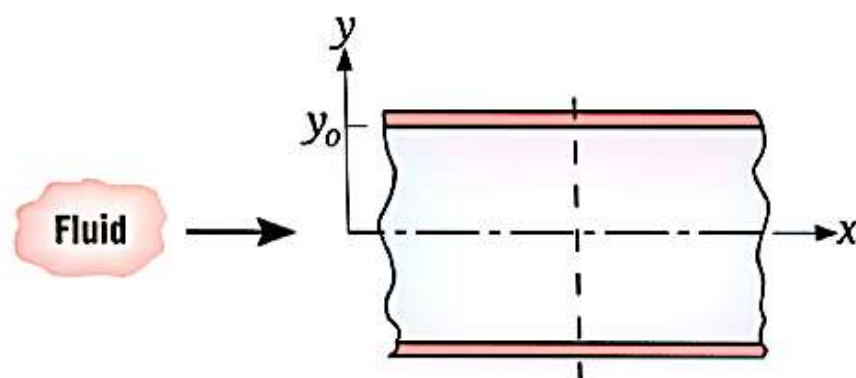


8.8 At a particular axial station, velocity and temperature profiles for laminar flow in a parallel plate channel have the form

$$u(y) = 0.75[1 - (y/y_o)^2]$$

$$T(y) = 5.0 + 95.66(y/y_o)^2 - 47.83(y/y_o)^4$$

with units of m/s and °C, respectively.



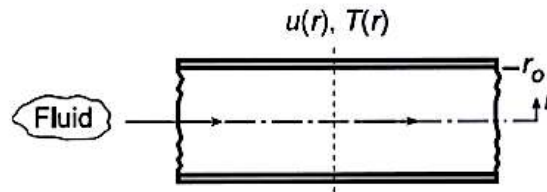
Determine corresponding values of the mean velocity, u_m and mean (or bulk) temperature, T_m . Plot the velocity and temperature distributions. Do your values of u_m and T_m appear reasonable?

PROBLEM 8.8

KNOWN: Velocity and temperature profiles for laminar flow in a tube of radius $r_o = 10 \text{ mm}$

FIND: Mean (or bulk) temperature, T_m at this axial position.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar incompressible flow, (2) Constant properties.

ANALYSIS: The prescribed velocity and temperature profiles, (m/s and K, respectively) are

$$u(r) = 0.1 [1 - (r/r_o)^2] \quad T(r) = 344.8 + 75.0 (r/r_o)^2 - 18.8 (r/r_o)^4 \quad (1,2)$$

For incompressible flow with constant c_p in a circular tube, from Eq. 8.27, the mean temperature and u_m , the mean velocity, from Eq. 8.8 are, respectively,

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} u(r) T(r) r \, dr \quad u_m = \frac{2}{r_o^2} \int_0^{r_o} u(r) r \, dr \quad (3,4)$$

Substituting the velocity profile, Eq. (1), into Eq. (4) and integrating, find

$$u_m = \frac{2}{r_o^2} \int_0^{r_o} 0.1 [1 - (r/r_o)^2] r \, dr = 2 \cdot 0.1 \left[\frac{1}{2} (r/r_o)^2 - \frac{1}{4} (r/r_o)^4 \right]_0^{r_o} = 0.05 \text{ m/s}$$

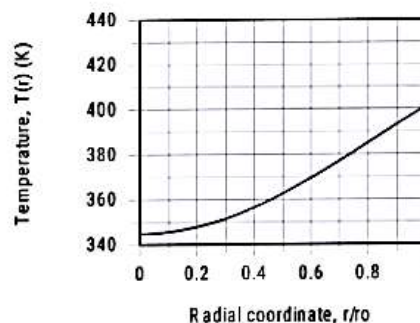
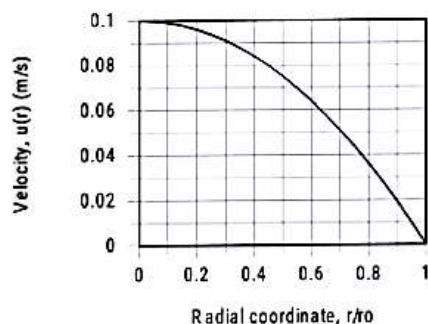
Substituting the profiles and u_m into Eq. (3), find

$$T_m = \frac{2}{0.05 \text{ m/s} \cdot r_o^2} \int_0^{r_o} 0.1 [1 - (r/r_o)^2] [344.8 + 75.0 (r/r_o)^2 - 18.8 (r/r_o)^4] r \, dr$$

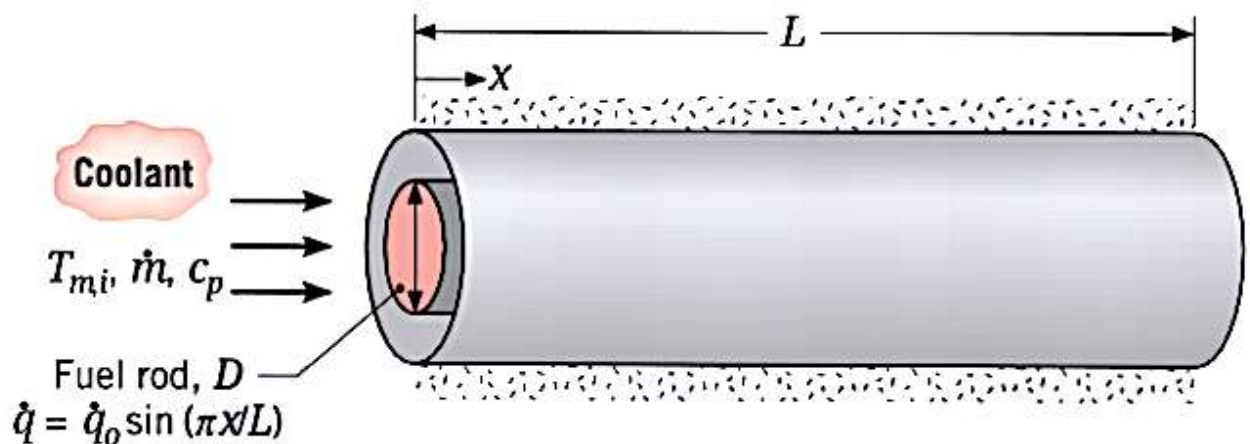
$$T_m = \frac{1}{4} \int_0^1 [344.8 r/r_o + 75.0 r/r_o^3 - 18.8 r/r_o^5] \, d(r/r_o)$$

$$T_m = \frac{1}{4} [172.40 r/r_o + 18.75 (r/r_o)^3 - 86.20 (r/r_o)^5]_0^1 = 12.50 + 2.35 = 367 \text{ K}$$

The velocity and temperature profiles appear as shown below. Do the values of u_m and T_m found above compare with their respective profiles as you thought? Is the fluid being heated or cooled?



8.13 Consider a cylindrical nuclear fuel rod of length L and diameter D that is encased in a concentric tube. Pressurized water flows through the annular region between the rod and the tube at a rate \dot{m} , and the outer surface of the tube is well insulated. Heat generation occurs within the fuel rod, and the volumetric generation rate is known to vary sinusoidally with distance along the rod. That is, $\dot{q}(x) = \dot{q}_0 \sin(\pi x/L)$, where \dot{q}_0 (W/m^3) is a constant. A uniform convection coefficient h may be assumed to exist between the surface of the rod and the water.



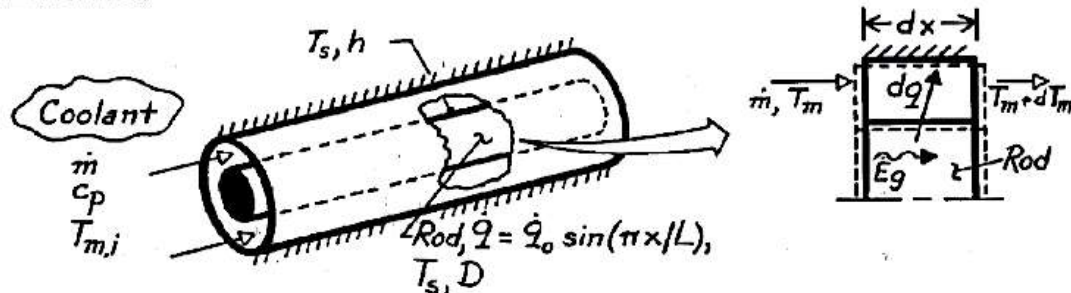
- Obtain expressions for the local heat flux $q''(x)$ and the total heat transfer q from the fuel rod to the water.
- Obtain an expression for the variation of the mean temperature $T_m(x)$ of the water with distance x along the tube.
- Obtain an expression for the variation of the rod surface temperature $T_s(x)$ with distance x along the tube. Develop an expression for the x location at which this temperature is maximized.

PROBLEM 8.13

KNOWN: Geometry and coolant flow conditions associated with a nuclear fuel rod. Axial variation of heat generation within the rod.

FIND: (a) Axial variation of local heat flux and total heat transfer rate, (b) Axial variation of mean coolant temperature, (c) Axial variation of rod surface temperature and location of maximum temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant fluid properties, (3) Uniform surface convection coefficient, (4) Negligible axial conduction in rod and fluid, (5) Negligible kinetic energy, potential energy and flow work changes, (6) Outer surface is adiabatic.

ANALYSIS: (a) Performing an energy balance for a control volume about the rod,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \quad - dq + \dot{E}_g = 0$$

or

$$- q''(p D dx) + \dot{q}_0 \sin(p x/L) \left(p D^2 / 4 \right) dx = 0 \quad q'' = \dot{q}_0 (D/4) \sin(p x/L). \quad <$$

The total heat transfer rate is then

$$q = \int_0^L q'' p D dx = \left(p D^2 / 4 \right) \dot{q}_0 \int_0^L \sin(p x/L) dx$$

$$q = \frac{p D^2}{4} \dot{q}_0 \left[-\frac{L}{p} \cos \frac{p x}{L} \right]_0^L = \frac{D^2 \dot{q}_0 L}{4} (1+1)$$

$$q = \frac{D^2 L}{2} \dot{q}_0. \quad (1) <$$

(b) Performing an energy balance for a control volume about the coolant,

$$\dot{m} c_p T_m + dq = \dot{m} c_p (T_m + dT_m) = 0.$$

Hence

$$\dot{m} c_p dT_m = dq = (p D dx) q''$$

$$\frac{dT_m}{dx} = \frac{p D}{\dot{m} c_p} \frac{\dot{q}_0 D}{4} \sin \frac{p x}{L}$$

Continued.....

PROBLEM 8.13 (Cont.)

Integrating,

$$T_m(x) - T_{m,i} = \frac{p D^2}{4} \frac{\dot{q}_0}{\dot{m} c_p} \int_0^x \sin \frac{p x}{L} dx$$

$$T_m(x) = T_{m,i} + \frac{L D^2}{4} \frac{\dot{q}_0}{\dot{m} c_p} \left[1 - \cos \frac{p x}{L} \right]$$

(2) <

(c) From Newton's law of cooling,

$$q'' = h(T_s - T_m).$$

Hence

$$T_s = \frac{q''}{h} + T_m$$

$$T_s = \frac{\dot{q}_0 D}{4h} \sin \frac{p x}{L} + T_{m,i} + \frac{L D^2}{4} \frac{\dot{q}_0}{\dot{m} c_p} \left[1 - \cos \frac{p x}{L} \right]$$

<

To determine the location of the maximum surface temperature, evaluate

$$\frac{dT_s}{dx} = 0 = \frac{\dot{q}_0 D p}{4hL} \cos \frac{p x}{L} + \frac{L D^2}{4} \frac{\dot{q}_0}{\dot{m} c_p} \frac{p}{L} \sin \frac{p x}{L}$$

or

$$\frac{1}{hL} \cos \frac{p x}{L} + \frac{D}{\dot{m} c_p} \sin \frac{p x}{L} = 0.$$

Hence

$$\tan \frac{p x}{L} = - \frac{\dot{m} c_p}{D h L}$$

$$x = \frac{L}{p} \tan^{-1} \left(- \frac{\dot{m} c_p}{D h L} \right) = x_{\max}.$$

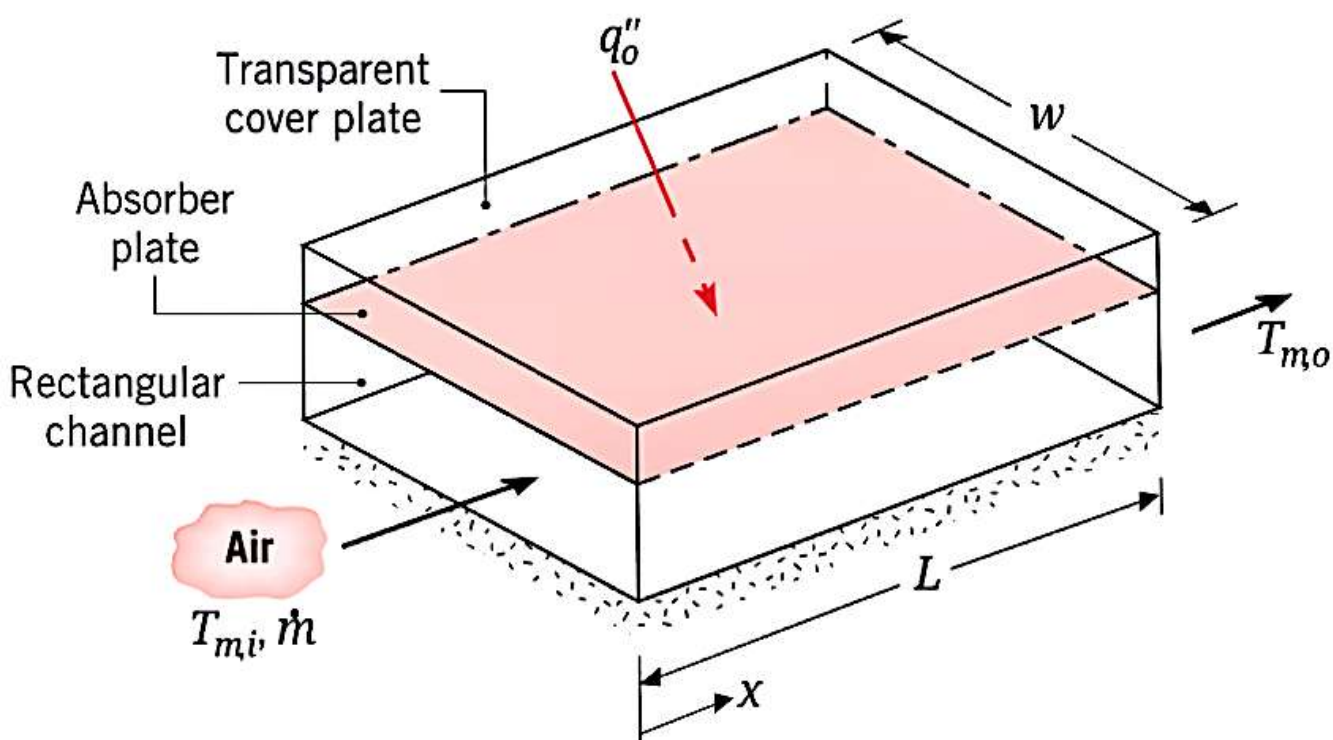
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COMMENTS: Note from Eq. (2) that

$$T_{m,o} = T_m(L) = T_{m,i} + \frac{L D^2 \dot{q}_0}{2 \dot{m} c_p}$$

which is equivalent to the result obtained by combining Eq. (1) and Eq. 8.37.

- 8.15** A flat-plate solar collector is used to heat atmospheric air flowing through a rectangular channel. The bottom surface of the channel is well insulated, while the top surface is subjected to a uniform heat flux q_o'' , which is due to the net effect of solar radiation absorption and heat exchange between the absorber and cover plates.
- (a) Beginning with an appropriate differential control volume, obtain an equation that could be used to determine the mean air temperature $T_m(x)$ as a function of distance along the channel. Solve this equation to obtain an expression for the mean temperature of the air leaving the collector.



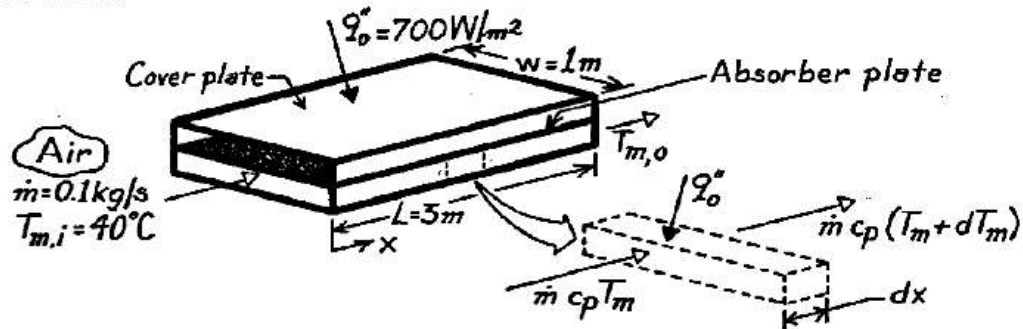
- (b) With air inlet conditions of $\dot{m} = 0.1 \text{ kg/s}$ and $T_{mi} = 40^\circ\text{C}$, what is the air outlet temperature if $L = 3 \text{ m}$, $w = 1 \text{ m}$, and $q''_o = 700 \text{ W/m}^2$? The specific heat of air is $c_p = 1008 \text{ J/kg} \cdot \text{K}$.

PROBLEM 8.15

KNOWN: Surface heat flux for air flow through a rectangular channel.

FIND: (a) Differential equation describing variation in air mean temperature, (b) Air outlet temperature for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible change in kinetic and potential energy of air, (2) No heat loss through bottom of channel, (3) Uniform heat flux at top of channel.

PROPERTIES: Table A-4, Air ($T \approx 50^\circ\text{C}$, 1 atm): $c_p = 1008 \text{ J/kg}\cdot\text{K}$.

ANALYSIS: (a) For the differential control volume about the air,

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m} c_p T_m + q_o'' (w \times dx) = \dot{m} c_p (T_m + dT_m)$$

$$\frac{dT_m}{dx} = \frac{q_o'' \times w}{\dot{m} c_p}$$

Separating and integrating between the limits of $x = 0$ and x , find

$$T_m(x) = T_{m,i} + \frac{q_o'' (w \times x)}{\dot{m} c_p}$$

$$T_{m,o} = T_{m,i} + \frac{q_o'' (w \times L)}{\dot{m} c_p}$$

(b) Substituting numerical values, the air outlet temperature is

$$T_{m,o} = 40^\circ\text{C} + \frac{(700 \text{ W/m}^2) (1 \times 3) \text{ m}^2}{0.1 \text{ kg/s} (1008 \text{ J/kg}\cdot\text{K})}$$

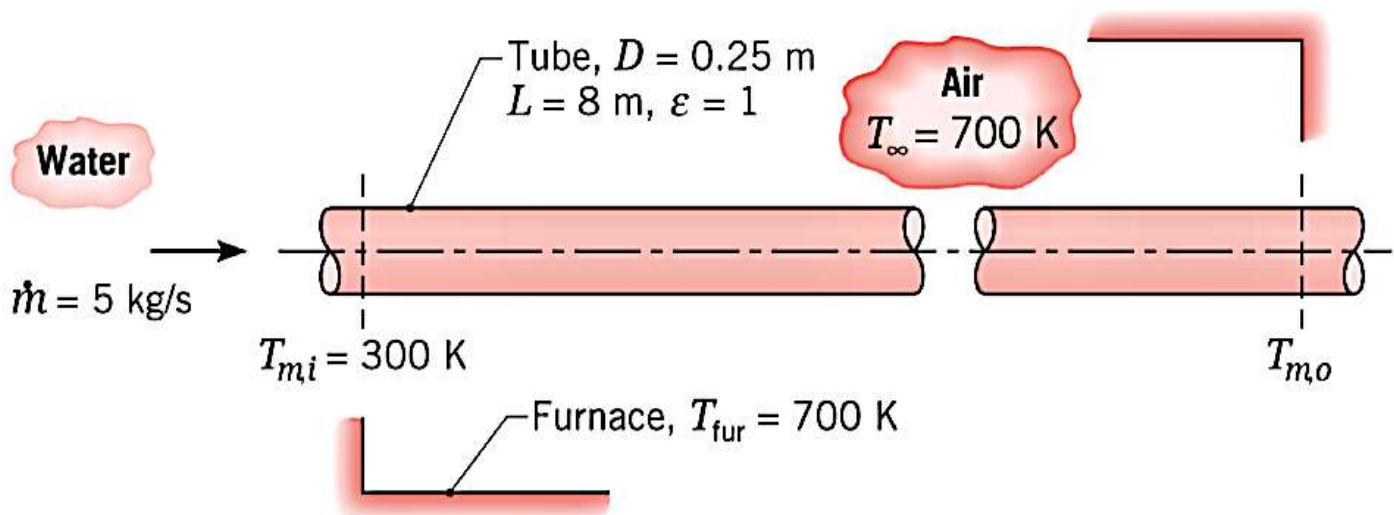
$$T_{m,o} = 60.8^\circ\text{C}.$$

COMMENTS: Due to increasing heat loss with increasing T_m , the net flux q_o'' will actually decrease slightly with increasing x .

8.17 Water at 300 K and a flow rate of 5 kg/s enters a black, thin-walled tube, which passes through a large furnace whose walls and air are at a temperature of 700 K. The diameter and length of the tube are 0.25 m and 8 m, respectively. Convection coefficients associated with

■ Problems

water flow through the tube and air flow over the tube are $300 \text{ W/m}^2 \cdot \text{K}$ and $50 \text{ W/m}^2 \cdot \text{K}$, respectively.



8.21

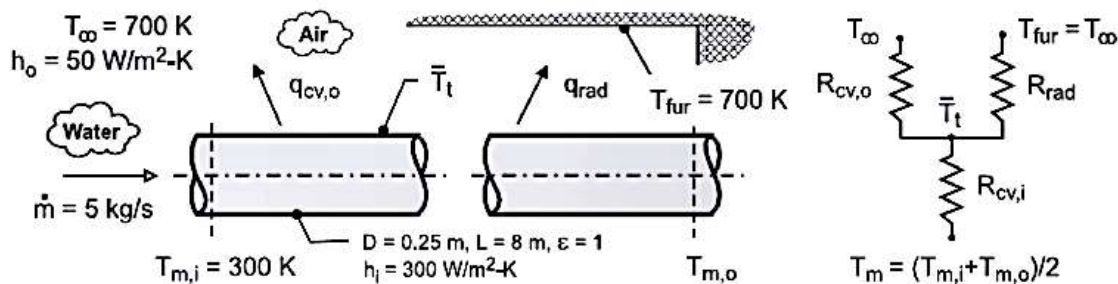
- Write an expression for the linearized radiation coefficient corresponding to radiation exchange between the outer surface of the pipe and the furnace walls. Explain how to calculate this coefficient if the surface temperature of the tube is represented by the arithmetic mean of its inlet and outlet values.
- Determine the outlet temperature of the water, T_{mo} .

PROBLEM 8.17

KNOWN: Water at prescribed temperature and flow rate enters a 0.25 m diameter, black thin-walled tube of 8-m length, which passes through a large furnace whose walls and air are at a temperature of $T_{\text{fur}} = T_{\infty} = 700 \text{ K}$. The convection coefficients for the internal water flow and external furnace air are $300 \text{ W/m}^2 \cdot \text{K}$ and $50 \text{ W/m}^2 \cdot \text{K}$, respectively.

FIND: (a) An expression for the linearized radiation coefficient for the radiation exchange process between the outer surface of the pipe and the furnace walls; represent the tube by an average temperature and explain how to calculate this value, and (b) determine the outlet temperature of the water, $T_{m,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions; (2) Tube is small object with large, isothermal surroundings; (3) Furnace air and walls are at the same temperature; and (3) Tube is thin-walled with black surface.

PROPERTIES: Table A-6, Water ($T_m = (T_{m,i} + T_{m,o})/2 = 331 \text{ K}$): $c_p = 4192 \text{ J/kg} \cdot \text{K}$.

ANALYSIS: (a) The linearized radiation coefficient follows from Eq. 1.9 with $\epsilon = 1$,

$$\bar{h}_{\text{rad}} = \frac{\bar{T}_t^2 + \bar{T}_{\text{fur}}^2}{\bar{T}_t - \bar{T}_{\text{fur}}}$$

where \bar{T}_t represents the average tube wall surface temperature, which can be evaluated from an energy balance on the tube as represented by the thermal circuit above.

$$T_m = \frac{T_{m,i} + T_{m,o}}{2}$$

$$R_{\text{tot}} = R_{cv,i} + \frac{1}{\frac{1}{R_{cv,o}} + \frac{1}{R_{\text{rad}}}}$$

$$\frac{T_m - \bar{T}_t}{R_{cv,i}} = \frac{\bar{T}_t - T_{\text{fur}}}{\frac{1}{R_{cv,o}} + \frac{1}{R_{\text{rad}}}}$$

The thermal resistances, with $A_s = PL = \pi DL$, are

$$R_{cv,i} = \frac{1}{h_i A_s} \quad R_{cv,o} = \frac{1}{h_o A_s} \quad R_{\text{rad}} = \frac{1}{\bar{h}_{\text{rad}}}$$

(b) The outlet temperature can be calculated using the energy balance relation, Eq. 8.46b, with $T_{\text{fur}} = T_{\infty}$,

$$\frac{T_{\infty} - T_{m,o}}{T_{\infty} - T_{m,i}} = \exp \left(\frac{1}{mc_p R_{\text{tot}}} \right)$$

where c_p is evaluated at T_m . Using IHT, the following results were obtained.

$$R_{cv,i} = 6.631 \times 10^5 \text{ K/W} \quad R_{cv,o} = 3.978 \times 10^4 \text{ K/W} \quad R_{\text{rad}} = 4.724 \times 10^4 \text{ K/W}$$

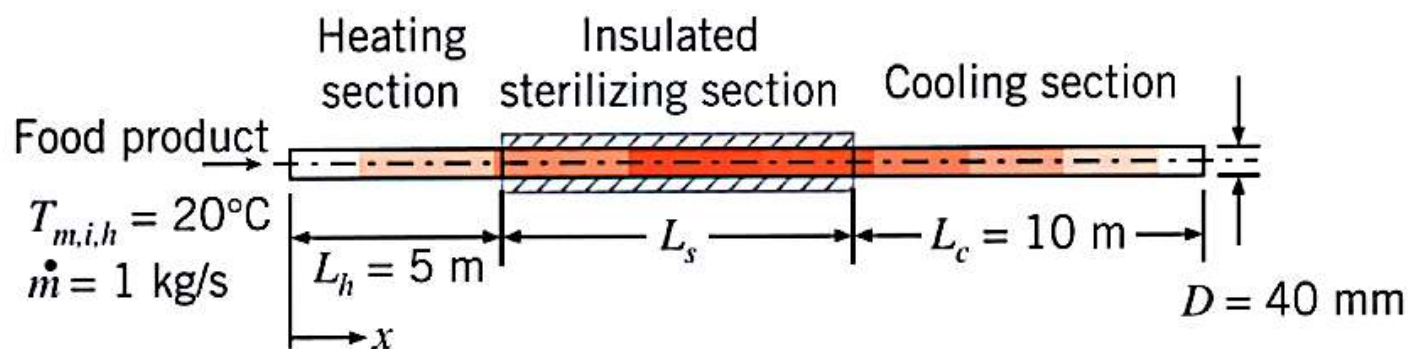
$$T_m = 331 \text{ K} \quad \bar{T}_t = 418 \text{ K}$$

$$T_{m,o} = 362 \text{ K}$$

COMMENTS: Since $T_{\infty} = T_{\text{fur}}$, it was possible to use Eq. 8.46b with R_{tot} . How would you write the energy balance relation if $T_{\infty} \neq T_{\text{fur}}$?

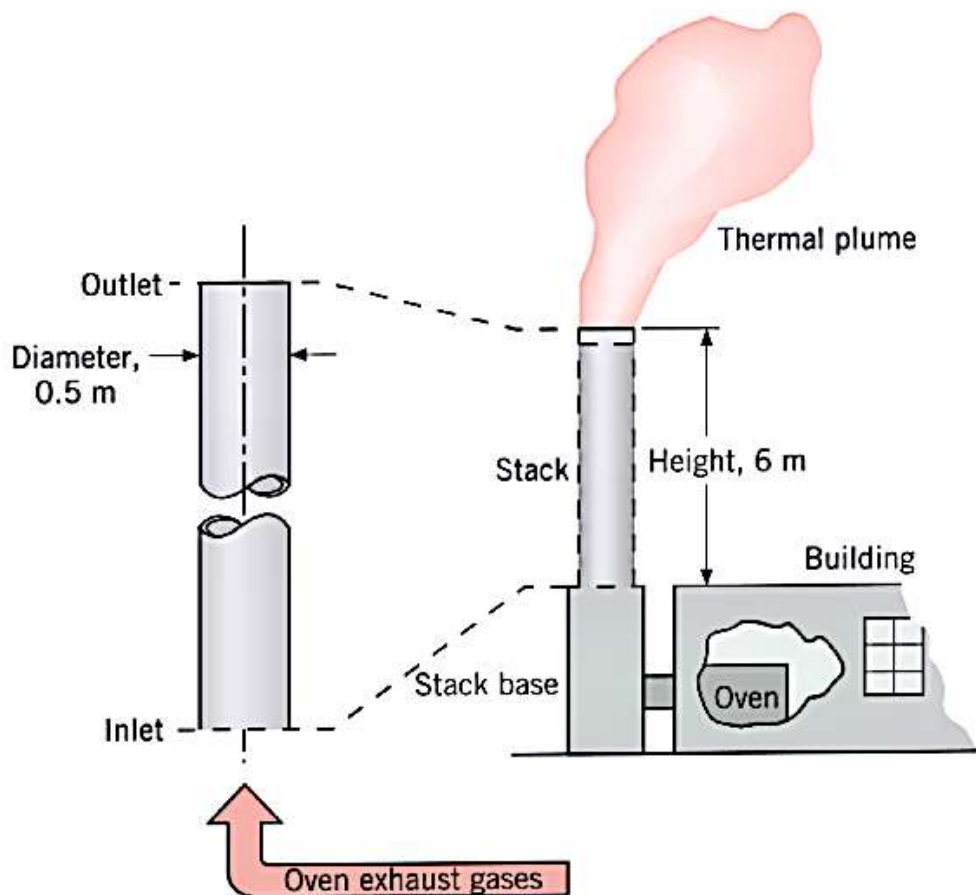
8.34 A liquid food product is processed in a continuous-flow sterilizer. The liquid enters the sterilizer at a temperature and flow rate of $T_{m,i,h} = 20^\circ\text{C}$, $\dot{m} = 1 \text{ kg/s}$, respectively. A time-at-temperature constraint requires that the product be held at a mean temperature of $T_m = 90^\circ\text{C}$ for 10 s to kill bacteria, while a second constraint is that the local product temperature cannot exceed $T_{\max} = 230^\circ\text{C}$ in order to preserve a pleasing taste. The sterilizer consists of an upstream, $L_h = 5 \text{ m}$ heating section characterized by a uniform heat flux,

an intermediate insulated sterilizing section, and a downstream cooling section of length $L_c = 10$ m. The cooling section is composed of an uninsulated tube exposed to a quiescent environment at $T_\infty = 20^\circ\text{C}$. The thin-walled tubing is of diameter $D = 40$ mm. Food properties are similar to those of liquid water at $T = 330$ K.



- What heat flux is required in the heating section to ensure a maximum mean product temperature of $T_m = 90^\circ\text{C}$?
- Determine the location and value of the maximum local product temperature. Is the second constraint satisfied?
- Determine the minimum length of the sterilizing section needed to satisfy the time-at-temperature constraint.
- Sketch the axial distribution of the mean, surface, and centerline temperatures from the inlet of the heating section to the outlet of the cooling section.

8.52 Exhaust gases from a wire processing oven are discharged into a tall stack, and the gas and stack surface temperatures at the outlet of the stack must be estimated. Knowledge of the outlet gas temperature $T_{m,o}$ is useful for predicting the dispersion of effluents in the thermal plume, while knowledge of the outlet stack surface temperature $T_{s,o}$ indicates whether condensation of the gas products will occur. The thin-walled, cylindrical stack is 0.5 m in diameter and 6.0 m high. The exhaust gas flow rate is 0.5 kg/s, and the inlet temperature is 600°C.



(a) Consider conditions for which the ambient air temperature and wind velocity are 4°C and 5 m/s, respectively. Approximating the thermophysical properties of the gas as those of atmospheric air, estimate the outlet gas and stack surface temperatures for the given conditions.

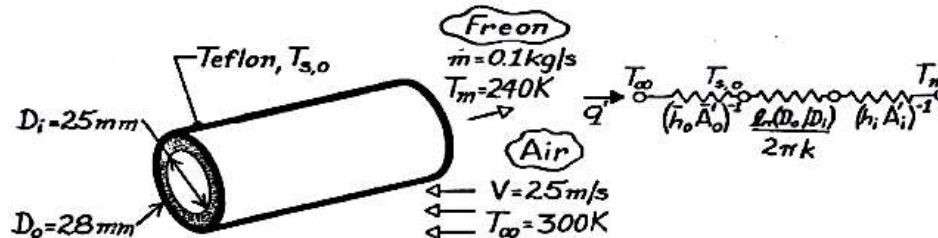
(b) The gas outlet temperature is sensitive to variations in the ambient air temperature and wind velocity. For $T_\infty = -25^\circ\text{C}$, 5°C , and 35°C , compute and plot the gas outlet temperature as a function of wind velocity for $2 \leq V \leq 10$ m/s.

PROBLEM 8.52

KNOWN: Flow rate and temperature of Freon passing through a Teflon tube of prescribed inner and outer diameter. Velocity and temperature of air in cross flow over tube.

FIND: Heat transfer per unit tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Fully developed flow.

PROPERTIES: Table A-4, Air ($T = 300\text{K}$, 1 atm): $\nu = 15.89 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.0263 \text{ W/m}\cdot\text{K}$, $\text{Pr} = 0.707$; Table A-5, Freon ($T = 240\text{K}$): $\mu = 3.85 \times 10^{-4} \text{ N}\cdot\text{s}/\text{m}^2$, $k = 0.069 \text{ W/m}\cdot\text{K}$, $\text{Pr} = 5.0$; Table A-3, Teflon ($T \approx 300\text{K}$): $k = 0.35 \text{ W/m}\cdot\text{K}$.

ANALYSIS: Considering the thermal circuit shown above, the heat rate is

$$q = \frac{T_F - T_m}{\left(\frac{1}{h_o \pi D_o} \right) + \frac{\ln(D_o/D_i)}{2\pi k} + \left(\frac{1}{h_i \pi D_i} \right)}$$

$$\text{Re}_{D,i} = \frac{4 \dot{m}}{\pi D_i \mu} = \frac{0.4 \text{ kg/s}}{\pi (0.025 \text{ m}) (3.85 \times 10^{-4} \text{ N}\cdot\text{s}/\text{m}^2)} = 13,228$$

and the flow is turbulent. Hence, from the Dittus-Boelter correlation

$$h_i = \frac{k}{D_i} 0.023 \text{Re}_{D,i}^{4/5} \text{Pr}^{0.4} = \frac{0.069 \text{ W/m}\cdot\text{K}}{0.025 \text{ m}} 0.023 (13,228)^{4/5} (5)^{0.4} = 240 \text{ W/m}^2\cdot\text{K}$$

$$\text{With } \text{Re}_{D,o} = \frac{VD_o}{\nu} = \frac{(2.5 \text{ m/s})(0.028 \text{ m})}{15.89 \times 10^{-6} \text{ m}^2/\text{s}} = 4.405 \times 10^4$$

it follows from Eq. 7.56 and Table 7.4 that

$$h_o = \frac{k}{D} 0.26 \text{Re}_{D,o}^{0.6} \text{Pr}^{0.37} = \frac{0.0263 \text{ W/m}\cdot\text{K}}{0.028 \text{ m}} 0.26 (4.405 \times 10^4)^{0.6} (0.707)^{0.37} = 131 \text{ W/m}^2\cdot\text{K}$$

Hence

$$q = \frac{T_F - T_m}{\left(\frac{1}{131 \text{ W/m}^2\cdot\text{K} \pi (0.028 \text{ m})} \right) + \frac{\ln(28/25)}{2\pi (0.350 \text{ W/m}\cdot\text{K})} + \left(\frac{1}{240 \text{ W/m}^2\cdot\text{K} \pi (0.025 \text{ m})} \right)}$$

$$q = \frac{(300 - 240) \text{ K}}{(0.087 + 0.052 + 0.053) \text{ K}\cdot\text{m/W}} = 312 \text{ W/m}$$

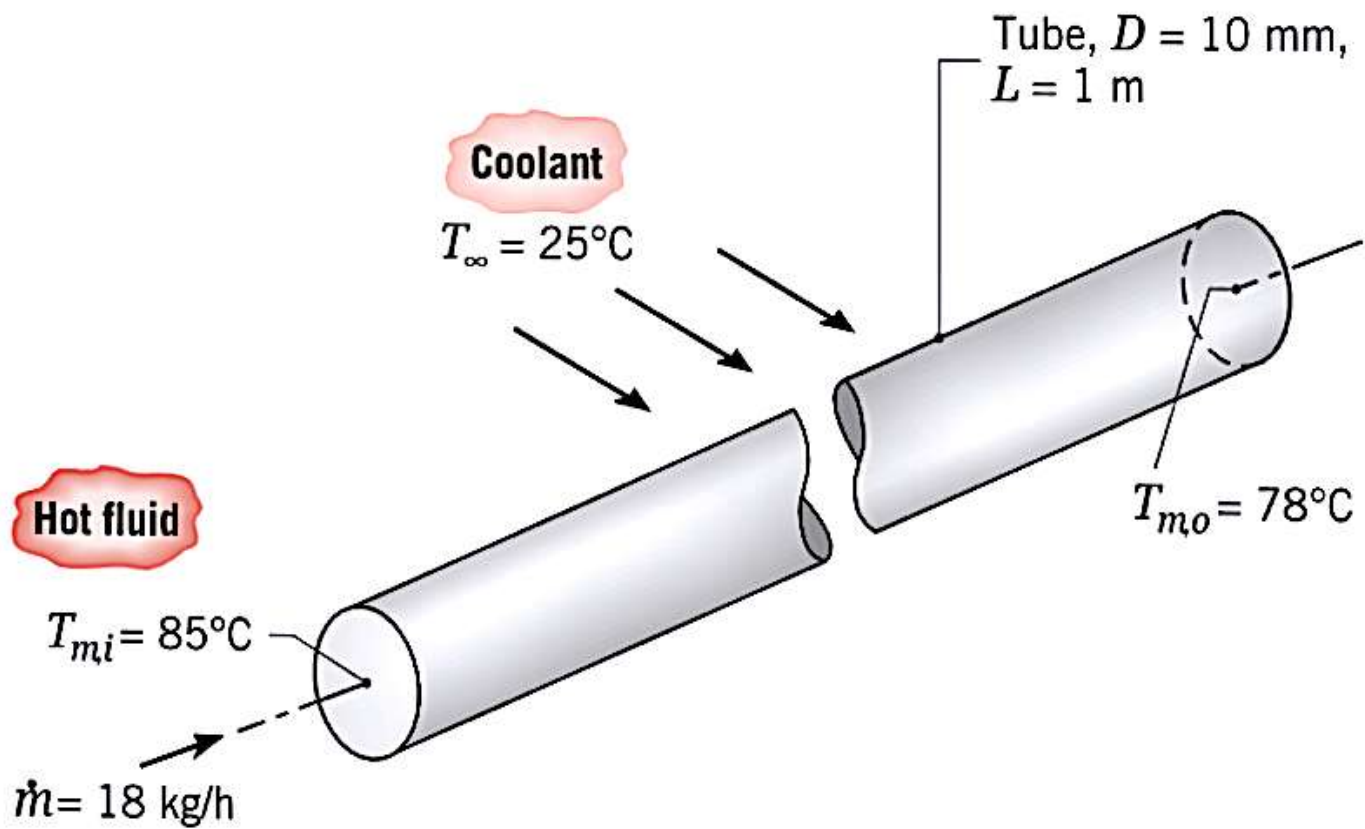
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COMMENTS: The three thermal resistances are comparable. Note that $T_{s,o} = T_F - q/h_o \pi D_o = 300\text{K} - 312 \text{ W/m} / (131 \text{ W/m}^2\cdot\text{K} \pi 0.028 \text{ m}) = 273 \text{ K}$.

8.53 A hot fluid passes through a thin-walled tube of 10-mm diameter and 1-m length, and a coolant at $T_\infty = 25^\circ\text{C}$ is in cross flow over the tube. When the flow rate is $\dot{m} = 18 \text{ kg/h}$ and the inlet temperature is $T_{mi} = 85^\circ\text{C}$, the outlet temperature is $T_{mo} = 78^\circ\text{C}$.

Assuming fully developed flow and thermal conditions in the tube, determine the outlet temperature, T_{mo} , if the flow rate is increased by a factor of 2. That is, $\dot{m} = 36 \text{ kg/h}$, with all other conditions the same. The thermophysical properties of the hot fluid are

$\rho = 1079 \text{ kg/m}^3$, $c_p = 2637 \text{ J/kg} \cdot \text{K}$, $\mu = 0.0034 \text{ N} \cdot \text{s/m}^2$, and $k = 0.261 \text{ W/m} \cdot \text{K}$.

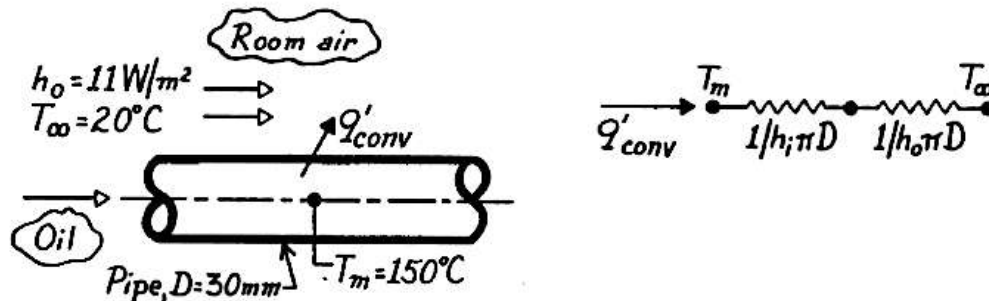


PROBLEM 8.53

KNOWN: Oil flowing slowly through a long, thin-walled pipe suspended in a room

FIND: Heat loss per unit length of the pipe, q'_{conv} .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Tube wall thermal resistance negligible, (3) Fully developed flow, (4) Radiation exchange between pipe and room negligible.

PROPERTIES: Table A-5, Unused engine oil ($T_m = 150^\circ\text{C} = 423\text{K}$): $k = 0.133 \text{ W/m}\cdot\text{K}$.

ANALYSIS: The rate equation, for a unit length of the pipe, can be written as

$$q'_{\text{conv}} = \frac{(T_m - T_\infty)}{R_{\text{ft}}}$$

where the thermal resistance is comprised of two elements,

$$R_{\text{ft}} = \frac{1}{h_i \pi D} + \frac{1}{h_o \pi D} = \frac{1}{\pi D} \left(\frac{1}{h_i} + \frac{1}{h_o} \right)$$

The convection coefficient for internal flow, h_i , must be estimated from an appropriate correlation. From practical considerations, we recognize that the oil flow rate cannot be large enough to achieve turbulent flow conditions. Hence, the flow is *laminar*, and if the pipe is very long, the flow will be *fully developed*. The appropriate correlation is

$$\text{Nu}_D = \frac{h_i D}{k} = 3.66$$

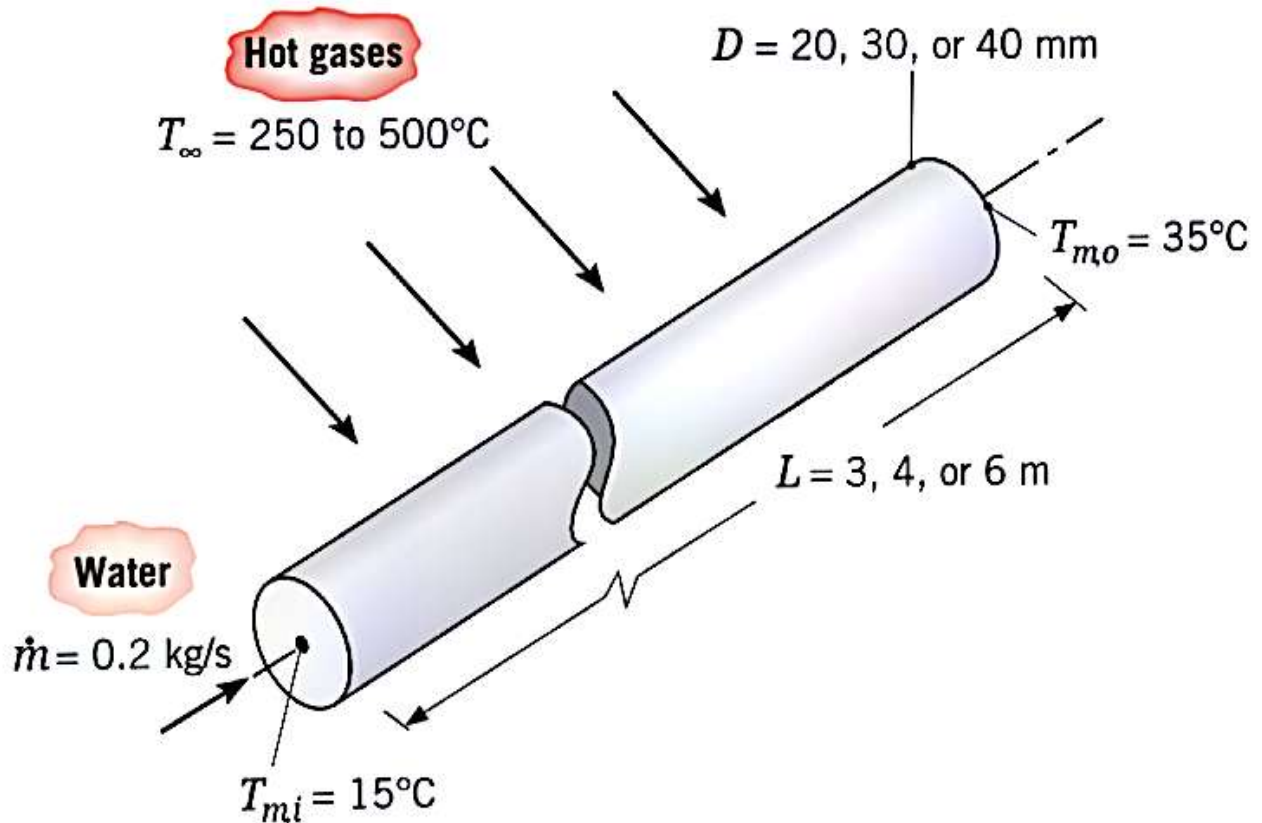
$$h_i = \text{Nu}_D \frac{k}{D} = 3.66 \cdot 0.133 \frac{\text{W}}{\text{m}\cdot\text{K}} / 0.030 \text{ m} = 16.2 \text{ W/m}^2\cdot\text{K}$$

The heat rate per unit length of the pipe is

$$q'_{\text{conv}} = \frac{(150 - 20)^\circ\text{C}}{\frac{1}{\pi (0.030 \text{ m})} \left(\frac{1}{16.2} + \frac{1}{11} \right) \frac{\text{m}^2\cdot\text{K}}{\text{W}}} = 80.3 \text{ W/m} \quad <$$

COMMENTS: This problem requires making a judgment that the oil flow will be laminar rather than turbulent. Why is this a reasonable assumption? Recognize that the correlation applies to a constant surface temperature condition.

8.59 A heating contractor must heat 0.2 kg/s of water from 15°C to 35°C using hot gases in cross flow over a thin-walled tube.



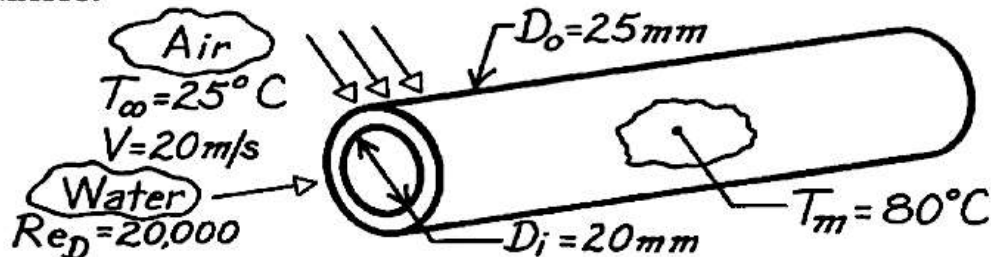
Your assignment is to develop a series of design graphs that can be used to demonstrate acceptable combinations of tube dimensions (D and L) and of hot gas conditions (T_∞ and V) that satisfy this requirement. In your analysis, consider the following parameter ranges: $D = 20, 30, \text{ or } 40$ mm; $L = 3, 4, \text{ or } 6$ m; $T_\infty = 250, 375, \text{ or } 500^\circ\text{C}$; and $20 \leq V \leq 40$ m/s.

PROBLEM 8.59

KNOWN: Thick-walled pipe of thermal conductivity 60 W/mK passing hot water with $Re_D = 20,000$, a mean temperature of 80°C , and cooled externally by air in cross-flow at 20 m/s and 25°C .

FIND: Heat transfer rate per unit pipe length, q_c

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Internal flow is turbulent and fully developed

PROPERTIES: Table A-6, Water ($T_m = 80^\circ\text{C} = 353\text{K}$): $k = 0.670 \text{ W/mK}$, $Pr = 2.20$; Table A-4, Air ($T_\infty = 25^\circ\text{C} \approx 300\text{K}$, 1 atm): $\nu = 15.89 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.0263 \text{ W/mK}$, $Pr = 0.707$.

ANALYSIS: The heat rate per unit length, considering thermal resistances to internal flow, wall conduction (Eq. 3.28) and external flow, with $A = pDL$, is

$$q_c = \frac{\dot{q}}{L} = \frac{1}{\frac{1}{h_i p D_i} + \frac{\ln(D_o/D_i)}{2\pi k} + \frac{1}{h_o p D_o}} (T_m - T_\infty).$$

Internal Flow: Using the Dittus-Boelter correlation with $n = 1/3$ for turbulent, fully developed flow, where $Re_{D_i} = 20,000$

$$h_i = (k/D_i) Nu_D = (k/D_i) 0.023 Re^{4/5} Pr^{1/3}$$

$$h_i = (0.670 \text{ W/mK} / 0.020 \text{ m}) 0.023 (20,000)^{4/5} 2.20^{1/3} = 2765 \text{ W/m}^2 \cdot \text{K}.$$

External Flow: Using the Zhukauskas correlation for cross-flow over a circular cylinder with $Pr/Pr_s \gg 1$, find first

$$Re_D = \frac{VD_o}{\nu} = \frac{20 \text{ m/s} \cdot 0.025 \text{ m}}{15.89 \times 10^{-6} \text{ m}^2/\text{s}} = 31,466$$

and from Table 7.4, $C = 0.26$ and $m = 0.6$, where $n = 0.37$,

$$Nu_D = \frac{h_o D}{k} = C Re_D^m Pr^n (Pr/Pr_s)^{1/4}$$

$$h_o = (0.0263 \text{ W/mK} / 0.025 \text{ m}) 0.26 (31,466)^{0.6} (0.707)^{0.37} = 120 \text{ W/m}^2 \cdot \text{K}.$$

Hence, the heat rate is

$$q_c = \frac{\dot{q}}{L} = \frac{1}{\frac{1}{2765 \text{ W/m}^2 \cdot \text{K} \cdot p \cdot 0.020 \text{ m}} + \frac{\ln(25/20)}{2\pi \cdot 60 \text{ W/mK}} + \frac{1}{120 \text{ W/m}^2 \cdot \text{K} \cdot p \cdot 0.025 \text{ m}}} (80 - 25)^\circ\text{C}$$

$$q_c = \frac{\dot{q}}{L} = 5.756 \times 10^{-3} + 5.919 \times 10^{-4} + 1.061 \times 10^{-1} \text{ W/mK} (80 - 25)^\circ\text{C}$$

$$q_c = 489 \text{ W/m}$$

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COMMENTS: Note that the external flow represents the major thermal resistance to heat transfer.