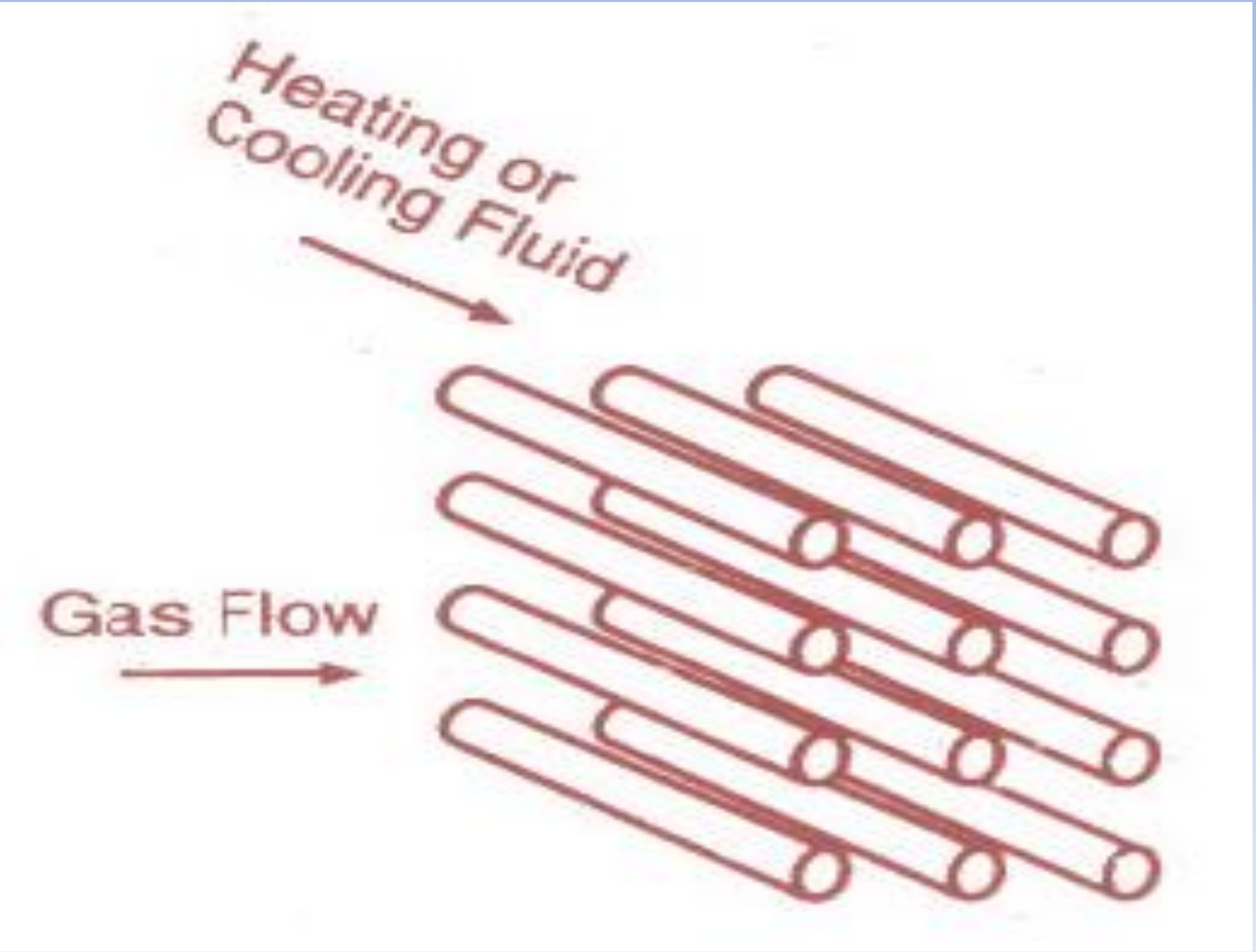
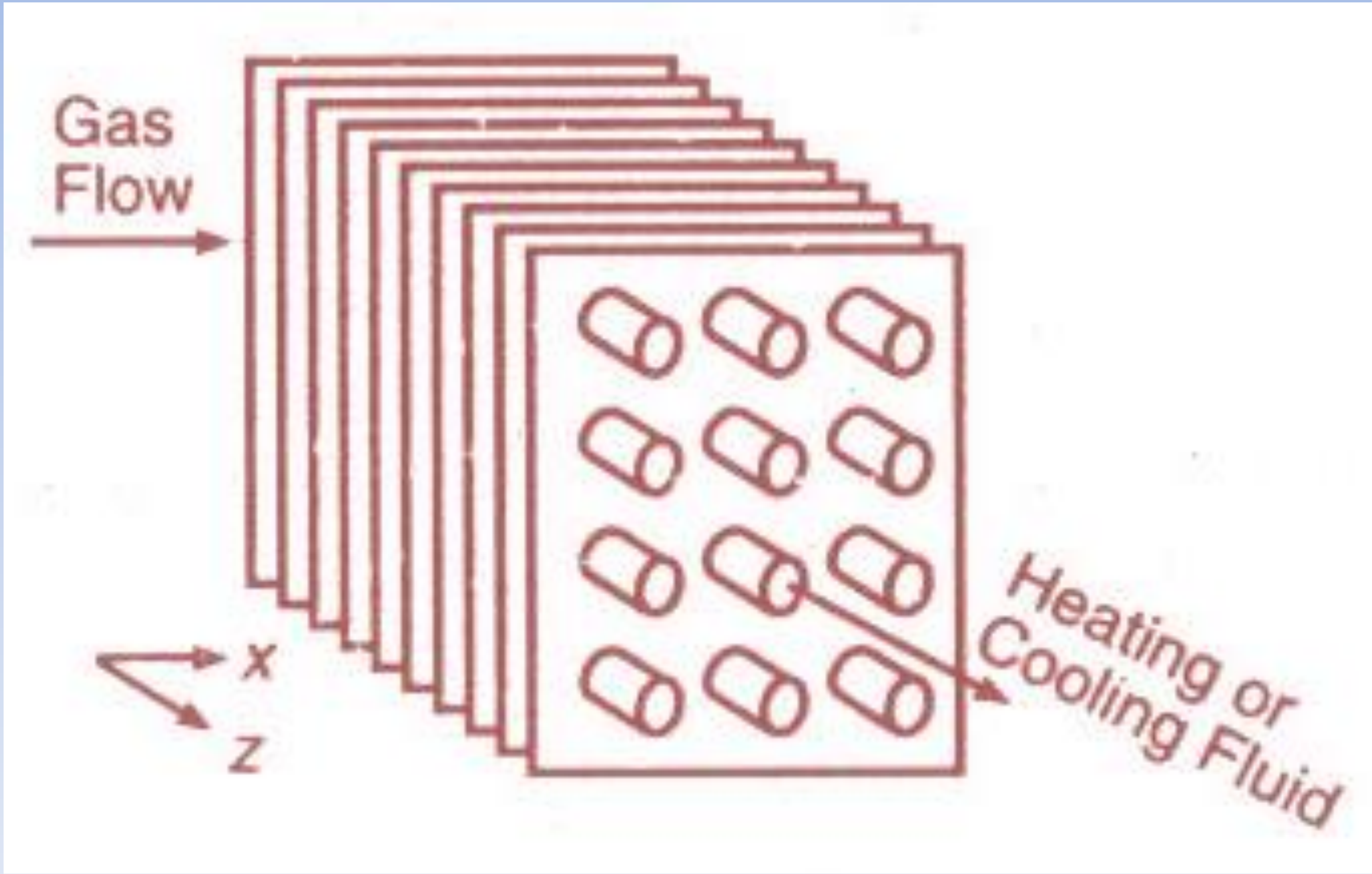
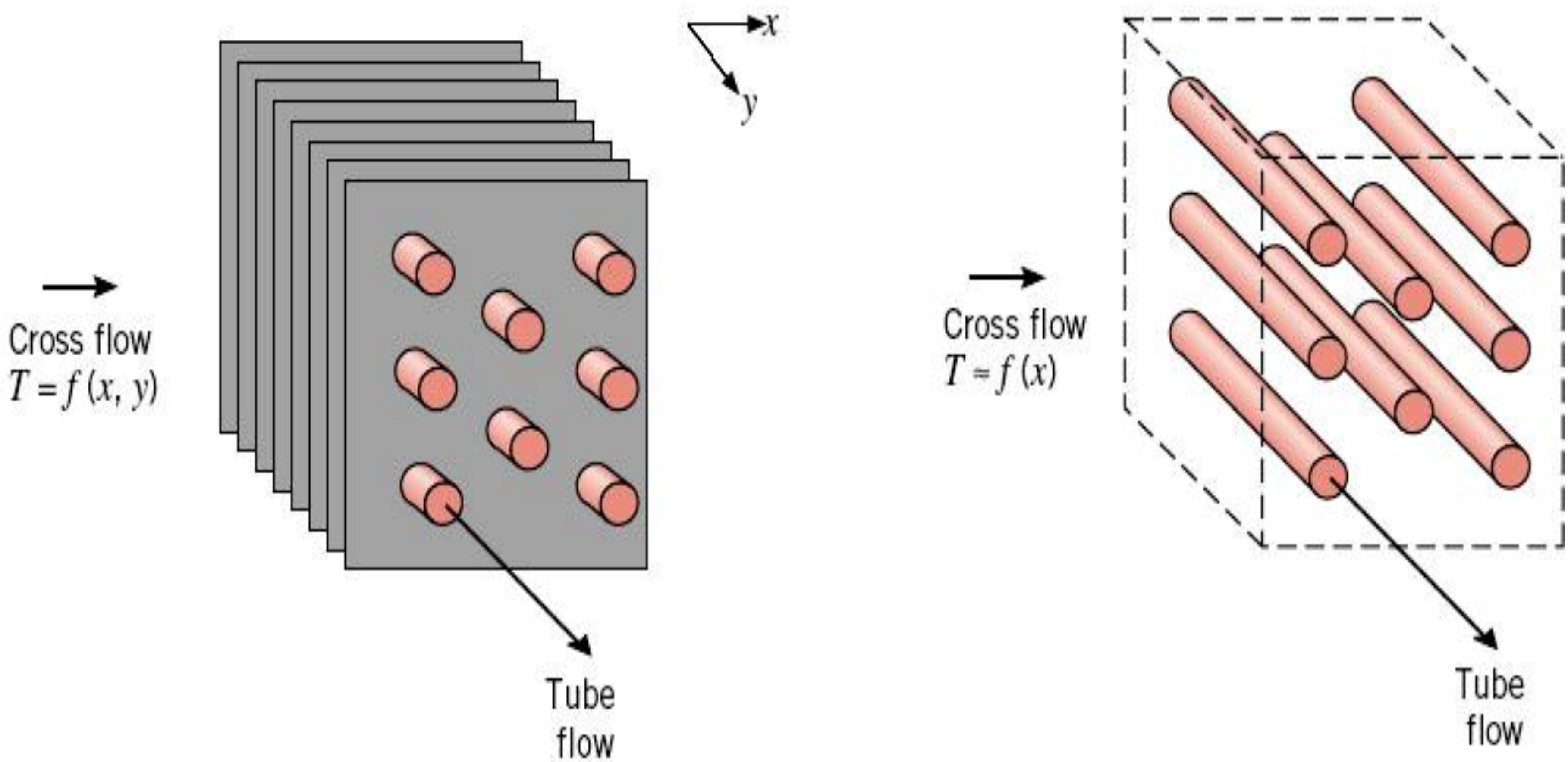
Cross Flow Exchanger and Exchanger Effectiveness

Cross flow mixed-unmixed flow



Cross flow exchanger both fluids unmixed



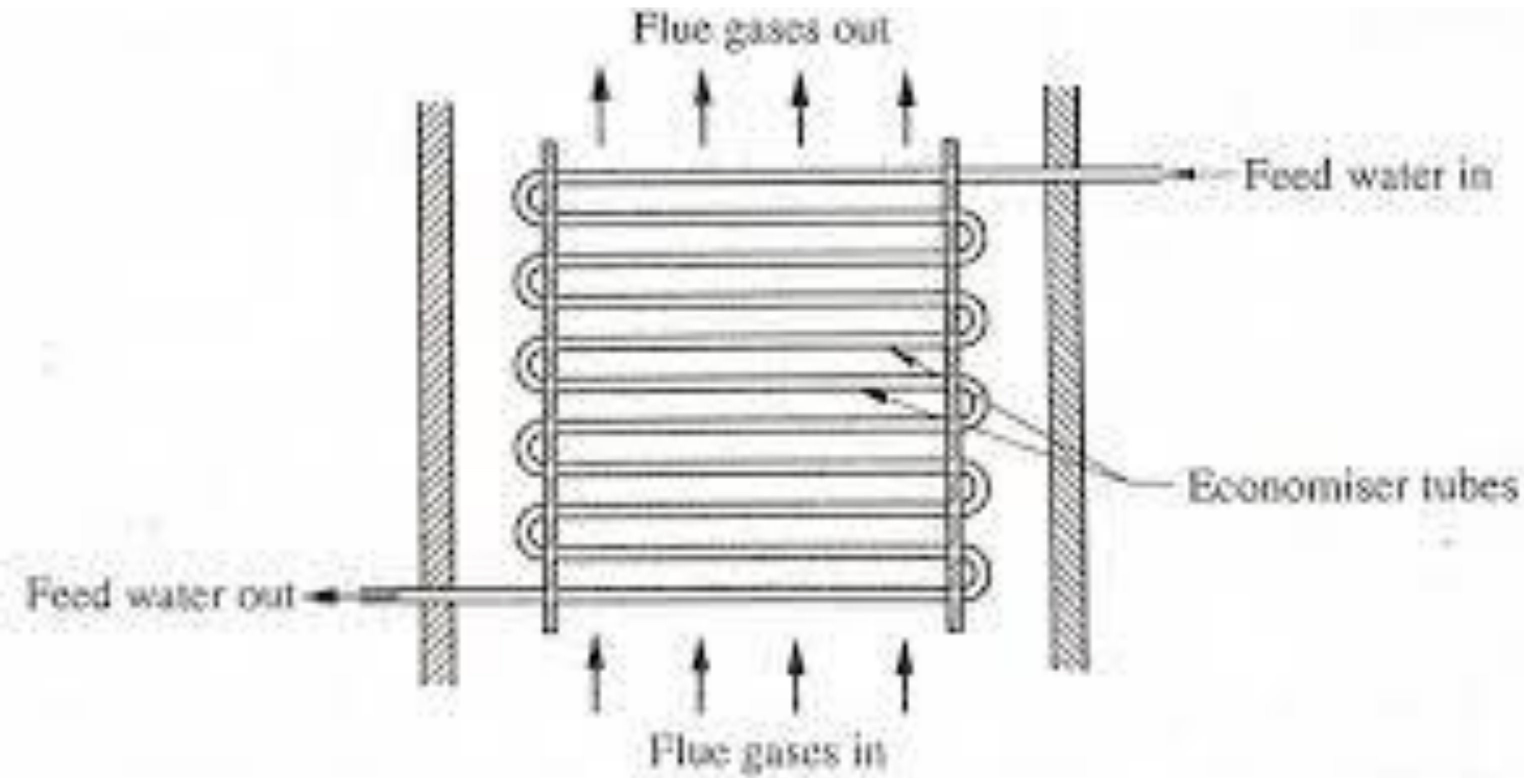


(a)

(b)

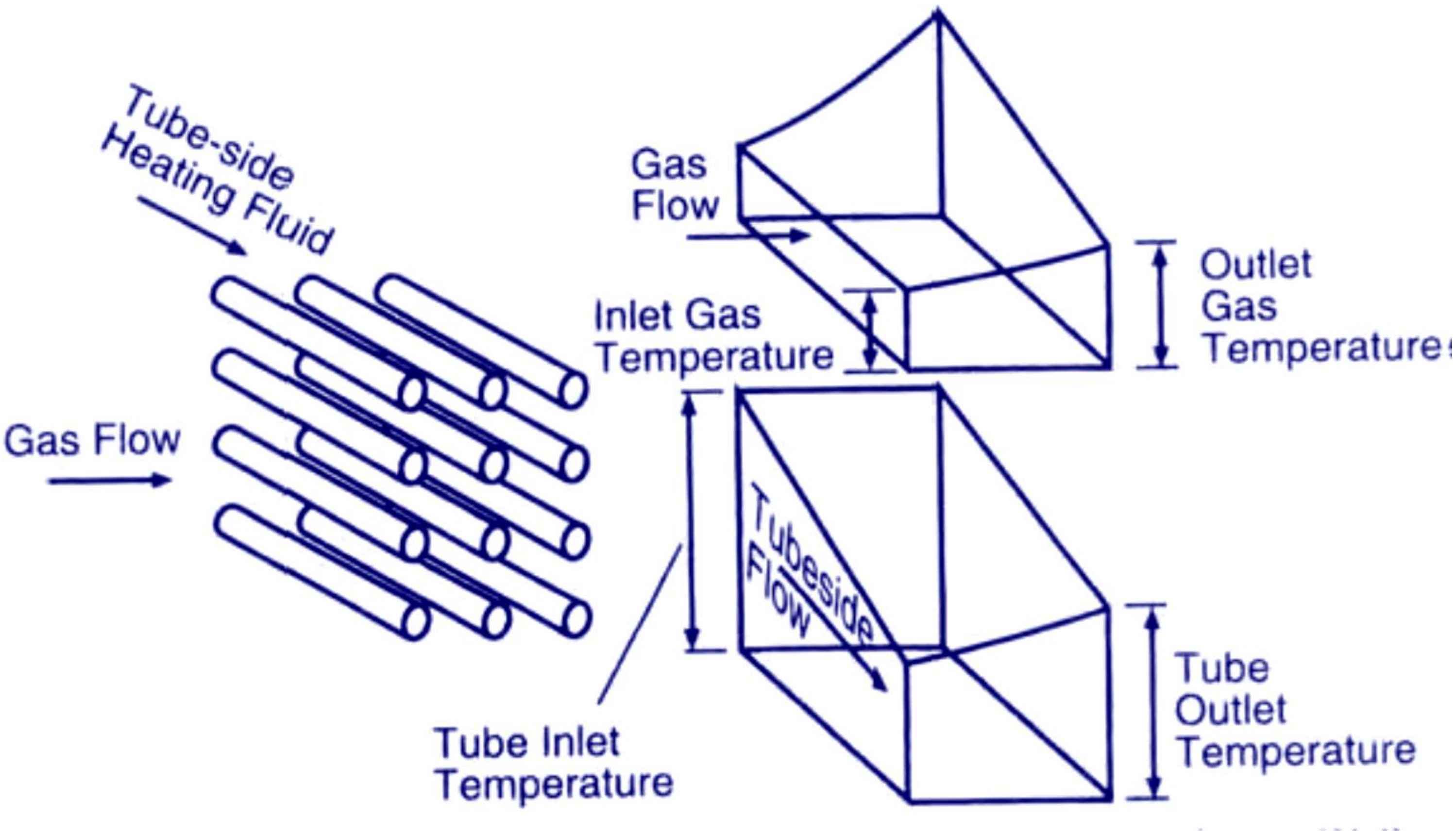
FIGURE 11.2 Cross-flow heat exchangers. (*a*) Finned with both fluids unmixed. (*b*) Unfinned with one fluid mixed and the other unmixed.

Applications _ Economizer

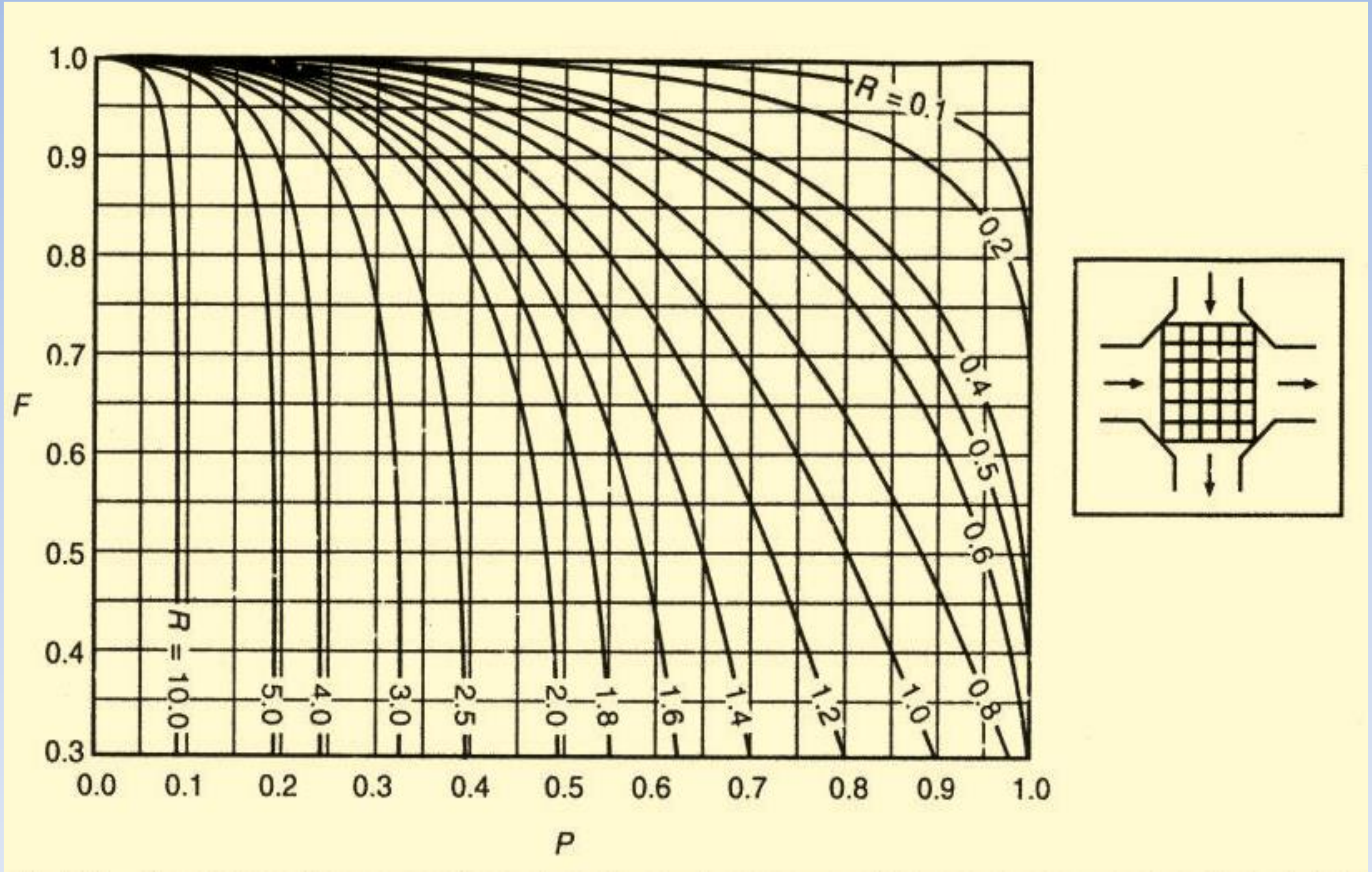


Economizer

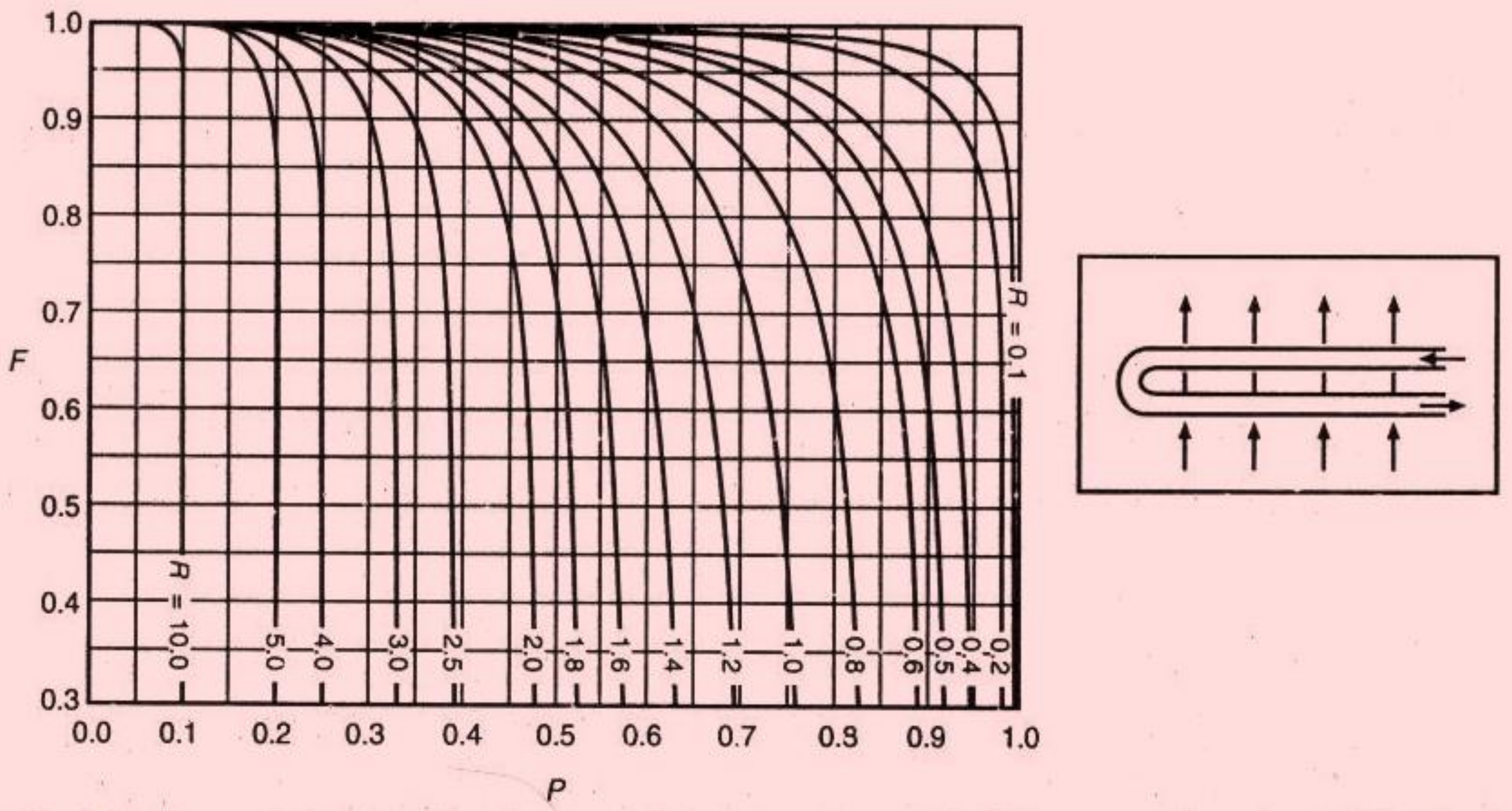
Temperature profiles in a cross-flow heat exchanger



Cross flow exchanger with both streams unmixed



Cross flow ‘mixed-unmixed streams’



Heat Exchanger analysis: The effectiveness-NTU Method

- This method is preferable when only inlet temperatures are known.
- Max. Possible heat transfer rate, q_{max} can be achieved in case of counter current flow exchanger of infinite length. But infinite length is …… !!!
- If $C_c < C_h$, $C_c \Delta T_c = C_h \Delta T_h \quad \therefore \Delta T_c > \Delta T_h$
- The cold fluid would be heated to inlet temperature of the hot fluid. Hence

$$q_{max} = C_c(T_{h,i} - T_{c,i})$$

- If $C_h < C_c$, $C_c \Delta T_c = C_h \Delta T_h \quad \therefore \ \Delta T_h > \Delta T_c$
- The hot fluid would be cooled to the inlet temperature of the cold fluid. Hence

$$q_{max} = C_h(T_{h,i} - T_{c,i})$$

- In general, $q_{max} = C_{min}(T_{h,i} - T_{c,i})$

  where C_{min} is equal to C_h or C_c whichever is smaller.
- Effectiveness, ε, is defined as the ratio of the actual heat transfer for exchanger to the maximum possible heat transfer rate:

$$\varepsilon = q_{act}/q_{max}$$

$$\varepsilon = C_h(T_{h,i} - T_{h,o}) / C_{min}(T_{h,i} - T_{c,i})$$

or

$$\varepsilon = C_c(T_{c,o} - T_{c,i}) / C_{min}(T_{h,i} - T_{c,i})$$

- Assume a parallel-flow for which $C_{min} = C_h$, hence

$$\varepsilon = (T_{h,i} - T_{h,o}) / (T_{h,i} - T_{c,i})$$

- If ε, $T_{h,i}$,and $T_{c,i}$ are known, the actual heat transfer:

$$q_{act} = \varepsilon\, q_{max}$$

$$q_{act} = \varepsilon\, C_{min}(T_{h,i} - T_{c,i})$$

- For any exchanger it can be shown that

$$\varepsilon = f(NTU, C_{min}/C_{max})$$

Where C_{min}/C_{max} is equal to C_c/C_h or C_h/C_c, depending on the relative values of heat capacity rates.

- The number of transfer unit, NTU, is defined as

$$NTU = UA/C_{min}$$

- ε and NTU can be obtained from equations or charts.

Table 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation		
Concentric tube			
Parallel flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterflow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - C_r)]}{1 - C_r \exp[-\text{NTU}(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{\text{NTU}}{1 + \text{NTU}}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\text{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n Shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-flow (single pass)			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\text{NTU})^{0.22}\{\exp[-C_r(\text{NTU})^{0.78}] - 1\}\right]$		(11.32)
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\text{NTU})]\})$		(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\text{NTU})]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\text{NTU})$		(11.35a)

Where $C_r = C_{min} / C_{max}$

Table 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation	
Concentric tube		
Parallel flow	$\mathrm{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$	(11.28b)
Counterflow	$\mathrm{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$	
	$\mathrm{NTU} = \dfrac{\varepsilon}{1 - \varepsilon} \quad (C_r = 1)$	(11.29b)
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1 + C_r^2)^{-1/2}\ln\left(\dfrac{E - 1}{E + 1}\right)$	(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$	(11.30c)
n Shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F - 1}{F - C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$	(11.31b, c, d)
Cross-flow (single pass)		
C_{max} (mixed), C_{min} (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1 - \varepsilon C_r)\right]$	(11.33b)
C_{min} (mixed), C_{max} (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r \ln(1 - \varepsilon) + 1]$	(11.34b)
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1 - \varepsilon)$	(11.35b)

Where $C_r = C_{min} / C_{max}$

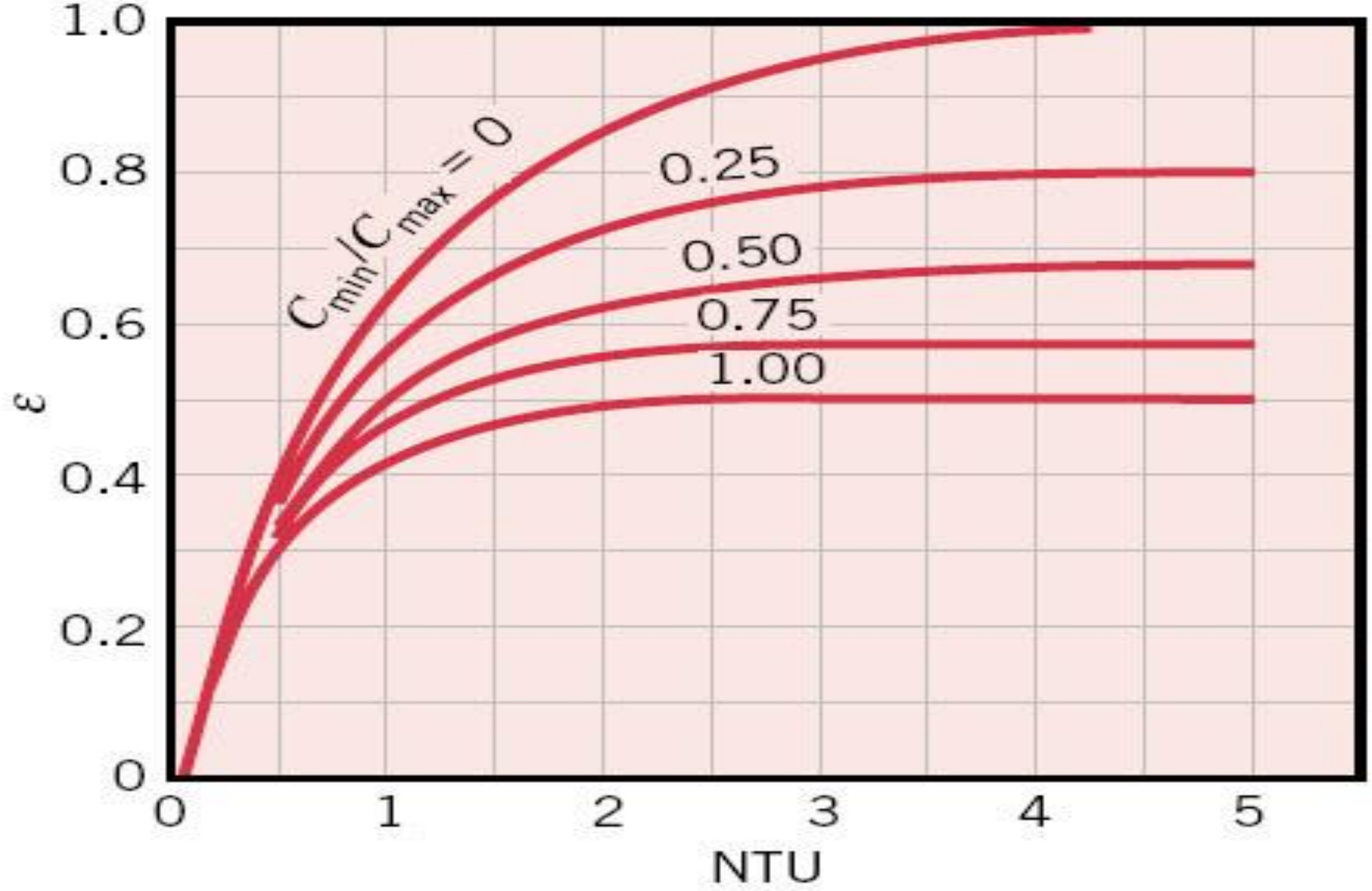


FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger (Equation 11.28).

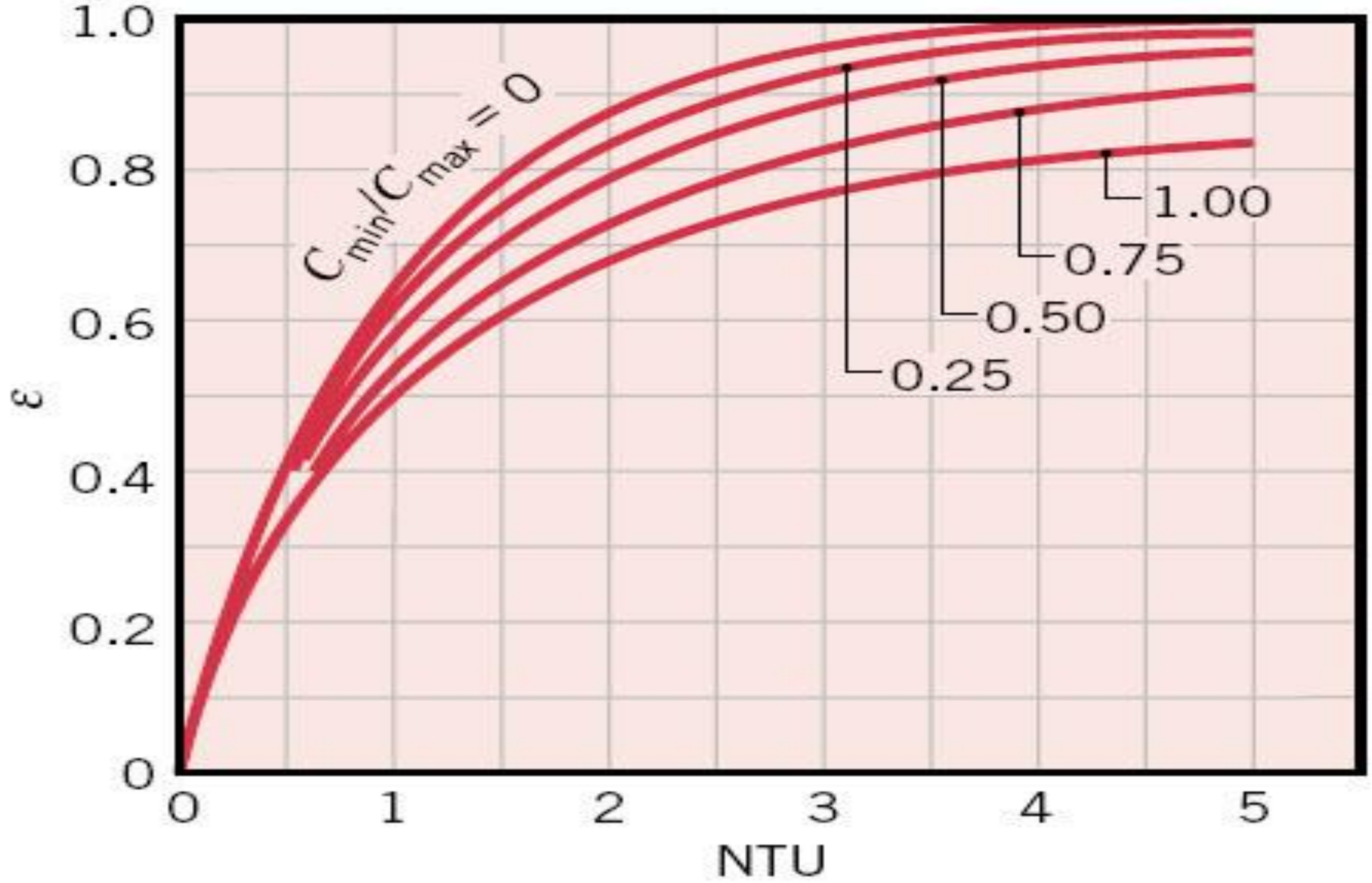


FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

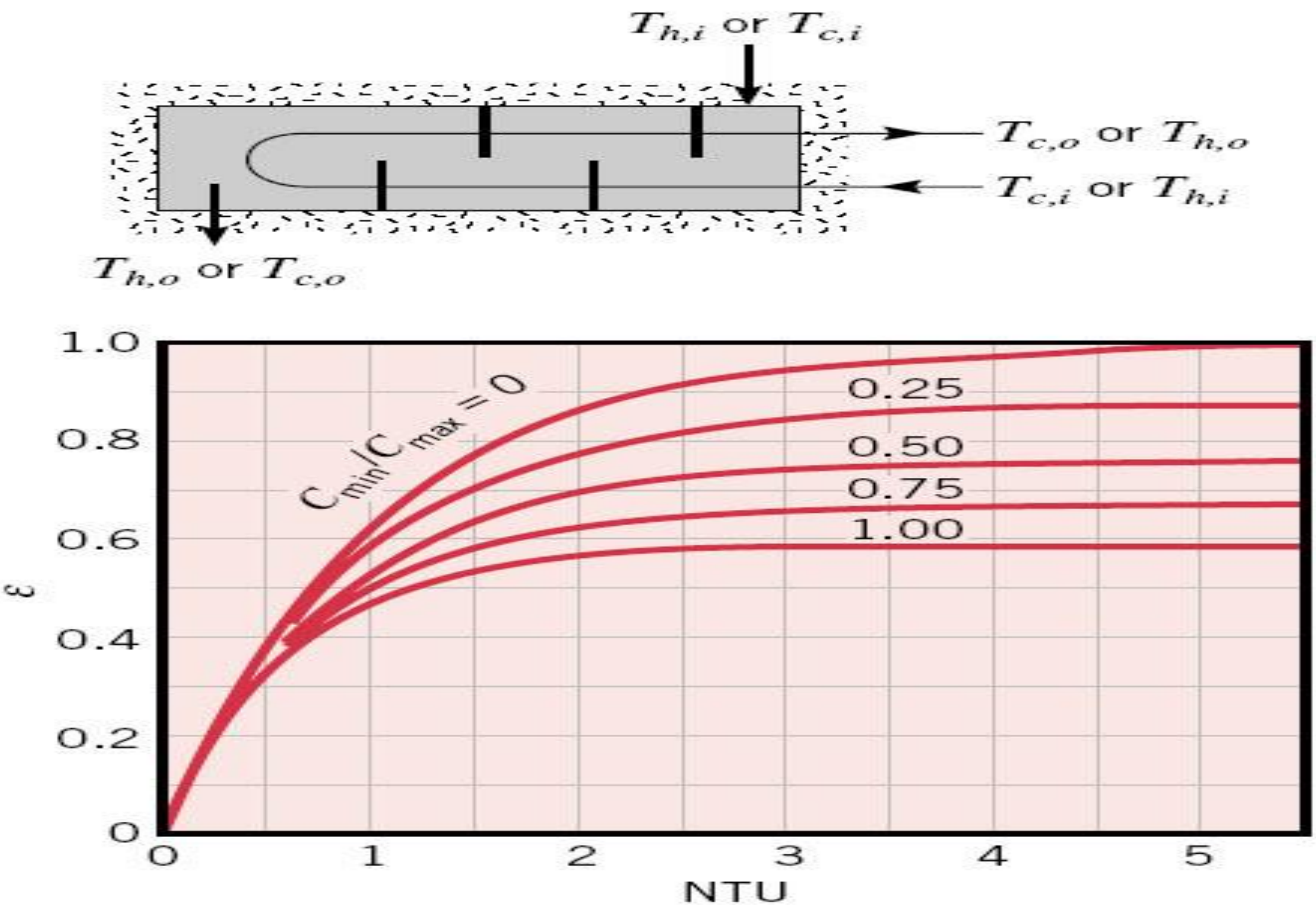


FIGURE 11.12 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes) (Equation 11.30).

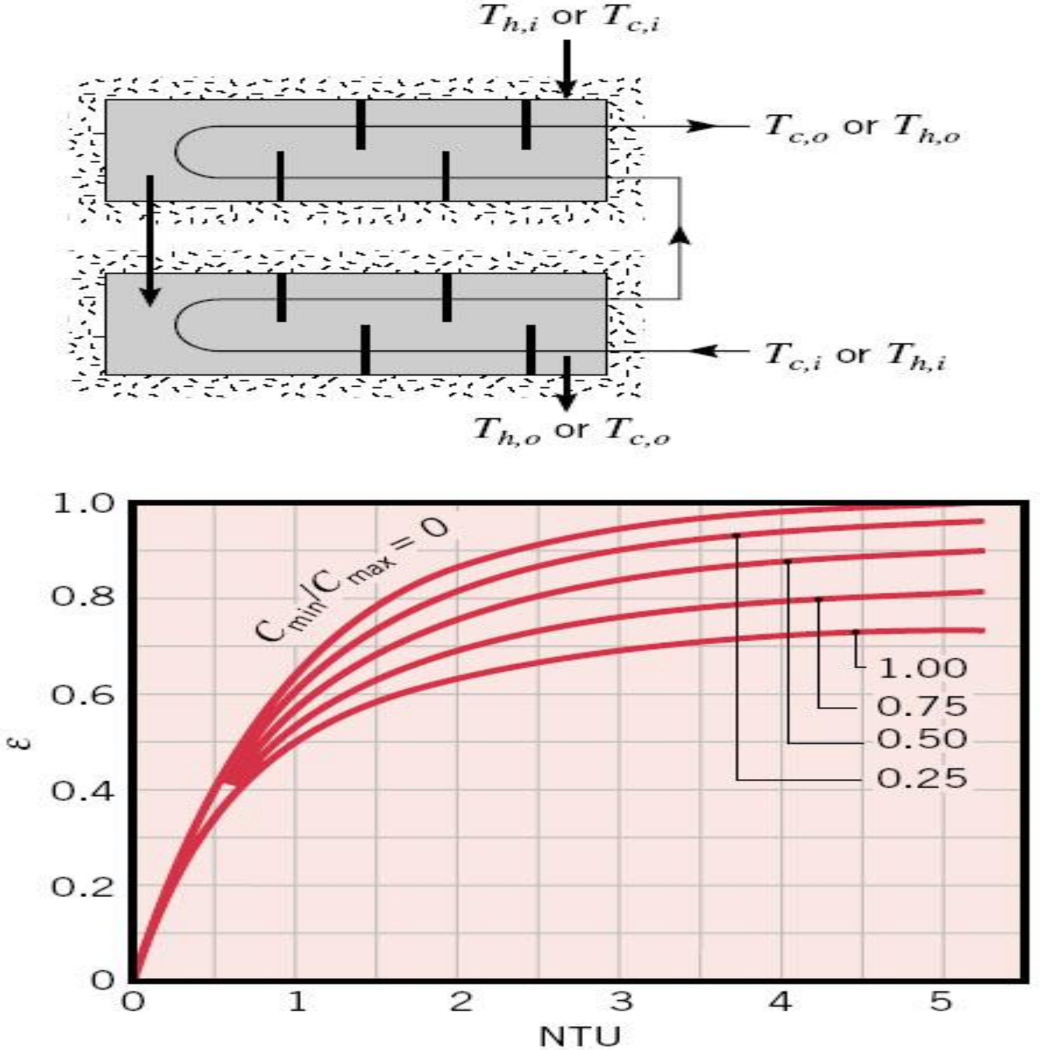
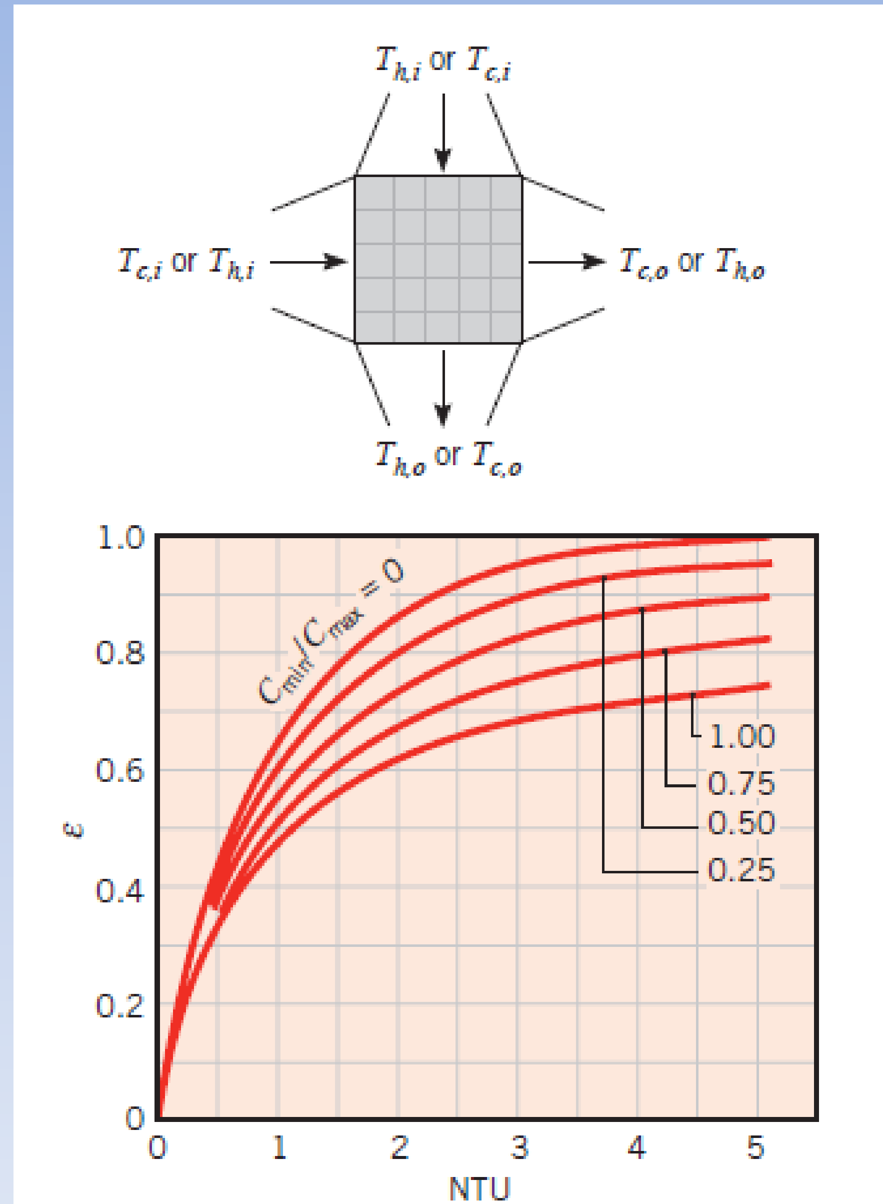
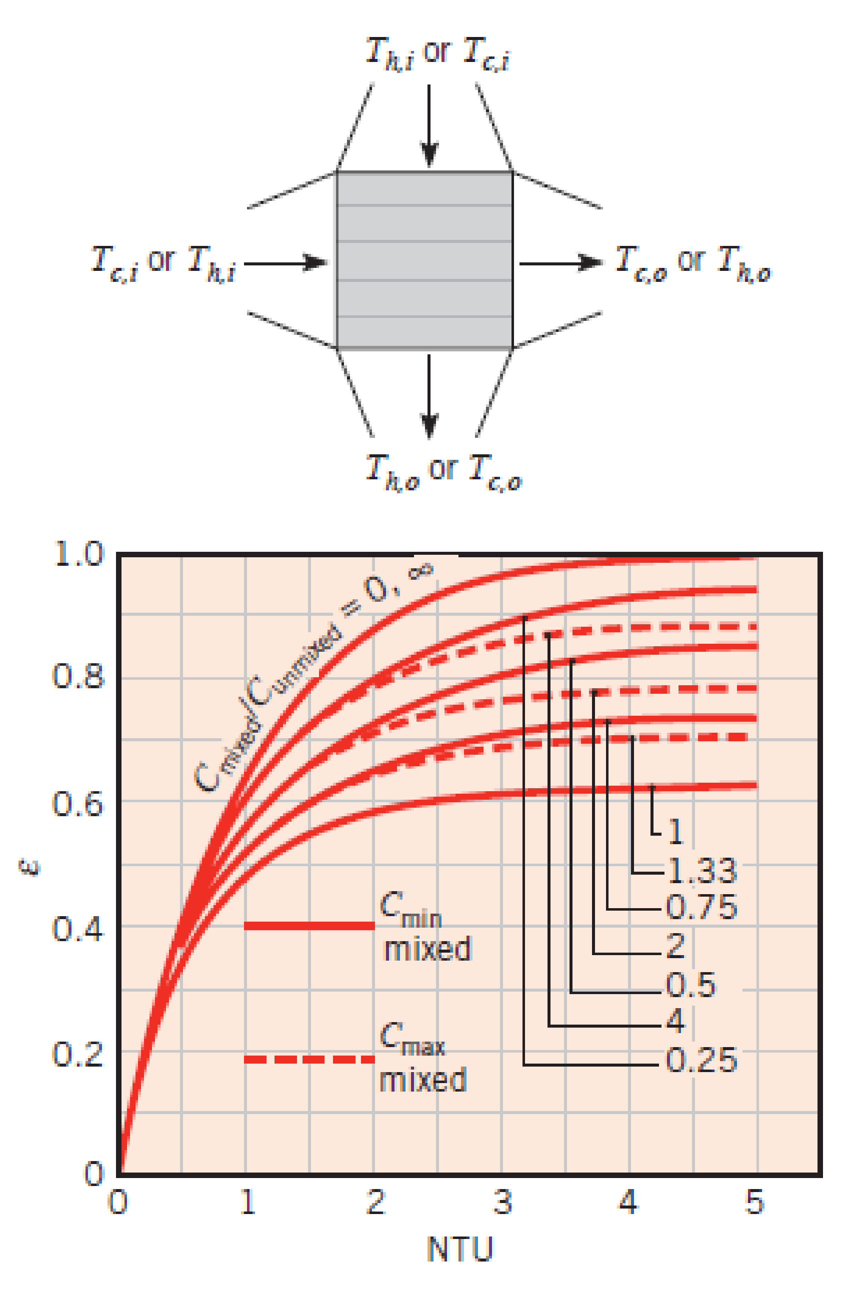


Figure 11.13 Effectiveness of a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes) (Equation 11.31 with $n = 2$).

Effectiveness of a single pass, cross-flow heat exchanger with both fluids unmixed (Equation 11.32).



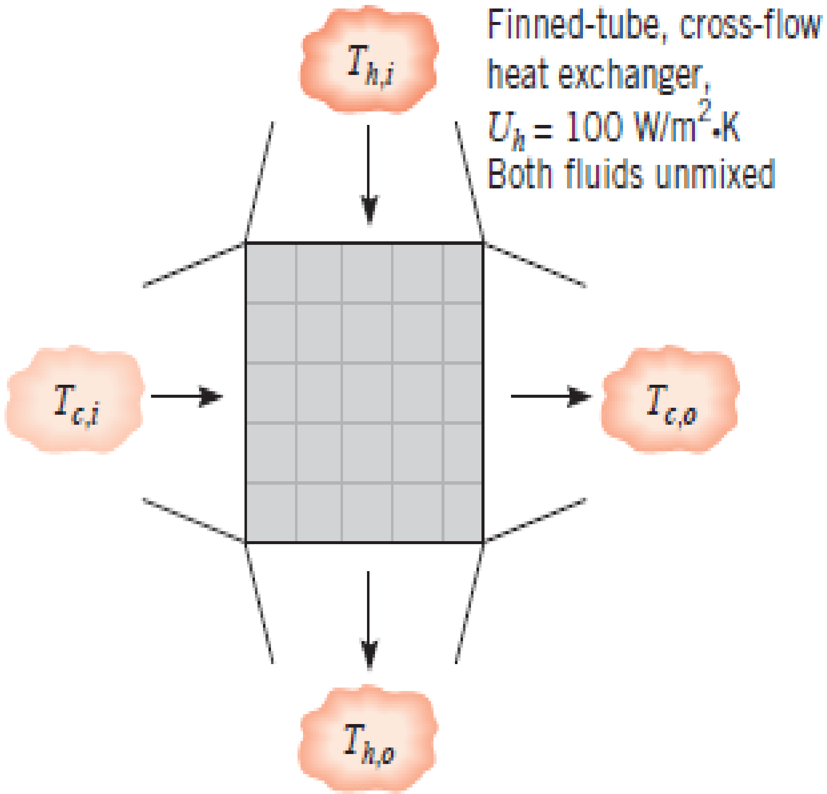
Effectiveness of a single pass, cross-flow heat exchanger with one fluid mixed and the other unmixed (Equations 11.33, 11.34).



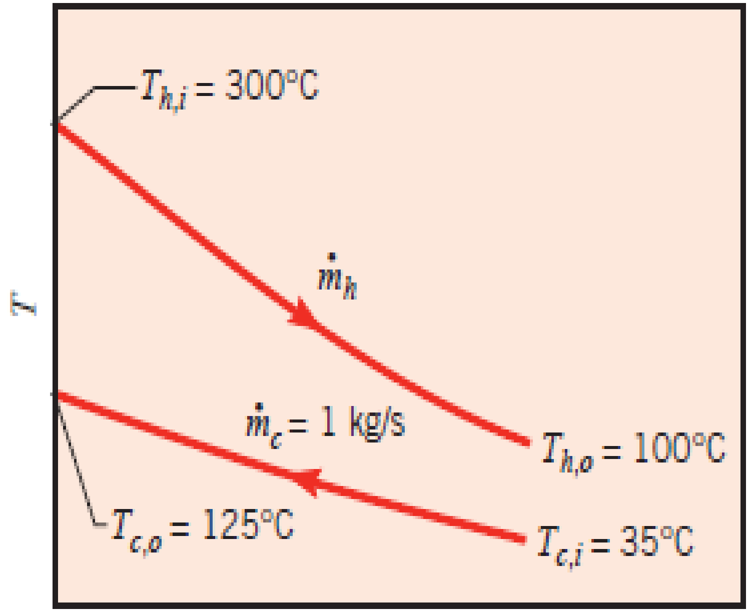
problem

Hot exhaust gases, which enter a finned-tube, cross-flow heat exchanger at 300°C and leave at 100°C, are used to heat pressurized water at a flow rate of 1 kg/s from 35 to 125°C. The overall heat transfer coefficient based on the gas-side surface area is $U_h = 100\ W/m^2 \cdot K$. Determine the required gas-side surface area A_h *using the NTU method.*

Schematic



Finned-tube, cross-flow heat exchanger,
U_h = 100 W/m²·K
Both fluids unmixed
$T_{h,i}$
$T_{c,i}$
$T_{c,o}$
$T_{h,o}$



$T_{h,i}$ = 300°C
$\dot{m}_h$
$\dot{m}_c$ = 1 kg/s
$T_{h,o}$ = 100°C
$T_{c,o}$ = 125°C
$T_{c,i}$ = 35°C
T

Try to solve the problem