Motion of particles in a Centrifugal Field

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Examples : Cyclones , Settlers

In Case of a particle is moving in a fluid under the action of Centrifugal field, the effects of gravitational is comparatively Small and can be neglected.

Assume stokes' regime

For spherical particle and stokes' regime

$$\frac{\pi}{6} d^3 (\rho_p - \rho_f) r\omega^2 - 3\pi \mu d \frac{dr}{dt} = \frac{\pi}{6} d^3 \rho_p \frac{d^2 r}{dt^2} \quad \ldots (1)$$

instantaneous velocity

inertial term

where

r : radius of rotation , m

w : angular velocity , rad/s

$r\omega^2$: Centrifugal acceleration

<u>Note</u>: As the particle moves outwards, the accelerating force increases and therefore it never acquires an equilibrium velocity in the fluid

$$\frac{d^2r}{dt^2} + \frac{18\mu}{d^2\rho_p} \cdot \frac{dr}{dt} - \frac{\rho_p - \rho_f}{\rho_p} \omega^2 r = 0 \quad \ldots\ldots (2)$$

OR

$$\frac{d^2r}{dt^2} + a\frac{dr}{dt} - nr = 0 \quad \ldots\ldots (3)$$

The Solution is

$$r = B_1 e^{-\left[a/2 + \sqrt{a^2/4 + n}\right]t} + B_2 e^{-\left[a/2 - \sqrt{a^2/4 + n}\right]t}$$

OR

$$\therefore r = e^{-at/2} \left\{ B_1 e^{-kt} + B_2 e^{kt} \right\} \quad \ldots\ldots (4)$$

where $a = 18\mu/d^2\rho_p$, $n = (1 - \rho_f/\rho_p)\omega^2$, $k = \sqrt{a^2/4 + n}$

$B.C^s$

$$t = 0, \quad r = r_1, \quad dr/dt = 0 \qquad \ldots\ldots (5)$$

$\therefore$ Eq.(4):

$$\frac{dr}{dt} = e^{-at/2}\left\{-kB_1 e^{-kt} + kB_2 e^{kt}\right\} - \frac{a}{2} e^{-\frac{at}{2}}\left\{B_1 e^{-kt} + B_2 e^{kt}\right\}$$

$$= e^{-\frac{at}{2}}\left\{(k - \tfrac{a}{2}) B_2 e^{kt} - (k + \tfrac{a}{2}) B_1 e^{-kt}\right\} \qquad \ldots\ldots (6)$$

Substituting the BC^s into eqs. (4) and (6)

$$r_1 = B_1 + B_2 \quad \text{-------- (7)}$$

$$0 = \left(k - \frac{a}{2}\right) B_2 - \left(k + \frac{a}{2}\right) B_1 \quad \text{------ (8)}$$

from (8) $\Longrightarrow$ $$\frac{B_1}{B_2} = \frac{k - a/2}{k + a/2} \quad \text{—— (9)}$$

Sub. into (7)

$$r_1 = B_2\left(\frac{B_1}{B_2} + 1\right) = B_2\left(\frac{k - a/2}{k + a/2} + 1\right)$$

$$\therefore \quad r_1 = \frac{2k}{k + a/2} B_2 \quad \text{-------- (10)}$$

OR

$$B_2 = \frac{k + a/2}{2k} r_1 \quad \text{-------- (11)}$$

Sub. into (9)

$$\therefore \quad B_1 = \frac{k - a/2}{k + a/2} B_2 = \frac{k - a/2}{\cancel{k + a/2}} \cdot \frac{\cancel{(k + a/2)}}{2k} r_1$$

$$= \frac{k - a/2}{2k} r_1 \quad - - - - - - - (12)$$

Thus eq. (4) becomes

$$r = e^{-\frac{at}{2}} \left\{ \frac{k - a/2}{2k} r_1 e^{-kt} + \frac{k + a/2}{2k} r_1 e^{kt} \right\} \quad \ldots (13)$$

OR

$$\frac{r}{r_1} = e^{-\frac{at}{2}}\left\{\cosh kt + \frac{a}{2k}\sinh kt\right\} \quad \ldots\ldots\ldots (14)$$

$\therefore$ The ratio r/r_1 could be calculated at any time t, but numerical solution is required to obtain t for any particular r/r_1 value.

Suppose the effect of particle acceleration is neglected, then eq.(3) simplifies to:

$$a\,\frac{dr}{dt} - n\,r = 0 \Rightarrow \frac{dr}{dt} = \frac{n}{a}\,r$$

$$\int_{r_1}^{r} \frac{dr}{r} = \frac{n}{a}\int_{0}^{t} dt \Rightarrow \ln\frac{r}{r_1} = \frac{n}{a}\,t$$

$$\therefore \ln\frac{r}{r_1} = \frac{d^2(\rho_p - \rho_f)\,\omega^2}{18\mu}\,t$$

The time required for the particle to move to a radius r is

$$t = \frac{18\mu}{d^2 \omega^2 (\rho_p - \rho_f)} \ln \frac{r}{r_1} \quad \cdots\cdots (15)$$

If the particle is initially situated in the liquid surface ($r_1 = r_i$), the time taken to reach the wall of the basket ($r = R$).

$$\boxed{t = \frac{18\mu}{d^2 \omega^2 (\rho_p - \rho_f)} \ln \frac{R}{r_i}} \quad \cdots\cdots (16)$$

If h is the thickness of liquid layer at the wall: $h = R - r_i$

$$\therefore \ln\frac{R}{r_i} = \ln\frac{R}{R-h} = -\ln\frac{R-h}{R} = -\ln\left(1-\frac{h}{R}\right) =$$

$$= \frac{h}{R} + \frac{1}{2}\left(\frac{h}{R}\right)^2 + \cdots$$

If h is small compared with R

$$\therefore \ln\frac{R}{r_i} = \frac{h}{R}$$

$\therefore$ Eq. (16) becomes

$$t = \frac{18\mu h}{d^2\omega^2(\rho_p-\rho_f)R} \qquad ///$$

Form Drag regime "Newton's Law"

$$C_D = \text{cons} = 0.44$$

$$\frac{\pi}{6} d^3 (\rho_p - \rho_f) r\omega^2 - \frac{1}{2}(0.44)\frac{\pi}{4} d^2 \rho_f \left(\frac{dr}{dt}\right)^2 = \frac{\pi}{6} d^3 \rho_p \frac{d^2 r}{dt^2}$$

This equation can only be solved numerically

If the acceleration is neglected:

$$\left(\frac{dr}{dt}\right)^2 = 3 d \omega^2 \left(\frac{\rho_p-\rho_f}{\rho_f}\right) r$$

$$r^{-1/2}\frac{dr}{dt} = \left\{3 d\omega^2 \left(\frac{\rho_p-\rho_f}{\rho_f}\right)\right\}^{1/2}$$

on integration

$$2(r^{1/2} - r_1^{1/2}) = \left\{3d\omega^2\left(\frac{\rho_p-\rho_f}{\rho_f}\right)\right\}^{1/2} t \Rightarrow t = \left[\frac{\rho_f}{3d\omega^2(\rho_p-\rho_f)}\right] 2(r^{1/2}-r_1^{1/2})$$

Note

- Angular velocity $\omega = 2\pi N / 60$, rad/s

  where N is the revolutions per min

Problem
Bach Centrifugation

A suspension consistent of fine solid particles (1425 kg/m^3) and water was transferred to a centrifuge bowl. The surface of water in a bowl was 16.5 cm from the axis of rotation. After centrifuging for 360 sec at 360 revolutions per min a pipette was inserted into the bowl and a small sample of the suspension withdrawn from 1.0 cm below the surface.

Find the particle size of the solid material which was suspended in the sample. $\mu_{water} = 1.24 \times 10^{-3}$ $kg/m \cdot s$

Effect of boundaries on terminal velocity

- When a particle is falling through a fluid in the presence of a solid boundary the terminal Velocity reached by the particle is less than that for an infinite fluid.
- wall factor:

$$f_w = u_T/u_{T\infty}$$

$$f_w = \left(1 - \frac{x}{D}\right)^{2.25} \quad Re_p \leq 0.3; \quad x/D \leq 0.97$$

Look!!!

- Sand particles falling from rest in air (particle density, 2600 kg/m^3)

Size	Time to each 99% of U_T (s)	U_T (m/s)	Distance travelled in this time (m)
30 μm	0.033	0.07	0.00185
3 mm	3.5	14	35
3 cm	11.9	44	453

Try to solve this problem

A sphere of diameter 10 mm and density 7700 kg/m^3 falls under gravity at terminal conditions through a liquid of density 900 kg/m^3 in a tube of diameter 12 mm. The measured terminal velocity of the particle is 1.6 mm/s. Calculate the viscosity of the fluid. Verify that Stokes' law applies.