Process Heat Transfer

Lec 7: Principles of Convection

Introduction, Physical Mechanism of Forced Convection, Velocity Boundary Layer, Laminar Boundary Layer on Flat Plate, Thermal Boundary Layer, Relationship between fluid friction and heat transfer, Heat Transfer in Turbulent Flow, Turbulent-Boundary-Layer Thickness

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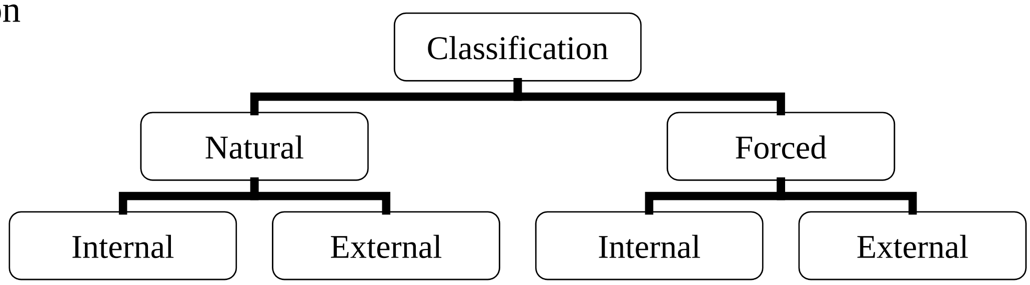
Content

- Physical Mechanism of Forced Convection
- Velocity Boundary Layer
- Laminar and Turbulent Flows
- Laminar Boundary Layers on Flat Plate: Analytical solution
- Heat transfer coefficient
- Heat Transfer in Turbulent Flow

Introduction

➢ **Convection:** mechanism of heat transfer through a fluid in the presence of bulk fluid motion



Objectives

- Physical description of the *convection* mechanism and the *velocity* and *thermal* boundary layers.
- Discussion of the dimensionless *Reynolds*, *Prandtl*, and *Nusselt* numbers and their physical significance.
- Present empirical relations for *friction* and *heat transfer coefficients* for flow over various geometries; flat plat, cylinder and sphere, for both laminar and turbulent flow conditions

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Physical Mechanism of Forced Convection

Conduction and Convection:

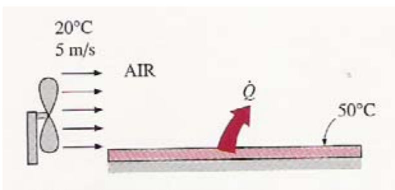
⇒ In solid, heat is transferred only by conduction, since molecules of the solid remain at relatively fixed position

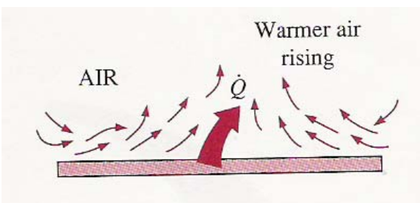
⇒ In a gas or liquid, heat can be transferred by conduction or convection, depending on the presence of any bulk fluid motion

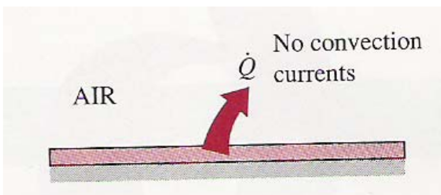
Convection: in the presence of bulk fluid motion

Conduction: in the absence of bulk fluid motion

∴ Conduction in a fluid is the limiting case convection







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Physical Mechanism of Forced Convection

- Convection is complicated because it involves both fluid motion and heat conduction:

  $\Rightarrow$ Fluid motion enhances heat transfer, since it brings hotter and cooler chunks into contact, initiating higher rates of conduction at a greater number of sites in a fluid

- Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction

  $\Rightarrow \uparrow$ fluid velocity $\Rightarrow \uparrow$ rate of heat transfer

- Consider the heat transfer between the two parallel plate shown in the drawing

  $\Rightarrow$ Assuming no fluid motion, energy can be imagined to transfer from one fluid layer to another from the hot plate to the cold plate

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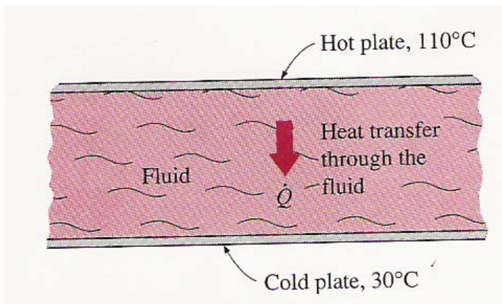
Physical Mechanism of Forced Convection

$\Rightarrow$ If part of the fluid near the hot plate is injected near the cold plate, this will speed up the rate of heat transfer process, since some energy is carried to the other side as a result of fluid motion

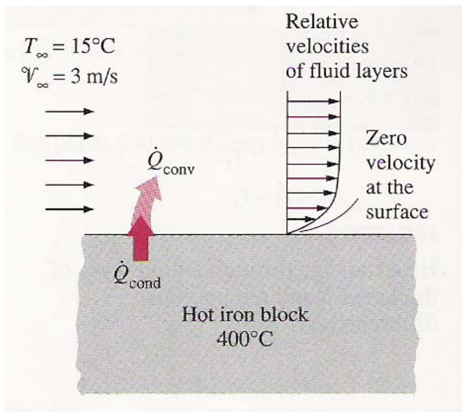


- Consider the cooling of a hot iron block with a fan blowing air over its top surface, as shown

  $\Rightarrow$ Heat will transfer from iron to air

  $\Rightarrow$ Cooling will be faster at higher fan speed

  $\Rightarrow$ Replacing air by water will enhance the convection heat transfer even more



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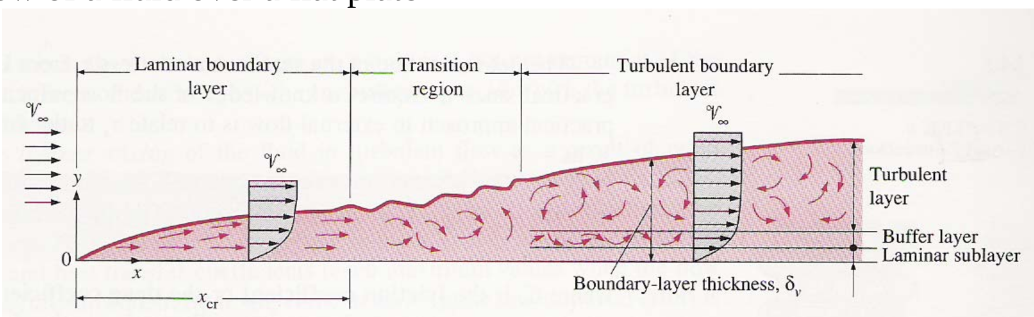
Physical Mechanism of Forced Convection

- Convection heat transfer strongly depends on the fluid velocity and the fluid properties:
  - Fluid properties: dynamic viscosity μ, thermal conductivity k, density ρ, and specific heat C_p
  - Fluid velocity, u
  - Geometry and roughness of the solid surface
  - Type of fluid flow (laminar or turbulent)
- Convection heat transfer relations are rather complex because of the dependence of convection on so many variables
- Despite the complexity of convection, the rate of convection heat transfer can be expressed by Newton's law of cooling

$$q = hA(T_w - T_\infty)$$

Velocity Boundary Layer

- Consider the flow of a fluid over a flat plate
  - *x*-direction
  - *y*-direction



- The fluid approaches the plate in the *x*-direction with uniform velocity, u_∞.
- Assume that the fluid consist of *adjacent layers* piled over each other:

  ⇒ Velocity of the first fluid layer adjacent to the plate becomes zero (no slip condition

  ⇒ Also, this motionless layer slow the particles of the neighboring fluid layer as a result of friction between the particles of these two adjoining fluid layers at different velocities

⇒ This fluid layer then slows down the molecules of the next layer, and so on.

- The friction between the two layers, exerting a drag force (or friction force).
- **Shear stress:** the drag force per unit area which is proportional to the velocity gradient and is given by,

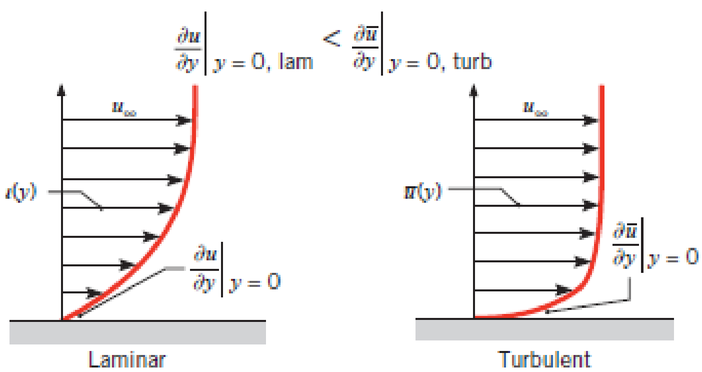
$$\tau = -\mu \frac{du}{dy} \qquad (\text{N/m}^2)$$

- The surface shear stress may be evaluated from knowledge of the velocity gradient at the surface

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

Velocity Boundary Layer

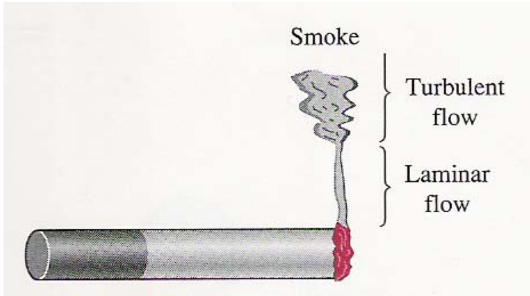
- As a result, the fluid velocity at any x-location will vary from 0 at $y = 0$ to nearly u_∞ at $y = \delta$
- **Boundary layer**, *δ:* The region of the flow above the plate bounded by δ in which the effects of the viscous sheering forces caused by fluid viscosity are felt.
- A region of the flow characterized by shear stresses and velocity gradients appears as a consequence of viscous effects associated with relative motion between a fluid and a surface



- The thickness of the boundary layer, δ, is arbitrary defined as the distance from he surface at which $u = 0.99u_\infty$

Laminar and Turbulent Flows

- The cigarette smoke rises in a smooth plum for the first few centimeters and then starts fluctuating randomly in all directions as it continues its journey toward the lung of nonsmokers



- This is similar to what happened for the flow of fluid over the flat plate:

$\Rightarrow$ Fluid flow in the boundary layer starts out as flat and streamlined but turns chaotic after some distance from the leading edge.

- **Laminar regime:** The flow regime at the beginning of the flow which is characterized by *smooth streamlines* and *highly ordered motion*.

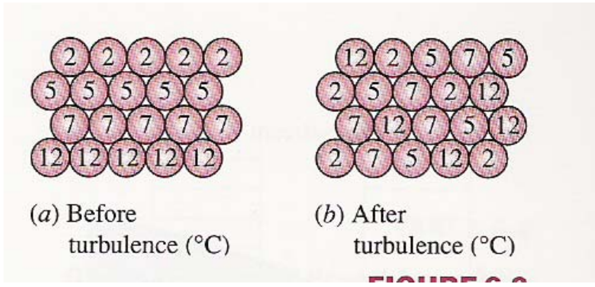
Laminar and Turbulent Flows

- **Turbulent regime:** The second flow regime which is characterized by *velocity fluctuations* and *highly disordered motion*.
- **Transition regime:** The transition from laminar to turbulent flow does not occur suddenly; rather it occurs over some region in which the flow hesitates between laminar and turbulent flows before it becomes fully turbulent.
- Velocity profile is approximately parabolic in laminar regime; but becomes flatter in turbulent flow with sharp drop near the surface
- Notice the three layers in the turbulent regime;
  - *Laminar sublayer*: viscous effects are dominant
  - *Buffer layer*: turbulent effects are significant but not dominant
  - *Overlab layer*: turbulent effects are more significant but not dominant
  - *Turbulent layer*: turbulent effects dominate

Laminar and Turbulent Flows

- The drawing shows how the intense mixing in turbulent flow brings fluid particles at different temperatures into close contact, and thus enhances heat transfer

Reynolds Number



- Transition from laminar to turbulent flow depends on:
  1. Surface geometry
  2. Surface roughness
  3. Free-stream velocity
  4. Type of fluid

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho u_\infty x}{\mu} = \frac{u_\infty x}{\nu}$$

- Obsorn Reynold (1880s) discovered that the flow regime depends mainly on the ratio of the *inertia forces to viscous forces* in the fluid. This ratio is called Reynolds number:

Laminar and Turbulent Flows

At large Re: inertia forces >> viscous forces

$\Rightarrow$ Viscous forces can not prevent the random and rapid fluctuations

At low Re: viscous forces >> inertia forces

$\Rightarrow$ Viscous forces are large enough to overcome the inertia forces and to keep the fluid in line

- **Critical Reynolds number:** the Reynolds number at which the flow becomes turbulent

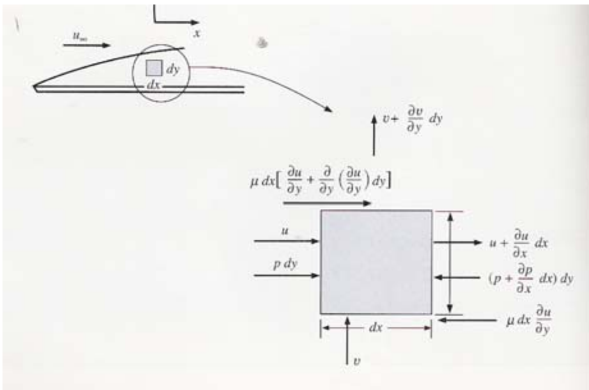
$$\text{Re}_{\text{critical, flat plate}} \approx 5\times10^5$$

$$\text{Transition}: \ 5\times10^5 < \text{Re} < 1\times10^5$$

Laminar Boundary Layer on Flat Plate

- Force and momentum balance on the elemental fluid shown in the drawing provide the following equation of motion for the boundary layer:

$$\rho\left(u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x}$$



- Following **Von Karman Approximation,** the following boundary conditions must be satisfied:

$u = 0$	@ $y = 0$	(a)
$u = u_\infty$	@ $y = \delta$	(b)
$du/dy = 0$	@ $y = \delta$	(c)

For constant pressure,

$d^2u/dy^2 = 0$ @ $y = 0$ (d) (Since $u = \upsilon = 0$ @ $y = 0$)

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Laminar Boundary Layer on Flat Plate

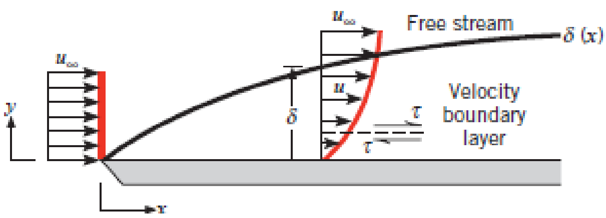
- The following equations can be obtained:

i. Velocity profile in the boundary layer

$$\frac{u}{u_\infty}=\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

ii. The wall shear stress

$$\tau_s=\mu\frac{\partial u}{\partial y}\bigg|_{y=0}=\frac{3}{2}\frac{\mu u_\infty}{\delta}$$



iii. boundary layer thickness

$$\delta=4.64\sqrt{\frac{\nu x}{u_\infty}}$$

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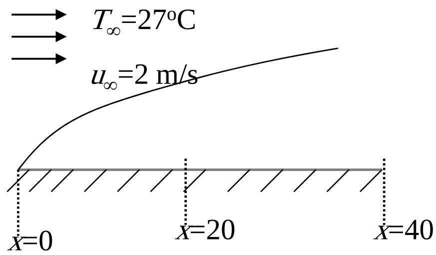
Laminar Boundary Layer on Flat Plate

In terms of Reynolds $\Rightarrow$ $$\frac{\delta}{x} = \frac{4.64}{\sqrt{\text{Re}_x}}$$

- Exact solution of boundary layer equation gives:

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{\text{Re}_x}}$$ (see appendix B)

Example:



Air at 27°C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate δ at x = 20, 40 cm from the leading edge.

At 27°C $\mu = 1.98\times10^{-5}$ kg/m.s. Assume unit depth in z-direction

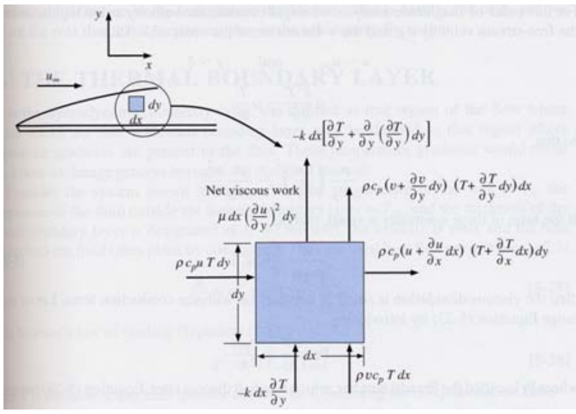
Example cont.

Example cont.

To calculate the mass flow which enters the boundary layer from the free stream between $x = 20$ cm and $x = 40$ cm, we can take the difference between the mass flow in the boundary layer at the two x-positions.

Energy Equation of the Boundary layer

- Energy balance on the elemental fluid shown in the drawing provide the energy equation.



Assumptions:

- Incompressible steady flow
- Constant μ, κ and C_p
- Negligible flow-axial direction
- small u $\Rightarrow$ then dissipative effect is low

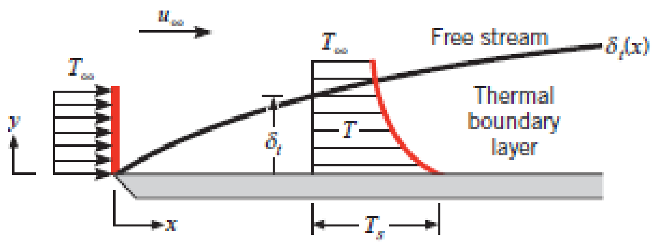
$\Rightarrow$ Energy balance results in the following equation:

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Thermal Boundary Layer

- ***Thermal boundary layer:*** A region of the flow characterized by temperature gradients and heat fluxes

  ⇒ A consequence of heat transfer (heat-exchange)between the surface and fluid



Thermal boundary layer development on an isothermal flat plate.

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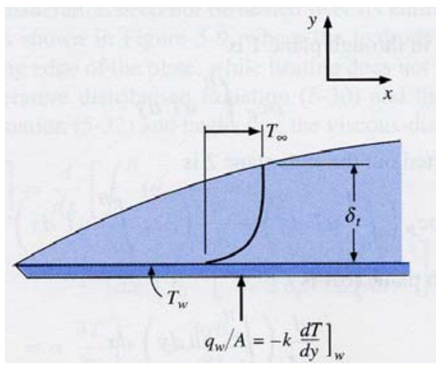
Thermal Boundary Layer

- Solving the energy equation using the following B.Cs:

$T = T_w$	@ $y = 0$	(a)
$T = T_\infty$	@ $y = \delta_t$	(b)
$dT/dy = 0$	@ $y = \delta_t$	(c)
$d^2T/dy^2 = 0$	@ $y = 0$	(d)



- Then, temperature distribution in the boundary is

$$\frac{\theta}{\theta_\infty} = \frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

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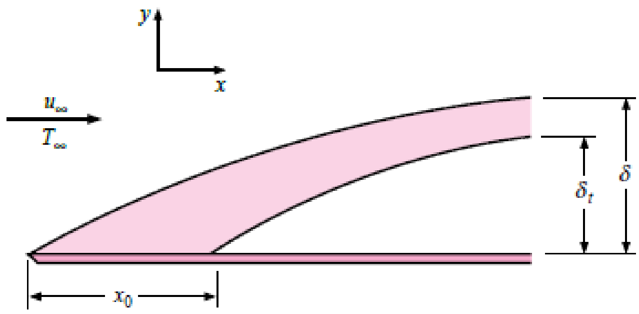
Thermal Boundary Layer

- For the system shown in the drawing.

At the wall, $u = 0$ $\Rightarrow$ Heat transfer to the fluid is by convection, or

$$q/A = q'' = -k\left.\frac{\partial T}{\partial y}\right|_{wall} = h(T_w - T_\infty)$$

or, $$h = \frac{-k(\partial T/\partial y)_w}{T - T_\infty}$$ $\Rightarrow$ Allows determination of h if temperature gradient is available



Determination of δ_t:

Assuming $\zeta = \dfrac{\delta_t}{\delta}$

$\delta_t = 0$ @ $x = x_0$

or $\zeta = 0$ @ $x = x_0$

Thermal Boundary Layer

- Using the integral analysis of the energy equation for the boundary layer, The final solution becomes:

$$\zeta = \frac{\delta_t}{\delta} = \frac{1}{1.026}\Pr^{-1/3}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

Where,

$$\Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{c_p \mu}{k} = \text{Prandtl number} = \frac{\text{hydrodynamic layer thickness}}{\text{thermal boundary layer thickness}}$$

- If heating is over the entire plate ($x_0 = 0$): $$\zeta = \frac{\delta_t}{\delta} = \frac{1}{1.026}\Pr^{-1/3}$$

  - The assumption of $\delta_t < \delta$ is justified for $\Pr > 0.7$.
  - Pr for most gases fall within this value

Heat Transfer Coefficient

If $$h = \frac{-k(\partial T/\partial y)_w}{T - T_\infty} = \frac{3}{2}\frac{k}{\delta_t}$$

- Then, substituting δ_t and δ, the local convection heat transfer coefficient is:

$$h_x = 0.332 k \mathrm{P_r}^{1/3} \left(\frac{u_\infty}{x\nu}\right)^{1/2} \left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

Define The Local Nusselt number is given as

$$\mathrm{Nu}_x = \frac{h_x x}{k} \quad = \text{Nusselt number} = \frac{\text{convective heat}}{\text{conductive heat}}$$

$$\Rightarrow \quad \mathrm{Nu}_x = 0.332 \mathrm{P_r}^{1/3} \mathrm{Re}_x^{1/2} \left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3} \qquad x_0 > 0.0$$

- If heating starts from the leading edge, i.e. $x_0 = 0.0$ $\Rightarrow$ $\mathrm{Nu}_x = 0.332 \mathrm{P_r}^{1/3} \mathrm{Re}_x^{1/2}$

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Heat Transfer Coefficient

- For a plate of length L, we can get an average value for h, since the above equation gives h at a given x.

- For the case, when $x_0 = 0$

$$\bar{h} = \frac{\int_0^L h_x dx}{\int_0^L dx} \quad \Rightarrow \overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k}$$

- Performing the integration: $\overline{\mathrm{Nu}} = 0.664 \mathrm{Re}^{1/2} \mathrm{P_r}^{1/3}$ $= 2\,\mathrm{Nu}_{x=L}$

❖ Note: fluid properties are normally calculated at the film average temperature:

$$T_f = \frac{T_w + T_\infty}{2}$$

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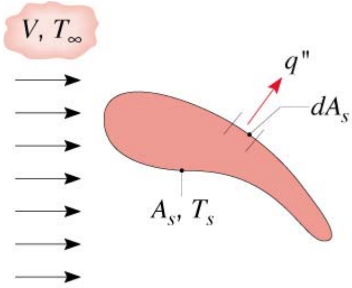
Distinction between Local and Average Heat Transfer Coefficients

- Local Heat Flux and Coefficient:

$$q_s'' = h(T_s - T_\infty)$$

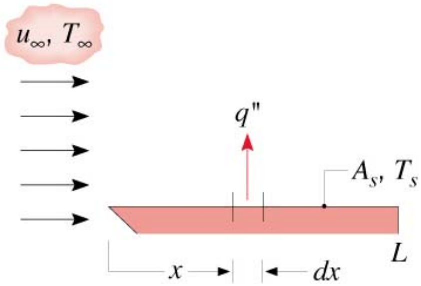


- Average Heat Flux and Coefficient for a Uniform Surface Temperature:

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty)\int_{A_s} h dA_s$$

$$\Rightarrow \quad q = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$



- For a flat plate in parallel flow: $\bar{h} = \frac{1}{L}\int_0^L h dx$

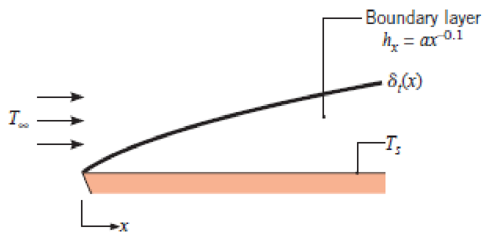
Example

Experimental results for the local heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$$h_x(x) = ax^{-0.1}$$

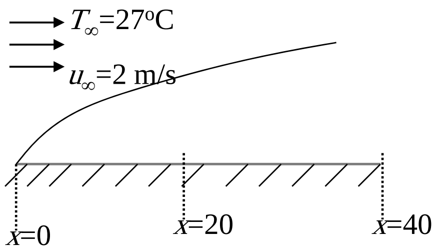
where a is a coefficient (W/m$^{1.9}\cdot$K) and x (m) is the distance from the leading edge of the plate.

1. Develop an expression for the ratio of the average heat transfer coefficient $\bar{h}_x$ for a plate of length x to the local heat transfer coefficient h_x at x.



Example

For the previous example, if heating starts from the leading edge, calculate q in the first 20 cm.



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Relationship between fluid friction and heat transfer

The shear stress at the wall may be related to the friction coefficient C_f by:

$$\tau_w = C_f \frac{\rho u_\infty^2}{2}$$ [from fluid mechanics for the definition of C_f]

We know that $\tau_w = \mu \left.\frac{du}{dy}\right|_w$ (for laminar)

The equation: $\mathrm{Nu}_x = 0.332 \mathrm{P_r}^{1/3} \mathrm{Re}_x^{1/2}$ can be written as:

$$\frac{\mathrm{Nu}_x}{\mathrm{Re}_x \mathrm{Pr}} = \frac{h_x}{\rho C_p u_\infty} = 0.332 \mathrm{P_r}^{-2/3} \mathrm{Re}_x^{-1/2}$$

where $\frac{\mathrm{Nu}_x}{\mathrm{Re}_x \mathrm{Pr}} = \mathrm{St}_x = \text{Stanton number}$

$\Rightarrow \mathrm{St}_x \mathrm{Pr}^{2/3} = 0.332 \mathrm{Re}_x^{-1/2}$

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Relationship between fluid friction and heat transfer

⇨ $St_x Pr^{2/3} = C_{fx}/2$ **Colburn analogy**

- Relates fluid friction to heat transfer for laminar flow on flat plate

⇨ Conclusion: h_x can be determined by measuring C_f.

Heat Transfer in Turbulent Flow

- The following correlations have bee verified for turbulent regime:

$$St_x Pr^{2/3} = 0.0296 Re_x^{-0.2} \qquad 5\times10^5 < Re_x < 10^7$$

$$St_x Pr^{2/3} = 0.185[\log Re_x]^{-2.585} \qquad 10^7 < Re_x < 10^9$$

- The average heat transfer over the entire laminar-turbulent layer is

$$\overline{St}\, Pr^{2/3} = \overline{C}_f / 2$$

If $Re_{crit} = 5\times10^5$ and $Re_L < 10^7$, then $\overline{St}\, Pr^{2/3} = 0.037 Re_L^{-1/5} - 871 Re_L^{-1}$

Recalling that, $\overline{St} = \dfrac{\overline{Nu}}{Re\, Pr}$, then

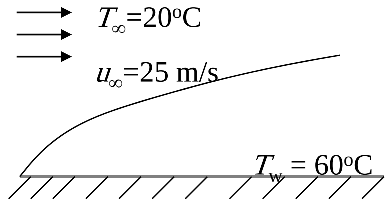
While,

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = Pr^{1/3}[0.037 Re_L^{0.8} - 871] \qquad Re_L < 10^7$$

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = Pr^{1/3}[0.228(\log Re)^{-2.584} - 871] \qquad 10^7 < Re_L < 10^9$$

Example

Air at 20°C and 1 atm flows over a flat plate at a 25 m/s. The plate is 75 cm long and is maintained at 60°C. For unit depth, calculate the heat transfer from the plate.



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Turbulent-Boundary-Layer Thickness

- The following velocity profile has been experimentally verified by a number of investigators:

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

where δ is the turbulent boundary layer,

for $Re_L < 10^7$

$$C_f = 0.0592\,\mathrm{Re}_L^{-1/5} = 0.0592\left[\frac{\rho u_\infty x}{\mu}\right]^{-1/5} \Rightarrow \tau_w = 0.0296\left(\frac{\nu}{u_\infty x}\right)^{1/5} \rho u_\infty^2$$

- If the boundary layer is fully turbulent from the leading edge, then

I.C. $\quad x = 0 \quad \delta = 0$

$$\Rightarrow \frac{\delta}{x} = 0.381\,\mathrm{Re}_x^{-1/5}$$

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Turbulent-Boundary-Layer Thickness

- If the boundary layer follows a laminar growth pattern up to $Re_{crit} = 5\times10^5$ and a turbulent growth thereafter, then

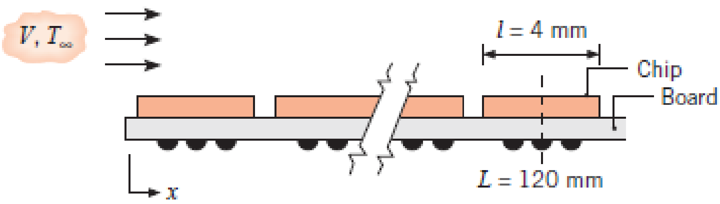
I.C. $x_{crit} = 5\times105\ \nu/u_\infty \qquad \delta = \delta_{laminar}$

$$\frac{\delta}{x} = 0.381 Re_x^{-1/5} - 10256 Re_x^{-1} \qquad \text{For } 5\times10^5 < Re_x < 10^7$$

Example

6.39 Forced air at $T_\infty = 25°C$ and $V = 10$ m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm × 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

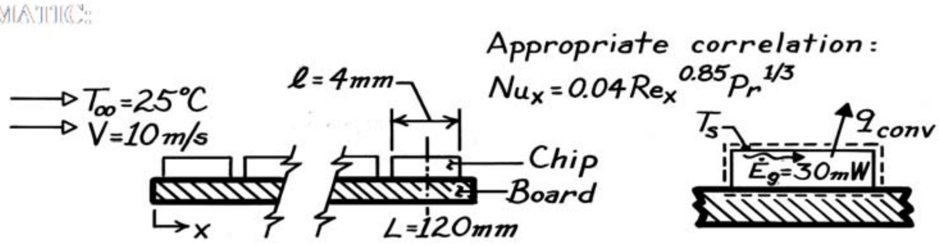
$$Nu_x = 0.04\, Re_x^{0.85}\, Pr^{1/3}$$



Estimate the surface temperature of the chip if it is dissipating 30 mW.

Example cont.

SCHEMATIC:



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Example cont.

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Example cont.

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Example

6.3 In a particular application involving airflow over a heated surface, the boundary layer temperature distribution may be approximated as

$$\frac{T - T_s}{T_\infty - T_s} = 1 - \exp\left(-Pr\frac{u_\infty y}{\nu}\right)$$

where y is the distance normal to the surface and the Prandtl number, $Pr = c_p\mu/k = 0.7$, is a dimensionless fluid property. If $T_\infty = 400\,\text{K}$, $T_s = 300\,\text{K}$, and $u_\infty/\nu = 5000\,\text{m}^{-1}$, what is the surface heat flux?

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