Faculty of Engineering and Technology
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Process Heat Transfer

Lec 8: External Forced Convection

Parallel Flow Over Flat Plates, Flow across cylinder and spheres, Flow Across Tube Bank

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Content

- Parallel Flow Over Flat Plates: laminar and turbulent flow,
- The average heat transfer coefficient with flow across cylinders and spheres , and
- The average heat transfer coefficient associated with flow across a tube bank

Introduction

- It was shown in the previous lecture (was mainly analytical in character) that the local and average Nusselt numbers have the functional form:

$$Nu_x = f_1\left(x, \mathrm{Re}_x, \mathrm{Pr}\right) \quad \text{and} \quad Nu = f_2\left(\mathrm{Re}_L, \mathrm{Pr}\right)$$

- This part deals almost entirely with empirical correlations that may be used to calculate convection heat transfer.

- The experimental data for heat transfer is often expressed by a simple power-law relation of the form:

$$Nu = C\,\mathrm{Re}_L^m \cdot \mathrm{Pr}^n$$

where m and n are constant exponents, and the value of the constant C depends on the geometry and flow.

Introduction

- The local drag and convection coefficients vary along the surface as a result of the changes in the velocity boundary layers in the flow direction.

- The *average* convection coefficients for the entire surface can be determined by

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx \qquad h = \frac{1}{L}\int_0^L h_x\,dx$$

- When the average drag and convection coefficients are available, the drag force and rate of heat transfer can be determined:

$$F_D = C_D A \frac{\rho u_\infty^2}{2} \qquad \dot{Q} = hA_s\left(T_s - T_\infty\right)$$

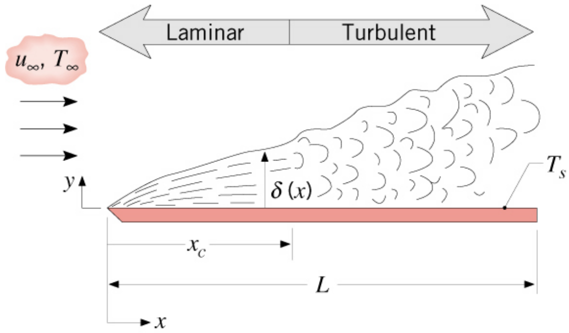
- The fluid temperature in the thermal boundary layer varies from T_s at the surface to about T_∞ at the outer edge of the boundary layer.

$\Rightarrow$ The fluid properties are evaluated at the film temperature: $T_f = \dfrac{T_s + T_\infty}{2}$

Parallel Flow Over Flat Plates

Physical Features

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction as shown in the drawing



- As with all external flows, the boundary layers develop freely without constraint.
- Boundary layer conditions may be entirely laminar, laminar and turbulent, or entirely turbulent.
- To determine the conditions, compute $\mathrm{Re} = \dfrac{\rho u_\infty x}{\mu} = \dfrac{u_\infty x}{\nu}$

and compare with the critical Reynolds number for transition to turbulence,

Parallel Flow Over Flat Plates

$\mathrm{Re}_L < \mathrm{Re}_{x,c}$ Laminar flow throughout

$\mathrm{Re}_L > \mathrm{Re}_{x,c}$ Transition to turbulent flow at $x_c / L = \mathrm{Re}_{x,c} / \mathrm{Re}_L$

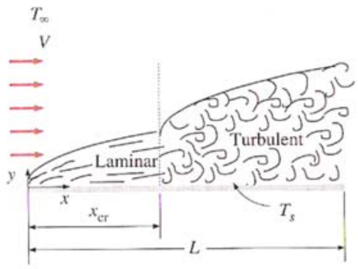
- For flow over a flat plat, $\mathrm{Re}_{\text{critical, flat plate}} \approx 5 \times 10^5$
- If boundary layer is tripped at the leading edge

$$\mathrm{Re}_{x,c} = 0$$

and the flow is turbulent throughout.

Parallel Flow Over Flat Plates

Friction Coefficient



Based on the analysis presented in the previous Chapter:

Laminar: $\delta_{v,x} = \dfrac{4.91x}{\sqrt{\text{Re}_x}}$ & $C_{f,x} = \dfrac{0.664}{\sqrt{\text{Re}_x}}$ $\quad \text{Re}_x < 5 \times 10^5$

Turbulent: $\delta_{v,x} = \dfrac{0.38x}{\text{Re}_x^{1/3}}$ & $C_{f,x} = \dfrac{0.059}{\text{Re}^{1/5}}$ $\quad 5 \times 10^5 \leq \text{Re} \leq 10^7$

Average friction coefficient when the flow is *laminar* over the entire plate: $C_f = \dfrac{1.33}{\text{Re}_L^{1/2}}$ $\quad \text{Re}_x < 5 \times 10^5$

Average friction coefficient when the flow is *turbulent* over the entire plate: $C_f = \dfrac{0.074}{\text{Re}_L^{1/5}}$ $\quad 5 \times 10^5 \leq \text{Re} \leq 10^7$

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Parallel Flow Over Flat Plates

- If the plate is long enough for the flow to be turbulent, but not long enough to disregard the laminar region, then

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x\ \text{laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,x\ \text{turbulent}}\,dx\right)$$

- Using $\text{Re}_{cr} = 5 \times 10^5$, the average friction coefficient over the entire plate is:

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L^{1/5}} \qquad 5 \times 10^5 \leq \text{Re} \leq 10^7$$

Remarks

- The above equation varies depending on the value of Re_{cr}
- The surface assumes to be smooth, and fee stream to be turbulent free
  - For laminar, C_f depends only on Re and is independent on surface roughness
  - For turbulent, C_f depends only on Re and the surface roughness:

$$C_f = \left(1.89 - 1.62\log\frac{\varepsilon}{L}\right)^{-2.5} \qquad \text{Re} > 10^6,\ \varepsilon/L > 10^{-4}$$

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Heat Transfer Coefficient

- Based on the analysis presented in the previous Chapter for isothermal flat plate:

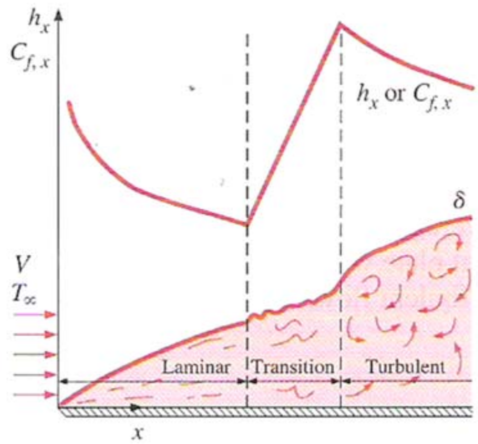
Laminar: $\mathrm{Nu}_x = \dfrac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$ $\quad \mathrm{Pr} > 0.6$

Turbulent: $\mathrm{Nu}_x = \dfrac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$, $\quad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$

$\Rightarrow$ h_x depends on $\mathrm{Re}_x^{0.5}$ for laminar and on $\mathrm{Re}_x^{0.8}$ for turbulent

$\Rightarrow$ h_x is proportional to $x^{-0.5}$ for laminar and to $x^{-0.2}$ for turbulent

$\Rightarrow$ h_x is infinite at the leading edge and is higher for turbulent than that of laminar



Heat Transfer Coefficient

- Average heat transfer coefficient when the flow is *laminar* over the entire isothermal plate:

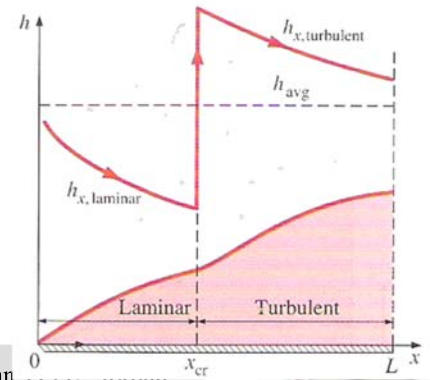
$$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re} < 5 \times 10^5\ ,\ 60 \le \mathrm{Pr} \qquad \mathrm{Re}_L = \frac{\rho u_\infty L}{\mu}$$

- Average heat transfer coefficient when the flow is *turbulent* over the entire isothermal plate:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad 5 \times 10^5 \le \mathrm{Re} \le 10^7,\quad 0.6 \le \mathrm{Pr} \le 60$$

- If the plate is long enough for the flow to be turbulent, but not long enough to disregard the laminar region, then

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x\ \text{laminar}}\,dx + \int_{x_{cr}}^{L} h_{x\ \text{turbulent}}\,dx\right)$$



Heat Transfer Coefficient

- The average Nusselt over the entire plate is

$$\overline{Nu}_L = (0.037\, Re_L^{4/5} - A)\, Pr^{1/3} \quad \begin{bmatrix} 0.6 \lesssim Pr \lesssim 60 \\ Re_{x,c} \lesssim Re_L \lesssim 10^8 \end{bmatrix}$$

Where $A = 0.037\, Re_{x,c}^{4/5} - 0.664\, Re_{x,c}^{1/2}$

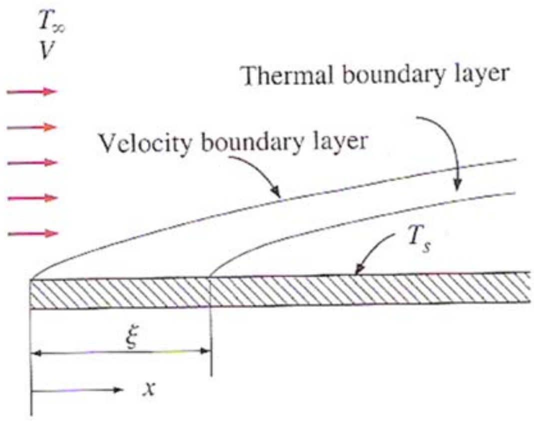
- Using $Re_{cr} = 5 \times 10^5$, the average heat transfer coefficient over the entire plate is :

$$Nu = \frac{hL}{k} = \left(0.037\, Re_L^{0.8} - 871\right) Pr^{1/3} \qquad 5 \times 10^5 \le Re \le 10^7\ ,\ 0.6 \le Pr \le 60$$

The equation varies depending on the value of Re_{cr}

Parallel Flow Over Flat Plates

- Surface thermal conditions are commonly idealized as being of uniform temperature or uniform heat flux .
- Thermal boundary layer development may be delayed by an unheated starting length.
- Many practical applications involve surfaces with an unheated starting sections of length ξ. This is illustrated in the drawing.



Flat Plate with Unheated Starting Length

- For such cases, and using the integral solution method, the local Nu numbers for both Laminar and turbulent flows are determined as.

Laminar:

$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for}\,\xi=0)}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}}$$

Turbulent:

$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for}\,\xi=0)}}{\left[1-(\xi/x)^{9/10}\right]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{\left[1-(\xi/x)^{9/10}\right]^{1/9}}$$

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Flat Plate with Unheated Starting Length

- Average Nu numbers requires integration of local Nu which can not be determined analytically.
- This has been done using numerical integration and been correlated as:

Laminar:

$$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$$

Turbulent:

$$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\,h_{x=L}$$

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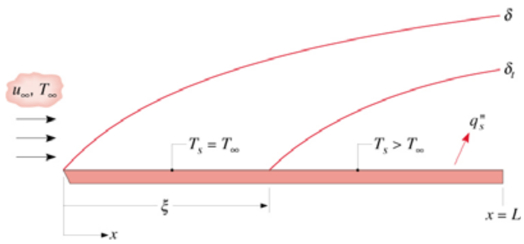

- For both uniform surface temperature (UST) and uniform surface heat flux (USF), the effect of the USL on the local Nusselt number may be represented as follows:

$$Nu_x = \frac{Nu_x|_{\xi=0}}{\left[1-(\xi/x)^a\right]^b}$$

$$Nu_x|_{\xi=0} = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^{1/3}$$

	Laminar		Turbulent	
	UST	USF	UST	USF
a	3/4	3/4	9/10	9/10
b	1/3	1/3	1/9	1/9
C	0.332	0.453	0.0296	0.0308
m	1/2	1/2	4/5	4/5

- For a plate of total length L, with laminar *or* turbulent flow over the entire surface, the expressions for **uniform plate temperature** (UST) with an unheated starting length are of the form

$$\overline{Nu}_L = \overline{Nu}_L|_{\xi=0}\frac{L}{L-\xi}\left[1-(\xi/L)^{(p+1)/(p+2)}\right]^{p/(p+1)}$$

where $p = 2$ for laminar flow

$p = 8$ for turbulent flow.

For Constant Heat Flux Conditions (USF)

$$T_s(x) = T_\infty + \frac{q_s''}{h_x} \qquad \text{and} \qquad q = q_s'' A_s$$

- Note: Properties are evaluated at the film temperature.

Constant Heat Flux Conditions (USF)

- Also $\mathrm{Nu}_x = \dfrac{hx}{k}$ which may be expressed in terms of the wall heat flux and temperature difference

Hence

$$\mathrm{Nu}_x = \frac{q_s'' x}{k(T_s - T_\infty)}$$

- The average temperature difference along the plate, for the constant-heat-flux condition, may be obtained by performing the integration

$$\overline{(T_s - T_\infty)} = \frac{1}{L}\int_0^L (T_s - T_\infty)dx = \frac{q_s''}{L}\int_0^L \frac{x}{k\,Nu_x}dx$$

$$\Rightarrow \quad \overline{(T_s - T_\infty)} = \frac{q_s'' L}{k\overline{Nu}_L}$$

where Nu_x is obtained from the appropriate convection correlation.

Other Relations

Liquid Metals

- Fluids as *liquid metals* usually have small Prandtl number.
- However, for this case the thermal boundary layer development is much more rapid than that of the velocity boundary layer

$$\Rightarrow \quad (\delta_t \gg \delta),$$

It is reasonable to assume uniform velocity throughout the thermal boundary layer, i.e. $u = u_\infty$

$$\Rightarrow \quad Nu_x = 0.564\, Pe_x^{1/2} \qquad Pr \lesssim 0.05, \quad Pe_x \gtrsim 100$$

where $Pe_x \equiv Re_x Pr$ is the *Peclet number*

Peclet number (Pe_L)	$\dfrac{VL}{\alpha} = Re_L\, Pr$	Ratio of advection to conduction heat transfer rates

Other Relations

- For laminar flow over an isothermal plate, the local convection coefficient may be obtained from A single correlation, which **applies for all Prandtl numbers**

$$Nu_x = \frac{0.3387\, Re_x^{1/2}\, Pr^{1/3}}{\left[1+\left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}} \quad \text{for } Re_x\, Pr > 100$$

Churchill and Ozoe correlation

with $\overline{Nu}_x = 2Nu_x$.

For the constant-heat-flux case, 0.3387 is changed to 0.4637 and 0.0468 is changed to 0.0207.

- **Note: Properties are evaluated at the film temperature.**

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Summary of equations for flow over flat plates

Flow regime	Restrictions	Equation
		Heat transfer
Laminar, local	$T_w = \text{const}$, $Re_x < 5\times10^5$, $0.6 < Pr < 50$	$Nu_x = 0.332\, Pr^{1/3} Re_x^{1/2}$
Laminar, local	$T_w = \text{const}$, $Re_x < 5\times10^5$, $Re_x\, Pr > 100$	$Nu_x = \frac{0.3387\, Re_x^{1/2}\, Pr^{1/3}}{\left[1+\left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}}$
Laminar, local	$q_w = \text{const}$, $Re_x < 5\times10^5$, $0.6 < Pr < 50$	$Nu_x = 0.453\, Re_x^{1/2}\, Pr^{1/3}$
Laminar, local	$q_w = \text{const}$, $Re_x < 5\times10^5$	$Nu_x = \frac{0.4637\, Re_x^{1/2}\, Pr^{1/3}}{\left[1+\left(\frac{0.0207}{Pr}\right)^{2/3}\right]^{1/4}}$
Laminar, average	$Re_L < 5\times10^5$, $T_w = \text{const}$	$\overline{Nu}_L = 2\, Nu_{x=L} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$
Laminar, local	$T_w = \text{const}$, $Re_x < 5\times10^5$, $Pr \ll 1$ (liquid metals)	$Nu_x = 0.564 (Re_x\, Pr)^{1/2}$
Laminar, local	$T_w = \text{const}$, starting at $x = x_0$, $Re_x < 5\times10^5$, $0.6 < Pr < 50$	$Nu_x = 0.332\, Pr^{1/3}\, Re_x^{1/2} \left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$

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Summary of equations for flow over flat plates

Flow regime	Restrictions	Equation
		Heat transfer
Turbulent, local	$T_w = \text{const}$, $5 \times 10^5 < Re_x < 10^7$	$St_x\, Pr^{2/3} = 0.0296\, Re_x^{-0.2}$
Turbulent, local	$T_w = \text{const}$, $10^7 < Re_x < 10^9$	$St_x\, Pr^{2/3} = 0.185(\log Re_x)^{-2.584}$
Turbulent, local	$q_w = \text{const}$, $5 \times 10^5 < Re_x < 10^7$	$Nu_x = 1.04\, Nu_{x T_w=\text{const}}$
Laminar-turbulent, average	$T_w = \text{const}$, $Re_x < 10^7$, $Re_{crit} = 5 \times 10^5$	$\overline{St}\, Pr^{2/3} = 0.037\, Re_L^{-0.2} - 871\, Re_L^{-1}$ $\overline{Nu}_L = Pr^{1/3}(0.037\, Re_L^{0.8} - 871)$
Laminar-turbulent, average	$T_w = \text{const}$, $Re_x < 10^7$, liquids, μ at T_∞, μ_w at T_w	$\overline{Nu}_L = 0.036\, Pr^{0.43}(Re_L^{0.8} - 9200)\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$
High-speed flow	$T_w = \text{const}$, $q = hA(T_w - T_{aw})$ $r = (T_{aw} - T_\infty)/(T_o - T_\infty)$ $=$ recovery factor $= Pr^{1/2}$ (laminar) $= Pr^{1/3}$ (turbulent)	Same as for low-speed flow with properties evaluated at $T^* = T_\infty + 0.5(T_w - T_\infty) + 0.22(T_{aw} - T_\infty)$

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Summary of equations for flow over flat plates

Flow regime	Restrictions	Equation
		Boundary-layer thickness
Laminar	$Re_x < 5 \times 10^5$	$\frac{\delta}{x} = 5.0\, Re_x^{-1/2}$
Turbulent	$Re_x < 10^7$, $\delta = 0$ at $x = 0$	$\frac{\delta}{x} = 0.381\, Re_x^{-1/5}$
Turbulent	$5 \times 10^5 < Re_x < 10^7$, $Re_{crit} = 5 \times 10^5$, $\delta = \delta_{lam}$ at Re_{crit}	$\frac{\delta}{x} = 0.381\, Re_x^{-1/5} - 10{,}256\, Re_x^{-1}$
		Friction coefficients
Laminar, local	$Re_x < 5 \times 10^5$	$C_{fx} = 0.332\, Re_x^{-1/2}$
Turbulent, local	$5 \times 10^5 < Re_x < 10^7$	$C_{fx} = 0.0592\, Re_x^{-1/5}$
Turbulent, local	$10^7 < Re_x < 10^9$	$C_{fx} = 0.37(\log Re_x)^{-2.584}$
Turbulent, average	$Re_{crit} < Re_x < 10^9$	$\overline{C}_f = \frac{0.455}{(\log Re_L)^{2.584}} - \frac{A}{Re_L}$ A from Table 5-1

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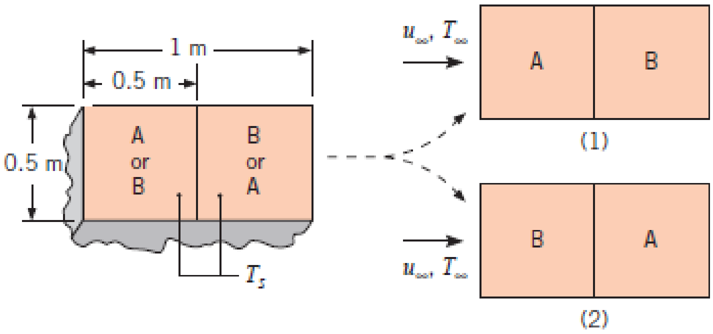
Example

7.21 The top surface of a heated compartment consists of very smooth (A) and highly roughened (B) portions, and the surface is placed in an atmospheric airstream.

In the interest of minimizing total convection heat transfer from the surface, which orientation, (1) or (2), is preferred? If $T_s = 100°C$, $T_\infty = 20°C$, and $u_\infty = 20$ m/s, what is the convection heat transfer from the entire surface for this orientation?



Example cont.

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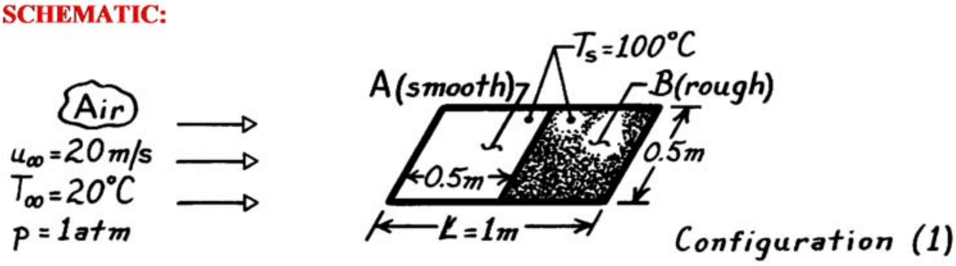
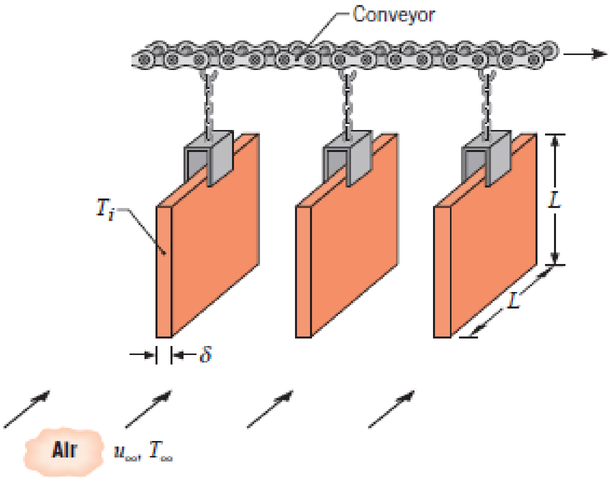


Example cont.

Example

7.24 Steel (AISI 1010) plates of thickness $\delta = 6$ mm and length $L = 1$ m on a side are conveyed from a heat treatment process and are concurrently cooled by atmospheric air of velocity $u_\infty = 10$ m/s and $T_\infty = 20°C$ in parallel flow over the plates.



For an initial plate temperature of $T_i = 300°C$, what is the rate of heat transfer from the plate? What is the corresponding rate of change of the plate temperature? The velocity of the air is much larger than that of the plate.

Example cont.

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Example cont.

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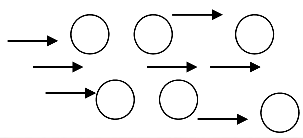
Example cont.

Flow across cylinder and spheres

- Flow across cylinder, rather than inside, has important practical applications
- Boundary-layer developed on the cylinder determines heat transfer characteristics.
- Conditions depend on special features of boundary layer development, including onset at a stagnation point and separation, as well as transition to turbulence.



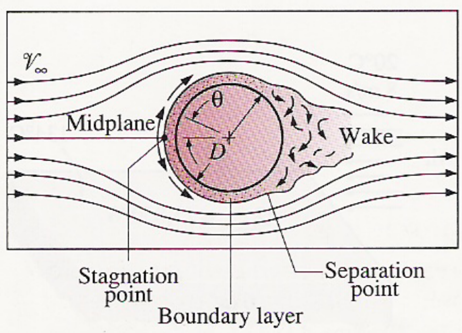
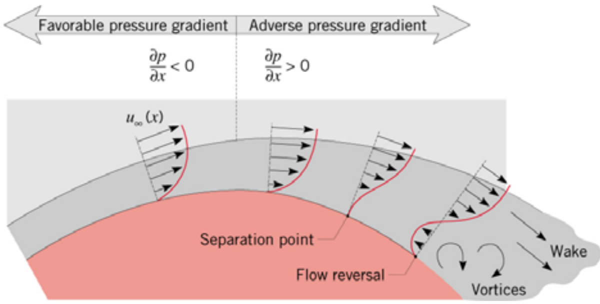
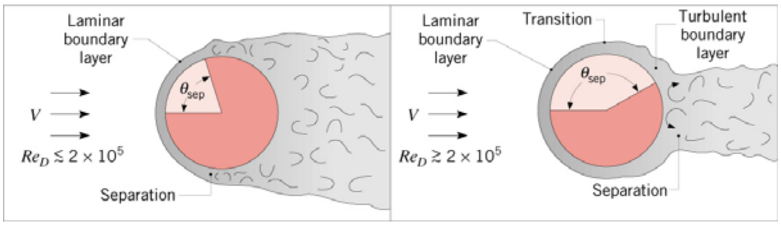


FIGURE 6-18
Typical flow patterns in cross flow over a cylinder.

Flow across cylinder and spheres



- Location of separation depends on boundary layer transition.



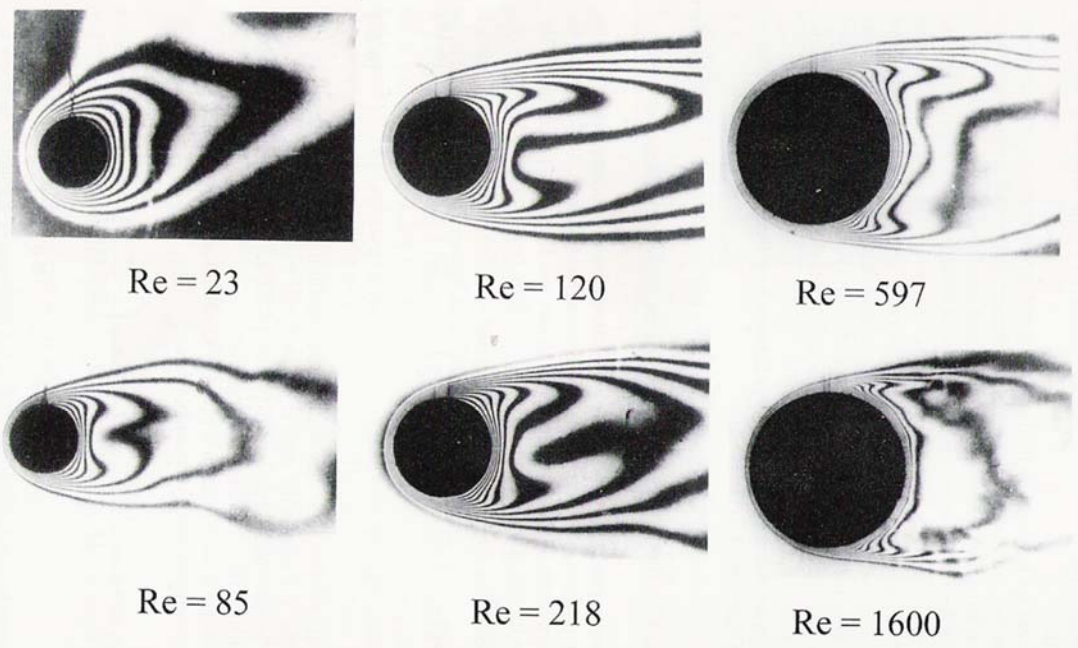
$$Re_D \equiv \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

Flow across cylinder and spheres

- The principal difference in flow over cylinder or a sphere, compared to the flow over a flat plate, is that the boundary layer in flow over cylinder or sphere may not undergo transition from laminar to turbulent flow, but also usually somewhere in the rear from the interface between the object and the fluid.
- The reason for this separation is the increasing pressure in the direction of flow, which causes a separated flow region to develop on the back of the cylinder or a sphere if the free-stream velocity is sufficiently large.
- The development of a separated flow region in flow over a cylinder is shown in the drawing in the previous slide.
- Obviously, regions in which the boundary layer has separated from the surface will exhibit considerably different Nusselt number characteristics from the region where the boundary layer is attached.

Flow across cylinder and spheres

Figure 6-13 | Interferometer photograph showing isotherms around heated horizontal cylinders placed in a transverse airstream. Re = $\rho u_\infty d/\mu$ (*Photograph courtesy E. Soehngen.*)



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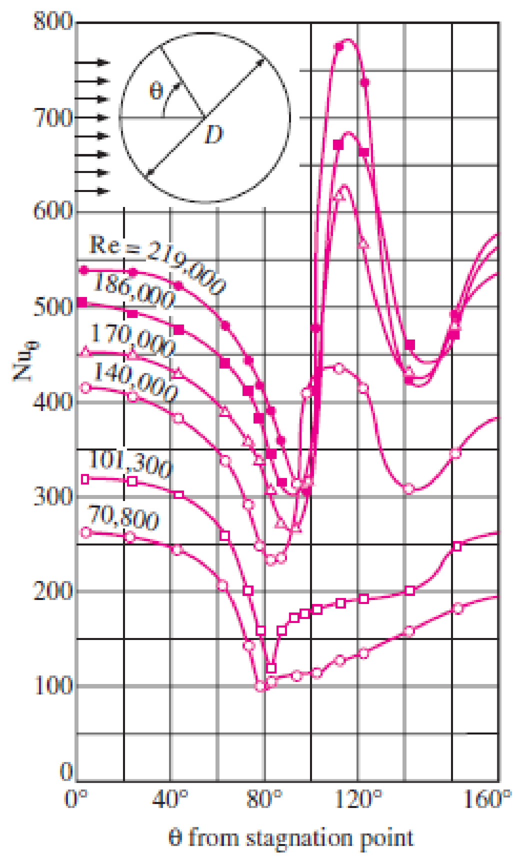
Flow across cylinder and spheres

- The local Nusselt number is given by

$$\mathrm{Nu}_\theta = \frac{h_{c,\theta}\theta}{k}$$

θ: angular distance from the stagnation point

- Note that: $\mathrm{Nu}_\theta \downarrow$ with θ
- But in the back of the cylinder, the flow is separated and Nu $\uparrow$ again



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Nusselt Number Correlations

- In normal engineering practice, it is not necessary to evaluate a local value of Nu; average values would be useful
- The average Nu can be related to the free-stream Reynold number, $\rho u_\infty d/\mu$, and Pr number, by an empirical correlation:
- For gases and ordinary liquids, the following correlation can be used:

$$\overline{Nu}_d = \frac{\overline{h}d}{k_f} = C\left(\frac{u_\infty d}{\nu_f}\right)^m \mathrm{Pr}^{1/3}$$

For $Pr \gtrsim 0.7$,

Table 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

- Subscript for properties at $T_f = \dfrac{T_w + T_\infty}{2}$

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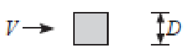
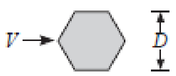
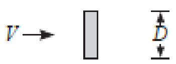
Nusselt Number Correlations

Table 7.3 Constants of Equation 7.52 for noncircular cylinders in cross flow of a gas [13, 14][a]

Geometry		Re_D	C	m
Square				
		6000–60,000	0.304	0.59
		5000–60,000	0.158	0.66
Hexagon				
		5200–20,400	0.164	0.638
		20,400–105,000	0.039	0.78
		4500–90,700	0.150	0.638
Thin plate perpendicular to flow				
	Front	10,000–50,000	0.667	0.500
	Back	7000–80,000	0.191	0.667

[a]These tabular values are based on the recommendations of Sparrow et al. [14] for air, with extension to other fluids through the $Pr^{1/3}$ dependence of Equation 7.52. A Prandtl number of $Pr = 0.7$ was assumed for the experimental results for air that are described in [14].

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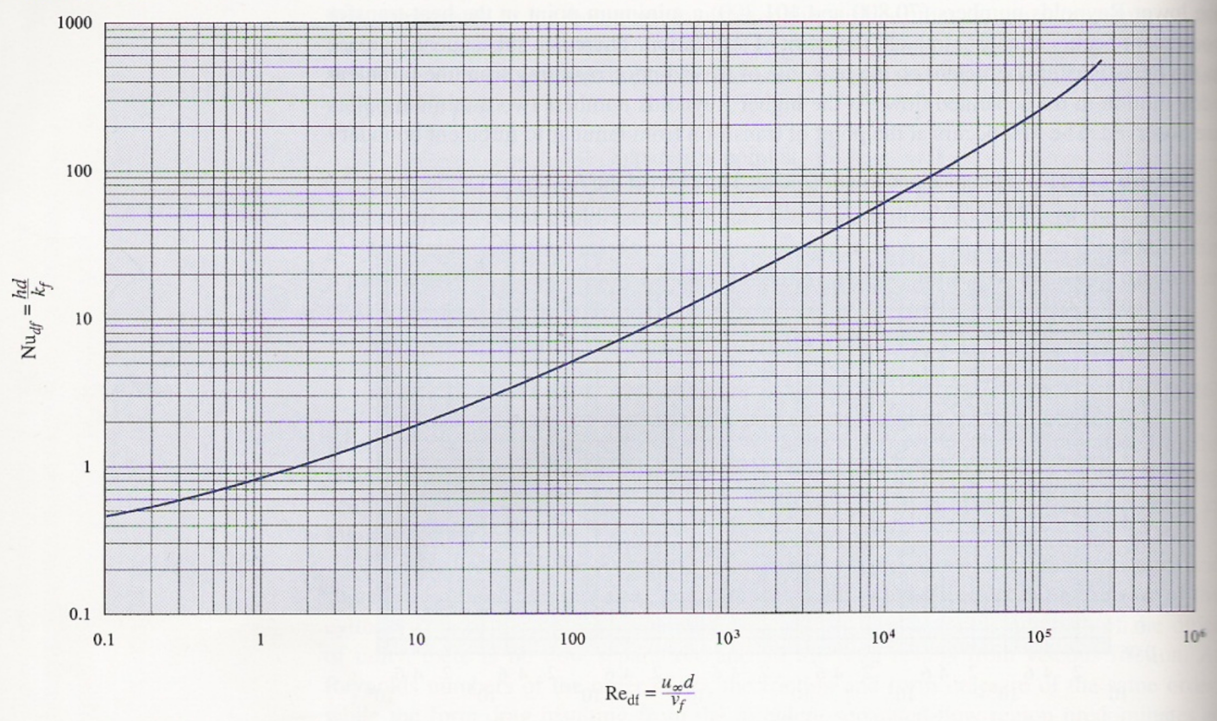
Nusselt Number Correlations

Figure 6-12 | Correlation for heating and cooling of air in cross flow over circular cylinders.



Nusselt Number Correlations

- It has also been shown that *h* from liquids to cylinders in cross flow may be better represented:

$$Nu_f = \left(0.35 + 0.56\, Re_f^{0.52}\right) Pr^{0.3}$$

• $10^{-1} < Re_f < 10^5$

- The following more complicated correlations are also suggested for heat transfer from tubes in cross flow:

$$Nu = \left(0.43 + 0.5\, Re^{0.52}\right) Pr^{0.38} \left(\frac{Pr_f}{Pr_w}\right)^{0.25}$$

• $1 < Re < 10^3$

$$Nu = 0.25\, Re^{0.6}\, Pr^{0.38} \left(\frac{Pr_f}{Pr_w}\right)^{0.25}$$

• $10^3 < Re < 10^5$

Nusselt Number Correlations

- For gases Pr_f/Pr_w may be dropped
- Pr_f evaluated at T_f and Pr_w evaluated at T_w.
- For liquids, Pr_f/Pr_w is retained and fluid properties are evaluated at the free stream temperature, T_∞

➢ The following relation is more comprehensive and is applicable over a complete range of available data:

$$Nu_d = 0.3 + \frac{0.62\,Re^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

for $10^2 < Re < 10^7$

$Pr > 0.2$

Not good for mid-range of Re

Nusselt Number Correlations

$$Nu_d = 0.3 + \frac{0.62\,Re^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re}{282000}\right)^{1/2}\right]$$

for $2\times10^4 < Re < 4\times10^5$

$Pr > 0.2$

For mid-range of Re

➢ Another relation that can be used:

$$Nu_d = \frac{\bar{h}d}{k} = \left(0.4\,Re^{0.5} + 0.06\,Re^{2/3}\right)Pr^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{0.25}$$

for $40 < Re < 10^5$

$0.65 < Pr < 300$

$0.25 < \mu_\infty/\mu_w < 5.2$

Properties at T_∞

➢ For $Pe < 0.2$, use the following correlation:

$$Nu_d = \left[0.8237 - \ln(Pe)^{1/2}\right]^{-1}$$

Nusselt Number Correlations

- For the circular cylinder in cross flow, Zukauskas correlation can be used

$$\overline{Nu}_D = C\, Re_D^m\, Pr^n \left(\frac{Pr}{Pr_s}\right)^{1/4} \qquad \text{If } Pr \lesssim 10,\ n = 0.37;$$

$$\left[\begin{array}{l} 0.7 \lesssim Pr \lesssim 500 \\ 1 \lesssim Re_D \lesssim 10^6 \end{array}\right] \qquad \text{if } Pr \gtrsim 10,\ n = 0.36.$$

where all properties are evaluated at T_∞, except Pr_s, which is evaluated at T_s.

TABLE 7.4 Constants of Equation 7.53 for the circular cylinder in cross flow [17]

Re_D	C	m
1–40	0.75	0.4
40–1000	0.51	0.5
10^3–2×10^5	0.26	0.6
2×10^5–10^6	0.076	0.7

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Noncircular Cylinder

- For non-circular cylinders, the equation

$$Nu = C\, Re_f^n\, Pr^{1/3}$$

can be used.

- *C* and *n* are given in the table shown for different geometries

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zhukauskas, Ref. 18, and Jakob, Ref. 8)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330} Pr^{1/3}$ $Nu = 0.911Re^{0.385} Pr^{1/3}$ $Nu = 0.683Re^{0.466} Pr^{1/3}$ $Nu = 0.193Re^{0.618} Pr^{1/3}$ $Nu = 0.027Re^{0.805} Pr^{1/3}$
Square	Gas	5000–100,000	$Nu = 0.102Re^{0.675} Pr^{1/3}$
Square (tilted 45°)	Gas	5000–100,000	$Nu = 0.246Re^{0.588} Pr^{1/3}$
Hexagon	Gas	5000–100,000	$Nu = 0.153Re^{0.638} Pr^{1/3}$
Hexagon (tilted 45°)	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638} Pr^{1/3}$ $Nu = 0.0385Re^{0.782} Pr^{1/3}$
Vertical plate	Gas	4000–15,000	$Nu = 0.228Re^{0.731} Pr^{1/3}$
Ellipse	Gas	2500–15,000	$Nu = 0.248Re^{0.612} Pr^{1/3}$

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Spheres

- For heat transfer from sphere to a flowing gas, use:

$$\frac{hd}{k_f} = 0.37\left(\frac{u_\infty d}{\nu_f}\right)^{0.6}$$

- For air (Pr = 0.71) over a wide range of Re, use:

$$Nu = 2 + \left(0.25 + 3\times10^{-4}\,Re^{1.6}\right)^{1/2} \qquad 100 < Re < 3\times10^5$$

$$Nu = 430 + a\,Re + b\,Re^2 + c\,Re^3 \qquad 3\times10^5 < Re < 5\times10^6$$

$a = 5\times10^{-3}$ $\quad b = 0.25\times10^{-9}$ $\quad c = -3.1\times10^{-17}$

- For flow of liquids past spheres:

$$\frac{hd}{k_f}Pr_f^{-0.3} = 0.97 + 0.68\left(\frac{u_\infty d}{\nu_f}\right)^{0.5} \qquad 1 < Re < 2000$$

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Spheres

- Heat transfer from sphere to oil and water:

$$Nu\,Pr_f^{-0.3}\left(\frac{\mu_w}{\mu}\right)^{0.25} = 1.2 + 0.53\,Re^{0.54}$$

All properties are evaluated at the free stream temperature (T_∞) except μ_w.
for $1 < Re_d < 2\times10^5$; $0.7 < Pr < 380$

- All above equations can also be evaluated from the following single equation for gases and liquids past a sphere:

$$Nu = 2.0 + \left(0.4\,Re_d^{1/2} + 0.06\,Re_d^{2/3}\right)\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$$

All properties at T_∞ except μ_w.
for $3.5 < Re_d < 8\times10^4$; $0.7 < Pr < 380$
$1.0 \lesssim (\mu/\mu_s) \lesssim 3.2$

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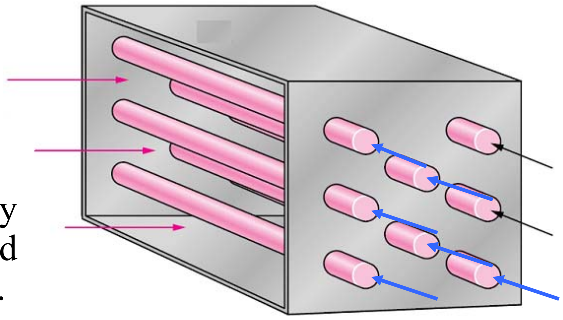
Flow Across Tube Bank

- Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such heat exchangers, air conditioners and refrigerators..

- In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.



- Flow *through* the tubes can be analyzed by considering flow through a single tube, and multiplying the results by the number of tubes.

- For flow *over* the tubes the tubes affect the flow pattern and turbulence level downstream, and thus heat transfer to or from them are altered.

- Typical arrangement
  - In-line arrangement
  - Staggered arrangement

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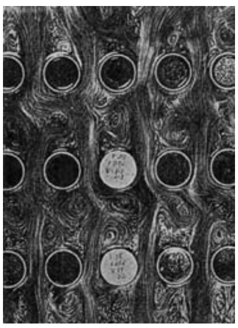
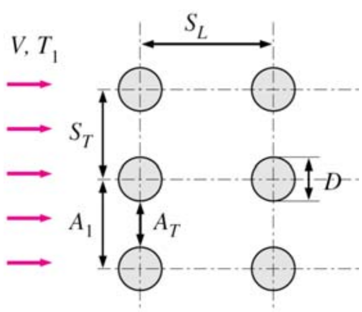
Flow Across Tube Bank

- The outer tube diameter *D* is the characteristic length.
- The arrangement of the tubes are characterized by the
  - *transverse pitch* S_T,
  - *longitudinal pitch* S_L, and the
  - *diagonal pitch* S_D between tube centers.

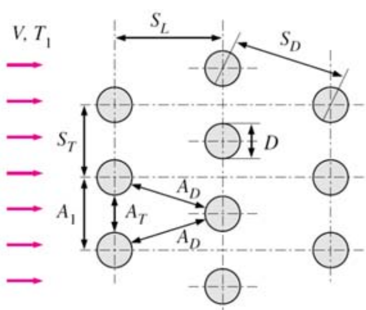
$$S_D = \sqrt{S_L^2 + (S_T/2)^2}$$

In-line





Staggered



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Flow Across Tube Bank

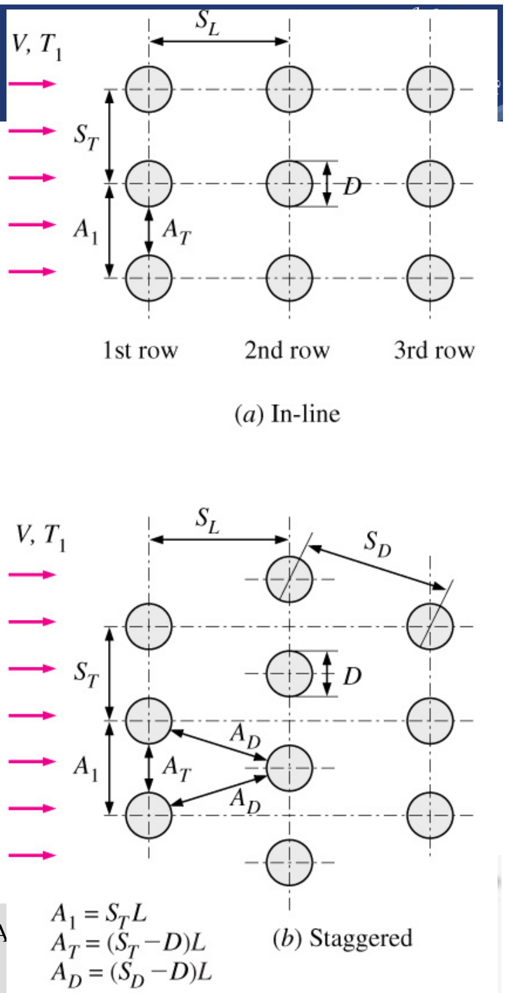
- As the fluid enters the tube bank, the flow area decreases from $A_1 = S_T L$ to A_T $(S_T - D)L$ between the tubes, and thus flow velocity increases.

  L = tube length.

- In tube banks, the flow characteristics are dominated by the maximum velocity V_{max} rather than the approach velocity V.

- The Reynolds number is defined on the basis of maximum velocity as

$$\mathrm{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{V_{\max} D}{\nu}$$

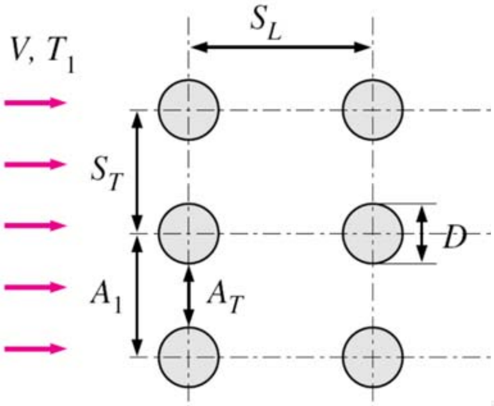


Flow Across Tube Bank

- For ***in-line*** arrangement, the maximum velocity occurs at the minimum flow area between the tubes (A_T).
- The conservation of mass:

$$\rho V A_1 = \rho V_{max} A_T \qquad or \qquad V S_T L = V_{max}\left(S_T - D\right) L$$

$$\Rightarrow \quad V_{\max} = \frac{S_T}{S_T - D} V$$



Flow Across Tube Bank

- In *staggered* arrangement:

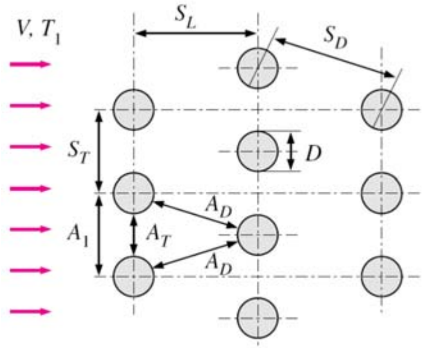
If $2A_D > A_T$ or $S_D > (S_T + D)/2$: V_{max} occurs at A_T:

$$V_{\max} = \frac{S_T}{S_T - D} V$$

If $2A_D < A_T$ or $S_D < (S_T + D)/2$: V_{max} occurs at the diagonal cross

$$\rho V A_1 = \rho V_{max} 2A_D$$

or $$V S_T L = V_{max} 2(S_D - D) L$$



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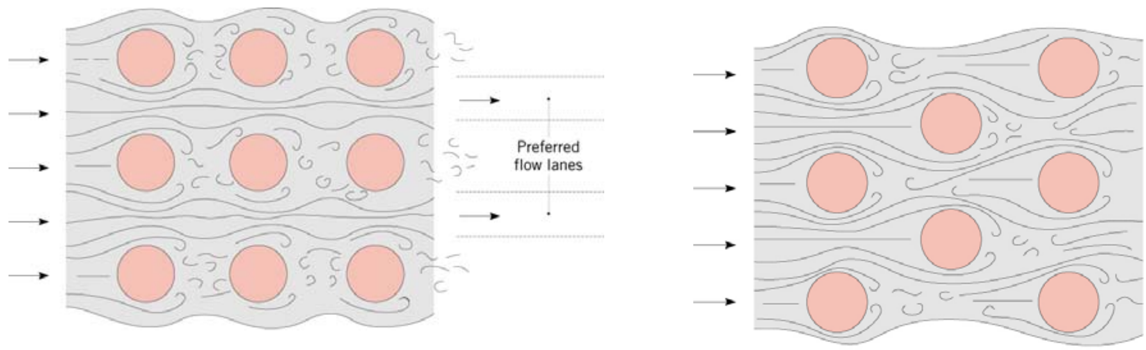
Flow Across Tube Bank



- The average Nusselt number is of the general form

$$Nu_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\,(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25} \qquad 0.7 < \mathrm{Pr} < 500$$

where the values of the constants *C*, *m*, and *n* depend on Reynolds number (Table 7–2).

- The average Nusselt number relations in Table 7–2 are for tube banks with 16 or more rows ($N_L > 16$).

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Flow Across Tube Bank

(Table 7–2). Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < Pr < 500$ (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\ Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\ Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.27\ Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.033\ Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\ Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\ Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.35 (S_T/S_L)^{0.2}\ Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.031 (S_T/S_L)^{0.2}\ Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

- Fluid properties are evaluated at: $T_m = (T_i - T_e)/2$

  except Pr_s at T_s. T_i and T_e are the fluid temperatures at the inlet and the exit of the tube bank.

Flow Across Tube Bank

- Those relations can also be used for tube banks with $N_L < 16$, provided that they are modified as

$$Nu_{D,N_L} = F \cdot Nu_D$$

- *The correction factor F* values are given in Table 7–3

TABLE 7–3

Correction factor F to be used in $Nu_{D,N_L} = F Nu_D$ for $N_L < 16$ and $Re_D > 1000$ (from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

Flow Across Tube Bank

- The heat transfer rate to or from a tube bank is:

$$\dot{Q} = hA_s \Delta T_{ln} = \dot{m} c_p (T_e - T_i)$$

A_s : the heat transfer surface area = $N\pi DL$

N: the number of tubes in the bank.

$$\dot{m} = \rho V (N_T S_T L)$$

N_T : the number of tubes in the transverse plane.

$\Delta T_{\ln}$: the logarithmic mean temperature difference:

$$\Delta T_{ln} = \frac{(T_s - T_e) - (T_s - T_i)}{ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{ln[\Delta T_e / \Delta T_i]}$$

- The exit temperature of the fluid is: $T_e = T_s - (T_s - T_i) exp\left(\frac{-A_s h}{\dot{m} c_p}\right)$

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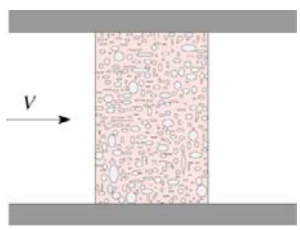
The Sphere and Packed Beds

Gas Flow through a Packed Bed



- Flow is characterized by tortuous paths through a bed of fixed particles.
- Large surface area per unit volume renders configuration desirable for the transfer and storage of thermal energy.
- The heat transfer coefficient can be calculated using

$$\varepsilon \overline{j}_H = 2.06 \, \text{Re}_D^{-0.575} \qquad 90 \le \text{Re}_D \le 4{,}000, \; \text{P} \approx 0.7$$

$\varepsilon \rightarrow$ void fraction ($0.3 < \varepsilon < 0.5$)

where $\overline{j}_H = \frac{\overline{h}}{\rho V c_p} \text{Pr}^{2/3}$

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The Sphere and Packed Beds

- The heat transfer coefficient for fluids as water, propylene glycol and water solutions) with moderate Pr (around 30), can be calculated from

$$\mathrm{Nu}_d = 1.27 + 2.66\,\mathrm{Re}_f^{0.56}\,\mathrm{Pr}_f^{-0.41}\left(\frac{1-\varepsilon}{\varepsilon}\right)^{0.29} \qquad 0 < \mathrm{Re} < 100$$

– $q = \bar{h} A_{p,t} \Delta T_{\ell m}$

$A_{p,t} \rightarrow$ total surface area of particles

– $\dfrac{T_s - T_o}{T_s - T_i} = \exp\left(-\dfrac{\bar{h} A_{p,t}}{\rho V A_{c,b} c_p}\right)$

$A_{c,b} \rightarrow$ cross-sectional area of bed

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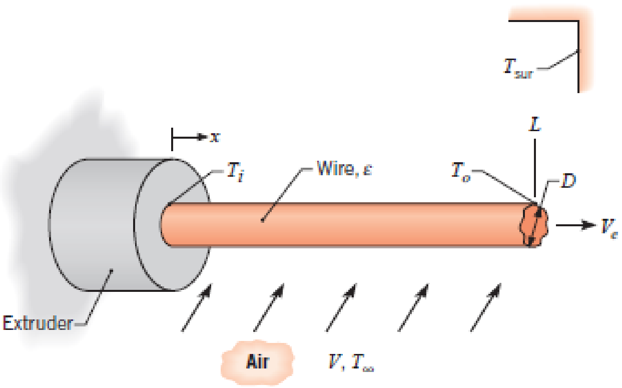
Example

7.72 In an extrusion process, copper wire emerges from the extruder at a velocity V_e and is cooled by convection heat transfer to air in cross flow over the wire, as well as by radiation to the surroundings.



(a) By applying conservation of energy to a differential control surface of length dx, which either moves with the wire *or* is stationary and through which the wire passes, derive a differential equation that governs the temperature distribution, $T(x)$, along the wire. In your derivation, the effect of axial conduction along the wire may be neglected. Express your result in terms of the velocity, diameter, and properties of the wire (V_e, D, ρ, c_p, ε), the convection coefficient associated with the cross flow ($\bar{h}$), and the environmental temperatures (T_∞, T_{sur}).

(b) Neglecting radiation, obtain a closed form solution to the foregoing equation. For $V_e = 0.2$ m/s, $D = 5$ mm, $V = 5$ m/s, $T_\infty = 25°$C, and an initial wire temperature of $T_i = 600°$C, compute the temperature T_o of the wire at $x = L = 5$ m. The density and specific heat of the copper are $\rho = 8900$ kg/m^3 and $c_p = 400$ J/kg·K, while properties of the air may be taken to be $k = 0.037$ W/m·K, $\nu = 3 \times 10^{-5}$ m^2/s, and $Pr = 0.69$.

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Example cont.

(c) Accounting for the effects of radiation, with $\varepsilon = 0.55$ and $T_{sur} = 25°C$, numerically integrate the differential equation derived in part (a) to determine the temperature of the wire at $L = 5$ m. Explore the effects of V_e and ε on the temperature distribution along the wire.

Example cont.

Example cont.

Example cont.

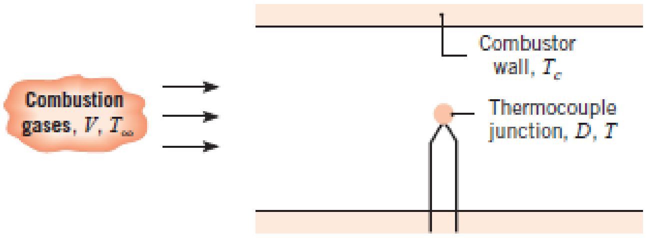
Hence, radiation makes a discernable contribution to cooling of the wire.

Example cont.

Example cont.

Example

7.85 A spherical thermocouple junction 1.0 mm in diameter is inserted in a combustion chamber to measure the temperature T_∞ of the products of combustion. The hot gases have a velocity of $V = 5$ m/s.



(a) If the thermocouple is at room temperature, T_i, when it is inserted in the chamber, estimate the time required for the temperature difference, $T_\infty - T$, to reach 2% of the initial temperature difference, $T_\infty - T_i$. Neglect radiation and conduction through the leads. Properties of the thermocouple junction are approximated as $k = 100$ W/m·K, $c = 385$ J/kg·K, and $\rho = 8920$ kg/m^3, while those of the combustion gases may be approximated as $k = 0.05$ W/m·K, $\nu = 50 \times 10^{-6}$ m^2/s, and $Pr = 0.69$.

(b) If the thermocouple junction has an emissivity of 0.5 and the cooled walls of the combustor are at $T_c = 400$ K, what is the steady-state temperature of the thermocouple junction if the combustion gases are at 1000 K? Conduction through the lead wires may be neglected.

(c) To determine the influence of the gas velocity on the thermocouple measurement error, compute the steady-state temperature of the thermocouple junction for velocities in the range $1 \le V \le 25$ m/s. The emissivity of the junction can be controlled through application of a thin coating. To reduce the measurement error, should the emissivity be increased or decreased? For $V = 5$ m/s, compute the steady-state junction temperature for emissivities in the range $0.1 \le \varepsilon \le 1.0$.

Example cont.

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Example cont.

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Example cont.