Process Heat Transfer

Lec 9- Internal Forced Convection

The Mean Velocity and Temperature, Hydrodynamic and Thermal Entry Lengths, Heat Transfer Correlations, Fully Developed Conditions, Determination of the Mean Temperature, The Concentric Tube Annulus

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Content

- The Mean Velocity and Temperature,
- Hydrodynamic and Thermal Entry Lengths,
- Heat Transfer Correlations,
- Fully Developed Conditions,
- Determination of the Mean Temperature,
- The Concentric Tube Annulus

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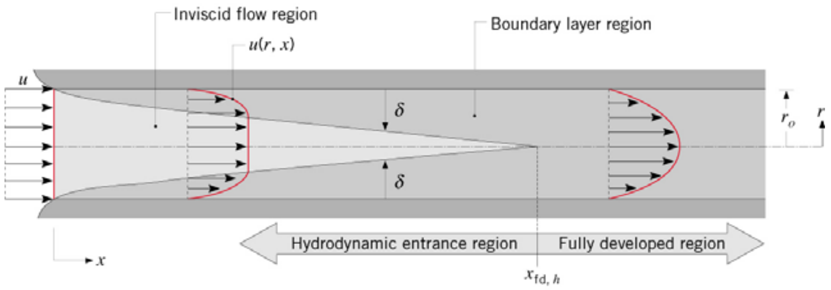
Flow Considerations in Pipe

- Must distinguish between entrance and fully developed regions.



– Velocity boundary layer develops on surface of tube and thickens with increasing *x*.

– Inviscid region of uniform velocity shrinks as boundary layer grows.

– Subsequent to boundary layer merger at the centerline, the velocity profile becomes parabolic and invariant with *x*.

⇨ The flow is then said to be hydrodynamically fully developed.

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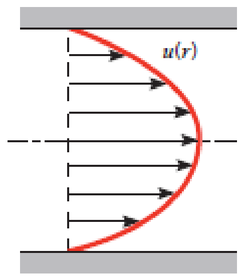
Flow Considerations in Pipe

- The velocity profile in the fully developed region is



$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

- The Reynolds number for flow in a circular tube is defined as

$$Re_D \equiv \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

where u_m is the mean fluid velocity over the tube cross section and D is the tube diameter.

- For steady, incompressible flow in a tube of uniform cross-sectional area $\dot{m}$ and u_m are constants and independent of x

$$Re_D = \frac{4\dot{m}}{\pi D \mu}$$

$$\dot{m} = \rho u_m A_c$$

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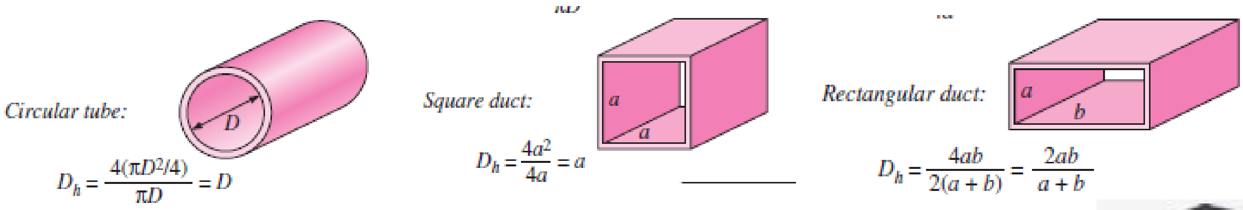
Flow Considerations in Pipe

- The Reynolds numbers for the different flow type are

$$\begin{aligned} Re &< 2300 && \text{laminar flow} \\ 2300 \le Re &\le 10{,}000 && \text{transitional flow} \\ Re &> 10{,}000 && \text{turbulent flow} \end{aligned}$$

- For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the **hydraulic diameter** D_h defined as

$$D_h = \frac{4A_c}{p}$$



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Flow Considerations in Pipe

For laminar flow ($Re_D \lesssim 2300$), the hydrodynamic entry length may be obtained from an expression of the form [1]

$$\left(\frac{x_{fd,h}}{D}\right)_{lam} \approx 0.05\, Re_D$$

- The entry length in turbulent flow, is approximately independent of Reynolds number and that, as a first approximation

$$10 \lesssim \left(\frac{x_{fd,h}}{D}\right)_{turb} \lesssim 60$$

or $$x_{fd,h\,turb} = 1.359\, Re^{1/4}$$

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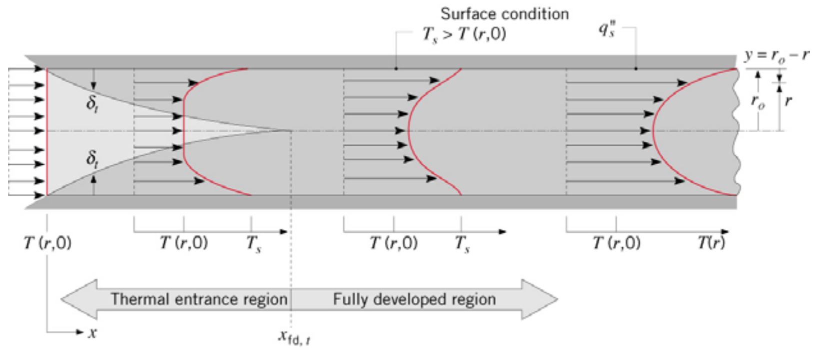
Thermal Considerations

- **Thermal Effects**: Assume laminar flow with uniform temperature, $T(r, 0) = T_i$ at inlet of circular tube with uniform temperature (T_s is constant) or a uniform heat flux (q''_s is constant),



  - Thermal boundary layer develops on surface of tube and thickens with increasing x.
  - Isothermal core shrinks as boundary layer grows.
  - Subsequent to boundary layer merger, dimensionless forms of the temperature profile for become independent of x.

⇨ Conditions are then said to be thermally fully developed.

Thermal Considerations

- For laminar flow the *thermal entry length* may be expressed as

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} \approx 0.05\, Re_D\, Pr$$

- Compared with the hydrodynamic entry length in laminar flow:

if $Pr > 1$, the hydrodynamic boundary layer develops more rapidly than the thermal boundary layer

- For turbulent flow, conditions are nearly independent of Prandtl number,

$$(x_{fd,t}/D) = 10$$

General Thermal Analysis

- Because the flow in a tube is completely enclosed,
  - An energy balance may be applied to determine how the mean temperature $T_m(x)$ (*mean* or *bulk temperature)* varies with position along the tube
  - The total convection heat transfer q_{conv} is related to the difference in temperatures at the tube inlet and outlet.

- Newton's law of cooling may be expressed as

$$q_s'' = h(T_s - T_m)$$

At the same time $$q_s'' = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = k\frac{\partial T}{\partial r}\bigg|_{r=r_o}$$

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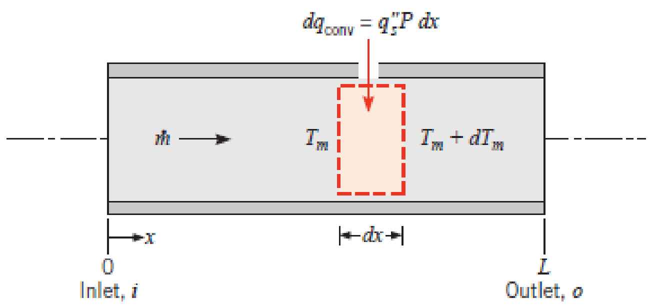
General Thermal Analysis

$$dq_{conv} = \dot{m}c_p[(T_m + dT_m) - T_m]$$

$$dq_{conv} = \dot{m}c_p dT_m$$



- Integrating from inlet to outlet

⇨ $$q_{conv} = \dot{m}c_p(T_{m,o} - T_{m,i})$$

- A differential equation from which $T_m(x)$ may be determined is obtained by substituting for

$$dq_{conv} = q_s''(P\,dx) = h(T_s - T_m)P\,dx. = \dot{m}c_p dT_m$$

where P is the surface perimeter ($P = \pi D$ for a circular tube).

⇨ $$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h(T_s - T_m)$$

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General Thermal Analysis

Special Case: **Uniform Surface Heat Flux**

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} \neq f(x) = \text{constant}$$

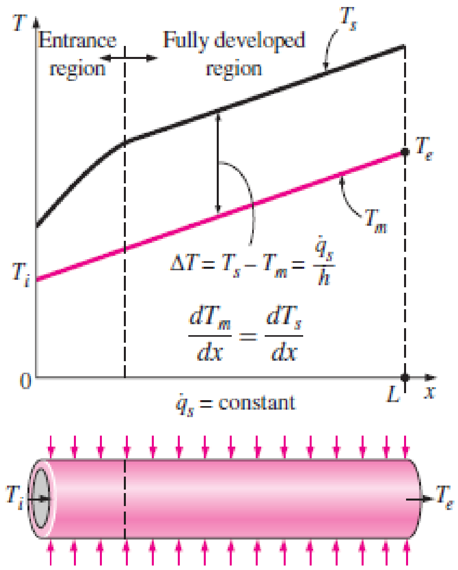
Integrating from $x = 0$,

⇨ $$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}c_p}x \qquad q_s'' = \text{constant}$$

⇨ the mean temperature varies *linearly* with *x* along the tube

Also

$$q_s'' = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{q_s''}{h}$$



$$\frac{dT_m}{dx} = \frac{dT_s}{dx} = \frac{q_s''P}{\dot{m}c_p}$$

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General Thermal Analysis

Special Case: **Uniform Surface Temperature**

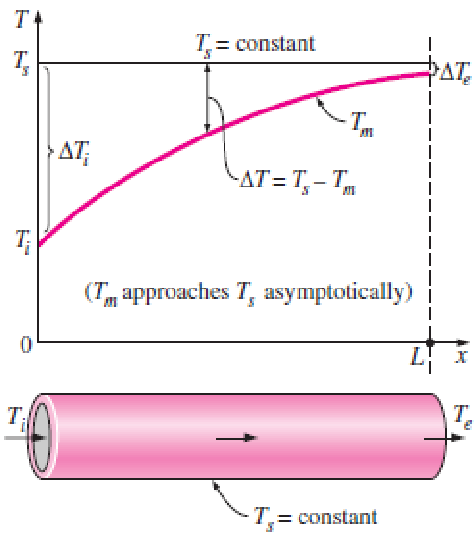
$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\, \Delta T$$

- Separating variables and integrating from the tube inlet to the outlet,

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m}c_p}\int_0^L h\, dx$$

⇨ $$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\left(\frac{1}{L}\int_0^L h\, dx\right)$$

⇨ $$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\bar{h}_L \qquad T_s = \text{constant}$$



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General Thermal Analysis

where $\bar{h}_L$, or simply $\bar{h}$, is the average value of h for the entire tube.

$$\Rightarrow \quad \frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right) \qquad T_s = \text{constant}$$

- Integrating from the tube inlet to some axial position x within the tube

$$\Rightarrow \quad \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right) \qquad T_s = \text{constant}$$

- Overall Conditions:

$$q_{conv} = \bar{h}A_s\Delta T_{lm} \qquad T_s = \text{constant} \qquad \text{where} \qquad \Delta T_{lm} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}$$

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General Thermal Analysis

Special Case: Uniform *External* Fluid Temperature

- In many applications, it is the temperature of an *external* fluid, rather than the tube surface temperature, that is fixed

$$\Rightarrow \quad \frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}c_p}\right) = \exp\left(-\frac{1}{\dot{m}c_p R_{tot}}\right)$$

$$\Rightarrow \quad q = \bar{U}A_s\,\Delta T_{\ell m} = \frac{\Delta T_{\ell m}}{R_{tot}}$$

$\Delta T_{\ell m} \rightarrow$ Eq. (3) with T_s replaced by T_∞.

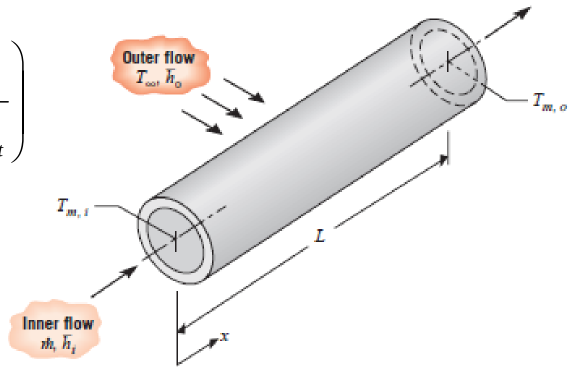

FIGURE 8.8 Heat transfer between fluid flowing over a tube and fluid passing through the tube.

Note: Replacement of T_∞ by $T_{s,o}$ if outer surface temperature is uniform.

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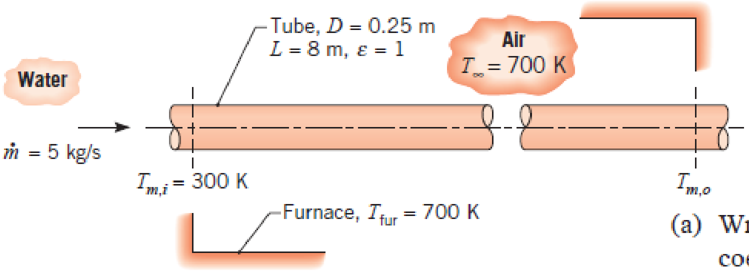
Example

8.20 Water at 300 K and a flow rate of 5 kg/s enters a black, thin-walled tube, which passes through a large furnace whose walls and air are at a temperature of 700 K. The diameter and length of the tube are 0.25 m and 8 m, respectively. Convection coefficients associated with water flow through the tube and airflow over the tube are 300 W/m$^2\cdot$K and 50 W/m$^2\cdot$K, respectively.



(a) Write an expression for the linearized radiation coefficient corresponding to radiation exchange between the outer surface of the pipe and the furnace walls. Explain how to calculate this coefficient if the surface temperature of the tube is represented by the arithmetic mean of its inlet and outlet values.

(b) Determine the outlet temperature of the water, $T_{m,o}$.

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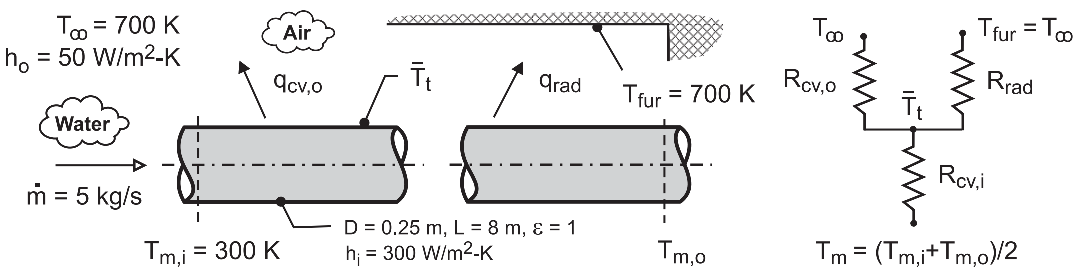
Example cont.



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Example cont.

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Example cont.

Fully Developed Flow

- **Laminar Flow in a Circular Tube:**

➢ The local Nusselt number is a constant throughout the fully developed region, but its value depends on the surface thermal condition.

– Uniform Surface Heat Flux (q_s''):

$$Nu_D = \frac{hD}{k} = 4.36$$

– Uniform Surface Temperature (T_s):

$$Nu_D = \frac{hD}{k} = 3.66$$

- **Turbulent Flow in a Circular Tube:**

➢ For a smooth surface and fully turbulent conditions $(\mathrm{Re}_D > 10{,}000)$ the Dittus – Boelter equation may be used as a first approximation:

$$Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n \quad \begin{cases} n = 0.3 & (T_s < T_m) \\ n = 0.4 & (T_s > T_m) \end{cases}$$

$$\begin{bmatrix} 0.6 \lesssim Pr \lesssim 160 \\ Re_D \gtrsim 10{,}000 \\ \frac{L}{D} \gtrsim 10 \end{bmatrix}$$

Fully Developed Flow

– The effects of wall roughness and transitional flow conditions $(\mathrm{Re}_D > 3000)$ may be considered by using the Gnielinski correlation:

$$Nu_D = \frac{(f/8)(\mathrm{Re}_D - 1000)\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)}$$

for $0.5 \lesssim Pr \lesssim 2000$ and $3000 \lesssim Re_D \lesssim 5 \times 10^6$

Smooth surface: $f = (0.790 \ln \mathrm{Re}_D - 1.64)^{-2}$

Surface of roughness $e > 0$: $f \rightarrow$ Figure 8.3

Fully Developed Flow

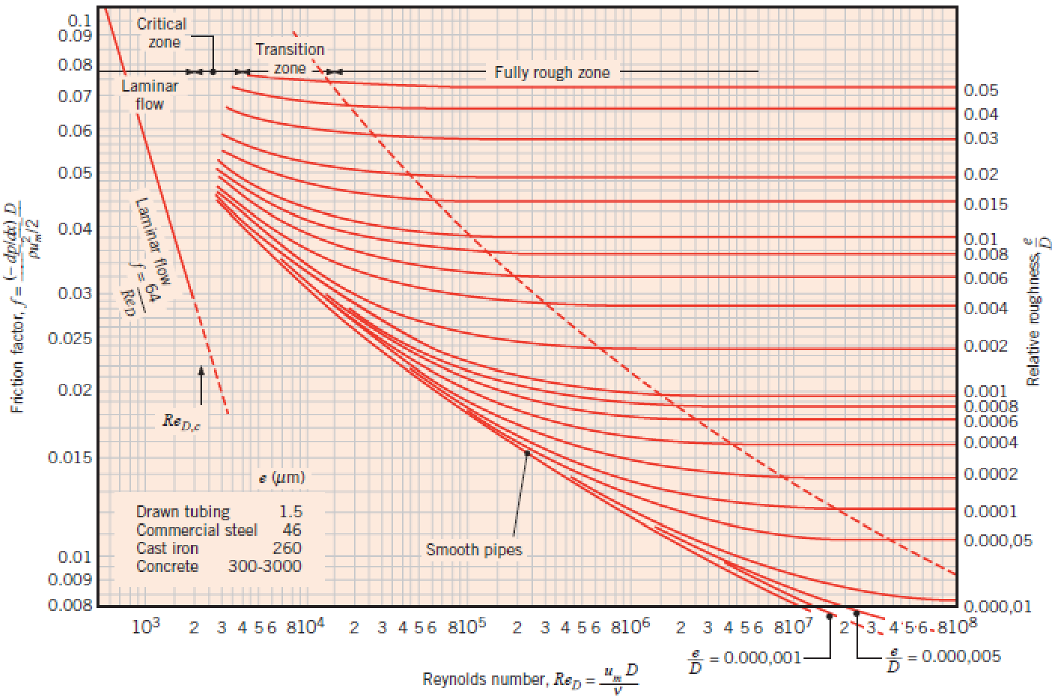


FIGURE 8.3 Friction factor for fully developed flow in a circular tube [6]. Used with permission.

Noncircular Tubes:

➢ Use of hydraulic diameter as characteristic length:

$$D_h \equiv \frac{4A_c}{P}$$

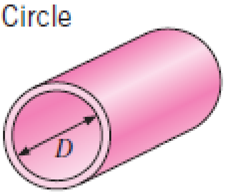
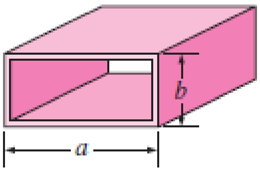
➢ Since the local convection coefficient varies around the periphery of a tube, approaching zero at its corners, correlations for the fully developed region are associated with convection coefficients averaged over the periphery of the tube.

o **Laminar Flow**:

➢ The local Nusselt number is a constant whose value (Table 8.1) depends on the surface thermal condition $(T_s \ or \ q_s'')$ and the duct aspect ratio.

Noncircular Tubes:

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = u_m D_h/\nu$, and $Nu = hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re

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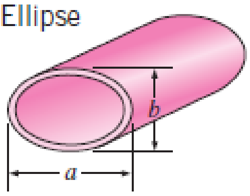
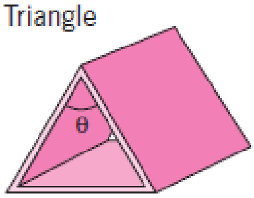
Noncircular Tubes:

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

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Noncircular Tubes:

- **Turbulent Flow**:

➢ As a first approximation, the Dittus-Boelter or Gnielinski correlation may be used with the hydraulic diameter, irrespective of the surface thermal condition.

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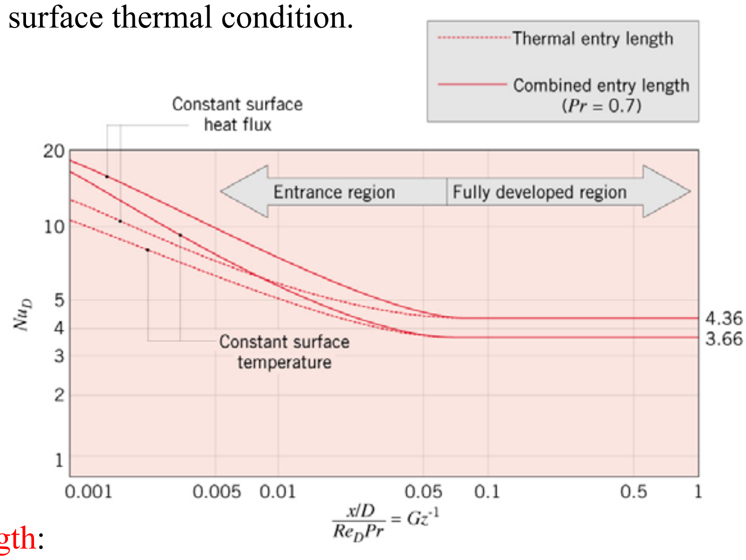
Effect of the Entry Region

➢ The manner in which the Nusselt decays from inlet to fully developed conditions for laminar flow depends on the nature of thermal and velocity boundary layer development in the entry region, as well as the surface thermal condition.

Laminar flow in a circular tube.



– Combined Entry Length:

➢ Thermal and velocity boundary layers develop concurrently from uniform profiles at the inlet.

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Effect of the Entry Region

– Thermal Entry Length:

- Velocity profile is fully developed at the inlet, and boundary layer development in the entry region is restricted to thermal effects.
- Such a condition may also be assumed to be a good approximation for a uniform inlet velocity profile if $\Pr >> 1$.

➢ **Average Nusselt Number for Laminar Flow in a Circular Tube with Uniform Surface Temperature:**

– Combined Entry Length:

$\left[\mathrm{Re}_D \Pr/(L/D)\right]^{1/3}(\mu/\mu_s)^{0.14} > 2:$

$$\overline{Nu}_D = 1.86\left(\frac{\mathrm{Re}_D \Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$\left[\mathrm{Re}_D \Pr/(L/D)\right]^{1/3}(\mu/\mu_s)^{0.14} < 2:$

$$\overline{Nu}_D = 3.66$$

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Effect of the Entry Region

– Thermal Entry Length:

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)\mathrm{Re}_D \Pr}{1 + 0.04\left[(D/L)\mathrm{Re}_D \Pr\right]^{2/3}}$$

- Average Nusselt Number for **Turbulent Flow** in a Circular Tube :

– Effects of entry and surface thermal conditions are less pronounced for turbulent flow and can be neglected.

– For long tubes $(L/D > 60)$:

$$\overline{Nu}_D \approx Nu_{D,fd}$$

– For short tubes $(L/D < 60)$:

$$\frac{\overline{Nu}_D}{Nu_{D,fd}} \approx 1 + \frac{C}{(L/D)^m}$$

$$C \approx 1$$
$$m \approx 2/3$$

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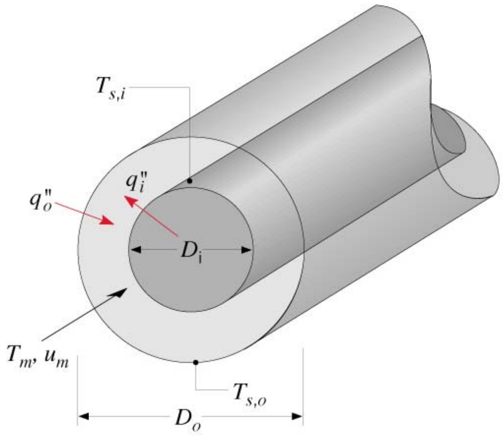
The Concentric Tube Annulus

- Fluid flow through region formed by concentric tubes.
- Convection heat transfer may be from or to inner surface of outer tube and outer surface of inner tube.
- Surface thermal conditions may be characterized by uniform temperature $(T_{s,i}, T_{s,o})$ or uniform heat flux (q_i'', q_o'')



- Convection coefficients are associated with each surface, where

$$q_i'' = h_i\left(T_{s,i} - T_m\right) \quad q_o'' = h_o\left(T_{s,o} - T_m\right)$$

and $Nu_i \equiv \dfrac{h_i D_h}{k}$ $\quad Nu_o \equiv \dfrac{h_o D_h}{k}$ $\quad$ where $D_h = D_o - D_i$

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The Concentric Tube Annulus

- Fully Developed Laminar Flow

Nusselt numbers depend on D_i / D_o and surface thermal conditions (Tables 8.2, 8.3)

TABLE 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	Comments
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
≈1.00	4.86	4.86	See Table 8.1, $b/a \rightarrow \infty$

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The Concentric Tube Annulus

TABLE 8.3 Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces

D_i/D_o	Nu_{ii}	Nu_{oo}	θ_i^*	θ_o^*
0	—	4.364[a]	∞	0
0.05	17.81	4.792	2.18	0.0294
0.10	11.91	4.834	1.383	0.0562
0.20	8.499	4.833	0.905	0.1041
0.40	6.583	4.979	0.603	0.1823
0.60	5.912	5.099	0.473	0.2455
0.80	5.58	5.24	0.401	0.299
1.00	5.385	5.385[b]	0.346	0.346

$$Nu_i = \frac{Nu_{ii}}{1 - (q_o''/q_i'')\theta_i^*} \qquad Nu_o = \frac{Nu_{oo}}{1 - (q_i''/q_o'')\theta_o^*}$$

- Fully Developed Turbulent Flow

Correlations for a circular tube may be used with D replaced by D_h .

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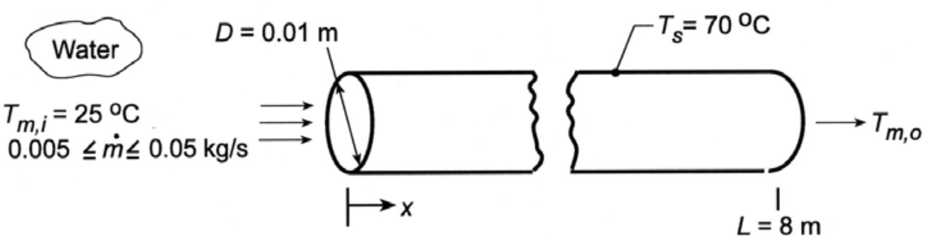
Example

Determine the effect of flow rate on outlet temperature and heat rate for water flow through the tube of a flat-plate solar collector.



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Example cont.

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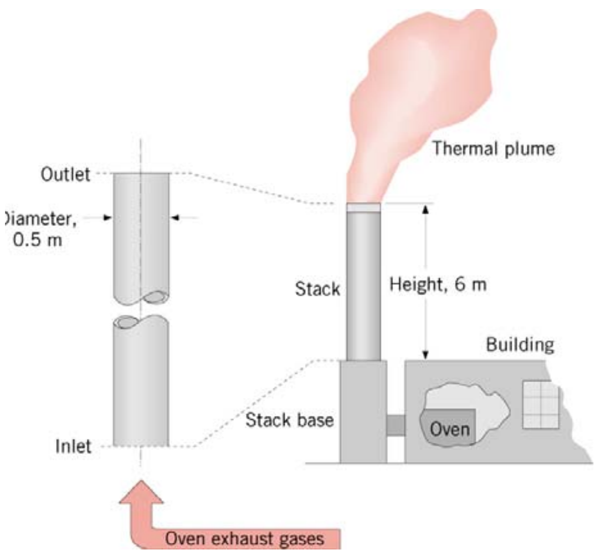
Example

8.59 Exhaust gases from a wire processing oven are discharged into a tall stack, and the gas and stack surface temperatures at the outlet of the stack must be estimated. Knowledge of the outlet gas temperature $T_{m,o}$ is useful for predicting the dispersion of effluents in the thermal plume, while knowledge of the outlet stack surface temperature $T_{s,o}$ indicates whether condensation of the gas products will occur. The thin-walled, cylindrical stack is 0.5 m in diameter and 6.0 m high. The exhaust gas flow rate is 0.5 kg/s, and the inlet temperature is 600°C.



(a) Consider conditions for which the ambient air temperature and wind velocity are 4°C and 5 m/s, respectively. Approximating the thermophysical properties of the gas as those of atmospheric air, estimate the outlet gas and stack surface temperatures for the given conditions.

(b) The gas outlet temperature is sensitive to variations in the ambient air temperature and wind velocity. For $T_\infty = -25°C$, 5°C, and 35°C, compute and plot the gas outlet temperature as a function of wind velocity for $2 \le V \le 10$ m/s.

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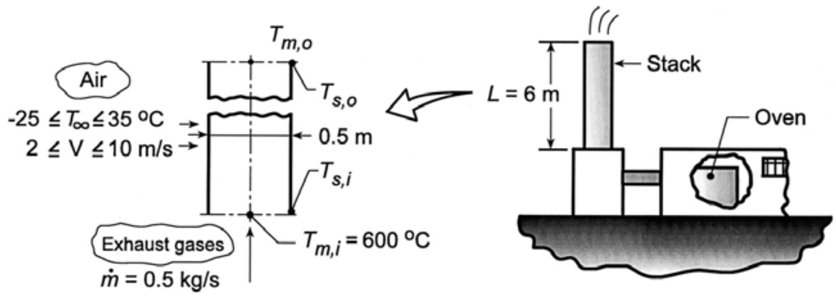
Example cont.

(b) Effect of wind temperature and velocity on $T_{m,o}$.



Example cont.

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Example cont.

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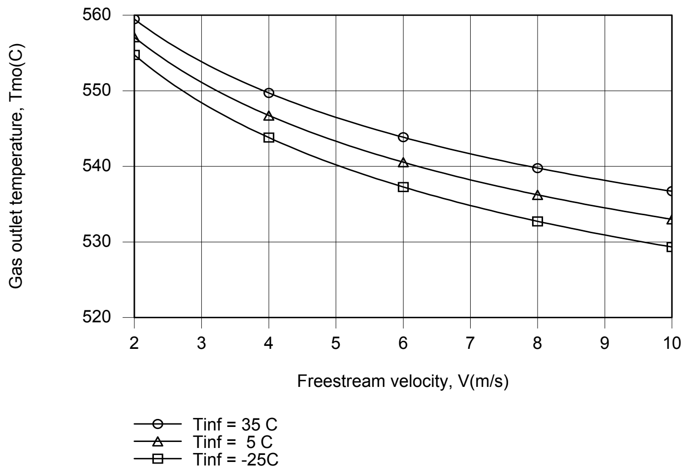
Example cont.

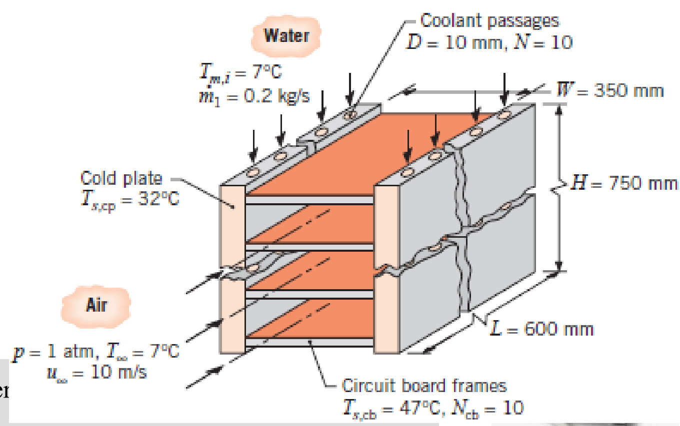


Example

8.56 One way to cool chips mounted on the circuit boards of a computer is to encapsulate the boards in metal frames that provide efficient pathways for conduction to supporting *cold plates*. Heat generated by the chips is then dissipated by transfer to water flowing through passages drilled in the plates. Because the plates are made from a metal of large thermal conductivity (typically aluminium or copper), they may be assumed to be at a temperature, $T_{s,cp}$.

(a) Consider circuit boards attached to cold plates of height $H = 750$ mm and width $L = 600$ mm, each with $N = 10$ holes of diameter $D = 10$ mm. If operating conditions maintain plate temperatures of $T_{s,cp} = 32°C$ with water flow at $\dot{m}_1 = 0.2$ kg/s per passage and $T_{m,i} = 7°C$, how much heat may be dissipated by the circuit boards?

(b) To enhance cooling, thereby allowing increased power generation without an attendant increase in system temperatures, a hybrid cooling scheme may be used. The scheme involves forced airflow over the encapsulated circuit boards, as well as water flow through the cold plates. Consider conditions for which $N_{cb} = 10$ circuit boards of width $W = 350$ mm are attached to the cold plates and their average surface temperature is $T_{s,cb} = 47°C$ when $T_{s,cp} = 32°C$. If air is in parallel flow over the plates with $u_\infty = 10$ m/s and $T_\infty = 7°C$, how much of the heat generated by the circuit boards is transferred to the air?



Example cont.

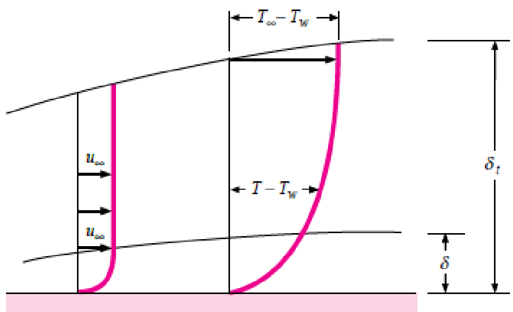
Example cont.

Example cont.

Excursion: Liquid-Metal Heat Transfer

- Considerable interest has been placed on liquid-metal heat transfer because of the high heat-transfer rates that may be achieved with these media.
- These high heat-transfer rates result from the high thermal conductivities of liquid metals as compared with other fluids; as a consequence, they are particularly applicable to situations where large energy quantities must be removed from a relatively small space, as in a nuclear reactor.
- In addition, the liquid metals remain in the liquid state at higher temperatures than conventional fluids like water and various organic coolants. This also makes more compact heat-exchanger design possible.
- Prandtl number for liquid metals is very low, of the order of 0.01, so that the thermal boundary- layer thickness should be substantially larger than the hydrodynamic-boundary layer thickness.

$$\frac{\delta}{\delta_t} \sim 0.16$$



Excursion: Liquid-Metal Heat Transfer

- For a liquid metal flowing across flat plate.

$$\frac{\theta}{\theta_\infty} = \frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

- The heat-transfer coefficient may be expressed

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.530(\mathrm{Re}_x \mathrm{Pr})^{1/2} = 0.530\mathrm{Pe}^{1/2}$$

- For calculation of heat-transfer coefficients in fully developed turbulent flow of liquid metals **in smooth tubes** with uniform heat flux at the wall, then

$$\mathrm{Nu}_d = \frac{hd}{k} = 0.625(\mathrm{Re}_d \mathrm{Pr})^{0.4} \qquad \text{for } 10^2 < \mathrm{Pe} < 10^4 \text{ and for } L/d > 60.$$

$$Nu_D = 4.82 + 0.0185\, Pe_D^{0.827} \qquad q_s'' = \text{constant} \qquad \begin{bmatrix} 3 \times 10^{-3} \lesssim Pr \lesssim 5 \times 10^{-2} \\ 3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5 \\ 10^2 \lesssim Pe_D \lesssim 10^4 \end{bmatrix}$$

Excursion: Liquid-Metal Heat Transfer

- The following relation for calculation of heat transfer to liquid metals **in tubes** with constant wall temperature:

$$\mathrm{Nu}_d = 5.0 + 0.025(\mathrm{Re}_d \mathrm{Pr})^{0.8} \qquad \mathrm{Pe} > 10^2 \text{ and } L/d > 60.$$

where all properties are evaluated at the bulk temperature

- The heat transfer from a **sphere** during forced convection

$$\mathrm{Nu} = 2 + 0.386(\mathrm{Re\,Pr})^{0.5} \qquad 3.56 \times 10^4 < \mathrm{Re} < 1.525 \times 10^5.$$

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SUMMARY

- The general calculation procedure is as follows:
  1. Establish the geometry of the situation.
  2. Make a preliminary determination of appropriate fluid properties.
  3. Establish the flow regime by calculating the Reynolds or Peclet number.
  4. Select an equation that fits the geometry and flow regime and reevaluate properties, if necessary, in accordance with stipulations and the equation.
  5. Proceed to calculate the value of h and/or the heat-transfer rate.

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Summary of forced-convection relations

Subscripts: b = bulk temperature, f = film temperature, ∞ = free stream temperature, w = wall temperature

Geometry	Equation	Restrictions
Tube flow	$\mathrm{Nu}_d = 0.023\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^n$	Fully developed turbulent flow, $n = 0.4$ for heating, $n = 0.3$ for cooling, $0.6 < \mathrm{Pr} < 100$, $2500 < \mathrm{Re}_d < 1.25 \times 10^5$
Tube flow	$\mathrm{Nu}_d = 0.0214(\mathrm{Re}_d^{0.8} - 100)\mathrm{Pr}^{0.4}$	$0.5 < \mathrm{Pr} < 1.5$, $10^4 < \mathrm{Re}_d < 5 \times 10^6$
	$\mathrm{Nu}_d = 0.012(\mathrm{Re}_d^{0.87} - 280)\mathrm{Pr}^{0.4}$	$1.5 < \mathrm{Pr} < 500$, $3000 < \mathrm{Re}_d < 10^6$
Tube flow	$\mathrm{Nu}_d = 0.027\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$	Fully developed turbulent flow
Tube flow, entrance region	$\mathrm{Nu}_d = 0.036\,\mathrm{Re}_d^{0.8}\mathrm{Pr}^{1/3}\left(\frac{d}{L}\right)^{0.055}$	Turbulent flow
	See also Figures 6-5 and 6-6	$10 < \frac{L}{d} < 400$
Tube flow	Petukov relation	Fully developed turbulent flow, $0.5 < \mathrm{Pr} < 2000$, $10^4 < \mathrm{Re}_d < 5 \times 10^6$, $0 < \frac{\mu_b}{\mu_w} < 40$

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Summary of forced-convection relations

Subscripts: b = bulk temperature, f = film temperature, ∞ = free stream temperature, w = wall temperature

Geometry	Equation	Restrictions
Tube flow	$\mathrm{Nu}_d = 3.66 + \dfrac{0.0668(d/L)\,\mathrm{Re}_d\mathrm{Pr}}{1+0.04[(d/L)\,\mathrm{Re}_d\mathrm{Pr}]^{2/3}}$	Laminar, T_w = const.
Tube flow	$\mathrm{Nu}_d = 1.86(\mathrm{Re}_d\mathrm{Pr})^{1/3}\left(\dfrac{d}{L}\right)^{1/3}\left(\dfrac{\mu}{\mu_w}\right)^{0.14}$	Fully developed laminar flow, T_w = const. $\mathrm{Re}_d\mathrm{Pr}\dfrac{d}{L} > 10$
Rough tubes	$\mathrm{St}_b\mathrm{Pr}_f^{2/3} = \dfrac{f}{8}$ or Equation (6-7)	Fully developed turbulent flow
Noncircular ducts	Reynolds number evaluated on basis of hydraulic diameter $D_H = \dfrac{4A}{P}$ A = flow cross-section area, P = wetted perimeter	Same as particular equation for tube flow
Flow across cylinders	$\mathrm{Nu}_f = C\,\mathrm{Re}_{df}^n\,\mathrm{Pr}^{1/3}$ C and n from Table 6-2	$0.4 < \mathrm{Re}_{df} < 400{,}000$
Flow across cylinders	$\mathrm{Nu}_{df} = 0.3 + \dfrac{0.62\,\mathrm{Re}_f^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\dfrac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\dfrac{\mathrm{Re}_f}{282{,}000}\right)^{5/8}\right]^{4/5}$	$10^2 < \mathrm{Re}_f < 10^7$, $\mathrm{Pe} > 0.2$

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Summary of forced-convection relations

Subscripts: b = bulk temperature, f = film temperature, ∞ = free stream temperature, w = wall temperature

Geometry	Equation	Restrictions
Flow across spheres	$\mathrm{Nu}_{df} = 0.37\,\mathrm{Re}_{df}^{0.6}$	$\mathrm{Pr} \sim 0.7$ (gases), $17 < \mathrm{Re} < 70{,}000$
	$\mathrm{Nu}_d\mathrm{Pr}^{-0.3}(\mu_w/\mu)^{0.25} = 1.2 + 0.53\,\mathrm{Re}_d^{0.54}$	Water and oils $1 < \mathrm{Re} < 200{,}000$ Properties at T_∞
	$\mathrm{Nu}_d = 2 + \left(0.4\,\mathrm{Re}_d^{1/2} + 0.06\,\mathrm{Re}_d^{2/3}\right)\mathrm{Pr}^{0.4}(\mu_\infty/\mu_w)^{1/4}$	$0.7 < \mathrm{Pr} < 380$, $3.5 < \mathrm{Re}_d < 80{,}000$, Properties at T_∞
Flow across tube banks	$\mathrm{Nu}_f = C\mathrm{Re}_{f,\max}^n\,\mathrm{Pr}_f^{1/3}$ C and n from Table 6-4	See text
Flow across tube banks	$\mathrm{Nu}_d = C\mathrm{Re}_{d,\max}^n\,\mathrm{Pr}^{0.36}\left(\dfrac{\mathrm{Pr}}{\mathrm{Pr}_w}\right)^{1/4}$	$0.7 < \mathrm{Pr} < 500$, $10 < \mathrm{Re}_{d,\max} < 10^6$
Flow across noncircular	$\mathrm{Nu} = C\,\mathrm{Re}_{df}^n\,\mathrm{Pr}^{1/3}$	

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