The Fin Equation: Solutions

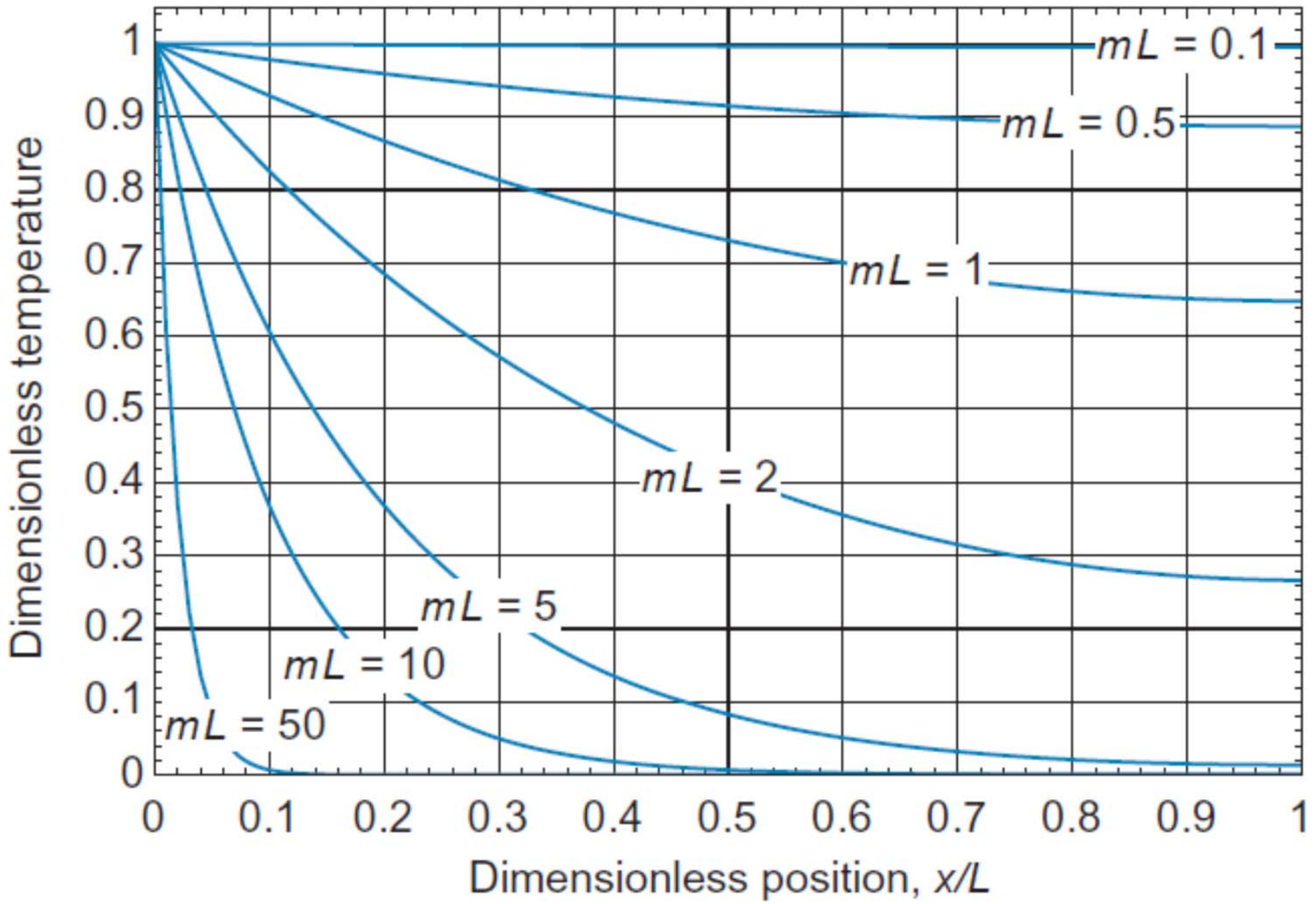
TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.80)	$M \tanh mL$ (3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.83)
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx} (3.84)	M (3.85)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

The Fin Equation: Solutions



Dimensionless fin temperature as a function of dimensionless position for various values of the parameter *mL* (for case B)

Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888

Efficiency and surface areas of common fin configurations

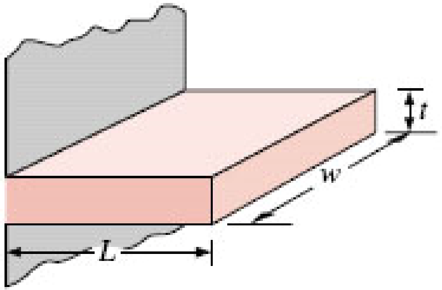
Straight Fins

Rectangular[a]

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$

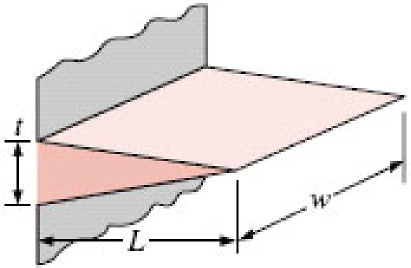


$$\eta_f = \frac{\tanh mL_c}{mL_c} \qquad m = \sqrt{2h/kt}$$

Triangular[a]

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$



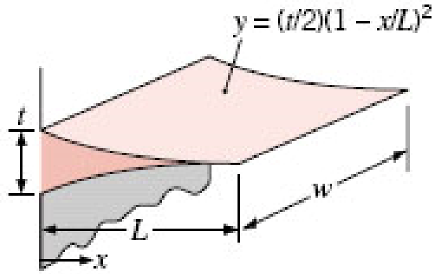
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} \qquad m = \sqrt{2h/kt}$$

Parabolic[a]

$A_f = w[C_1 L + (L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1} \qquad m = \sqrt{2h/kt}$$

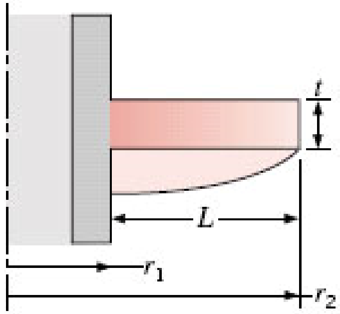
Circular Fin

Rectangular[a]

$A_f = 2\pi (r_{2c}^2 - r_1^2)$

$r_{2c} = r_2 + (t/2)$

$V = \pi (r_2^2 - r_1^2)t$



$$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

$$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$$

$$m = \sqrt{2h/kt}$$

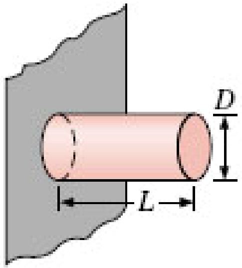
Pin Fins

Rectangular[b]

$A_f = \pi D L_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$



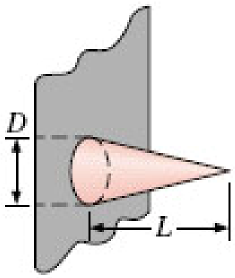
$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

$$m = \sqrt{4h/kD}$$

Triangular[b]

$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$

$V = (\pi/12)D^2L$



$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

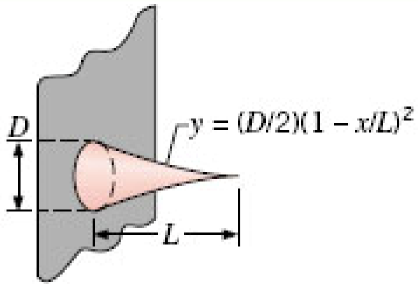
$$m = \sqrt{4h/kD}$$

Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888

Parabolic[b]

$$A_f = \frac{\pi L^3}{8D}\{C_3C_4 - \frac{L}{2D}\ln[(2DC_4/L) + C_3]\}$$

$$C_3 = 1 + 2(D/L)^2$$
$$C_4 = [1 + (D/L)^2]^{1/2}$$
$$V = (\pi/20)D^2 L$$



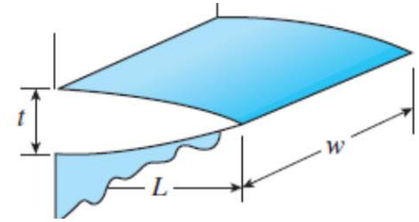
$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$$

$$m = \sqrt{4h/kD}$$

Straight parabolic fins

$$m = \sqrt{2h/kt}$$
$$A_{\text{fin}} = wL[C_1 + (L/t)\ln(t/L + C_1)]$$
$$C_1 = \sqrt{1 + (t/L)^2}$$

$$\eta_{\text{fin}} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$



Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888

Pin fins of parabolic profile

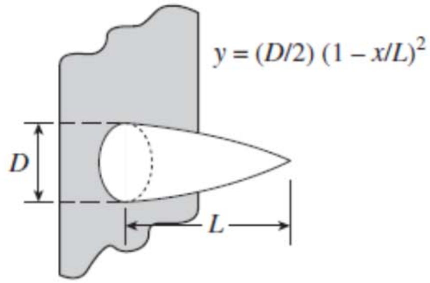
$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi L^3}{8D}\left[C_3 C_4 - \frac{L}{2D}\ln(2DC_4/L + C_3)\right]$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = \sqrt{1 + (D/L)^2}$$

$$\eta_{\text{fin}} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$

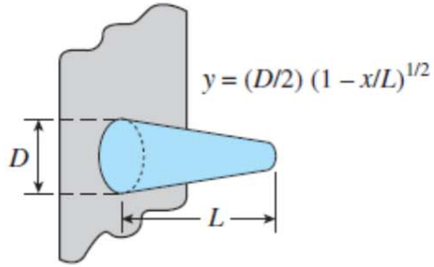


Pin fins of parabolic profile (blunt tip)

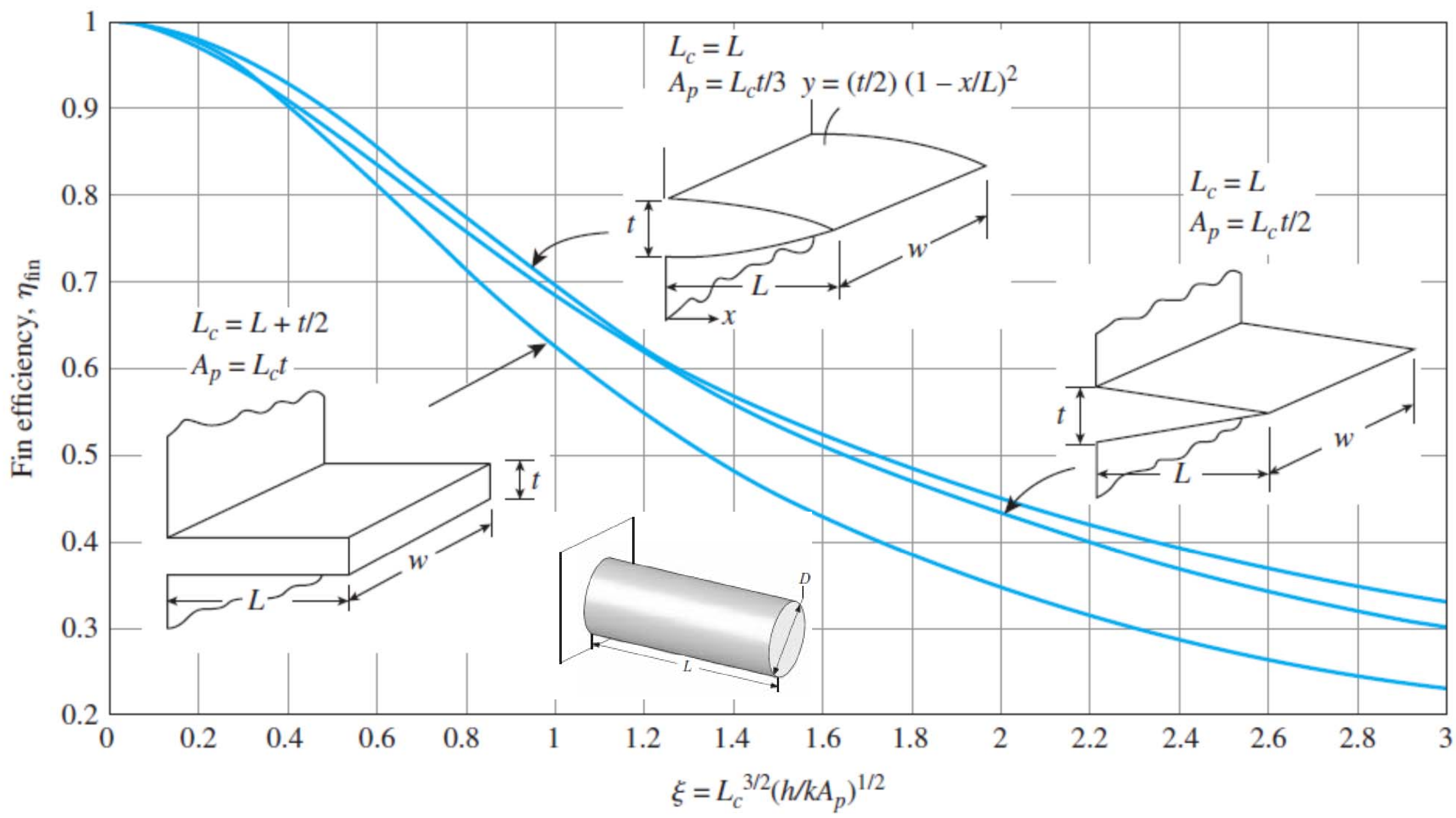
$$m = \sqrt{4h/kD}$$

$$A_{\text{fin}} = \frac{\pi D^4}{96L^2}\left\{[16(L/D)^2 + 1]^{3/2} - 1\right\}$$

$$\eta_{\text{fin}} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)}$$

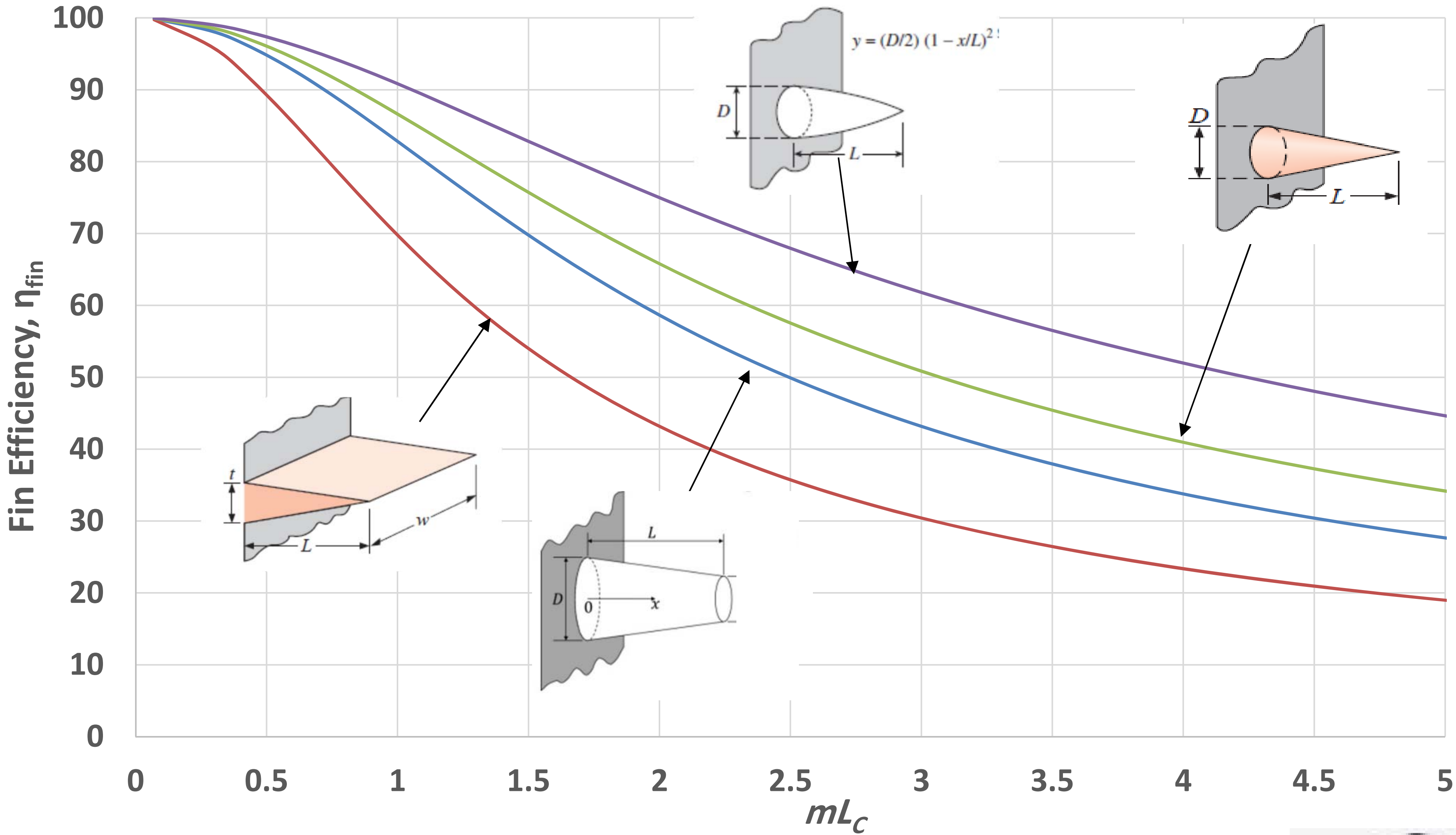


Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888

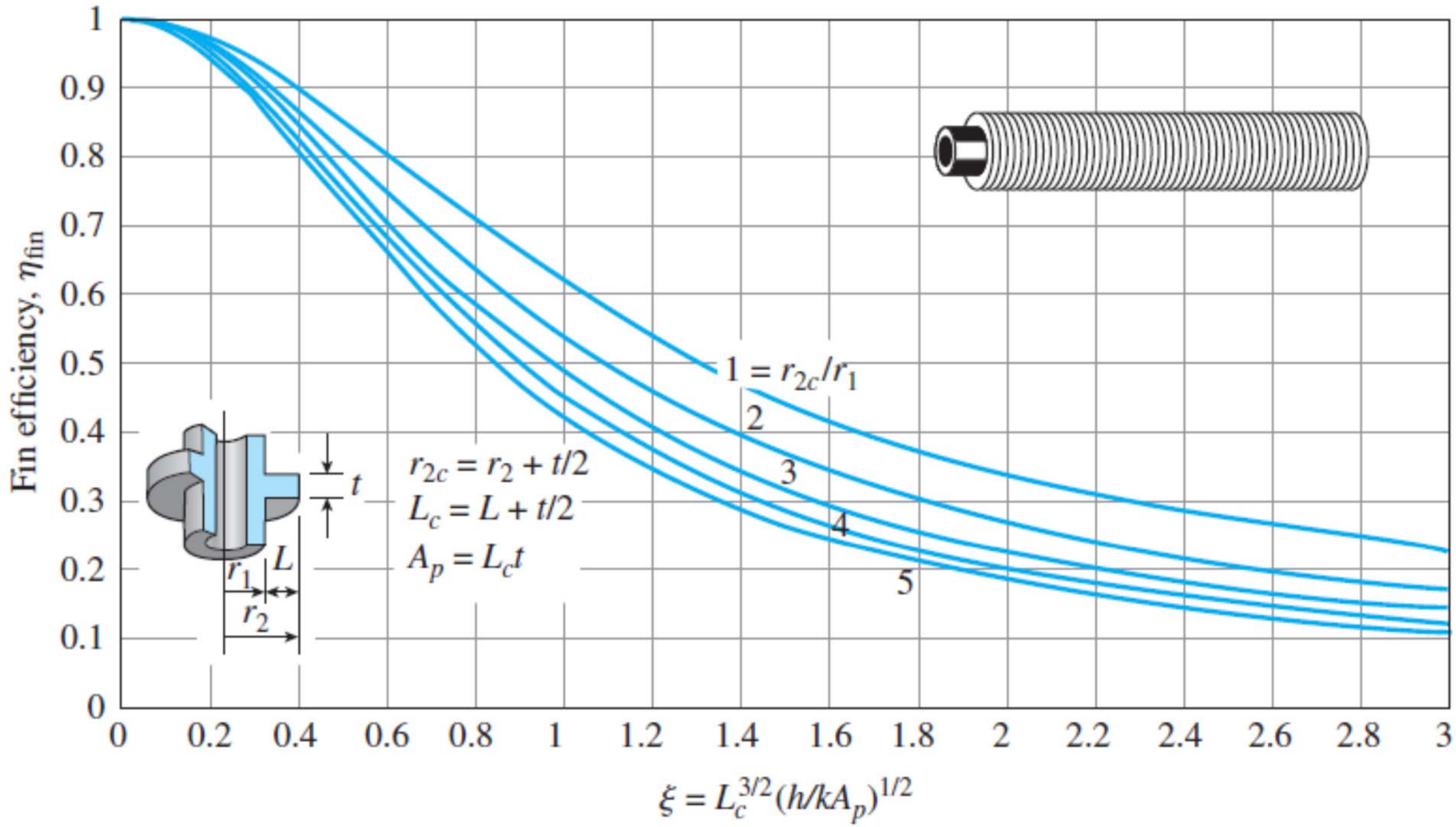


A_p is a corrected fin profile area,

Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888

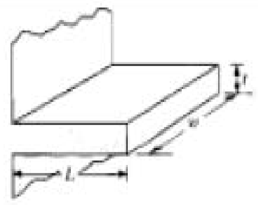
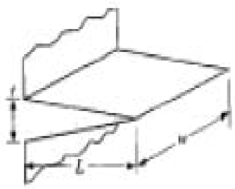
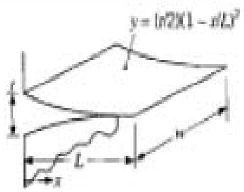
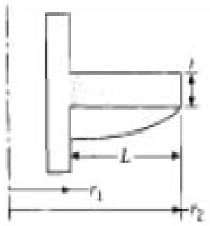
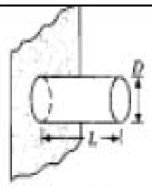
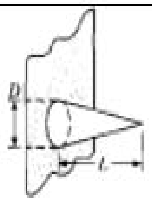
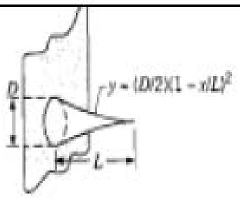


Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888



Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888

Table A. Temperature distribution equation for common fins

Straight fin, rectangular profile		$\theta = \frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$ $m = \sqrt{2h/kt}$ x is measured from the base
Straight fin, triangular profile		$\theta = \frac{T - T_\infty}{T_o - T_\infty} = \frac{I_o\left(2\sqrt{hL(L-x)/kt}\right)}{I_o\left(2\sqrt{hL^2/kt}\right)} = \frac{I_o\left(mL\sqrt{(1-x/L)}\right)}{I_o(mL)}$ $m = \sqrt{2h/kt}$ x is measured from the base
Straight fin, concave parabolic profile		$\theta = \frac{T - T_\infty}{T_o - T_\infty} = \left(1 - \frac{x}{L}\right)^{\left[\sqrt{\frac{1}{4}+(mL)^2} - \frac{1}{2}\right]}$ $m = \sqrt{2h/kt}$ x is measured from the base
Annular fin, rectangular profile		$\theta = \frac{T - T_\infty}{T_o - T_\infty} = \frac{K_1(mr_2)I_o(mr) + I_1(mr_2)K_o(mr)}{K_1(mr_2)I_o(mr_1) + I_1(mr_2)K_o(mr)}$ $m = \sqrt{2h/kt}$
Pin fin, rectangular profile		$\theta = \frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$ $m = 2\sqrt{h/kD}$
Pin fin, trianglular profile		$\theta = \frac{T - T_\infty}{T_o - T_\infty} = \left(\sqrt{\frac{L}{L-x}}\right)\frac{I_1\left(2m\sqrt{L-x}\right)}{I_1\left(2m\sqrt{L}\right)}$ $m = 2\sqrt{hL/kD}$ x is measured from the base
Pin fin, concave parabolic profile		$\theta = \frac{T - T_\infty}{T_o - T_\infty} = \left(1 - \frac{x}{L}\right)^{\left[\sqrt{9+4(mL)^2} - \frac{3}{2}\right]}$ $m = 2\sqrt{h/kD}$ x is measured from the base