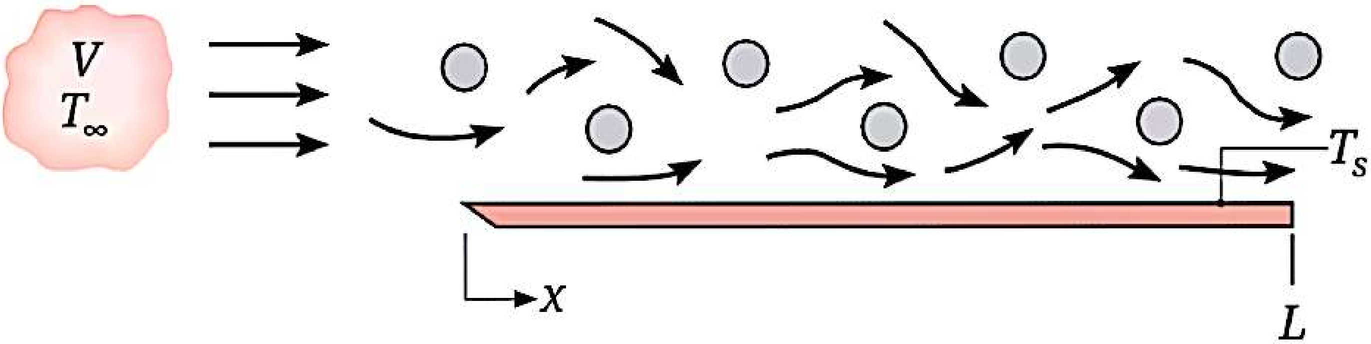
6.7 Parallel flow of atmospheric air over a flat plate of length $L = 3$ m is disrupted by an array of stationary rods placed in the flow path over the plate.



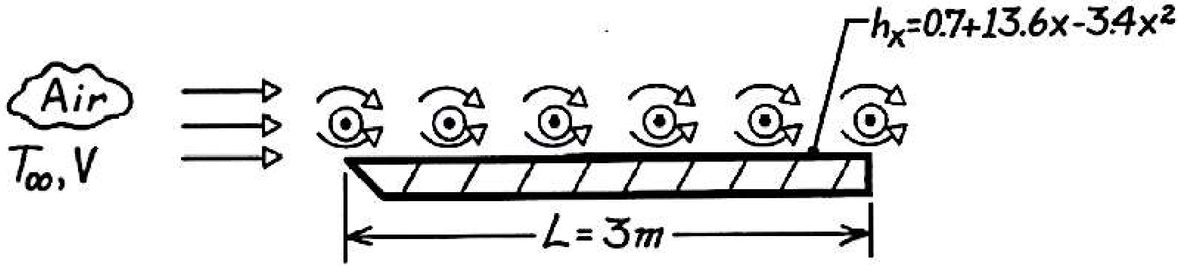
Laboratory measurements of the local convection coefficient at the surface of the plate are made for a prescribed value of V and $T_s > T_\infty$. The results are correlated by an expression of the form $h_x = 0.7 + 13.6x - 3.4x^2$, where h_x has units of $W/m^2 \cdot K$ and x is in meters. Evaluate the average convection coefficient $\bar{h}_L$ for the entire plate and the ratio $\bar{h}_L/h_L$ at the trailing edge.

PROBLEM 6.7

KNOWN: Distribution of local convection coefficient for obstructed parallel flow over a flat plate.

FIND: Average heat transfer coefficient and ratio of average to local at the trailing edge.

SCHEMATIC:



ANALYSIS: The average convection coefficient is

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left(0.7 + 13.6x - 3.4x^2\right) dx$$

$$\bar{h}_L = \frac{1}{L}\left(0.7L + 6.8L^2 - 1.13L^3\right) = 0.7 + 6.8L - 1.13L^2$$

$$\bar{h}_L = 0.7 + 6.8(3) - 1.13(9) = 10.9 \text{ W/m}^2 \cdot \text{K}. \qquad <$$

The local coefficient at x = 3m is

$$h_L = 0.7 + 13.6(3) - 3.4(9) = 10.9 \text{ W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h}_L / h_L = 1.0. \qquad <$$

COMMENTS: The result $\bar{h}_L / h_L = 1.0$ is unique to x = 3m and is a consequence of the existence of a maximum for $h_x(x)$. The maximum occurs at x = 2m, where $(dh_x / dx) = 0$ and $\left(d^2h_x / dx^2 < 0.\right)$

6.10 Experiments have been conducted to determine local heat transfer coefficients for flow perpendicular to a long, isothermal bar of rectangular cross section. The bar is of width c, parallel to the flow, and height d, normal to the flow. For Reynolds numbers in the range $10^4 \leq Re_d \leq 5 \times 10^4$, the *face-averaged* Nusselt numbers are well correlated by an expression of the form

$$Nu_d = hd / k = C Re_d^m Pr^{1/3}$$

The values of C and m for the front face, side faces, and back face of the rectangular rod are found to be the following:

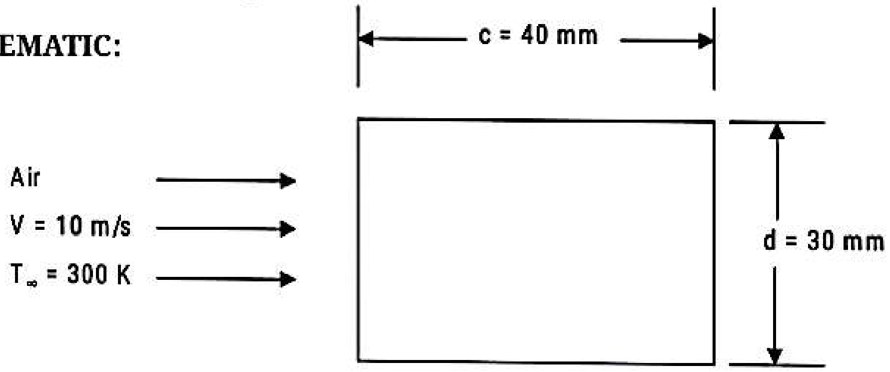
Face	c/d	C	m
Front	$0.33 \leq c/d \leq 1.33$	0.674	1/2
Side	0.33	0.153	2/3
Side	1.33	0.107	2/3
Back	0.33	0.174	2/3
Back	1.33	0.153	2/3

PROBLEM 6.10

KNOWN: Expression for face-averaged Nusselt numbers on a cylinder of rectangular cross section. Dimensions of the cylinder.

FIND: Average heat transfer coefficient over the entire cylinder. Plausible explanation for variations in the face-averaged heat transfer coefficients.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

PROPERTIES: Table A.4, air (300 K): k = 0.0263 W/m·K, $\nu = 1.589 \times 10^{-5}$ m²/s, Pr = 0.707.

ANALYSIS:

For the square cylinder, c/d = 40 mm/30 mm = 1.33

$$Re_d = \frac{Vd}{\nu} = \frac{10\text{ m/s} \times 30 \times 10^{-3}\text{ m}}{1.589 \times 10^{-5}\text{ m}^2\text{/s}} = 18{,}880$$

Therefore, for the front face C = 0.674, m = ½. For the sides, C = 0.107, m = 2/3 while for the back C = 0.153, m = 2/3.

Front face:

$$Nu_{d,f} = 0.674 \times 18{,}880^{1/2} \times 0.707^{1/3} = 82.44$$

$$\bar{h}_f = \frac{kNu_d}{d} = \frac{0.0263\text{ W/m}\cdot\text{K} \times 82.44}{30 \times 10^{-3}\text{ m}} = 72.27\text{ W/m}^2\cdot\text{K}$$

Side faces:

$$Nu_{d,s} = 0.107 \times 18{,}880^{2/3} \times 0.707^{1/3} = 67.36$$

$$\bar{h}_s = \frac{kNu_{d,s}}{d} = \frac{0.0263\text{ W/m}\cdot\text{K} \times 67.36}{30 \times 10^{-3}\text{ m}} = 59.05\text{ W/m}^2\cdot\text{K}$$

Back face:

$$Nu_{d,b} = 0.153 \times 18{,}880^{2/3} \times 0.707^{1/3} = 96.43$$

$$\bar{h}_b = \frac{kNu_{d,b}}{d} = \frac{0.0263\text{ W/m}\cdot\text{K} \times 96.43}{30 \times 10^{-3}\text{ m}} = 84.54\text{ W/m}^2\cdot\text{K}$$

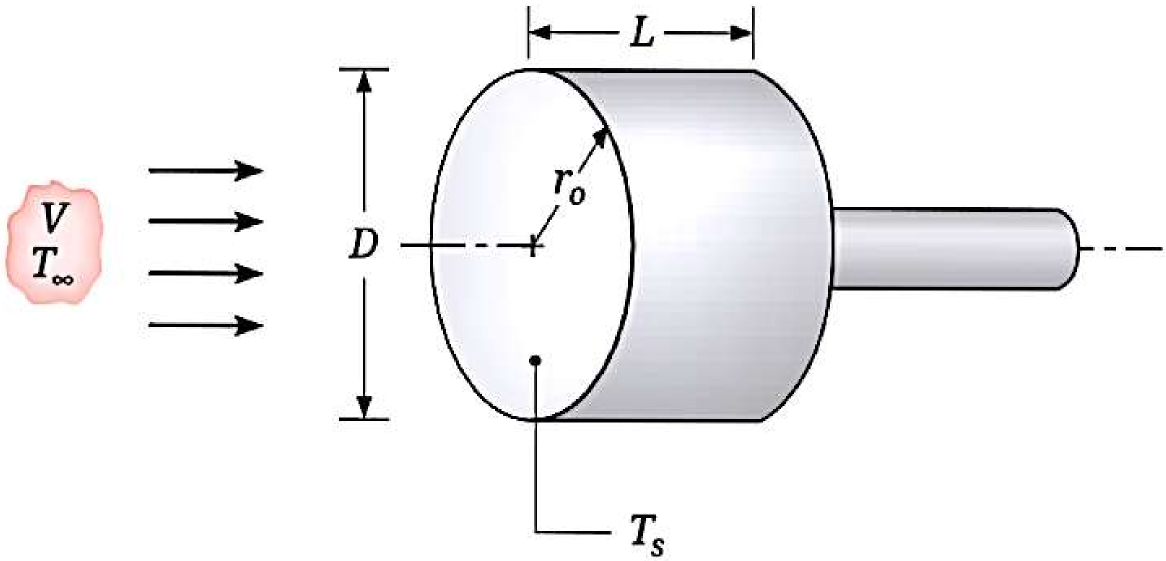
Continued...

6.11 Experiments to determine the local convection heat transfer coefficient for uniform flow normal to a heated circular disk have yielded a radial Nusselt number distribution of the form

$$Nu_D = \frac{h(r)D}{k} = Nu_o\left[1 + a\left(\frac{r}{r_o}\right)^n\right]$$

where both n and a are positive. The Nusselt number at the stagnation point is correlated in terms of the Reynolds ($Re_D = VD/\nu$) and Prandtl numbers

$$Nu_o = \frac{h(r = 0)D}{k} = 0.814 Re_D^{1/2} Pr^{0.36}$$



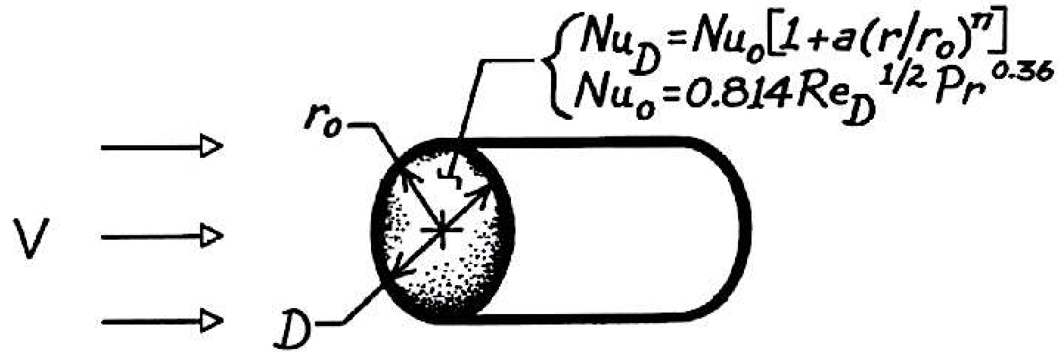
Obtain an expression for the average Nusselt number, $\overline{Nu}_D = \bar{h}D/k$, corresponding to heat transfer from an isothermal disk. Typically, boundary layer development from a stagnation point yields a decaying convection coefficient with increasing distance from the stagnation point. Provide a plausible explanation for why the opposite trend is observed for the disk.

PROBLEM 6.11

KNOWN: Radial distribution of local convection coefficient for flow normal to a circular disk.

FIND: Expression for average Nusselt number.

SCHEMATIC:



ASSUMPTIONS: Constant properties.

ANALYSIS: The average convection coefficient is

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{\pi r_o^2}\int_0^{r_o} \frac{k}{D} Nu_o\left[1+a(r/r_o)^n\right]2\pi\, r dr$$

$$\bar{h} = \frac{kNu_o}{r_o^3}\left[\frac{r^2}{2}+\frac{ar^{n+2}}{(n+2)r_o^n}\right]_0^{r_o}$$

where Nu_o is the Nusselt number at the stagnation point (r = 0). Hence,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2Nu_o\left[\frac{(r/r_o)^2}{2}+\frac{a}{(n+2)}\left(\frac{r}{r_o}\right)^{n+2}\right]_0^{r_o}$$

$$\overline{Nu}_D = Nu_o\left[1+2a/(n+2)\right]$$

$$\overline{Nu}_D = \left[1+2a/(n+2)\right]0.814 Re_D^{1/2} Pr^{0.36}. \qquad <$$

COMMENTS: The increase in h(r) with r may be explained in terms of the sharp turn which the boundary layer flow must make around the edge of the disk. The boundary layer accelerates and its thickness decreases as it makes the turn, causing the local convection coefficient to increase.

6.30 To assess the efficacy of different liquids for cooling an object of given size and shape by forced convection, it is convenient to introduce a *figure of merit*, F_F, which combines the influence of all pertinent fluid properties on the convection coefficient. If the Nusselt number is governed by an expression of the form, $Nu_L \sim Re_L^m Pr^n$, obtain the corresponding relationship between F_F and the fluid properties. For representative values of $m = 0.80$ and $n = 0.33$, calculate values of F_F for air ($k = 0.026$ W/m·K, $\nu = 1.6 \times 10^{-5}$ m²/s, $Pr = 0.71$), water ($k = 0.600$ W/m·K, $\nu = 10^{-6}$ m²/s, $Pr = 5.0$), and a dielectric liquid ($k = 0.064$ W/m·K, $\nu = 10^{-6}$ m²/s, $Pr = 25$). What fluid is the most effective cooling agent?

PROBLEM 6.30

KNOWN: Form of the Nusselt number correlation for forced convection and fluid properties.

FIND: Expression for figure of merit F_F and values for air, water and a dielectric liquid.

PROPERTIES: Prescribed. Air: $k = 0.026$ W/m·K, $\nu = 1.6 \times 10^{-5}$ m^2/s, Pr = 0.71. Water: $k = 0.600$ W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 5.0. Dielectric liquid: $k = 0.064$ W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 25

ANALYSIS: With $Nu_L \sim Re_L^m Pr^n$, the convection coefficient may be expressed as

$$h \sim \frac{k}{L}\left(\frac{VL}{\nu}\right)^m Pr^n \sim \frac{V^m}{L^{1-m}}\left(\frac{kPr^n}{\nu^m}\right)$$

The figure of merit is therefore

$$F_F = \frac{kPr^n}{\nu^m} \qquad <$$

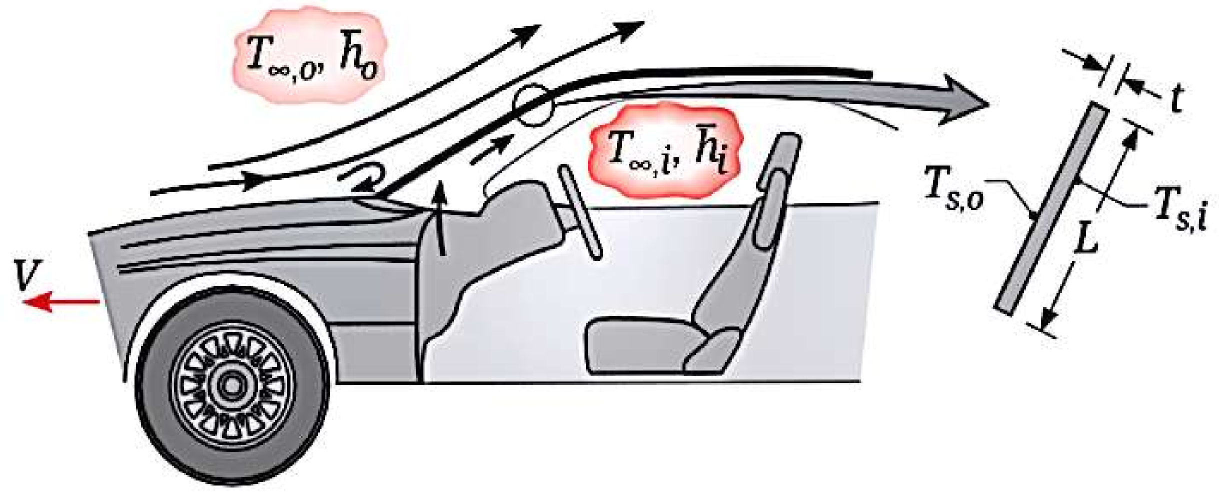
and for the three fluids, with m = 0.80 and n = 0.33,

	Air	Water	Dielectric	
$F_F\left(W \cdot s^{0.8}/m^{2.6} \cdot K\right)$	167	64,400	11,700	<

Water is clearly the superior heat transfer fluid, while air is the least effective.

COMMENTS: The figure of merit indicates that heat transfer is enhanced by fluids of large k, large Pr and small ν.

6.32 The defroster of an automobile functions by discharging warm air on the inner surface of the windshield. To prevent condensation of water vapor on the surface, the temperature of the air and the surface convection coefficient ($T_{\infty,i}$, $\overline{h}_i$) must be large enough to maintain a surface temperature $T_{s,i}$ that is at least as high as the dewpoint ($T_{s,i} \geq T_{dp}$).



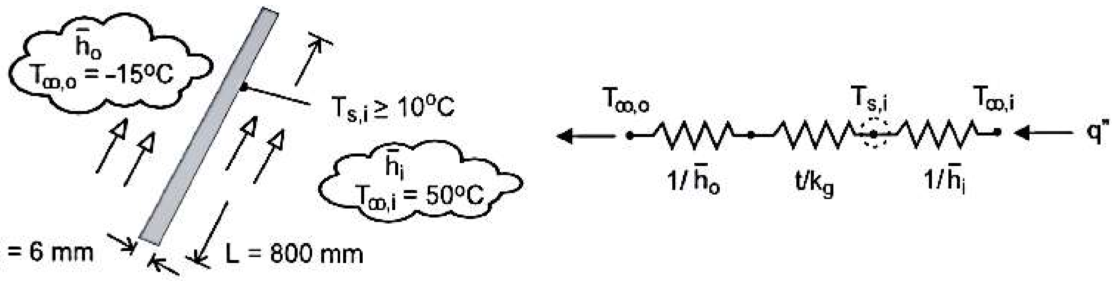
Consider a windshield of length $L = 800$ mm and thickness $t = 6$ mm and driving conditions for which the vehicle moves at a velocity of $V = 70$ mph in ambient air at $T_{\infty,o} = -15°\text{C}$. From laboratory experiments performed on a model of the vehicle, the average convection coefficient on the outer surface of the windshield is known to be correlated by an expression of the form $\overline{Nu}_L = 0.030\, Re_L^{0.8}\, Pr^{1/3}$, where $Re_L \equiv VL/\nu$. Properties of the ambient air may be approximated as $k = 0.023$ W/m·K, $\nu = 12.5 \times 10^{-6}$ m²/s, and $Pr = 0.71$. If $T_{dp} = 10°\text{C}$ and $T_{\infty,i} = 50°\text{C}$, what is the smallest value of $\overline{h}_i$ required to prevent condensation on the inner surface?

PROBLEM 6.32

KNOWN: Ambient, interior and dewpoint temperatures. Vehicle speed and dimensions of windshield. Heat transfer correlation for external flow.

FIND: Minimum value of convection coefficient needed to prevent condensation on interior surface of windshield.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

PROPERTIES: *Table A-3*, glass: k_g = 1.4 W/m·K. Prescribed, air: k = 0.023 W/m·K, $\nu = 12.5 \times 10^{-6}$ m^2/s, Pr = 0.70.

ANALYSIS: From the prescribed thermal circuit, conservation of energy yields

$$\frac{T_{\infty,i} - T_{s,i}}{1/\bar{h}_i} = \frac{T_{s,i} - T_{\infty,o}}{t/k_g + 1/\bar{h}_o}$$

where $\bar{h}_o$ may be obtained from the correlation

$$\overline{Nu}_L = \frac{\bar{h}_o L}{k} = 0.030\, Re_L^{0.8}\, Pr^{1/3}$$

With V = (70 mph × 1585 m/mile)/3600 s/h = 30.8 m/s, Re_D = (30.8 m/s × 0.800 m)/12.5 × 10^{-6} m^2/s = 1.97 × 10^6 and

$$\bar{h}_o = \frac{0.023\,W/m\cdot K}{0.800\ m} 0.030\left(1.97\times10^6\right)^{0.8}(0.70)^{1/3} = 83.1\,W/m^2\cdot K$$

From the energy balance, with $T_{s,i} = T_{dp}$ = 10°C

$$\bar{h}_i = \frac{\left(T_{s,i} - T_{\infty,o}\right)}{\left(T_{\infty,i} - T_{s,i}\right)}\left(\frac{t}{k_g} + \frac{1}{\bar{h}_o}\right)^{-1}$$

$$\bar{h}_i = \frac{(10+15)°C}{(50-10)°C}\left(\frac{0.006\ m}{1.4\,W/m\cdot K} + \frac{1}{83.1\,W/m^2\cdot K}\right)^{-1}$$

$$\bar{h}_i = 38.3\,W/m^2\cdot K$$ <

COMMENTS: The output of the fan in the automobile's heater/defroster system must maintain a velocity for flow over the inner surface that is large enough to provide the foregoing value of $\bar{h}_i$. In addition, the output of the heater must be sufficient to maintain the prescribed value of $T_{\infty,i}$ at this velocity.

6.50 An industrial process involves the evaporation of water from a liquid film that forms on a contoured surface. Dry air is passed over the surface, and from laboratory measurements the convection heat transfer correlation is of the form

$$\overline{Nu}_L = 0.43 Re_L^{0.58} Pr^{0.4}$$

(a) For an air temperature and velocity of 27°C and 10 m/s, respectively, what is the rate of evaporation from a surface of 1-m^2 area and characteristic length $L = 1$ m? Approximate the density of saturated vapor as $\rho_{A,sat} = 0.0077$ kg/m^3.

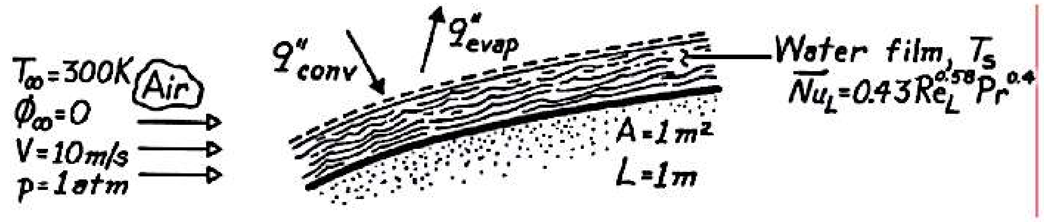
(b) What is the steady-state temperature of the liquid film?

PROBLEM 6.50

KNOWN: Convection heat transfer correlation for flow over a contoured surface.

FIND: (a) Evaporation rate from a water film on the surface, (b) Steady-state film temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (b) Constant properties, (c) Negligible radiation, (d) Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): k = 0.0263 W/mK, $\nu = 15.89\times10^{-6}\ m^2/s$, Pr = 0.707; *Table A-6*, Water ($T_s \approx 280K$): $v_g = 130.4\ m^3/kg$, $h_{fg} = 2485$ kJ/kg; *Table A-8*, Water-air ($T \approx 298K$): $D_{AB} = 0.26\times10^{-4}\ m^2/s$.

ANALYSIS: (a) The mass evaporation rate is

$$\dot{m}_{evap} = n_A = \bar{h}_m A\left[\rho_{A,sat}(T_s) - \phi_\infty\ \rho_{A,sat}(T_\infty)\right] = \bar{h}_m A\ \rho_{A,sat}(T_s).$$

From the heat and mass transfer analogy: $\overline{Sh}_L = 0.43\ Re_L^{0.58}\ Sc^{0.4}$

$$Re_L = \frac{VL}{\nu} = \frac{(10\ m/s)\ 1m}{15.89\times10^{-6} m^2/s} = 6.29\times10^5 \qquad Sc = \frac{\nu}{D_{AB}} = \frac{15.89\times10^{-6} m^2/s}{26\times10^{-6} m^2/s} = 0.61$$

$$\overline{Sh}_L = 0.43\left(6.29\times10^5\right)^{0.58}(0.61)^{0.4} = 814$$

$$\bar{h}_m = \frac{D_{AB}}{L}\overline{Sh}_L = \frac{0.26\times10^{-4} m^2/s}{1m}(814) = 0.0212\ m/s$$

$$\rho_{A,sat}(T_s) = v_g(T_s)^{-1} = 0.0077\ kg/m^3.$$

Hence, $\dot{m}_{evap} = 0.0212 m/s \times 1m^2 \times 0.0077 kg/m^3 = 1.63\times10^{-4} kg/s.$ <

(b) From a surface energy balance, $q''_{conv} = q''_{evap}$, or

$$\bar{h}_L(T_\infty - T_s) = \dot{m}''_{evap} h_{fg} \qquad T_s = T_\infty - \frac{\left(\dot{m}''_{evap} h_{fg}\right)}{\bar{h}_L}.$$

With $\overline{Nu}_L = 0.43\left(6.29\times10^5\right)^{0.58}(0.707)^{0.4} = 864$

$$\bar{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\ W/m\cdot K}{1m}864 = 22.7\ W/m^2\cdot K.$$

Hence, $T_s = 300K - \dfrac{1.63\times10^{-4} kg/s\cdot m^2\left(2.485\times10^6\ J/kg\right)}{22.7\ W/m^2\cdot K} = 282.2K.$ <

COMMENTS: The saturated vapor density, $\rho_{A,sat}$, is strongly temperature dependent, and if the initial guess of T_s needed for its evaluation differed from the above result by more than a few degrees, the density would have to be evaluated at the new temperature and the calculations repeated.