Process Heat Transfer

Lec 10: Natural Convection Systems

Free Convection Heat Transfer on a Vertical Plate, Empirical Relations for Free Convection for Vertical Plates and Cylinders, Free Convection from Different Geometries

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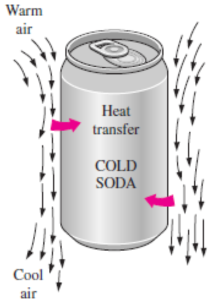
Introduction

- Natural convection is due to density difference in the same fluid (buoyancy force.
- The changes in density results in movement of the fluid; fluid property variation is due to the change in temperature or temperature gradient.
- For the fluid to move due to buoyancy force, an external force is necessary, such as gravity, or centrifugal force.
- Buoyancy forces may arise in a fluid for which there are density gradients and a body force that is proportional to density.
- In heat transfer, density gradients are due to temperature gradients and the body force is gravitational.

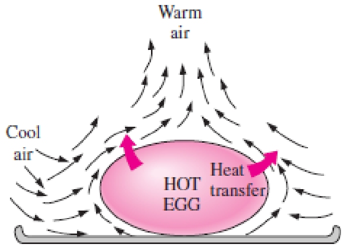
✓ **Body force:** buoyancy force which gives rise to the fee convection currents.

Introduction

Examples:

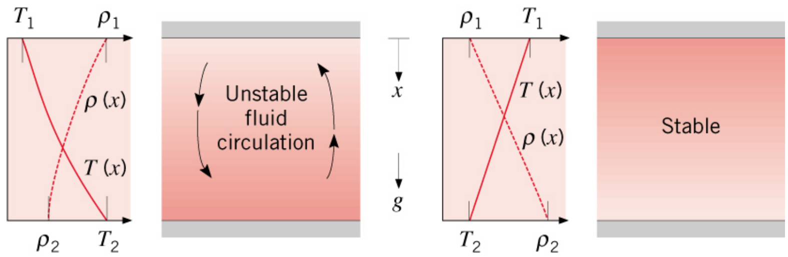


The warming up of a cold drink in a warmer environment by natural convection.



The cooling of a boiled egg in a cooler environment by natural convection.

- Stable and Unstable Temperature Gradients

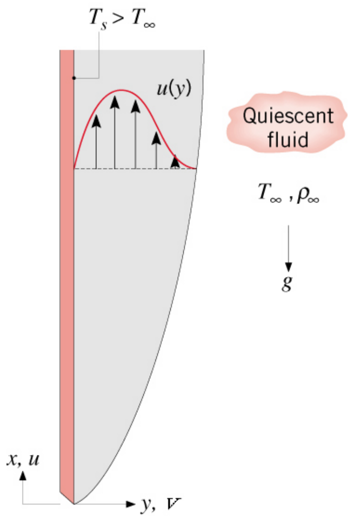


$\frac{dT}{dx} > 0, \frac{d\rho}{dx} < 0$ (*a*)

$\frac{dT}{dx} < 0, \frac{d\rho}{dx} > 0$ (*b*)

Free Convection Heat Transfer on a Vertical Plate

- Consider a flat plate whose temperature $T_w > T_\infty$.
  - ✓ Flow field develops due to natural convection
  - ✓ The velocity at the wall: $u(y=0) = 0$ (No slip condition)
  - ✓ $u\uparrow$ as we go away from the plate, however, at certain distance, the fluid will not be influenced by the heated plate $\Rightarrow$ No buoyancy force ($u = 0$)
- The *initial boundary-layer development is laminar*, but at some distance from the leading edge, depending on
- (1) the fluid properties, and
- (2) the temperature difference between the wall and the environment, turbulent eddies are formed and *turbulent boundary layer begins*



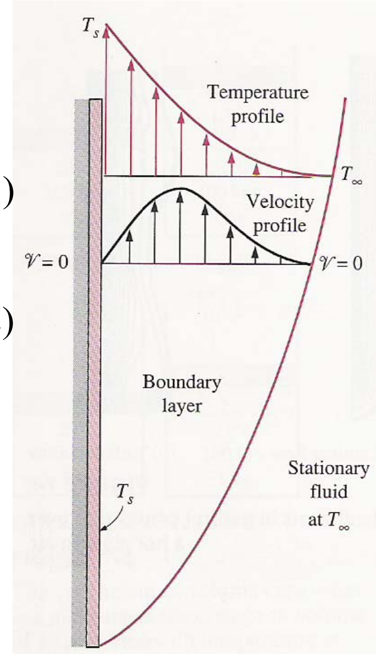
Free Convection Heat Transfer on a Vertical Plate

- It can be shown using the momentum equation and energy equation for natural convection that:

$$\boxed{\frac{u}{u_x} = \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2}$$

@ $y = 0$ $\quad u = 0$ $\qquad$ @ $y = \delta$ $\quad \partial u/\partial y = 0$ $\qquad$ (1)

@ $y = \delta$ $\quad u = 0$ $\qquad$ (2)



- The following boundary conditions apply for temperature distribution

@ $y = 0$ $\quad T = T_w$ $\qquad$ @ $y = \delta$ $\quad T = T_\infty$

@ $y = \delta$ $\quad \partial T/\partial y = 0$

$$\Rightarrow \quad \boxed{\frac{T - T_\infty}{T_w - T_\infty} = \left(1-\frac{y}{\delta}\right)^2} \qquad (3)$$

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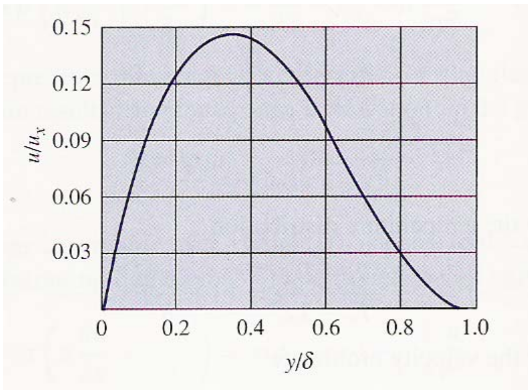
Free Convection Heat Transfer on a Vertical Plate

- The resultant expression for the boundary-layer thickness is:

$$\boxed{\frac{\delta}{x} = 3.93\,\mathrm{Pr}^{-1/2}(0.952+\mathrm{Pr})^{1/4}\mathrm{Gr}_x^{-1/4}}$$



where: $\boxed{\mathrm{Gr}_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}}$ **Grashof number**

where

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

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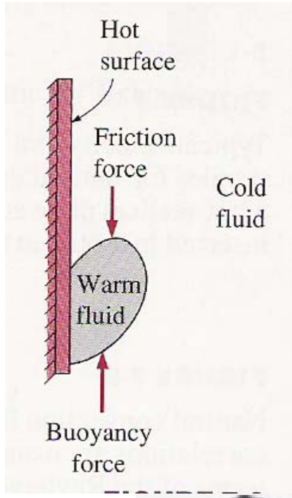
Free Convection Heat Transfer on a Vertical Plate

Now,

$$q_w = -kA\left.\frac{\partial T}{\partial y}\right|_{y=0} = hA(T_w - T_\infty) \longrightarrow h = \frac{2k}{\delta} \quad \text{or:} \quad \frac{hx}{k} = \mathrm{Nu}_x = 2\frac{x}{\delta}$$

$$\Rightarrow \boxed{\mathrm{Nu}_x = 0.508\,\mathrm{Pr}^{1/2}(0.952+\mathrm{Pr})^{-1/4}\mathrm{Gr}_x^{1/4}} \qquad 0 < \mathrm{Pr} < \infty$$

and $\bar{h} = \frac{1}{L}\int_0^L h_x dx = \frac{4}{3}h_{x=L}$



$$\mathrm{Gr} = \frac{\text{Buoyance force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$$

- *Gr* has a role similar to Re for forced convection

$$\mathrm{Re} = \frac{\text{momentum force}}{\text{viscous force}} = \frac{u_\infty x}{\nu}$$

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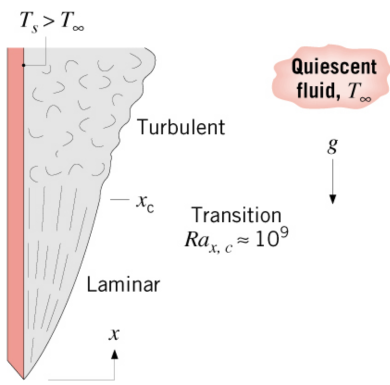
Free Convection Heat Transfer on a Vertical Plate

- Gr can be used as a criteria for the change from laminar to turbulent boundary layer flow, since it has similar physical meaning as Re.
- For air in free convection on a vertical plate, the critical Gr is 4×10^8; i.e. if Gr > $4\times10^8 \Rightarrow$ turbulent convection
- For other fluids, $\mathrm{Gr_c}$ ranges between 10^8 - 10^9.

Transition to Turbulence

- Amplification of disturbances depends on relative magnitudes of buoyancy and viscous forces.
- Transition occurs at a critical Rayleigh Number.



$$Ra_{x,c} = Gr_{x,c}\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} \approx 10^9$$

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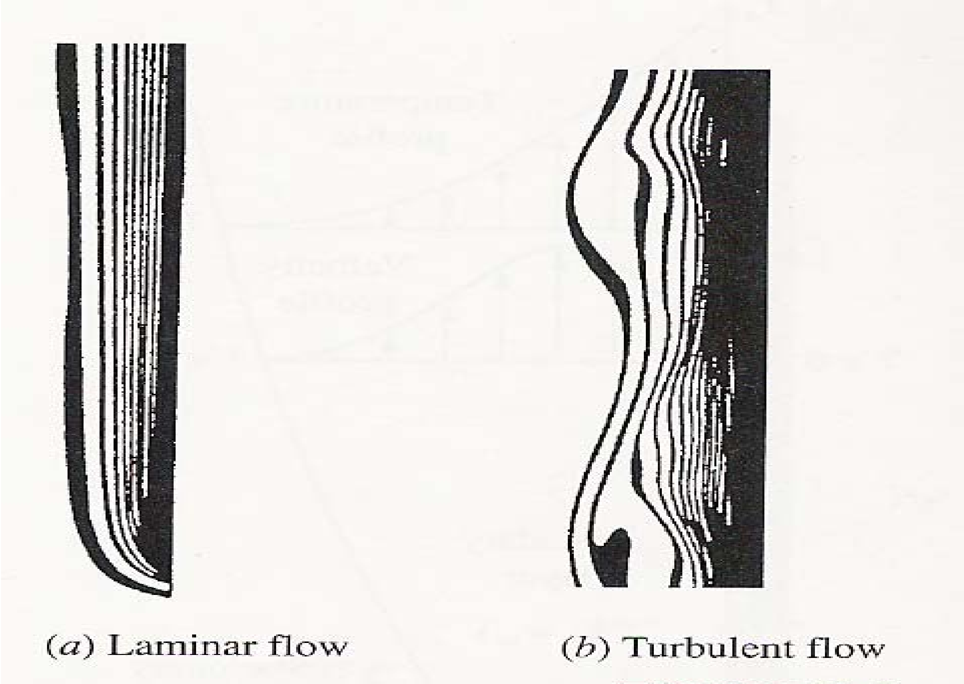

Empirical Relations for Free Convection for Vertical Plates and Cylinder

- Previous relation is useful for free convection heat transfer on a vertical plate and is the simples to be treated mathematically (i.e. expressed in terms of Gr)

- In some free-convection problems, experimental measurements must be relied due to:
  1. difficulty in prediction of *T*- and *u*-distributions, and
  2. turbulent free convection is difficult to be predicted analytically.

  $\Rightarrow$ Experimental data can be used to formulate empirical relations for free convections.

- Average heat transfer coefficient can be represented in the following functional form for a variety of circumstances:

$$\boxed{\overline{\mathrm{Nu}}_f = C\left(\mathrm{Gr}_f\,\mathrm{Pr}_f\right)^m} \quad \Rightarrow \text{properties at} \quad T_f = \frac{T_w + T_\infty}{2}$$

Define *Rayleigh number* as: $\boxed{\mathrm{Ra} = \mathrm{Gr}\,\mathrm{Pr}}$ $\qquad \mathrm{Ra} > 10^9 \Rightarrow$ turbulent flow

Empirical Relations for Free Convection

- Over the years it has been found that average free-convection heat-transfer coefficients can be represented in the following functional form for a variety of circumstances:

$$\overline{\mathrm{Nu}}_f = C(\mathrm{Gr}_f\,\mathrm{Pr}_f)^m$$

where the subscript f indicates that the properties in the dimensionless groups are evaluated at the film temperature

$$T_f = \frac{T_\infty + T_w}{2}$$

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Constants for use for isothermal surfaces

Geometry	$\mathrm{Gr}_f\,\mathrm{Pr}_f$	C	m
Vertical planes and cylinders	10^{-1}–10^4	Use Fig. 7-5	Use Fig. 7-5
	10^4–10^9	0.59	$\frac{1}{4}$
	10^9–10^{13}	0.021	$\frac{2}{5}$
	10^9–10^{13}	0.10	$\frac{1}{3}$
Horizontal cylinders	0–10^{-5}	0.4	0
	10^{-5}–10^4	Use Fig. 7-6	Use Fig. 7-6
	10^4–10^9	0.53	$\frac{1}{4}$
	10^9–10^{12}	0.13	$\frac{1}{3}$
	10^{-10}–10^{-2}	0.675	0.058
	10^{-2}–10^2	1.02	0.148
	10^2–10^4	0.850	0.188
	10^4–10^7	0.480	$\frac{1}{4}$
	10^7–10^{12}	0.125	$\frac{1}{3}$
Upper surface of heated plates or lower surface of cooled plates	2×10^4–8×10^6	0.54	$\frac{1}{4}$
Upper surface of heated plates or lower surface of cooled plates	8×10^6–10^{11}	0.15	$\frac{1}{3}$
Lower surface of heated plates or upper surface of cooled plates	10^5–10^{11}	0.27	$\frac{1}{4}$
Vertical cylinder, height = diameter characteristic length = diameter	10^4–10^6	0.775	0.21
Irregular solids, characteristic length = distance fluid particle travels in boundary layer	10^4–10^9	0.52	$\frac{1}{4}$

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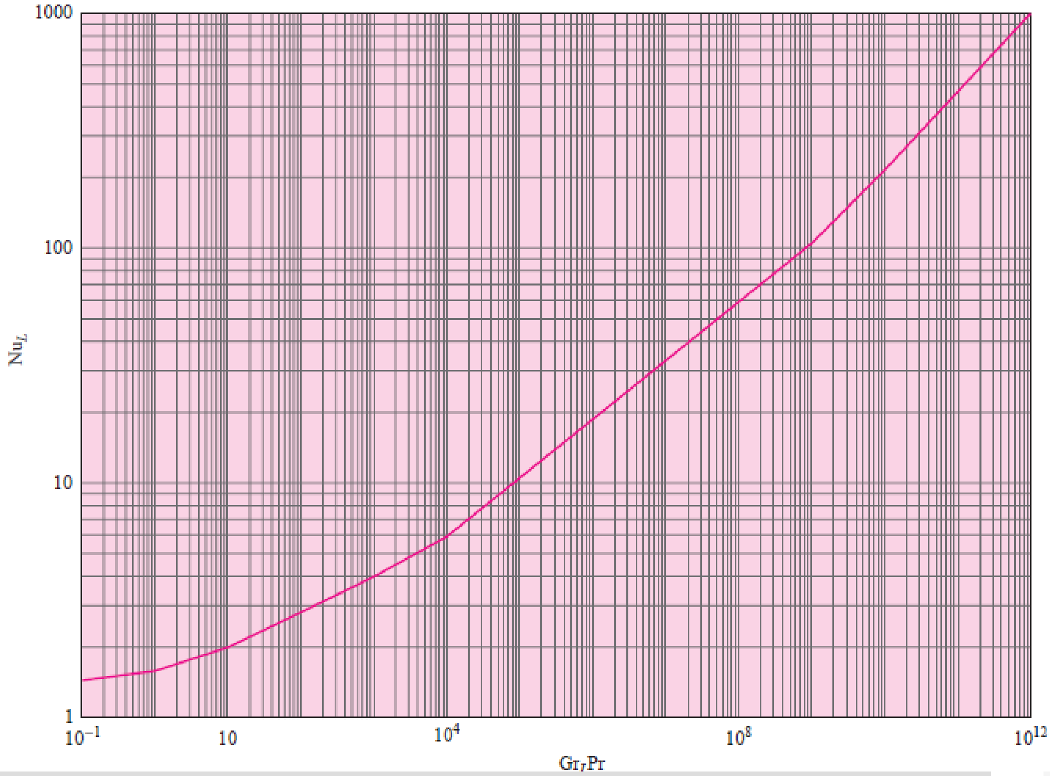
Free-convection heat transfer from vertical isothermal plates, Fig. 7.5



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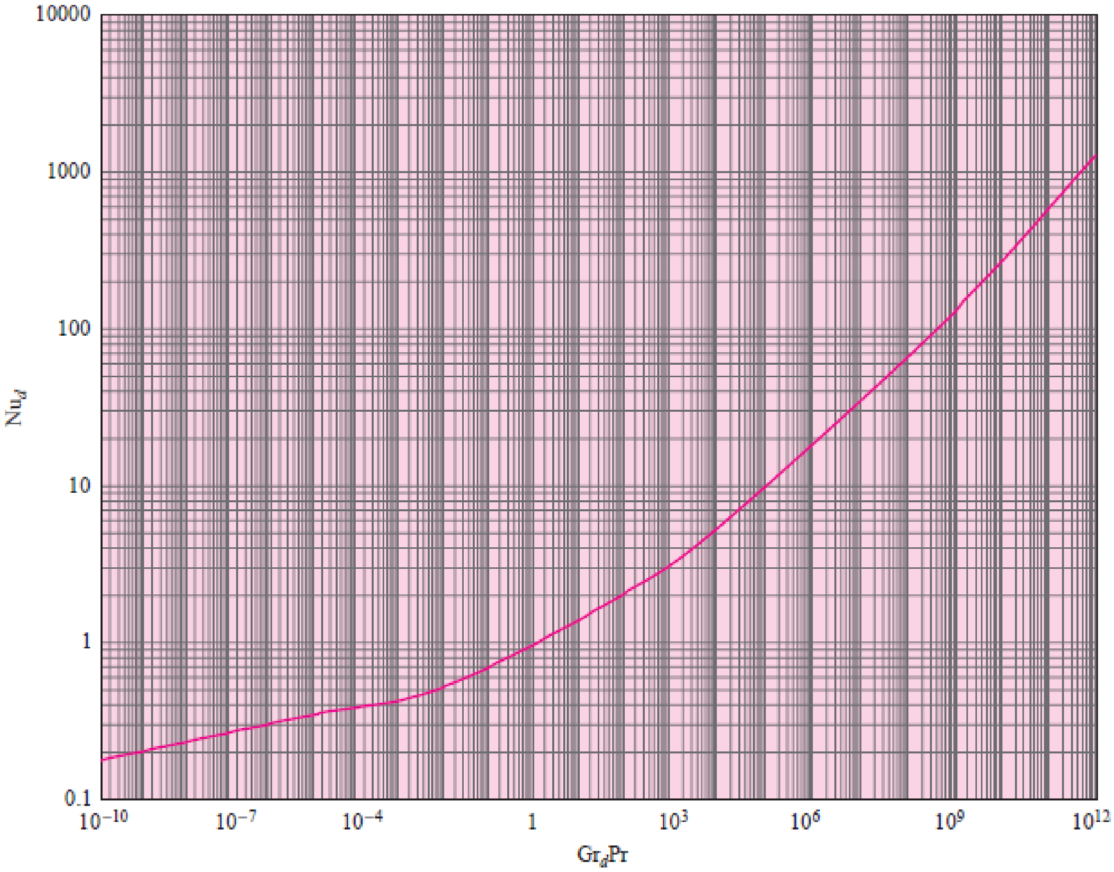
Free-convection heat transfer from horizontal isothermal cylinders, Fig. 7.6



Empirical Relations for Free Convection for Vertical Plates and Cylinder

Isothermal Surfaces

- Laminar Flow $\left(Ra_L < 10^9\right)$:

$$\overline{Nu}_L = 0.68 + \frac{0.670\, Ra_L^{1/4}}{\left[1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{4/9}}$$

- for $10^{-1} < Ra_L < 10^{12}$

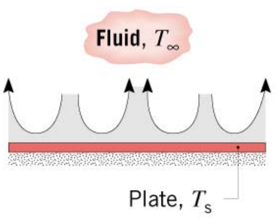
$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{4/9}}\right\}^2$$

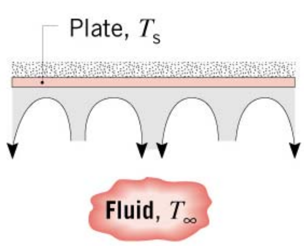
- The above relations may also be applied to *vertical* cylinders of height *L*, if the boundary layer thickness is much less than the cylinder diameter *D, or if*

$$\frac{D}{L} \gtrsim \frac{35}{Gr_L^{1/4}}$$

Horizontal Plates

- Buoyancy force is normal, instead of parallel, to the plate.
- Flow and heat transfer depend on whether the plate is heated or cooled and whether it is facing upward or downward.
- Heated Surface Facing Upward or Cooled Surface Facing Downward





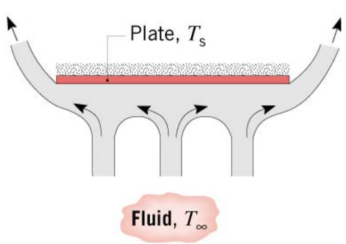
$T_s > T_\infty$ $\qquad$ $T_s < T_\infty$

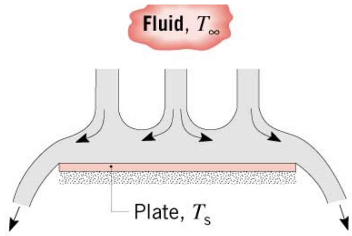
$$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \qquad \left(10^4 < Ra_L < 10^7\right)$$

$$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \qquad \left(10^7 < Ra_L < 10^{11}\right)$$

Horizontal Plates

➢ Heated Surface Facing Downward or Cooled Surface Facing Upward





$T_s > T_\infty$ $\qquad$ $T_s < T_\infty$

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \qquad \left(10^5 < Ra_L < 10^{10}\right)$$

Horizontal Plates

Constant-Heat-Flux Surfaces

➢ The experiments have produced the following correlations for constant heat flux on a horizontal plate: For the heated surface facing upward

$$\overline{\mathrm{Nu}}_L = 0.13(\mathrm{Gr}_L\,\mathrm{Pr})^{1/3} \qquad \text{for } \mathrm{Gr}_L\,\mathrm{Pr} < 2 \times 10^8$$

$$\overline{\mathrm{Nu}}_L = 0.16(\mathrm{Gr}_L\,\mathrm{Pr})^{1/3} \qquad \text{for } 2 \times 10^8 < \mathrm{Gr}_L\,\mathrm{Pr} < 10^{11}$$

For the heated surface facing downward,

$$\overline{\mathrm{Nu}}_L = 0.58(\mathrm{Gr}_L\,\mathrm{Pr})^{1/5} \qquad \text{for } 10^6 < \mathrm{Gr}_L\,\mathrm{Pr} < 10^{11}$$

In these equations all properties except β are evaluated at a temperature T_e defined by

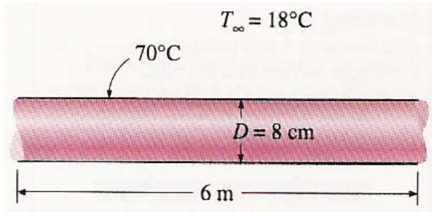
$$T_e = T_w - 0.25(T_w - T_\infty)$$

and T_w is the *average* wall temperature related, as before, to the heat flux by

$$\bar{h} = \frac{q_w}{T_w - T_\infty}$$

Example

6-m-long section of an 8-cm diamter horizontal hot water pipe shown in the drawing passes through a large room whose temperature is 18ºC. If the outer surface temperature of the pipe is 70ºC, determine the rate of heat loss from the pipe by natural convection



Example cont.

Example cont.

Remark:

The pipe will also lose heat to the surroundings by radiation as well as by natural convection. Assuming the outer surface of the pipe to be black body ($\varepsilon = 1$):

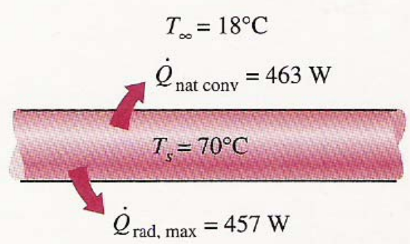


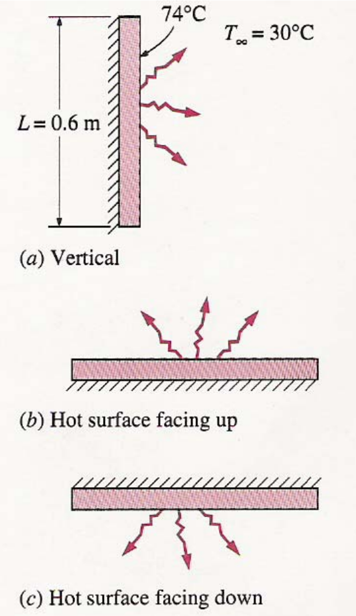
FIGURE 7-10

Radiation heat transfer is usually comparable to natural convection in magnitude and should be considered in heat transfer analysis.

Example

Cooling of a Plate in different orientations

Consider a 0.6-m × 0.6 m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 74°C, while the other side is insulated, as shown. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down.



Example cont.

Example cont.

Example cont.

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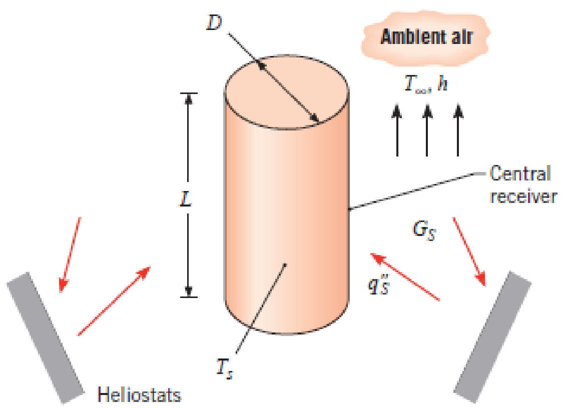
Example

9.33 In the *central receiver* concept of a solar power plant, many heliostats at ground level are used to direct a concentrated solar flux q_s'' to the receiver, which is positioned at the top of a tower. However, even with absorption of all the solar flux by the outer surface of the receiver, losses due to free convection and radiation reduce the collection efficiency below the maximum possible value of 100%. Consider a cylindrical receiver of diameter $D = 7$ m, length $L = 12$ m, and emissivity $\varepsilon = 0.20$.



(a) If all of the solar flux is absorbed by the receiver and a surface temperature of $T_s = 800$ K is maintained, what is the rate of heat loss from the receiver? The ambient air is quiescent at a temperature of $T_\infty = 300$ K, and irradiation from the surroundings may be neglected. If the corresponding value of the solar flux is $q_S'' = 10^5$ W/m^2, what is the collector efficiency?

(b) The surface temperature of the receiver is affected by design and operating conditions within the power plant. Over the range from 600 to 1000 K, plot the variation of the convection, radiation, and total heat rates as a function of T_s. For a fixed value of $q_S'' = 10^5$ W/m^2, plot the corresponding variation of the receiver efficiency.

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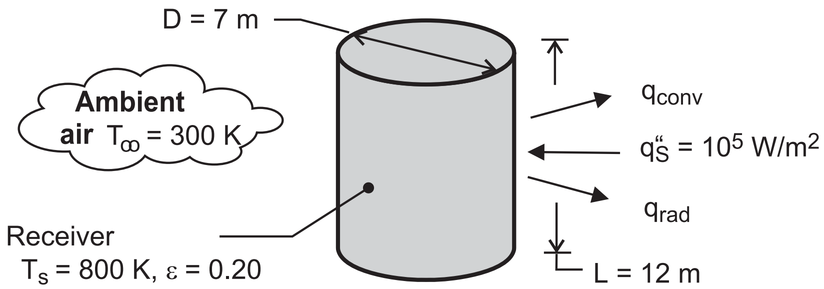
Example cont.



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Example cont.

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Example cont.

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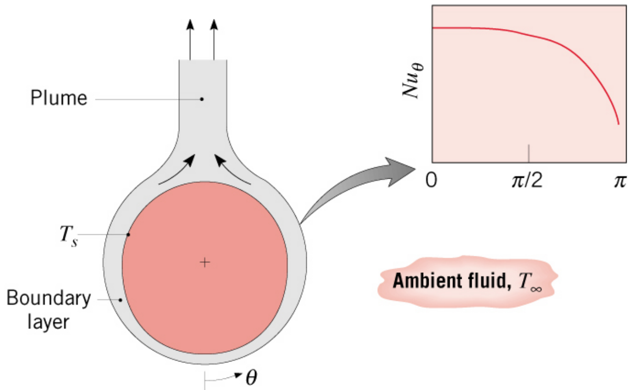
The Long Horizontal Cylinder

- Boundary Layer Development and Variation of the Local Nusselt Number for a Heated Cylinder:



- The Average Nusselt Number:

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}} \right\}^2 \qquad Ra_D < 10^{12}$$

Spheres

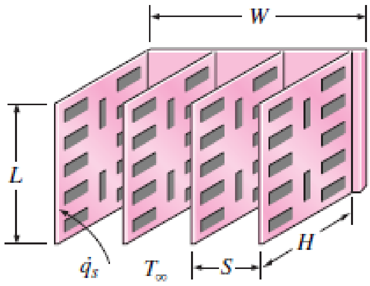
- The Average Nusselt Number:

$$\overline{Nu}_D = 2 + \frac{0.589\, Ra_D^{1/4}}{\left[1+\left(0.469/\mathrm{Pr}\right)^{9/16}\right]^{4/9}}$$

➤ In the limit as $Ra_D \to 0$, $\overline{Nu}_D = 2$, corresponds to heat transfer by conduction between a spherical surface and a stationary infinite medium

Natural Convection Cooling of Vertical PCBs (constant heat flux)

➤ The modified Rayleigh number for uniform heat flux on both plates is $\mathrm{Ra}_S^* = \frac{g\beta \dot{q}_s S^4}{k\nu^2}\mathrm{Pr}$



➤ The Nusselt number at the upper edge of the plate where maximum temperature occurs is determined from

$$\mathrm{Nu}_L = \frac{h_L S}{k} = \left[\frac{48}{\mathrm{Ra}_S^* S/L} + \frac{2.51}{(\mathrm{Ra}_L^* S/L)^{0.4}}\right]^{-0.5}$$

➤ The optimum fin spacing for the case of uniform heat flux on both plates is

q_s = constant: $$S_{\mathrm{opt}} = 2.12\left(\frac{S^4 L}{\mathrm{Ra}_S^*}\right)^{0.2}$$

➤ The total rate of heat transfer from the plates is $\dot{Q} = \dot{q}_s A_s = \dot{q}_s(2nLH)$

where $n = W/(S + t) \approx W/S$ is the number of plates. $\dot{q}_s = h_L(T_L - T_\infty)$

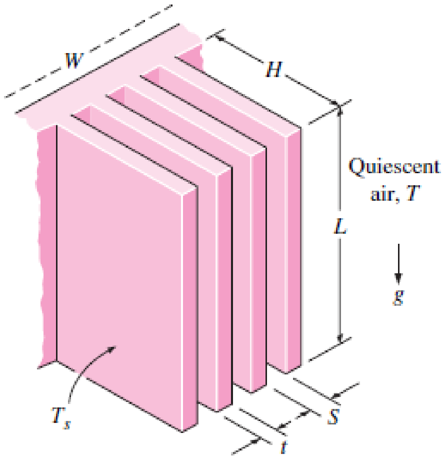
All fluid properties are to be evaluated at the average temperature $T_{\mathrm{ave}} = (T_L + T_\infty)/2$.

Natural Convection Cooling of Finned Surfaces (T_s constant)

$$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2} Pr \qquad \text{and} \qquad Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr = Ra_S \frac{L^3}{S^3}$$

- the average Nusselt number for vertical isothermal parallel plates is

T_s = constant:
$$Nu = \frac{hS}{k} = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.873}{(Ra_S S/L)^{0.5}}\right]^{-0.5}$$



- When the fins are essentially isothermal and the fin thickness t is small relative to the fin spacing S, the optimum fin spacing for a vertical heat sink is

T_s = constant:
$$S_{opt} = 2.714\left(\frac{S^3 L}{Ra_S}\right)^{0.25} = 2.714 \frac{L}{Ra_L^{0.25}}$$

$S = S_{opt}$:
$$Nu = \frac{hS_{opt}}{k} = 1.307$$

- The rate of heat transfer by natural convection from the fins $\dot{Q} = h(2nLH)(T_s - T_\infty)$

where $n = W/(S + t) \approx W/S$ is the number of fins on the heat sink and T_s is the surface temperature of the fins.

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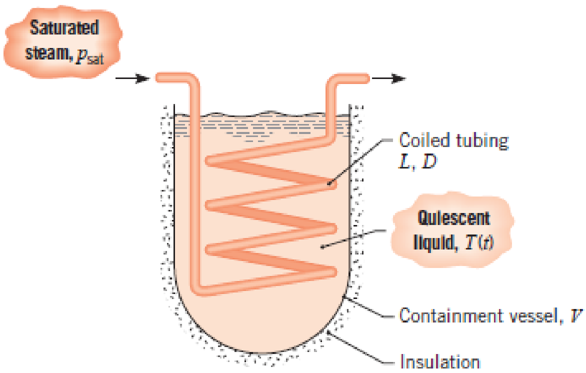
Example

9.73 Consider a batch process in which 200 L of a pharmaceutical are heated from 25°C to 70°C by saturated steam condensing at 2.455 bars as it flows through a coiled tube of 15-mm diameter and 15-m length. At any time during the process, the liquid may be approximated as an infinite, quiescent medium of uniform temperature and may be assumed to have constant properties of $\rho = 1100\ \text{kg/m}^3$, $c = 2000\ \text{J/kg}\cdot\text{K}$, $k = 0.25\ \text{W/m}\cdot\text{K}$, $\nu = 4.0 \times 10^{-6}\ \text{m}^2/\text{s}$, $Pr = 10$, and $\beta = 0.002\ \text{K}^{-1}$. The thermal resistances of the condensing steam and tube wall may be neglected.

(a) What is the initial rate of heat transfer to the pharmaceutical?

$\boxed{\text{(b)}}$ Neglecting heat transfer between the tank and its surroundings, how long does it take to heat the pharmaceutical to 70°C? Plot the corresponding variation with time of the fluid temperature and the convection coefficient at the outer surface of the tube. How much steam is condensed during the heating process?



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Example cont.

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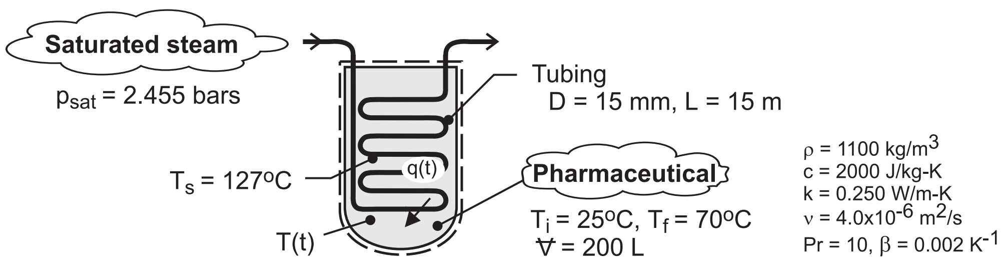

Example cont.

Example cont.

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Example cont.

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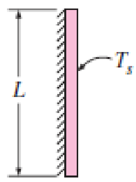
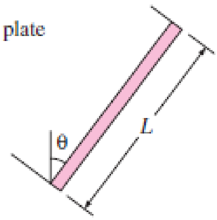
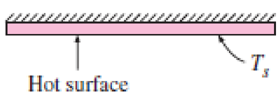
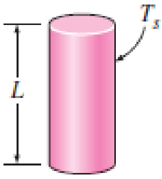
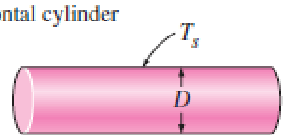
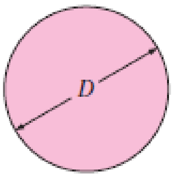
Example cont.

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $Ra < 10^9$
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$ $Nu = 0.27Ra_L^{1/4}$

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$