{2}.711$	$\bar{2}.810$
0	$\bar{2}.908$	$\bar{1}.007$	$\bar{1}.105$	$\bar{1}.203$	$\bar{1}.301$	$\bar{1}.398$	$\bar{1}.495$	$\bar{1}.591$	$\bar{1}.686$	$\bar{1}.781$
1	$\bar{1}.874$	$\bar{1}.967$	0.008	0.148	0.236	0.324	0.410	0.495	0.577	0.659
2	0.738	0.817	0.895	0.972	1.048	1.124	1.199	1.273	1.346	1.419
3	1.491	1.562	1.632	1.702	1.771	1.839	1.907	1.974	2.040	2.106
4	2.171	2.236	2.300	2.363	2.425	2.487	2.548	2.608	2.667	2.725
5	2.783	2.841	2.899	2.956	3.013	3.070	3.127	3.183	3.239	3.295

Table 3.5. Values of $\log Re'$ as a function of $\log[(R'/\rho u^2)Re'^{-1}]$ for spherical particles

$\log[(R'/\rho u^2)Re'^{-1}]$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{5}$										3.401
$\bar{4}$	3.316	3.231	3.148	3.065	2.984	2.903	2.824	2.745	2.668	2.591
$\bar{3}$	2.517	2.443	2.372	2.300	2.231	2.162	2.095	2.027	1.961	1.894
$\bar{2}$	1.829	1.763	1.699	1.634	1.571	1.508	1.496	1.383	1.322	1.260
$\bar{1}$	1.200	1.140	1.081	1.022	0.963	0.904	0.846	0.788	0.730	0.672
0	0.616	0.560	0.505	0.449	0.394	0.339	0.286	0.232	0.178	0.125
1	0.072	0.019	$\bar{1}.969$	$\bar{1}.919$	$\bar{1}.865$	$\bar{1}.811$	$\bar{1}.760$	$\bar{1}.708$	$\bar{1}.656$	$\bar{1}.605$
2	$\bar{1}.554$	$\bar{1}.503$	$\bar{1}.452$	$\bar{1}.401$	$\bar{1}.350$	$\bar{1}.299$	$\bar{1}.249$	$\bar{1}.198$	$\bar{1}.148$	$\bar{1}.097$
3	$\bar{1}.047$	$\bar{2}.996$	$\bar{2}.946$	$\bar{2}.895$	$\bar{2}.845$	$\bar{2}.794$	$\bar{2}.744$	$\bar{2}.694$	$\bar{2}.644$	$\bar{2}.594$
4	$\bar{2}.544$	$\bar{2}.493$	$\bar{2}.443$	$\bar{2}.393$	$\bar{2}.343$	$\bar{2}.292$				

Example 3

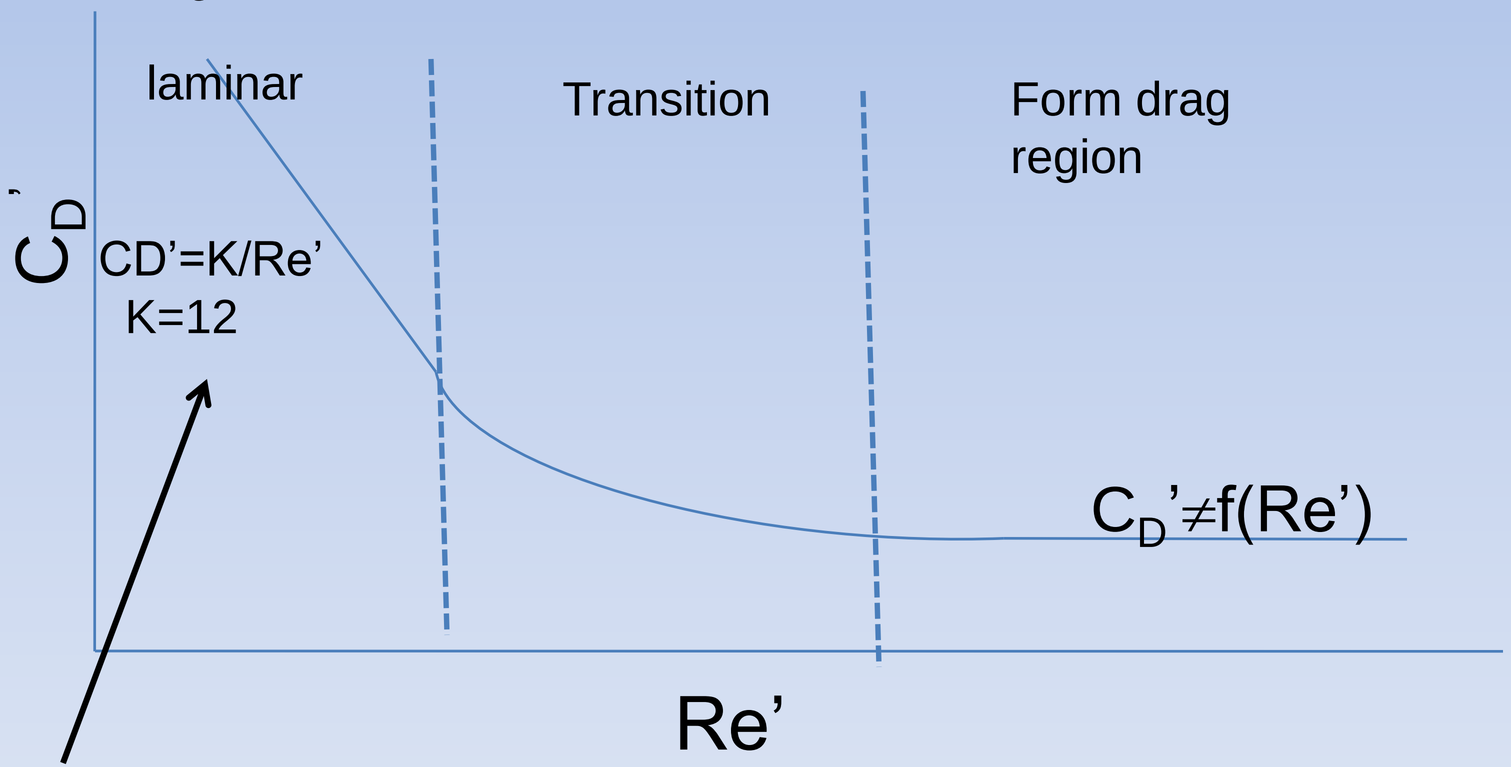
What is the terminal velocity of a spherical steel particle, 0.40 mm in diameter, settling in an oil of density 820 kg/m^3 and viscosity 10 mN s/m^2? The density of steel is 7870 kg/m^3.

Alternative solution

Non-spherical particles

- Since $C_D = f(Re', \text{shape})$
- There are a set of experimental curves (C_D against Re') for various shapes, similar to spherical particles.
- For non-spherical particles, the orientation must be specified before the drag force can be calculated.
- $Re' = ud'\rho/\mu$, *where* d' the diameter of the circle having the same area as the projected area of the particle

- The curve for *R'/ρu2 against Re' may be divided into three main regions, (a), (b), and (c) as before.*



In this region (a), a particle falling freely in the fluid under the action of gravity will normally move with its longest surface nearly parallel to the direction of motion.

- Region *(b) represents transition conditions and commences at a lower value* of *Re′, and a correspondingly higher value of $R'/\rho u^2$, than in the case of the sphere.*
- A freely falling particle will tend to change its orientation as the value of *Re′ changes* and some instability may be apparent.
- In region *(c) the particle tends to fall so that it is* presenting the maximum possible surface to the oncoming fluid.
- Typical values of $R'/\rho u^2$ for non-spherical particles in region *(c) are given in Table 3.6*

Table 3.6. Drag coefficients for Non - Spherical Particles

Configuration	Length/breadth	$R'/\rho u^2$
Thin rectangular plates with their planes Perpendicular to the direction of motion	1–5	0.6
	20	0.75
	∞	0.95
Cylinders with axes parallel to the direction of motion	1	0.45
Cylinders with axes perpendicular to the Direction of motion	1	0.3
	20	0.45
	∞	0.6

Free falling velocity for Non-spherical particles

$$F_D = mg\left(1 - \frac{\rho_f}{\rho_p}\right)$$

$$= k\, d_p^3\, \rho_p\, g\left(1 - \frac{\rho_f}{\rho_p}\right)$$

$$= k\, d_p^3\, g\,(\rho_p - \rho_f)$$

$$C_D \cdot \frac{1}{2}\rho_f u_o^2 \cdot \frac{\pi}{4} d_p^2 = k\, d_p^3\, g\,(\rho_p - \rho_f)$$

$$\frac{\cancel{2} R'}{\rho_f u_o^2}\,\frac{1}{\cancel{2}}\rho_f u_o^2\,\frac{\pi}{4} d_p^2 = k\, d_p^3\, g\,(\rho_p - \rho_f)$$

$$\therefore \frac{R'_o}{\rho_f u_o^2} = \frac{4 k\, d_p\, g}{\pi \rho_f u_o^2}(\rho_p - \rho_f)$$

Assume that dp is the same as the mean projected diameter d’

Note:
$V = k\, dp^3$
$\therefore k = V/\, dp^3$
For sphere $k = \pi/6$

Multiply by Re^2

$$C_D' \rightarrow \left(\frac{R_0'}{\rho u_0^2}\right) Re_0^2 = \frac{4 k \rho_f d_p^3 g}{\mu^2 \pi}(\rho_p - \rho_f)$$

free of u_0

confusion

and divide by Re

$$C_D' \rightarrow \left(\frac{R_0'}{\rho u_0^2}\right)\frac{1}{Re} = \frac{4 k \mu g}{\pi \rho_f^2 u_0^3}(\rho_p - \rho_f)$$

free of particle size "d"

OR

$$C_D Re_0^2 = \frac{8 k \rho_f d_p^3 g}{\mu^2 \pi}(\rho_p - \rho_f)$$

$$\frac{C_D}{Re} = \frac{8 k \mu g}{\pi \rho_f^2 u_0^3}(\rho_p - \rho_f)$$

Example 4

What will be the terminal velocities of mica plates, 1 mm thick and ranging in area from 6 to 600 mm^2, settling in an oil of density 820 kg/m^3 and viscosity 10 mN s/m^2? The density of mica is 3000 kg/m^3.

solution

Table 3.7. Corrections to $\log Re'$ as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for non-spherical particles

$\log\{(R'/\rho u^2)Re'^2\}$	$k' = 0.4$	$k' = 0.3$	$k' = 0.2$	$k' = 0.1$
$\bar{2}$	−0.022	−0.002	+0.032	+0.131
$\bar{1}$	−0.023	−0.003	+0.030	+0.131
0	−0.025	−0.005	+0.026	+0.129
1	−0.027	−0.010	+0.021	+0.122
2	−0.031	−0.016	+0.012	+0.111
2.5	−0.033	−0.020	0.000	+0.080
3	−0.038	−0.032	−0.022	+0.025
3.5	−0.051	−0.052	−0.056	−0.040
4	−0.068	−0.074	−0.089	−0.098
4.5	−0.083	−0.093	−0.114	−0.146
5	−0.097	−0.110	−0.135	−0.186
5.5	−0.109	−0.125	−0.154	−0.224
6	−0.120	−0.134	−0.172	−0.255