Internal flow

Laminar & Turbulent regimes

Laminar hydrodynamic B.L. development in a circular tube

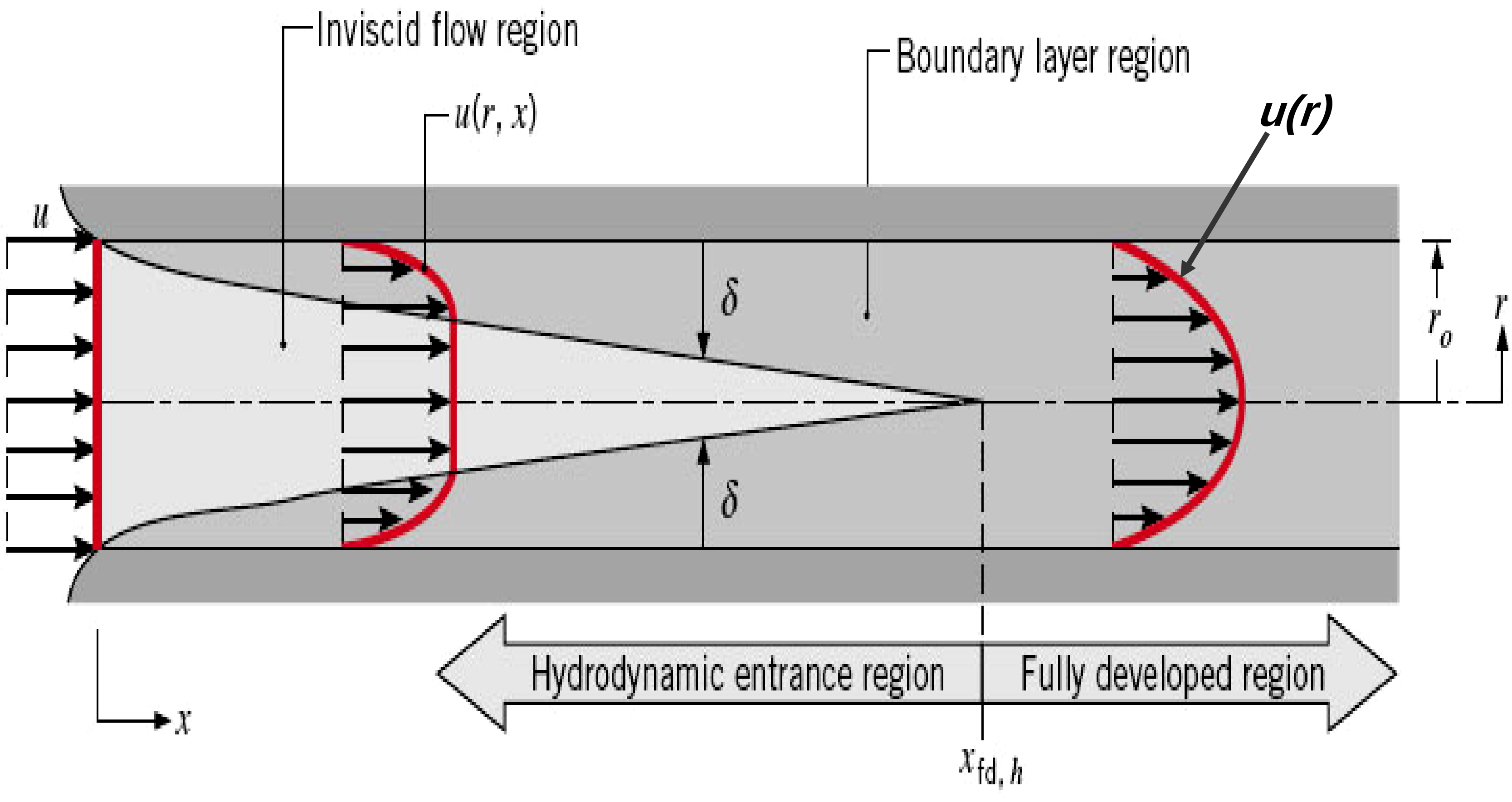


FIGURE 8.1 Laminar, hydrodynamic boundary layer development in a circular tube.

Velocity Boundary layer Laminar Flow

- For Laminar flow $Re_D \leq 2300$
- $Re = \rho u_m D/\mu = u_m D/\nu$
- The velocity profile is commonly flat near the entrance of the pipe and function of both x and r.
- The distance from the entrance to the fully developed is called the hydrodynamic entrance length, x_h.

- In general, the entrance length, $x_h = f(Re_D)$ and

$$(x/D)_{lam} \approx 0.05\ Re_D$$

- A force balance on a differential fluid element gives a parabolic velocity profile for fully developed laminar flow. See the text for more details.

Velocity Profile for laminar, Fully developed Region

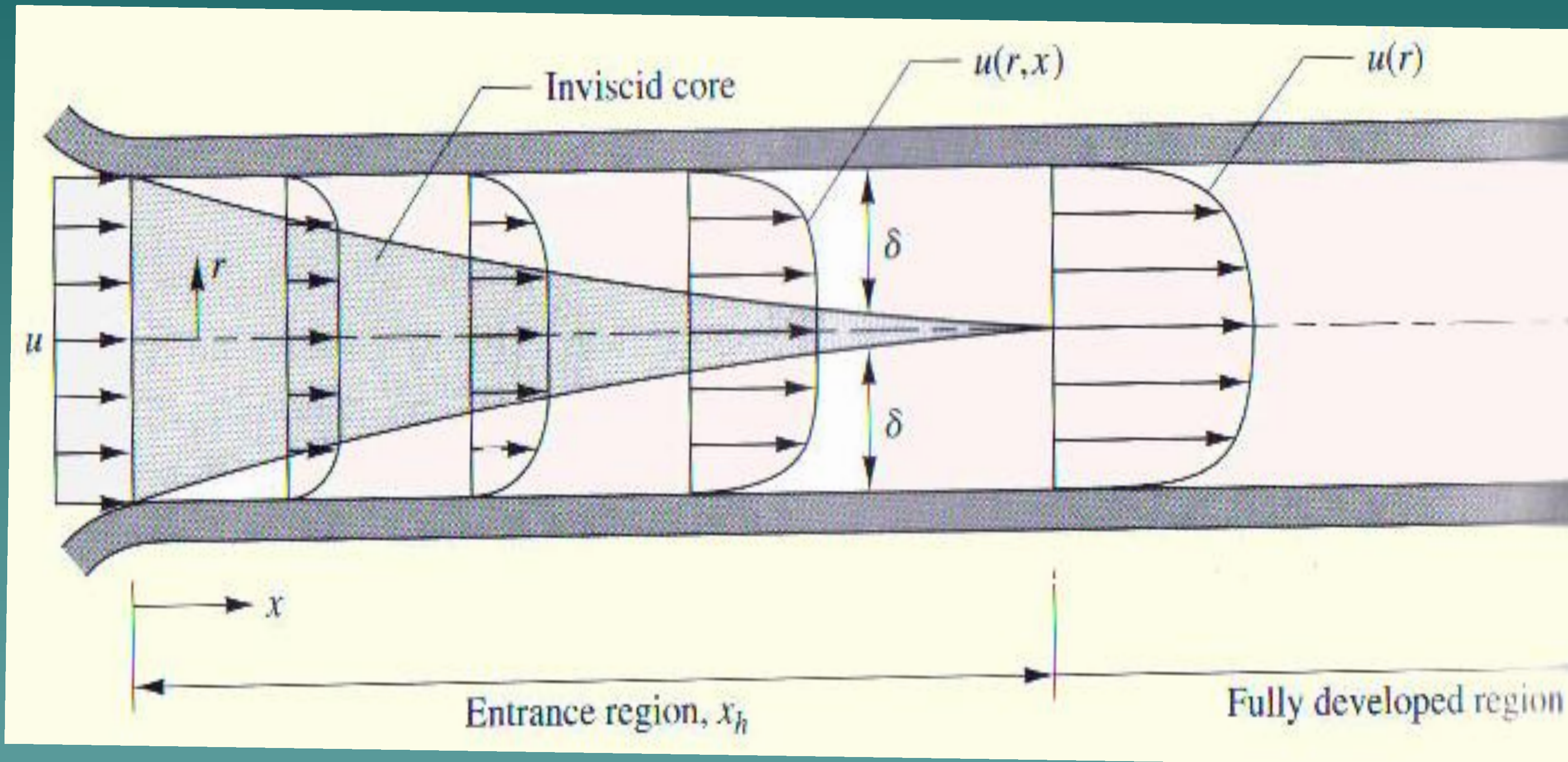
$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right)r_o^2\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx} \qquad \text{where } u_m \text{ is mean velocity}$$

$$\frac{u(r)}{u_m} = 2\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

For details see the text

Turbulent hydrodynamic B.L. in a circular tube



Turbulent condition

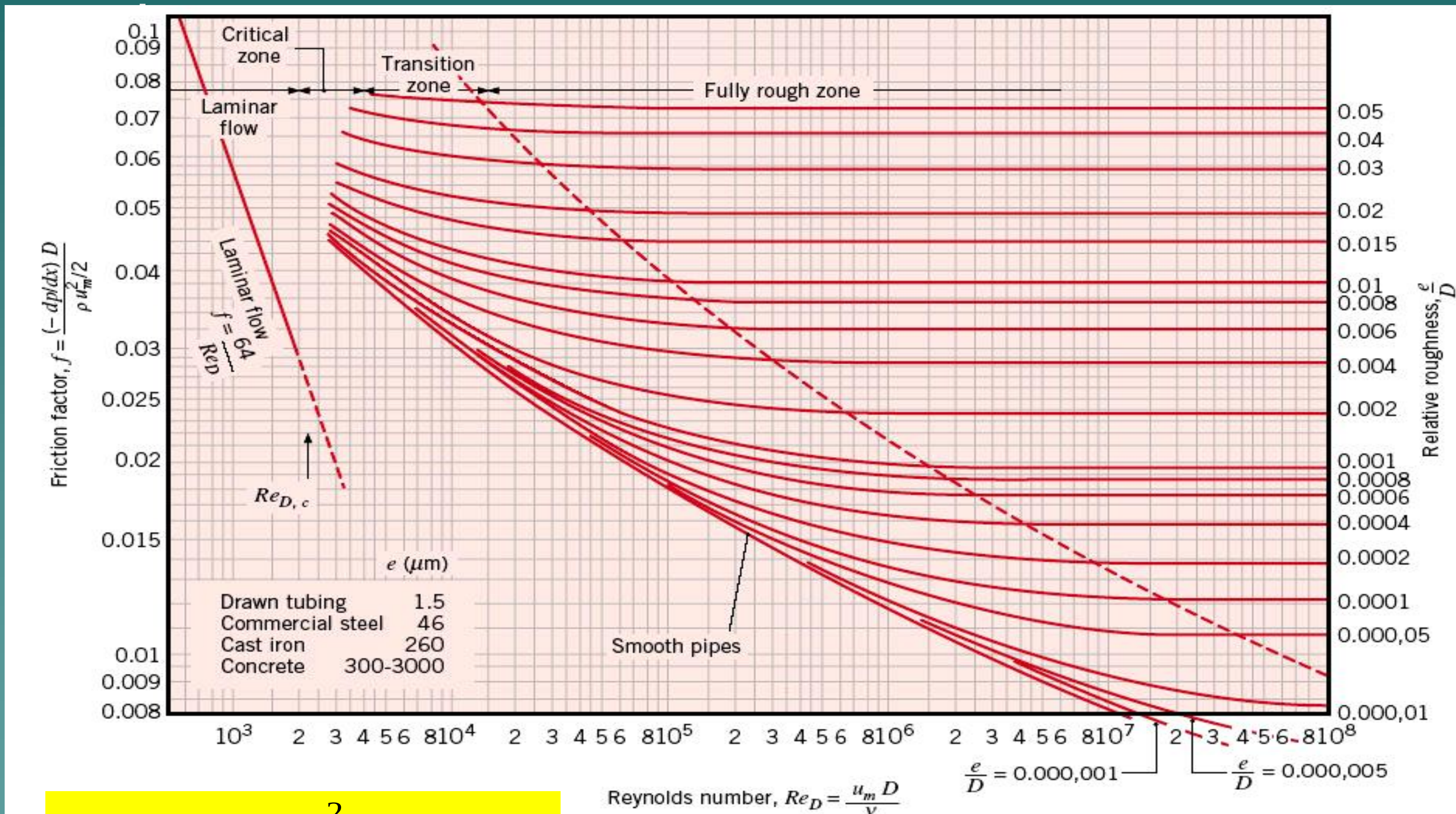
- Similar fashion as laminar B.L.
- Unlike laminar flow, the entrance length, X_h is not function in Re_D .
- Experiments have shown that

$$10 < \left(\frac{x_h}{D}\right) < 60$$

- Velocity profile for fully developed turbulent flow is not parabolic and is flatter due to turbulent mixing in the radial direction.
- Velocity profile is difficult to be obtained analytically, therefore it must be determined experimentally.
- We shall assume that fully developed occurs at $(x/D)>10$

Pressure gradient and friction factor

- Correlations are given in the course of fluid mechanics and also in the text. See



$$\Delta p = -f\,\frac{\rho u_m^2}{2D}(x_2 - x_1)$$

$$P = (\Delta p)(\dot{m}/\rho) \quad \text{Power}$$

Thermal B.L. development in a heated circular tube

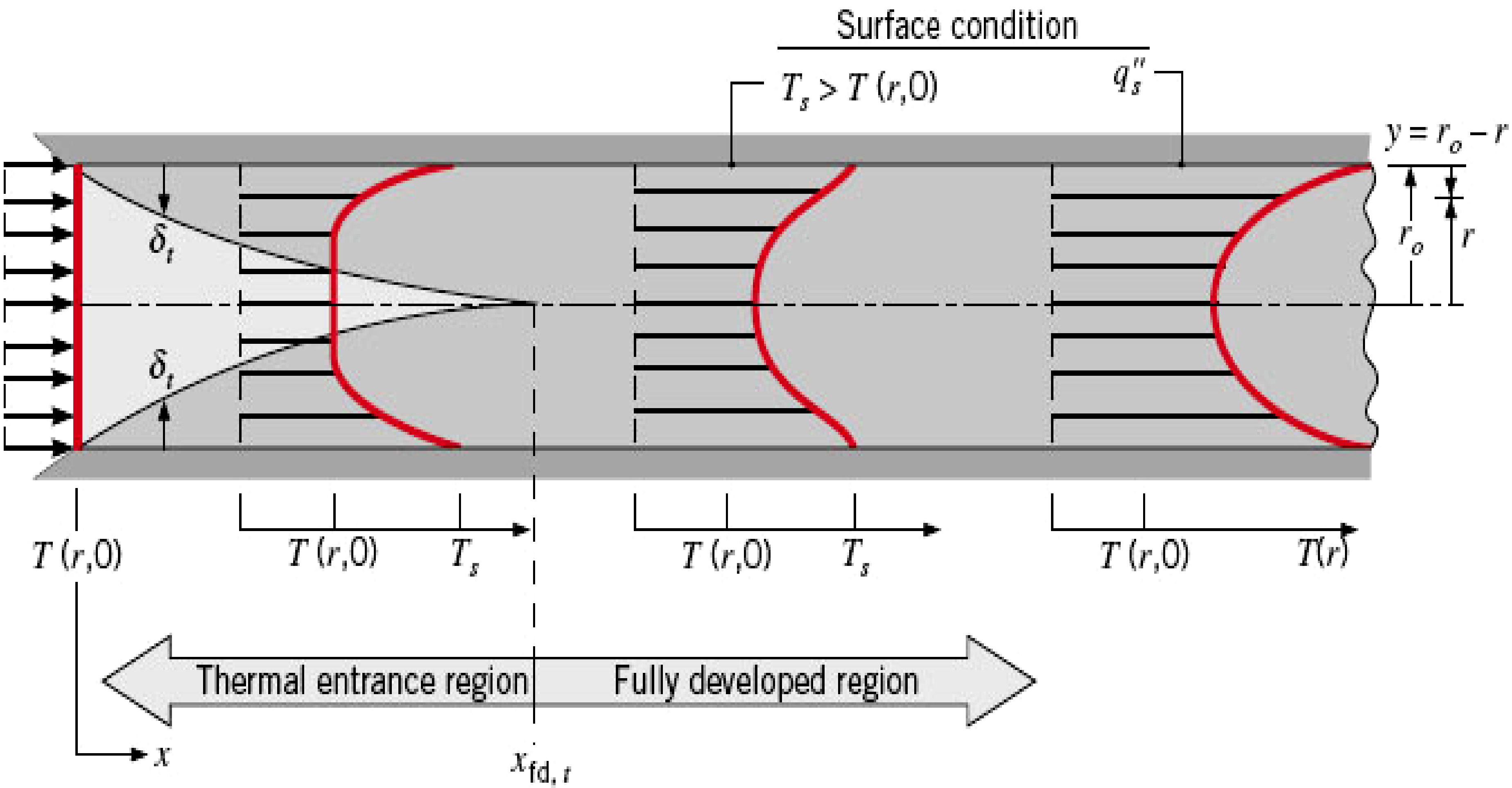


FIGURE 8.4 Thermal boundary layer development in a heated circular tube.

Thermal Boundary layer

- In general, the fluid temperature increases with x in both cases (constant surface temperature or uniform heat flux).
- For laminar flow, the thermal entrance length, x_t; $x_t = f(Re, Pr)$

$$\frac{x_t}{D} \approx 0.05\,\mathrm{Re}_D\,\mathrm{Pr}$$

- If Pr >1, the hydrodynamic B.L. developed more rapidly than the thermal B.L. while the inverse if Pr <1.
- For turbulent, $x_t \neq f(Re,Pr) \sim (x_t/D) > 10$ *or assume* $(x_t/D) = 10$

Thermal energy balance

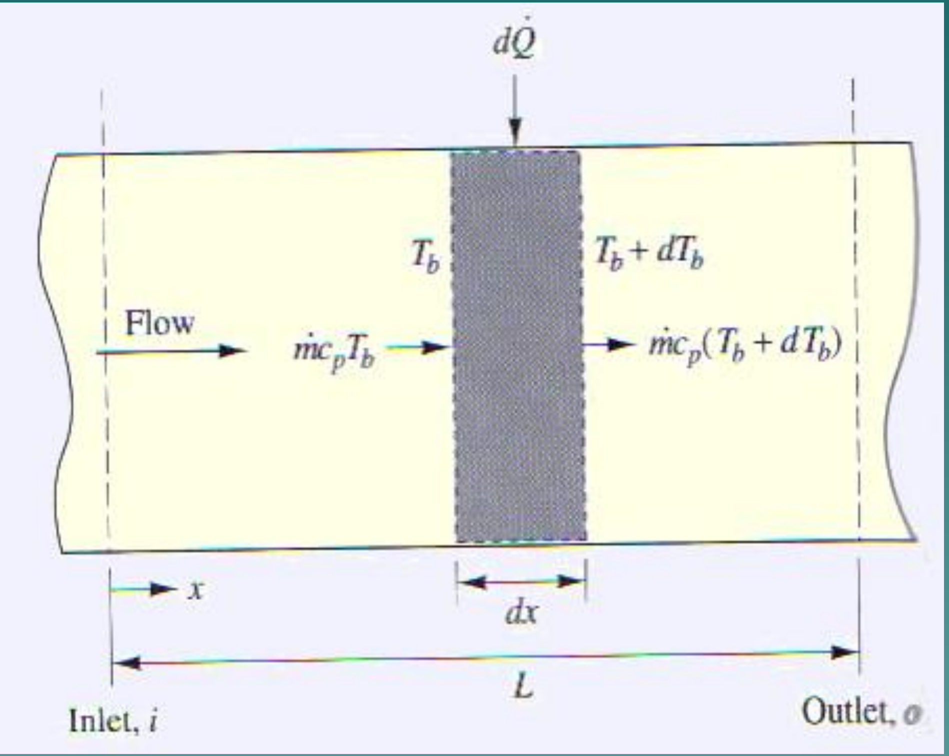
- Assume a differential control volume for internal flow as shown
- Energy balance

energy input=energy output

$$\dot{m}c_p T_b + d\dot{Q} = \dot{m}c_p(T_b + dT_b) \quad (i)$$

Where T_b : bulk or mean temp of fluid.

$$T_b = \frac{2}{u_m r_o^2}\int_0^{r_o} u(r)T(r)r\,dr$$



•Integrating eq. (i) over the length L

$$\dot{Q} = \dot{m}c_p(T_{b,o} - T_{b,i}) \quad (ii)$$

According to Newton's Law of cooling,

$$q_x^{''}=h_x(T_s-T_b) \qquad \text{(iii)}$$

Eq. (iii) can be rewritten as

$$dq = q_x^{''}dA = hPdx(T_s-T_b) \qquad \text{(iv)}$$

Or

$$\dot{m}c_p\, dT_b = hPdx(T_s-T_b)$$

$$\therefore \quad \frac{dT_b}{dx} = \frac{P}{\dot{m}c_p} h(T_s-T_b) \qquad \text{(v)}$$

Where P is the perimeter of inside pipe surface; for circular pipe $P = \pi D$

Note: The variation of T_b with x depends on whether the flow is fully developed or not and whether the pipe surface is isothermal or has a uniform heat flux.

Isothermal surface

Integrating Eq. (v), we obtain

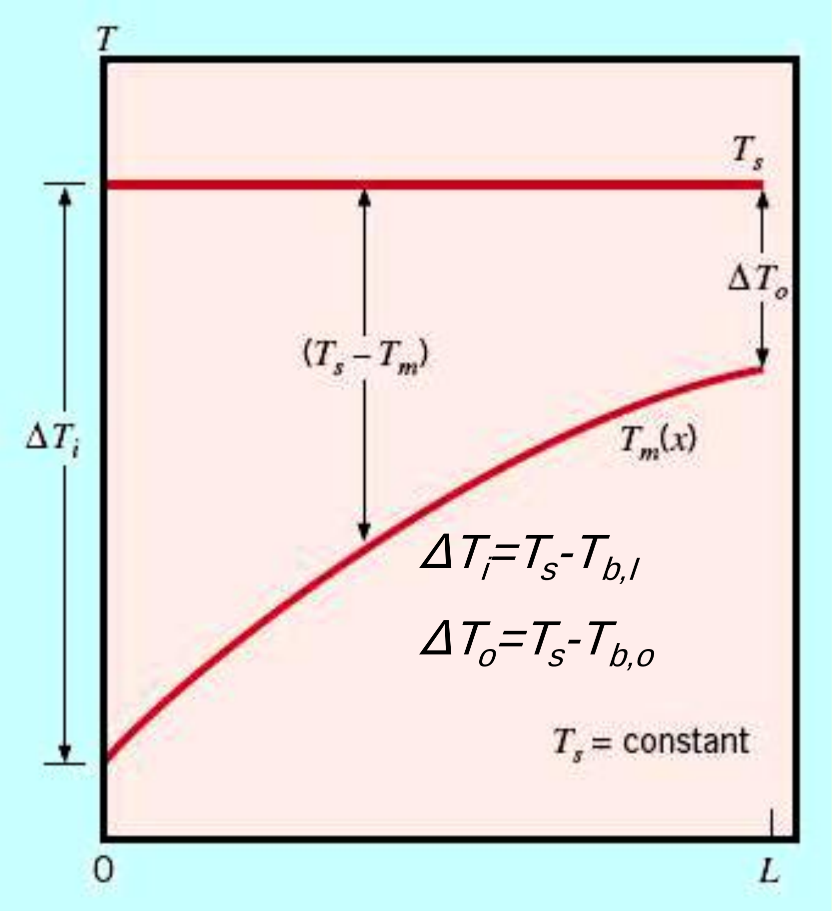
$$\int_{T_{b,i}}^{T_{b,o}} \frac{dT_b}{(T_s - T_b)} = \frac{P\bar{h}}{\dot{m}c_p}\int_0^x dx$$

$$\ln\left(\frac{T_s - T_{b,o}}{T_s - T_{b,i}}\right) = -\frac{P\bar{h}x}{\dot{m}c_p}$$

$$\therefore \frac{T_s - T_{b,o}}{T_s - T_{b,i}} = \exp\left(-\frac{P\bar{h}L}{\dot{m}c_p}\right) \qquad \text{(vi)}$$

Axial temperature variation for heat transfer in a tube **for constant surface temperature**

- $\Delta T = T_s - T_b$ decreases exponentially with distance from the pipe inlet as suggested by the eq.(vi).



- The convection heat transfer given by Newton's Law of cooling is:

$$q = \bar{h} A_i \Delta T_{lm}$$

$$\text{where} \quad \Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln[\Delta T_o / \Delta T_i]}$$

A_i : inside surface area of pipe

- **<u>Note:</u>** Special case but frequently occurring case.

  Suppose the temp. of the fluid on the outside of the duct is constant rather than the duct surface itself. In this case T_s is replaced by T_∞, the free-stream fluid temperature and the average H.T.C., $\bar{h}$, is replaced by $\bar{U}$, the average overall H.T.C. Hence eq. (vi) becomes

$$\frac{T_\infty - T_{b,o}}{T_\infty - T_{b,i}} = \exp\left(-\frac{A_i \bar{U}}{\dot{m} c_p}\right) \qquad \text{(vii)}$$

$$q = \bar{U} A \Delta T_{lm} \qquad \text{(viii)}$$

where $\bar{\text{U}}$ is based on either the inside surface area or outside surface area, A, and

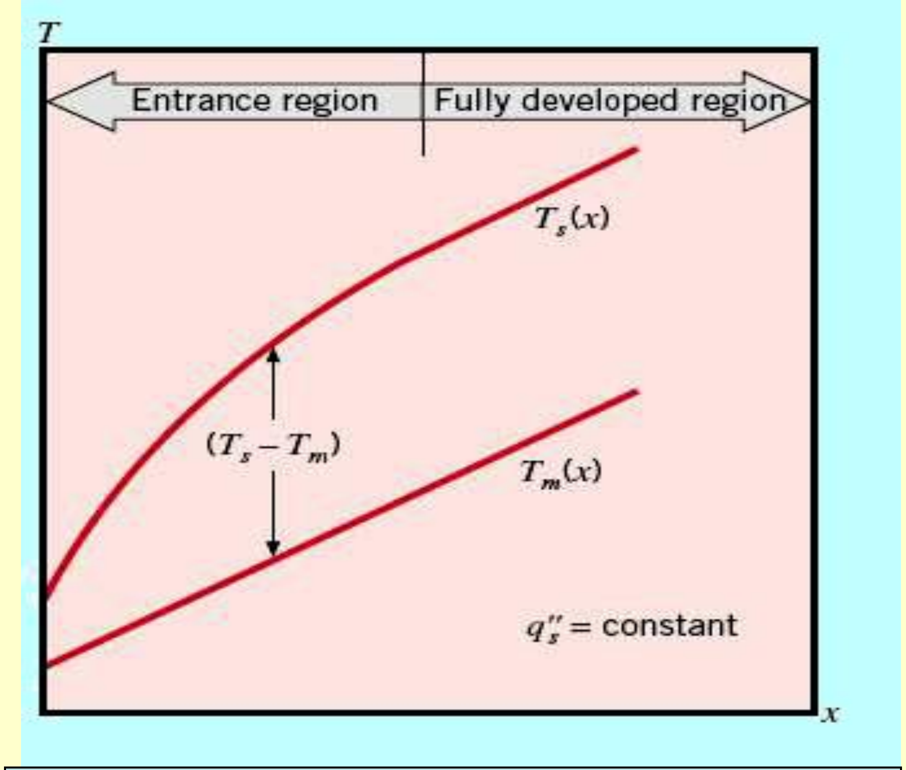
$$\Delta T_{lm} = \frac{(T_\infty - T_{b,o}) - (T_\infty - T_{b,i})}{\ln[(T_\infty - T_{b,o})/(T_\infty - T_{b,i})]}$$

Uniform Heat flux

From eq. (v) $\dot{m}c_p dT_b = q_s^{"} P dx$

Integrating this eq. yields:

$$T_{b,x} = T_{b,i} + \frac{q_s^{"} P x}{\dot{m} c_p}$$



- The temp. difference $(T_s\text{-}T_b)$ varies with x , but only in entrance region where the thermal B.L. is developing.
- In fully developed region, h is independent of x. Hence,

$(T_s\text{-}T_b)$ is also independent of x. Hence, $(T_s\text{-}T_b)$ is constant in fully Developed region.

Fully developed Laminar Flow

- H.T.C for fully developed laminar flow in a pipe can be found analytically by solving the following energy balance equation:

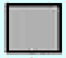
$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

- Subjected to proper B.C.[s] The eq. can be solved for two conditions: (i) isothermal surface and (ii) uniform heat flux surface.
- In general, for fully developed velocity and thermal B.L. in a duct of a given cross section, the Nu is constant, independent of axial position in the duct, Re and Pr.

- Hence, for fully developed laminar flow

$$Nu_D = \frac{hD_h}{k} = \text{constant}$$

Table 8.1 Nu and friction factors for fully developed laminar flow in tubes of differing cross sections

Cross Section	$\frac{b}{a}$	$Nu_D = \frac{hD_h}{k}$ (Uniform q_s'')	$Nu_D = \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
 a, b	1.0	3.61	2.98	57
 a, b	1.43	3.73	3.08	59
 a, b	2.0	4.12	3.39	62

The entry region for a Laminar flow in a circular tube

- The following correlation can be used:

$$\overline{Nu}_D = 1.86\left(\frac{Re_D \Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$\left[\left(\frac{Re_D \Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}\right] > 2$$

$$0.0044 \le \left(\frac{\mu}{\mu_s}\right) \le 9.75$$

$$0.48 < \Pr < 16{,}700$$

- The correlation applies to circular ducts with an isothermal surface.
- Fluid properties are evaluated at the average bulk temperature except μ_s which is evaluated at surface temperature.

Heat Transfer coefficients for Turbulent Flow

- For complexity of turbulent regime, empirical methods are most common.
- Sine the hydrodynamic and thermal entrance length are relatively short, it can be assumed that fully developed flow exists over the entire length of the duct.
- Hence, $\overline{N}u_D\Big|_{\text{entire length}} = \overline{N}u_D\Big|_{\text{fully developed region}}$
- The following formula for fully turbulent flow in a smooth pipe can be used:

$$\overline{Nu}_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n$$

Where $n = 0.4$ fluid heated $(T_s > T_b)$

$n = 0.3$ fluid cooled $(T_s < T_b)$

$0.7 < \mathrm{Pr} < 160 \qquad L/D \geq 10$

$\mathrm{Re} > 10{,}000$

- Note 1: the formula should be used only when the duct surface temp. and fluid temp. do not differ greatly
- Note 2: the formula is valid for both isothermal surface and uniform heat flux surface.

- For appreciable difference between T_s and T_b , the following formula is recommended:

$$Nu_D = 0.027 \mathrm{Re}_D^{4/5} \mathrm{Pr}^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$0.7 < \mathrm{Pr} < 16{,}700 \qquad \mathrm{Re} > 10{,}000$$

The formula is valid for isothermal surface and uniform heat flux.

- More accurate correlation for rough and smooth ducts is:

$$Nu_D = \frac{(f/8)(\mathrm{Re}_D - 1000)\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)}\left[1 + \left(\frac{D}{L}\right)^{2/3}\right]$$

$$0.6 < \mathrm{Pr} < 2000, \qquad \mathrm{Re} > 2300$$

- Where f is the friction factor (from Moody diagram)
- If large temperature differences exist between the fluid and the surface the Nu_D in the above equation must be multiplied by the following correction factor:

Gases	$\left(\frac{\bar{T}_b}{T_s}\right)^{0.04}$
Liquid	$\left(\frac{Pr}{Pr_s}\right)^{0.11}$

- In order to use the previous eq. for fully developed flow, set (D/L) to be zero.

- The previous eq. can be used for circular duct and noncircular ducts as well as pipes and tubes. On other words, no effect of cross-section shape on Nu.
- For Liquid metals, it is recommended

$$Nu_D = 4.8 + 0.0156\,\mathrm{Re}_D^{0.85}\,\mathrm{Pr}^{0.93} \qquad \mathrm{Pr} << 1$$

for the isothermal surface

$$Nu_D = 6.3 + 0.0167\,\mathrm{Re}_D^{0.85}\,\mathrm{Pr}^{0.93} \qquad \mathrm{Pr} << 1$$

for the uniform heat flux.