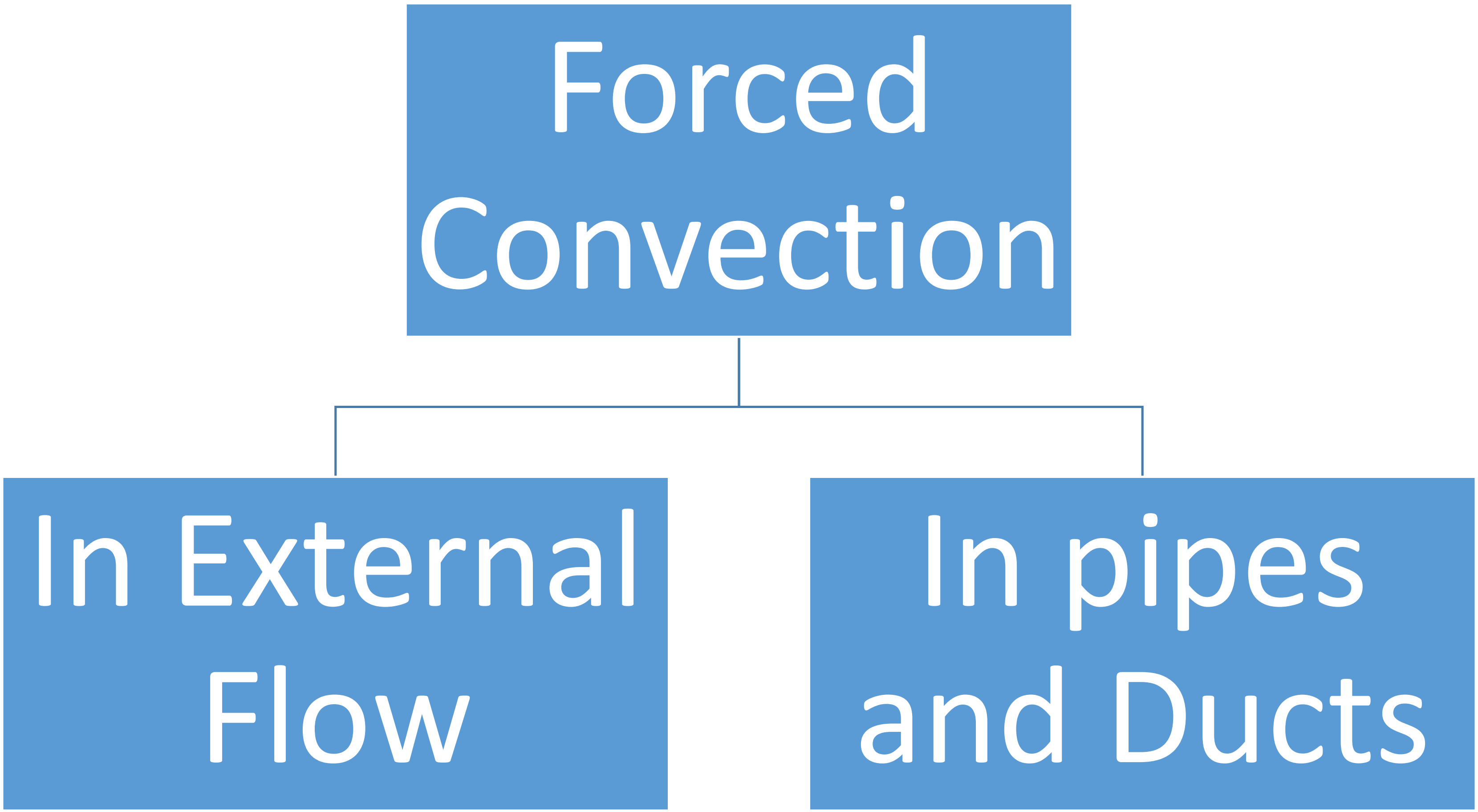
Forced convection

External and Internal & Design Problems


Forced Convection
In External Flow
In pipes and Ducts

Forced convection in external flow

- Many problems of engineering interest involve heat transfer to fluids flowing over objects such as pipes, tanks, buildings or other structures.
- In this section, correlations for several such systems are presented in order to illustrate the method of calculation.
- Flow over a fiat plate is illustrated in fig.1.

The undisturbed fluid velocity and temperature upstream of the plate are V_∞ and T_∞, respectively.

The surface temperature of the plate is T_s and L is the length of the plate in the direction of flow.

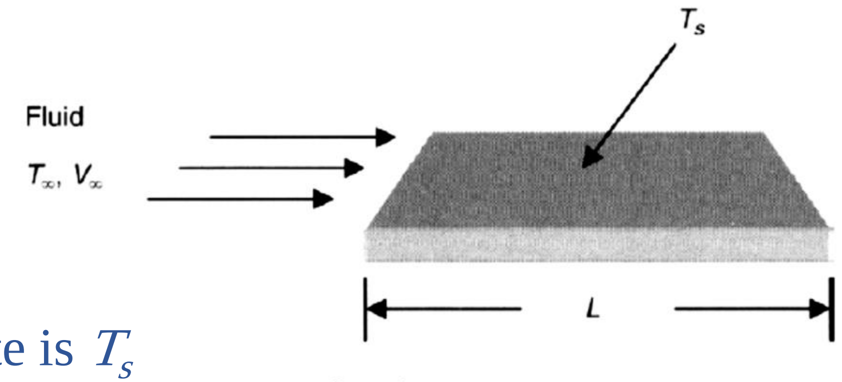


fig.1

Notes

- The heat-transfer coefficient is obtained from the following correlations (See next slides).
- Where

$$Nu = \frac{hL}{k} \quad \text{and} \quad Re = \frac{LV_\infty \rho}{\mu}$$

- In these equations all fluid properties are evaluated at the film temperature, T_f, defined by:

$$T_f = \frac{T_\infty + T_s}{2}$$

- Hence, the rate of heat transfer between the plate and the fluid is given by:

$$q = hA|T_s - T_\infty|$$

where A is the total surface area contacted by the fluid.

Laminar Flow over isothermal Plate

Correlation	Geometry	Conditions[b]
$\delta = 5x\,Re_x^{-1/2}$	Flat plate	Laminar, T_f
$C_{f,x} = 0.664 Re_x^{-1/2}$	Flat plate	Laminar, local, T_f
$Nu_x = 0.332 Re_x^{1/2}\,Pr^{1/3}$	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$
$\delta_t = \delta Pr^{-1/3}$	Flat plate	Laminar, T_f
$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$	Flat plate	Laminar, average, T_f
$\overline{Nu}_x = 0.664 Re_x^{1/2}\,Pr^{1/3}$ for $Re < 5 \times 10^5$	Flat plate	Laminar, average, T_f, $Pr \gtrsim 0.6$
$Nu_x = 0.565 Pe_x^{1/2}$ ***For Liquid Metals***	Flat plate	Laminar, local, T_f, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$

$Pe_x = Re_x\,Pr$ ***"Peclet number"***

Notes

Liquid Metals: Despite the corrosive and reactive nature of liquid metals, their unique properties (low melting point and vapor pressure, as well as high thermal capacity and conductivity) render them attractive as **coolants** in applications requiring high heat transfer rates.

TABLE A.7 Thermophysical Properties of Liquid Metals[a]

Composition	Melting Point (K)	T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\nu \cdot 10^7$ (m^2/s)	k (W/m · K)	$\alpha \cdot 10^5$ (m^2/s)	Pr
Bismuth	544	589	10,011	0.1444	1.617	16.4	1.138	0.0142
		811	9739	0.1545	1.133	15.6	1.035	0.0110
		1033	9467	0.1645	0.8343	15.6	1.001	0.0083
Lead	600	644	10,540	0.159	2.276	16.1	1.084	0.024
		755	10,412	0.155	1.849	15.6	1.223	0.017
		977	10,140	—	1.347	14.9	—	—
Potassium	337	422	807.3	0.80	4.608	45.0	6.99	0.0066
		700	741.7	0.75	2.397	39.5	7.07	0.0034
		977	674.4	0.75	1.905	33.1	6.55	0.0029
Sodium	371	366	929.1	1.38	7.516	86.2	6.71	0.011
		644	860.2	1.30	3.270	72.3	6.48	0.0051
		977	778.5	1.26	2.285	59.7	6.12	0.0037
NaK, (45%/55%)	292	366	887.4	1.130	6.522	25.6	2.552	0.026
		644	821.7	1.055	2.871	27.5	3.17	0.0091
		977	740.1	1.043	2.174	28.9	3.74	0.0058
NaK, (22%/78%)	262	366	849.0	0.946	5.797	24.4	3.05	0.019
		672	775.3	0.879	2.666	26.7	3.92	0.0068
		1033	690.4	0.883	2.118	—	—	—
PbBi, (44.5%/55.5%)	398	422	10,524	0.147	—	9.05	0.586	—
		644	10,236	0.147	1.496	11.86	0.790	0.189
		922	9835	—	1.171	—	—	—
Mercury	234			See Table A.5				

Mercury Properties

Saturated Liquids **Mercury (Hg)**

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^2$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^7$ (m^2/s)	Pr	$\beta \cdot 10^3$ (K^{-1})
273	13,595	0.1404	0.1688	0.1240	8180	42.85	0.0290	0.181
300	13,529	0.1393	0.1523	0.1125	8540	45.30	0.0248	0.181
350	13,407	0.1377	0.1309	0.0976	9180	49.75	0.0196	0.181
400	13,287	0.1365	0.1171	0.0882	9800	54.05	0.0163	0.181
450	13,167	0.1357	0.1075	0.0816	10,400	58.10	0.0140	0.181
500	13,048	0.1353	0.1007	0.0771	10,950	61.90	0.0125	0.182
550	12,929	0.1352	0.0953	0.0737	11,450	65.55	0.0112	0.184
600	12,809	0.1355	0.0911	0.0711	11,950	68.80	0.0103	0.187

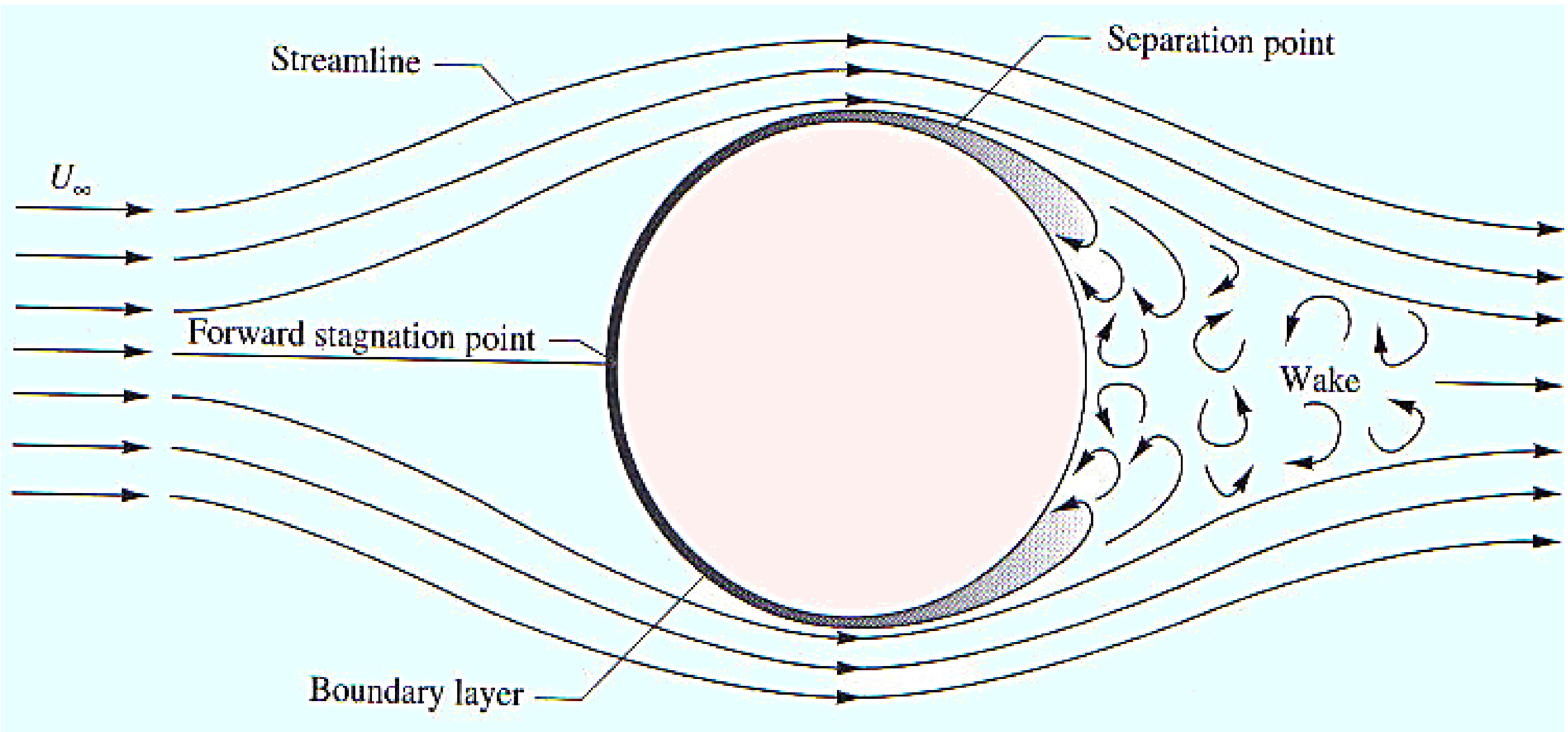
Turbulent and Mixed Flow over isothermal Plate

$C_{f,x} = 0.0592 Re_x^{-1/5}$	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\, Re_x^{-1/5}$	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296 Re_x^{4/5}\, Pr^{1/3}$	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$ Or $Nu = (0.037\, Re^{0.8} - 870) Pr^{1/3}$ for $Re > 5 \times 10^5$	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$

Flow past cylinder

- For flow perpendicular to a circular cylinder of diameter D, the average heat-transfer coefficient can be obtained from the correlations given in the next tables.
- The third correlation is widely use. The correlation is valid for $Re\ Pr > 0.2$, and all fluid properties are evaluated at the film temperature, T_f.

The Cylinder in Cross Flow



Empirical Correlation for cylinder

$\overline{Nu}_D = C\, Re_D^m\, Pr^{1/3}$ **Table 1**	**Eq. 1**	Cylinder	Average, T_f, $0.4 \lesssim Re_D \lesssim 4 \times 10^5$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C\, Re_D^m\, Pr^n (Pr/Pr_s)^{1/4}$ **Table 3**	**Eq. 2**	Cylinder	Average, T_∞, $1 \lesssim Re_D \lesssim 10^6$, $0.7 \lesssim Pr \lesssim 500$ If $Pr \lesssim 10$, $n = 0.37$; if $Pr \gtrsim 10$, $n = 0.36$
$\overline{Nu}_D = 0.3 + [0.62 Re_D^{1/2} Pr^{1/3}$ $\times [1 + (0.4/Pr)^{2/3}]^{-1/4}]$ $\times [1 + (Re_D/282{,}000)^{5/8}]^{4/5}$		Cylinder	Average, T_f, $Re_D\, Pr \gtrsim 0.2$

It can also be used for noncircular cylinder table 2

Note

$$\overline{Nu}_D = \frac{\bar{h} D}{k}$$

TABLE 1 Constants of Equation (1) for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

TABLE 2 Constants of Equation **(1)** for noncircular cylinders in cross flow of a gas [13]

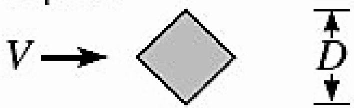
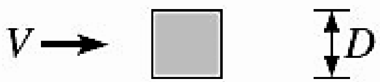
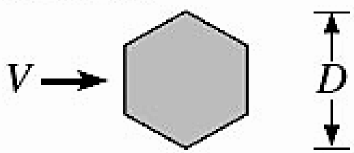
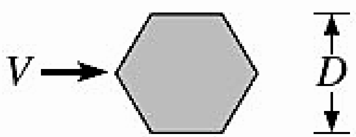
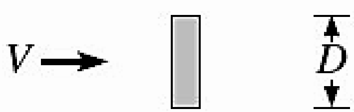
Geometry	Re_D	C	m
Square			
	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon			
	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
	5×10^3–10^5	0.153	0.638
Vertical plate			
	4×10^3–1.5×10^4	0.228	0.731

TABLE 3 Constants of Equation (2) for the circular cylinder in cross flow [16]

Re_D	C	m
1–40	0.75	0.4
40–1000	0.51	0.5
10^3–2×10^5	0.26	0.6
2×10^5–10^6	0.076	0.7

For flow over a sphere

For flow over a sphere of diameter D, the following correlation is recommended

$$Nu = 2 + (0.4\,Re^{1/2} + 0.06\,Re^{2/3})Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{0.25}$$

where

$$Nu = \frac{hD}{k}$$

and

$$Re = \frac{DV_\infty \rho}{\mu}$$

Notes:

- All fluid properties, except μ_s, are evaluated at the free-stream temperature, T_∞. The viscosity, μ_s, is evaluated at the surface temperature, *Ts. The above e*quation is valid for Reynolds numbers between 3.5 and 80,000, and Prandtl numbers between 0.7 and 380.
- The following table summarizes the correlations for flow over a sphere and for falling droplet.

Empirical Correlation for Sphere

$\overline{Nu}_D = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4} \times (\mu/\mu_s)^{1/4}$	Sphere	Average, T_∞, $3.5 \leq Re_D \leq 7.6 \times 10^4$, $0.71 \leq Pr \leq 380$
$\overline{Nu}_D = 2 + 0.6Re_D^{1/2}\, Pr^{1/3}$	Falling drop	Average, T_∞

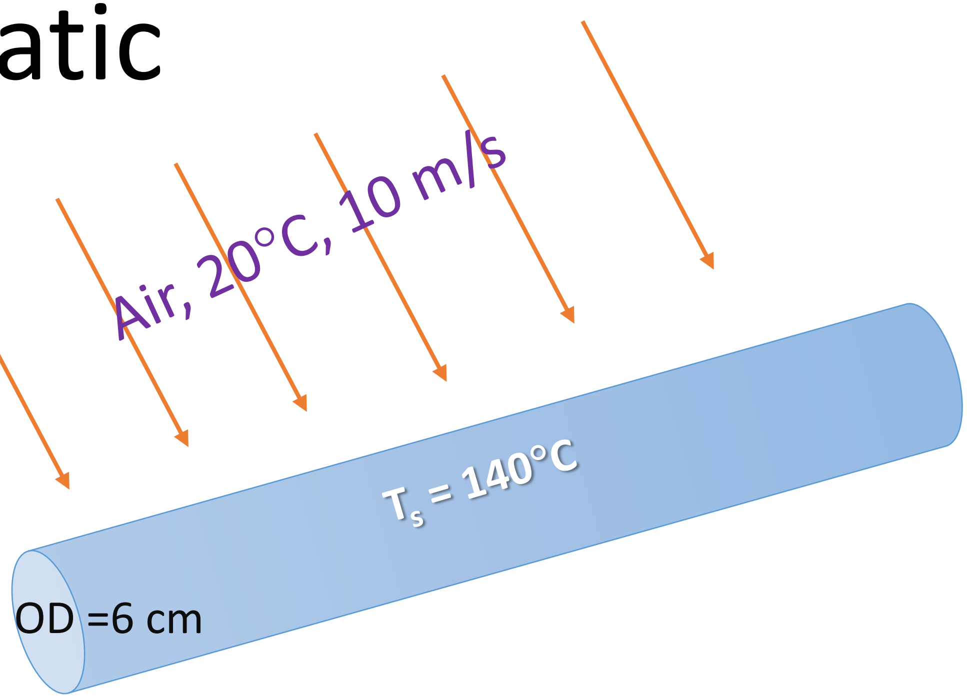
Note

- The equations presented in this (and the following) section, as well as similar correlations which appear in the literature, are not highly accurate. In general, one should not expect the accuracy of computed values to be better than $\pm 25\%$ to $\pm 30\%$ when using these equations.
- This limitation should be taken into consideration when interpreting the results of heat-transfer calculations.

Example 1

Air at 20°C is blown over a 6 cm OD pipe that has a surface temperature of 140°C The free-stream air velocity is 10 m/s What is the rate of heat transfer per meter of pipe?

Schematic



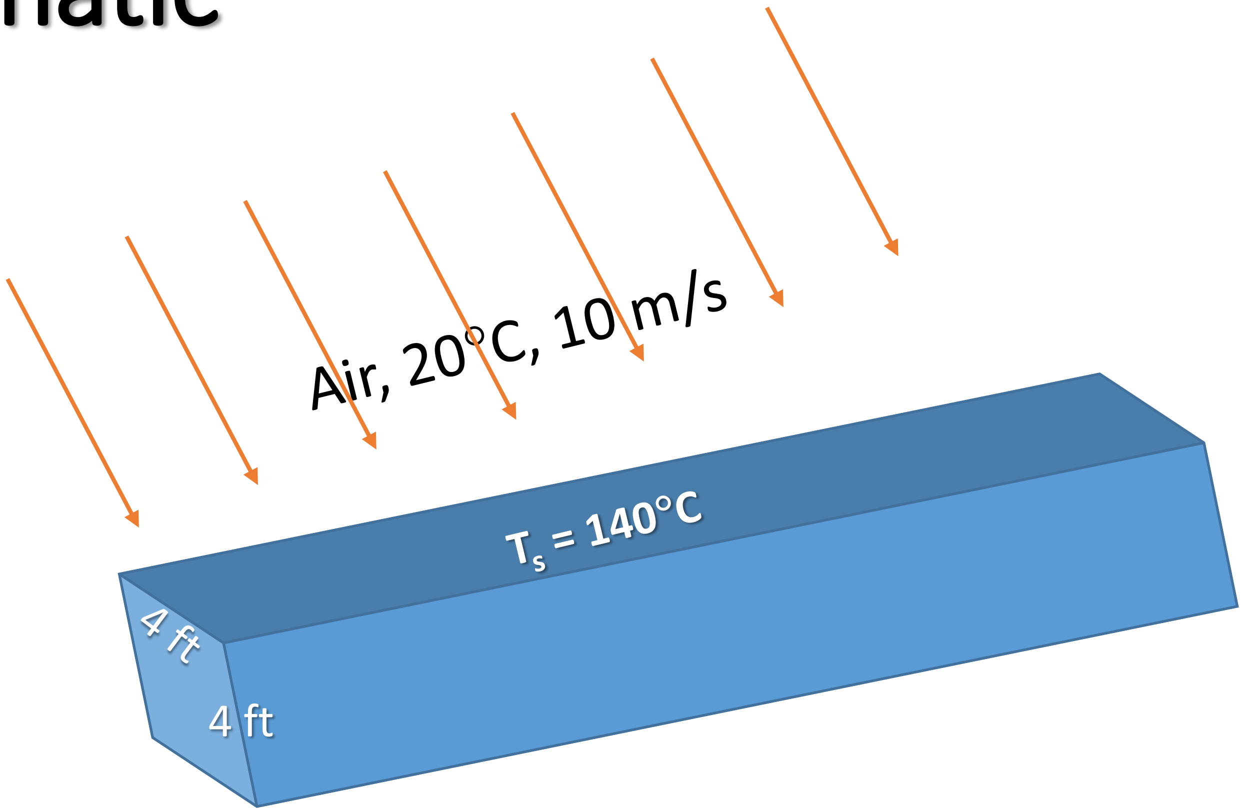
Find q per unit length.

Solution

Example 2

A duct carries hot waste gas from a process unit to a pollution control device. The duct cross section is 4 ft x 4 ft and it has a surface temperature of 140°C. Ambient air at 20°C blows across the duct with a wind speed of 10 m/s. Estimate the rate of heat loss per meter of duct length.

Schematic



Find q per unit length.

Solution

Check the range

$$Nu =$$

$$=$$

$$Nu =$$

$$h =$$

$$h =$$

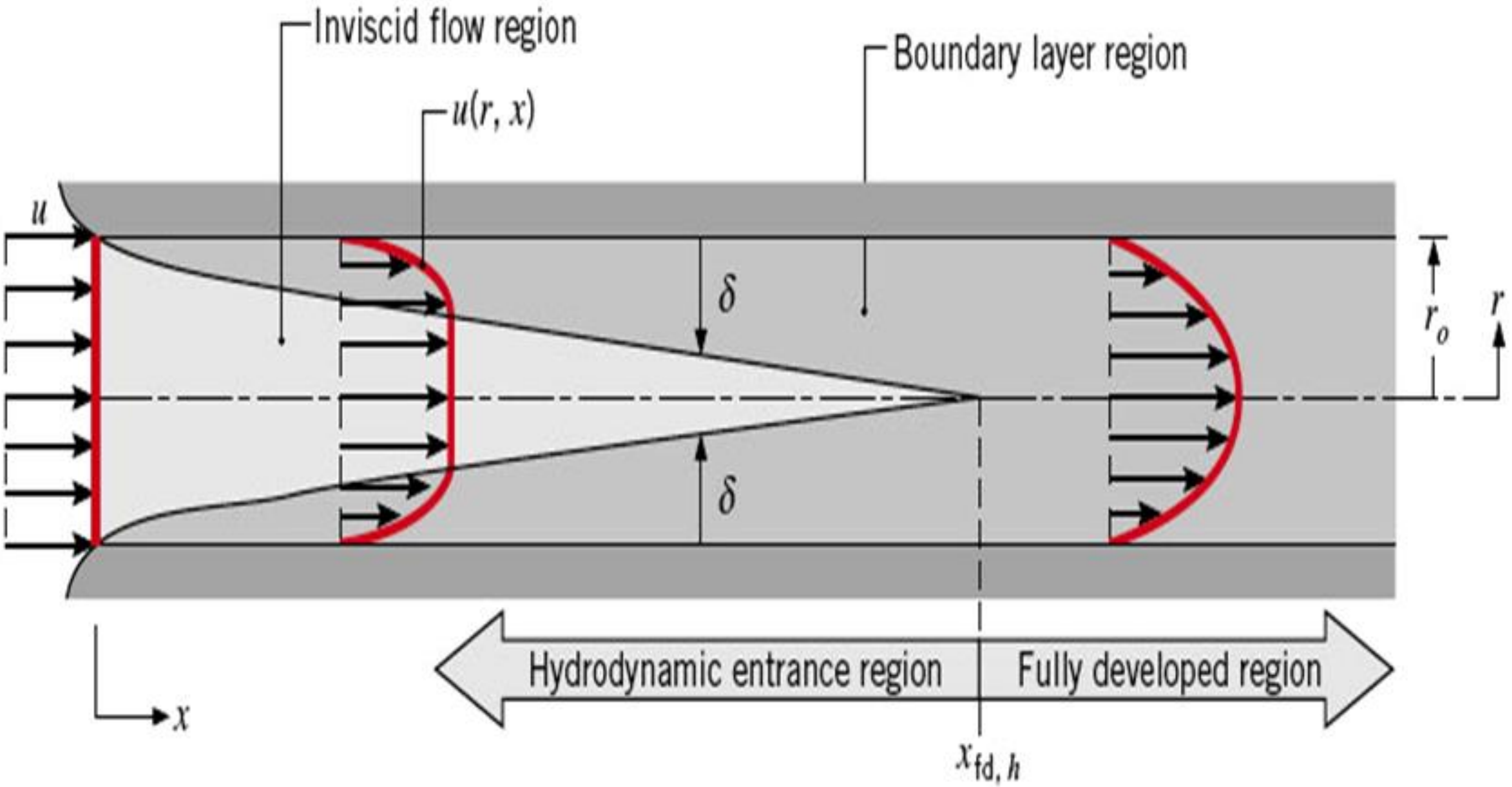
$$A =$$

Finally, the rate of heat loss is:

INTERNAL FLOW

Laminar & Turbulent regimes

Laminar hydrodynamic B.L. development in a circular tube



Laminar, hydrodynamic boundary layer development in a circular tube.

Notes

- In general, the entrance length, $x_h=f(Re_D)$ and

  $$(x/D)_{lam} \approx 0.05\ Re_D$$

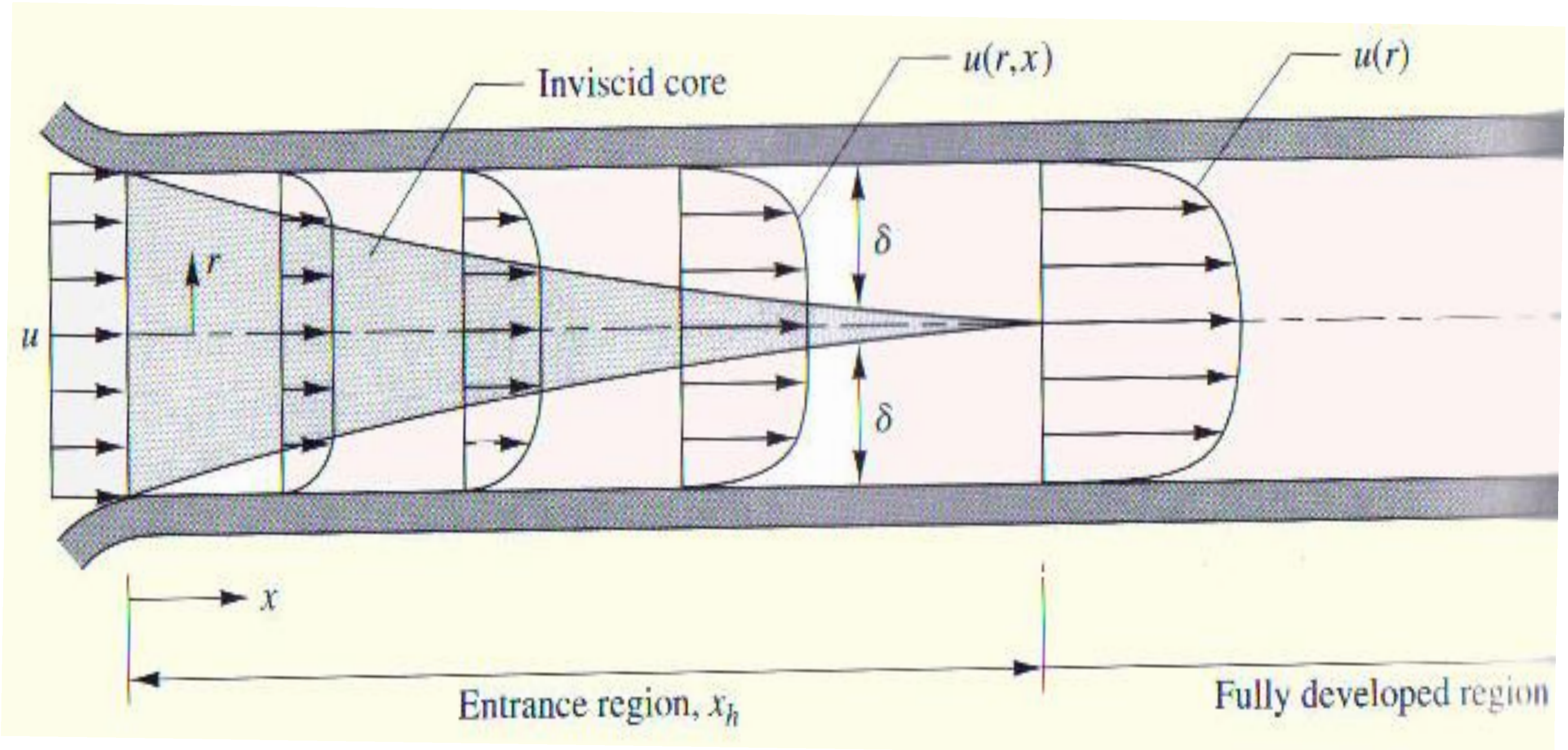
- A force balance on a differential fluid element gives a parabolic velocity profile for fully developed laminar flow. See the Fluid mechanics or Transport Phenomena 1 Course for more details.

$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right) r_o^2 \left[1-\left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx} \quad \text{where } u_m \text{ is mean velocity}$$

$$\frac{u(r)}{u_m} = 2\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

Turbulent hydrodynamic B.L. in a circular tube



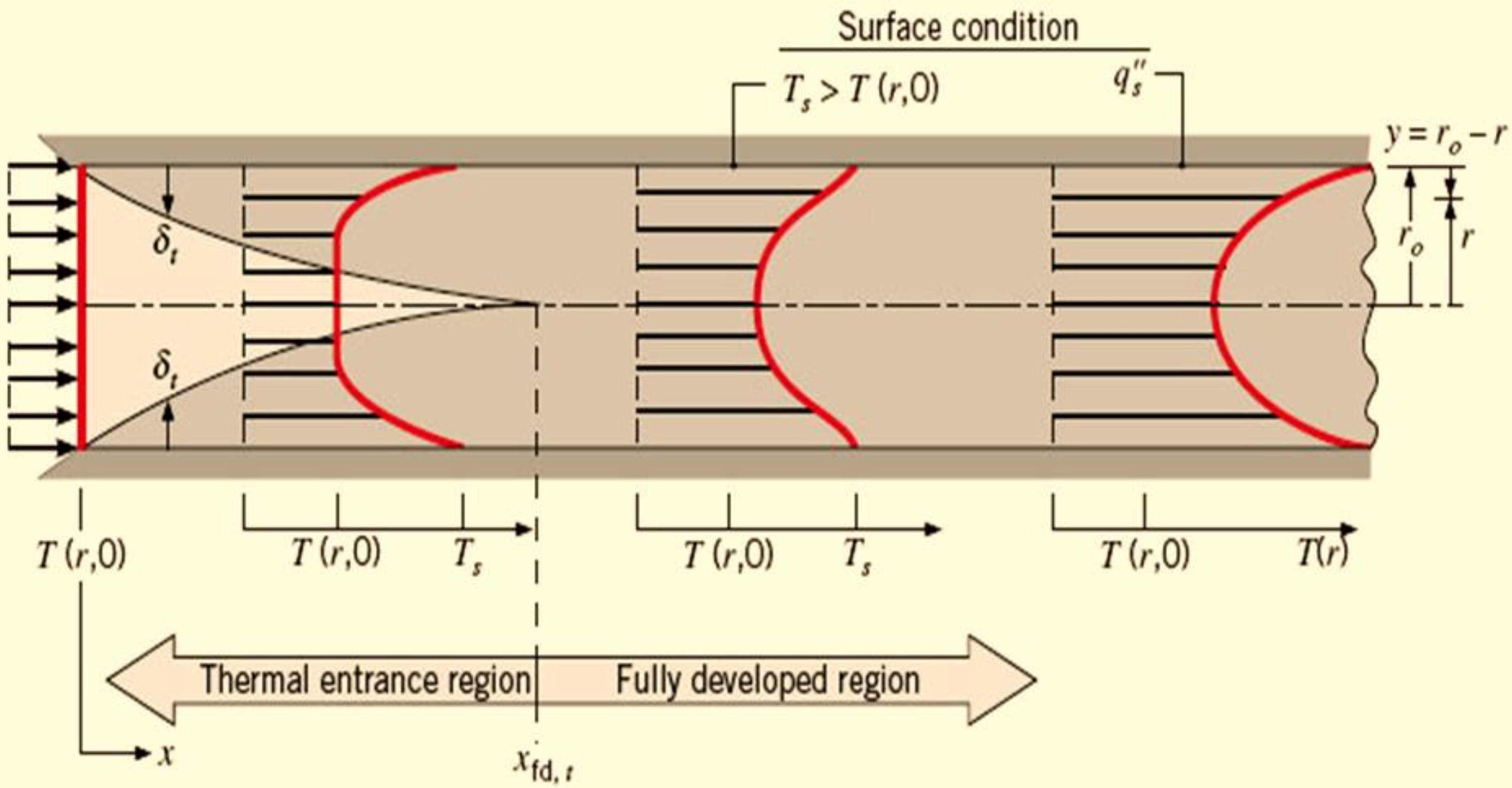
Notes

- Unlike laminar flow, the entrance length, X_h is not function in Re_D .
- Experiments have shown that

$$10 < (\frac{x_h}{D}) < 60$$

- Velocity profile for fully developed turbulent flow is not parabolic and is flatter due to turbulent mixing in the radial direction.
- For simplicity, assume that fully developed occurs at (x/D)>10

Thermal B.L. development in a heated circular tube



Thermal boundary layer development in a heated circular tube.

Notes

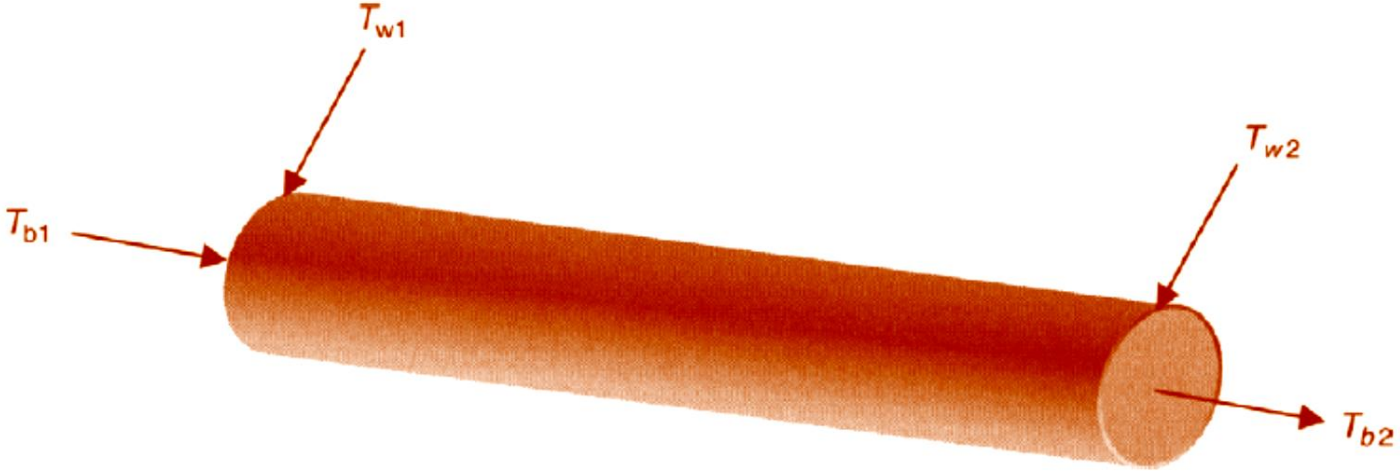
- For laminar flow, the thermal entrance length, x_t; $x_t = f(Re, Pr)$

$$\frac{x_t}{D} \approx 0.05 \, \mathrm{Re}_D \, \mathrm{Pr}$$

- For turbulent, $x_t \neq f(Re, Pr)$ ~ $(x_t/D) > 10$ *or assume* $(x_t/D) = 10$

Correlations for Forced Convection in Pipes and Ducts

Correlations are given for each of the three flow regimes; turbulent, transition, and laminar. The physical situation described by the correlations is given in Figure below. Fluid enters the pipe at an average temperature T_{b1} and leaves at an average temperature T_{b2} (T_b stands for bulk temperature. It is the fluid temperature averaged over the flow area.) The pipe wall temperature is T_{w1} at the entrance and T_{w2} at the exit.



Turbulent Region

With respect to heat transfer in circular pipes, fully developed turbulent flow is achieved at a Reynolds number of approximately 10^4. For this flow regime ($Re \geq 10^4$), the following correlation is widely used:

$$Nu = 0.027\, Re^{0.8} Pr^{1/3} (\mu / \mu_w)^{0.14}$$ "Seider-Tate correlation"

........................(i)

where

$Nu \equiv$ *Nusselt Number* $\equiv hD/k$

$\mathrm{Re} \equiv$ *Reynolds Number* $\equiv DV\rho/\mu$

$\mathrm{Pr} \equiv$ *Prandtl Number* $= C_P\mu/k$

$D =$ *inside pipe diameter*

$V =$ *average fluid velocity*

$C_P, \mu, \rho, k =$ *fluid properties evaluated at the average bulk fluid temperature*

$\mu_w =$ *fluid viscosity evaluated at average wall temperature*

- The average bulk fluid temperature and average wall temperature are given by:

$$T_{b,ave} = \frac{T_{b1} + T_{b2}}{2}$$

$$T_{w,ave} = \frac{T_{w1} + T_{w2}}{2}$$

- The equation is frequently written with the coefficient 0.027 replaced by 0.023. The latter value gives a somewhat more conservative estimate for the heat-transfer coefficient, which is often desirable for design purposes.
- The previous correlation is valid for fluids with Prandtl numbers between 0.5 and 17,000, and for pipes with $L/D > 10$.
- For short pipes with $10 < L/D < 60$, the right-hand side of the equation is often multiplied by the factor $[1 + (D/L)^{2/3}]$ to correct for entrance and exit effects.

$$Nu = 0.027\, Re^{0.8} Pr^{1/3} (\mu/\mu_w)^{0.14} [1 + (D/L)^{2/3}]$$

Notes

- The correlation is generally accurate to within $\pm 20\%$ to $\pm 40\%$. It is most accurate for fluids with low to moderate Prandtl numbers ($0.5 \le Pr \le 100$), which includes all gasses and low-viscosity process liquids such as water, organic solvents, light hydrocarbons, etc.
- It is less accurate for highly viscous liquids, which have correspondingly large Prandtl numbers.

For laminar flow in circular pipes *(Re* < 2100), the Seider-Tate correlation takes the form:

$$Nu = 1.86[Re\,Pr\,D/L]^{1/3}(\mu/\mu_w)^{0.14}$$

- This equation is valid for 0.5 <Pr < 17,000 and $(RePrD/L)^{1/3}(\mu/\mu_w)^{0.14} > 2$, and is generally accurate to within ±25%.
- Fluid properties are evaluated at $T_{b,ave}$ except for μ_w, which is evaluated at $T_{w,ave}$.
- *For* $(RePrD/L)^{1/3}(\mu/\mu_w)^{0.14} < 2$, the Nusselt number should be set to 3.66, which is the theoretical value for laminar flow in an infinitely long pipe with constant wall temperature.
- Also, at low Reynolds numbers heat transfer by natural convection can be significant (see Natural convection), and this effect is not accounted for in the Seider-Tate correlation.

Transition Region

- For flow in the transition region ($2100 < Re < 10^4$), the Hausen correlation is recommended:

$$Nu = 0.116[Re^{2/3} - 125]Pr^{1/3}(\mu/\mu_w)^{0.14}[1 + (D/L)^{2/3}] \quad \text{............(ii)}$$

- Heat-transfer calculations in the transition region are subject to a higher degree of uncertainty than those in the laminar and fully developed turbulent regimes.
- Although industrial equipment is sometimes designed to operate in the transition region, it is generally recommended to avoid working in this flow regime if possible.
- An alternative equation for the transition and turbulent regimes has been proposed by Gnielinski

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}[1 + (\frac{D}{L})^{2/3}] \quad \text{..............(iii)}$$

$$0.6 < Pr < 2000, \qquad Re > 2300$$

- Here, f is the Darcy friction factor, which can be computed from the following explicit approximation of the Colebrook equation

$$f = (0.782 \ln Re - 1.51)^{-2} \quad \text{.......................(iv)}$$

ducts and conduits with non-circular cross-sections

- For flow in ducts and conduits with non-circular cross-sections, Equations (i) and (ii) - (iv), can be used if the diameter is everywhere replaced by the equivalent diameter, *De,* where

De = 4 x hydraulic radius = 4 x flow area/wetted perimeter

- This approximation generally gives reliable results for turbulent flow. However, it is not recommended for laminar flow.
- The most frequently encountered case of laminar flow in non-circular ducts is flow in the annulus of a double-pipe heat exchanger. For laminar annular flow, the following equation given by Gnielinski is recommended:

$$Nu = 3.66 + 1.2(D_2/D_1)^{0.8} + \frac{0.19[1+0.14(D_2/D_1)^{0.5}][Re\,Pr\,D_e/L]^{0.8}}{1+0.117[Re\,Pr\,D_e/L]^{0.467}}$$
........................(v)

where

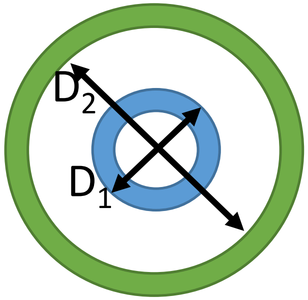
D_1 = outside diameter of inner pipe

D_2 = inside diameter of outer pipe

D_e = equivalent diameter = $D_2 - D_1$

$$D_h = \frac{4A}{P}$$

$$D_h = \frac{4\frac{\pi}{4}(D_2^2 - D_1^2)}{\pi(D_2 + D_1)} = D_2 - D_1$$



- The Nusselt number in the previous equation is based on the equivalent diameter, *De.*

Note: All of the above correlations give average values of the heat-transfer coefficient over the entire length, L, of the pipe.

- Hence, the total rate of heat transfer between the fluid and the pipe wall can be calculated from the following Equation:

$$q = hA\,\Delta T_{\ln}$$

- In this equation, A is the total heat-transfer surface area (πDL for a circular pipe) and ΔT_{Ln} is an average temperature difference between the fluid and the pipe wall. A logarithmic average is used; it is termed the logarithmic mean temperature difference (LMTD) and is defined by:

$$\Delta T_{\ln} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

where

$$\Delta T_1 = |T_{w1} - T_{b1}|$$
$$\Delta T_2 = |T_{w2} - T_{b2}|$$

Notes

- Like any mean value, the LMTD lies between the extreme values, ΔT_1 and ΔT_2. Hence, when ΔT_1 and ΔT_2 are not greatly different, the LMTD will be approximately equal to the arithmetic mean temperature difference, by virtue of the fact that they both lie between ΔT_1 and ΔT_2.
- The arithmetic mean temperature difference, ΔT_{ave} , is given by:

$$\Delta T_{ave} = \frac{\Delta T_1 + \Delta T_2}{2}$$

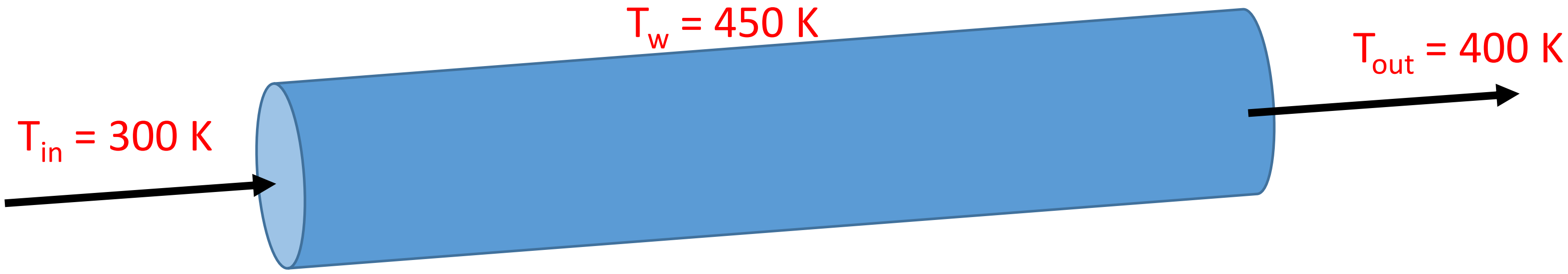
- The difference between ΔT_{ln} and ΔT_{ave} is small when the flow is laminar and $Re\,Pr\,D/L > 10$.

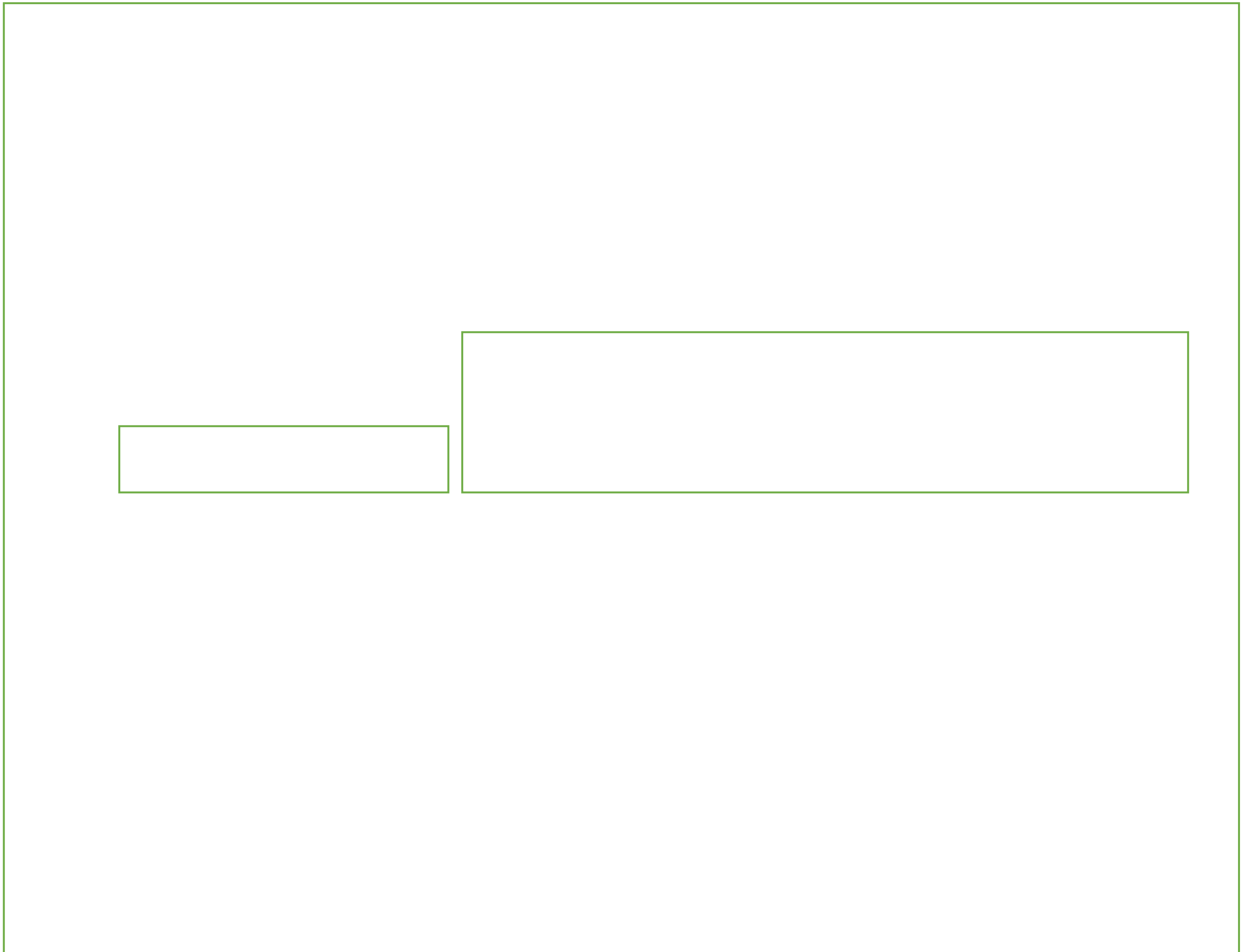
Example 3

Carbon dioxide at 300 K and 1 atm is to be pumped through a 5 cm ID pipe at a rate of 50 kg/h. The pipe wall will be maintained at a temperature of 450 K in order to raise the carbon dioxide temperature to 400 K. What length of pipe will be required?

Solution

- Schematic diagram and assumptions





- Obtain LMTD

- Find q from energy balance equation

- From the 1st eq. find the required length.

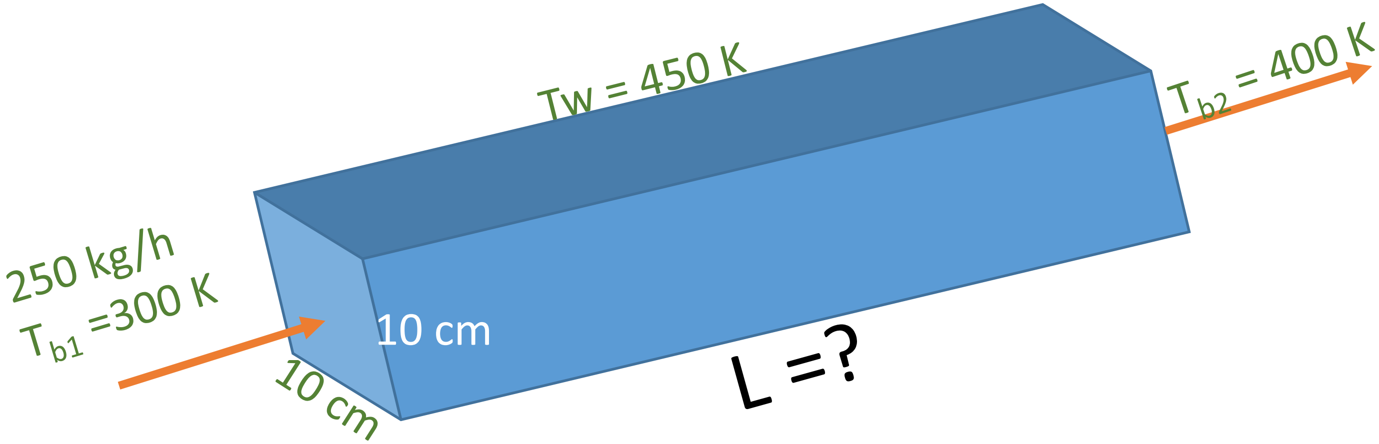
Check & comments

Example 4

Carbon dioxide at 300 K and 1 atm is to be pumped through a duct with a 10 cm x 10 cm square cross-section at a rate of 250 kg/h. The walls of the duct will be at a temperature of 450 K. What distance will the CO_2 travel through the duct before its temperature reaches 400 K?

Solution

- Schematic and properties



Steps of calculations

Don’t forget the check