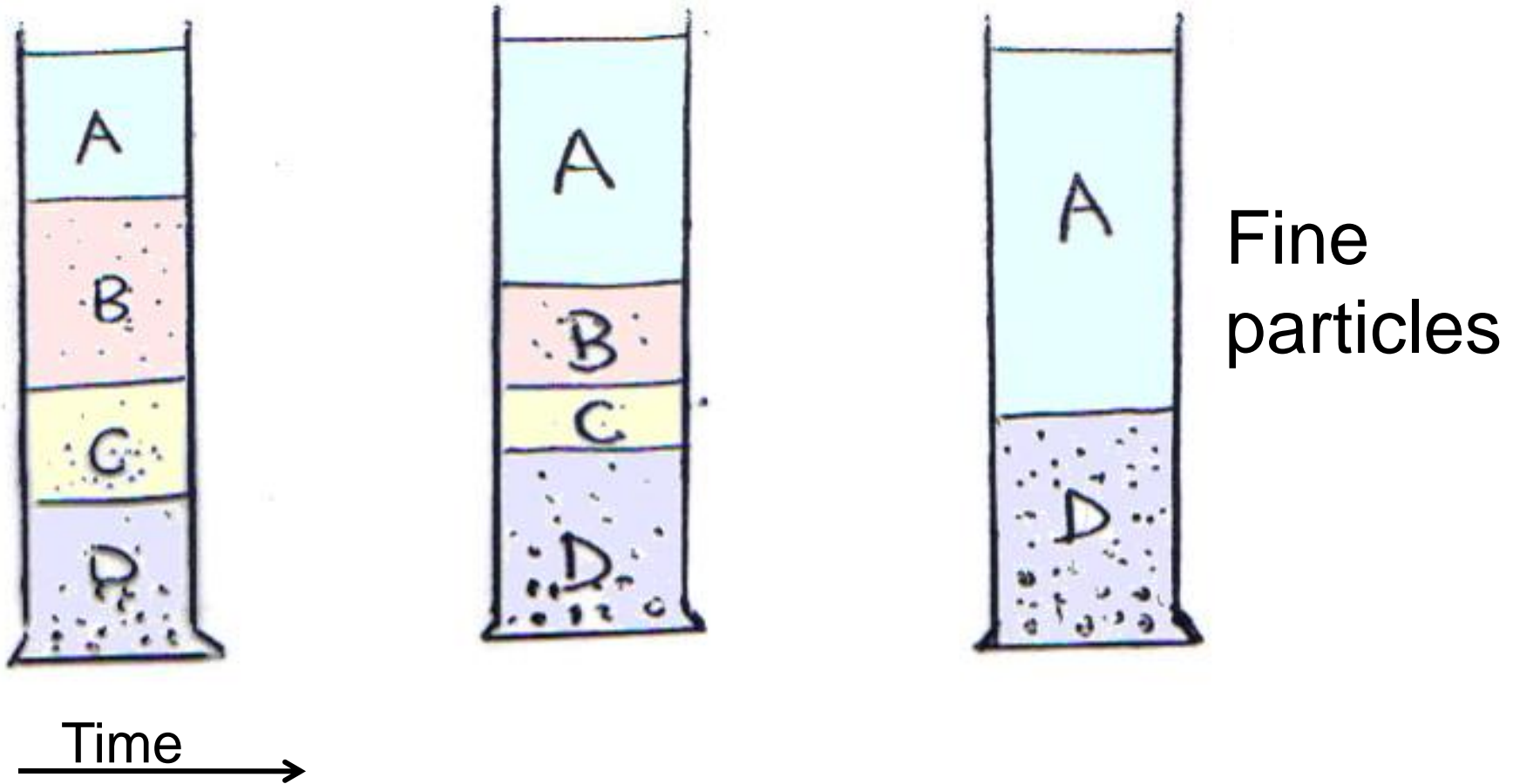


Sedimentation

Sedimentation

- Gravitational sedimentation
- Meaning
- The batch settling test 'Solids settle from a slurry in a glass cylinder'.
- 4 zones can be observed:
clear liquid 'A'; uniform concentration 'B';
transition 'C'; coarse solids 'D'.
- Critical settling point \Rightarrow only zones A and D.

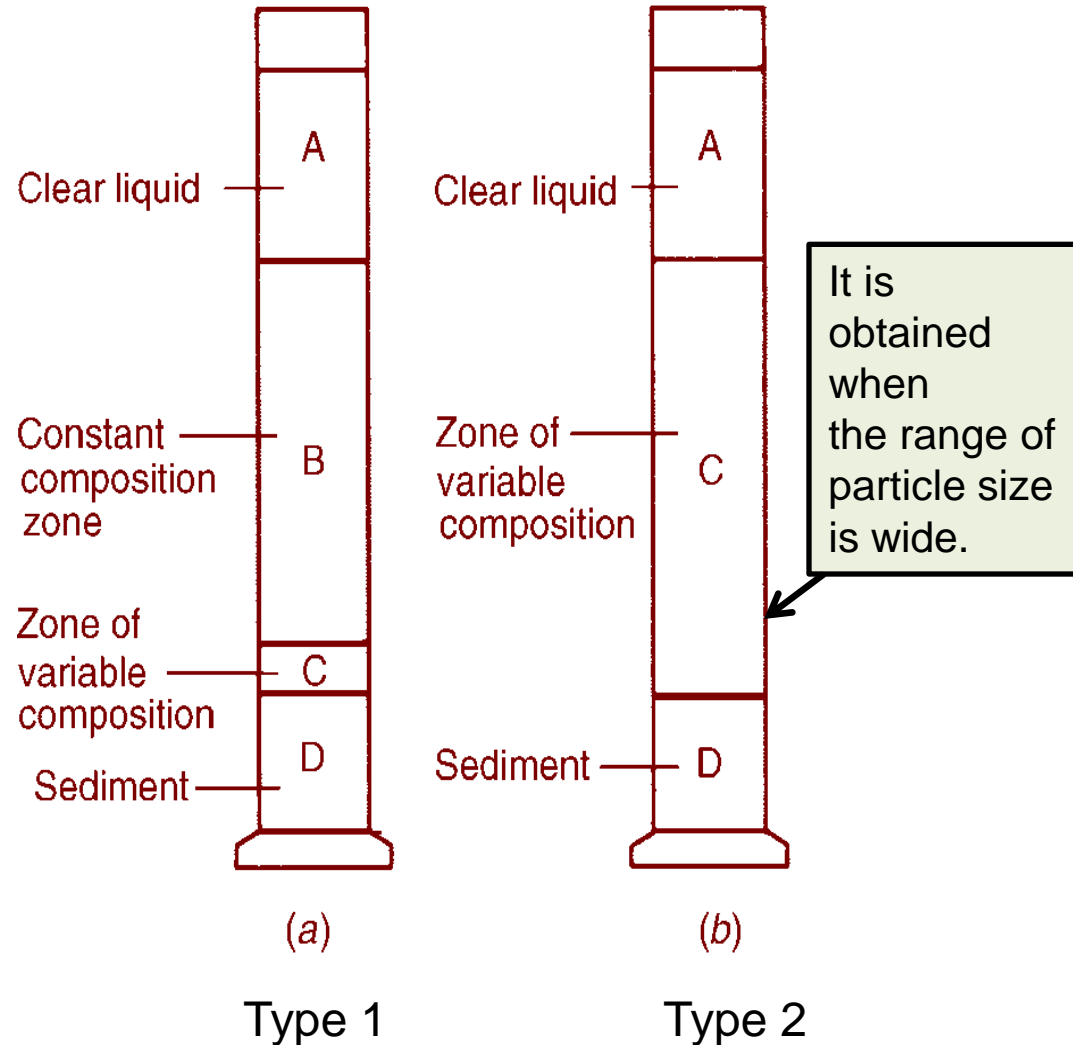
Batch settling experiment



Comparison between Type 1 and Type 2

Settling can be divided into two types according to the zones:

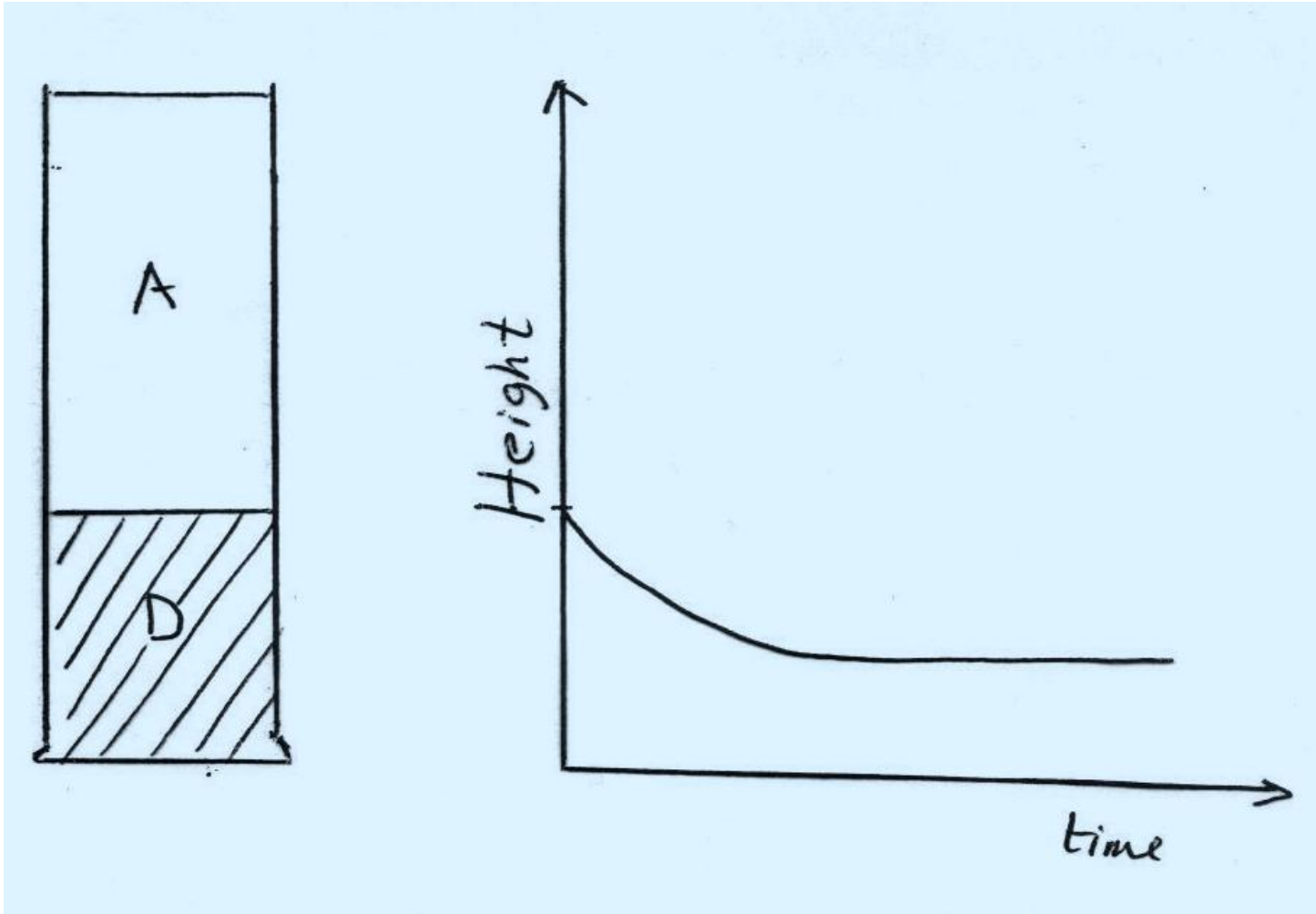
- Type 1 contains zones A, B, C and D;
- Type 2 contains zones A, C and D.



Gravity sedimentation

- Suspension can also be divided into concentrated suspension and dilute suspension. The first is called **‘sludge line settling’** and the later **‘selective settling’**.
- After critical settling pt., compression and consolidation of solid layer ‘D’ will take place with liquid being forced upwards into the clear liquid zone ‘A’. Hence the height of layer D will be decreased.

Compression and consolidation of solid layer



Stokes' law

For a suspension of particles in a fluid, Stokes' law is assumed to apply but an effective viscosity and effective average density are used.

The modified settling or sedimentation velocity in suspension is:

$$u_c = K'' \frac{d^2 (\rho_s - \rho_c) g}{\mu_c} \dots\dots\dots(1)$$

where

K'' : cons.

$$(\rho_s - \rho_c) = \rho_s - \{\rho_s(1 - e) + \rho e\} = e(\rho_s - \rho) \dots\dots\dots(2)$$

$$\mu_c = \mu(1 + k''c) \quad \leftrightarrow \quad c \text{ up to } 0.02 \quad \dots\dots\dots(3)$$

where

k'' : cons. for a given shape of particles(2.5 for spheres)

c : volumetric conc. of particles.

μ : fluid viscosity

But

$$\mu_c = \mu e^{k''c/(1+a'c)} \quad \leftrightarrow \quad c > 0.02 \quad \dots\dots\dots(4)$$

where a' : cons. = 0.609 for spheres

It has been shown for a suspension of uniform particles, the velocity of particle relative to the fluid, u_p , is given by:

$$u_p = \frac{d^2 (\rho_s - \rho_c) g}{18\mu} f(e) \dots\dots\dots(5)$$

or

$$u_p = \frac{u_c}{e} \dots\dots\dots(6)$$

f(e) could be as

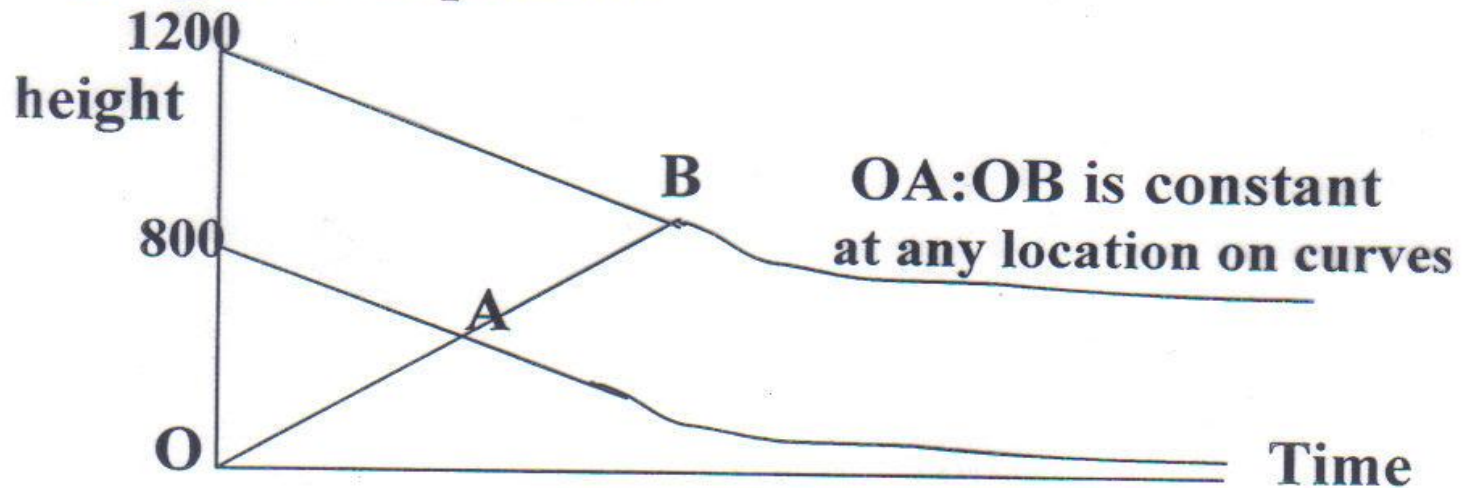
$$f(e) = 10^{-1.82(1-e)} \dots\dots\dots(7)$$

Substituting 6,7,2, into 5

$$u_c = \frac{e^2 d^2 (\rho_s - \rho) g}{18\mu} 10^{-1.82(1-e)} \dots\dots\dots(8)$$

Main factors which affect sedimentation

1. Height of suspension



Suspension height has no effect on rate of sedimentation

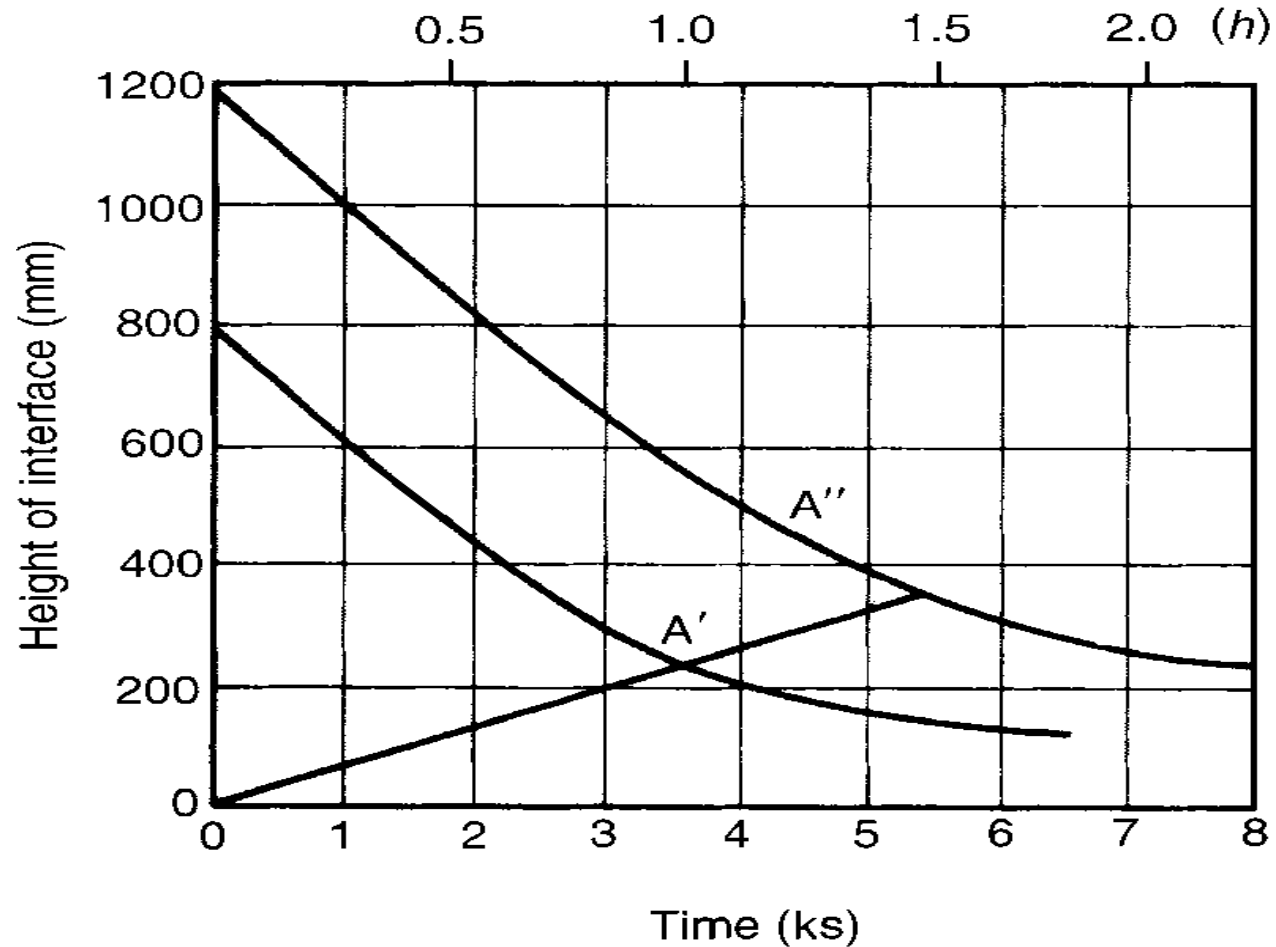
2. Diameter of Vessel

If $D_c/D_p > 100$, the walls have no effect on sedimentation rate.

If $D_c/D_p < 100$, walls^r reduce the sedimentation rate

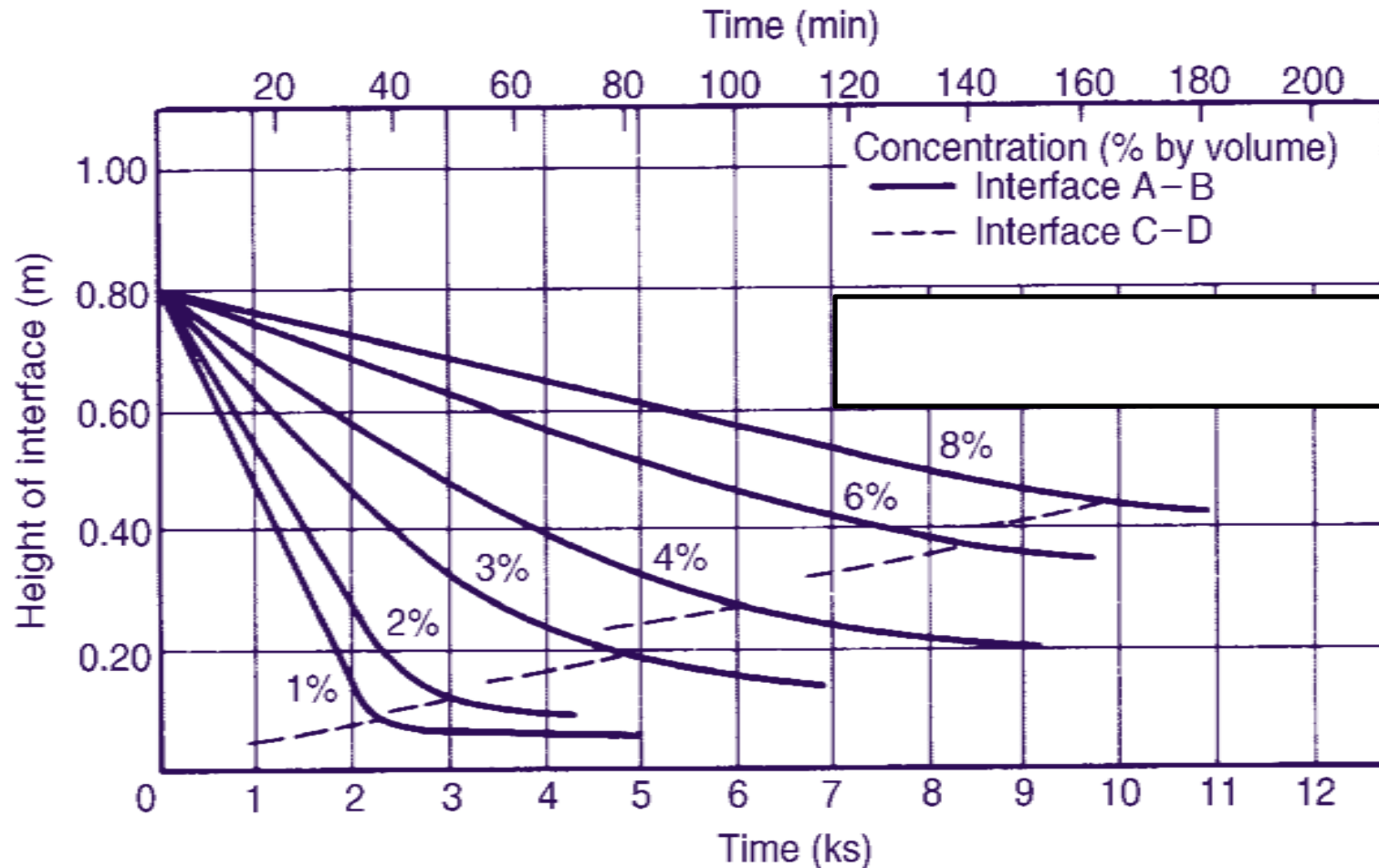
3. Concentration of suspension

Height of Suspension



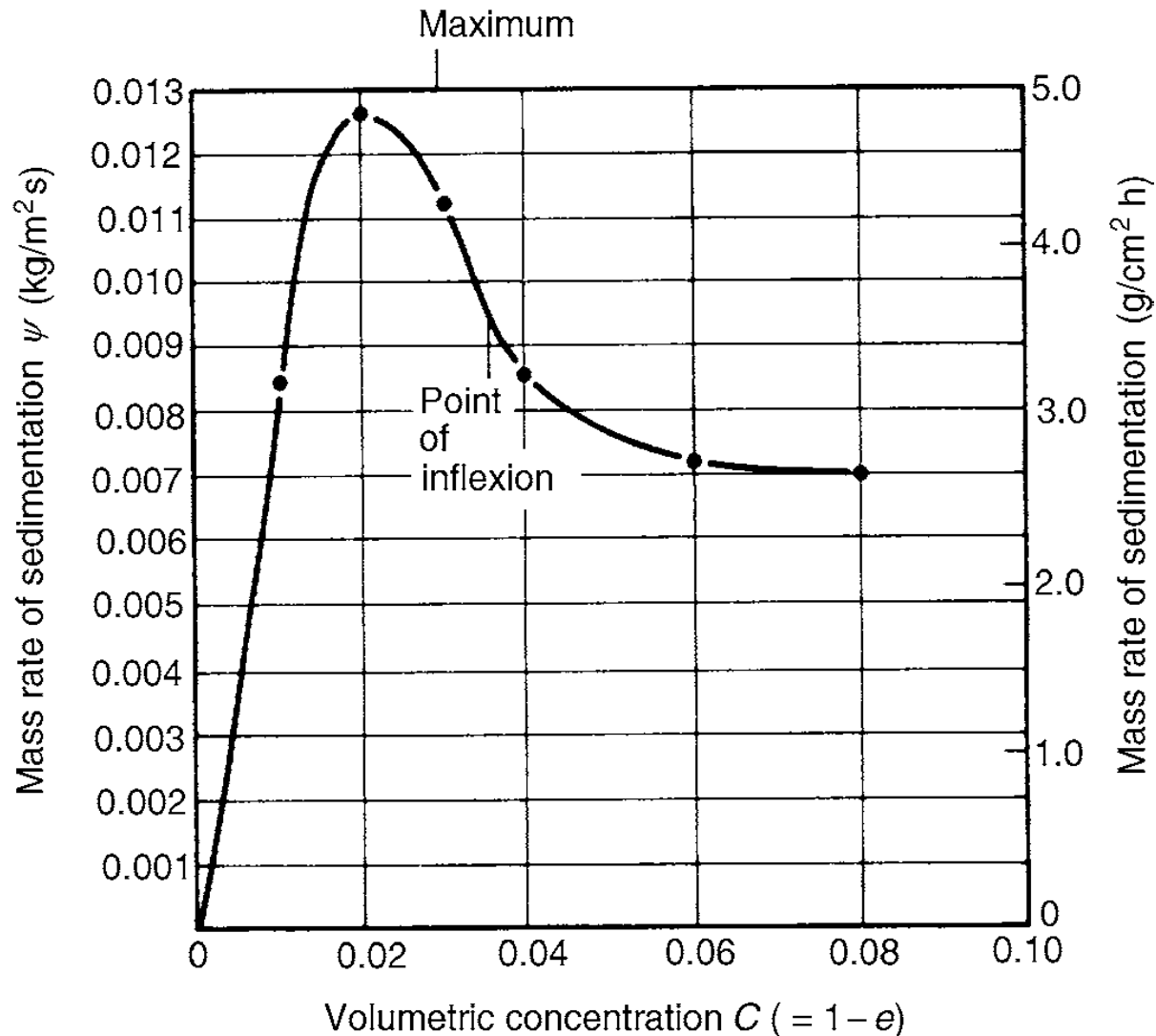
Effect of height on sedimentation of a 3 per cent (by volume) suspension of calcium carbonate

Effect of concentration on the sedimentation of calcium carbonate suspensions



\therefore Conc. \uparrow Rate of sedimentation \downarrow

Effect of concentration on mass rate of sedimentation of calcium carbonate



Note

- Rate of sedimentation during consolidation or solid-compression period is

$$-\frac{dH}{dt} = b(H - H_{\infty})$$

On integration with B.C^S : $t = 0, H = H_C$; $t = t, H = H$

$$\ln(H - H_{\infty}) - \ln(H_C - H_{\infty}) = -bt$$

Where H_C : the height of sluge at critical settling pt.

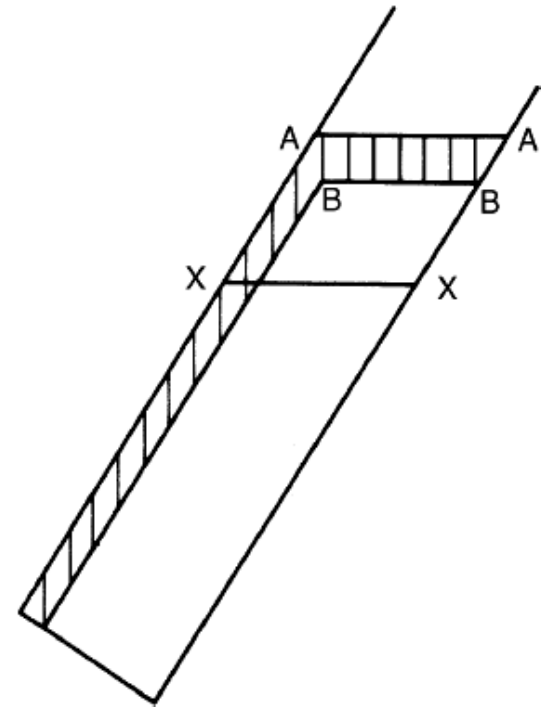
H_{∞} : the final height of the sediment

b : cons. For a given suspension

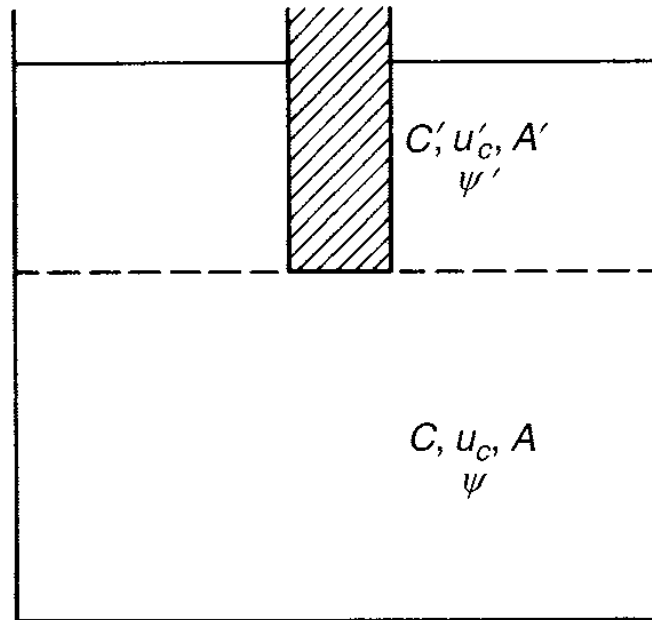
- If $\ln(H-H_\infty)$ is plotted against t , hence b can be found from the slope of the given line.

4. Shape of vessel

- It has little effect
- If the container has a slope with horizontal, after a short time interval, the system becomes to a stable condition 'XX state'. Hence,
Area $AAXX$ = shaded area



- **Obstructed vessel**



Sedimentation in
partially
obstructed vessel

$$A \psi = A c u_c$$

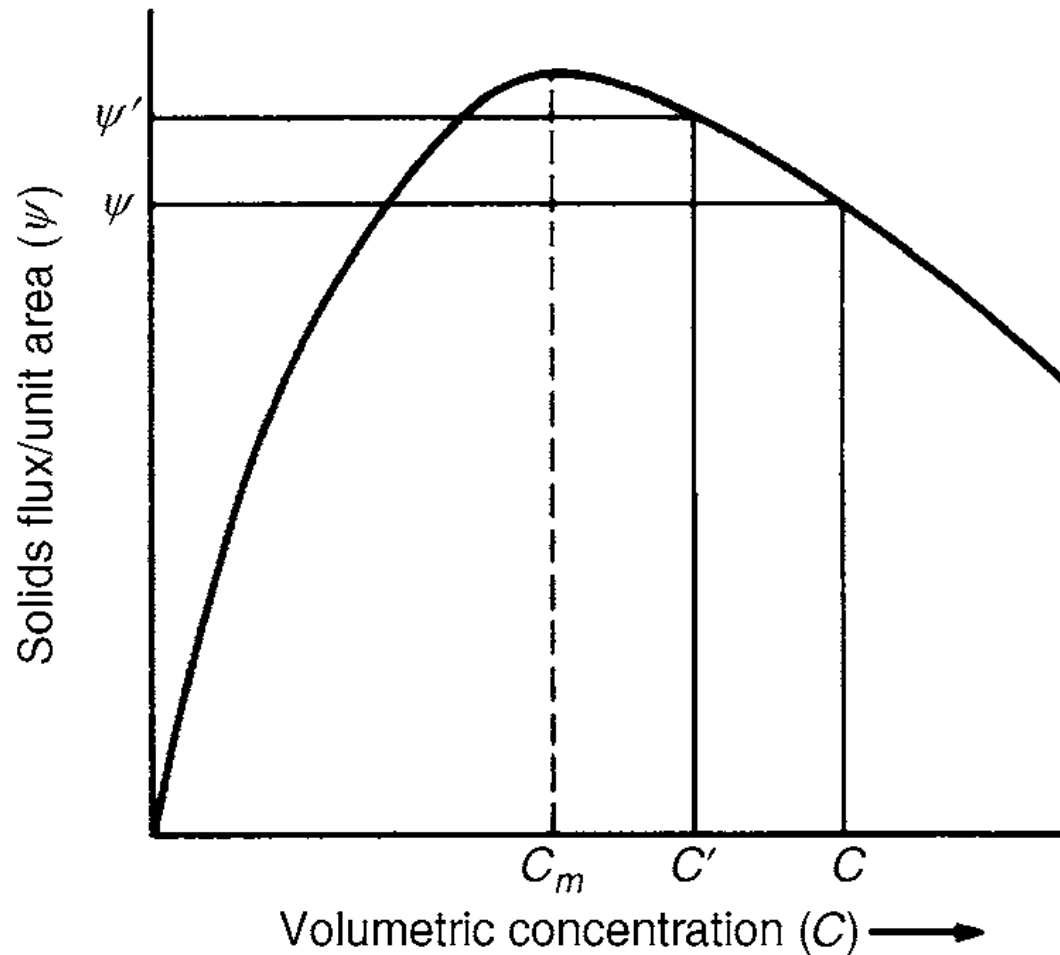
$$A' \psi' = A' c' u'_c$$

For continuity at the bottom of construction

$$\psi' = \frac{A}{A'} \psi \quad \therefore \psi' > \psi \quad \text{since } A' < A$$

In sum as $A \downarrow \quad \psi \uparrow$

Solids flux per unit area as a function of volumetric concentration



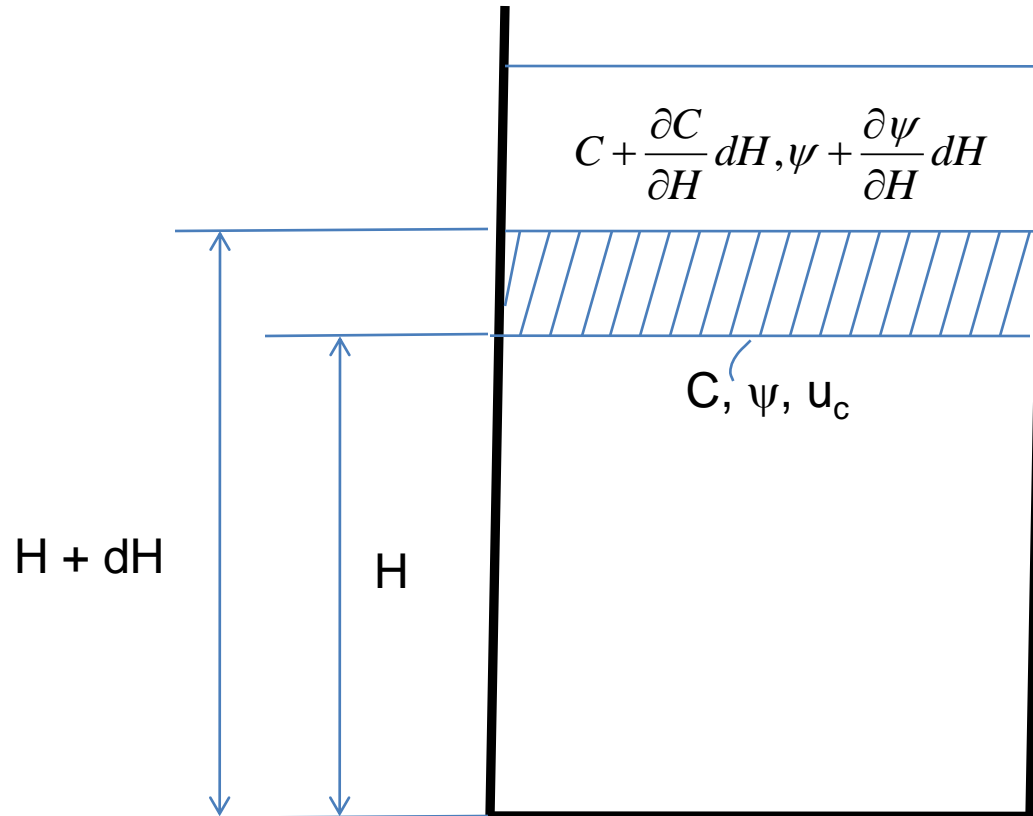
Kynch Theory

The mechanism of sedimentation of concentrated suspension can be studied via using the concept of continuity.

Some basic assumption:

- Uniform conc. at any level.
- No wall effects
- Velocity of fall of particles depends on the local conc. of particles.
- There is no differential settling
- Sedimentation velocity = 0.0 at the sediment layer at the bottom.

Kynch Theory



- In general, $\psi = c u_c$

Making a material balance between the height h and the height $H+dH$

$$\left\{ \left(\psi + \frac{\partial \psi}{\partial H} dH \right) - \psi \right\} dt = \frac{\partial}{\partial t} (c dH) dt$$

$$\frac{\partial \psi}{\partial H} = \frac{\partial c}{\partial t} \dots\dots\dots (1)$$

note : Right term $\frac{\partial dH}{\partial t} c + \frac{\partial c}{\partial t} dH$

$$\frac{\partial \psi}{\partial H} = \frac{\partial \psi}{\partial c} \cdot \frac{\partial c}{\partial H} = \frac{d\psi}{dc} \cdot \frac{\partial c}{\partial H} \text{ since } \psi \text{ depends only on } c$$

Substituting in (1)

$$\frac{\partial c}{\partial t} - \frac{d\psi}{dc} \cdot \frac{\partial c}{\partial H} = 0 \dots\dots\dots (2)$$

But $c = f(H, t)$

$$dc = \frac{\partial c}{\partial H} dH + \frac{\partial c}{\partial t} dt$$

Assume constant conc.

$$\frac{\partial c}{\partial H} dH + \frac{\partial c}{\partial t} dt = 0$$

$$\frac{\partial c}{\partial H} = - \cancel{\frac{\partial c}{\partial t}} \bigg/ \frac{dH}{dt}$$

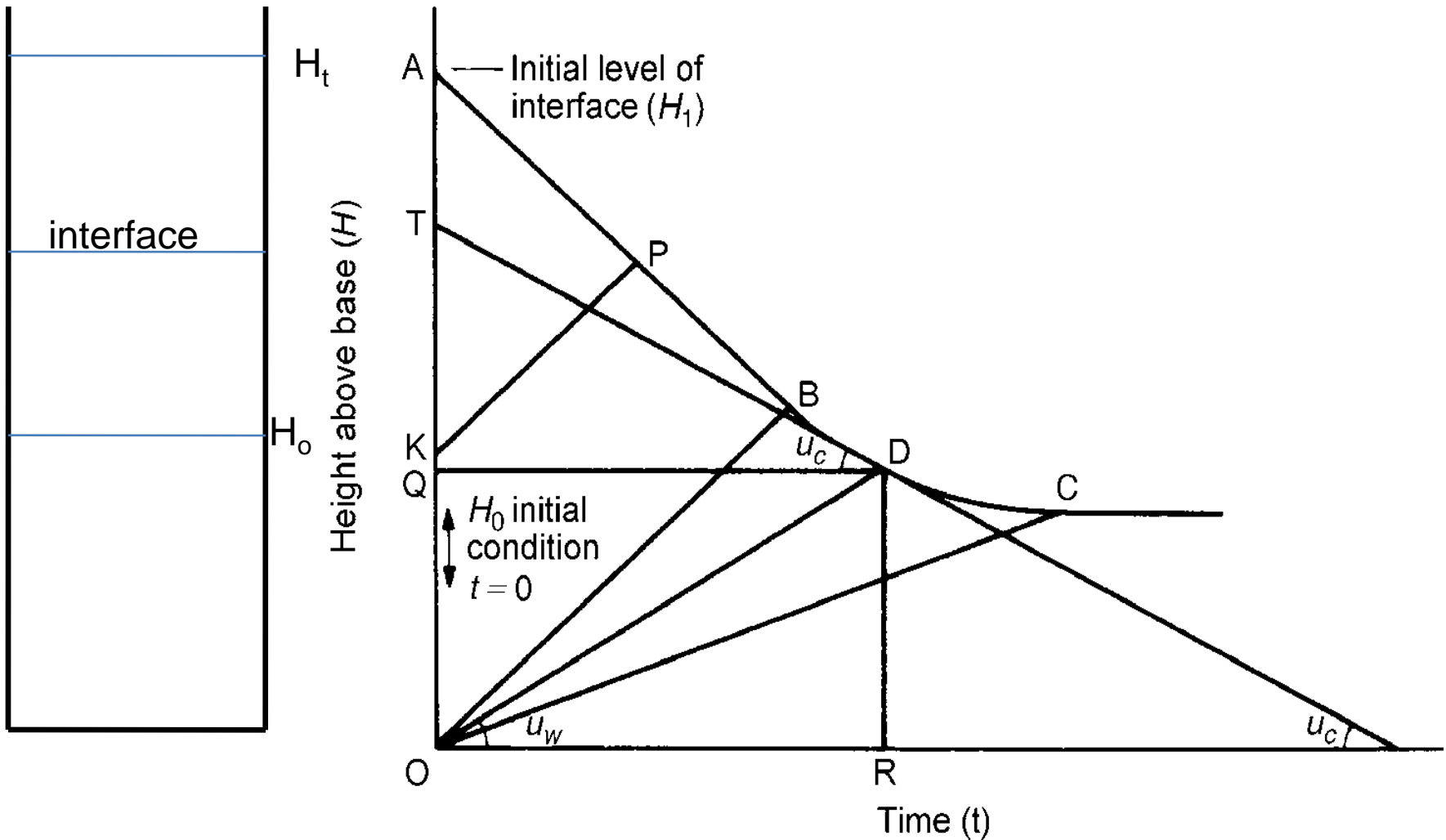
sub. in (2)

$$\frac{\partial c}{\partial t} - \frac{d\psi}{dc} \left\{ - \cancel{\frac{\partial c}{\partial t}} \bigg/ \frac{dH}{dt} \right\} = 0$$

$$\therefore - \frac{d\psi}{dc} = \frac{dH}{dt} = u_w \dots\dots\dots(3)$$

Where u_w velocity of propagation of a zone of constant concentration 'velocity of propagation of concentration wave'. Note u_w is constant for any concentration.

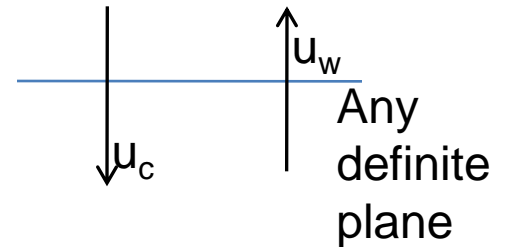
The relationship between flux ' Ψ ' and concentration ' c '



- lines such as KP and OB represent constant concentrations with slopes = $dH/dt = u_w$
- line AB represents the fall down of interface 'between clear liquid and suspension' with fall down velocity 'sedimentation velocity' , u_c = slope of this line = $- dH/dt$
- the total volume of particles that pass through a certain plane in time

't' per unit area is:

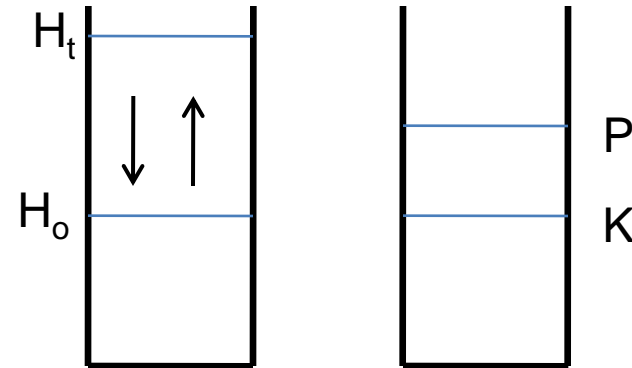
$$V = C_o (u_c + u_w) t$$



- Total volume, V , must equal to the total volume of particles was originally above the K , for example.

$$C_o (u_c + u_w) t = C_o (H_t - H_o)$$

$$(u_c + u_w) t = (H_t - H_o)$$



- For intermediate concentration curve BC $C_o < C < C_{\max}$
- Consider pt. D and apply the same previous approach:

- Here the line OD represents a wave starts from pt. O 'the base of the container' and terminates at pt. D 'the interface between clear liquid and suspension' on the sedimentation curve.

$$C (u_c + u_w) t = C_o H_t(i)$$

But $H_t = OA$ 'see the fig.'

$$u_c = QT/t$$

or $u_c t = QT$ (ii)

and $u_w t = RD = OQ \dots\dots\dots \text{...(iii)}$

add (iii) to (ii)

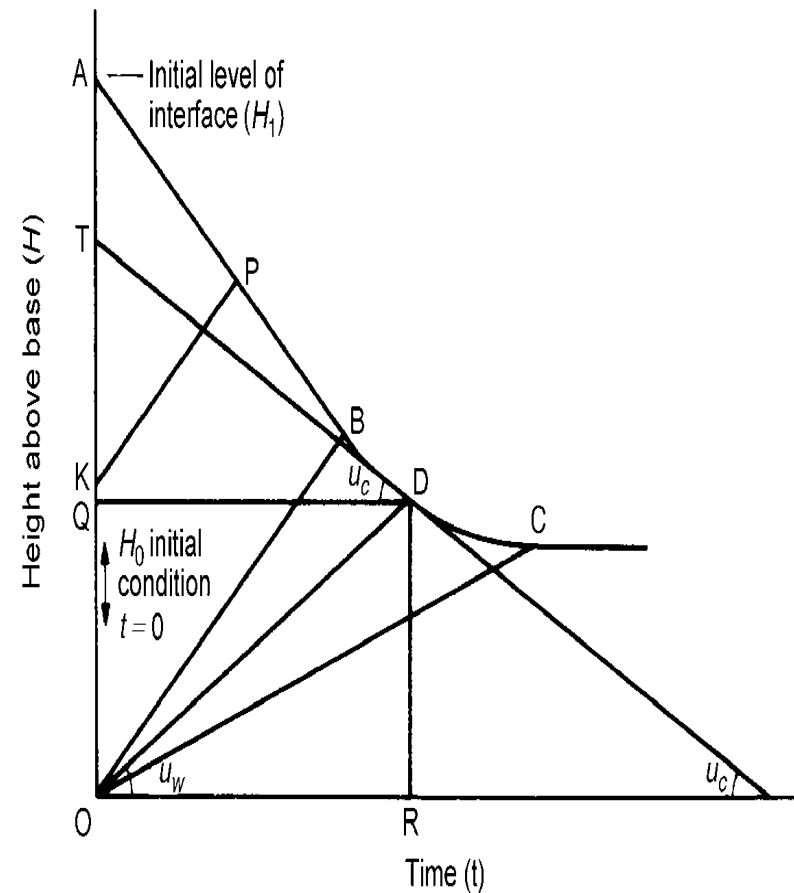
$$(u_c + u_w) t = OT \dots\dots\dots (iv)$$

divide (i) by (iv)

$$C = C_0 (OA/OT) \quad ###$$

The corresponding Flux is

$$\Psi = C u_c = C_o (OA/OT) u_c \quad \text{###}$$



Thickener design

Thickening zone

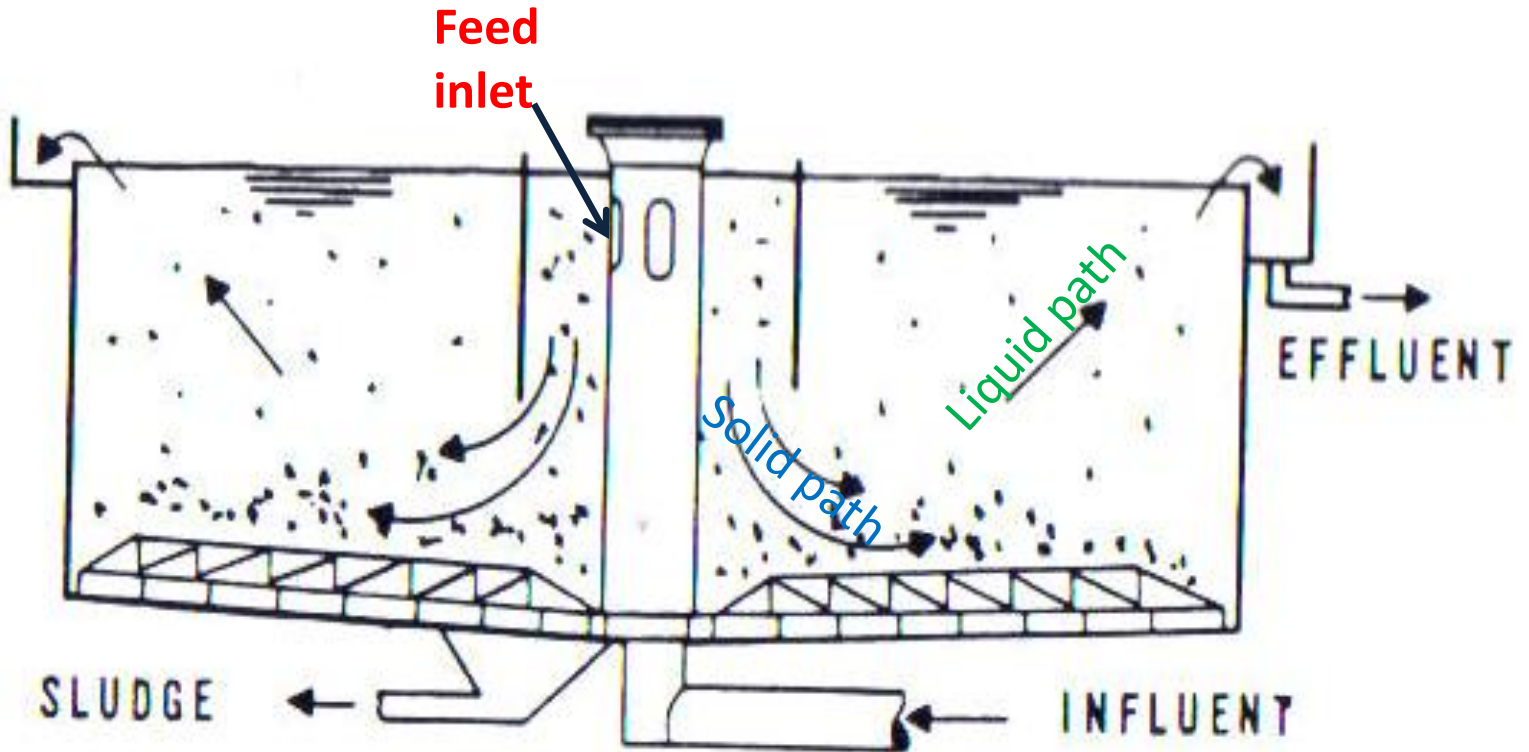
Thickening zone

- **Good design requires:**
 1. adequate or suitable diameter to obtain reasonable clarification.
 2. Enough depth to obtain the required degree of thickening.

Constrain

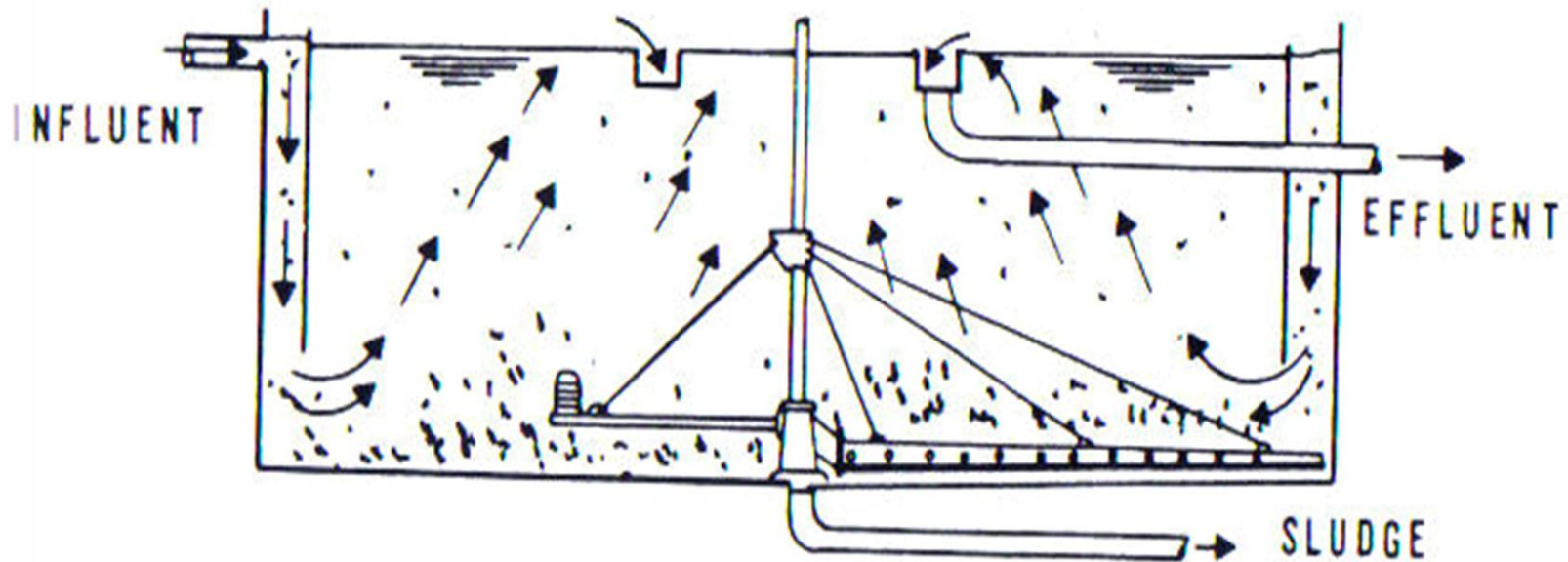
- In a continuous thickener, **the area required for thickening** must be such that the total solids flux (volumetric flow rate per unit area) at any level does not exceed the rate at which the solids can be transmitted downwards.
- If this condition is not met, solids will build up and unsteady-state operation will be possible.

Clarifier or thickener



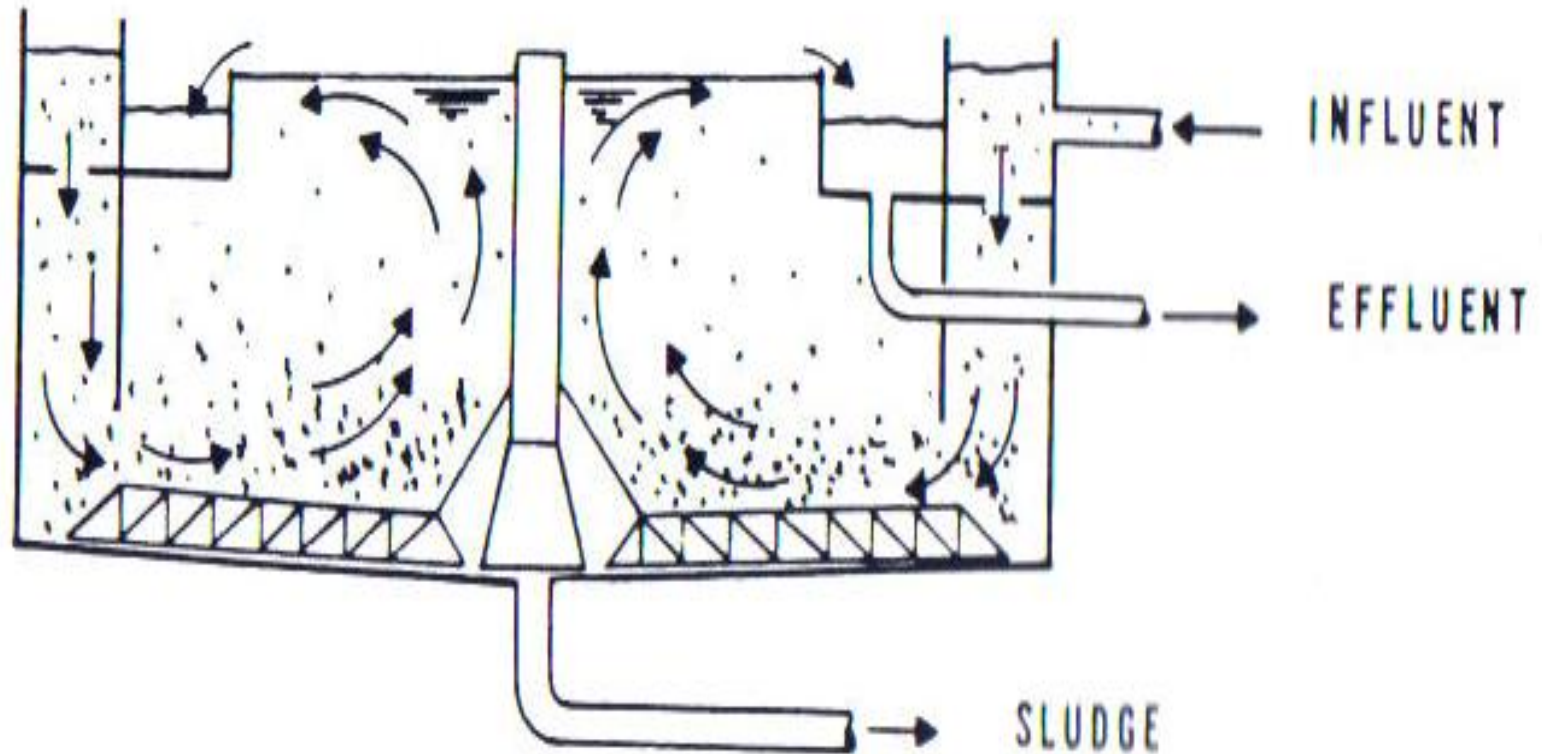
(a) CIRCULAR CENTER-FEED CLARIFIER WITH
A SCRAPER SLUDGE REMOVAL SYSTEM

Clarifier or thickener



(b) CIRCULAR RIM-FEED, CENTER TAKE-OFF CLARIFIER WITH A
HYDRAULIC SUCTION SLUDGE REMOVAL SYSTEM

Clarifier or thickener



(c) CIRCULAR RIM-FEED. RIM TAKE-OFF CLARIFIER
WITH A SCRAPER SLUDGE REMOVAL SYSTEM

Required Area

- To calculate the required area , it is very important to obtain the concentration at which the rate of sedimentation or the flux is a minimum.
- Total flux, ψ_t , consists of two componets:
 - a. Due to the sedimentation of solid in the liquid (as measured by batch sedimentation experiment, $\psi = u_c C$).
 - b. Arising from the removal of the underflow at the base of the thickener, $\psi_u = u_u C$.

∴ Total flux is,

$$\begin{aligned}\Psi_T &= \Psi + \Psi_u \\ &= \Psi + u_u C\end{aligned}$$

This flux equals the volumetric flow rate per unit area at which the solids are fed to the thickener

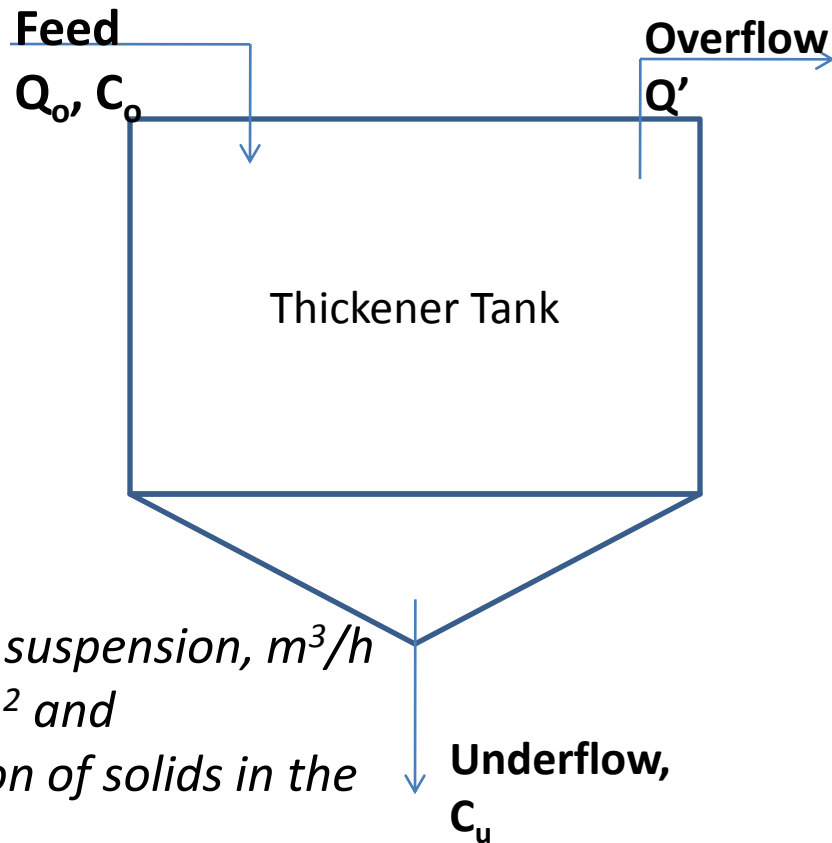
$$\Psi_T = \left(\frac{Q_0}{A} \right) C_o$$

Where;

Q_0 is the volumetric feed rate of suspension, m^3/h

A is the area of the thickener, m^2 and

C_o is the volumetric concentration of solids in the feed.



$$\frac{\partial \psi_T}{\partial C} = \frac{\partial \psi}{\partial C} + u_u$$

The minimum value of $\psi_T (= \psi_{TL})$ occurs when $\frac{\partial \psi_T}{\partial C} = 0$;

i.e. when:

$$\frac{\partial \psi}{\partial C} = -u_u$$

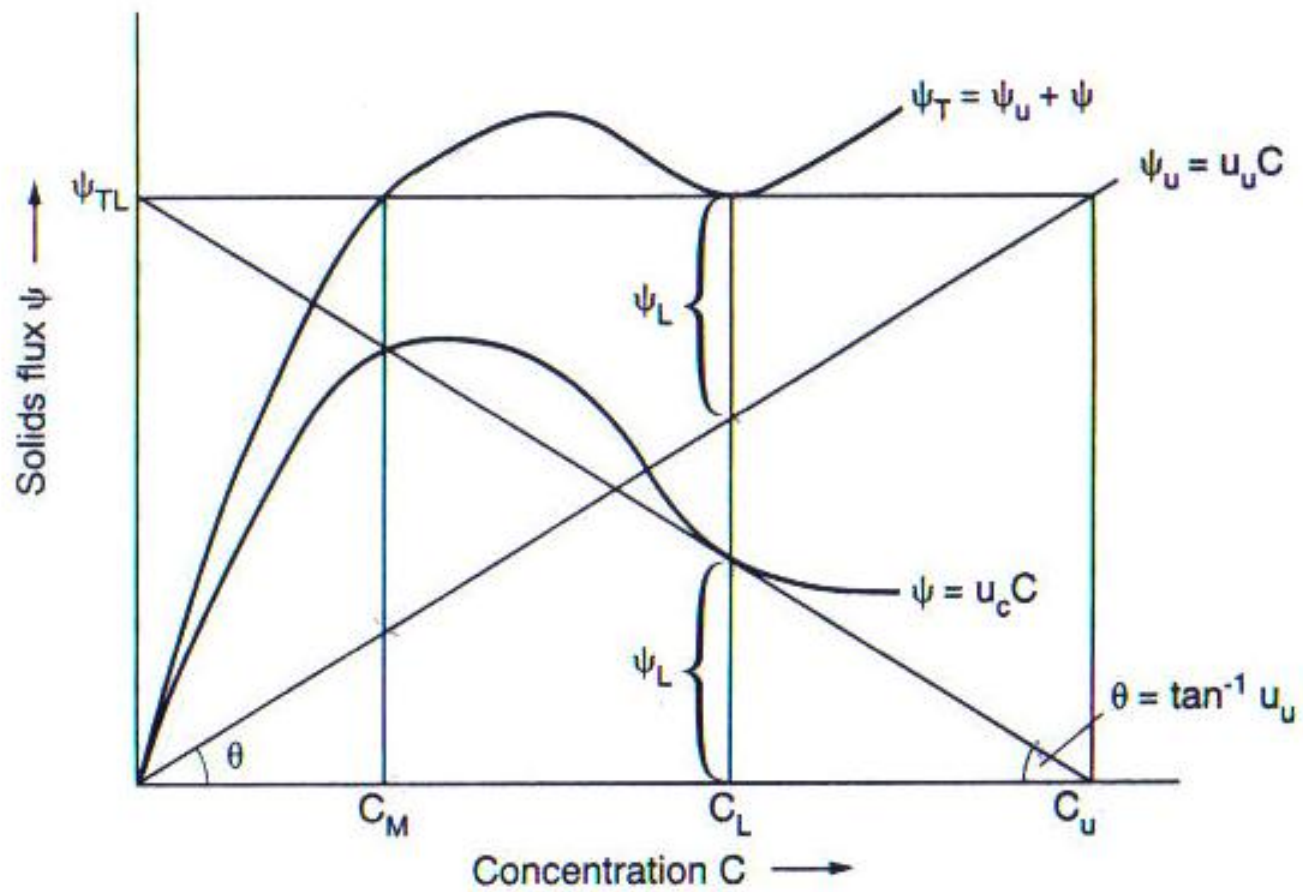


FIG. 5.13. Solids fluxes as functions of concentration and Yoshioka construction.

$$\frac{\psi_{TL}}{C_u} = \frac{\psi_L}{C_u - C_L}$$

where ψ_L is the value of ψ at the concentration C_L .

Since $\psi_L = u_{cL} C_L$

$$\begin{aligned} A &= \frac{Q_0 C_0}{\psi_{TL}} \left[\frac{1}{C_L} - \frac{1}{C_u} \right] \\ &= Q_0 C_0 \left[\frac{\frac{1}{C_L} - \frac{1}{C_u}}{u_{cL}} \right] \end{aligned}$$

where u_{cL} is the value of u_c at the concentration C_L . Thus, the minimum necessary area of the thickener may be obtained from the maximum value of

$$\left[\frac{\frac{1}{C} - \frac{1}{C_u}}{u_c} \right] \text{ which will be designated } \left[\frac{\frac{1}{C} - \frac{1}{C_u}}{u_c} \right]_{\max}.$$

Concentrations can also be expressed as mass per unit volume (using c in place of C) to give:

$$A = Q_0 c_0 \left[\frac{[(1/c) - (1/c_u)]}{u_c} \right]_{\max}$$

Overflow

The liquid flowrate in the overflow (Q') is the difference between that in the feed and in the underflow.

Thus:

$$Q' = Q_0 (1 - C_0) - (Q_0 - Q') (1 - C_u)$$

or:

$$\frac{Q'}{Q_0} = 1 - \frac{C_0}{C_u} \quad (5.52)$$

At any depth below the feed point, the upward liquid velocity must not exceed the settling velocity of the particles. (u_c) Where the concentration is C , the required area is therefore given by:

$$A = Q_0 \frac{1}{u_c} \left[1 - \frac{C}{C_u} \right] \quad (5.53)$$

It is therefore necessary to calculate the *maximum* value of A for all the values of C which may be encountered.

Equation 5.53 can usefully be rearranged in terms of the mass ratio of liquid to solid in the feed (Y) and the corresponding value (U) in the underflow:

$$Y = \frac{1 - C}{C} \frac{\rho}{\rho_s} \quad \text{and} \quad U = \frac{1 - C_u}{C_u} \frac{\rho}{\rho_s}$$

Then:

$$C = \frac{1}{1 + Y(\rho_s/\rho)} \quad C_u = \frac{1}{1 + U(\rho_s/\rho)}$$

$$A = \frac{Q_0}{u_c} \left\{ 1 - \frac{1 + U(\rho_s/\rho)}{1 + Y(\rho_s/\rho)} \right\}$$
$$= \frac{Q_0(Y - U)C\rho_s}{u_c\rho}$$

The values of A should be calculated for the whole range of concentrations present in the thickener, and the design should then be based on the maximum value so obtained.

Depth of the thickener Zone, H_t

The required depth of the thickening region is

$$H_t = \left\{ \frac{W t_R}{A \rho_s} + W \frac{t_R}{A \rho} X \right\} = \frac{W t_R}{A \rho_s} \left(1 + \frac{\rho_s}{\rho} X \right)$$

where:

t_R is the required time of retention of the solids, as determined experimentally,

W is the mass rate of feed of solids to the thickener,

X is the average value of the mass ratio of liquid to solids in the thickening portion, and

ρ and ρ_s are the densities of the liquid and solid respectively.

NOTE

- Take the ratio $H/D=1.5$ for deep tank up to 19 m height.

