Radiation

Thermal Radiation

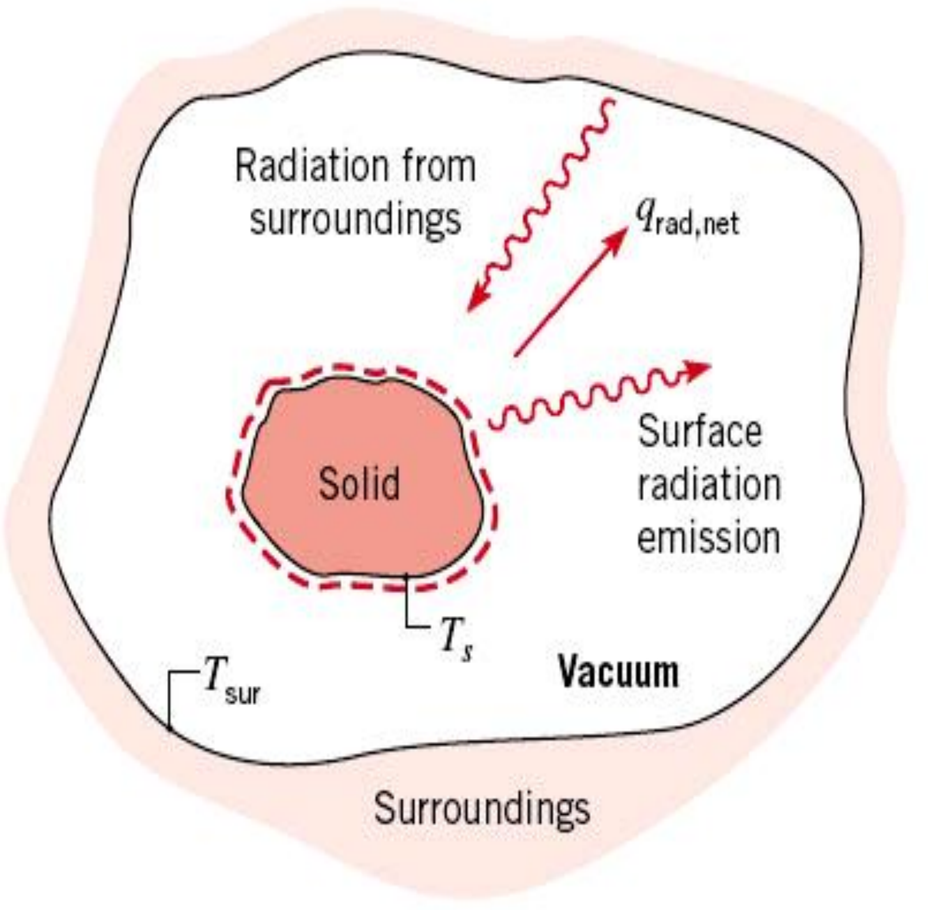
Radiation Exchange between Surfaces

Radiation Fundamentals

- Radiation Properties
- Radiation concepts and laws
- Blackbody surface
- Blackbody model
- Plank distribution
- Wien’s Displacement Law
- Spectral Emissive power
- Stefan-Boltzman law
- Kirchhoff’s Law
- Types of radiative Bodies and Gray surface model
- Black Body Radiation Function
- Radiation Intensity & Hemispherical Emission model

All objects with a temperature above absolute zero radiate

Radiation propagates in vacuum



Radiation cooling of a heated solid.

Physical concepts

- **Mechanism of radiation**
  1) Electromagnetic waves ~ ‘solar radiation, X-ray, radio waves’
  2) Photons ‘a discrete packets of energy’.

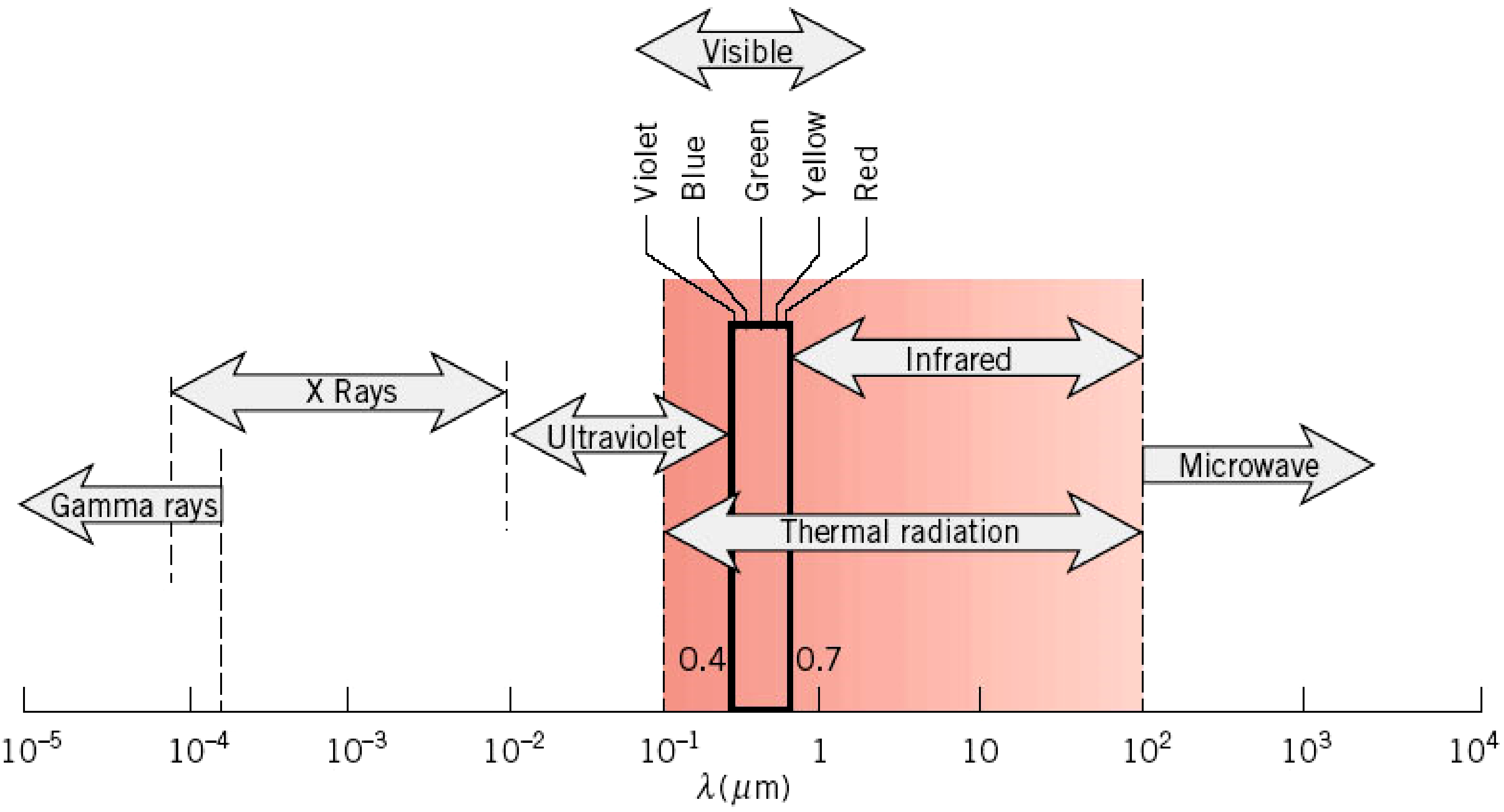
- **The wave nature of thermal radiation** implies that the wavelength, λ, should be associated with the frequency of radiation, *ν*. The relation between λ and *ν* is given by

$$\lambda = c / \nu$$

Where c is speed of propagation in the medium. In case of vacuum the speed of propagation equals the speed of light, 3×10^8 m/s.

- **<u>Thermal radiation is the radiation due to the temperature of body.</u>**

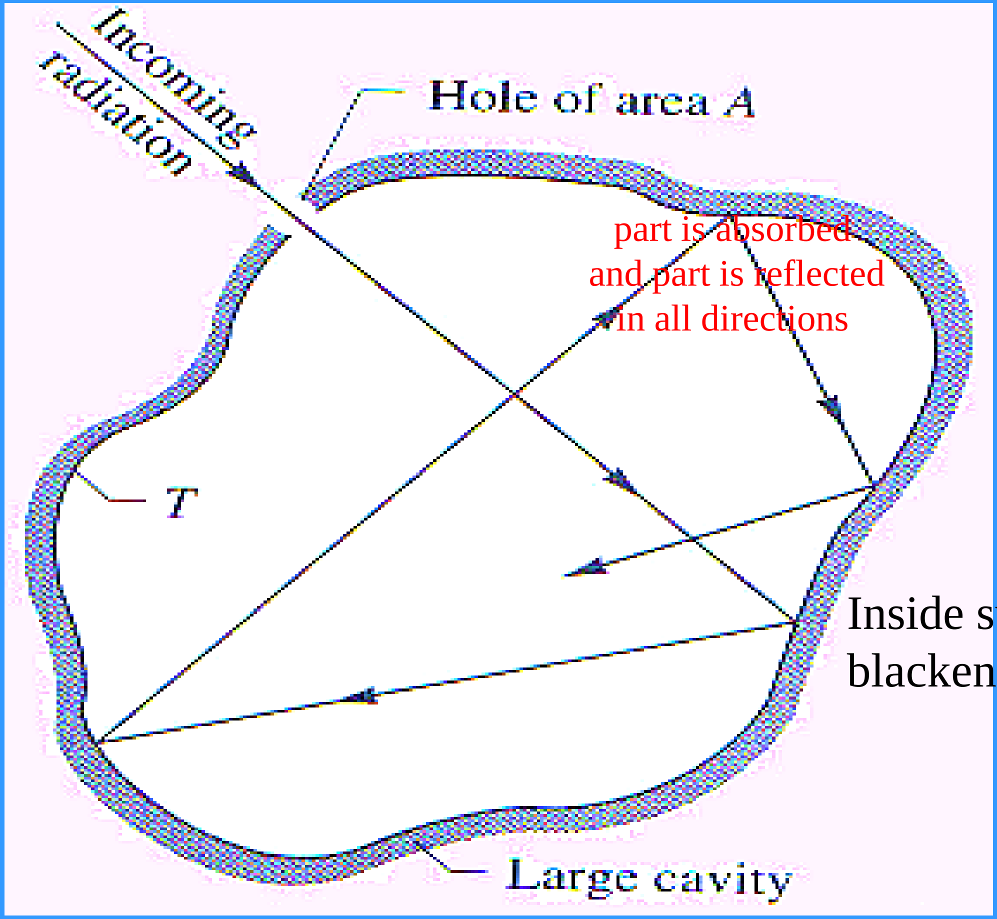
Spectrum of electromagnetic radiation



Characteristics of a blackbody

1. For a given surface temperature and wavelength, no surface can emit more radiation than a blackbody: i.e., a blackbody is a perfect emitter.
2. A blackbody absorbs all incident radiation regardless of its direction and wavelength: i.e., a blackbody is a perfect absorber.
3. A blackbody emits radiation in all directions: i.e., it is a diffuse emitter.

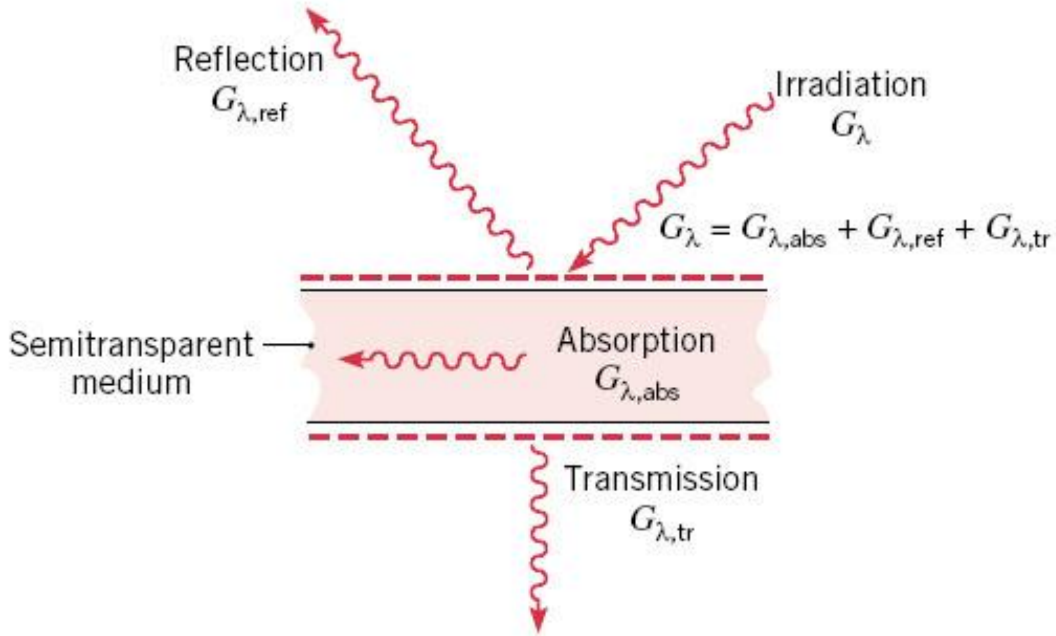
Cavity model of a blackbody



Inside surface area is blackened by charcoal

Note: The process continues until all of the energy entering is absorbed and the area of the hole acts as a perfect black body.

Absorption, reflection, and Transmission by Real surface



Absorption, reflection, and transmission processes associated with a semitransparent medium.

Blackbody Emissive Power

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 \{\exp[C_2/(\lambda T)] - 1\}} \quad \frac{W}{m^2 \cdot \mu m}$$

or $W_{b\lambda}$

where

$C_1 = 2\pi h c^2 = 3.743 \times 10^8 \ W \mu m^4/m^2$

$C_2 = 1.4387 \times 10^4 \ \mu m \cdot K$

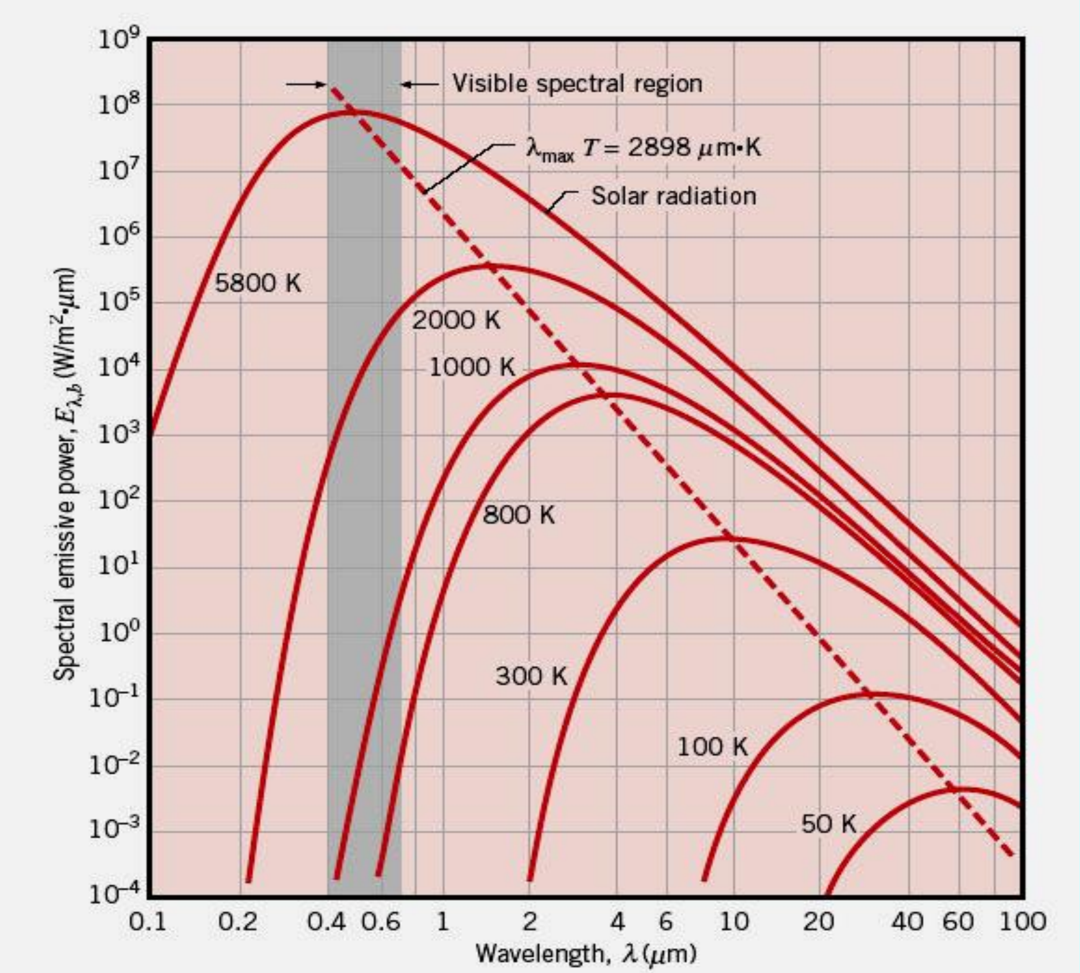
T = absolute temp. K

c = Speed of propagation, m/s

h = Plank's cons

λ = wavelength, μm

Spectral blackbody emissive power



Wien's Displacement Law

$$(\lambda T)_{max} = 2897.6\ \mu m \cdot K$$

The Stefan-boltzmann Law

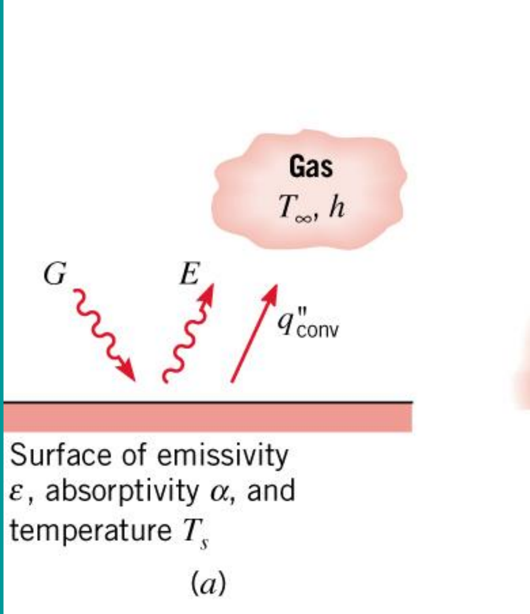
$$E_b(T) = \int_{\lambda=0}^{\lambda=\infty} E_{b\lambda}(T)\, d\lambda$$

$$= \sigma T^4$$

$E_b(T) \equiv$ blackbody emissive power

$\sigma \equiv 5.67 \times 10^{-8}\ W/m^2 K^4$

Heat Transfer Rates: Radiation



Heat transfer at a gas/surface interface involves radiation emission from the surface and may also involve the absorption of radiation incident from the surroundings (irradiation, G), as well as convection (if $T_s \neq T_\infty$).

Energy outflow due to emission:

$$E = \varepsilon E_b = \varepsilon \sigma T_s^4$$

E : Emissive power $\left(\mathrm{W/m^2}\right)$

ε : Surface emissivity $\left(0 \le \varepsilon \le 1\right)$

E_b : Emissive power of a blackbody (the perfect emitter)

σ : Stefan-Boltzmann constant $\left(5.67\times10^{-8}\,\mathrm{W/m^2 \cdot K^4}\right)$

Energy absorption due to irradiation:

$$G_{abs} = \alpha G$$

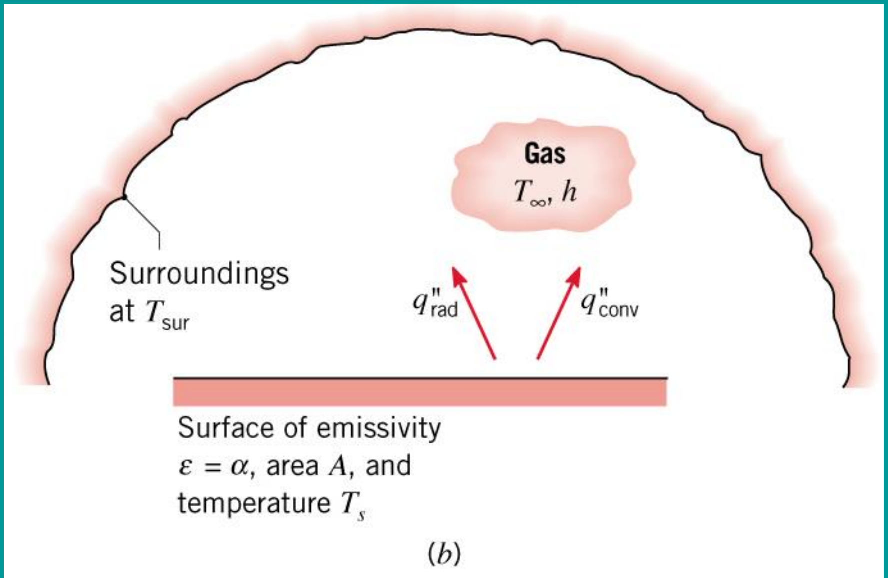
G_{abs} : Absorbed incident radiation $(\mathrm{W/m^2})$

α : Surface absorptivity $\left(0 \le \alpha \le 1\right)$

G : Irradiation $\left(\mathrm{W/m^2}\right)$

Heat Transfer Rates

Irradiation: **Special case of surface exposed to large surroundings of uniform temperature,** T_{sur}



$$G = G_{sur} = \sigma T_{sur}^4$$

If $\alpha = \varepsilon$, the net radiation heat flux from the surface due to exchange with the surroundings is:

$$q''_{rad} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) \qquad \text{.......(A)}$$

Heat Transfer Rates

Alternatively,

$$q''_{rad} = \mathrm{h}_r \left(T_s - T_{sur}\right) \qquad \text{.....................(B)}$$

h_r: Radiation heat transfer coefficient $\left(\mathrm{W/m^2 \cdot K}\right)$

$$\mathrm{h}_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) \qquad \text{.....................(C)}$$

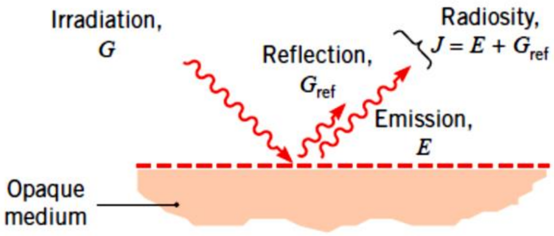
For combined convection and radiation,

$$q'' = q''_{conv} + q''_{rad} = h\left(T_s - T_\infty\right) + h_r\left(T_s - T_{sur}\right)$$

.....................(D)

Types of Radiative fluxes (over all wavelengths and in all directions)

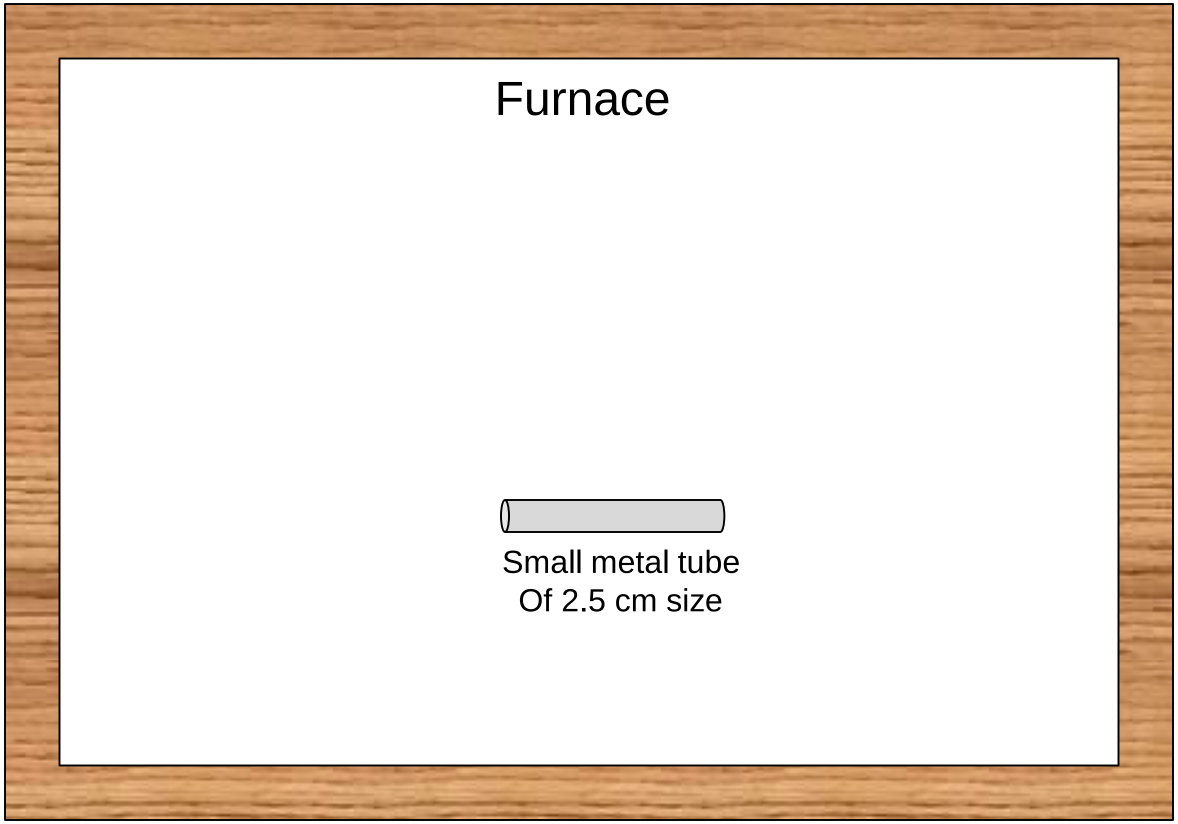
Flux (W/m^2)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon \sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$ See fig below
Net radiative flux, $q''_{rad} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{rad} = \varepsilon \sigma T_s^4 - \alpha G$



Example 1

A small oxidized metal tube of diam = 2.5 cm and long = 60 cm, Ts = 500 K is in a very large furnace enclosure with fire-brick walls and surrounding air at 1100 K. Assume oxidized metal tube emissivity = 0.75. Calculate the heat transfer to the metal tube.

Schematic



Example 2

Recalculate Example 1 for combined radiation plus natural convection to the

Note: ***Simplified Equations for Natural Convection from Various Surfaces***

Physical Geometry	$N_{Gr}N_{Pr}$	Equation: $h = btu/h \cdot ft^2 \cdot °F$, $L = ft$, $\Delta T = °F$, $D = ft$	Equation: $h = W/m^2 \cdot K$, $L = m$, $\Delta T = K$, $D = m$
Air at 101.32 kPa (1 atm) abs pressure			
Vertical planes and cylinders	10^4–10^9	$h = 0.28(\Delta T/L)^{1/4}$	$h = 1.37(\Delta T/L)^{1/4}$
	$>10^9$	$h = 0.18(\Delta T)^{1/3}$	$h = 1.24\ \Delta T^{1/3}$
Horizontal cylinders	10^3–10^9	$h = 0.27(\Delta T/D)^{1/4}$	$h = 1.32(\Delta T/D)^{1/4}$
	$>10^9$	$h = 0.18(\Delta T)^{1/3}$	$h = 1.24\ \Delta T^{1/3}$
Horizontal plates			
Heated plate facing upward or cooled plate facing downward	10^5–2×10^7	$h = 0.27(\Delta T/L)^{1/4}$	$h = 1.32(\Delta T/L)^{1/4}$
	2×10^7–3×10^{10}	$h = 0.22(\Delta T)^{1/3}$	$h = 1.52\ \Delta T^{1/3}$
Heated plate facing downward or cooled plate facing upward	3×10^5–3×10^{10}	$h = 0.12(\Delta T/L)^{1/4}$	$h = 0.59(\Delta T/L)^{1/4}$
Water at 70°F (294 K)			
Vertical planes and cylinders	10^4–10^9	$h = 26(\Delta T/L)^{1/4}$	$h = 127(\Delta T/L)^{1/4}$
Organic liquids at 70°F (294 K)			
Vertical planes and cylinders	10^4–10^9	$h = 12(\Delta T/L)^{1/4}$	$h = 59(\Delta T/L)^{1/4}$

Blackbody Radiation Function

$$E_{b,0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(T)\, d\lambda$$

($0-\lambda$ ↑ band)

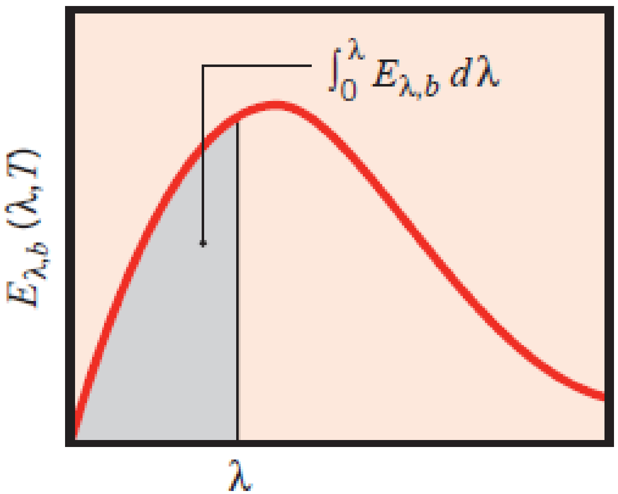
$$f_{0\lambda}(T) = \frac{\int_0^{\lambda} E_{b\lambda}(T)\, d\lambda}{\int_0^{\infty} E_{b\lambda}(T)\, d\lambda} = \frac{\int_0^{\lambda} E_{b\lambda}(T)\, d\lambda}{\sigma T^4}$$ ////

$f_{0\lambda}(T) \equiv$ blackbody Radiation function
$\equiv$ given in tables

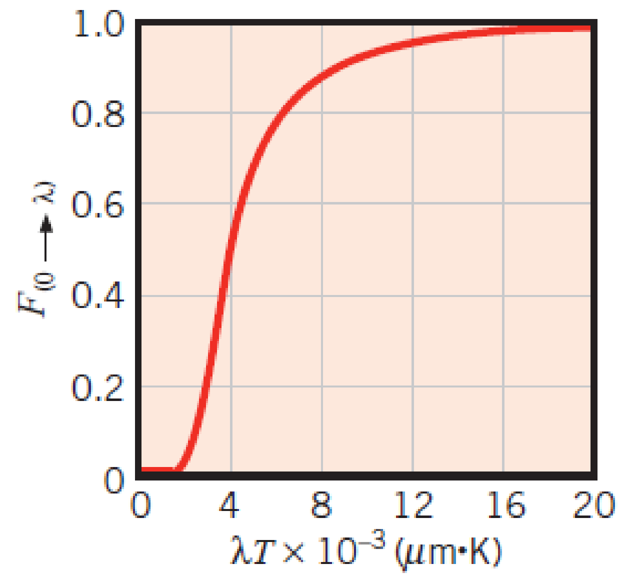
Over a finite-wavelength band $\lambda_1 - \lambda_2$
i.e

$$f_{\lambda_1-\lambda_2}(T) = f_{0-\lambda_2}(T) - f_{0-\lambda_1}(T)$$ /////

Blackbody Radiation Function



Radiation emission from a blackbody in the spectral band 0 to λ .



Fraction of the total blackbody emission in the spectral band from 0 to λ as a function of λ*T*.

Blackbody Radiation Function

λT $(\mu m \cdot K)$	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000

λT $(\mu m \cdot K)$	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801

Blackbody Radiation Function

Table *Continued*

λT $(\mu m \cdot K)$	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

Example 3

A light bulb radiates like a black body at 2800 K. find the percentage of total emitted energy that lies in the visible range.

RADIATION PROPERTIES OF SURFACES

- Absorption and emission of radiation depend on :
  - a. Type of material
  - b. Temp. or wavelength

Type of MATERIALS

* metals, wood stone ...	* glass, water
* Opaque to thermal radiation	* Semitrasparent to radiation
* Emission or absorption within a very short distance	* penetrate into the depths

Note

Most engineering materials are opaque to thermal radiation.

Emissivity, 'ϵ'

$$\epsilon = \frac{\text{Energy emitted by a real surface}}{\text{" \quad " \quad " a blackbody at the same temp.}}$$

$\epsilon = f($T, Wavelengths λ, Direction$)$

Hemispherical emissivity $\epsilon(T)$

Definition:

$$\epsilon(T) = \frac{\text{Rad. flux by real body over all wavelength}}{\text{Energy emitted by blackbody at same } T}$$

"in to a hemispherical" space

$$= \frac{q_r(T)}{E_b(T)}$$

OR

$$q_r(T) = \epsilon(T)\, E_b(T) = \epsilon(T)\, \sigma\, T^4 \ \text{W/m}^2$$

Note (1) Oxidation ↑ $\epsilon(T)$

Note (2) Spectral hemispherical emissivity ϵ_λ

$\equiv$ emissivity at a specific wavelength λ at certain temp.

* determined experimentally
* Can be used to obtain the average value of ϵ like this

$$\epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{b\lambda}(T)\, d\lambda}{\int_0^\infty E_{b\lambda}(T)\, d\lambda} = \frac{\int_0^\infty \epsilon_\lambda E_{b\lambda}\, d\lambda}{\sigma T^4}$$

look $\int_0^\infty \epsilon_\lambda E_{b\lambda}(T)\, d\lambda$ can be treated as band of cons. "ϵ"

Example

From 0 to λ_1: $\epsilon_\lambda = \epsilon_1 = $ Cons.
From λ_1 to λ_2: $\epsilon_\lambda = \epsilon_2 = $ Cons.
From λ_2 to ∞: $\epsilon_\lambda = \epsilon_3 = $ Cons.

$$\epsilon = \epsilon_1 \frac{\int_0^{\lambda_1} E_{b\lambda}(T)\, d\lambda}{\sigma T^4} + \epsilon_2 \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{\sigma T^4} + \epsilon_3 \frac{\int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{\sigma T^4}$$

$$= \epsilon_1 f_{0-\lambda_1}(T) + \epsilon_2 \left[f_{0-\lambda_2}(T) - f_{0-\lambda_1}(T) \right] + \epsilon_3 \left[1 - f_{0-\lambda_2}(T) \right]$$

$\therefore$ Knowing ϵ_1, ϵ_2 and ϵ_3, we can obtain ϵ from the previous Eq. by using the table of black body radiation functions "$f_{0\lambda}(T)$"

Emissivity values for various substances

Surface	*t*, °F*	Emissivity*
Aluminum		
Highly polished plate, 98.3 % pure	440–1070	0.039–0.057
Polished plate	73	0.040
Rough plate	78	0.055
Oxidized at 1110°F	390–1110	0.11–0.19
Al-surfaced roofing	100	0.216
Calorized surfaces, heated at 1110°F		
Copper	390–1110	0.18–0.19
Steel	390–1110	0.52–0.57
Brass		
Highly polished		
73.2 % Cu, 26.7 % Zn	476–674	0.028–0.031
62.4 % Cu, 36.8 % Zn, 0.4 % Pb, 0.3 % Al	494–710	0.033–0.037
82.9 % Cu, 17.0 % Zn	530	0.030
Hard rolled, polished, but direction of polishing visible	70	0.038
but somewhat attacked	73	0.043
but traces of stearin from polish left on	75	0.053
Polished	100–600	0.096–0.096
Rolled plate, natural surface	72	0.06
Rubbed with coarse emery	72	0.20
Dull plate	120–660	0.22
Oxidized by heating at 1110°F	390–1110	0.61–0.59
Chromium (see Nickel alloys for Ni-Cr steels)	100–1000	0.08–0.26
Copper		
Carefully polished electrolytic copper	176	0.018
Commercial emeried, polished, but pits remaining	66	0.030
Commercial, scraped shiny but not mirrorlike	72	0.072
Polished	242	0.023
Plate, heated long time, covered with thick oxide layer	77	0.78
Plate heated at 1110°F	390–1110	0.57–0.57
Cuprous oxide	1470–2010	0.66–0.54
Molten copper	1970–2330	0.16–0.13
Gold		
Pure, highly polished	440–1160	0.018–0.035
Iron and steel		
Metallic surfaces (or very thin oxide layer)		
Electrolytic iron, highly polished	350–440	0.052–0.064
Polished iron	800–1880	0.144–0.377
Iron freshly emeried	68	0.242
Cast iron, polished	392	0.21
Wrought iron, highly polished	100–480	0.28
Cast iron, newly turned	72	0.435
Polished steel casting	1420–1900	0.52–0.56
Ground sheet steel	1720–2010	0.55–0.61
Smooth sheet iron	1650–1900	0.55–0.60
Cast iron, turned on lathe	1620–1810	0.60–0.70

Emissivity values for various substances (continue)

Surface	t, °F*	Emissivity*
Iron and steel—(*Continued*)		
Oxidized surfaces		
Iron plate, pickled, then rusted red	68	0.612
Completely rusted	67	0.685
Rolled sheet steel	70	0.657
Oxidized iron	212	0.736
Cast iron, oxidized at 1100°F	390–1110	0.64–0.78
Steel, oxidized at 1100°F	390–1110	0.79–0.79
Smooth oxidized electrolytic iron	260–980	0.78–0.82
Iron oxide	930–2190	0.85–0.89
Rough ingot iron	1700–2040	0.87–0.95
Sheet steel, strong, rough oxide layer	75	0.80
Dense, shiny oxide layer	75	0.82
Cast plate, smooth	73	0.80
Rough	73	0.82
Cast iron, rough, strongly oxidized	100–480	0.95
Wrought iron, dull oxidized	70–680	0.94
Steel plate, rough	100–700	0.94–0.97
High-temperature alloy steels (see Nickel alloys)		
Molten metal		
Cast iron	2370–2550	0.29–0.29
Mild steel	2910–3270	0.28–0.28
Lead		
Pure (99.96%), unoxidized	260–440	0.057–0.075
Gray oxidized	75	0.281
Oxidized at 390°F	390	0.63
Mercury	32–212	0.09–0.12
Molybdenum filament	1340–4700	0.096–0.292
Monel metal, oxidized at 1110°F	390–1110	0.41–0.46
Nickel		
Electroplated on polished iron, then polished	74	0.045
Technically pure (98.9% Ni, + Mn), polished	440–710	0.07–0.087
Electroplated on pickled iron, not polished	68	0.11
Wire	368–1844	0.096–0.186
Plate, oxidized by heating at 1110°F	390–1110	0.37–0.48
Nickel oxide	1200–2290	0.59–0.86
Nickel alloys		
Chromnickel	125–1894	0.64–0.76
Nickelin (18–32 Ni; 55–68 Cu; 20 Zn), gray oxidized	70	0.262
KA-2S alloy steel (8% Ni; 18% Cr), light silvery, rough, brown, after heating	420–914	0.44–0.36
after 42 hr. heating at 980°F	420–980	0.62–0.73
NCT-3 alloy (20 Ni; 25 Cr). Brown, splotched, oxidized from service	420–980	0.90–0.97
NCT-6 alloy (60 Ni; 12 Cr). Smooth, black, firm adhesive oxide coat from service	520–1045	0.89–0.82
Platinum		
Pure, polished plate	440–1160	0.054–0.104
Strip	1700–2960	0.12–0.17
Filament	80–2240	0.036–0.192
Wire	440–2510	0.073–0.182

Emissivity values for refractories, building materials, paints and miscellaneous

	Temp, °F	Emissivity, ε
Asbestos		
Board	74	0.96
Paper	100–700	0.93–0.945
Brick		
Red, rough, but no gross irregularities	70	0.93
Silica, unglazed, rough	1832	0.80
Silica, glazed, rough	2012	0.85
Grog brick, glazed	2012	0.75
(See Refractory materials)		
Carbon		
T-carbon (Gebruder Siemens) 0.9 % ash	260–1160	0.81–0.79
This started with emissivity at 260°F of 0.72, but on heating changed to values given		
Carbon filament	1900–2560	0.526
Candle soot	206–520	0.952
Lampblack-water-glass coating	209–362	0.959–0.947
Same	260–440	0.957–0.952
Thin layer on iron plate	69	0.927
Thick coat	68	0.967
Lampblack, 0.003 in. or thicker	100–700	0.945
Enamel, white fused, on iron	66	0.897
Glass, smooth	72	0.937
Gypsum, 0.02 in. thick on smooth or blackened plate	70	0.903
Marble, light gray, polished	72	0.931
Oak, planed	70	0.895
Oil layers on polished nickel (lub. oil)	68	
Polished surface, alone	....	0.045
+0.001 in. oil	....	0.27
+0.002 in. oil	....	0.46
+0.005 in. oil	....	0.72
∞ thick oil layer	....	0.82

Emission of Radiation

* Monochromatic radiation ≡ radiation of a single wavelength.

* A beam of thermal radiation is not monochromatic but a wide or a band of wavelengths.

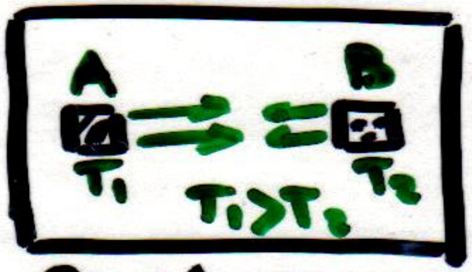
* At temp. above 500 °C ~ heat radiation in the visible spectrum becomes significant. 'Red heat' and 'white heat' are commonly used.

* **State of aggregation and Molecular Structure of the Substance affect radiation**

- Monoatomic and diatomic gases 'O_2, N_2, Ar' radiate weakly even at high temp.
- Polyatomic gases (H_2O, CO_2) emit and absorb radiation at several wavelengths.
- Solids & liquids absorb and emit radiation over the entire spectrum in thin Layers.

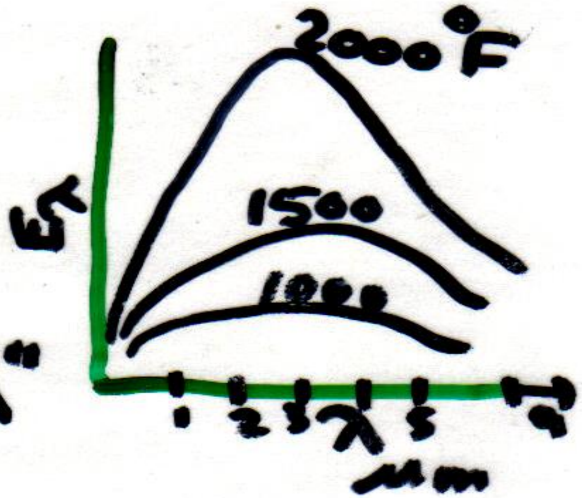
* Radiation travels in a straight line ; " beam " .

* Net energ transfer from hoter body A to cold body B .



enclosure

* Monochromatic energy emitted by an object



depens on: "Temp" + "λ"

$$E_T = \int_0^\infty E_\lambda \, d\lambda$$

Graphically , E, is the entire area under any curve .

* ## Types of Bodies:

1. Opaque body : no transmission of electromagnetic radiation is possible. $t=0$, $\alpha+r=1$
2. White body : any body whose Surface relects incident rays of radiation. $\alpha=0$, $r=1$
3. Black body : $\alpha=1$, $r=0$
4. Gray body : $0<\alpha<1$

Absorptivity

- Hemispherical absorptivity α
- Spectral hemispherical absorptivity α_λ

$$\alpha = f(T_{source}, \text{Material}, \lambda)$$

Example

	Source Temp, K	α
White Fireclay	300	0.9
" "	6 000	0.1 ↓
Aluminum	300	0.08
"	6 000	0.3 ↑ ⁈

Graybody Approximation

- In general, α or ϵ depends on the wavelength of radiation.
- It is not practical to consider heat transfer at each wavelength.
- To solve this problem assume a uniform emissivity ϵ over the entire wavelength spectrum. This is called the graybody approx. Commonly used in engineering applications.

look! Graybody $\neq f(\lambda)$

Relation between ϵ & α

- Suppose an evacuated a hollow sphere at cons. temp. and a body of A_1 inside it.

spherical enclosure

- After equil. $\Rightarrow$
  $T_{body} = T_{enclosure}$
- Let, I, the intensity of rad. falling upon the body A_1
- The fraction absorbed α_1 by the body
- Let E_1 is the total emissive power by the body.
- The energy emitted by the body A_1 = the energy received.

$$E_1 A_1 = I \alpha_1 A_1$$

$$\therefore E_1 = I \alpha_1 \quad \ldots\ldots (1)$$

- If the body is replaced by another (of identical shape and equil. is again attained

$$\therefore E_2 = I \alpha_2 \quad \ldots\ldots (2)$$

- Now, If a blackbody is placed instead of the later

$$\therefore E_b = I \quad \ldots\ldots (3)$$

from the previous eqs :

$\therefore \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = E_b$ at thermal equil.
This is Kirchhoff's Law.

$\therefore E_1 = \alpha_1 E_b$, $E_2 = \alpha_2 E_b$ ..... (4)

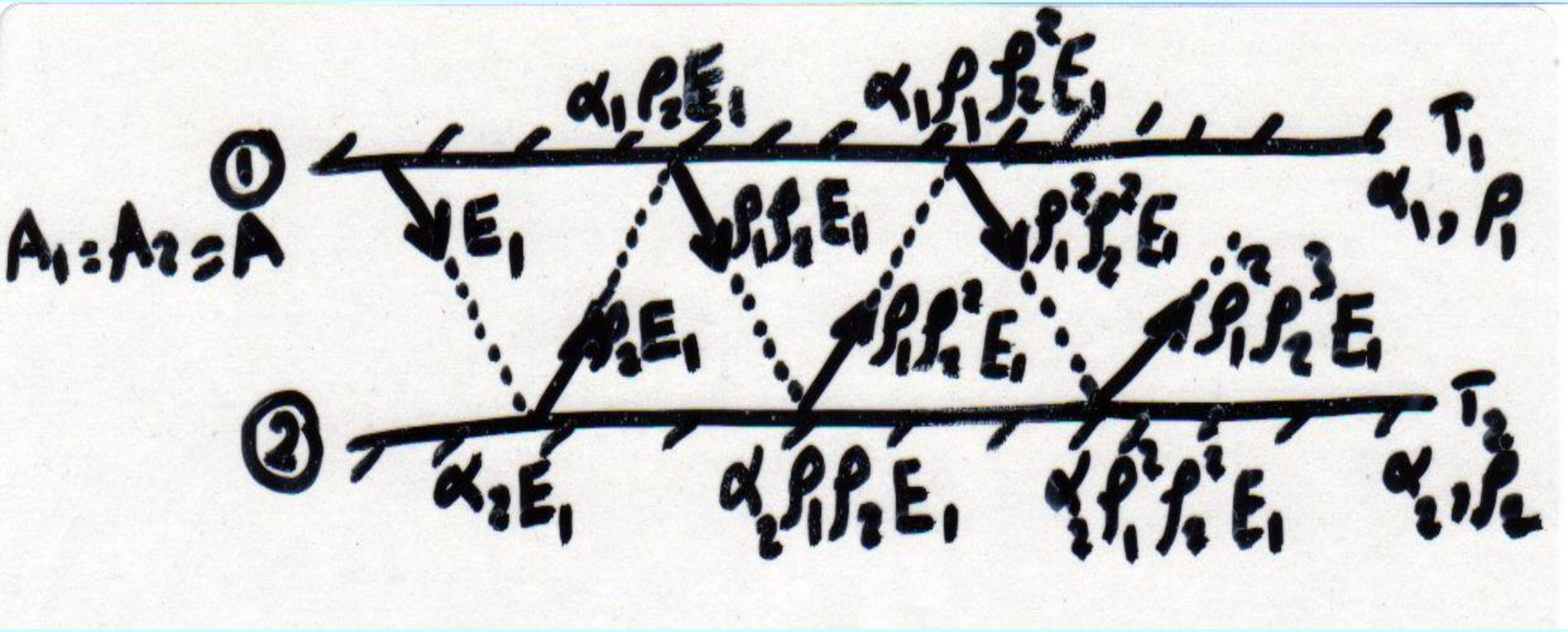
But from the definition of emissivity ϵ or

$$\epsilon_1 = \frac{E_1}{E_b} = \alpha_1$$

..... (5)

and $\epsilon_2 = \frac{E_2}{E_b} = \alpha_2$ at equil.

Application



The rate of energy from 1 to 2

$$\frac{q_{1-2}}{A} = \alpha_2 E_1 + \alpha_2 \rho_1 \rho_2 E_1 + \alpha_2 \rho_1^2 \rho_2^2 E_1 +$$

$$+ \alpha_2 \rho_1^n \rho_2^n E_1 + \cdots$$

Consider only the 1st two terms

$$\frac{q'_{1-2}}{A} = \alpha_2 E_1 (1 + \rho_1 \rho_2)$$

$$= \alpha_2 E_1 (1 + \rho_1 \rho_2) \frac{(1 - \rho_1 \rho_2)}{(1 - \rho_1 \rho_2)}$$

$$= \frac{\alpha_2 E_1 (1 - \rho_1^2 \rho_2^2)}{1 - \rho_1 \rho_2} \simeq \frac{\alpha_2 E_1}{1 - \rho_1 \rho_2} \quad //$$

likewise, the rate of energy transfer from 2 to 1

$$\frac{q'_{2-1}}{A} = \frac{\alpha_1 E_2}{1 - \rho_2 \rho_1} \qquad \text{III}$$

The net rate of energy exchange from 1-2

$$\therefore \ _{net}\frac{q_{1-2}}{A} = \frac{q'_{1-2}}{A} - \frac{q'_{2-1}}{A} = \frac{\alpha_2 E_1 - \alpha_1 E_2}{1 - \rho_2 \rho_1}$$

For Gray Surfaces with zero transmissivity ;

$$\rho = 1 - \alpha = 1 - \epsilon$$

and

$$E = \epsilon \sigma T^4$$

$$\therefore \text{Net } \frac{q_{1-2}}{A} = \frac{\epsilon_2 \epsilon_1 \sigma T_1^4 - \epsilon_1 \epsilon_2 \sigma T_2^4}{1 - (1 - \epsilon_1)(1 - \epsilon_2)}$$

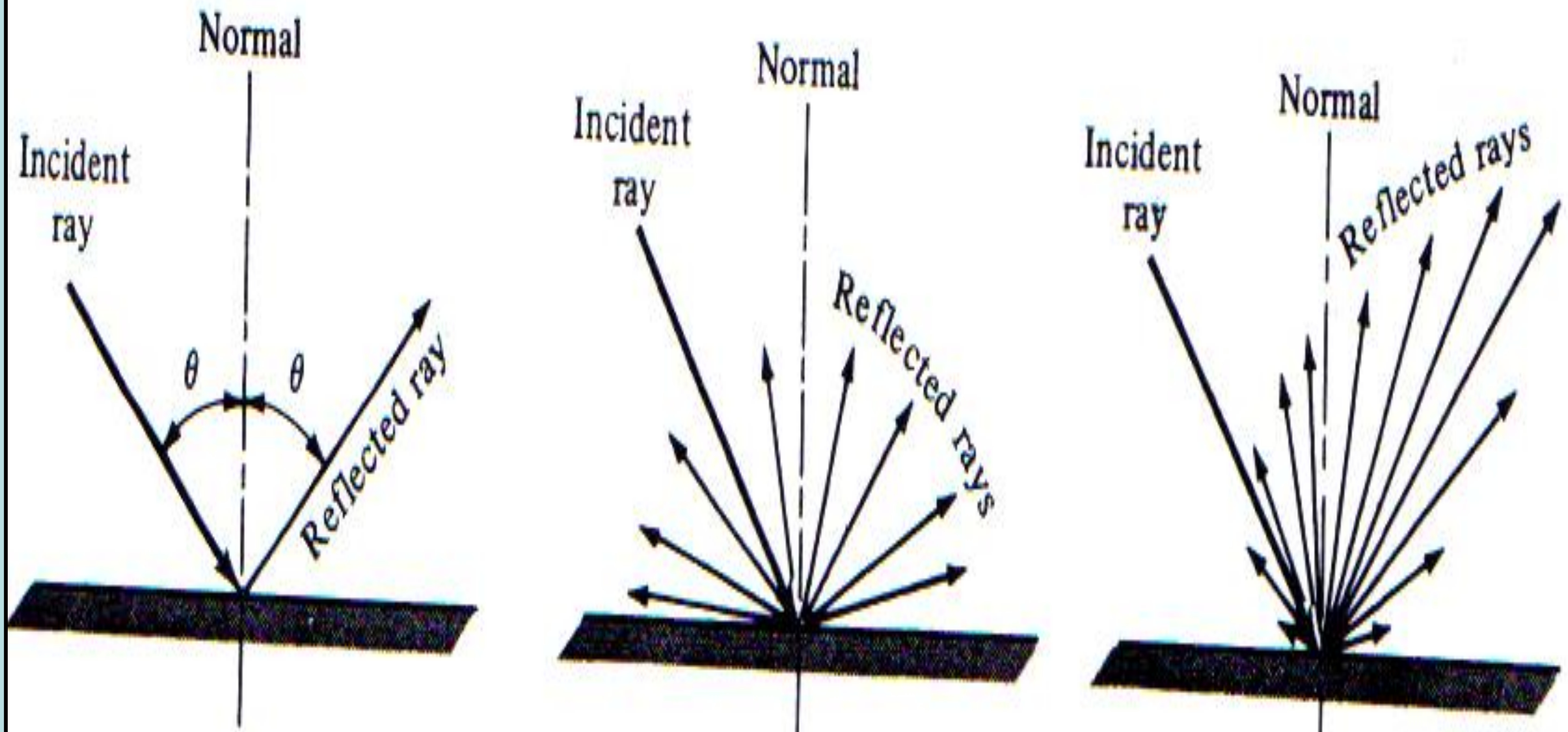
$$= \frac{\epsilon_2 \epsilon_1 [\sigma (T_1^4 - T_2^4)]}{1 - 1 + \epsilon_2 + \epsilon_1 - \epsilon_1 \epsilon_2}$$

$$= \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \quad ///$$

Radiation Intensity

Definitions & Expressions

Reflection from surfaces

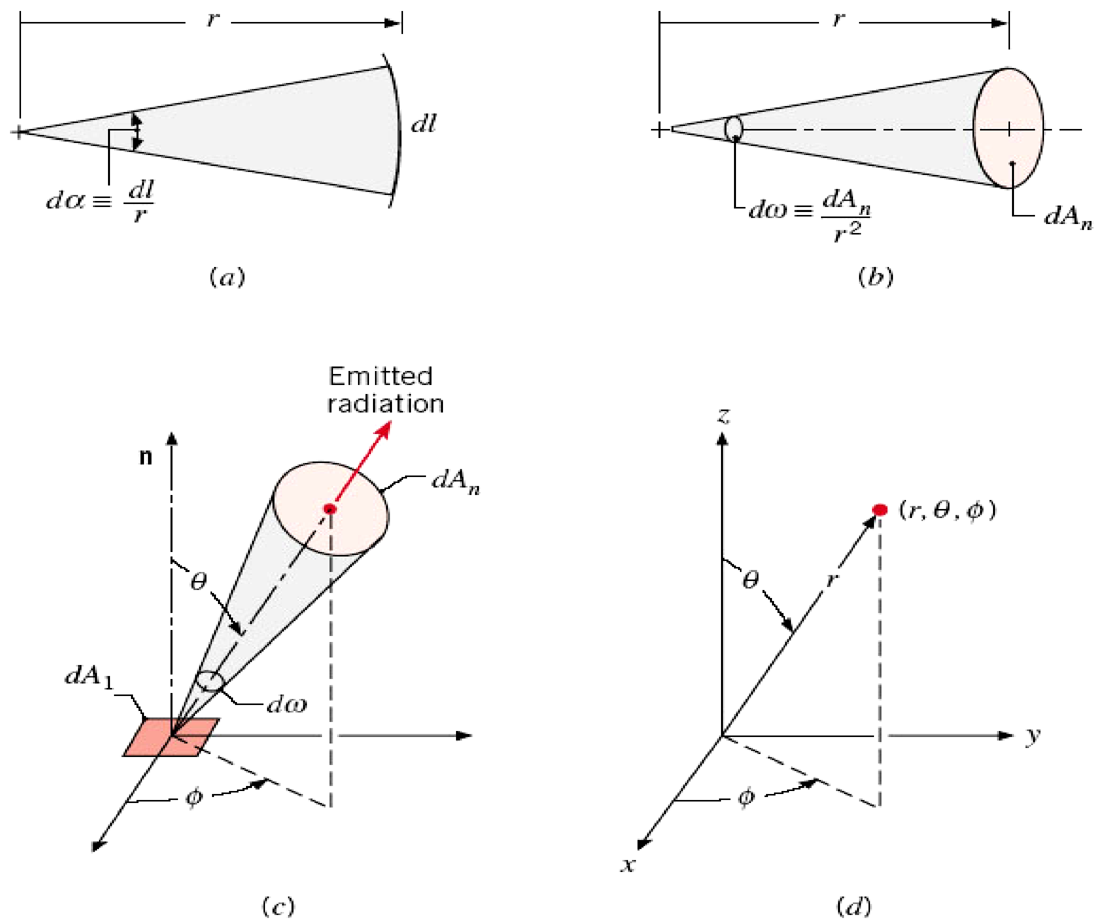


Specular reflection

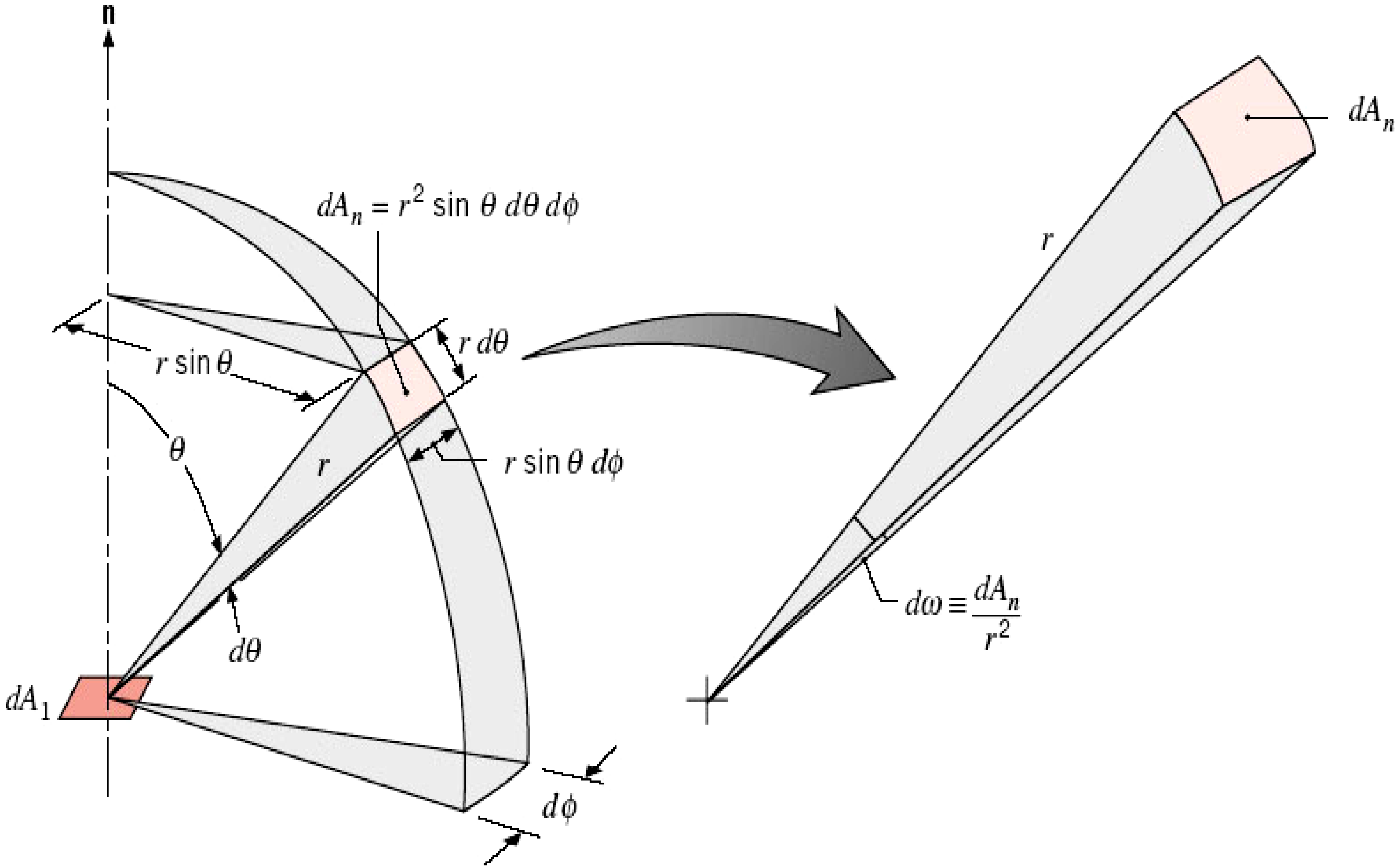
Diffuse reflection

Irregular reflection

Mathematical Definitions



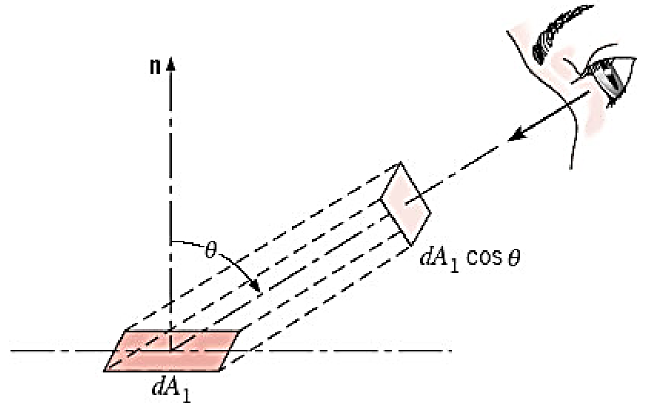
Mathematical definitions. (*a*) Plane angle. (*b*) Solid angle. (*c*) Emission of radiation from a differential area dA_1 into a solid angle $d\boldsymbol{\omega}$ subtended by dA_n at a point on dA_1. (*d*) The spherical coordinate system.



The solid angle subtended by dA_n at a point on dA_1 in the spherical coordinate system.

Radiation Intensity, I

$$I_{\lambda,e}(\lambda, \theta, \phi) \equiv \frac{dq}{dA_1 \cos\theta \cdot d\omega \cdot d\lambda}$$



The projection of dA_1 normal to the direction of radiation.

where $(dq/d\lambda)$ dq_λ is the rate at which radiation of wavelength λ leaves dA_1 and passes through dA_n

$$dq_\lambda = I_{\lambda,e}(\lambda, \theta, \phi) dA_1 \cos\theta \, d\omega$$

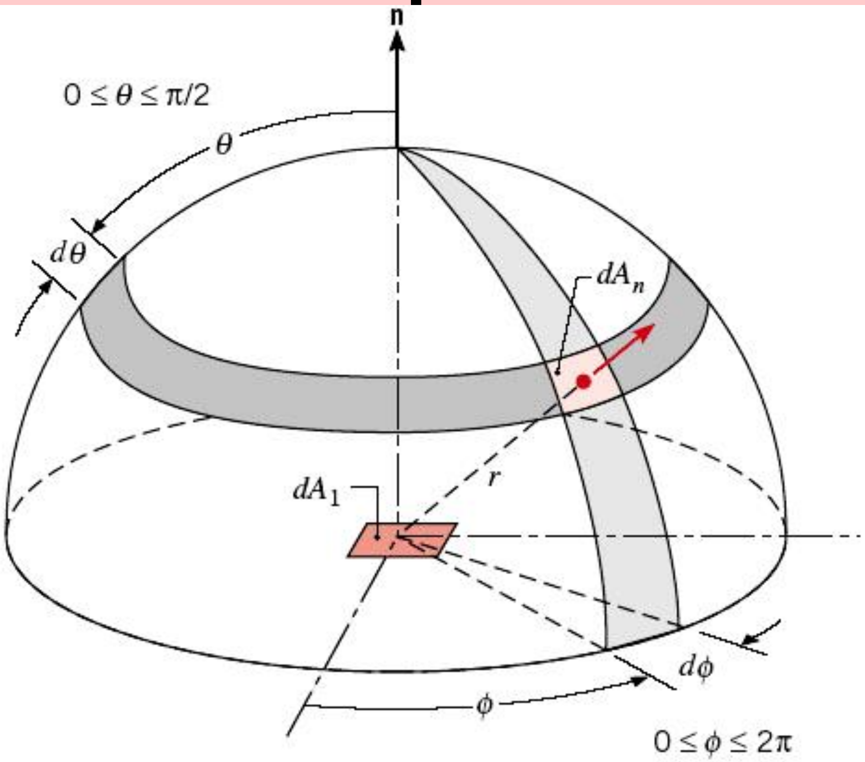
But solid angle is

$$d\omega = \sin\theta \, d\theta \, d\phi$$

Therefore, the Heat flux from dA_1 becomes

$$dq''_\lambda = I_{\lambda,e}(\lambda, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi$$

Hemispherical direction



Emission from a differential element of area dA_1 into a hypothetical hemisphere centered at a point on dA_1.

$$E_\lambda(\lambda) = q''_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

- The total, hemispherical emissive power, E (W/m^2) is the rate at which radiation is emitted per unit area at all possible wavelengths and in all possible directions:

$$E = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

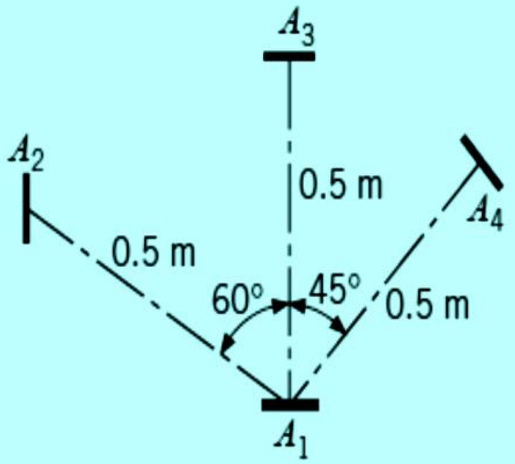
- Suppose a diffuse surface (emitted radiation is independent of direction), intensity, I, does not depend on direction ϴ and φ ~ $I_{\lambda,e}(\lambda, \theta, \varphi) = I_{\lambda,e}(\lambda)$. Hence, the previous eq. becomes

$$E = \pi I_e$$

- The constant in the above eq. is π and not 2π and has the unit steradians.

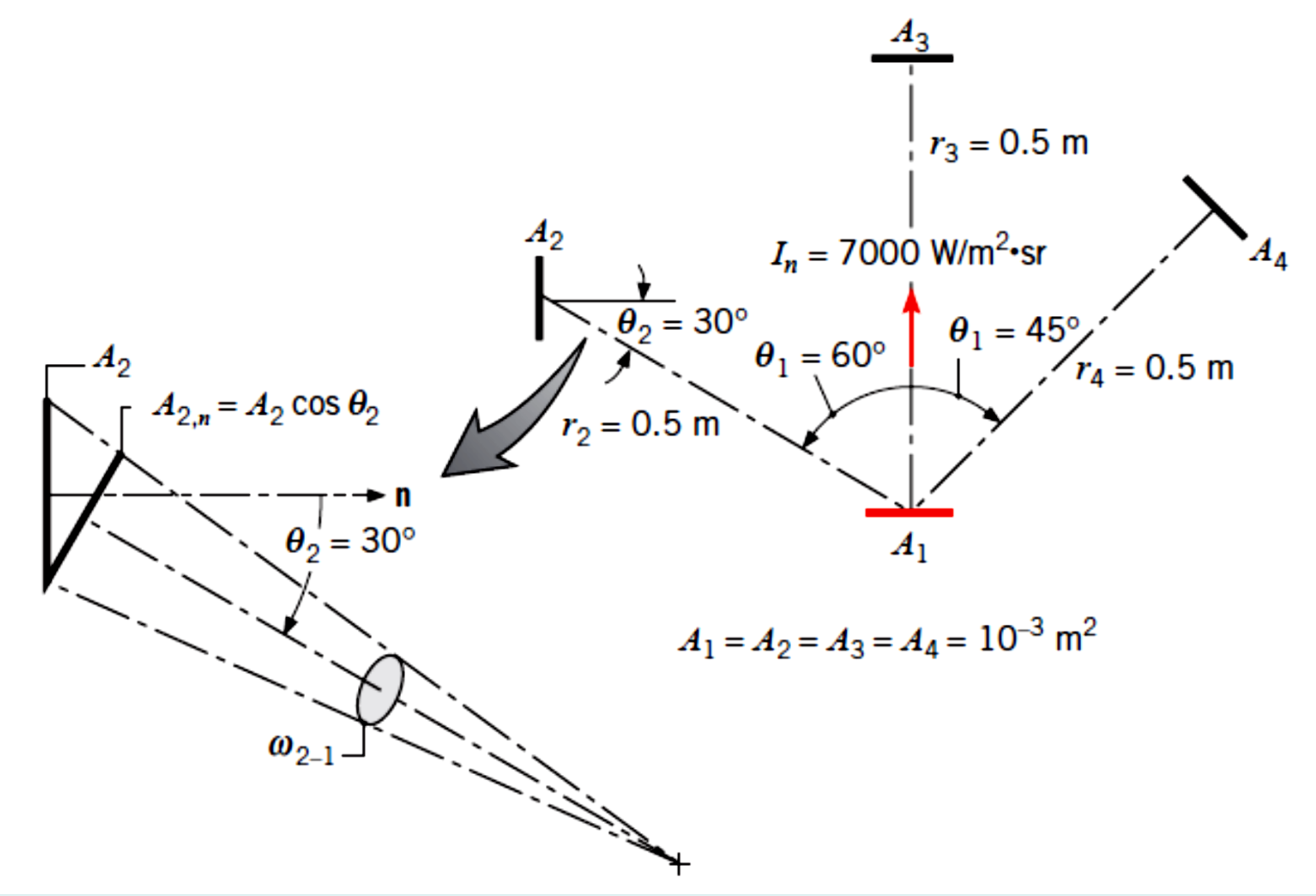
Example

A small surface of area $A_1 = 10^{-3}\ \text{m}^2$ is known to emit diffusely, and from measurements the total intensity associated with emission in the normal direction is $I_n = 7000\ \text{W/m}^2 \cdot \text{sr}$.



Radiation emitted from the surface is intercepted by three other surfaces of area $A_2 = A_3 = A_4 = 10^{-3}\ \text{m}^2$, which are 0.5 m from A_1 and are oriented as shown. What is the intensity associated with emission in each of the three directions? What are the solid angles subtended by the three surfaces when viewed from A_1? What is the rate at which radiation emitted by A_1 is intercepted by the three surfaces?

Schematic



Surface A_1 emits diffusely.

A_1, A_2, A_3, and A_4 may be approximated as differential surfaces, $(A_j/r_j^2) \ll 1$

Solution

- From the definition of a diffuse emitter, we know that the intensity of the emitted radiation is independent of direction. Hence

$$I = 7000\ \mathrm{W/m^2 \cdot sr}$$

for each of the three directions.

Continue solution

- Treating A_2, A_3, and A_4 as differential surface areas, the solid angles may be computed from Equation

$$d\omega \equiv \frac{dA_n}{r^2}$$

- where dA_n is the projection of the surface normal to the direction of the radiation. Since surfaces A_3 and A_4 are normal to the direction of radiation, the solid angles subtended by these surfaces can be directly found from this equation as

$$\omega_{3-1} = \omega_{4-1} = \frac{A_3}{r^2} = \frac{10^{-3}\ \text{m}^2}{(0.5\ \text{m})^2} = 4.00 \times 10^{-3}\ \text{sr}$$

Continue solution

Since surface A_2 is not normal to the direction of radiation, we use $dA_{n,2} = dA_2 \cos\theta_2$, where θ_2 is the angle between the surface normal and the direction of the radiation. Thus

$$\omega_{2-1} = \frac{A_2 \cos\theta_2}{r^2} = \frac{10^{-3}\,\text{m}^2 \times \cos 30°}{(0.5\,\text{m})^2} = 3.46 \times 10^{-3}\,\text{sr}$$

- Approximating A_1 as a differential surface, the rate at which radiation is intercepted by each of the three surfaces may be found from Intensity Equation,

$$dq_\lambda = I_{\lambda,e}(\lambda, \theta, \phi)\,dA_1 \cos\theta\, d\omega$$

which, for the total radiation, may be expressed as

$$q_{1-j} = I \times A_1 \cos\theta_1 \times \omega_{j-1}$$

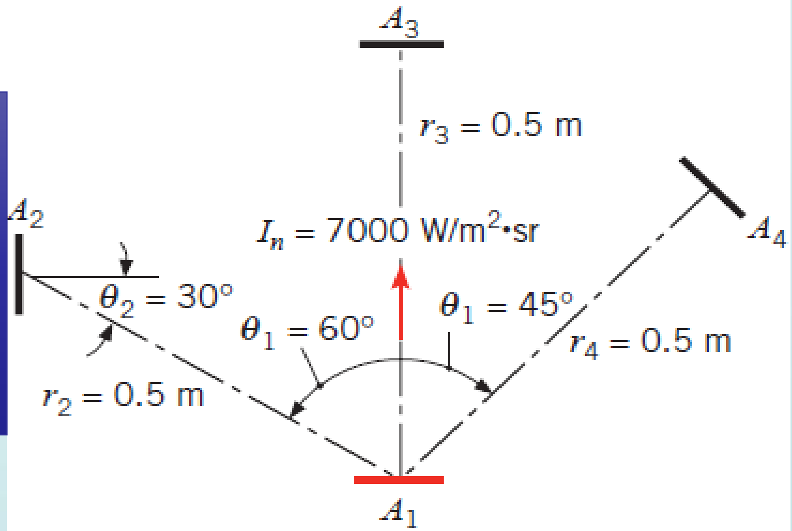
Continue solution

$$q_{1-2} = 7000\ \text{W/m}^2 \cdot \text{sr}\ (10^{-3}\ \text{m}^2 \times \cos 60°)\ 3.46 \times 10^{-3}\ \text{sr}$$
$$= 12.1 \times 10^{-3}\ \text{W}$$

$$q_{1-3} = 7000\ \text{W/m}^2 \cdot \text{sr}\ (10^{-3}\ \text{m}^2 \times \cos 0°)\ 4.00 \times 10^{-3}\ \text{sr}$$
$$= 28.0 \times 10^{-3}\ \text{W}$$

$$q_{1-4} = 7000\ \text{W/m}^2 \cdot \text{sr}\ (10^{-3}\ \text{m}^2 \times \cos 45°)\ 4.00 \times 10^{-3}\ \text{sr}$$
$$= 19.8 \times 10^{-3}\ \text{W}$$

Where θ in the eq. is the angle between the normal to surface and the direction of the radiation.

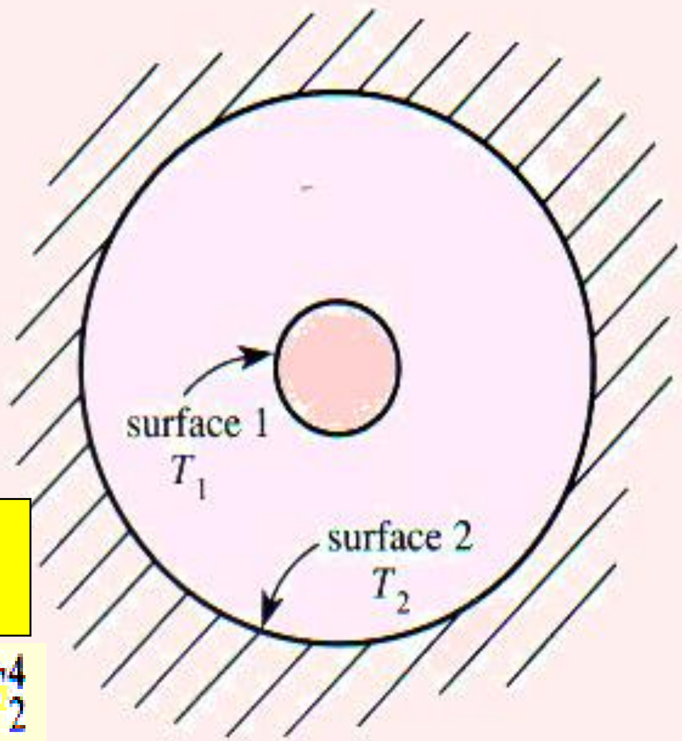


General Comment
The Net Heat transfer between Two Surfaces

$$\dot{Q}_{net} = A_1\sigma(T_2^4 - T_1^4)$$

If T_1 is more than T_2, the blackbody cools down because of the net loss of heat until T_1 and T_2 are equal. If T_1 is less than T_2, net heat will be added to the blackbody until T_1 and T_2 are again equal.

Blackbody
inside another blackbody



Note: For gray body inside another gray body

$$\dot{Q}_{net} = A_1\epsilon_1\sigma T_1^4 - A_1\alpha_1\sigma T_2^4$$