

## Controller Tuning: A Motivational Example

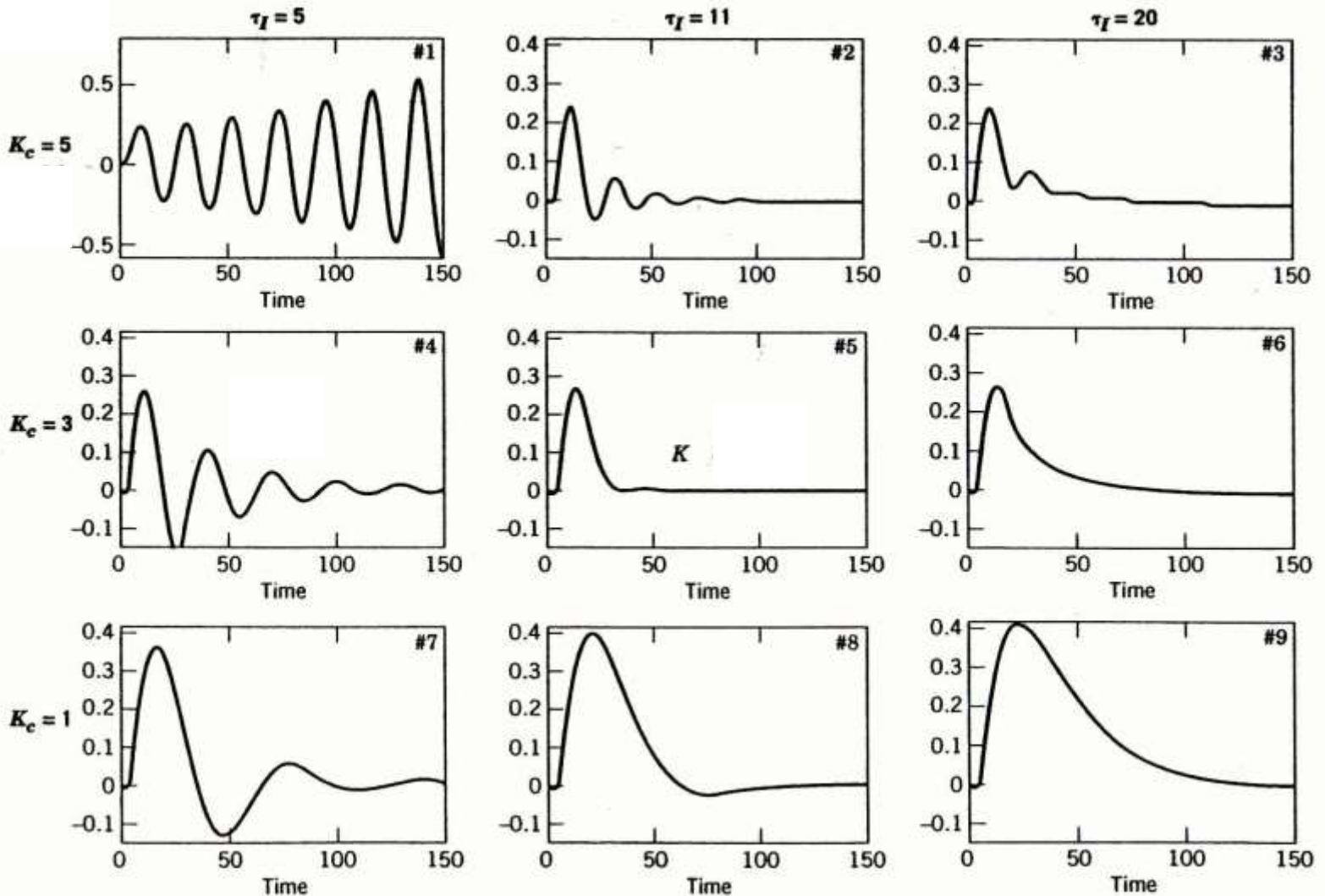


Fig. 12.1. Unit-step disturbance responses for the candidate controllers (FOPTD Model:  $K = 1$ ,  $\theta = 4$ ,  $\tau = 20$ ).

# PID Controller Design, Tuning, and Troubleshooting

## Performance Criteria For Closed-Loop Systems

- The function of a feedback control system is to ensure that the closed loop system has desirable dynamic and steady-state response characteristics.
- Ideally, we would like the closed-loop system to satisfy the following performance criteria:
  1. The closed-loop system must be stable.
  2. The effects of disturbances are minimized, providing good *disturbance rejection*.
  3. Rapid, smooth responses to set-point changes are obtained, that is, good *set-point tracking*.

4. Steady-state error (offset) is eliminated.
5. Excessive control action is avoided.
6. The control system is robust, that is, insensitive to changes in process conditions and to inaccuracies in the process model.

PID controller settings can be determined by a number of alternative techniques:

1. Direct Synthesis (DS) method
2. Internal Model Control (IMC) method
3. Controller tuning relations
4. Frequency response techniques
5. Computer simulation
6. *On-line tuning* after the control system is installed.

# Direct Synthesis Method

- In the Direct Synthesis (DS) method, the controller design is based on a process model and a desired closed-loop transfer function.
- The latter is usually specified for set-point changes, but responses to disturbances can also be utilized (Chen and Seborg, 2002).
- Although these feedback controllers do not always have a PID structure, the DS method does produce PI or PID controllers for common process models.
- As a starting point for the analysis, consider the block diagram of a feedback control system in Figure 12.2. The closed-loop transfer function for set-point changes was derived in Section 11.2:

$$\frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} \quad (12-1)$$

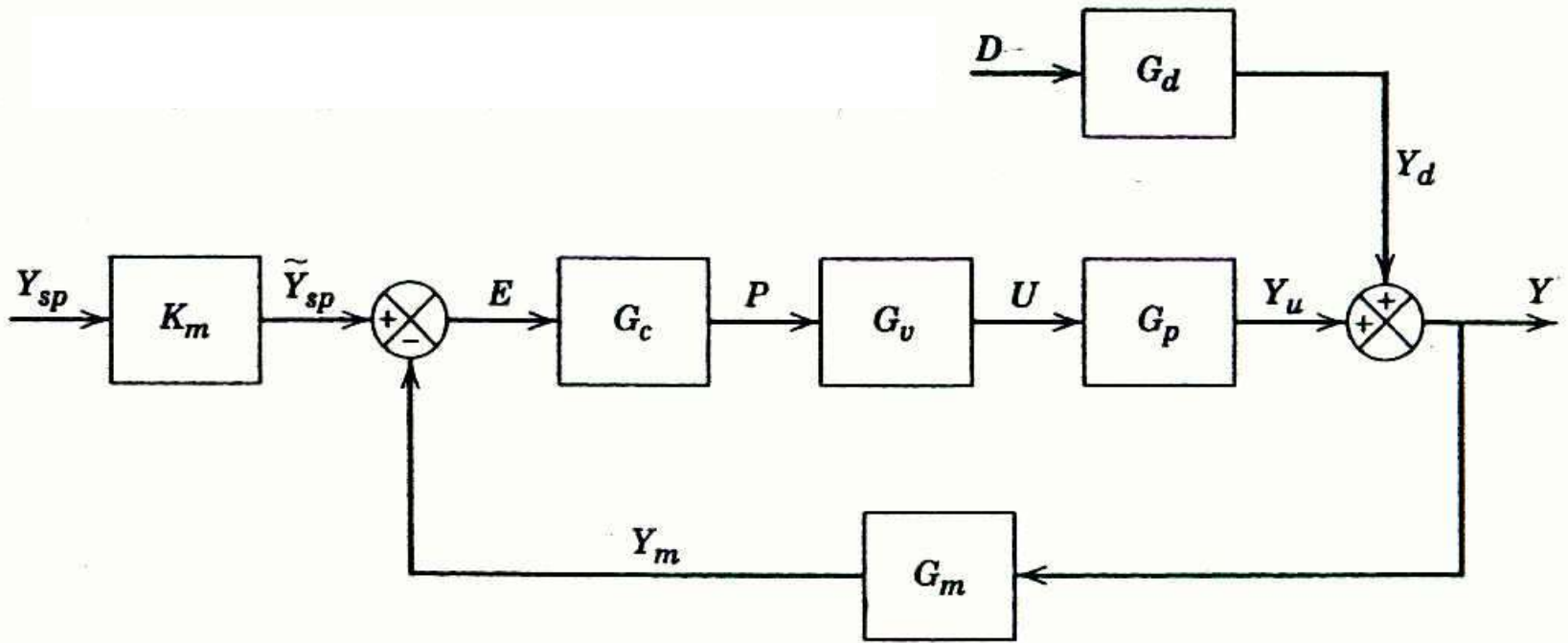


Fig. 12.2. Block diagram for a standard feedback control system.

For simplicity, let  $G \square G_v G_p G_m$  and assume that  $G_m = K_m$ . Then Eq. 12-1 reduces to

$$\frac{Y}{Y_{sp}} = \frac{G_c G}{1 + G_c G} \quad (12-2)$$

Rearranging and solving for  $G_c$  gives an expression for the feedback controller:

$$G_c = \frac{1}{G} \left( \frac{Y / Y_{sp}}{1 - Y / Y_{sp}} \right) \quad (12-3a)$$

- Equation 12-3a cannot be used for controller design because the closed-loop transfer function  $Y/Y_{sp}$  is not known *a priori*.
- Also, it is useful to distinguish between the actual process  $G$  and the model,  $\tilde{G}$ , that provides an approximation of the process behavior.
- A practical design equation can be derived by replacing the unknown  $G$  by  $\tilde{G}$ , and  $Y/Y_{sp}$  by a *desired closed-loop transfer function*,  $(Y/Y_{sp})_d$ :

$$G_c = \frac{1}{\tilde{G}} \left[ \frac{(Y/Y_{sp})_d}{1 - (Y/Y_{sp})_d} \right] \quad (12-3b)$$

- The specification of  $(Y/Y_{sp})_d$  is the key design decision and will be considered later in this section.
- Note that the controller transfer function in (12-3b) contains the inverse of the process model owing to the  $1/\tilde{G}$  term.
- This feature is a distinguishing characteristic of model-based control.

## Desired Closed-Loop Transfer Function

For processes without time delays, the first-order model in Eq. 12-4 is a reasonable choice,

$$\left( \frac{Y}{Y_{sp}} \right)_d = \frac{1}{\tau_c s + 1} \quad (12-4)$$

- The model has a settling time of  $\sim 4\tau_c$ , as shown in Section 5. 2.
- Because the steady-state gain is one, no offset occurs for set-point changes.
- By substituting (12-4) into (12-3b) and solving for  $G_c$ , the controller design equation becomes:

$$G_c = \frac{1}{\tilde{G}} \frac{1}{\tau_c s} \quad (12-5)$$

- The  $1/\tau_c s$  term provides integral control action and thus eliminates offset.
- Design parameter  $\tau_c$  provides a convenient controller tuning parameter that can be used to make the controller more aggressive (small  $\tau_c$ ) or less aggressive (large  $\tau_c$ ).



- If the process transfer function contains a known time delay  $\theta$ , a reasonable choice for the desired closed-loop transfer function is:

$$\left( \frac{Y}{Y_{sp}} \right)_d = \frac{e^{-\theta s}}{\tau_c s + 1} \quad (12-6)$$

- The time-delay term in (12-6) is essential because it is physically impossible for the controlled variable to respond to a set-point change at  $t = 0$ , before  $t = \theta$ .
- If the time delay is unknown,  $\theta$  must be replaced by an estimate.
- Combining Eqs. 12-6 and 12-3b gives:

$$G_c = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{\tau_c s + 1 - e^{-\theta s}} \quad (12-7)$$

- Although this controller is not in a standard PID form, it is physically realizable.
- Next, we show that the design equation in Eq. 12-7 can be used to derive PID controllers for simple process models.
- The following derivation is based on approximating the time-delay term in the denominator of (12-7) with a truncated Taylor series expansion:

$$e^{-\theta s} \approx 1 - \theta s \quad (12-8)$$

Substituting (12-8) into the denominator of Eq. 12-7 and rearranging gives

$$G_c = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{(\tau_c + \theta)^s} \quad (12-9)$$

Note that this controller also contains integral control action.

## First-Order-plus-Time-Delay (FOPTD) Model

Consider the standard FOPTD model,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (12-10)$$

Substituting Eq. 12-10 into Eq. 12-9 and rearranging gives a PI controller,  $G_c = K_c (1 + 1/\tau_I s)$ , with the following controller settings:

$$K_c = \frac{1}{K} \frac{\tau}{\theta + \tau_c}, \quad \tau_I = \tau \quad (12-11)$$

## Second-Order-plus-Time-Delay (SOPTD) Model

Consider a SOPTD model,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (12-12)$$

Substitution into Eq. 12-9 and rearrangement gives a PID controller in parallel form,

$$G_c = K_c \left( 1 + \frac{1}{\tau_I s} + \tau_D s \right) \quad (12-13)$$

where:

$$K_c = \frac{1}{K} \frac{\tau_1 + \tau_2}{\tau_c + \theta}, \quad \tau_I = \tau_1 + \tau_2, \quad \tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \quad (12-14)$$

### Example 12.1

Use the DS design method to calculate PID controller settings for the process:

$$G = \frac{2e^{-s}}{(10s + 1)(5s + 1)}$$

Consider three values of the desired closed-loop time constant:  $\tau_c = 1, 3$ , and  $10$ . Evaluate the controllers for unit step changes in both the set point and the disturbance, assuming that  $G_d = G$ .

Repeat the evaluation for two cases:

- The process model is perfect ( $\tilde{G} = G$ ).
- The model gain is  $\tilde{K} = 0.9$ , instead of the actual value,  $K = 2$ .

Thus,

$$\tilde{G} = \frac{0.9e^{-s}}{(10s+1)(5s+1)}$$

The controller settings for this example are:

	$\tau_c = 1$	$\tau_c = 3$	$\tau_c = 10$
$K_c (\tilde{K} = 2)$	3.75	1.88	0.682
$K_c (\tilde{K} = 0.9)$	8.33	4.17	1.51
$\tau_I$	15	15	15
$\tau_D$	3.33	3.33	3.33

The values of  $K_c$  decrease as  $\tau_c$  increases, but the values of  $\tau_I$  and  $\tau_D$  do not change, as indicated by Eq. 12-14.

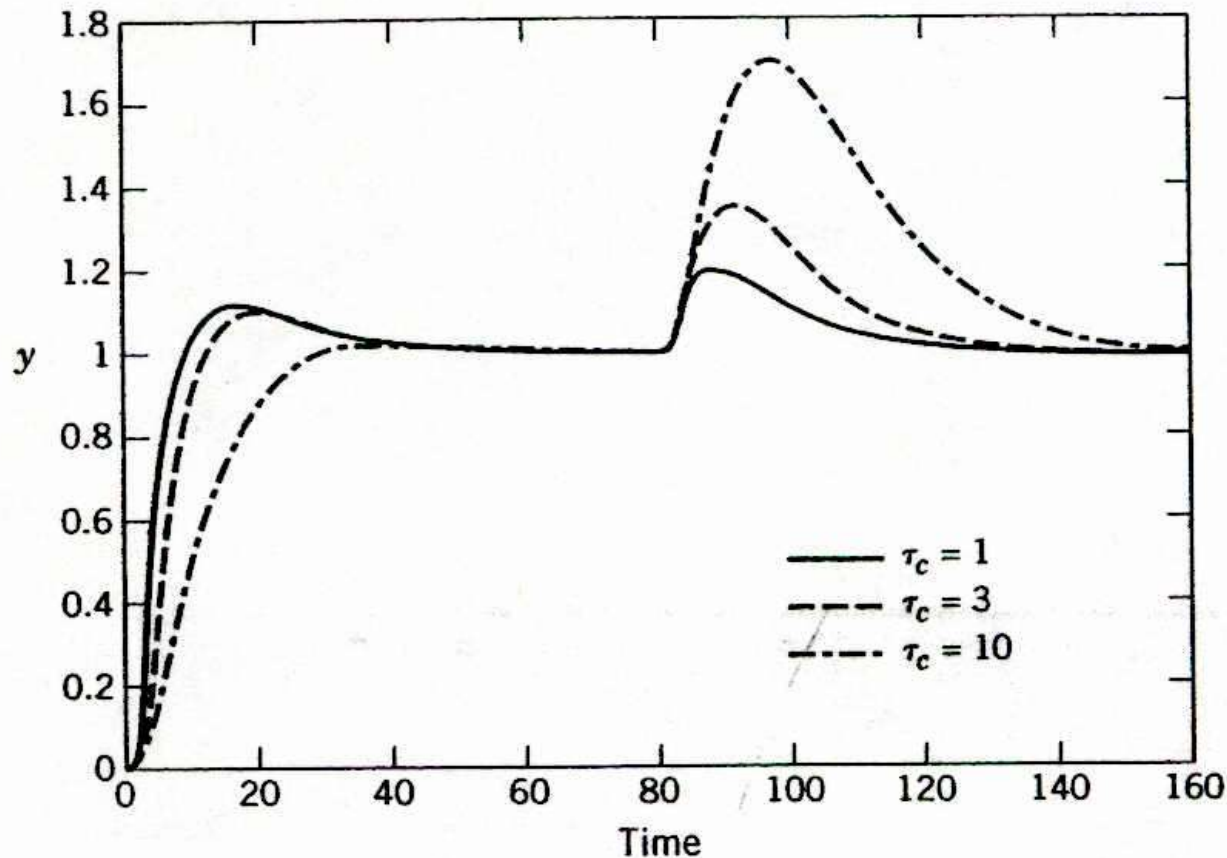


Figure 12.3 Simulation results for Example 12.1 (a): correct model gain.

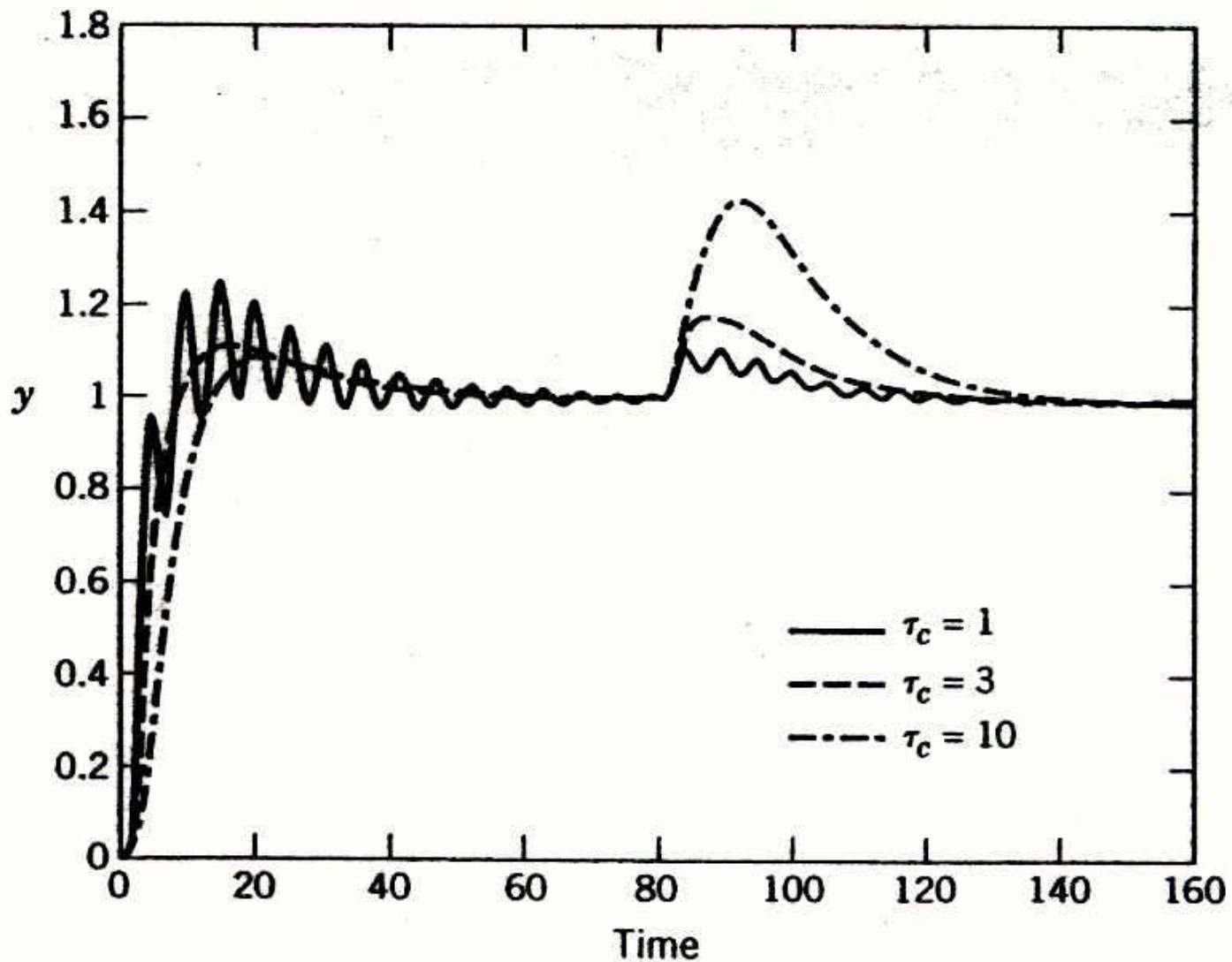
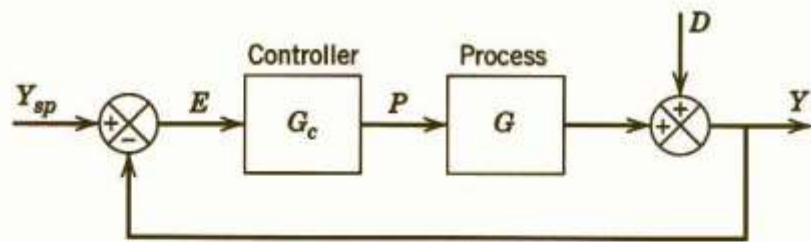


Fig. 12.4 Simulation results for Example 12.1 (b): incorrect model gain.

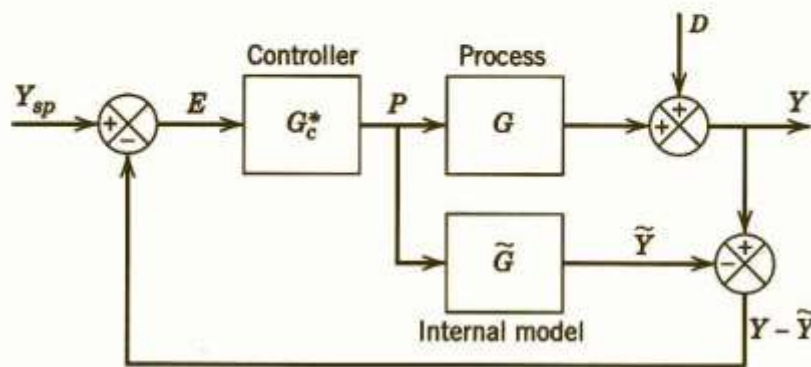
# Internal Model Control (IMC)

- A more comprehensive model-based design method, *Internal Model Control (IMC)*, was developed by Morari and coworkers (Garcia and Morari, 1982; Rivera et al., 1986).
- The IMC method, like the DS method, is based on an assumed process model and leads to analytical expressions for the controller settings.
- These two design methods are closely related and produce identical controllers if the design parameters are specified in a consistent manner.
- The IMC method is based on the simplified block diagram shown in Fig. 12.6b. A process model  $\tilde{G}$  and the controller output  $P$  are used to calculate the model response,  $\tilde{Y}$ .





(a) Classical feedback control



(b) Internal model control

Figure 12.6.  
Feedback control  
strategies

- The model response is subtracted from the actual response  $Y$ , and the difference,  $Y - \tilde{Y}$  is used as the input signal to the IMC controller,  $G_c^*$ .
- In general,  $Y \neq \tilde{Y}$  due to modeling errors ( $\tilde{G} \neq G$ ) and unknown disturbances ( $D \neq 0$ ) that are not accounted for in the model.
- The block diagrams for conventional feedback control and IMC are compared in Fig. 12.6.

- It can be shown that the two block diagrams are identical if controllers  $G_c$  and  $G_c^*$  satisfy the relation

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} \quad (12-16)$$

- Thus, any IMC controller  $G_c^*$  is equivalent to a standard feedback controller  $G_c$ , and vice versa.
- The following closed-loop relation for IMC can be derived from Fig. 12.6b using the block diagram algebra of Chapter 11:

$$Y = \frac{G_c^* G}{1 + G_c^* (G - \tilde{G})} Y_{sp} + \frac{1 - G_c^* \tilde{G}}{1 + G_c^* (G - \tilde{G})} D \quad (12-17)$$

For the special case of a perfect model,  $\tilde{G} = G$ , (12-17) reduces to

$$Y = G_c^* G Y_{sp} + (1 - G_c^* G) D \quad (12-18)$$

The IMC controller is designed in two steps:

**Step 1.** The process model is factored as

$$\tilde{G} = \tilde{G}_+ \tilde{G}_- \quad (12-19)$$

where  $\tilde{G}_+$  contains any time delays and right-half plane zeros.

- In addition,  $\tilde{G}_+$  is required to have a steady-state gain equal to one in order to ensure that the two factors in Eq. 12-19 are unique.

**Step 2.** The controller is specified as

$$G_c^* = \frac{1}{\tilde{G}_-} f \quad (12-20)$$

where  $f$  is a *low-pass filter* with a steady-state gain of one. It typically has the form:

$$f = \frac{1}{(\tau_c s + 1)^r} \quad (12-21)$$

In analogy with the DS method,  $\tau_c$  is the desired closed-loop time constant. Parameter  $r$  is a positive integer. The usual choice is  $r = 1$ .

For the ideal situation where the process model is perfect ( $\tilde{G} = G$ ), substituting Eq. 12-20 into (12-18) gives the closed-loop expression

$$Y = \tilde{G}_+ f Y_{sp} + (1 - f \tilde{G}_+) D \quad (12-22)$$

Thus, the closed-loop transfer function for set-point changes is

$$\frac{Y}{Y_{sp}} = \tilde{G}_+ f \quad (12-23)$$

### *Selection of $\tau_c$*

- The choice of design parameter  $\tau_c$  is a key decision in both the DS and IMC design methods.
- In general, increasing  $\tau_c$  produces a more conservative controller because  $K_c$  decreases while  $\tau_I$  increases.

- Several IMC guidelines for  $\tau_c$  have been published for the model in Eq. 12-10:
  1.  $\tau_c / \theta > 0.8$  and  $\tau_c > 0.1\tau$  (Rivera et al., 1986)
  2.  $\tau > \tau_c > \theta$  (Chien and Fruehauf, 1990)
  3.  $\tau_c = \theta$  (Skogestad, 2003)

## Controller Tuning Relations

In the last section, we have seen that model-based design methods such as DS and IMC produce PI or PID controllers for certain classes of process models.

### IMC Tuning Relations

The IMC method can be used to derive PID controller settings for a variety of transfer function models.

Table 12.1 IMC-Based PID Controller Settings for  $G_c(s)$  (Chien and Fruehauf, 1990). See the text for the rest of this table.

Case	Model	$K_c K$	$\tau_I$	$\tau_D$
A	$\frac{K}{\tau s + 1}$	$\frac{\tau}{\tau_c}$	$\tau$	—
B	$\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
C	$\frac{K}{\tau^2 s^2 + 2\zeta \tau s + 1}$	$\frac{2\zeta \tau}{\tau_c}$	$2\zeta \tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta \tau s + 1}, \beta > 0$	$\frac{2\zeta \tau}{\tau_c + \beta}$	$2\zeta \tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s + 1)}$	$\frac{2\tau_c + \tau}{\tau_c^2}$	$2\tau_c + \tau$	$\frac{2\tau_c \tau}{2\tau_c + \tau}$
G	$\frac{K e^{-\theta s}}{\tau s + 1}$	$\frac{\tau}{\tau_c + \theta}$	$\tau$	—
H	$\frac{K e^{-\theta s}}{\tau s + 1}$	$\frac{\tau + \frac{\theta}{2}}{\tau_c + \frac{\theta}{2}}$	$\tau + \frac{\theta}{2}$	$\frac{\tau \theta}{2\tau + \theta}$

**Table 12.2** Equivalent PID Controller Settings  
for the Parallel and Series Forms

Parallel Form	Series Form
$G_c(s) = K_c \left( 1 + \frac{1}{\tau_I s} + \tau_D s \right)$	$G_c(s) = K'_c \left( 1 + \frac{1}{\tau'_I s} \right) (1 + \tau'_D s)^\dagger$
$K_c = K'_c \left( 1 + \frac{\tau'_D}{\tau'_I} \right)$	$K'_c = \frac{K_c}{2} (1 + \sqrt{1 - 4\tau_D/\tau_I})$
$\tau_I = \tau'_I + \tau'_D$	$\tau'_I = \frac{\tau_I}{2} (1 + \sqrt{1 - 4\tau_D/\tau_I})$
$\tau_D = \frac{\tau'_D \tau'_I}{\tau'_I + \tau'_D}$	$\tau'_D = \frac{\tau_I}{2} (1 - \sqrt{1 - 4\tau_D/\tau_I})$

<sup>†</sup>These conversion equations are only valid if  $\tau_D/\tau_I \leq 0.25$ .



## Tuning for Lag-Dominant Models

- First- or second-order models with relatively small time delays ( $\theta/\tau \ll 1$ ) are referred to as *lag-dominant models*.
- The IMC and DS methods provide satisfactory set-point responses, but very slow disturbance responses, because the value of  $\tau_I$  is very large.
- Fortunately, this problem can be solved in three different ways.

### Method 1: Integrator Approximation

$$\text{Approximate } \tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \text{ by } \tilde{G}(s) = \frac{K^* e^{-\theta s}}{s}$$

where  $K^* \approx K/\tau$ .

- Then can use the IMC tuning rules (Rule M or N) to specify the controller settings.

## Method 2. Limit the Value of $\tau_I$

- For lag-dominant models, the standard IMC controllers for first-order and second-order models provide sluggish disturbance responses because  $\tau_I$  is very large.
- For example, controller  $G$  in Table 12.1 has  $\tau_I = \tau$  where  $\tau$  is very large.
- As a remedy, Skogestad (2003) has proposed limiting the value of  $\tau_I$ :

$$\tau_I = \min \left\{ \tau_1, 4(\tau_c + \theta) \right\} \quad (12-34)$$

where  $\tau_1$  is the largest time constant (if there are two).

## Method 3. Design the Controller for Disturbances, Rather Set-point Changes

- The desired CLTF is expressed in terms of  $(Y/D)_{\text{des}}$ , rather than  $(Y/Y_{sp})_{\text{des}}$
- *Reference:* Chen & Seborg (2002)

## Example 12.4

Consider a lag-dominant model with  $\theta / \tau = 0.01$ :

$$\tilde{G}(s) = \frac{100}{100s + 1} e^{-s}$$

**Design four PI controllers:**

- a) IMC ( $\tau_c = 1$ )
- b) IMC ( $\tau_c = 2$ ) based on the integrator approximation
- c) IMC ( $\tau_c = 1$ ) with Skogestad's modification (Eq. 12-34)
- d) Direct Synthesis method for disturbance rejection (Chen and Seborg, 2002): The controller settings are  $K_c = 0.551$  and  $\tau_I = 4.91$ .

Evaluate the four controllers by comparing their performance for unit step changes in both set point and disturbance. Assume that the model is perfect and that  $G_d(s) = G(s)$ .

### Solution

The PI controller settings are:

Controller	$K_c$	$\tau_I$
(a) IMC	0.5	100
(b) Integrator approximation	0.556	5
(c) Skogestad	0.5	8
(d) DS-d	0.551	4.91

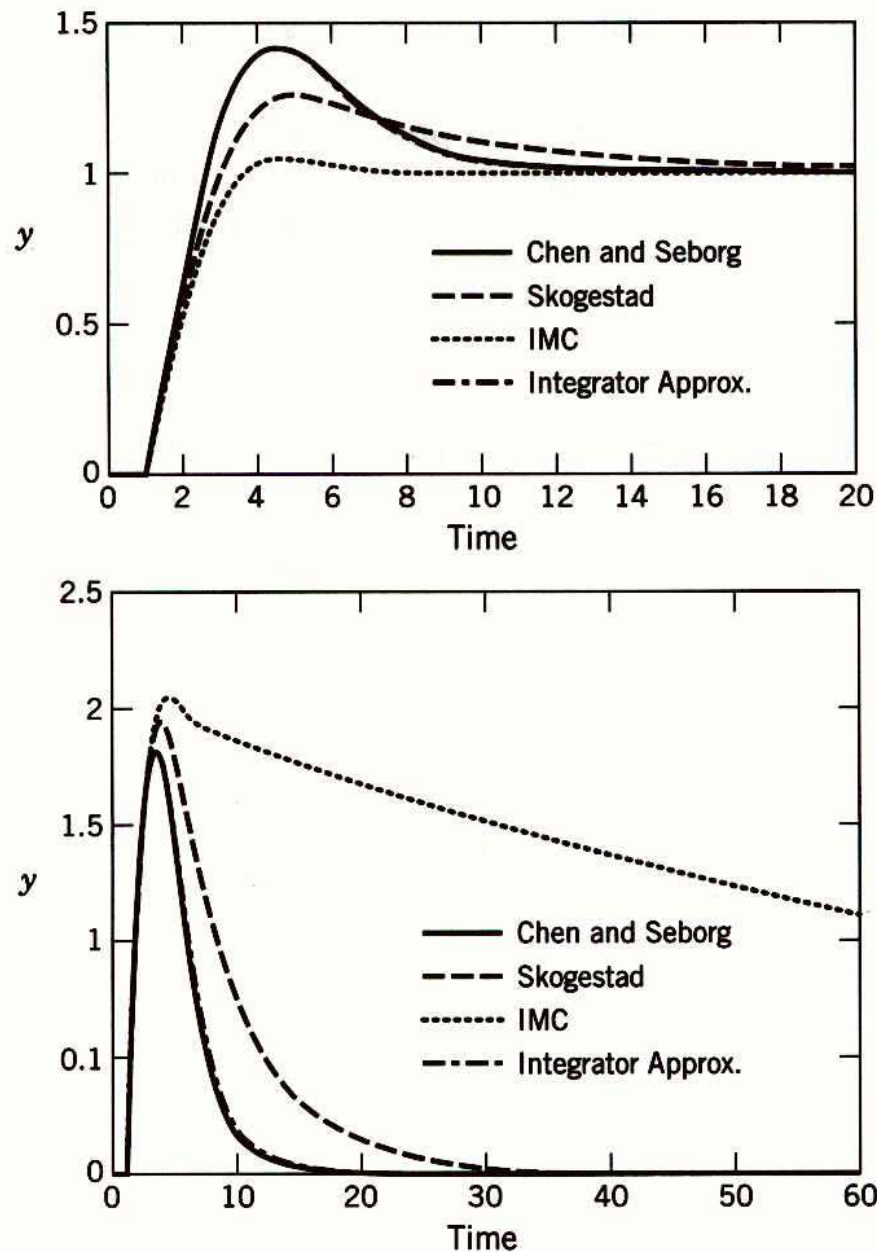


Figure 12.8. Comparison of set-point responses (top) and disturbance responses (bottom) for Example 12.4. The responses for the Chen and Seborg and integrator approximation methods are essentially identical.

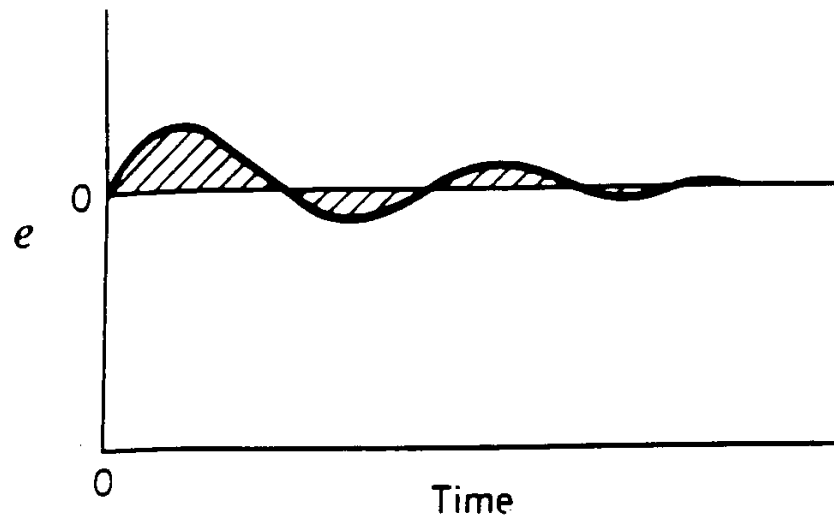
# Tuning Relations Based on Integral Error Criteria

- Controller tuning relations have been developed that optimize the closed-loop response for a simple process model and a specified disturbance or set-point change.
- The optimum settings minimize an *integral error criterion*.
- Three popular integral error criteria are:

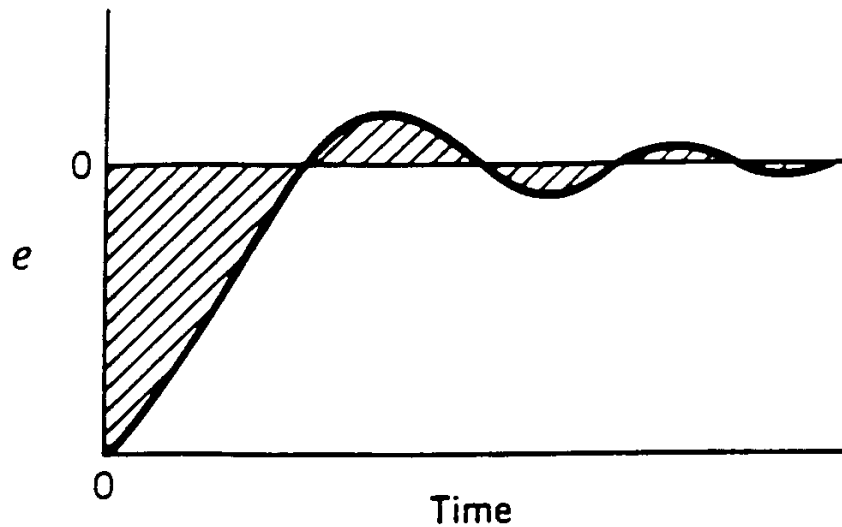
## 1. *Integral of the absolute value of the error (IAE)*

$$\text{IAE} = \int_0^{\infty} |e(t)| dt \quad (12-35)$$

where the error signal  $e(t)$  is the difference between the set point and the measurement.



(a) Load change



(b) Set-point change

Figure 12.9. Graphical interpretation of IAE. The shaded area is the IAE value.

## 2. *Integral of the squared error (ISE)*

$$\text{ISE} = \int_0^{\infty} [e(t)]^2 dt \quad (12-36)$$

## 3. *Integral of the time-weighted absolute error (ITAE)*

$$\text{ITAE} = \int_0^{\infty} t |e(t)| dt \quad (12-37)$$

See text for ITAE controller tuning relations.

# Comparison of Controller Design and Tuning Relations

Although the design and tuning relations of the previous sections are based on different performance criteria, several general conclusions can be drawn:



1. The controller gain  $K_c$  should be inversely proportional to the product of the other gains in the feedback loop (i.e.,  $K_c \propto 1/K$  where  $K = K_v K_p K_m$ ).
2.  $K_c$  should decrease as  $\theta / \tau$ , the ratio of the time delay to the dominant time constant, increases. In general, the quality of control decreases as  $\theta / \tau$  increases owing to longer settling times and larger maximum deviations from the set point.
3. Both  $\tau_I$  and  $\tau_D$  should increase as  $\theta / \tau$  increases. For many controller tuning relations, the ratio,  $\tau_D / \tau_I$ , is between 0.1 and 0.3. As a rule of thumb, use  $\tau_D / \tau_I = 0.25$  as a first guess.
4. When integral control action is added to a proportional-only controller,  $K_c$  should be reduced. The further addition of derivative action allows  $K_c$  to be increased to a value greater than that for proportional-only control.

# Controllers With Two Degrees of Freedom

- The specification of controller settings for a standard PID controller typically requires a tradeoff between set-point tracking and disturbance rejection.
- These strategies are referred to as *controllers with two-degrees-of-freedom*.
- The first strategy is very simple. Set-point changes are introduced gradually rather than as abrupt step changes.
- For example, the set point can be ramped as shown in Fig. 12.10 or “filtered” by passing it through a first-order transfer function,

$$\frac{Y_{sp}^*}{Y_{sp}} = \frac{1}{\tau_f s + 1} \quad (12-38)$$

where  $Y_{sp}^*$  denotes the *filtered set point* that is used in the control calculations.

- The filter time constant,  $\tau_f$  determines how quickly the filtered set point will attain the new value, as shown in Fig. 12.10.

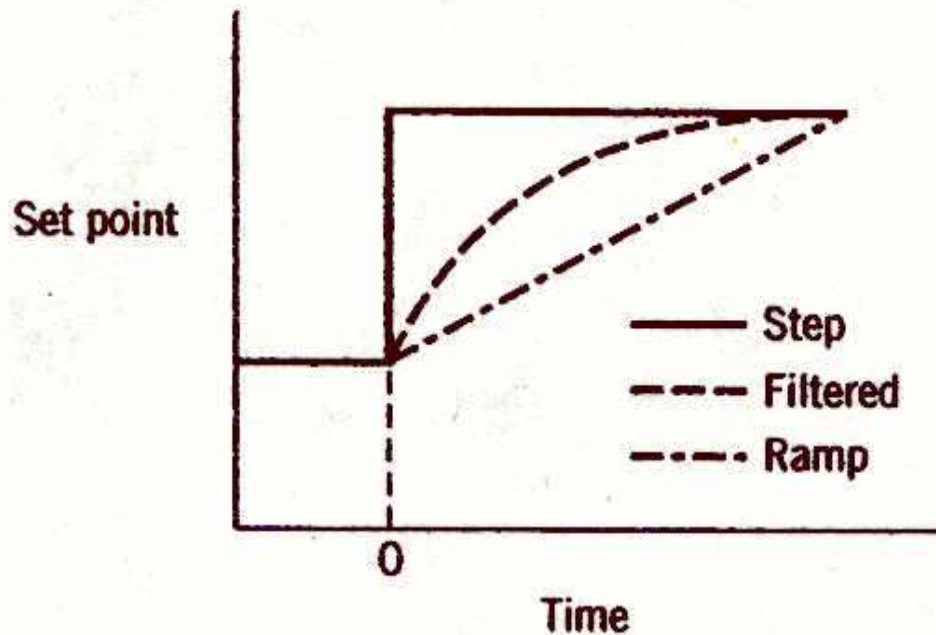


Figure 12.10 Implementation of set-point changes.

- A second strategy for independently adjusting the set-point response is based on a simple modification of the PID control law in Chapter 8,

$$p(t) = \bar{p} + K_c \left[ e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* - \tau_D \frac{dy_m}{dt} \right] \quad (8-7)$$

where  $y_m$  is the measured value of  $y$  and  $e$  is the error signal.  
 $e = y_{sp} - y_m$

- The control law modification consists of multiplying the set point in the proportional term by a *set-point weighting factor*,  $\beta$ :

$$p(t) = \bar{p} + K_c \left[ \beta y_{sp}(t) - y_m(t) \right] + K_c \left[ \frac{1}{\tau_I} \int_0^t e(t^*) dt^* - \tau_D \frac{dy_m}{dt} \right] \quad (12-39)$$

The set-point weighting factor is bounded,  $0 < \beta < 1$ , and serves as a convenient tuning factor.

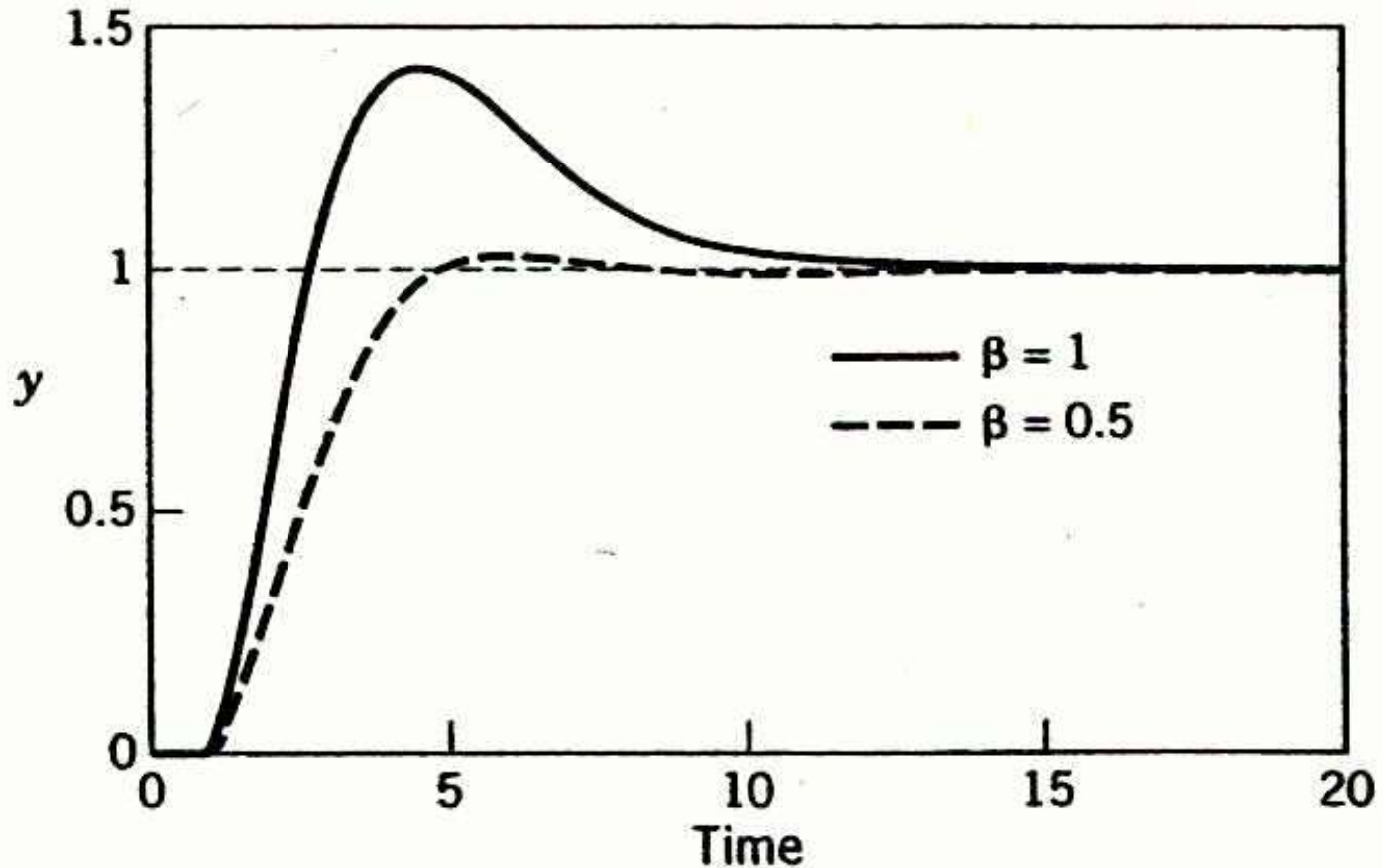


Figure 12.11 Influence of set-point weighting on closed-loop responses for Example 12.6.

# On-Line Controller Tuning

1. *Controller tuning inevitably involves a tradeoff between performance and robustness.*
2. *Controller settings do not have to be precisely determined.* In general, a small change in a controller setting from its best value (for example,  $\pm 10\%$ ) has little effect on closed-loop responses.
3. *For most plants, it is not feasible to manually tune each controller.* Tuning is usually done by a control specialist (engineer or technician) or by a plant operator. Because each person is typically responsible for 300 to 1000 control loops, it is not feasible to tune every controller.
4. *Diagnostic techniques for monitoring control system performance are available.*

# Continuous Cycling Method

Over 60 years ago, Ziegler and Nichols (1942) published a classic paper that introduced the *continuous cycling method* for controller tuning. It is based on the following trial-and-error procedure:

**Step 1.** After the process has reached steady state (at least approximately), eliminate the integral and derivative control action by setting  $\tau_D$  to zero and  $\tau_I$  to the largest possible value.

**Step 2.** Set  $K_c$  equal to a small value (e.g., 0.5) and place the controller in the automatic mode.

**Step 3.** Introduce a small, momentary set-point change so that the controlled variable moves away from the set point. Gradually increase  $K_c$  in small increments until continuous cycling occurs. The term *continuous cycling* refers to a sustained oscillation with a constant amplitude. The numerical value of  $K_c$  that produces

continuous cycling (for proportional-only control) is called the *ultimate gain*,  $K_{cu}$ . The period of the corresponding sustained oscillation is referred to as the *ultimate period*,  $P_u$ .

**Step 4.** Calculate the PID controller settings using the Ziegler-Nichols (Z-N) tuning relations in Table 12.6.

**Step 5.** Evaluate the Z-N controller settings by introducing a small set-point change and observing the closed-loop response. Fine-tune the settings, if necessary.

The continuous cycling method, or a modified version of it, is frequently recommended by control system vendors. Even so, the continuous cycling method has several major disadvantages:

1. It can be quite time-consuming if several trials are required and the process dynamics are slow. The long experimental tests may result in reduced production or poor product quality.



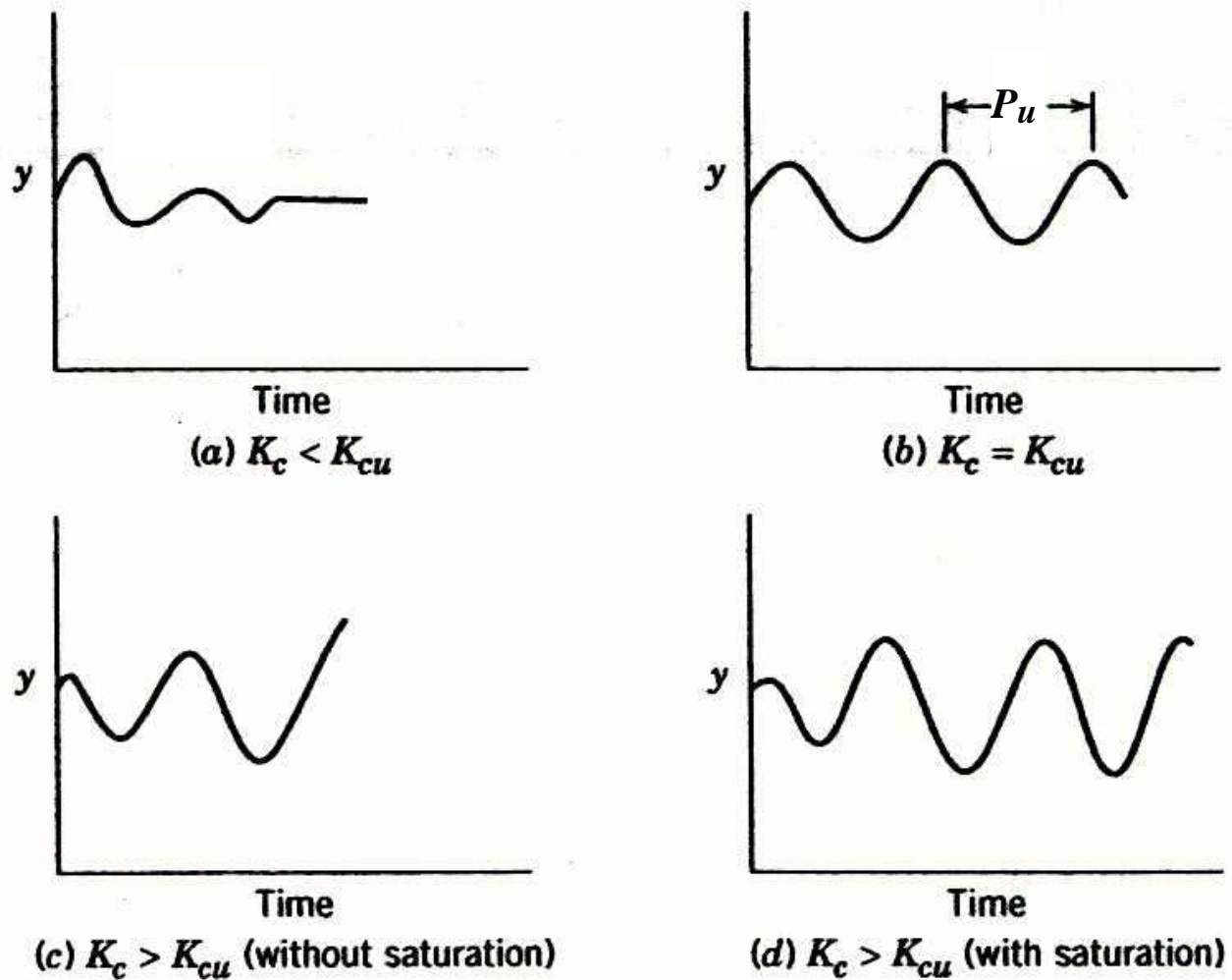


Figure 12.12 Experimental determination of the ultimate gain  $K_{cu}$ .

**Table 12.6** Controller Settings based on the Continuous Cycling Method

Ziegler-Nichols	$K_c$	$\tau_I$	$\tau_D$
P	$0.5K_{cu}$	—	—
PI	$0.45K_{cu}$	$P_u/1.2$	—
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$
Tyres-Luyben†	$K_c$	$\tau_I$	$\tau_D$
PI	$0.31K_{cu}$	$2.2P_u$	—
PID	$0.45K_{cu}$	$2.2P_u$	$P_u/6.3$

† Luyben and Luyben (1997).

2. In many applications, continuous cycling is objectionable because the process is pushed to the stability limits.
3. This tuning procedure is not applicable to integrating or open-loop unstable processes because their control loops typically are unstable at both high and low values of  $K_c$ , while being stable for intermediate values.
4. For first-order and second-order models without time delays, the ultimate gain does not exist because the closed-loop system is stable for all values of  $K_c$ , providing that its sign is correct. However, in practice, it is unusual for a control loop not to have an ultimate gain.

# Relay Auto-Tuning

- Åström and Hägglund (1984) have developed an attractive alternative to the continuous cycling method.
- In the *relay auto-tuning* method, a simple experimental test is used to determine  $K_{cu}$  and  $P_u$ .
- For this test, the feedback controller is temporarily replaced by an on-off controller (or *relay*).
- After the control loop is closed, the controlled variable exhibits a sustained oscillation that is characteristic of on-off control (cf. Section 8.4). The operation of the relay auto-tuner includes a *dead band* as shown in Fig. 12.14.
- The dead band is used to avoid frequent switching caused by measurement noise.

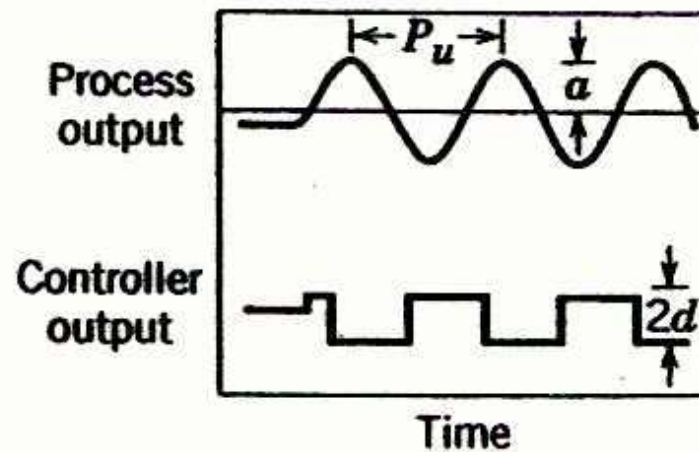
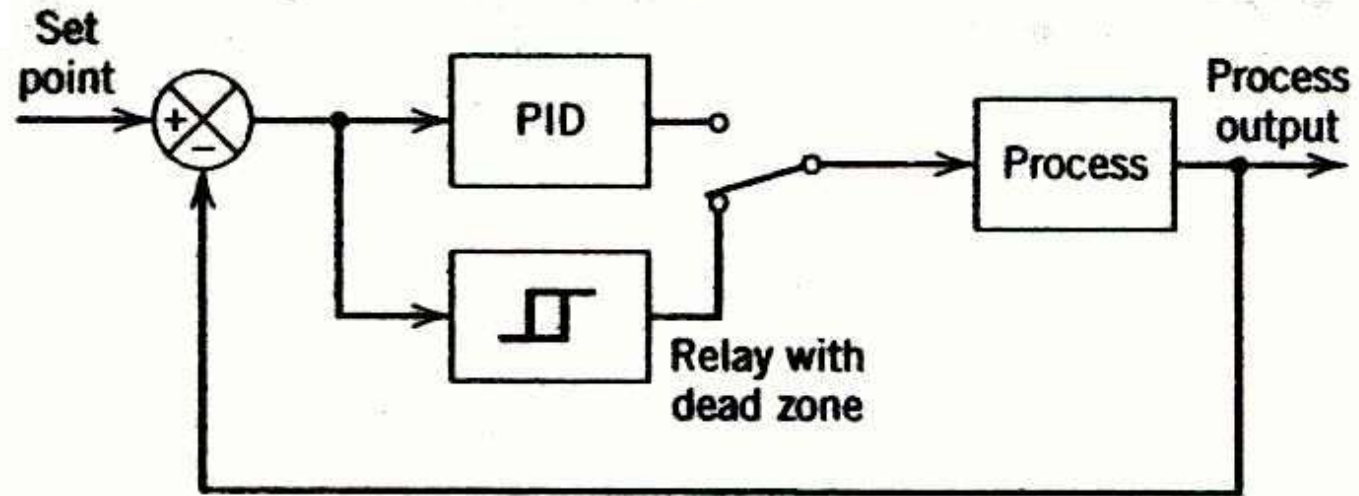


Figure 12.14 Auto-tuning using a relay controller.

- The relay auto-tuning method has several important advantages compared to the continuous cycling method:
  1. Only a single experiment test is required instead of a trial-and-error procedure.
  2. The amplitude of the process output  $a$  can be restricted by adjusting relay amplitude  $d$ .
  3. The process is not forced to a stability limit.
  4. The experimental test is easily automated using commercial products.

# Step Test Method

- In their classic paper, Ziegler and Nichols (1942) proposed a second on-line tuning technique based on a single step test. The experimental procedure is quite simple.
- After the process has reached steady state (at least approximately), the controller is placed in the manual mode.
- Then a small step change in the controller output (e.g., 3 to 5%) is introduced.
- The controller settings are based on the *process reaction curve* (Section 7.2), the open-loop step response.
- Consequently, this on-line tuning technique is referred to as *the step test method* or *the process reaction curve method*.

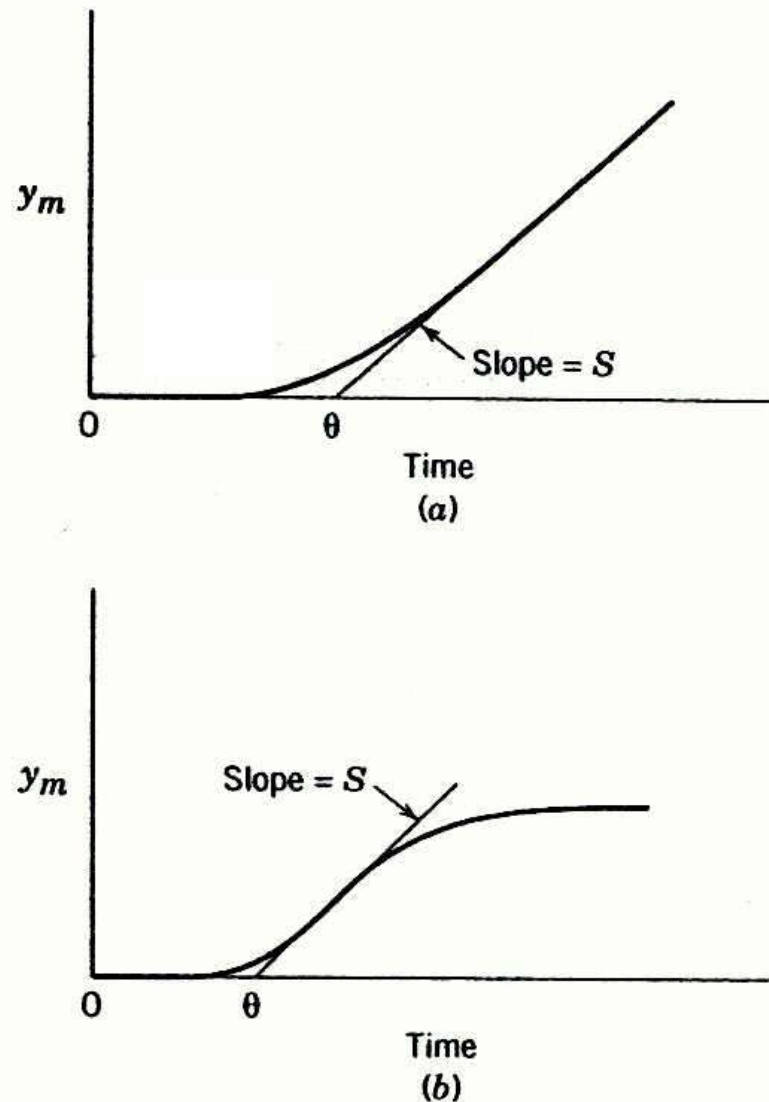


Figure 12.15 Typical process reaction curves: (a) non-self-regulating process, (b) self-regulating process.



An appropriate transfer function model can be obtained from the step response by using the parameter estimation methods of Chapter 7.

The chief advantage of the step test method is that only a single experimental test is necessary. But the method does have four disadvantages:

1. The experimental test is performed under open-loop conditions. Thus, if a significant disturbance occurs during the test, no corrective action is taken. Consequently, the process can be upset, and the test results may be misleading.
2. For a nonlinear process, the test results can be sensitive to the magnitude and direction of the step change. If the magnitude of the step change is too large, process nonlinearities can influence the result. But if the step magnitude is too small, the step response may be difficult to distinguish from the usual fluctuations due to noise and disturbances. The direction of the step change (positive or negative) should be chosen so that

the controlled variable will not violate a constraint.

3. The method is not applicable to open-loop unstable processes.
4. For analog controllers, the method tends to be sensitive to controller calibration errors. By contrast, the continuous cycling method is less sensitive to calibration errors in  $K_c$  because it is adjusted during the experimental test.

### *Example 12.8*

Consider the feedback control system for the stirred-tank blending process shown in Fig. 11.1 and the following step test. The controller was placed in manual, and then its output was suddenly changed from 30% to 43%. The resulting process reaction curve is shown in Fig. 12.16. Thus, after the step change occurred at  $t = 0$ , the measured exit composition changed from 35% to 55% (expressed as a percentage of the measurement span), which is equivalent to the mole fraction changing from 0.10 to 0.30. Determine an appropriate process model for  $G \square G_{IP}G_vG_pG_m$ .

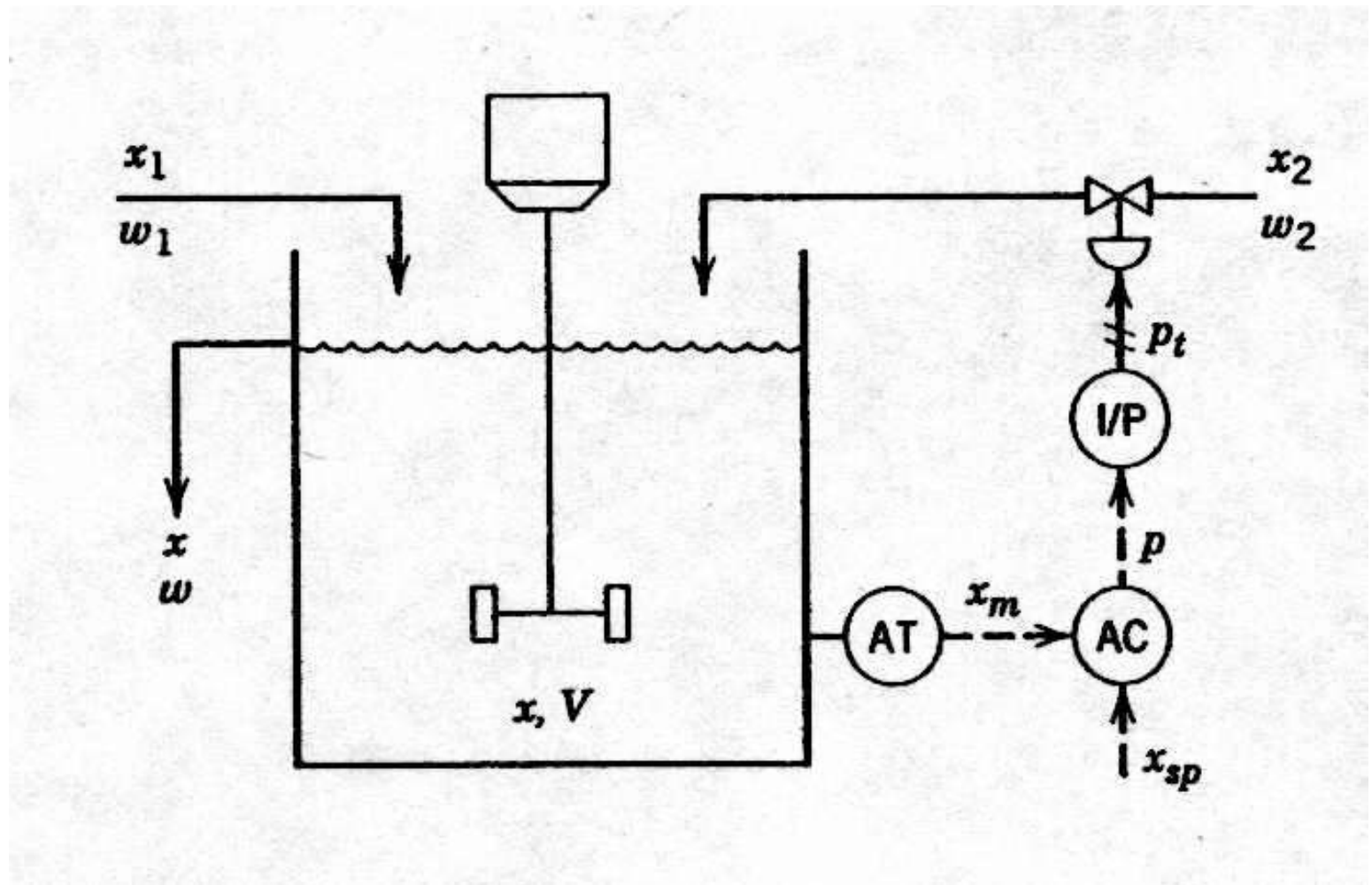


Figure 11.1 Composition control system for a stirred-tank blending process.

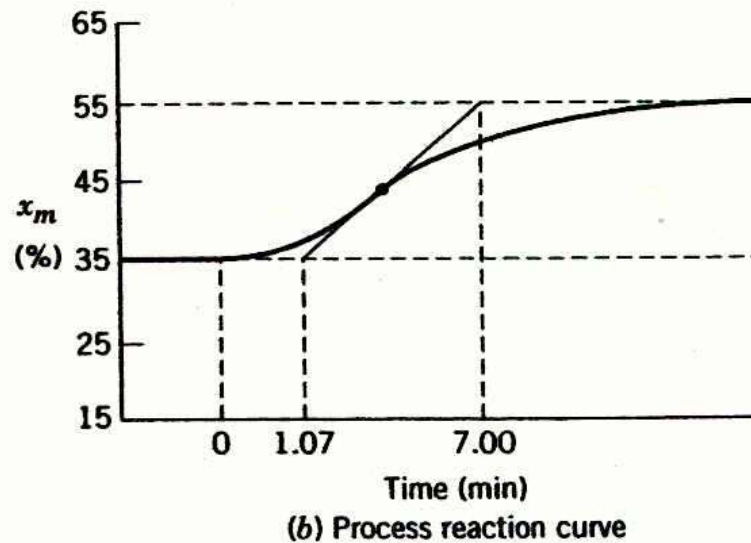
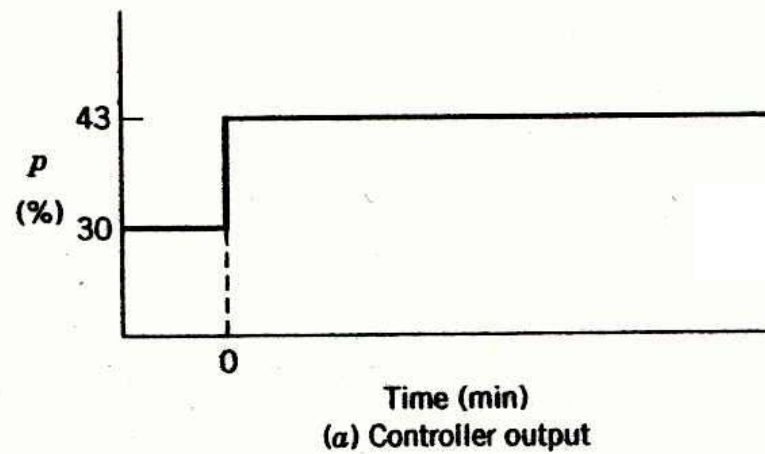


Figure 12.16 Process reaction curve for Example 12.8.

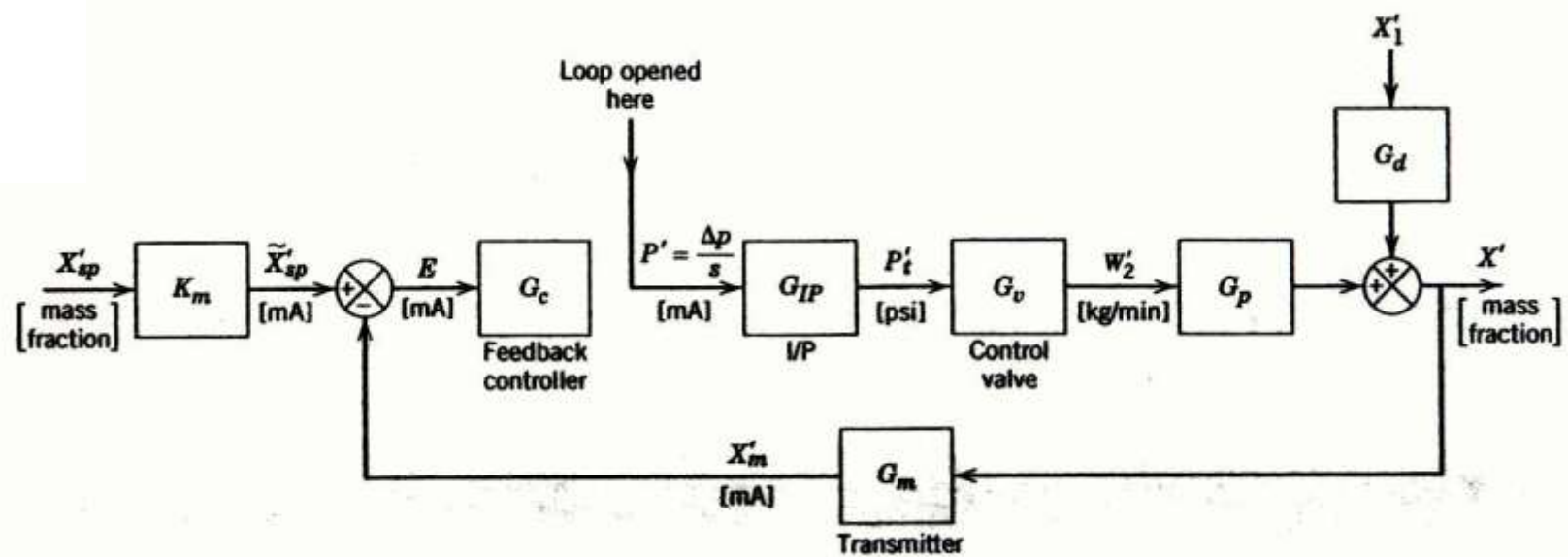


Figure 12.17 Block diagram for Example 12.8.

## Solution

A block diagram for the closed-loop system is shown in Fig. 12.17. This block diagram is similar to Fig. 11.7, but the feedback loop has been broken between the controller and the current-to-pressure (I/P) transducer. A first-order-plus-time-delay model can be developed from the process reaction curve in Fig. 12.16 using the graphical method of Section 7.2. The tangent line through the inflection point intersects the horizontal lines for the initial and final composition values at 1.07 min and 7.00 min, respectively. The slope of the line is

$$S = \left( \frac{55 - 35\%}{7.00 - 1.07 \text{ min}} \right) = 3.37\% / \text{min}$$

and the normalized slope is

$$R = \frac{S}{\Delta p} = \frac{3.37\% / \text{min}}{43\% - 30\%} = 0.259 \text{ min}^{-1}$$

The model parameters can be calculated as

$$K = \frac{\Delta x_m}{\Delta p} = \frac{55\% - 35\%}{43\% - 30\%} = 1.54 \text{ (dimensionless)}$$

$$\theta = 1.07 \text{ min}$$

$$\tau = 7.00 - 1.07 \text{ min} = 5.93 \text{ min}$$

The apparent time delay of 1.07 min is subtracted from the intercept value of 7.00 min for the  $\tau$  calculation.

The resulting empirical process model can be expressed as

$$\frac{X'_m(s)}{P'(s)} = G(s) = \frac{1.54e^{-1.07s}}{5.93s + 1}$$

Example 12.5 in Section 12.3 provided a comparison of PI controller settings for this model that were calculated using different tuning relations.

# Guidelines For Common Control Loops

(see text)

## Troubleshooting Control Loops

- If a control loop is not performing satisfactorily, then troubleshooting is necessary to identify the source of the problem.
- Based on experience in the chemical industry, he has observed that a control loop that once operated satisfactorily can become either unstable or excessively sluggish for a variety of reasons that include:
  - a. Changing process conditions, usually changes in throughput rate.
  - b. Sticking control valve stem.



- c. Plugged line in a pressure or differential pressure transmitter.
- d. Fouled heat exchangers, especially reboilers for distillation columns.
- e. Cavitating pumps (usually caused by a suction pressure that is too low).

The starting point for troubleshooting is to obtain enough background information to clearly define the problem. Many questions need to be answered:

1. What is the process being controlled?
2. What is the controlled variable?
3. What are the control objectives?
4. Are closed-loop response data available?
5. Is the controller in the manual or automatic mode? Is it reverse or direct acting?

6. If the process is cycling, what is the cycling frequency?
7. What control algorithm is used? What are the controller settings?
8. Is the process open-loop stable?
9. What additional documentation is available, such as control loop summary sheets, piping and instrumentation diagrams, etc.?