7.6 Consider a steady, turbulent boundary layer on an isothermal flat plate of temperature T_s. The boundary layer is "tripped" at the leading edge $x = 0$ by a fine wire. Assume constant physical properties and velocity and temperature profiles of the form

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \quad \text{and} \quad \frac{T - T_\infty}{T_s - T_\infty} = 1 - \left(\frac{y}{\delta_t}\right)^{1/7}$$

(a) From experiment it is known that the surface shear stress is related to the boundary layer thickness by an expression of the form

$$\tau_s = 0.0228 \rho u_\infty^2 \left(\frac{u_\infty \delta}{\nu}\right)^{-1/4}$$

Beginning with the momentum integral equation (Appendix F), show that

$$\delta/x = 0.376 Re_x^{-1/5}.$$

Determine the average friction coefficient $\overline{C}_{f,x}$.

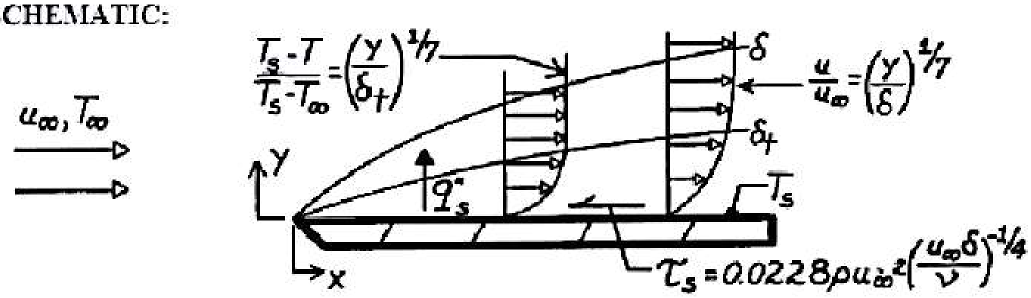
(b) Beginning with the energy integral equation, obtain an expression for the local Nusselt number Nu_x and use this result to evaluate the average Nusselt number $\overline{Nu}_x$.

PROBLEM 7.6

KNOWN: Velocity and temperature profiles and shear stress-boundary layer thickness relation for turbulent flow over a flat plate.

FIND: (a) Expressions for hydrodynamic boundary layer thickness and average friction coefficient, (b) Expressions for local and average Nusselt numbers.

SCHEMATIC:



ASSUMPTIONS: (1) Steady flow, (2) Constant properties, (3) Fully turbulent boundary layer, (4) Incompressible flow, (5) Isothermal plate, (6) Negligible viscous dissipation, (7) $\delta \approx \delta_t$.

ANALYSIS: (a) The momentum integral equation is

$$\rho u_\infty^2 \frac{d}{dx} \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) \frac{u}{u_\infty} dy = \tau_s.$$

Substituting the expression for the wall shear stress

$$\rho u_\infty^2 \frac{d}{dx} \int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] \left(\frac{y}{\delta}\right)^{1/7} dy = 0.0228 \rho u_\infty^2 \left(\frac{u_\infty \delta}{\nu}\right)^{-1/4}$$

$$\frac{d}{dx} \int_0^\delta \left[\left(\frac{y}{\delta}\right)^{1/7} - \left(\frac{y}{\delta}\right)^{2/7}\right] dy = \frac{d}{dx}\left(\frac{7}{8}\frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{y^{9/7}}{\delta^{2/7}}\right)\Bigg|_0^\delta$$

$$\frac{d}{dx}\left(\frac{7}{8}\delta - \frac{7}{9}\delta\right) = 0.0228 \left(\frac{u_\infty \delta}{\nu}\right)^{-1/4}$$

$$\frac{7}{72}\frac{d\delta}{dx} = 0.0228 \left(\frac{\nu}{u_\infty}\right)^{1/4} \delta^{-1/4} \qquad \frac{7}{72}\int_0^\delta \delta^{1/4} d\delta = 0.0228 \left(\frac{\nu}{u_\infty}\right)^{1/4} \int_0^x dx$$

$$\frac{7}{72} \times \frac{4}{5} \delta^{5/4} = 0.0228 \left(\frac{\nu}{u_\infty}\right)^{1/4} x, \qquad \delta = 0.376 \left(\frac{\nu}{u_\infty}\right)^{1/5} x^{4/5}, \quad \frac{\delta}{x} = 0.376 Re_x^{-1/5}. <$$

Knowing δ, it follows

$$\tau_s = 0.0228 \rho u_\infty^2 \left(\frac{u_\infty}{\nu}\right)^{-1/4} \left[0.376 \, x \, Re_x^{-1/5}\right]^{-1/4}$$

$$C_{f,x} = \frac{\tau_s}{\rho u_\infty^2 / 2} = 0.0456 \left[0.376 \frac{u_\infty}{\nu} \left(\frac{u_\infty}{\nu}\right)^{-1/5} x \, x^{-1/5}\right]^{-1/4} = 0.0582 \, Re_x^{-1/5}.$$

Continued .....

The average friction coefficient is then

$$\overline{C}_{f,x} = \frac{1}{x}\int_0^x C_{f,x}\,dx = \frac{1}{x}0.0582\left(\frac{u_\infty}{\nu}\right)^{-1/5}\int_0^x x^{-1/5}\,dx$$

$$\overline{C}_{f,x} = \frac{1}{x}0.0582\left(\frac{u_\infty}{\nu}\right)^{-1/5} x^{4/5}\left(\frac{5}{4}\right) = 0.073\,Re_x^{-1/5}. \qquad <$$

(b) The energy integral equation for turbulent flow is

$$\frac{d}{dx}\int_0^{\delta_t} u\left(T_\infty - T\right)dy = \frac{q_s''}{\rho c_p} = -\frac{h}{\rho c_p}\left(T_s - T_\infty\right).$$

Hence,

$$u_\infty \frac{d}{dx}\int_0^{\delta_t}\frac{u}{u_\infty}\frac{T-T_\infty}{T_s-T_\infty}\,dy = u_\infty\frac{d}{dx}\int_0^{\delta_t}(y/\delta)^{1/7}\left[1-(y/\delta_t)^{1/7}\right]dy = \frac{h}{\rho c_p}$$

$$u_\infty \frac{d}{dx}\left[\frac{7}{8}\frac{\delta_t^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{\delta_t^{8/7}}{\delta^{1/7}}\right] = \frac{h}{\rho c_p}$$

or, with $\xi \equiv \delta_t/\delta$,

$$u_\infty \frac{d}{dx}\left[\frac{7}{8}\delta\xi^{8/7} - \frac{7}{9}\delta\xi^{8/7}\right] = \frac{h}{\rho c_p} \qquad u_\infty\frac{d}{dx}\left[\frac{7}{72}\delta\xi^{8/7}\right] = \frac{h}{\rho c_p}.$$

Hence, with $\xi \approx 1$ and $\delta/x = 0.376\,Re_x^{-1/5}$,

$$\frac{7}{72}u_\infty(0.376)\left(\frac{u_\infty}{\nu}\right)^{-1/5}\frac{d\left(x^{4/5}\right)}{dx} = \frac{h}{\rho c_p}$$

$$h = 0.0292\,\rho c_p u_\infty Re_x^{-1/5} = 0.0292\,\frac{k}{x}\frac{\nu}{\alpha}\frac{u_\infty x}{\nu}Re_x^{-1/5}$$

$$Nu_x = \frac{hx}{k} = 0.0292\,Re_x^{4/5}\,Pr. \qquad <$$

Hence,

$$\overline{h}_x = \frac{1}{x}\int_0^x h\,dx = \frac{0.0292\,Pr}{x}k\left(\frac{u_\infty}{\nu}\right)^{4/5}\int_0^x x^{-1/5}\,dx = 0.0292\,\frac{k}{x}Pr\left(\frac{u_\infty x}{\nu}\right)^{4/5}\frac{5}{4}$$

$$\overline{Nu}_x = \frac{\overline{h}_x x}{k} = 0.037\,Re_x^{4/5}\,Pr. \qquad <$$

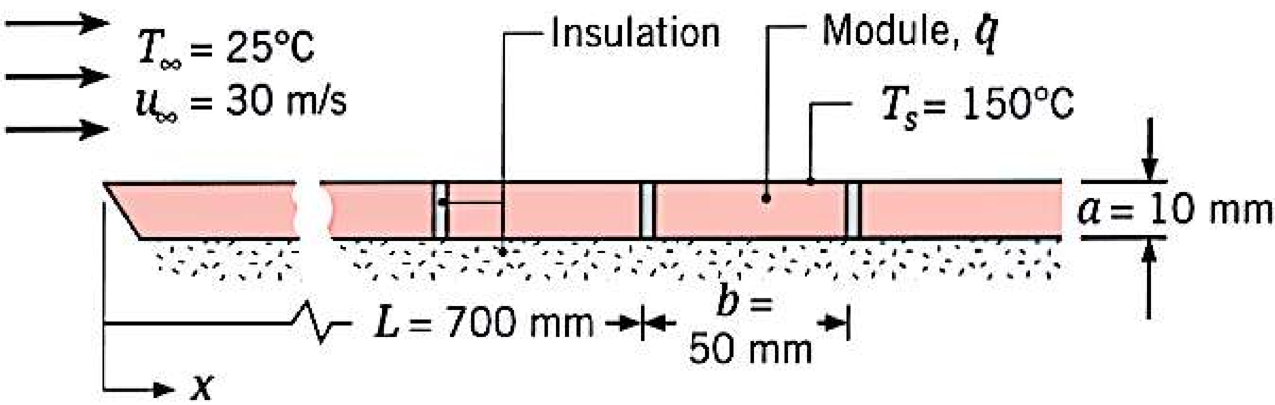
COMMENTS: (1) The foregoing results are in excellent agreement with empirical correlations, except that use of $Pr^{1/3}$ instead of Pr, would be more appropriate. This result arose because of the assumption $\delta \approx \delta_t$, which is only valid for $Pr \approx 1$.

(2) Note that the 1/7 profile breaks down at the surface. For example,

$$\left.\frac{\partial(u/u_\infty)}{\partial y}\right)_{y=0} = \frac{1}{7}\delta^{-1/7}y^{-6/7} = \infty$$

or $\tau_s = \infty$. Despite this unrealistic characteristic of the profile, its use with integral methods provides excellent results.

7.8 A flat plate of width 1 m is maintained at a uniform surface temperature of $T_s = 150°C$ by using independently controlled, heat-generating rectangular modules of thickness $a = 10$ mm and length $b = 50$ mm. Each module is insulated from its neighbors, as well as on its back side. Atmospheric air at 25°C flows over the plate at a velocity of 30 m/s. The thermophysical properties of the module are $k = 5.2$ W/m · K, $c_p = 320$ J/kg · K, and $\rho = 2300$ kg/m³.



(a) Find the required power generation, $\dot{q}$ (W/m³), in a module positioned at a distance 700 mm from the leading edge.

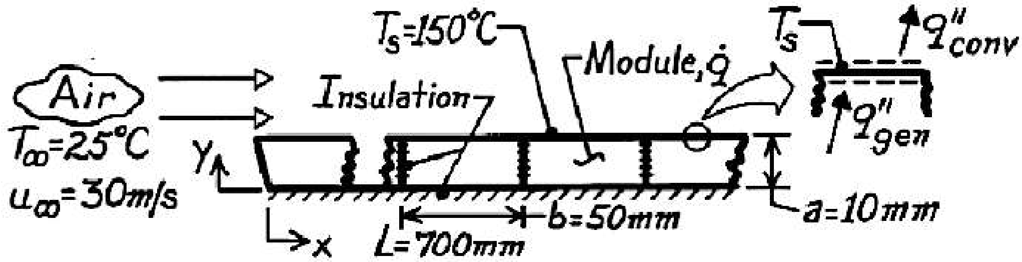
(b) Find the maximum temperature T_{max} in the heat-generating module.

PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s, thickness a and length b cooled by air at 25°C and a velocity of 30 m/s. Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5, (3) Heat transfer is one-dimensional in y-direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): $k = 5.2$ W/m·K, $c_p = 320$ J/kg·K, $\rho = 2300$ kg/m^3; *Table A-4*, Air $(\overline{T}_f = (T_s + T_\infty)/2 = 360$ K, 1 atm$)$: $k = 0.0308$ W/m·K, $\nu = 22.02 \times 10^{-6}$ m^2/s, Pr = 0.698.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q''_{conv} = q''_{gen}$$

$$\overline{h}(T_s - T_\infty) = \dot{q} \cdot a \qquad \text{or} \qquad \dot{q} = \frac{\overline{h}(T_s - T_\infty)}{a}.$$

To select a convection correlation for estimating $\overline{h}$, first find the Reynolds numbers at x = L.

$$Re_L = \frac{u_\infty L}{\nu} = \frac{30 \text{ m/s} \times 0.70 \text{ m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.537 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\overline{h} \approx h_x \; (L + b/2)$ is appropriate, with

$$Re_{L+b/2} = \frac{30 \text{ m/s} \times (0.700 + 0.050/2)\text{m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.877 \times 10^5.$$

Using the turbulent flow correlation with x = L + b/2 = 0.725 m,

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0296\left(9.877 \times 10^5\right)^{4/5} (0.698)^{1/3} = 1640$$

$$\overline{h} \approx h_x = \frac{Nu_x k}{x} = \frac{1640 \times 0.0308 \text{ W/m-K}}{0.725} = 69.7 \text{ W/m}^2 \cdot \text{K}.$$

Continued .....

Hence,

$$\dot{q} = \frac{69.7\ \text{W/m}^2\cdot\text{K}(150-25)\text{K}}{0.010\ \text{m}} = 8.713\times10^5\ \text{W/m}^3.$$ <

(b) The maximum temperature within the module occurs at the surface next to the insulation (y = 0). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T(0) = \frac{\dot{q}a^2}{2k} + T_s = \frac{8.713\times10^5\ \text{W/m}^3\times(0.010\ \text{m})^2}{2\times5.2\ \text{W/m}\cdot\text{K}} + 150^\circ\text{C} = 158.4^\circ\text{C}.$$ <

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$q_{module} = q_{0\to L+b} - q_{0\to L}$$

$$\bar{h}\cdot b = \bar{h}_{L+b}\cdot(L+b) - \bar{h}_L\cdot L \qquad \text{or} \qquad \bar{h} = \bar{h}_{L+b}\frac{L+b}{b} - \bar{h}_L\frac{L}{b}.$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{Nu}_x = \left(0.037Re_x^{4/5} - 871\right)Pr^{1/3}$$

With x = L + b and x = L, find

$$\bar{h}_{L+b} = 54.79\ \text{W/m}^2\cdot\text{K} \qquad \text{and} \qquad \bar{h}_L = 53.73\ \text{W/m}^2\cdot\text{K}.$$

Hence,

$$\bar{h} = \left[54.79\frac{0.750}{0.050} - 53.73\frac{0.700}{0.05}\right]\ \text{W/m}^2\cdot\text{K} = 69.7\ \text{W/m}^2\cdot\text{K}.$$

which is in excellent agreement with the approximate result employed in part (a).

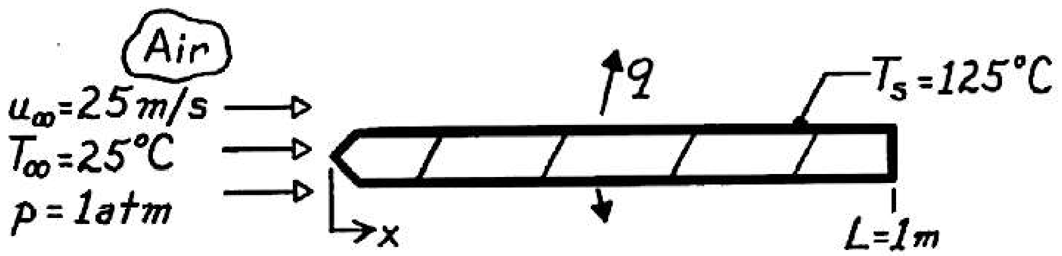
7.10 Consider atmospheric air at 25°C and a velocity of 25 m/s flowing over both surfaces of a 1-m-long flat plate that is maintained at 125°C. Determine the rate of heat transfer per unit width from the plate for values of the critical Reynolds number corresponding to 10^5, 5×10^5, and 10^6.

PROBLEM 7.10

KNOWN: Speed and temperature of atmospheric air flowing over a flat plate of prescribed length and temperature.

FIND: Rate of heat transfer corresponding to $Re_{x,c} = 10^5$, 5×10^5 and 10^6.

SCHEMATIC:



ASSUMPTIONS: (1) Flow over top and bottom surfaces.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): $\rho = 1.00$ kg/m^3, $\nu = 20.72 \times 10^{-6}$ m^2/s, k = 0.0299 W/m·K, Pr = 0.700.

ANALYSIS: With

$$Re_L = \frac{u_\infty L}{\nu} = \frac{25\ \text{m/s} \times 1\text{m}}{20.72 \times 10^{-6} \text{m}^2/\text{s}} = 1.21 \times 10^6$$

the flow becomes turbulent for each of the three values of $Re_{x,c}$. Hence,

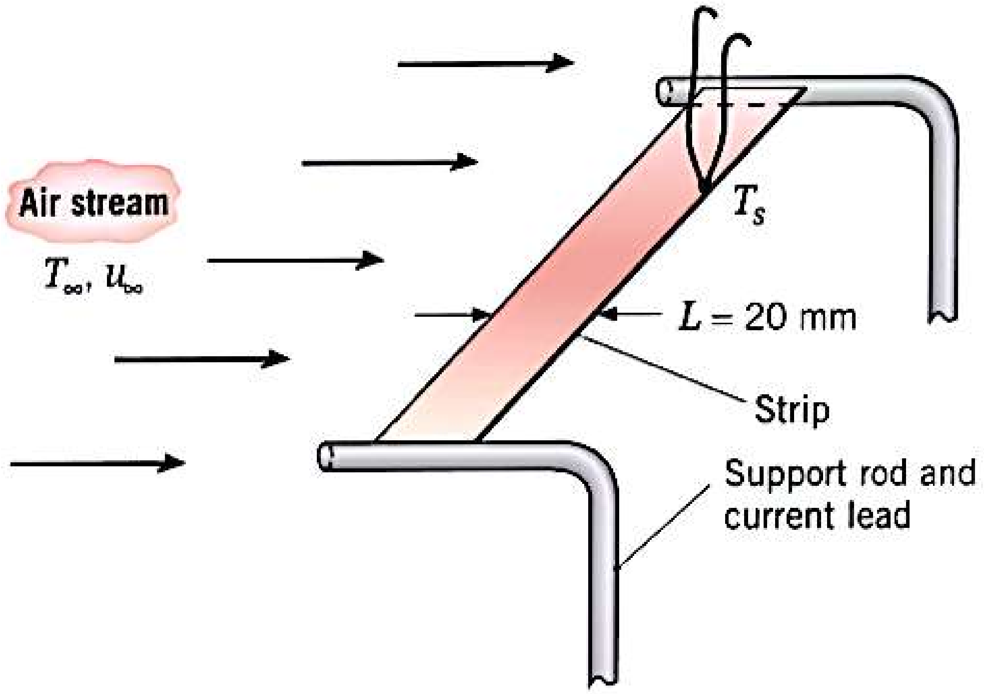
$$\overline{Nu}_L = \left(0.037\, Re_L^{4/5} - A\right) Pr^{1/3}$$
$$A = 0.037\, Re_{x,c}^{4/5} - 0.664\, Re_{x,c}^{1/2}$$

$Re_{x,c}$	10^5	5×10^5	10^6
A	160	871	1671
$\overline{Nu}_L$	2267	1635	926
$\bar{h}_L\left(W/m^2 \cdot K\right)$	67. 8	48.9	27.7
$q'(W/m)$	13,560	9780	5530

where $q' = 2\,\bar{h}_L L\left(T_s - T_\infty\right)$ is the total heat loss per unit width of plate.

COMMENTS: Note that $\bar{h}_L$ decreases with increasing $Re_{x,c}$, as more of the surface becomes covered with a laminar boundary layer.

7.23 The proposed design for an anemometer to determine the velocity of an airstream in a wind tunnel is comprised of a thin metallic strip whose ends are supported by stiff rods serving as electrodes for passage of current used to heat the strip. A fine-wire thermocouple is attached to the trailing edge of the strip and serves as the sensor for a system that controls the power to maintain the strip at a constant operating temperature for variable airstream velocities. Design conditions pertain to an airstream at $T_\infty = 25°C$ and $1 \le u_\infty \le 50$ m/s, with a strip temperature of $T_s = 35°C$.



7.25

(a) Determine the relationship between the electrical power dissipation per unit width of the strip in the transverse direction, P' (mW/mm), and the airstream velocity. Show this relationship graphically for the specified range of u_∞.

(b) If the accuracy with which the temperature of the operating strip can be measured and maintained constant is $\pm 0.2°C$, what is the uncertainty in the airstream velocity?

(c) The proposed design operates in a strip constant-temperature mode for which the airstream velocity is related to the measured power. Consider now an alternative mode wherein the strip is provided with a constant power, say, 30 mW/mm, and the airstream velocity is related to the measured strip temperature T_s. For this mode of operation, show the graphical relationship between the strip temperature and airstream velocity. If the temperature can be measured with an uncertainty of $\pm 0.2°C$, what is the uncertainty in the airstream velocity?

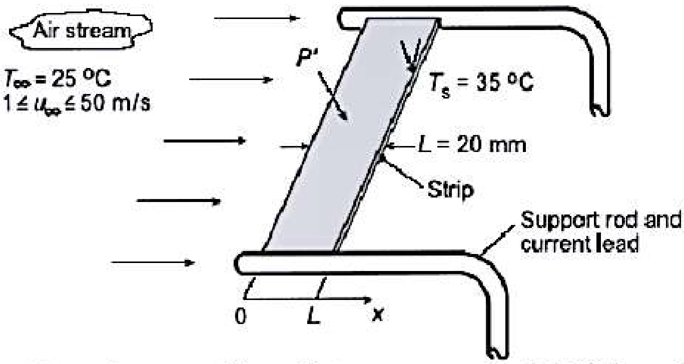
(d) Compare the features associated with each of the anemometer operating modes.

PROBLEM 7.23

KNOWN: Design of an anemometer comprised of a thin metallic strip supported by stiff rods serving as electrodes for passage of heating current. Fine-wire thermocouple on trailing edge of strip.

FIND: (a) Relationship between electrical power dissipation per unit width of the strip in the transverse direction, P' (mW/mm), and airstream velocity u_∞ when maintained at constant strip temperature, T_s; show the relationship graphically; (b) The uncertainty in the airstream velocity if the accuracy with which the strip temperature can be measured and maintained constant is ±0.2°C; (c) Relationship between strip temperature and airstream velocity u_∞ when the strip is provided with a constant power, P' = 30 mW/mm; show the relationship graphically. Also, find the uncertainty in the airstream velocity if the accuracy with which the strip temperature can be measured is ±0.2°C; (d) Compare features associated with each of the operating nodes.

SCHEMATIC:

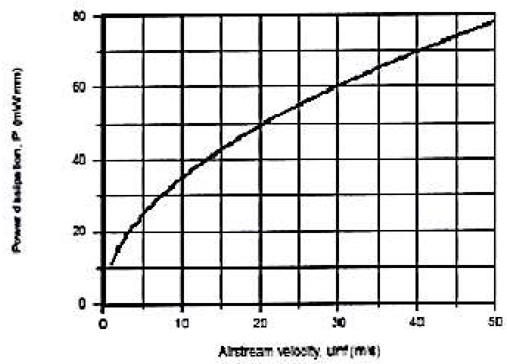


ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Strip has uniform temperature in the midspan region of the strip, (4) Negligible conduction in the transverse direction in the midspan region, and (5) Airstream over strip approximates parallel flow over two sides of a smooth flat plate.

ANALYSIS: (a) In the midspan region of uniform temperature T_s with no conduction in the transverse direction, all the dissipated electrical power is transferred by convection to the airstream,

$$P' = 2\bar{h}_L L(T_s - T_\infty) \qquad (1)$$

where P' is the power per unit width (transverse direction). Using the *IHT Correlation Tool* for *External Flow-Flat Plate* the power as a function of airstream velocity was determined and is plotted below. The IHT tool uses the flat plate correlation, Eq. 7.30 since the flow is laminar over this velocity range.



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(b) By differentiation of Eq. (1), the relative uncertainties of the convection coefficient and strip temperature are, assuming the power remains constant,

$$\frac{\Delta\bar{h}_L}{\bar{h}} = -\frac{\Delta T_s}{T_s - T_\infty} \qquad (2)$$

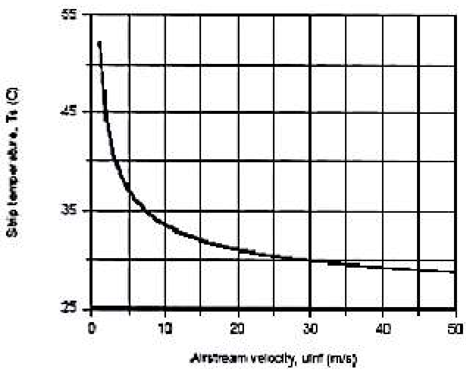
Since the flow was laminar for the range of airstream velocities, Eq. 7.30,

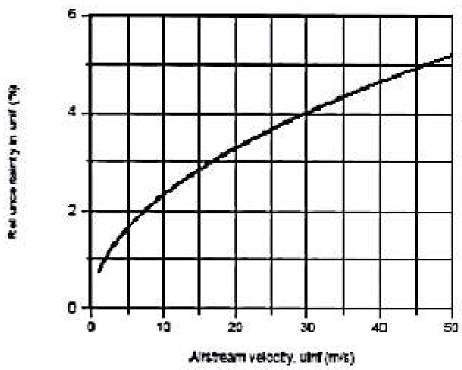
$$\bar{h}_L \sim u_\infty^{1/2} \qquad \text{or} \qquad \frac{\Delta\bar{h}_L}{\bar{h}_L} = 0.5\frac{\Delta u_\infty}{u_\infty} \qquad (3)$$

Hence, the relative uncertainty in the air velocity due to uncertainty in T_s, $\Delta T_s = \pm 0.2^\circ C$

$$\frac{\Delta u_\infty}{u_\infty} = 2\frac{\Delta T_s}{T_s - T_\infty} = 2\frac{\pm 0.2^\circ C}{(35-25)^\circ C} = \pm 4\% \qquad (4)$$ <

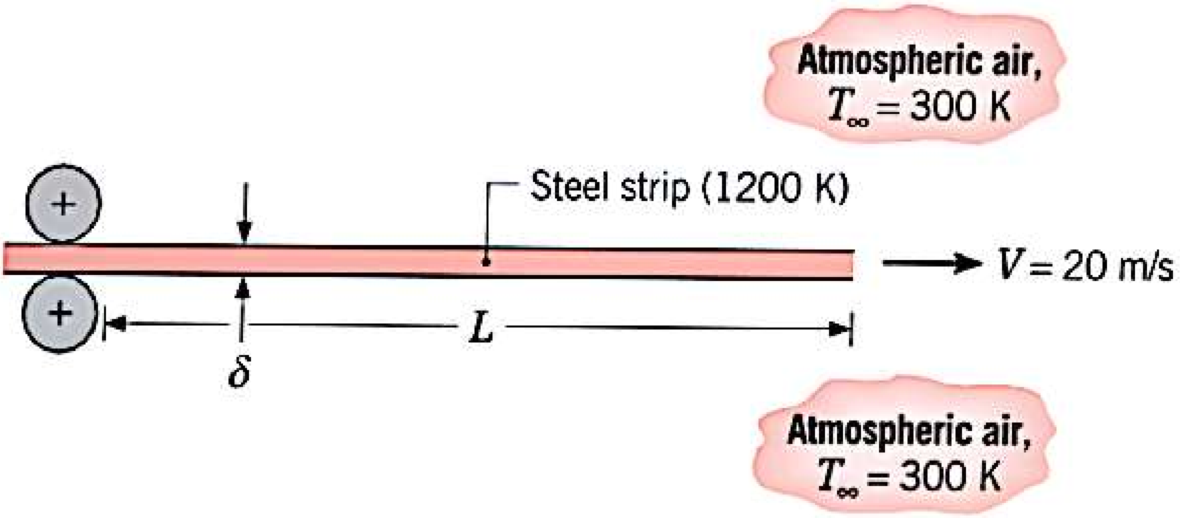
(c) Using the IHT workspace setting $P' = 30$ mW/mm, the strip temperature T_s as a function of the airstream velocity was determined and plotted. Note that the slope of the T_s vs. u_∞ curve is steep for low velocities and relatively flat for high velocities. That is, the technique is more sensitive at lower velocities. Using Eq. (4), but with T_s dependent upon u_∞, the relative uncertainty in u_∞ can be determined.





(d) For the constant power mode of operation, part (a), the uncertainty in u_∞ due to uncertainty in temperature measurement was found as 4%, independent of the magnitude u_∞. For the constant-temperature mode of operation, the uncertainty in u_∞ is less than 4% for velocities less than 30 m/s, with a value of 1% around 2 m/s. However, in the upper velocity range, the error increases to 5%.

7.27 A steel strip emerges from the hot roll section of a steel mill at a speed of 20 m/s and a temperature of 1200 K. Its length and thickness are $L = 100$ m and $\delta = 0.003$ m, respectively, and its density and specific heat are 7900 kg/m^3 and 640 J/kg · K, respectively.



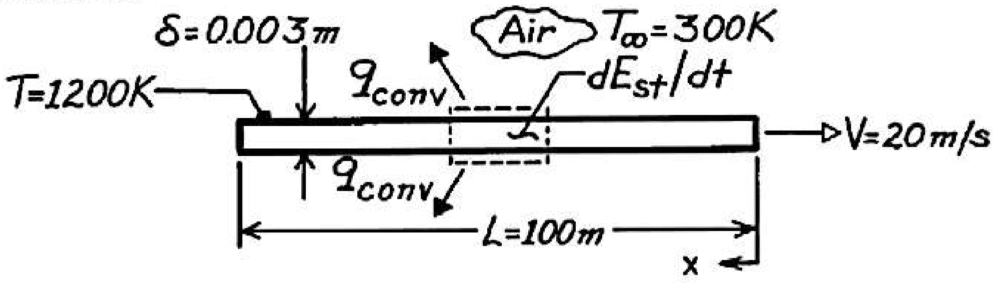
Accounting for heat transfer from the top and bottom surfaces and neglecting radiation and strip conduction effects, determine the time rate of change of the strip temperature at a distance of 1 m from the leading edge and at the trailing edge. Determine the distance from the leading edge at which the minimum cooling rate is achieved.

PROBLEM 7.27

KNOWN: Length, thickness, speed and temperature of steel strip.

FIND: Rate of change of strip temperature 1 m from leading edge and at trailing edge. Location of minimum cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation, (3) Negligible longitudinal conduction in strip, (4) Critical Reynolds number is 5×10^5.

PROPERTIES: Steel (given): $\rho = 7900\ kg/m^3$, $c_p = 640\ J/kg\cdot K$. *Table A-4*, Air $\left(\bar{T} = 750K,\ 1\ atm\right)$: $\nu = 76.4 \times 10^{-6}\ m^2/s$, $k = 0.0549\ W/m\cdot K$, $Pr = 0.702$.

ANALYSIS: Performing an energy balance for a control mass of unit surface area A_s riding with the strip,

$$-\dot{E}_{out} = dE_{st}/dt$$

$$-2h_x A_s \left(T - T_\infty\right) = \rho\delta A_s c_p \left(dT/dt\right)$$

$$dT/dt = \frac{-2h_x\left(T - T_\infty\right)}{\rho\delta c_p} = -\frac{2(900K)h_x}{7900\ kg/m^3(0.003\ m)640\ J/kg\cdot K} = -0.119h_x\ (K/s).$$

At x = 1 m, $\quad Re_x = \frac{Vx}{\nu} = \frac{20\ m/s(1m)}{76.4\times10^{-6}\ m^2/s} = 2.62\times10^5 < Re_{x,c}$. Hence,

$$h_x = (k/x)0.332Re_x^{1/2}Pr^{1/3} = \frac{0.0549\ W/m\cdot K}{1\ m}(0.332)\left(2.62\times10^5\right)^{1/2}(0.702)^{1/3} = 8.29 W/m^2\cdot K$$

and at x = 1m, $dT/dt = -0.987$ K/s. <

At the trailing edge, $Re_x = 2.62\times10^7 > Re_{x,c}$. Hence

$$h_x = (k/x)0.0296Re_x^{4/5}Pr^{1/3} = \frac{0.0549\ W/m\cdot K}{100\ m}(0.0296)\left(2.62\times10^7\right)^{4/5}(0.702)^{1/3} = 12.4 W/m^2\cdot K$$

and at x = 100 m, $dT/dt = -1.47$ K/s. <

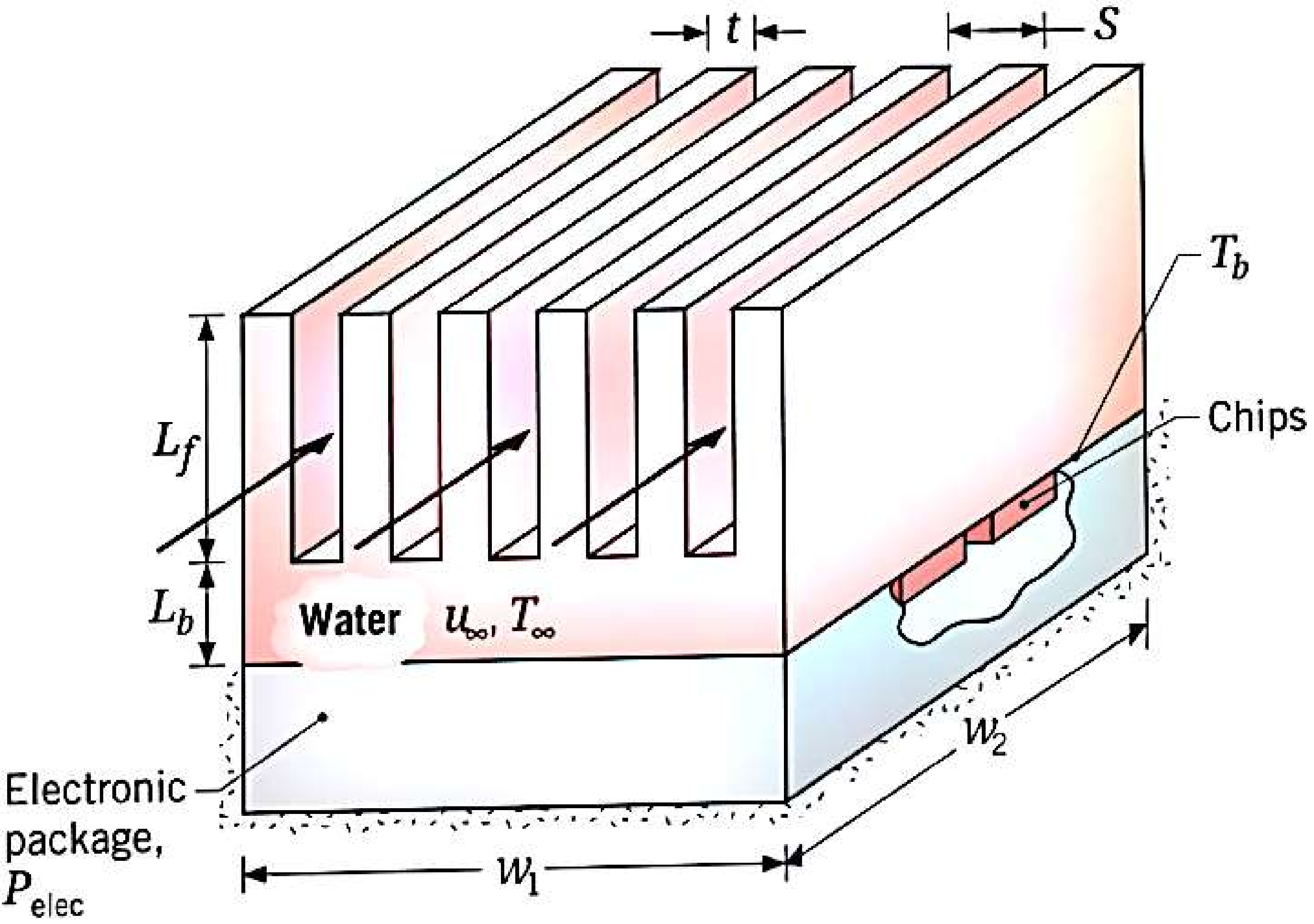
The minimum cooling rate occurs just before transition; hence, for $Re_{x,c} = 5\times10^5$

$$x_c = 5\times10^5(\nu/V) = \frac{5\times10^5\times76.4\times10^{-6}m^2/s}{20\ m/s} = 1.91\ m$$ <

COMMENTS: The cooling rates are very low and would remain low even if radiation were considered. For this reason, hot strip metals are quenched by water and not by air.

7.29 An array of electronic chips is mounted within a sealed rectangular enclosure, and cooling is implemented by attaching an aluminum heat sink ($k = 180$ W/m·K). The base of the heat sink has dimensions of $w_1 = w_2 = 100$ mm, while the 6 fins are of thickness $t = 10$ mm and pitch $S = 18$ mm. The fin length is $L_f = 50$ mm, and the base of the heat sink has a thickness of $L_b = 10$ mm.

If cooling is implemented by water flow through the heat sink, with $u_\infty = 3$ m/s and $T_\infty = 17°C$, what is the base temperature T_b of the heat sink when power dissipation by the chips is $P_{elec} = 1800$ W? The average convection coefficient for surfaces of the fins and the exposed base may be estimated by assuming parallel flow over a flat plate. Properties of the water may be approximated as $k = 0.62$ W/m·K, $\nu = 7.73 \times 10^{-7}$ m^2/s, and $Pr = 5.2$.



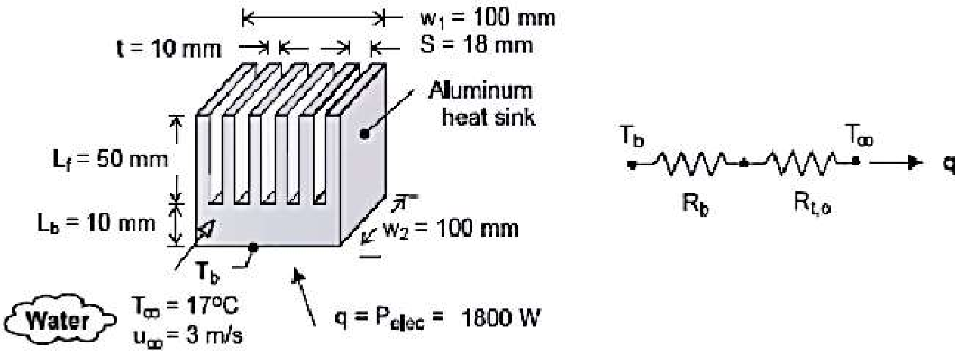
t
S
T_b
Chips
L_f
L_b
Water
u_∞, T_∞
W_2
Electronic package, P_{elec}
W_1

PROBLEM 7.29

KNOWN: Dimensions of aluminum heat sink. Temperature and velocity of coolant (water) flow through the heat sink. Power dissipation of electronic package attached to the heat sink.

FIND: Base temperature of heat sink.

SCHEMATIC:



ASSUMPTIONS: (1) Average convection coefficient association with flow over fin surfaces may be approximated as that for a flat plate in parallel flow, (2) All of the electric power is dissipated by the heat sink, (3) Transition Reynolds number of $Re_{x,c} = 5 \times 10^5$, (4) Constant properties.

PROPERTIES: Given. Aluminum: $k_{hs} = 180$ W/m·K. Water: $k_w = 0.62$ W/m·K, $\nu = 7.73 \times 10^{-7}$ m^2/s, Pr = 5.2.

ANALYSIS: From the thermal circuit,

$$q = P_{elec} = \frac{T_b - T_\infty}{R_b + R_{t,o}}$$

where $R_b = L_b / k_{hs}(w_1 \times w_2) = 0.01\text{ m}/180\text{ W/m}\cdot\text{K}(0.10\text{ m})^2 = 5.56 \times 10^{-3}\text{ K/W}$ and, from Eqs. 3.102 and 3.103,

$$R_{t,o} = \left\{\bar{h}A_t\left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\right\}^{-1}$$

The fin and total surface area of the array are $A_f = 2w_2(L_f + t/2) = 0.2\text{ m}(0.055\text{ m}) = 0.011\text{ m}^2$ and

$A_t = NA_f + A_b = NA_f + (N-1)(S-t)w_2 = 6\left(0.011\text{ m}^2\right) + 5(0.008\text{ m})0.1\text{ m} = (0.066 + 0.004) = 0.070\text{ m}^2$.

With $Re_{w_2} = u_\infty w_2/\nu = 3\text{ m/s} \times 0.10\text{ m}/7.73 \times 10^{-7}\text{ m}^2/\text{s} = 3.88 \times 10^5$, laminar flow may be assumed over the entire surface. Hence

$$\bar{h} = \left(\frac{k_w}{w_2}\right)0.664\,Re_{w_2}^{1/2}\,Pr^{1/3} = \left(\frac{0.62\text{ W/m}\cdot\text{K}}{0.10\text{ m}}\right)0.664\left(3.88 \times 10^5\right)^{1/2}(5.2)^{1/3} = 4443\text{ W/m}^2\cdot\text{K}$$

With $m = (2\bar{h}/k_{hs}t)^{1/2} = \left(2 \times 4443\text{ W/m}^2\cdot\text{K}/180\text{ W/m}\cdot\text{K} \times 0.01\text{ m}\right)^{1/2} = 70.3\text{ m}^{-1}$, $mL_c = 70.3\text{ m}^{-1}$ $(0.055\text{ m}) = 3.86$ and $\tanh mL_c = 0.9991$, Eq. 3.89 yields

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \frac{0.9991}{3.86} = 0.259$$

Continued .....

PROBLEM 7.29 (Cont.)

Hence,

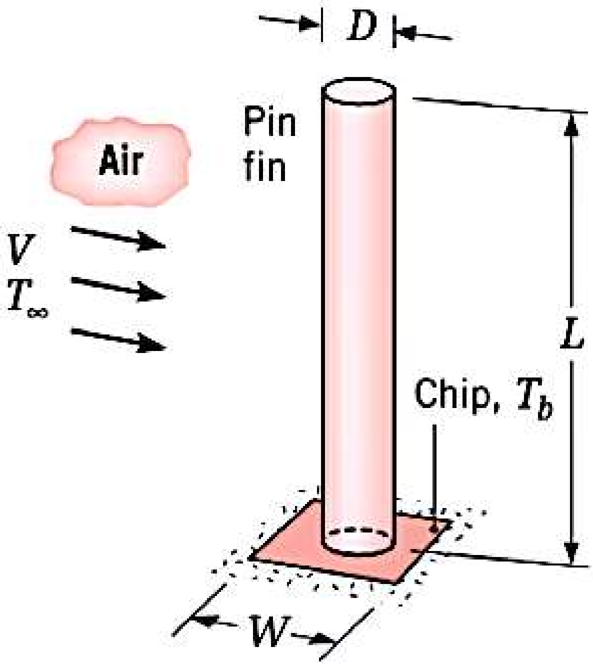
$$R_{t,o} = \left\{ 4443\ \mathrm{W/m^2 \cdot K} \times 0.070\ \mathrm{m^2} \left[1 - \frac{0.066\ \mathrm{m^2}}{0.070\ \mathrm{m^2}} (1 - 0.259) \right] \right\}^{-1} = 0.0107\ \mathrm{K/W} \qquad <$$

and

$$T_b = T_\infty + P_{elec}\left(R_b + R_{t,o}\right) = 17°\mathrm{C} + 1800\ \mathrm{W}\left(5.56 \times 10^{-3} + 0.0107\right)\mathrm{K/W} = 46.2°\mathrm{C} \qquad <$$

COMMENTS: (1) The boundary layer thickness at the trailing edge of the fin is $\delta = 5w_2/\left(Re_{w_2}\right)^{1/2} = 0.80\ \mathrm{mm} \ll (S - t)$. Hence, the assumption of parallel flow over a flat plate is reasonable. (2) If a finned heat sink is not employed and heat transfer is simply by convection from the $w_2 \times w_2$ base surface, the corresponding convection resistance would be 0.0225 K/W, which is only twice the resistance associated with the fin array. The small enhancement by the array is attributable to the large value of $\overline{h}$ and the correspondingly small value of η_f. Were a fluid such as air or a dielectric liquid used as the coolant, the much smaller thermal conductivity would yield a smaller $\overline{h}$, a larger η_f and hence a larger effectiveness for the array.

7.47 To enhance heat transfer from a silicon chip of width $W = 4$ mm on a side, a copper pin fin is brazed to the surface of the chip. The pin length and diameter are $L = 12$ mm and $D = 2$ mm, respectively, and atmospheric air at $V = 10$ m/s and $T_\infty = 300$ K is in cross flow over the pin. The surface of the chip, and hence the base of the pin, are maintained at a temperature of $T_b = 350$ K.



(a) Assuming the chip to have a negligible effect on flow over the pin, what is the average convection coefficient for the surface of the pin?

(b) Neglecting radiation and assuming the convection coefficient at the pin tip to equal that calculated in part (a), determine the pin heat transfer rate.

(c) Neglecting radiation and assuming the convection coefficient at the exposed chip surface to equal that calculated in part (a), determine the total rate of heat transfer from the chip.

(d) Independently determine and plot the effect of increasing velocity ($10 \leq V \leq 40$ m/s) and pin diameter ($2 \leq D \leq 4$ mm) on the total rate of heat transfer from the chip. What is the heat rate for $V = 40$ m/s and $D = 4$ mm?

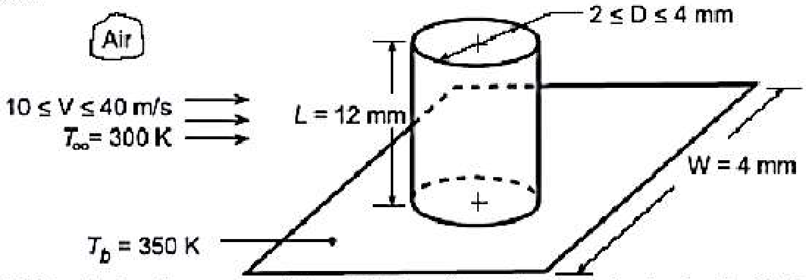
7

PROBLEM 7.47

KNOWN: Dimensions of chip and pin fin. Chip temperature. Free stream velocity and temperature of air coolant.

FIND: (a) Average pin convection coefficient, (b) Pin heat transfer rate, (c) Total heat rate, (d) Effect of velocity and pin diameter on total heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pin, (3) Constant properties, (4) Convection coefficients on pin surface (tip and side) and chip surface correspond to single cylinder in cross flow, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Copper (350 K): k = 399 W/m·K; *Table A.4*, Air ($T_f \approx 325$ K, 1 atm): ν = 18.41×10^{-6} m²/s, k = 0.0282 W/m·K, Pr = 0.704.

ANALYSIS: (a) With V = 10 m/s and D = 0.002 m,

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.002\,m}{18.41 \times 10^{-6}\,m^2/s} = 1087$$

Using the Churchill and Bernstein correlations, Eq. (7.54),

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 16.7$$

$$\bar{h} = \left(\overline{Nu}_D k/D\right) = \left(16.7 \times 0.0282\,W/m\cdot K/0.002\,m\right) = 235\,W/m^2\cdot K$$ <

(b) For the fin with tip convection and

$$M = \left(\bar{h}\pi D k \pi D^2/4\right)^{1/2}\theta_b = (\pi/2)\left[235\,W/m^2\cdot K(0.002\,m)^3\,399\,W/m\cdot K\right]^{1/2} 50K = 2.15\,W$$

$$m = \left(\bar{h}P/kA_c\right)^{1/2} = \left(4 \times 235\,W/m^2\cdot K/399\,W/m\cdot K \times 0.002\,m\right)^{1/2} = 34.3\,m^{-1}$$

$$mL = 34.3\,m^{-1}(0.012\,m) = 0.412$$

$$(\bar{h}/mk) = \left(235\,W/m^2\cdot K/34.3\,m^{-1} \times 399\,W/m\cdot K\right) = 0.0172.$$

The fin heat rate is

$$q_f = M\frac{\sinh mL + (\bar{h}/mk)\cosh mL}{\cosh mL + (\bar{h}/mk)\sinh mL} = 0.868\,W.$$ <

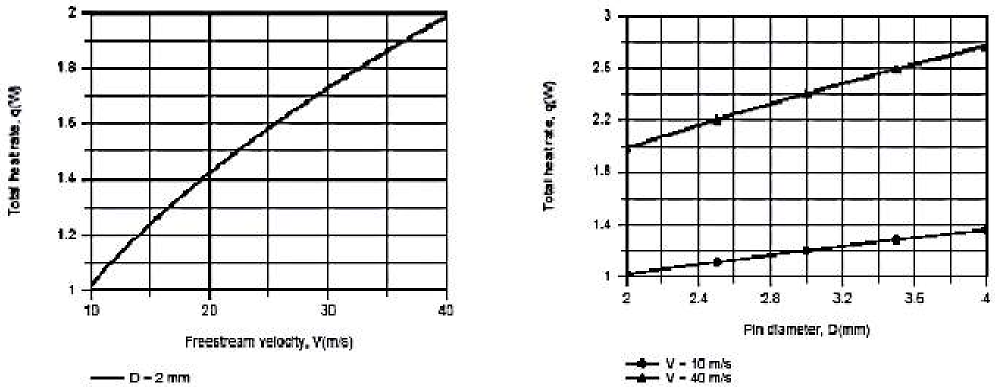
Continued...

PROBLEM 7.47 (Cont.)

(c) The total heat rate is that from the base and through the fin,

$$q = q_b + q_f = \overline{h}\left(W^2 - \pi D^2/4\right)\theta_b + q_f = (0.151 + 0.868)\,W = 1.019\,W \,. \qquad <$$

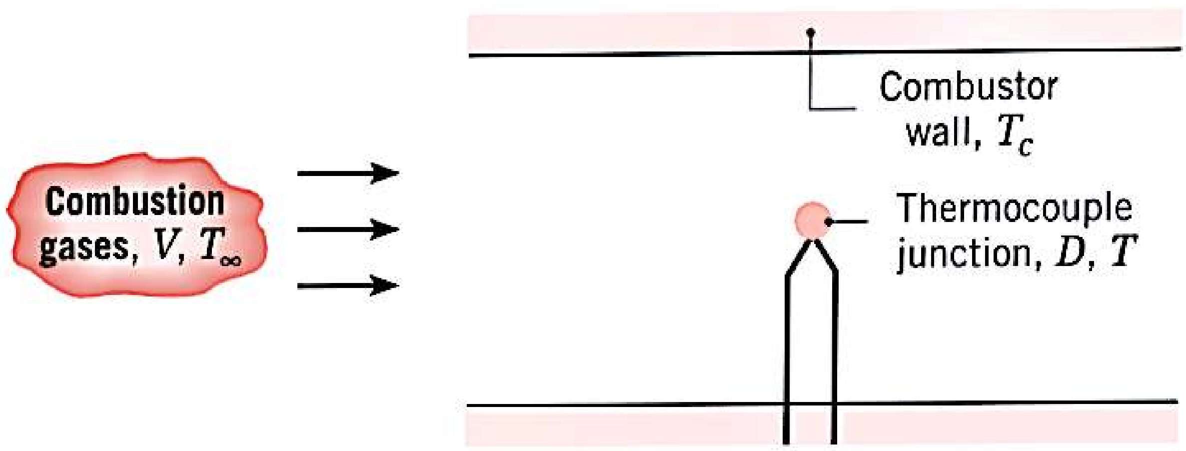
(d) Using the IHT Extended Surface Model for a Pin Fin with the Correlations Tool Pad for a Cylinder in crossflow and Properties Tool Pad for Air, the following results were generated.



Clearly, there is significant benefit associated with increasing V which increases the convection coefficient and the total heat rate. Although the convection coefficient decreases with increasing D, the increase in the total heat transfer surface area is sufficient to yield an increase in q with increasing D. The maximum heat rate is q = 2.77 W for V = 40 m/s and **D = 4 mm**.

COMMENTS: Radiation effects should be negligible, although tip and base convection coefficients will differ from those calculated in parts (a) and (d).

7.78 A spherical thermocouple junction 1.0 mm in diameter is inserted in a combustion chamber to measure the temperature T_∞ of the products of combustion. The hot gases have a velocity of $V = 5$ m/s.



(a) If the thermocouple is at room temperature, T_i, when it is inserted in the chamber, estimate the time required for the temperature difference, $T_\infty - T$, to reach 2% of the initial temperature difference, $T_\infty - T_i$. Neglect radiation and conduction through the leads. Properties of the thermocouple junction are approximated as $k = 100$ W/m · K, $c = 385$ J/kg · K, and $\rho = 8920$ kg/m^3, while those of the combustion gases may be approximated as $k = 0.05$ W/m · K, $\nu = 50 \times 10^{-6}$ m^2/s, and $Pr = 0.69$.

(b) If the thermocouple junction has an emissivity of 0.5 and the cooled walls of the combustor are at

$T_c = 400$ K, what is the steady-state temperature of the thermocouple junction if the combustion gases are at 1000 K? Conduction through the lead wires may be neglected.

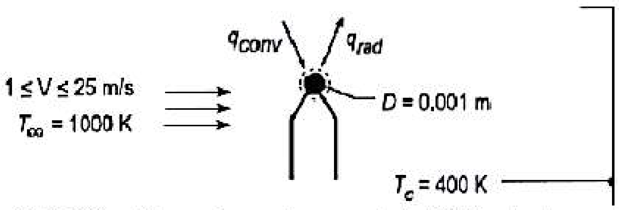
(c) To determine the influence of the gas velocity on the thermocouple measurement error, compute the steady-state temperature of the thermocouple junction for velocities in the range $1 \leq V \leq 25$ m/s. The emissivity of the junction can be controlled through application of a thin coating. To reduce the measurement error, should the emissivity be increased or decreased? For $V = 5$ m/s, compute the steady-state junction temperature for emissivities in the range $0.1 \leq \varepsilon \leq 1.0$.

PROBLEM 7.78

KNOWN: Velocity and temperature of combustion gases. Diameter and emissivity of thermocouple junction. Combustor temperature.

FIND: (a) Time to achieve 98% of maximum thermocouple temperature rise, (b) Steady-state thermocouple temperature, (c) Effect of gas velocity and thermocouple emissivity on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Validity of lumped capacitance analysis, (2) Constant properties, (3) Negligible conduction through lead wires, (4) Radiation exchange between small surface and a large enclosure (parts b and c).

PROPERTIES: Thermocouple (given): $0.1 \le \varepsilon \le 1.0$, $k = 100$ W/m·K, $c = 385$ J/kg·K, $\rho = 8920$ kg/m^3; Gases (given): $k = 0.05$ W/m·K, $\nu = 50 \times 10^{-6}$ m^2/s, Pr = 0.69.

ANALYSIS: (a) If the lumped capacitance analysis may be used, it follows from Equation 5.5 that

$$t = \frac{\rho V c}{\bar{h} A_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{D\rho c}{6\bar{h}} \ln(50).$$

Neglecting the viscosity ratio correlation for variable property effects, use of V = 5 m/s with the Whitaker correlation yields

$$\overline{Nu}_D = (\bar{h}D/k) = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Pr^{0.4} \qquad Re_D = \frac{VD}{\nu} = \frac{5\,\text{m/s}(0.001\,\text{m})}{50 \times 10^{-6}\,\text{m}^2/\text{s}} = 100$$

$$\bar{h} = \frac{0.05\,\text{W/m}\cdot\text{K}}{0.001\,\text{m}} \left[2 + \left(0.4(100)^{1/2} + 0.06(100)^{2/3}\right)(0.69)^{0.4}\right] = 328\,\text{W/m}^2\cdot\text{K}$$

Since Bi = $\bar{h}(r_o/3)/k$ = 5.5×10^{-4}, the lumped capacitance method may be used. Hence,

$$t = \frac{0.001\,\text{m}\left(8920\,\text{kg/m}^3\right)385\,\text{J/kg}\cdot\text{K}}{6 \times 328\,\text{W/m}^2\cdot\text{K}} \ln(50) = 6.83\,\text{s}$$ <

(b) Performing an energy balance on the junction and evaluating radiation exchange from Equation 1.7, $q_{conv} = q_{rad}$. Hence, with $\varepsilon = 0.5$,

$$\bar{h}A_s(T_\infty - T) = \varepsilon A_s \sigma \left(T^4 - T_c^4\right)$$

$$(1000 - T)\text{K} = \frac{0.5 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4}{328\,\text{W/m}^2\cdot\text{K}} \left[T^4 - (400)^4\right]\text{K}^4.$$

$$T = 936\,\text{K}$$ <

(c) Using the *IHT First Law Model* for a *Solid Sphere* with the appropriate *Correlation* for external flow from the Tool Pad, parametric calculations were performed to determine the effects of V and ε_g, and the following results were obtained.

Continued...

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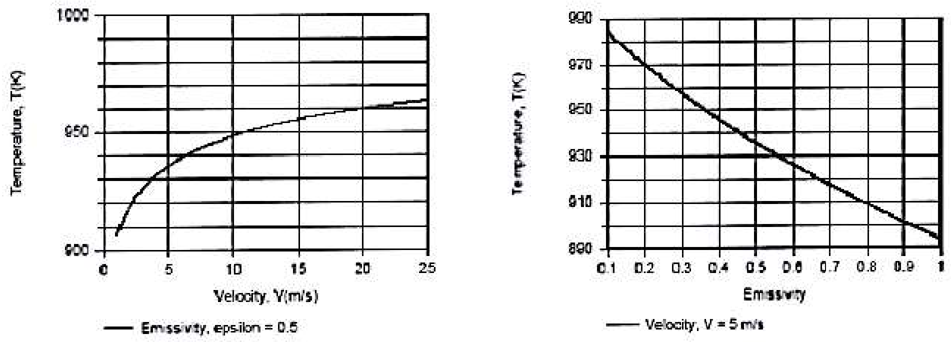
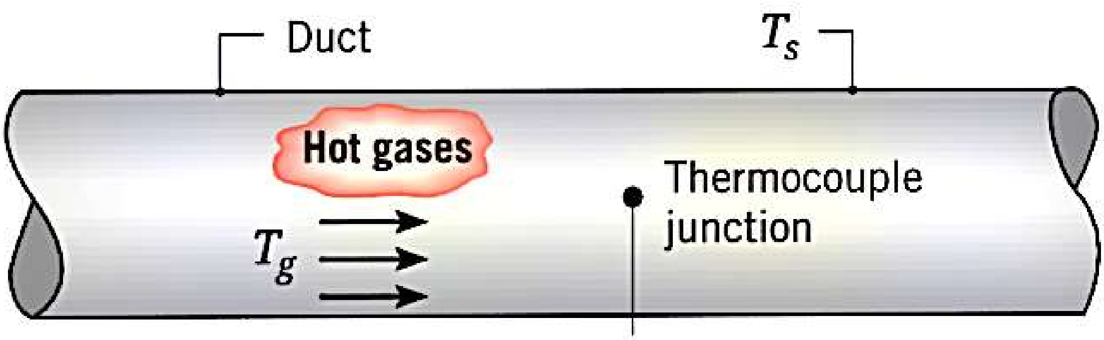
PROBLEM 7.78 (Cont.)



Since the temperature recorded by the thermocouple junction increases with increasing V and decreasing ε, the measurement error, $T_\infty - T$, decreases with increasing V and decreasing ε. The error is due to net radiative transfer from the junction (which depresses T) and hence should decrease with decreasing ε. For a prescribed heat loss, the temperature difference ($T_\infty - T$) decreases with decreasing convection resistance, and hence with increasing h(V).

COMMENTS: To infer the actual gas temperature (1000 K) from the measured result (936 K), correction would have to be made for radiation exchange with the cold surroundings.

7.79 A thermocouple junction is inserted in a large duct to measure the temperature of hot gases flowing through the duct.



(a) If the duct surface temperature T_s is less than the gas temperature T_g, will the thermocouple sense a temperature that is less than, equal to, or greater than T_g? Justify your answer on the basis of a simple analysis.

(b) A thermocouple junction in the shape of a 2-mm-diameter sphere with a surface emissivity of 0.60 is placed in a gas stream moving at 3 m/s. If the thermocouple senses a temperature of 320°C when the duct surface temperature is 175°C, what is the actual gas temperature? The gas may be assumed to have the properties of air at atmospheric pressure.

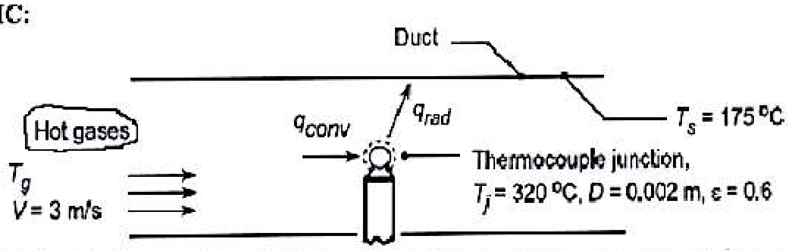
(c) How would changes in velocity and emissivity affect the temperature measurement error? Determine the measurement error for velocities in the range $1 \leq V \leq 25$ m/s ($\varepsilon = 0.6$) and for emissivities in the range $0.1 \leq \varepsilon \leq 1.0$ ($V = 3$ m/s).

PROBLEM 7.79

KNOWN: Diameter, emissivity and temperature of a thermocouple junction exposed to hot gases flowing through a duct of prescribed surface temperature.

FIND: (a) Relative magnitudes of gas and thermocouple temperatures if the duct surface temperature is less than the gas temperature, (b) Gas temperature for prescribed conditions, (c) Effect of Velocity and emissivity on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Junction is diffuse-gray, (3) Duct forms a large enclosure about the junction, (4) Negligible heat transfer by conduction through the thermocouple leads, (5) Gas properties are those of atmospheric air.

PROPERTIES: *Table A-4*, Air ($T_f \approx 650$ K, 1 atm): $\nu = 60.21 \times 10^{-6}\ m^2/s$, k = 0.0497 W/m·K, Pr = 0.690, $\mu = 322.5 \times 10^{-7}\ N\cdot s/m^2$; Air ($T_j$ = 593 K, 1 atm): $\mu = 304 \times 10^{-7}\ N\cdot s/m^2$.

ANALYSIS: (a) From an energy balance on the thermocouple junction, $q_{conv\,(g\to j)} = q_{rad\,(j\to s)}$. Hence,

$$\overline{h}A\left(T_g - T_j\right) = \varepsilon\sigma A\left(T_j^4 - T_s^4\right) \qquad \text{or} \qquad T_g - T_j = \frac{\varepsilon}{\overline{h}}\sigma\left(T_j^4 - T_s^4\right).$$

If $T_s < T_j$, it follows that $T_j < T_g$. <

(b) Neglecting the variable property correction, $(\mu/\mu_s)^{1/4} = (322.5/304)^{1/4} = \mathbf{1.01} \approx 1.00$, and using

$$Re_D = \frac{VD}{\nu} = \frac{3\,m/s\,(0.002\,m)}{60.21 \times 10^{-6}\,m^2/s} = 100$$

the Whitaker correlation for a sphere gives

$$\overline{h} = \frac{0.0497\,W/m\cdot K}{0.002\,m}\left\{2 + \left[0.4(100)^{1/2} + 0.06(100)^{2/3}\right](0.69)^{0.4}\right\} = 163\,W/m^2\cdot K.$$

Hence

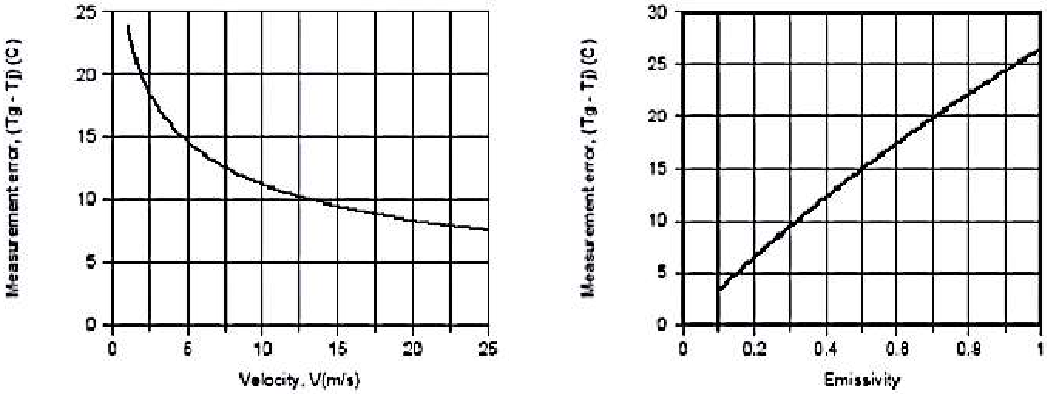
$$\left(T_g - 593K\right) = \frac{0.6}{163\,W/m^2\cdot K}5.67 \times 10^{-8}\,W/m^2\cdot K^4\left[(593\,K)^4 - (448\,K)^4\right] = 17K$$

$$T_g = 610\,K = 337^\circ C.$$ <

(c) With T_g fixed at 610 K, the IHT *First Law* Model was used with the *Correlations* and *Properties* Tool Pads to compute the measurement error as a function of V and ε.

Continued...

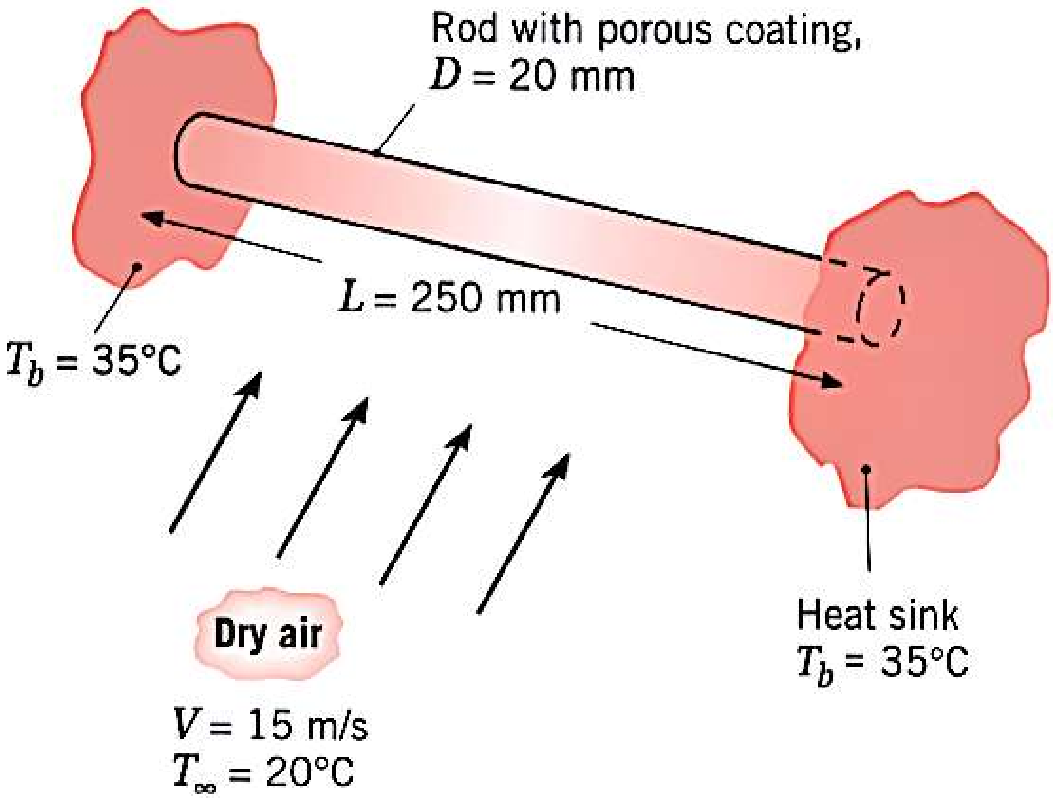
PROBLEM 7.79 (Cont.)



Since the convection resistance decreases with increasing V, the junction temperature will approach the gas temperature and the measurement error will decrease. Since the depression in the junction temperature is due to radiation losses from the junction to the duct wall, a reduction in ε will reduce the measurement error.

COMMENTS: In part (b), calculations could be improved by evaluating properties at 610 K (instead of 650 K).

7.125 Dry air at 20°C and a velocity of 15 m/s flows over a 20-mm-diameter rod covered with a thin porous coating that is saturated with water. The rod ($k = 175$ W/m · K) is 250 mm long and its ends are attached to heat sinks maintained at 35°C.



Perform a steady-state, finite-difference analysis of the rod–porous coating system, considering conduction in the rod as well as energy transfer from the surface by convection heat and mass transfer. Use the analysis to estimate the temperature at the midspan of the rod and the evaporation rate from the surface. (*Suggestions:* Use 10 nodes to represent the half-length of the system.

Estimate the overall average convection heat transfer coefficient based on an average film temperature for the system, and use the heat–mass transfer analogy to determine the average convection mass transfer coefficient. Validate your code by using it to predict a temperature distribution that agrees with the analytical solution for a fin without evaporation.)

PROBLEM 7.125 (Cont.)

where all properties are evaluated at $\overline{T}_f$. The density of water vapor, $\rho_{A,s,m}$, as well as the heat of vaporization, $h_{fg,m}$, must be evaluated at the nodal temperature T_m.

Using the *IHT Correlation Tool, External Flow, Cylinder*, an estimate of $\overline{h}_D = 101\ W/m^2\cdot K$ was obtained with $\overline{T}_f = 298.5\ K$ (based upon assumed value of $T_1 = 27°C$). From the analogy, Eq. (4), find that $\overline{h}_{D,m} = 0.0772\ m/s$. Using the *IHT Workspace*, the finite-difference equations, Eq. (1), were entered and the temperature distribution (K, Case 1) determined as tabulated below. Using this same code with $\overline{h}_{D,m} = 1.0 \times 10^{-10}\ m/s$, the temperature distribution (K, Case 2) was obtained. The results compared identically with the analytical solution for a fin with an adiabatic tip using the *IHT Model, Extended Surface, Rectangular Pin Fin*.

Case	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_b
1	287	287.2	287.6	288.3	289.4	290.9	292.9	295.4	298.6	302.7	308
2	300.3	300.4	300.6	300.9	301.4	302.1	302.8	303.8	305	306.4	308

<

The evaporation rate obtained by summing rates from each nodal element including node b is

$$n_{A,tot} = 1.08 \times 10^{-5}\ kg/s \quad <$$

COMMENTS: A copy of the *IHT Workspace* used to perform the above analysis is shown below.

```
// Nodal finite-difference equations (Only Nodes 1, 2 and 10 shown):
k * Ac * (T2 - T1) / delx - mdot1 * hfg1 + hbar * P * delx * (Tinf - T1) + k * Ac * (T2 - T1) / delx = 0
mdot1 = hmbar * P * delx * rhoA1
k * Ac * (T3 - T2) / delx - mdot2 * hfg2 + hbar * P * delx * (Tinf - T2) + k * Ac * (T1 - T2) / delx = 0
mdot2 = hmbar * P * delx * rhoA2
........

........
k * Ac * (Tb - T10) / delx - mdot10 * hfg10 + hbar * P * delx * (Tinf - T10) + k * Ac * (T9 - T10) / delx = 0
mdot10 = hmbar * P * delx * rhoA10

// Evaporation Rate:
mtot = mdot1/2 + mdot2 + mdot3 + mdot4 + mdot5 + mdot6 + mdot7 + mdot8 + mdot9 + mdot10 + mdotb
mdotb = hmbar * P * delx/2 * rhoAb

// Properties Tool - Water Vapor, rhoAm and hfgm
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 1                                  // Quality (0=sat liquid or 1=sat vapor)
rhoA1 = rho_Tx("Water",T1,x)           // Density, kg/m^3
hfg1 = hfg_T("Water",T1)               // Heat of vaporization, J/kg
rhoA2 = rho_Tx("Water",T2,x)           // Density, kg/m^3
hfg2 = hfg_T("Water",T2)               // Heat of vaporization, J/kg
.....

.....
rhoA10 = rho_Tx("Water",T10,x)         // Density, kg/m^3
hfg10 = hfg_T("Water",T10)             // Heat of vaporization, J/kg
rhoAb = rho_Tx("Water",Tb,x)           // Density, kg/m^3
hfgb = hfg_T("Water",Tb)               // Heat of vaporization, J/kg

// Assigned Variables
Ac = pi * D^2 /4                       // Cross-sectional area, m^2
P = pi * D                             // Perimeter, m
D = 0.020                              // Diameter, m
delx = 0.125 /10                       // Spatial increment, m
k = 175                                // Thermal conductivity, W/m.K
Tb = 35 + 273                          // Base temperature, K
Tinf = 20 + 273                        // Fluid temperature, K
hmbar = 0.07719                        // Average mass transfer coefficient, m/s
hbar = 101                             // Average heat transfer coefficient, W/m^2.K
```