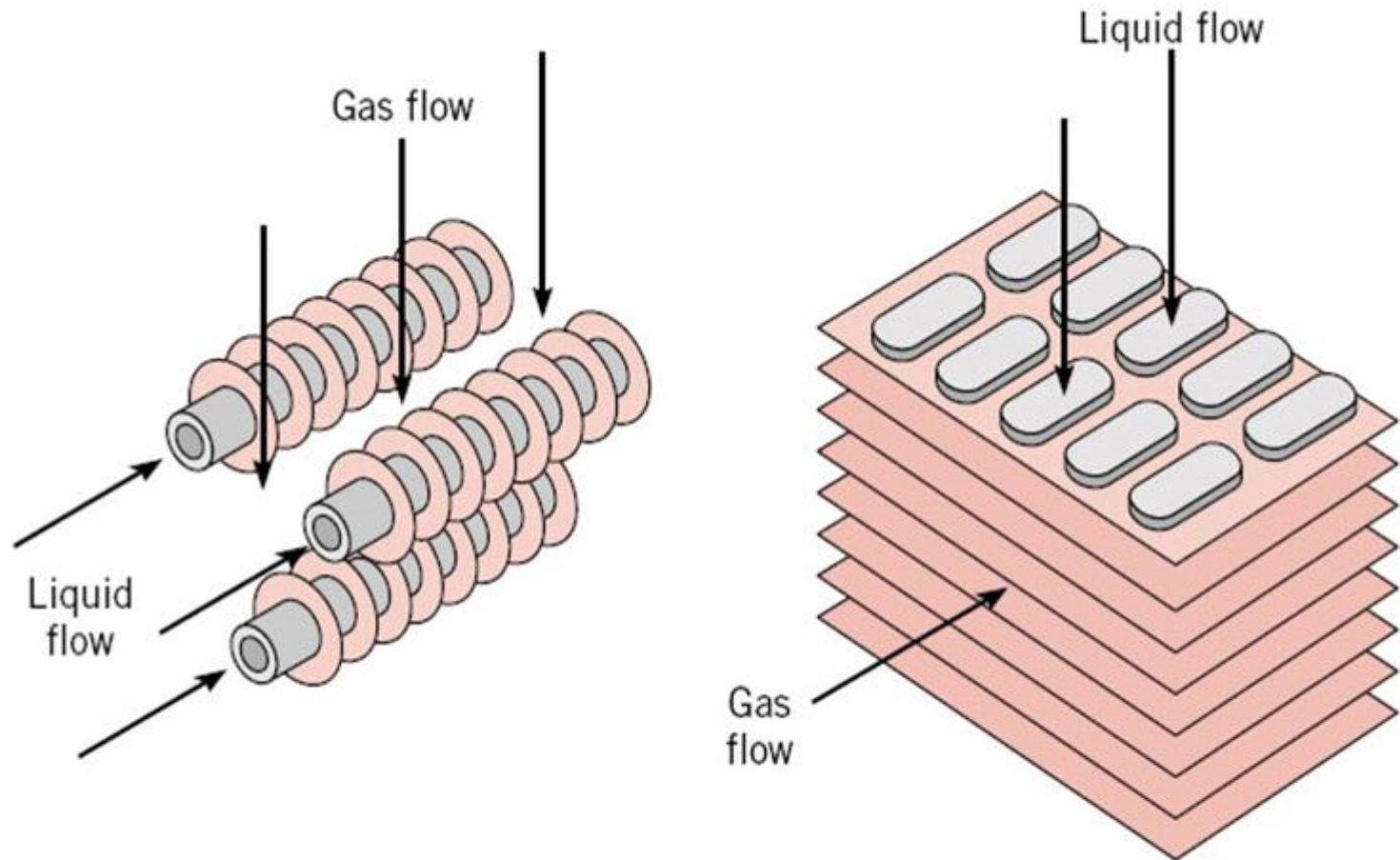
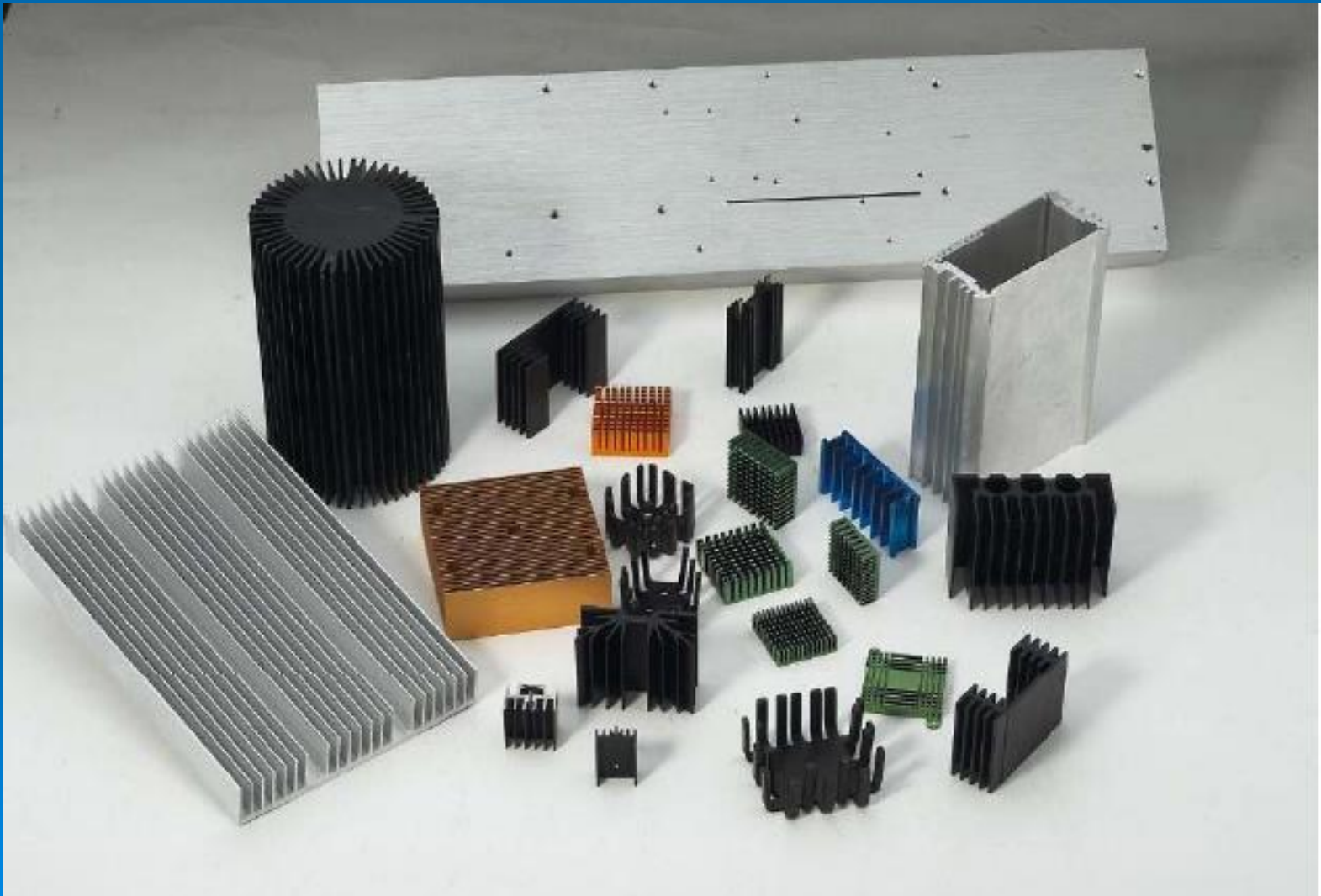


Extended Surfaces



Schematic of typical finned-tube heat exchangers.

Heat Sinks

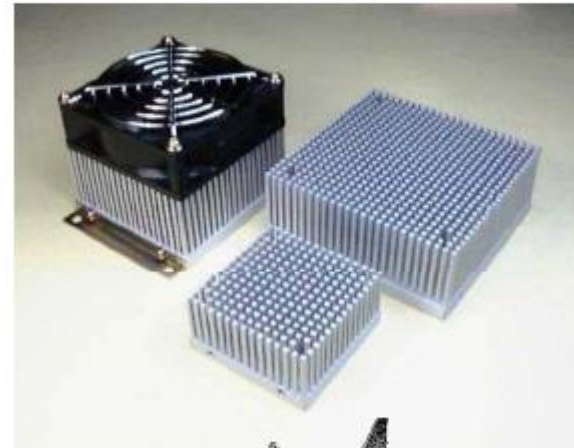
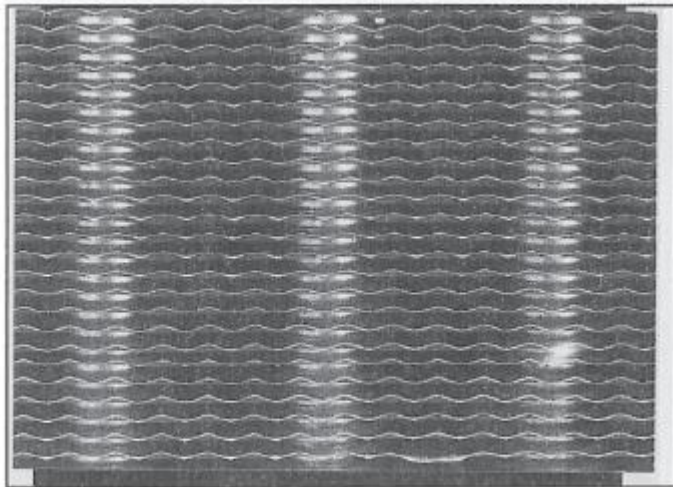


Pin fins



A splayed pin fin heat sink.

The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air

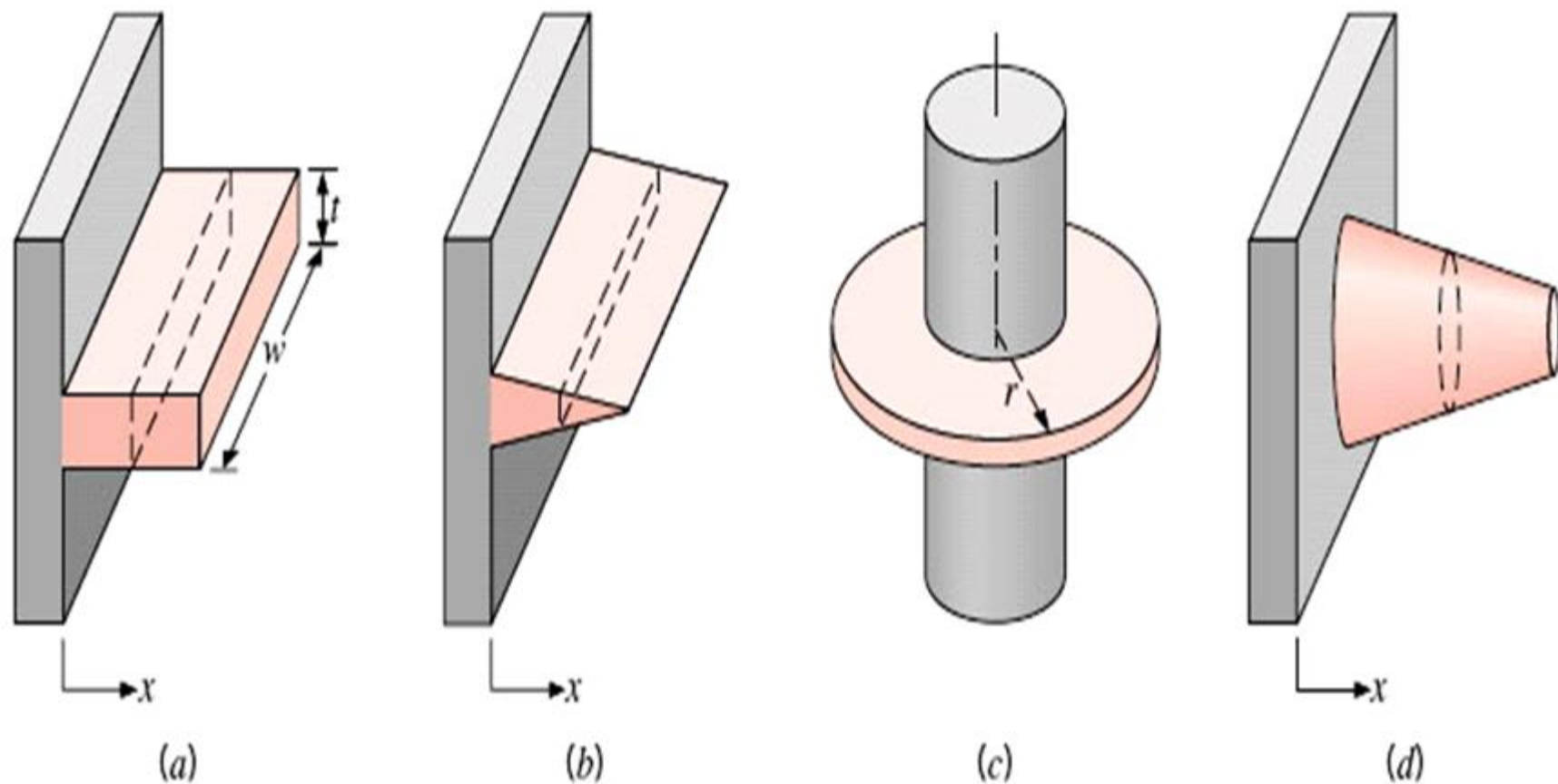


Fins from nature

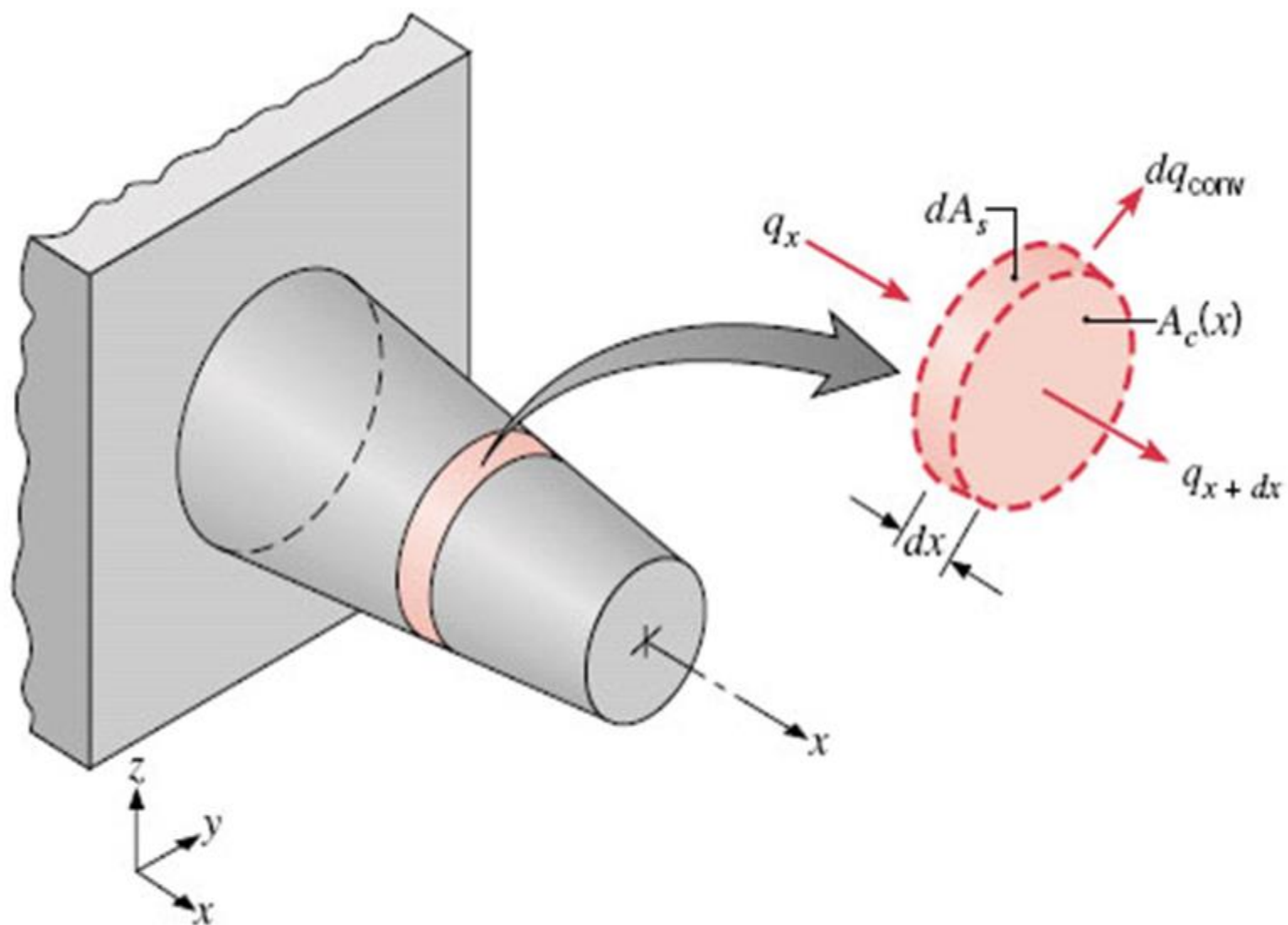
Examples



Radial fin coffee cup



Fin configurations. (a) Straight fin of uniform cross section. (b) Straight fin of nonuniform cross section. (c) Annular fin. (d) Pin fin.



Energy balance for an extended surface.

General Energy equation for fin

Applying the conservation of energy over the differential element

$$q_x = q_{x+dx} + dq_{\text{conv}}$$

From Fourier's law we know that

$$q_x = -kA_c \frac{dT}{dx}$$

where A_c is the *cross-sectional* area, which may vary with x . Since the conduction heat rate at $x + dx$ may be expressed as

$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

it follows that

$$q_{x+dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx} \left(A_c \frac{dT}{dx} \right) dx \quad (3.59)$$

The convection heat transfer rate may be expressed as

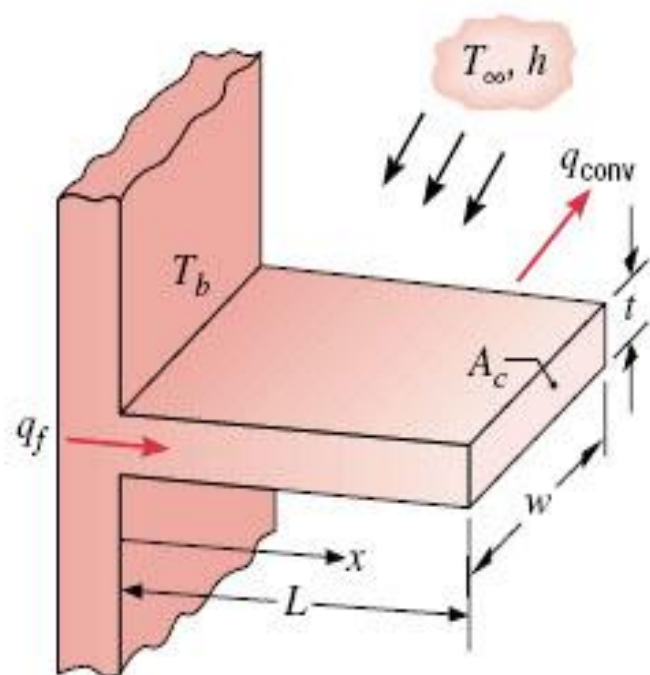
$$dq_{\text{conv}} = h dA_s (T - T_\infty) \quad (3.60)$$

where dA_s is the *surface* area of the differential element. Substituting the foregoing rate equations into the energy balance, Equation 3.56, we obtain

$$\frac{d}{dx} \left(A_c \frac{dT}{dx} \right) - \frac{h}{k} \frac{dA_s}{dx} (T - T_\infty) = 0$$

or

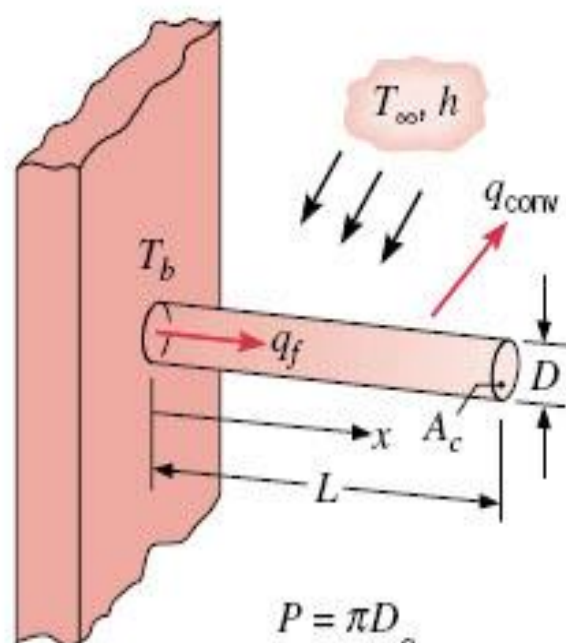
$$\frac{d^2 T}{dx^2} + \left(\frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left(\frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0 \quad (3.61)$$



$$P = 2w + 2t$$

$$A_c = wt$$

(a)



$$P = \pi D$$

$$A_c = \pi D^2/4$$

(b)

FIGURE 3.16 Straight fins of uniform cross section. (a) Rectangular fin. (b) Pin fin.

Fins of Uniform Cross-Sectional Area

Each fin is attached to a base surface of temperature $T(0) = T_b$ and extends into a fluid of temperature T_∞ .

For the prescribed fins, A_c is a constant and $A_s = Px$, where A_s is the surface area measured from the base to x and P is the fin perimeter. Accordingly, with $dA_c/dx = 0$ and $dA_s/dx = P$, previous Equation reduces to

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

To simplify the form of this equation, we transform the dependent variable by defining an *excess temperature* θ as

$$\theta(x) \equiv T(x) - T_\infty$$

where, since T_∞ is a constant, $d\theta/dx = dT/dx$. Substituting Equation 3.63 into Equation 3.62, we then obtain

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \quad (\text{A})$$

where

$$m^2 \equiv \frac{hP}{kA_c}$$

Equation (A) is a linear, homogeneous, second-order differential equation with constant coefficients. Its general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \quad (\text{B}) \quad (3.66)$$

To evaluate the constants C_1 and C_2 of Equation (B) it is necessary to specify appropriate boundary conditions. One such condition may be specified in terms of the temperature at the *base* of the fin ($x = 0$)

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

C

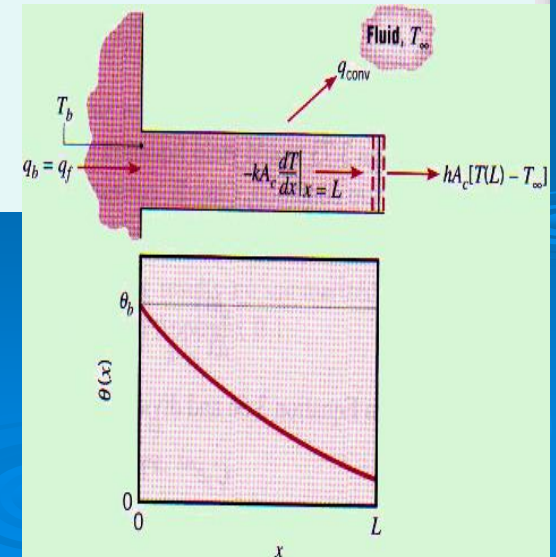
The second condition, specified at the fin tip ($x = L$), may correspond to one of four different physical situations. **See note next slide**

The first condition, Case A, considers convection heat transfer from the fin tip. Applying an energy balance to a control surface about this tip, we obtain

$$hA_c[T(L) - T_\infty] = -kA_c \left. \frac{dT}{dx} \right|_{x=L}$$

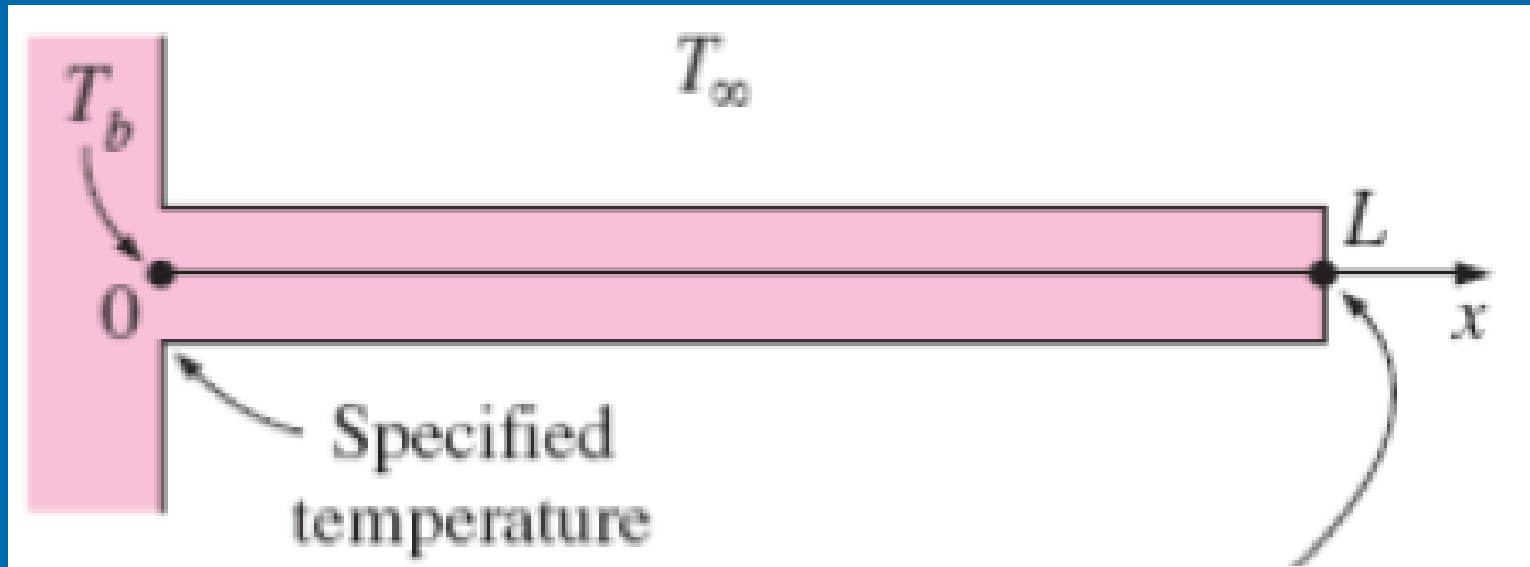
or

$$h\theta(L) = -k \left. \frac{d\theta}{dx} \right|_{x=L} \quad (D)$$



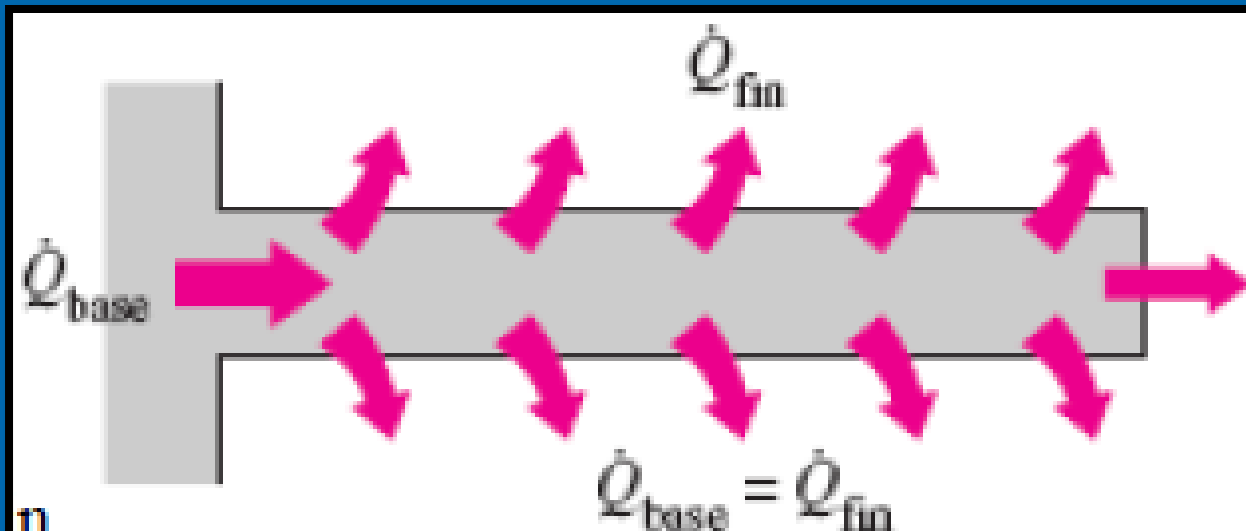
Note

Boundary Conditions



- a) Convection heat transfer
- b) Prescribed temperature
- c) Adiabatic
- d) Infinite length ($L \rightarrow \infty$)

Note



Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

Using the B.Cs we obtain (one at the base, the other at the tip, C & D

$$\theta_b = C_1 + C_2$$

and

$$h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})$$

Solving for C_1 and C_2 , it may be shown, after some manipulation, that

Temp.

Distribution

See also fig 3.17

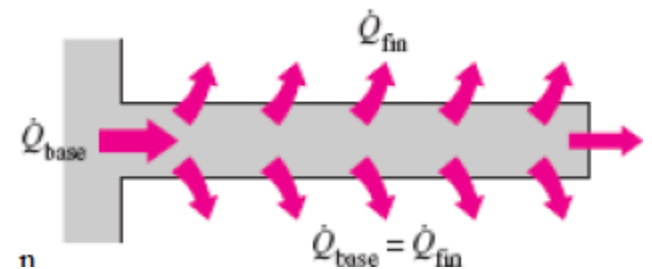
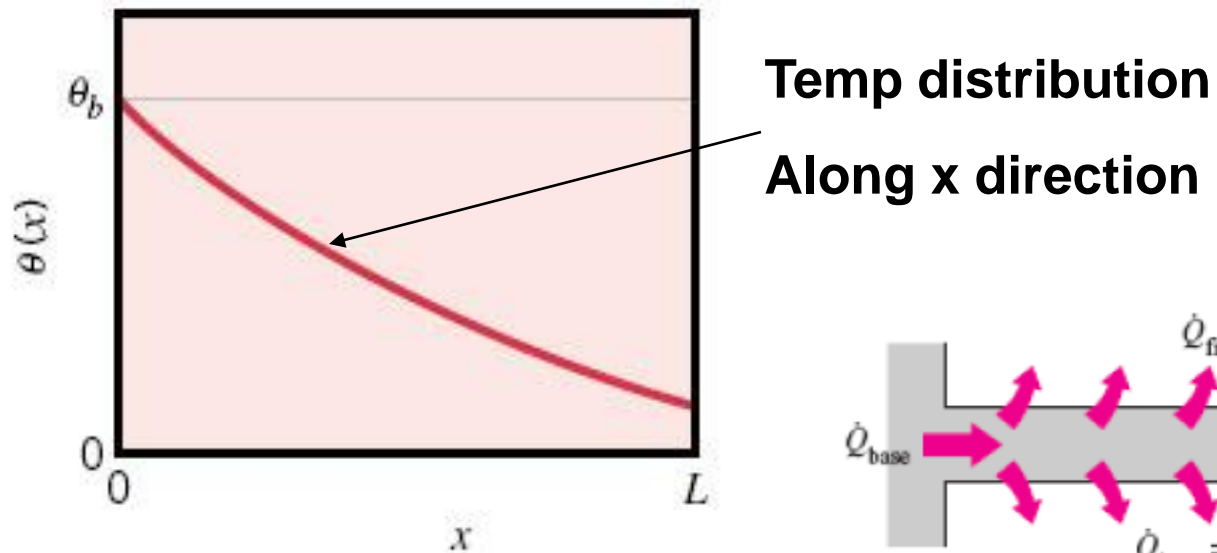
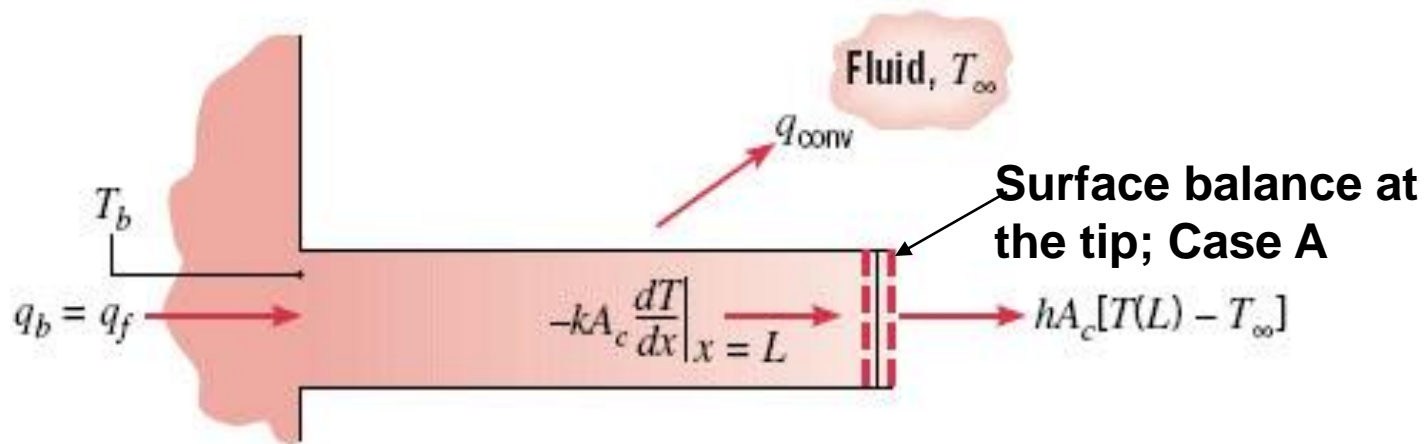
$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$$

**Heat rate; applying
Fourier's Law at
the base**

$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

Hence, knowing the temperature distribution, $\theta(x)$, q_f may be evaluated, giving

$$q_f = \sqrt{hPkA_c} \theta_b \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$$



Conduction and convection in a fin of uniform cross section.

Other cases of tip conditions

Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx _{x=L}$	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$	$M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$
B	Adiabatic $d\theta/dx _{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	M

$$\theta \equiv T - T_\infty \quad m^2 \equiv hP/kA_c$$

$$\theta_b = \theta(0) = T_b - T_\infty \quad M \equiv \sqrt{hPkA_c}\theta_b$$

**SPECIAL
NOTE**

Insulated tip

General Sol: $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$

BC 1: $\theta(0) = T_b - T_\infty \equiv \theta_b$

BC 2: $\left. \frac{d\theta}{dx} \right|_{x=L} = 0$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} = -kA_c \left. \frac{d\theta}{dx} \right|_{x=0}$$

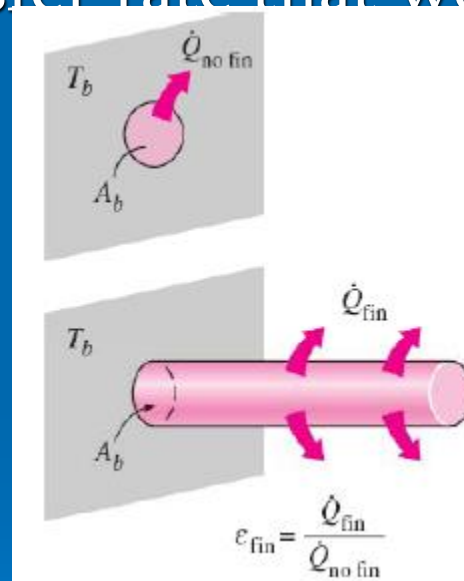
$$q_f = \sqrt{hPkA_c} \theta_b \tanh mL$$

Fine Performance

➤ Fine effectiveness, ε

It is defined as the ratio of the fine heat transfer rate to the heat transfer rate that would exist without the fin.

$$\varepsilon = \frac{q_f}{hA_{c,b}\theta_b}$$



(3.81)

Where $A_{c,b}$ is the fine cross-sectional area at the base.

➤ Assume infinite fin $L \rightarrow \infty$, $q_f = M = (hPkA_c)^{1/2} \theta_b$

$$\varepsilon = \frac{(hPkA_c)^{1/2} \theta_b}{hA_{c,b} \theta_b} = \left(\frac{kP}{hA_c} \right)^{1/2}$$

- Notes:**
1. k should be as high as possible, (copper, aluminum, iron).
Aluminum is preferred: low cost and weight, resistance to corrosion.
 2. p/A_c should be as high as possible. (Thin plate fins and slender pin fins)
 3. Most effective in applications where h is low. (Use of fins justified if when the medium is gas and heat transfer is by natural convection).
 4. max heat rate could be achieved by using very long fins.
However, it is not reasonable to use very long fins to achieve near max. heat transfer.

How to obtain a reasonable length?

Since there is no heat transfer from the tip of an infinitely long fin, it more appropriate to compare it with adiabatic tip fin (also no heat loss). Therefore, assume adiabatic tip fin

$$q_f = \sqrt{hPkA_c} \theta_b \tanh mL$$

Assume 98% of the max possible heat transfer $q_{f,max} = M = (hPkA_c)^{1/2} \theta_b$

$$\therefore 0.98q_{f,max} = q_{f, \text{adiabatic}}$$

$$0.98(hPkA_c)^{1/2} \theta_b = (hPkA_c)^{1/2} \theta_b \tanh mL$$

➤ Hence,

$$mL=2.3 \quad \text{or} \quad L=2.3/m$$

Conclusions

It is more suitable to use fin with $L=2.3/m$ which yield 98% max possible heat transfer rather than to use $L > 2.3/m$ or infinite length.

Effectiveness and thermal resistance

$$q_f = \sqrt{hPkA_c} \theta_b$$

$$= \frac{\theta_b}{\frac{1}{\sqrt{hPkA_c}}} = \frac{\theta_b}{R_{t,f}}$$

$$\therefore R_{t,f} = \frac{\theta_b}{q_f} = \frac{1}{\sqrt{hPkA_c}}$$

➤ Also,

$$\begin{aligned} q_{f,b} \Big|_{\text{without fin}} &= hA_{c,b}\theta_b \\ &= \frac{\theta_b}{\frac{1}{hA_{c,b}}} = \frac{\theta_b}{R_{t,b}} \end{aligned}$$

$$\therefore \varepsilon_f = \frac{q_f}{q_{f,b}} = \frac{\theta_b}{R_{t,f}} \frac{R_{t,b}}{\theta_b} = \frac{R_{t,b}}{R_{t,f}} = \frac{\text{Conduction}}{\text{Convection}}$$

Conclusion

If $\varepsilon > 1$ adding fins enhance heat transfer

$\varepsilon = 1$ adding fins has no effect

$\varepsilon < 1$ adding fins decrease heat transfer
(as insulator)

Example1

Turbine-Blade

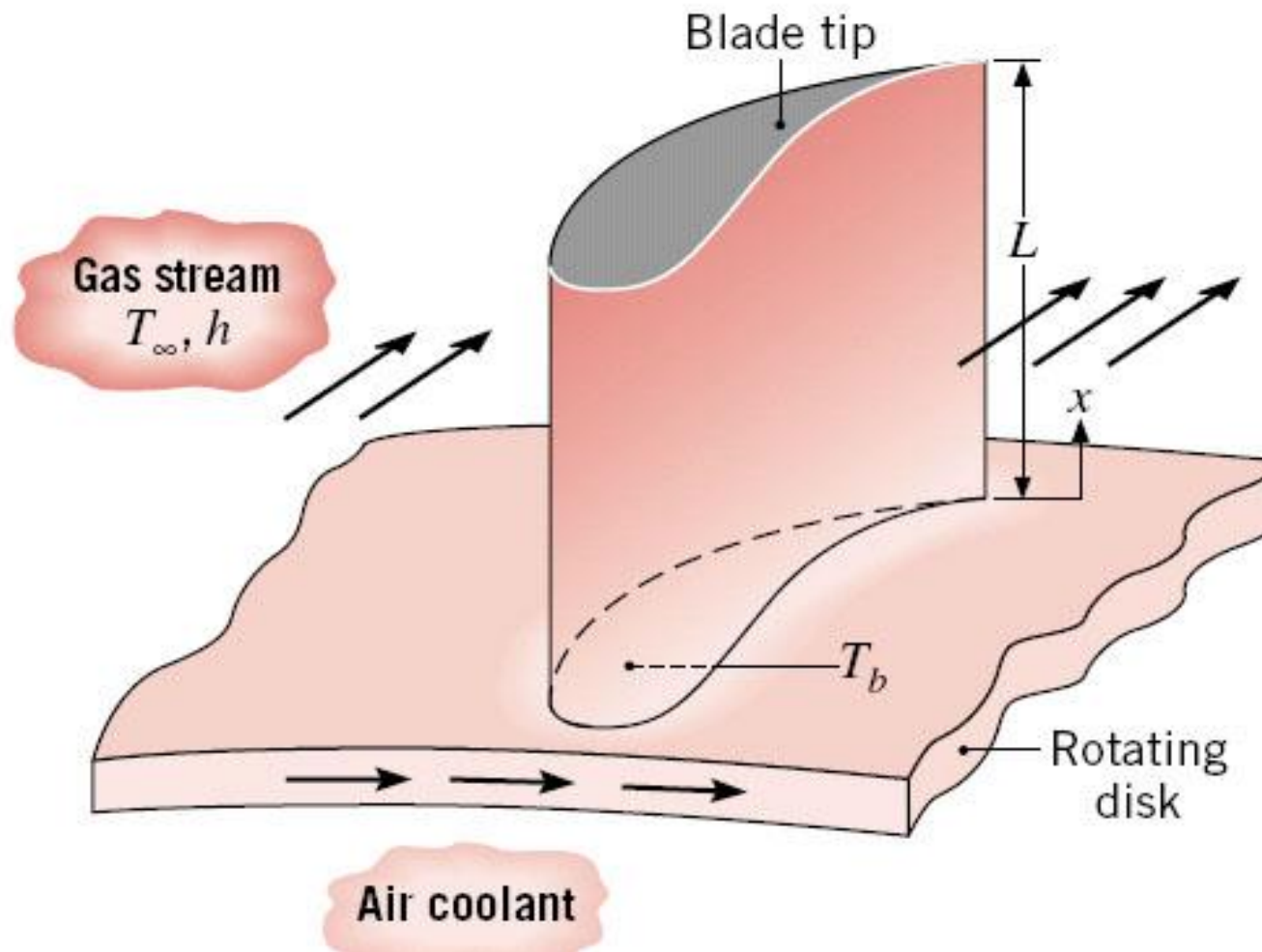
Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at $T_\infty = 1200^\circ\text{C}$ and maintains a convection coefficient of $h = 250 \text{ W/m}^2 \cdot \text{K}$ over the blade.

The blades, which are fabricated from Inconel, $k \approx 20 \text{ W/m} \cdot \text{K}$, have a length of $L = 50 \text{ mm}$. The blade profile has a uniform cross-sectional area of $A_c = 6 \times 10^{-4} \text{ m}^2$ and a perimeter of $P = 110 \text{ mm}$. A proposed blade-cooling scheme, which involves rout-

ing air through the supporting disc, is able to maintain the base of each blade at a temperature of $T_b = 300^\circ\text{C}$.

- (a) If the maximum allowable blade temperature is 1050°C and the blade tip may be assumed to be adiabatic, is the proposed cooling scheme satisfactory?
- (b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?

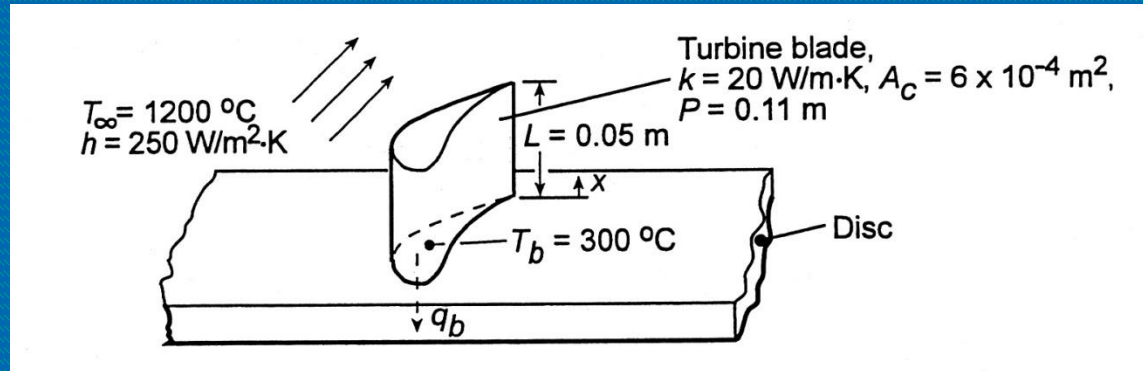
Example 1



Assessment of cooling scheme for gas turbine blade.

Determination of whether blade temperatures are less than the maximum allowable value ($1050\text{ }^{\circ}\text{C}$) for prescribed operating conditions and evaluation of blade cooling rate.

Schematic:



Assumptions: (1) One-dimensional, steady-state conduction in blade, (2) Constant k , (3) Adiabatic blade tip, (4) Negligible radiation.

Analysis: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at $x=L$, Eq. 3.75 yields

$$\frac{T(L) - T_{\infty}}{T_b - T_{\infty}} = \frac{1}{\cosh mL}$$

$$m = (hP/kA_c)^{1/2} =$$

$$mL =$$

From Table B.1, $\cosh mL = 5.51$ Hence,

$$T(L) = 1200^\circ\text{C} + (300 - 1200)^\circ\text{C}/5.51 = 1037^\circ\text{C}$$

and, *subject to the assumption of an adiabatic tip*, the operating conditions are acceptable.

(b) With $M = (hPkA_c)^{1/2} \theta_b = \left(250\text{W/m}^2 \cdot \text{K} \times 0.11\text{m} \times 20\text{W/m} \cdot \text{K} \times 6 \times 10^{-4}\text{m}^2\right)^{1/2} (-900^\circ\text{C}) = -517\text{W}$,

Eq. 3.76 and Table B.1 yield

$$q_f = M \tanh mL = -517\text{W} (0.983) = -508\text{W}$$

Hence,

$$q_b = -q_f = 508\text{W}$$

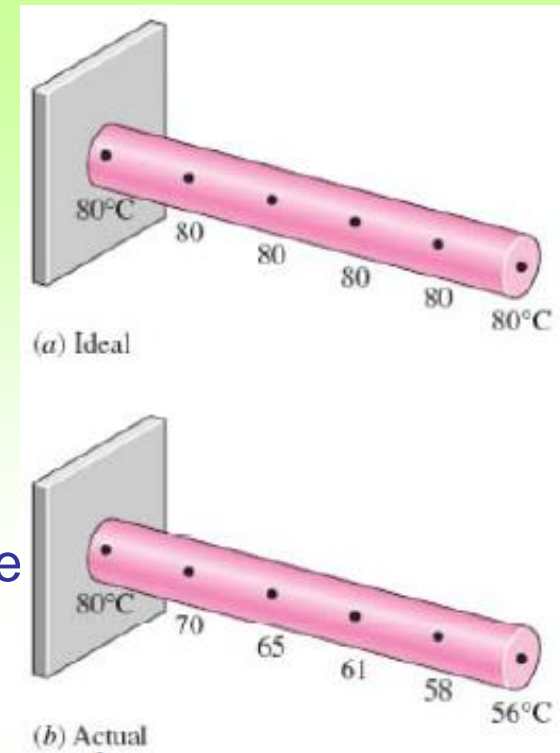
Calculate the tip temperature allowing for convection from the gas.

Efficiency of Fins, η_f

- Definition:
$$\eta_f = \frac{q_f}{q_{\max}} = \frac{q_f}{hA_f\theta_b}$$

where A_f is the surface area of the fin.

Look! Max heat transfer takes place when the surface temp. of the fin equals the base temperature.



Assume adiabatic tip fin, the previous eq. becomes

$$\eta_f = \frac{M \tanh mL}{hPL\theta_b} = \frac{\sqrt{hPkA_c}\theta_b \tanh mL}{hPL\theta_b} = \frac{\tanh mL}{mL}$$

$\eta_f \rightarrow \max$ as $L \rightarrow 0$; $\eta_f \rightarrow \min$ as $L \rightarrow \infty$

Approximation for heat transfer from a convection tip fin

- The heat transfer, q_f , of a convection tip fin; eq. 3.72, can be calculated via using adiabatic tip eq.3.76 by making a correction for the length; $L_c=L+(t/2)$ for a rectangular fin and $L_c=L+(D/4)$ for a pin fin.
- Therefore, with tip convection, the fin heat transfer rate may be approximated as

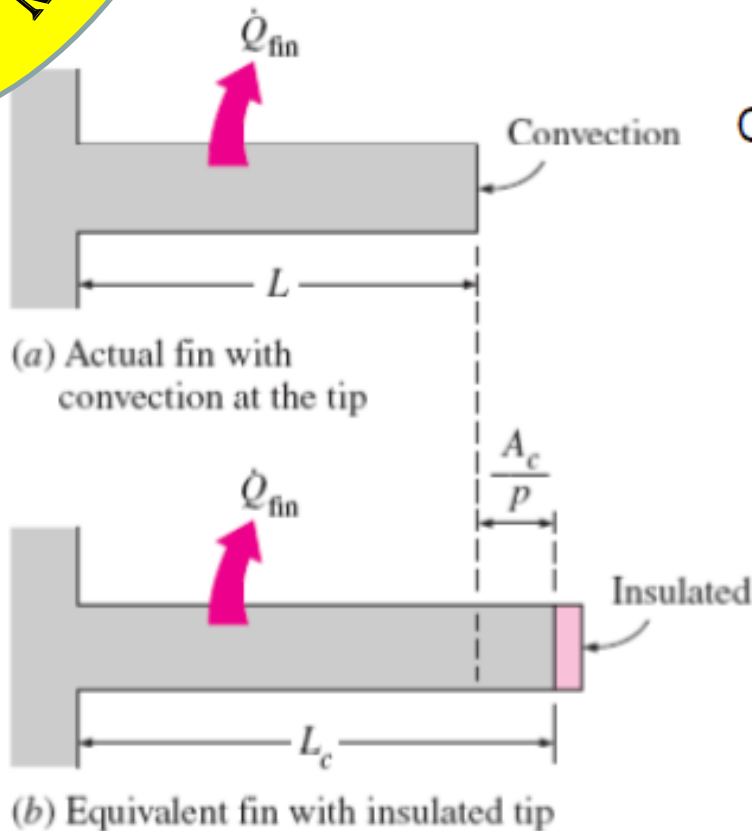
$$q_f = M \tanh mL_c$$

$$\text{where } M = \sqrt{hPkA_c} \theta_b$$

$$\text{and } \eta_f = \frac{\tanh mL_c}{mL_c}$$

**SPECIAL
NOTE**

Corrected fin length



Corrected fin length:

$$L_c = L + \frac{A_c}{P}$$

Multiplying the relation above by the perimeter gives

$$A_{\text{corrected}} = A_{\text{fin (lateral)}} + A_{\text{tip}}$$

$$L_{c, \text{rectangular fin}} = L + \frac{t}{2}$$

$$L_{c, \text{cylindrical fin}} = L + \frac{D}{4}$$

Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

Notes

1. Errors associated with the approximation are negligible if (ht/k) or $(hD/2k) \leq 0.0625$

2. If $w \gg t$ for rectangular fin

$$\therefore P \approx 2w$$

Pin fin

Rectangular fin

$$mL_c = \left(\frac{hp}{kA_c}\right)^{1/2} L_c = \left(\frac{h2w}{kwt}\right)^{1/2} L_c = \left(\frac{2h}{kt}\right)^{1/2} L_c$$

Introducing a corrected fin profile area, $A_p = L_c t$

$$\therefore mL_c = \left(\frac{2h}{kt}\right)^{1/2} \left(\frac{L_c}{L_c}\right)^{1/2} L_c = \left(\frac{2h}{ktL_c}\right)^{1/2} L_c^{3/2} = \left(\frac{2h}{kA_p}\right)^{1/2} L_c^{3/2}$$

See Figure 3.18 and figure 3.19 $\left\{ \eta \text{ vis } \left(\frac{h}{kA_p}\right)^{1/2} L_c^{3/2} \right\}$

Fig. is developed from this eq. $\eta_f = \frac{\tanh mL_c}{mL_c}$

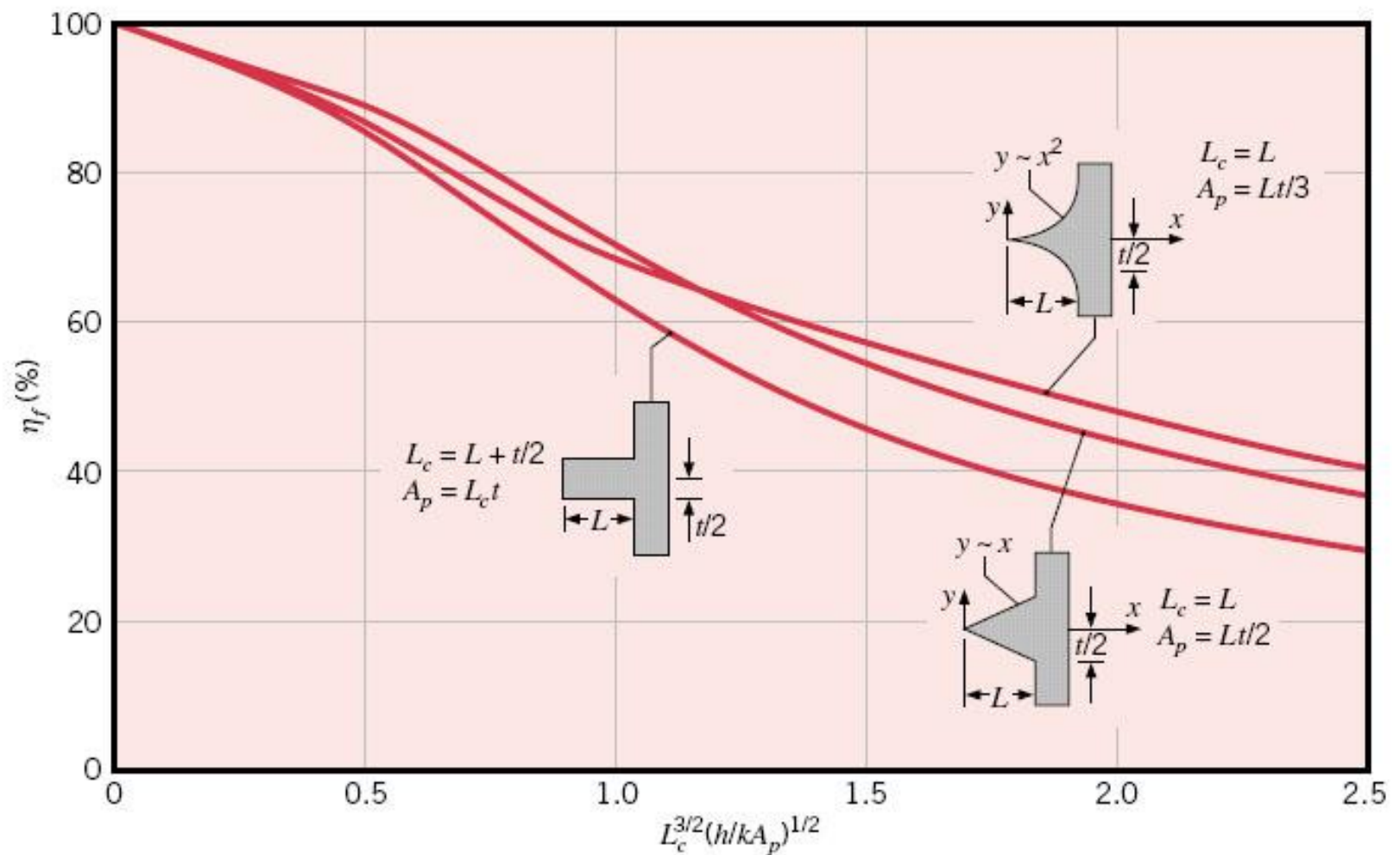


FIGURE 3.18 Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

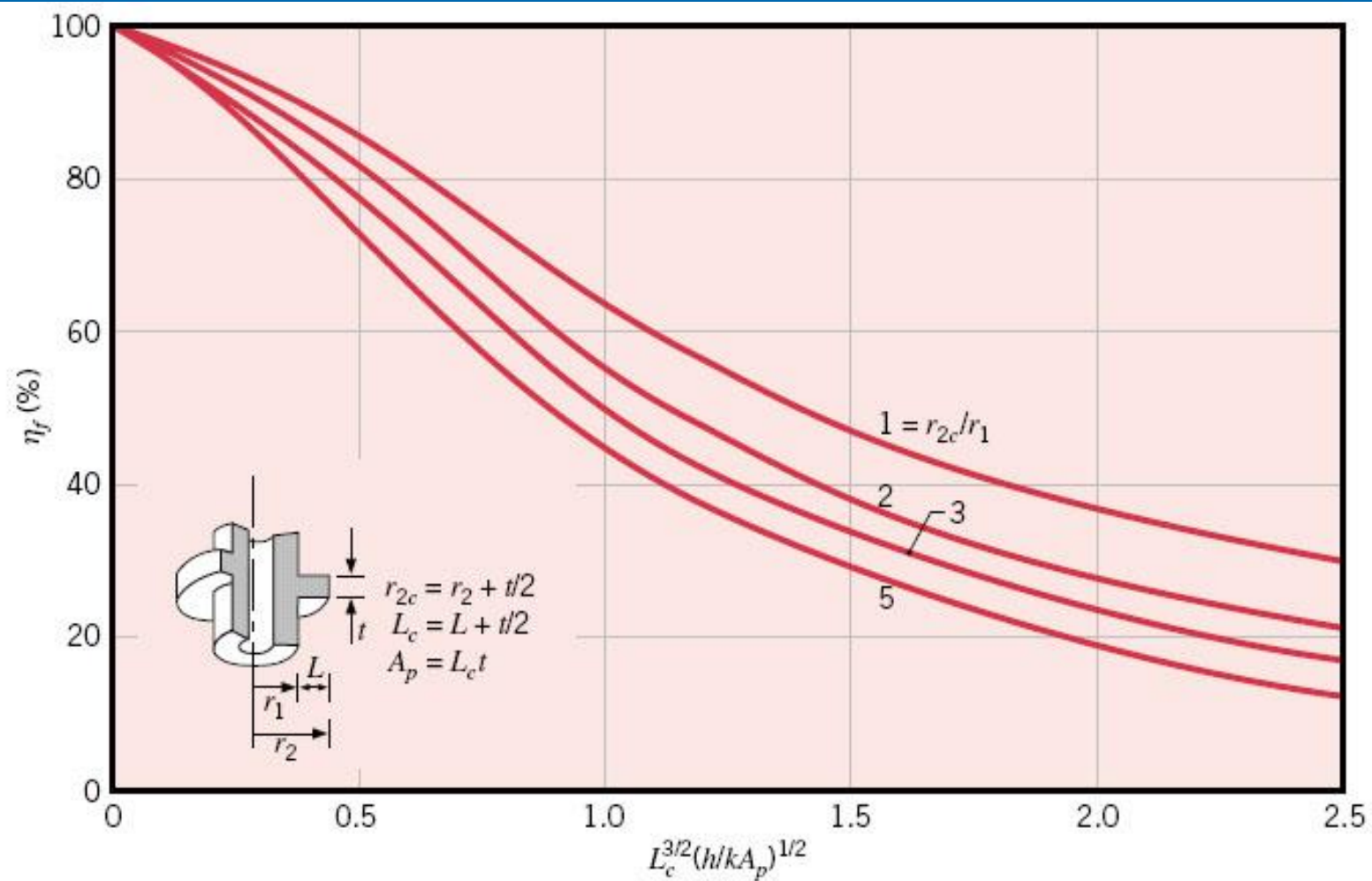


FIGURE 3.19 Efficiency of annular fins of rectangular profile.

summary

$$\therefore \eta_f = \frac{q_f}{q_{\max}} = \frac{q_f}{hA_f \theta_b}$$

$$q_f = \eta_f q_{\max} = \eta_f A_f \theta_b$$

η_f is obtained from charts or equations

A_f : fin surface area

For example: Pin fin~ $A_f = PL_c = \pi D L_c$

See next table

TABLE 3.5 Efficiency of common fin shapes

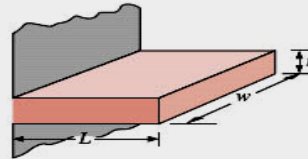
Straight Fins

Rectangular^a

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

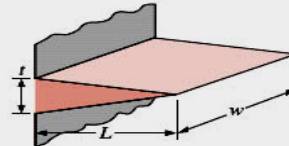


$$\eta_f = \frac{\tanh mL_c}{mL_c} \quad (3.89)$$

Triangular^a

$$A_f = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



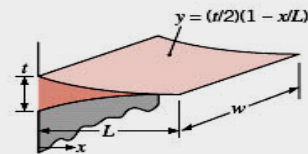
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)} \quad (3.93)$$

Parabolic^a

$$A_f = w[C_1L + \frac{(L^2/t)\ln(t/L + C_1)}{(L^2/t)\ln(t/L + C_1)}]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1} \quad (3.94)$$

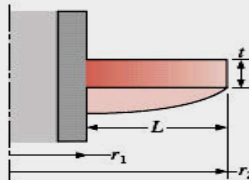
Circular Fin

Rectangular^a

$$A_f = 2\pi(r_{2c}^2 - r_1^2)$$

$$r_{2c} = r_2 + (t/2)$$

$$V = \pi(r_{2c}^2 - r_1^2)t$$



$$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})} \quad (3.91)$$

$$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$$

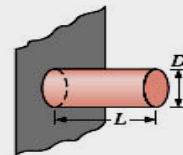
Pin Fins

Rectangular^b

$$A_f = \pi DL_c$$

$$L_c = L + (D/4)$$

$$V = (\pi D^2/4)L$$

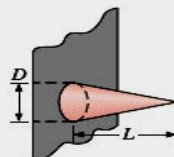


$$\eta_f = \frac{\tanh mL_c}{mL_c} \quad (3.95)$$

Triangular^b

$$A_f = \frac{\pi D}{2} [L^2 + (D/2)^2]^{1/2}$$

$$V = (\pi/12)D^2L$$



$$\eta_f = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)} \quad (3.96)$$

TABLE 3.5 *Continued*

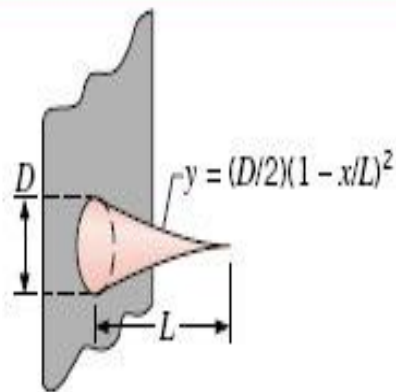
Parabolic^b

$$A_f = \frac{\pi L^3}{8D} \{ C_3 C_4 - \frac{L}{2D} \ln [(2DC_4/L) + C_3] \}$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = [1 + (D/L)^2]^{1/2}$$

$$V = (\pi/20) D^2 L$$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1} \quad (3.97)$$

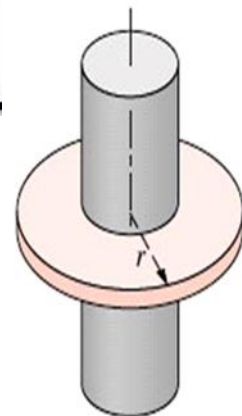
$$^a m = (2h/kt)^{1/2},$$

$$^b m = (4h/kD)^{1/2},$$

Suggested Problem Plus Solution

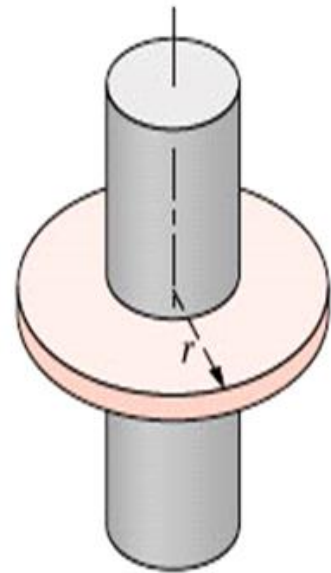
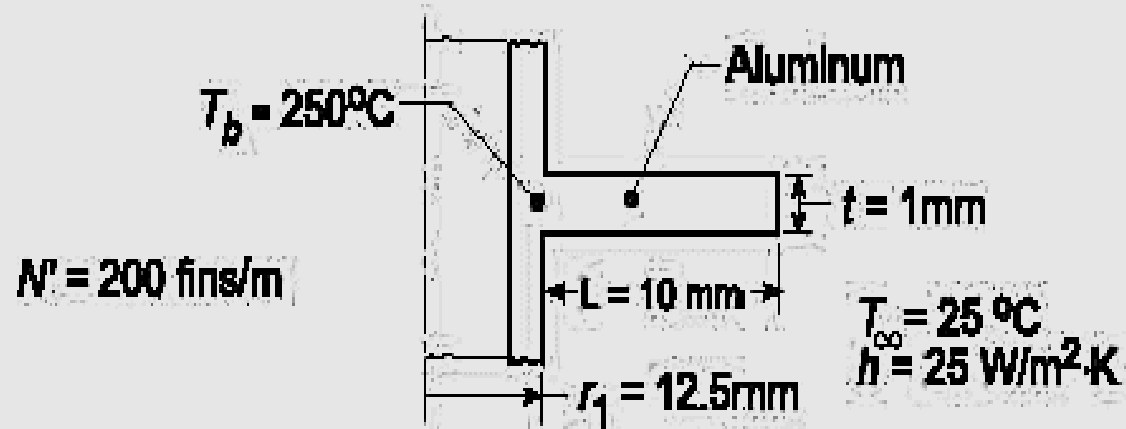
An annular aluminum fin of rectangular profile is attached to a circular tube having an outside diameter of 25 mm and a surface temperature of 250°C . The fin is 1 mm thick and 10 mm long, and the temperature and the convection coefficient associated with the adjoining fluid are 25°C and $25 \text{ W/m}^2 \cdot \text{K}$, respectively.

What is the heat loss per fin?



Solution

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure ($T \approx 400 \text{ K}$): $k = 240 \text{ W/m}\cdot\text{K}$

Solution

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = (12.5 \text{ mm} + 10 \text{ mm}) + 0.5 \text{ mm} = 23 \text{ mm} = 0.023 \text{ m}$$

$$r_{2c}/r_1 = 1.84$$

$$L_c = L + t/2 = 10.5 \text{ mm} = 0.0105 \text{ m}$$

$$A_p = L_c t = 0.0105 \text{ m} \times 0.001 \text{ m} = 1.05 \times 10^{-5} \text{ m}^2$$

$$L_c^{3/2} (h/kA_p)^{1/2} = (0.0105 \text{ m})^{3/2} \left(\frac{25 \text{ W/m}^2 \cdot \text{K}}{240 \text{ W/m} \cdot \text{K} \times 1.05 \times 10^{-5} \text{ m}^2} \right)^{1/2} = 0.15$$

Hence, the fin effectiveness is $\eta_f \approx 0.97$, and from Eq. 3.86 and Fig. 3.5, the fin heat rate is

$$q_f = \eta_f q_{\max} = \eta_f h A_{f(\text{ann})} \theta_b = 2\pi \eta_f h (r_{2c}^2 - r_1^2) \theta_b$$

$$q_f = 2\pi \times 0.97 \times 25 \text{ W/m}^2 \cdot \text{K} \times \left[(0.023 \text{ m})^2 - (0.0125 \text{ m})^2 \right] 225^\circ \text{C} = 12.8 \text{ W}$$

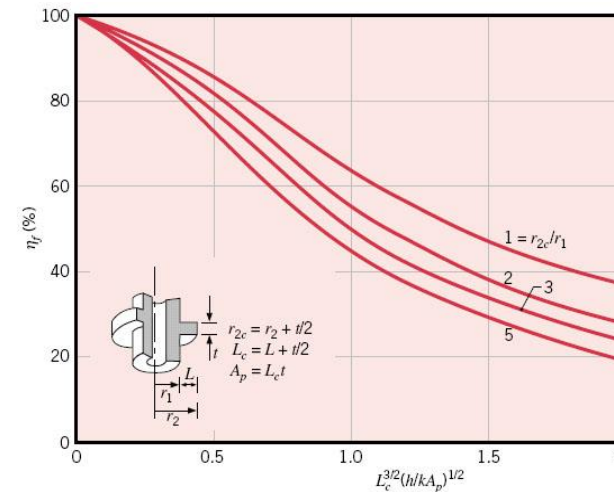
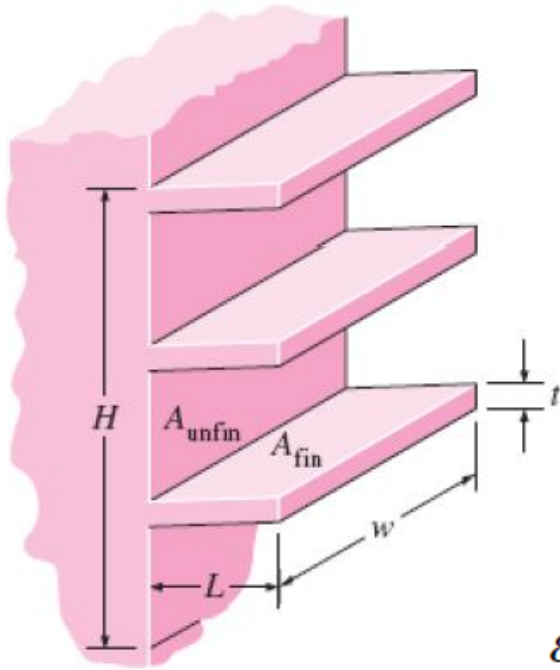


FIGURE 3.19 Efficiency of annular fins of rectangular profile.

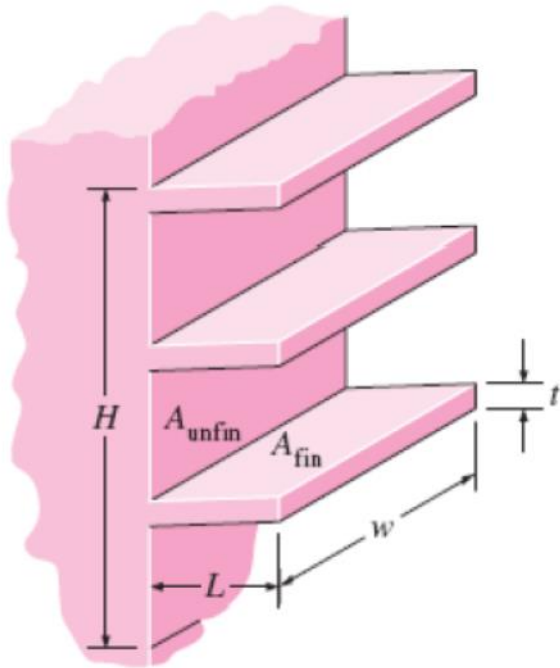
Overall Fin Efficiency



$$\begin{aligned}\dot{Q}_{total, fin} &= \dot{Q}_{unfin} + \dot{Q}_{fin} \\ &= hA_{unfin} (T_b - T_\infty) + \eta_{fin} hA_{fin} (T_b - T_\infty) \\ &= h(A_{unfin} + \eta_{fin} A_{fin})(T_b - T_\infty)\end{aligned}$$

$$\varepsilon_{fin, overall} = \frac{\dot{Q}_{total, fin}}{\dot{Q}_{total, nofin}} = \frac{h(A_{unfin} + \eta_{fin} A_{fin})(T_b - T_\infty)}{hA_{nofin} (T_b - T_\infty)}$$

Note



$$A_{no\ fin} = w \times H$$

$$A_{unfin} = w \times H - 3 \times (t \times w)$$

$$A_{fin} = 2 \times L \times w + t \times w \text{ (one fin)}$$

$$\approx 2 \times L \times w$$

Fin Arrays

- Representative arrays of
 - (a) rectangular and
 - (b) annular fins.

$$\eta_o = q_t / q_{\max} = q_t / h A_t \theta_b$$

Overall efficiency

- Total surface area:

$$A_t = N A_f + A_b$$

Number of fins

Area of exposed base (*prime* surface)

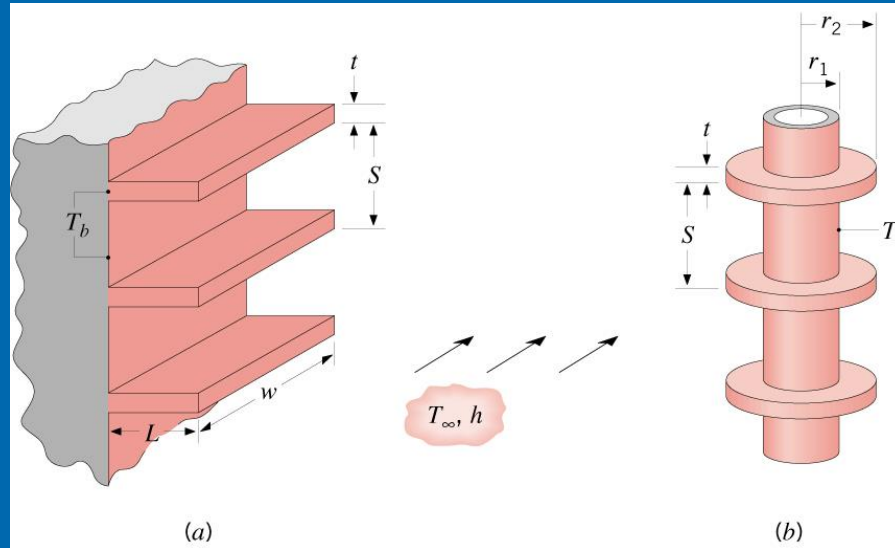
- Total heat rate:

$$q_t = N \eta_f h A_f \theta_b + h A_b \theta_b \equiv \eta_o h A_t \theta_b = \frac{\theta_b}{R_{t,o}} \quad (3.100)$$

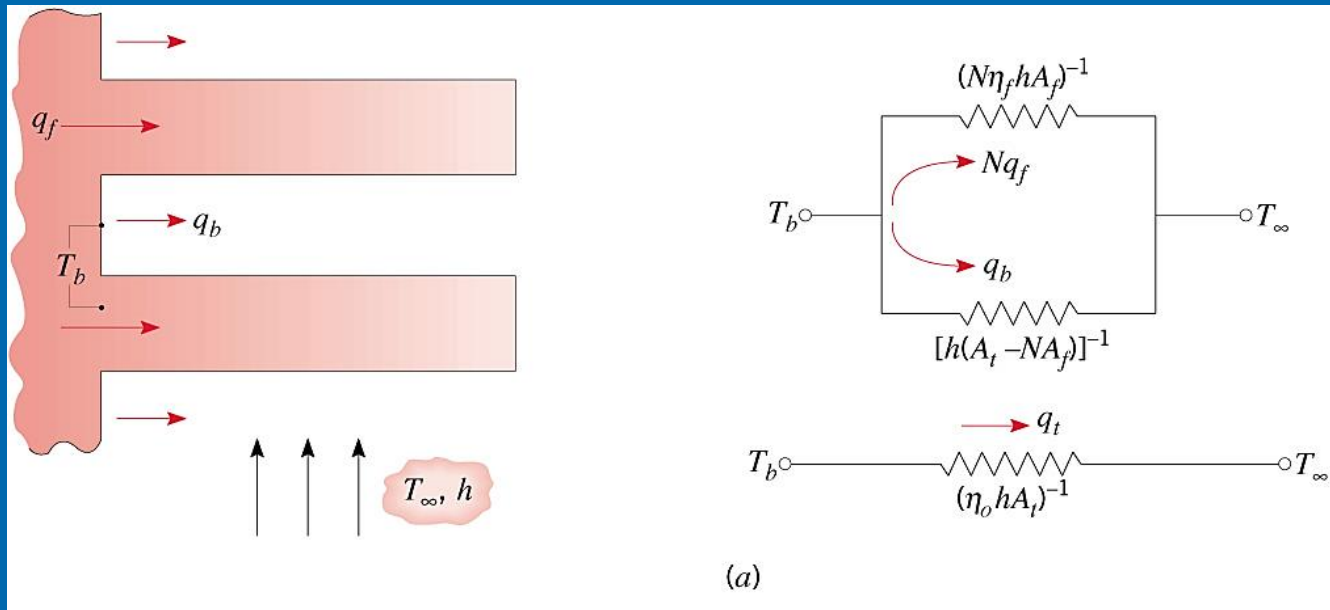
- Overall surface efficiency and resistance:

$$\eta_o = 1 - \frac{N A_f}{A_t} (1 - \eta_f) \quad \text{Give the proof !!} \quad (3.102)$$

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t} \quad (3.103)$$



Equivalent Thermal Circuit



Suggested Exercise

Summary

Proper length of a fin

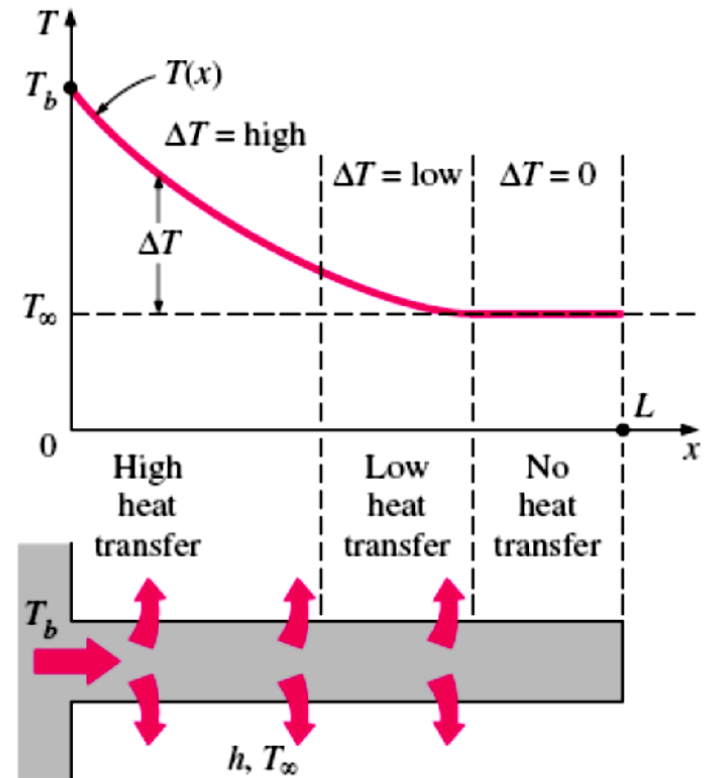
Heat transfer ratio:

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpKA_c} (T_b - T_\infty) \tanh aL}{\sqrt{hpKA_c} (T_b - T_\infty)} = \tanh aL$$

“a” means “m”

The variation of heat transfer from a fin relative to that from an infinitely long fin

aL	$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

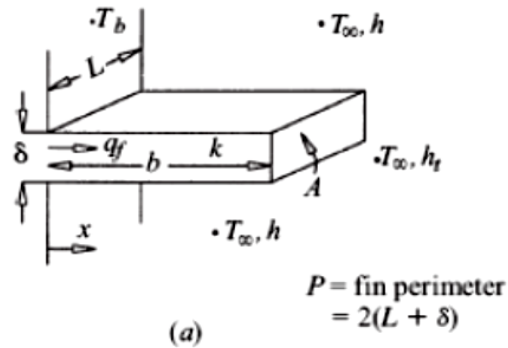


Comments

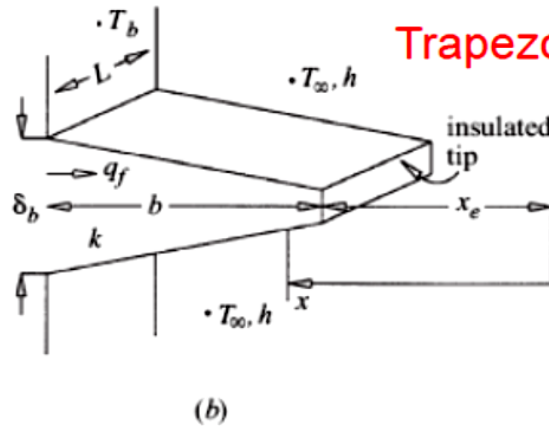
- Fins with triangular and parabolic profiles contain less material and are more efficient requiring minimum weight
- An important consideration is the selection of the proper *fin length* L . Increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.
- The efficiency of most fins used in practice is above 90 percent

Longitudinal fins

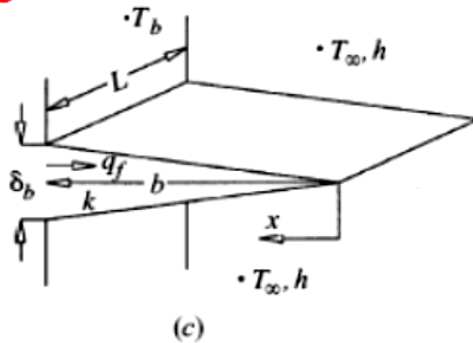
Rectangular



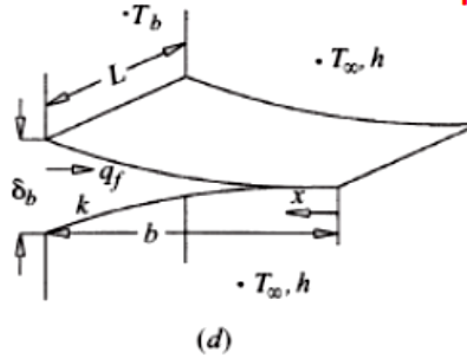
Trapezoidal



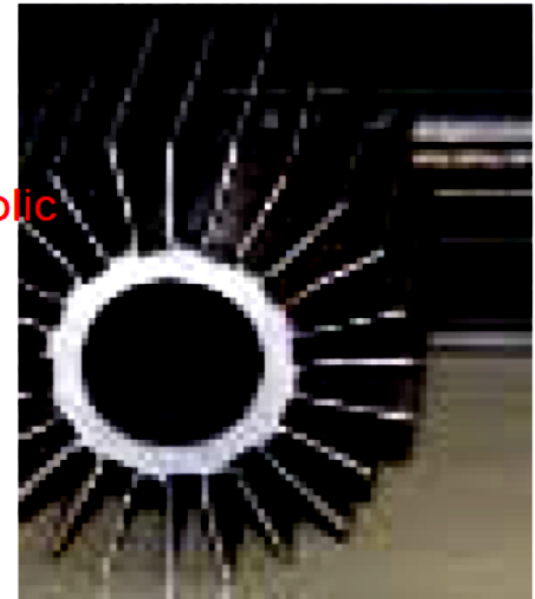
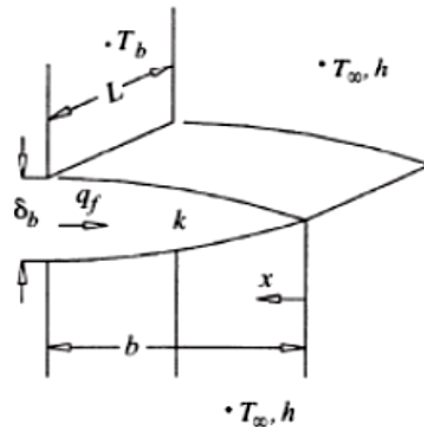
Triangular



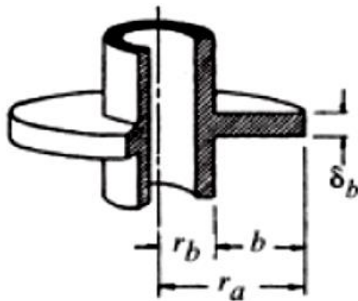
Concave parabolic



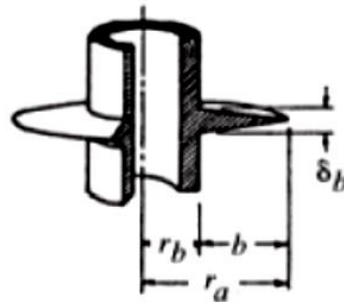
Convex parabolic



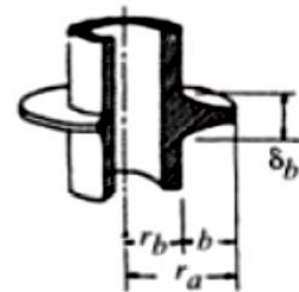
Radial fins:



Rectangular profile

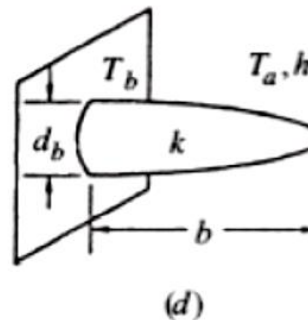
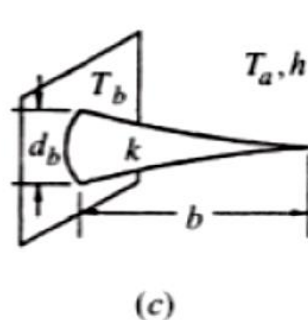
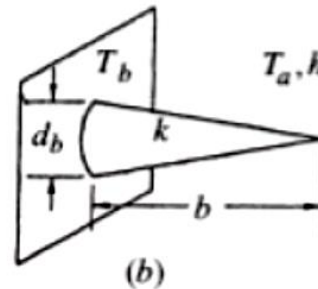
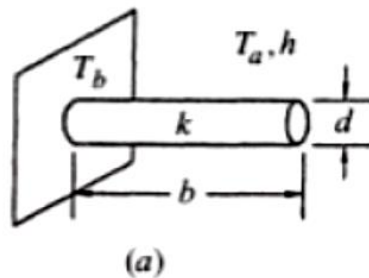


Triangular profile



Hyperbolic profile

Pins:



Radial fin coffee cup

(a) Cylindrical (b) conical (c) concave parabolic (d) convex parabolic