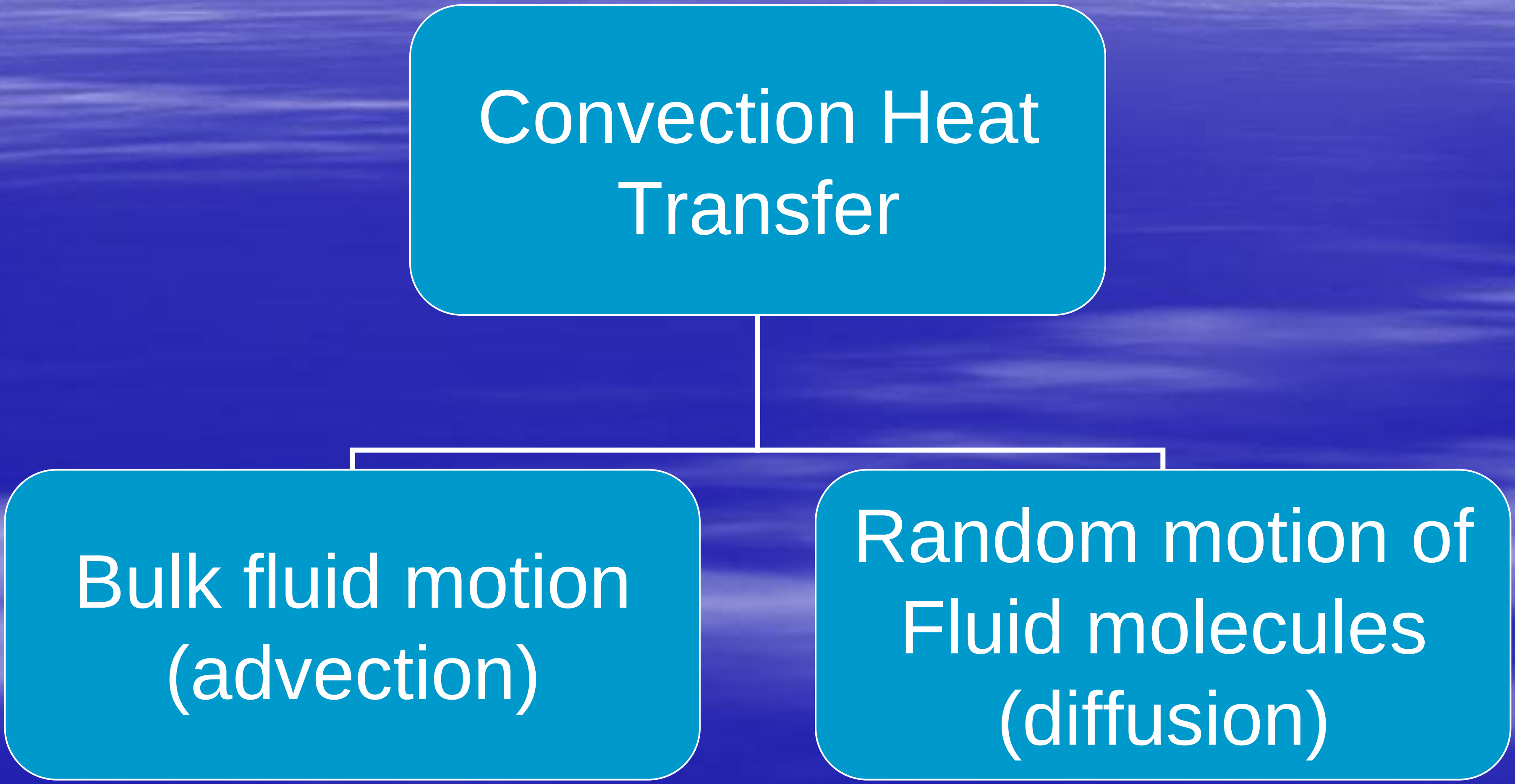
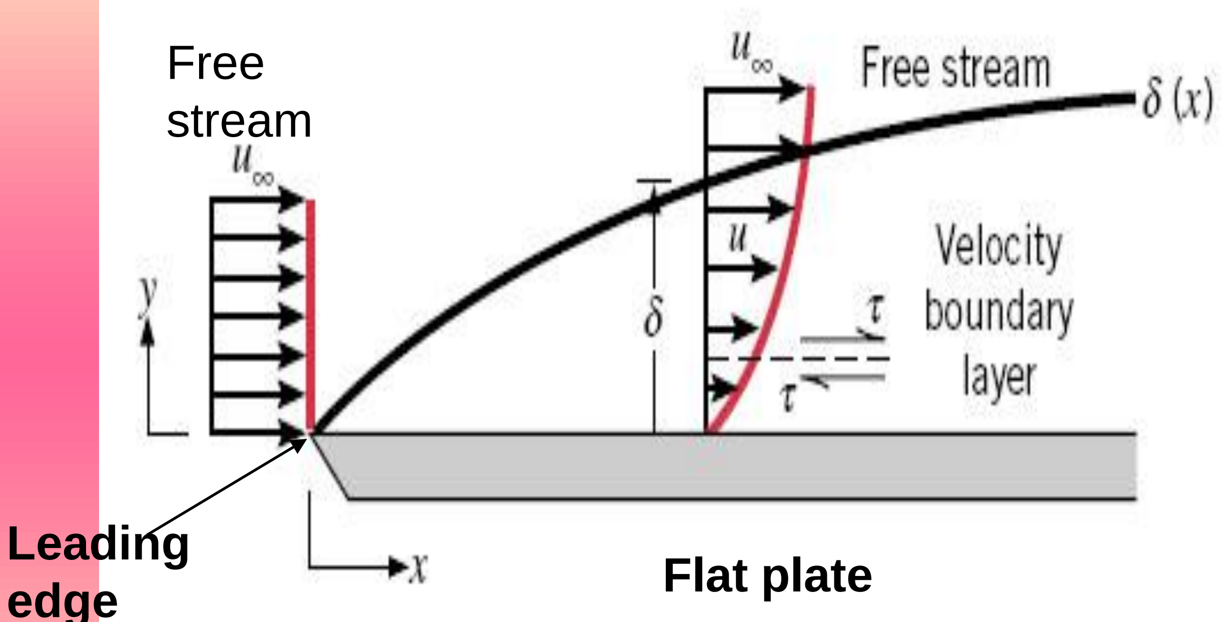
Convection Heat Transfer

Chapter 6


Convection Heat Transfer
Bulk fluid motion (advection)
Random motion of Fluid molecules (diffusion)

The velocity boundary layer



δ : Boundary layer thickness≡ the value of y for which $u=0.99u_\infty$

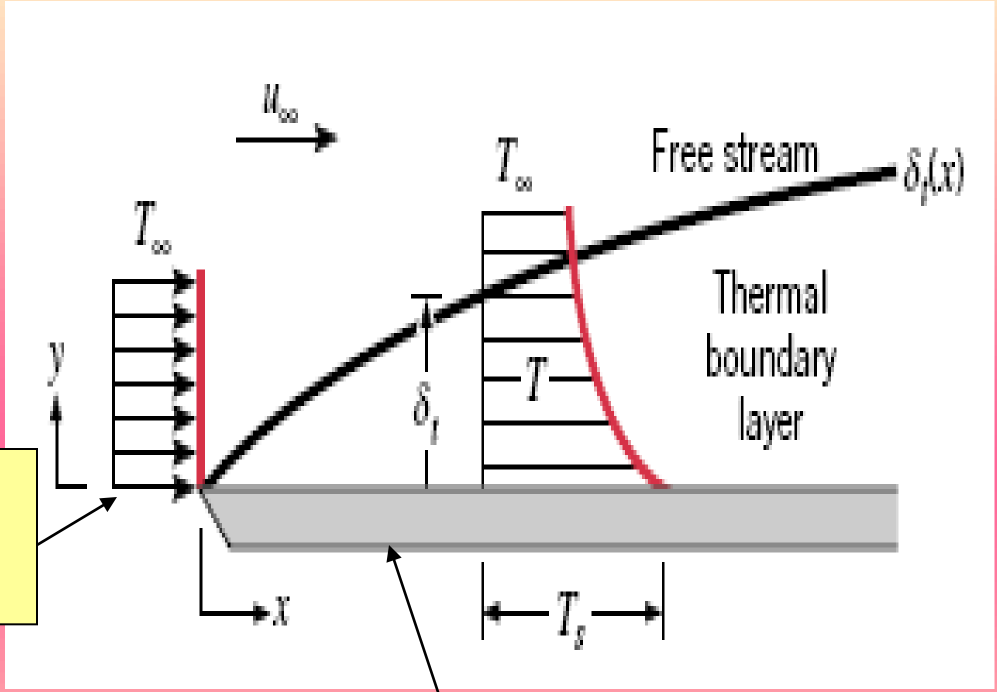
The local friction coefficient is $C_f = \dfrac{\tau_s}{\rho u_\infty^2/2}$

The surface shear stress is evaluated at the wall surface

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

The Thermal Boundary Layer

- Thermal B.L is created as a temperature difference is found between surface and a fluid.
- The region of the fluid in which temperature gradients develop is the thermal B.L.



Fluid of Uniform temp

Isothermal plate

Thermal B.L. thickness δ_t is the value y for the ratio $[(T_s-T)/(T_s-T_\infty)]=0.99$

The local heat flux at any distance x from the leading edge is obtained by applying Fourier's law to the fluid at y=0.

$$q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

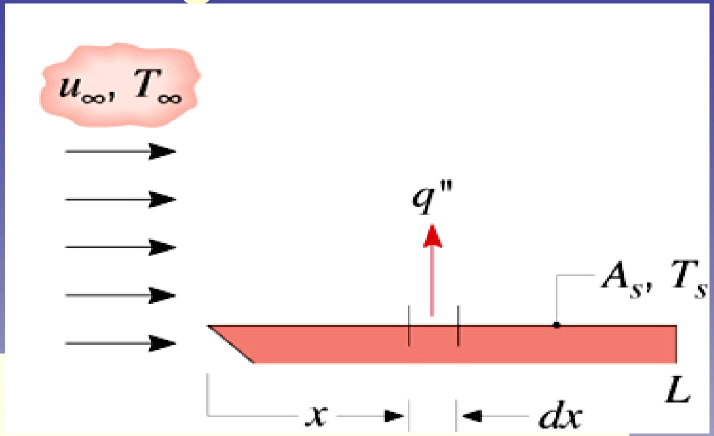
We can also apply Newton's law of cooling

$$q_s'' = h(T_s - T_\infty)$$

combing the previous two equations

$$h = \frac{-k_f \left.\partial T / \partial y\right|_{y=0}}{T_s - T_\infty}$$

The relation between local and average convection coefficients



$$q = \int_{A_s} q'' dA_s$$

$$q = (T_s - T_\infty)\int_{A_s} h dA_s$$

Defining an *average convection coefficient* $\bar{h}$ for the entire surface, the total heat transfer rate may also be expressed as

$$q = \bar{h} A_s(T_s - T_\infty) \qquad (6.12)$$

Equating Equations 6.11 and 6.12, it follows that the average and local convection coefficients are related by an expression of the form

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s \qquad (6.13)$$

Note that for the special case of flow over a flat plate (Figure 6.4*b*), h varies only with the distance x from the leading edge and Equation 6.13 reduces to

$$\bar{h} = \frac{1}{L}\int_0^L h dx \qquad (6.14)$$

Summary of the Boundary layers

	Velocity B.L **$\delta(x)$**	Thermal B.L **$\delta_t(x)$**
Characterized by	velocity gradients and shear stress	temperature gradients and heat transfer
Engineering applications	Surface friction	Convection heat transfer
Key parameter	Friction coeff. 'C_f'	Convection heat transfer coeff. 'h'

Laminar and turbulent Flow

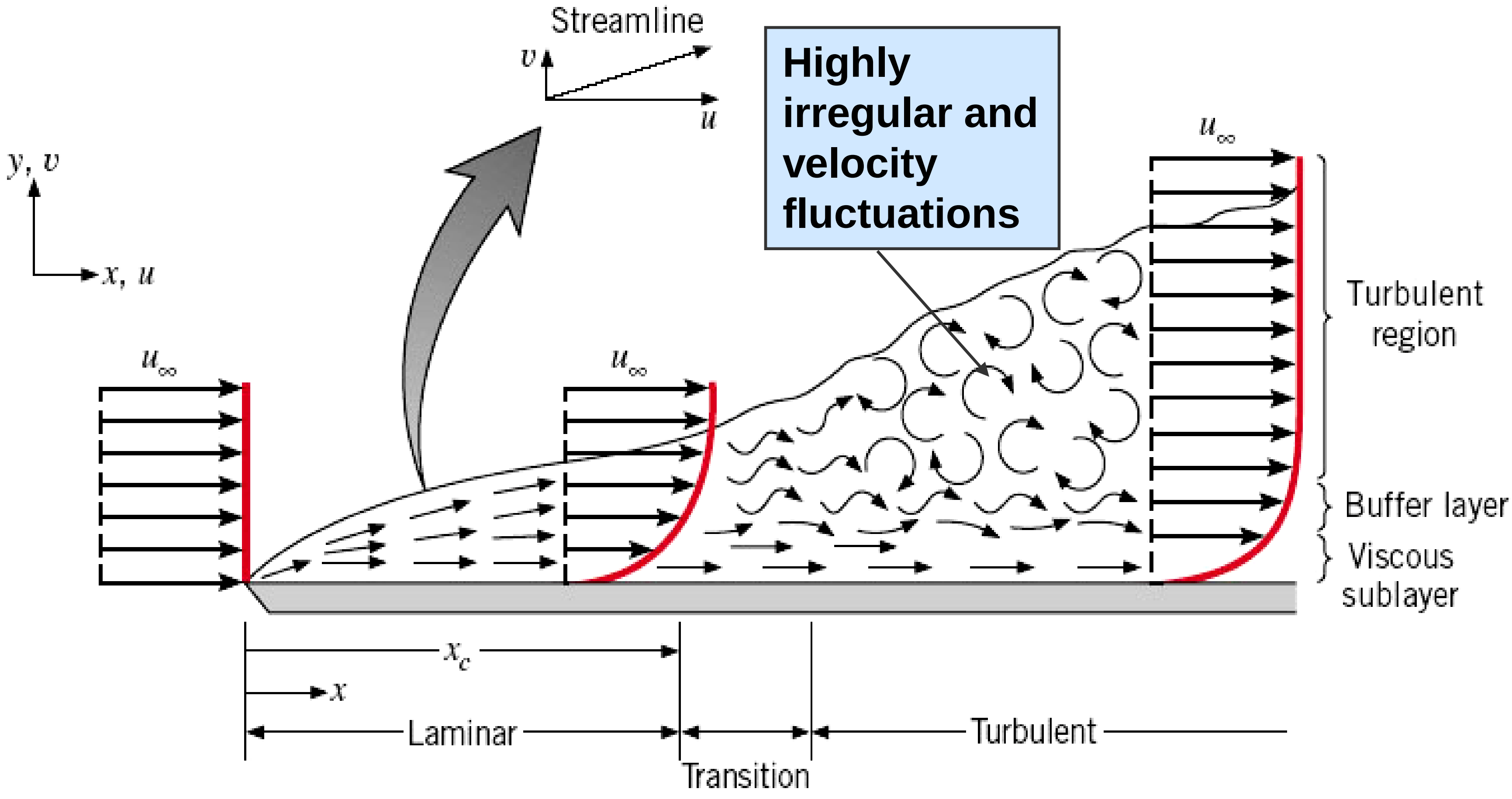


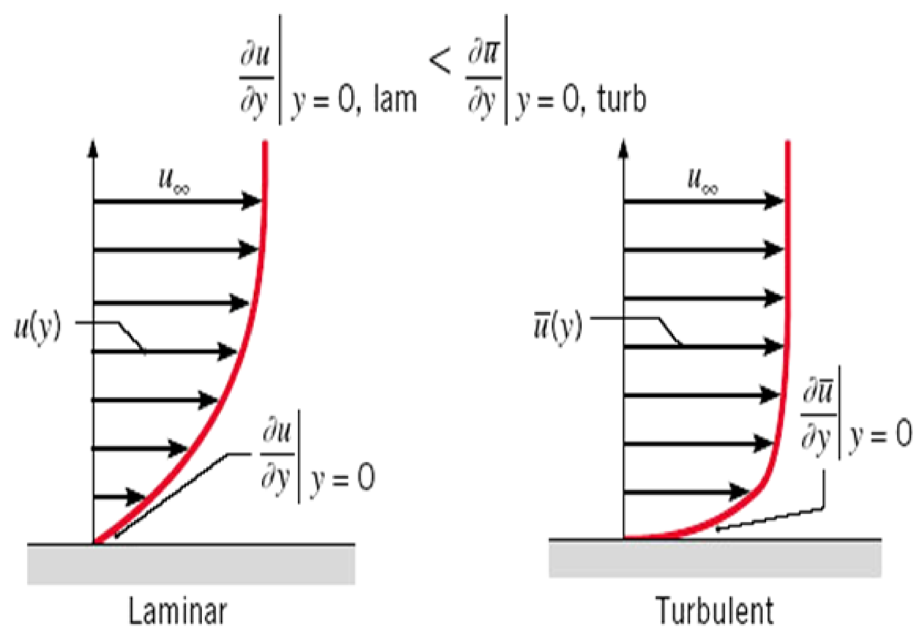
FIGURE 6.6 Velocity boundary layer development on a flat plate.

Summary of the turbulent layer

Regions	Transport by	Velocity profile
1. Laminar sublayer	diffusion	Nearly linear
2. Buffer layer	diffusion + turbulent mixing	Not linear
3. Turbulent zone	Turbulent mixing	Not linear

Comparison of Laminar and turbulent velocity B.L. profiles for the same free stream velocity

The turbulent velocity profile is relatively flat due to mixing that occurs within the buffer and turbulent layers giving rise to large velocity gradients within the viscous sublayer.



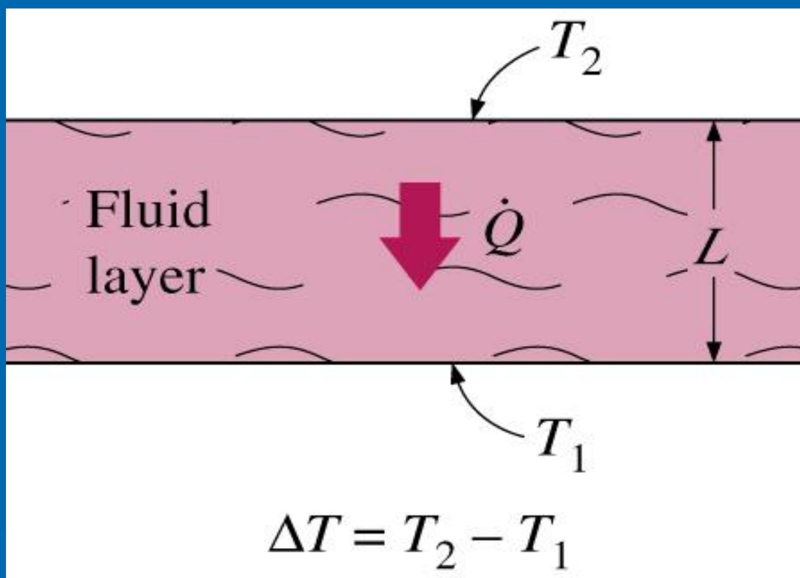
$$\mathrm{Re}_x = \frac{\rho u_\infty x}{\mu}$$

Transition begins at a critical distance, x_c defined by

$$\mathrm{Re}_{x,c} = \frac{\rho u_\infty x_c}{\mu} = 5x10^5$$

$$\tau_s\big|_{\mathrm{turb}} > \tau_s\big|_{\mathrm{lam}}$$

Nusselt Number



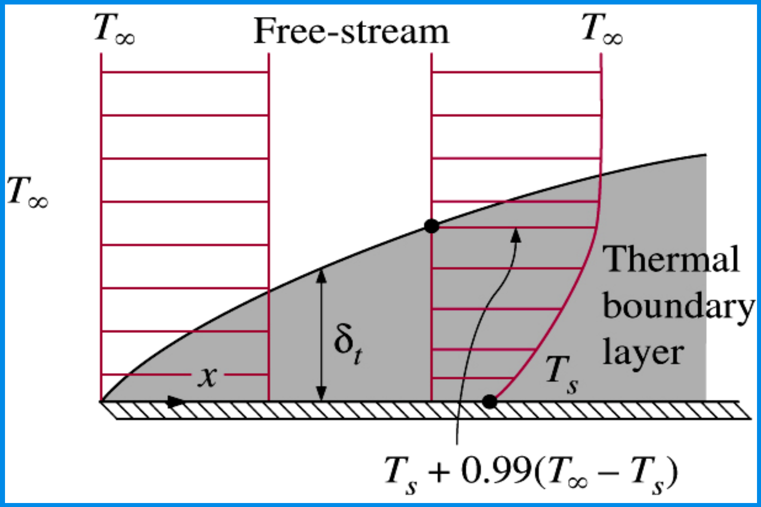
$$\dot{q}_{conv} = h\Delta T$$

$$\dot{q}_{cond} = k\frac{\Delta T}{L}$$

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

The larger the Nusselt number, the more effective the convection. A Nusselt number of Nu 1 for a fluid layer represents heat transfer across the layer by pure conduction.

Boundary layer – temperature & Prandtl Number



The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the thermal boundary layer.

$$\Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

$\mu = \rho \nu$,

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

Boundary Layer Similarity

The normalized Boundary Layer Equations

With the foregoing simplification and approximations, the overall continuity equation and the x-momentum equation reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Also, the energy equation reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

And the species continuity equation becomes

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

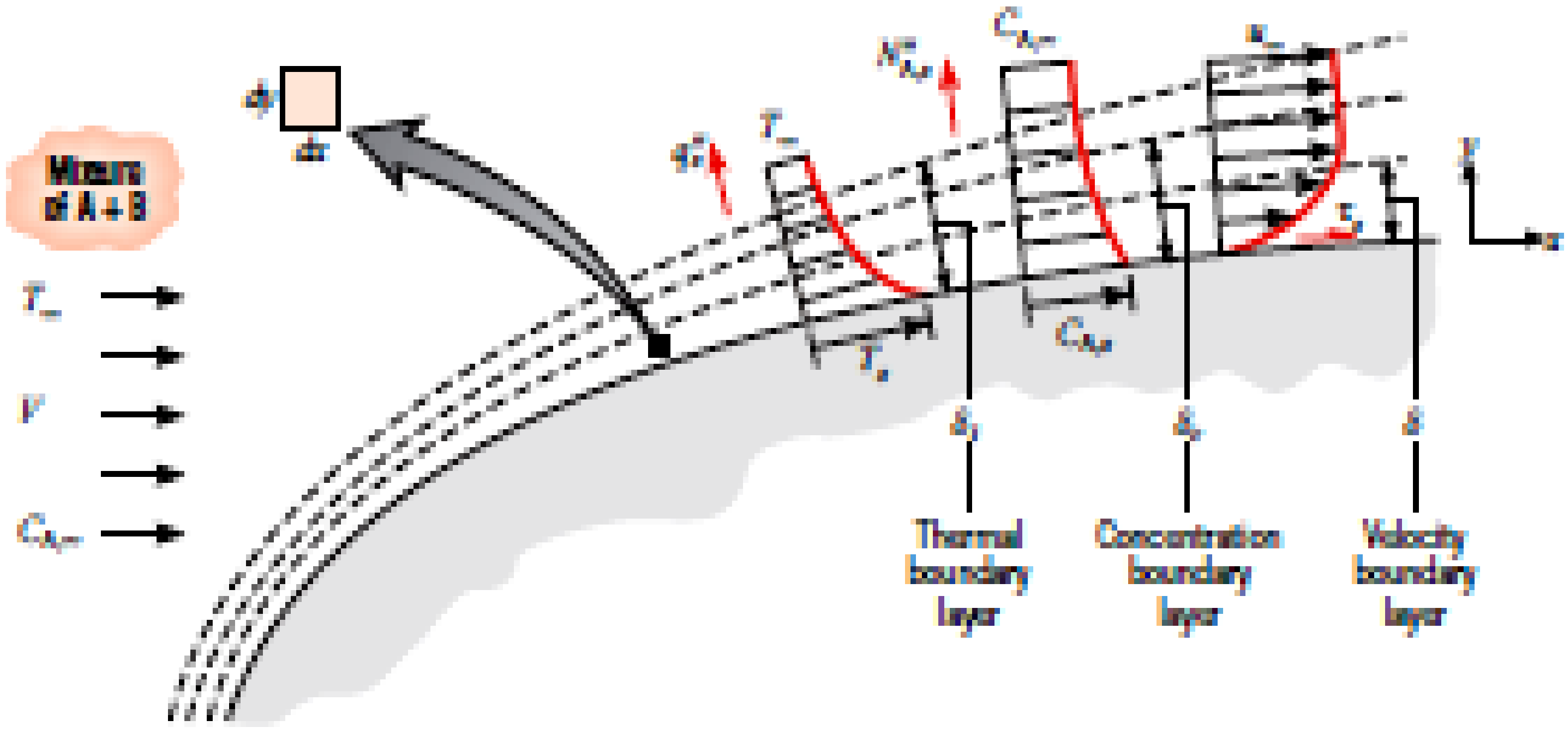


FIGURE 6.9 Development of the velocity, thermal, and concentration boundary layers for an arbitrary surface.

Boundary Layer Similarity Parameter

- Define the following dimensionless variables:

$$x^* = \frac{x}{L} \quad , \quad y^* = \frac{y}{L} \; , \; P^* = \frac{P_\infty}{\rho V^2}$$

$$u^* = \frac{u}{V} \quad , \quad v^* = \frac{v}{V}$$

$$T^* = \frac{T - T_s}{T_\infty - T_s} \quad , \quad C_A^* = \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}}$$

Where L is the characteristic length of the surface, and V is the velocity upstream of the surface.

- Using the above definitions, the velocity and temperature equations become as shown in the next table. Neglect viscous dissipation term.

Similarity Parameters and the dimensionless form of the B.L. Equations

TABLE 6.1 The boundary layer equations and their y-direction boundary conditions in nondimensional form

Boundary Layer	Conservation Equation		Boundary Conditions: Wall	Boundary Conditions: Free Stream		Similarity Parameter(s)	
Velocity	$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$	(6.35)	$u^*(x^*,0)=0$ $v^*(x^*,0)=0$	$u^*(x^*,\infty)=\frac{u_\infty(x^*)}{V}$	(6.38)	$Re_L=\frac{VL}{\nu}$	(6.41)
Thermal	$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$	(6.36)	$T^*(x^*,0)=0$	$T^*(x^*,\infty)=1$	(6.39)	$Re_L,\ Pr=\frac{\nu}{\alpha}$	(6.42)
Concentration	$u^*\frac{\partial C_A^*}{\partial x^*}+v^*\frac{\partial C_A^*}{\partial y^*}=\frac{1}{Re_L Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$	(6.37)	$C_A^*(x^*,0)=0$	$C_A^*(x^*,\infty)=1$	(6.40)	$Re_L,\ Sc=\frac{\nu}{D_{AB}}$	(6.43)

- The following dimensionless parameters are given in table 6.1

Re_L	Reynolds No.
Pr	Prandtl No.
Sc	Schmidt No.

- These parameters allow us to apply results obtained for a surface experiencing one set of convective conditions to geometrically similar surfaces experiencing entirely different conditions.

Functional form of the Solution

- The velocity eq. suggest the following functional forms of solution

Depends on geometry

$$u^* = f\left(x^*, y^*, \mathrm{Re}_L, \tfrac{dp^*}{dx^*}\right) \qquad (i)$$

$$\therefore \tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(\frac{\mu V}{L}\right) \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$\text{and} \qquad C_f = \frac{\tau_s}{\rho V^2/2} = \frac{2}{\mathrm{Re}_L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} \qquad (ii)$$

From eq. (i)

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = f\left(x^*, \mathrm{Re}_L, \tfrac{dp^*}{dx^*}\right)$$

Assume prescribed geometry, eq. (ii) becomes

$$C_f = \frac{2}{\mathrm{Re}_L} f\left(x^*, \mathrm{Re}_L\right)$$

- The thermal eq. suggests the following functional forms of solution

$$T^* = f\left(x^*, y^*, \mathrm{Re}_L, \Pr, \frac{dp^*}{dx^*}\right) \qquad \text{(i)}$$

- Where the dependence on *dp*/dx** originates from the effect of the geometry on the fluid motion (u* and *v**),which, hence, affects the thermal conditions.

$$\therefore\ h = \frac{-k_f\ \partial T/\partial y|_{y=0}}{T_s - T_\infty}$$

$$= -\frac{k_f}{L}\frac{T_\infty - T_s}{T_s - T_\infty}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = \frac{k_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} \qquad \text{(ii)}$$

$\therefore$ equation (ii) can be rearranged as

$$\frac{hL}{k_f} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \quad \text{this called Nusselt number}$$

$$\therefore \; Nu = \frac{hL}{k_f} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \quad \text{dimensionless temp gradient at surface}$$

For a prescribed geometry equqtion (i) becomes

$$Nu = f\,(x^*, \mathrm{Re}_L, \mathrm{Pr}) \qquad \text{(iii)}$$

Equation (iii) shows that Nu is a function of x*, Re_L, and Pr. If this function is known, hence Nu can be computed for various fluids and for various values of V and L. Consequently, the coefficient h can be found from the computed value of Nu.

Average Nusselt number

- As given before the average value for heat transfer coefficient h is evaluated by integrating over the entire surface.
- Therefore, the average coefficient is independent of the spatial variable x*.
- The functional dependence of the average Nusselt number is

$$\overline{Nu} = \frac{\overline{h}L}{k_f} = f(\mathrm{Re}_L, \mathrm{Pr})$$

Physical interpretation of Prandtal number

Since Pr = $c_p \mu / k = \nu / \alpha$

=momentum /thermal diffusivity

This number gives a measure of the relative effectiveness of momentum in the velocity B.L. and energy transport by diffusion in the thermal B.L.

For gases Pr ≈ 1.0 , this means momentum transfer=energy transfer

- For liquid metal Pr <<1 ; this means energy diffusion rate exceeds the momentum diffusion rate
- For oil Pr >>1; this means momentum diffusion rate exceeds the energy diffusion rate.
- In sum, the value of Pr number influences the relative growth rate of the velocity and thermal boundary layers.

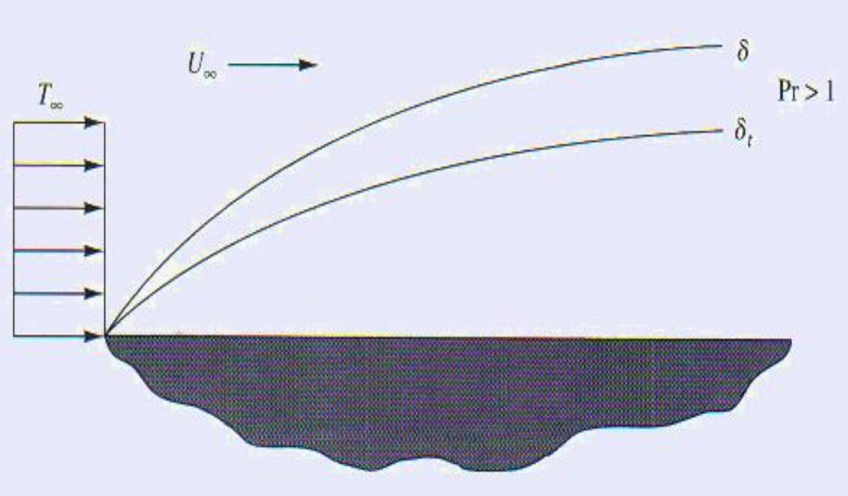
conclusion

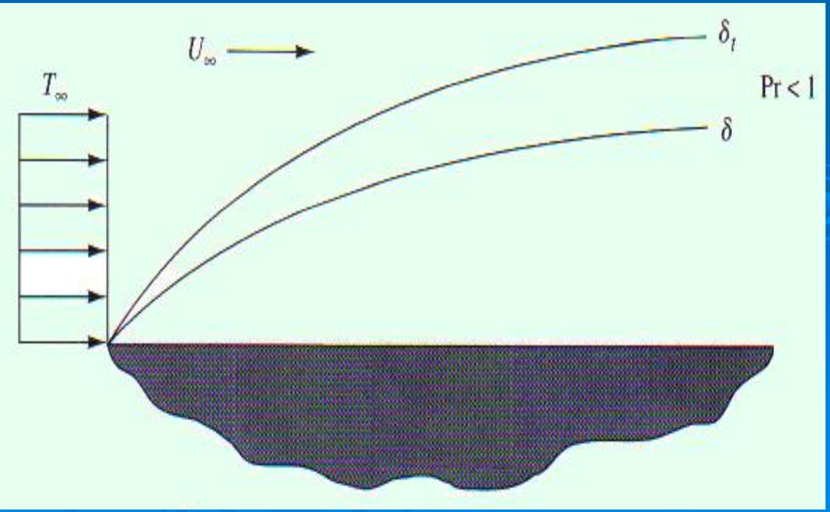
- For Laminar flow (transport by diffusion) it is reasonable to assume that

$$\frac{\delta}{\delta_t} \approx \mathrm{Pr}^n$$

where n is a positive exponent. for a gas $\delta_t \approx \delta$

for a liquid metal $\delta_t >> \delta$, for an oil $\delta_t << \delta$





Boundary Layer analogies
Heat and Mass transfer Analogy

- Definition:

  'If two or more processes are governed by dimensionless equations of the same form, the processes are said to be analogous'.

- The next table (6.3) shows the analogies between Heat and Mass transfer via eq.^{s} (6.47&6.51), (6.48&6.52), (6.49&6.53) and (6.50&6.54)

Summary of the functional relations and B.L. analogies

TABLE 6.3 Functional relations pertinent to the boundary layer analogies

Fluid Flow		Heat Transfer		Mass Transfer	
$u^* = f\left(x^*, y^*, Re_L, \dfrac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \dfrac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \dfrac{dp^*}{dx^*}\right)$	(6.51)
$C_f = \dfrac{2}{Re_L} \dfrac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.45)	$Nu = \dfrac{hL}{k} = +\dfrac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.48)	$Sh = \dfrac{h_m L}{D_{AB}} = +\dfrac{\partial C_A^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.52)
$C_f = \dfrac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$	(6.53)
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$	(6.54)

Conclusion

- If one has performed a set of heat experiments to find the functional form of equation 6.49, for example, the results may be used for the convective mass transfer involving the same geometry. This could be obtained by replacing Nu with Sh and Pr with Sc.
- In general, Nu and Sh are proportional to Pr^n and Sc^n , respectively.

- Use the following analogy equations:

  Nu= $f(x^*, Re_L)$ Pr^n, Sh= $f(x^*, Re_L)$ Sc^n

  in which case, with equivalent functions, $f(x^*, Re_L)$,

$$\frac{Nu}{\Pr^n} = \frac{Sh}{Sc^n}$$

$$\frac{hL/k}{\Pr^n} = \frac{h_m L/D_{AB}}{Sc^n}$$

Or

$$\frac{h}{h_m} = \frac{k}{D_{AB} Le^n} = \rho c_p Le^{1-n}$$

Note

For most engineering applications, assume a value of n=1/3

$$Le = \frac{\alpha}{D_{AB}} = \frac{\rho C_p}{k D_{AB}}$$

$$Sc = \frac{\nu}{D_{AB}}$$

Reynolds Analogy

- This analogy assumes the following:
  $dp^*/dx^* = 0$ and $Pr = Sc = 1$.
  and for a flat surface $u_\infty = V$
- Hence, the velocity, the thermal and the concentration Equations and boundary conditions become analogous and the functional form of the solutions for u^*, T^*, and C^*, eqs. 6.44, 6.47, and 6.51 are equivalent.

- From eqs. 6.45, 6.48 and 6.52 it follows that (see table 6.3)

$$C_f \frac{\mathrm{Re}_L}{2} = Nu = Sh \qquad (6.66)$$

- Replacing Nu and Sh by the Stanton number, St, and the mass transfer Stanton number, St_m, respectively,

$$St = \frac{h}{\rho V c_p} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}}$$

$$St_m = \frac{h_m}{V} = \frac{Sh}{\mathrm{Re}\,Sc}$$

TABLE 6.3 Functional relations pertinent to the boundary layer analogies

Fluid Flow		Heat Transfer		Mass Transfer	
$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$	(6.51)
$C_f = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.45)	$Nu = \frac{hL}{k} = +\frac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.48)	$Sh = \frac{h_m L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.52)
$C_f = \frac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$	(6.53)
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$	(6.54)

- Eq. 6.66 may be expressed as

$$\frac{C_f}{2} = St = St_m \qquad \Pr = Sc = 1$$

$$\text{and} \quad dp^*/dx^* = 0$$

- The modified Reynolds, or Chilton-Colburn, analogies

$$\frac{C_f}{2} = St\,Pr^{2/3} = j_H \qquad 0.6 < Pr < 60$$

$$\frac{C_f}{2} = St_m\,Sc^{2/3} = j_m \qquad 0.6 < Sc < 3000$$

TABLE 6.2 Selected dimensionless groups of heat and mass transfer

Group	Definition	Interpretation
Biot number (Bi)	$\frac{hL}{k_s}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance.
Mass transfer Biot number (Bi_m)	$\frac{h_m L}{D_{AB}}$	Ratio of the internal species transfer resistance to the boundary layer species transfer resistance.
Bond number (Bo)	$\frac{g(\rho_l - \rho_v)L^2}{\sigma}$	Ratio of gravitational and surface tension forces.
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho V^2/2}$	Dimensionless surface shear stress.
Eckert number (Ec)	$\frac{V^2}{c_p(T_s - T_\infty)}$	Kinetic energy of the flow relative to the boundary layer enthalpy difference.
Fourier number (Fo)	$\frac{\alpha t}{L^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid. Dimensionless time.
Mass transfer Fourier number (Fo_m)	$\frac{D_{AB} t}{L^2}$	Ratio of the species diffusion rate to the rate of species storage. Dimensionless time.
Friction factor (f)	$\frac{\Delta p}{(L/D)(\rho u_m^2/2)}$	Dimensionless pressure drop for internal flow.
Grashof number (Gr_L)	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	Measure of the ratio of buoyancy forces to viscous forces.
Colburn j factor (j_H)	$St\, Pr^{2/3}$	Dimensionless heat transfer coefficient.
Colburn j factor (j_m)	$St_m\, Sc^{2/3}$	Dimensionless mass transfer coefficient.
Jakob number (Ja)	$\frac{c_p(T_s - T_{sat})}{h_{fg}}$	Ratio of sensible to latent energy absorbed during liquid–vapor phase change.
Lewis number (Le)	$\frac{\alpha}{D_{AB}}$	Ratio of the thermal and mass diffusivities.
Nusselt number (Nu_L)	$\frac{hL}{k_f}$	Ratio of convection to pure conduction heat transfer.
Peclet number (Pe_L)	$\frac{VL}{\alpha} = Re_L Pr$	Ratio of advection to conduction heat transfer rates.
Prandtl number (Pr)	$\frac{c_p \mu}{k} = \frac{\nu}{\alpha}$	Ratio of the momentum and thermal diffusivities.

Table 6.2 *Continued*

Group	Definition	Interpretation
Reynolds number (Re_L)	$\frac{VL}{\nu}$	Ratio of the inertia and viscous forces.
Schmidt number (Sc)	$\frac{\nu}{D_{AB}}$	Ratio of the momentum and mass diffusivities.
Sherwood number (Sh_L)	$\frac{h_m L}{D_{AB}}$	Dimensionless concentration gradient at the surface.
Stanton number (St)	$\frac{h}{\rho V c_p} = \frac{Nu_L}{Re_L\, Pr}$	Modified Nusselt number.
Mass transfer Stanton number (St_m)	$\frac{h_m}{V} = \frac{Sh_L}{Re_L Sc}$	Modified Sherwood number.
Weber number (We)	$\frac{\rho V^2 L}{\sigma}$	Ratio of inertia to surface tension forces.