View Factor Concept

Introduction

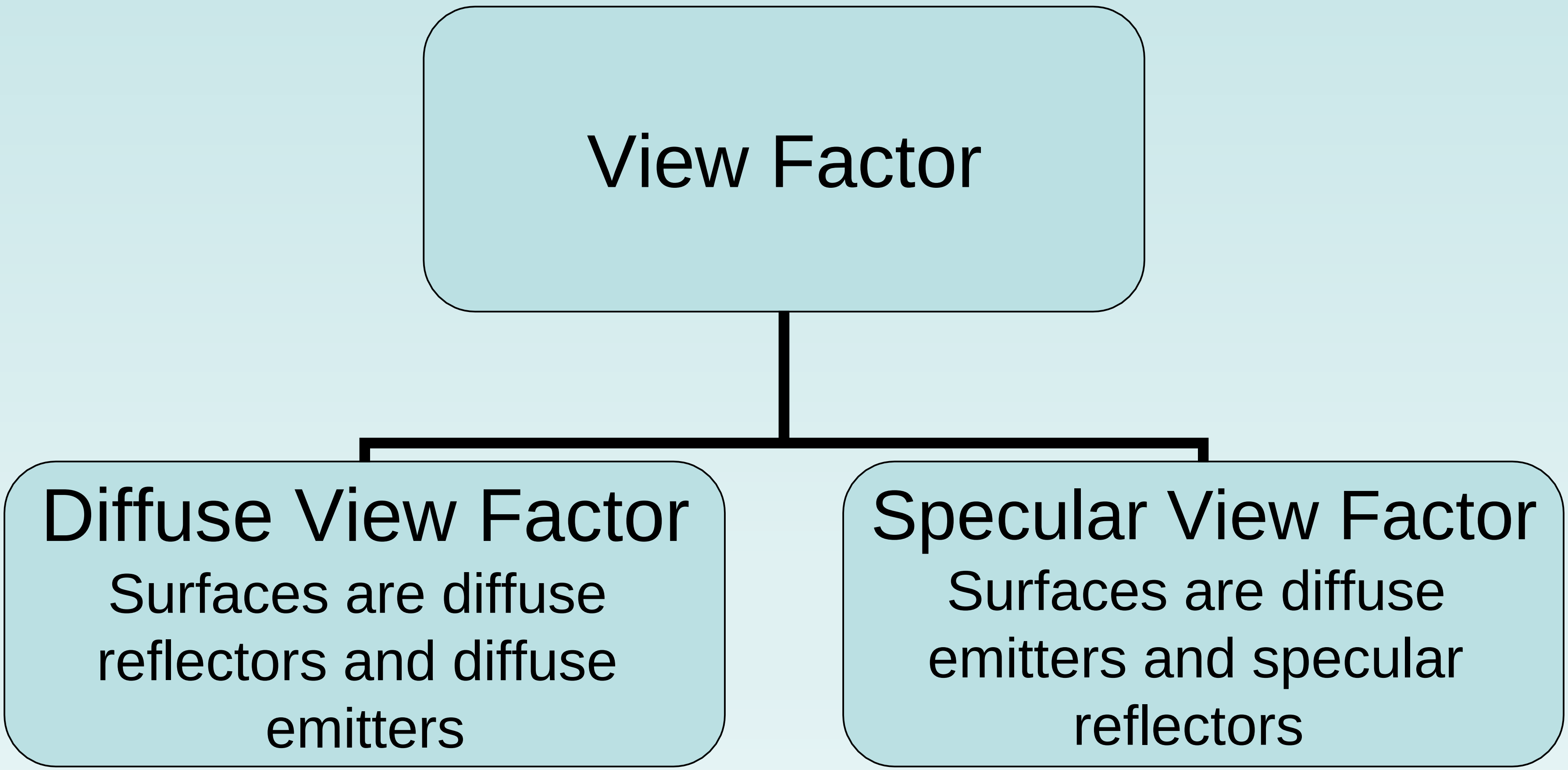
- Radiation Between surfaces are strongly affected by :

  a. Orientation

  b. size of the surfaces relative to each other.

- In order to formulate the effects of orientation in the analysis of radiation heat exchange among surfaces, the concept of view factor must be considered.

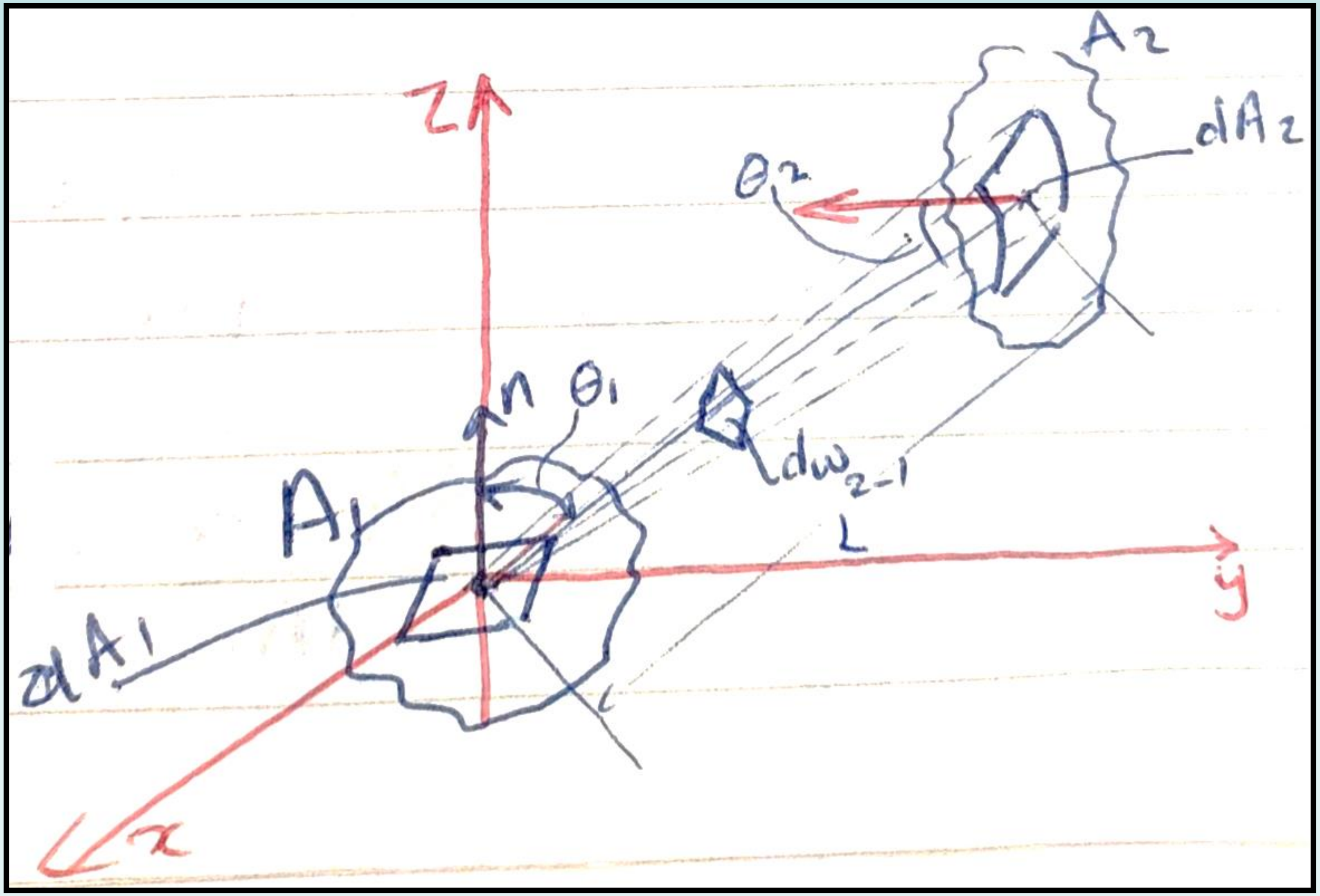
- View Factor ~ Shape Factor ~ angle Factor ~ Configuration Factor have been used in literature.



- We are going to consider only the surfaces of diffuse emitters and diffuse reflectors. Thus, we simply use the term view factor, and it will imply the diffuse view factor.
- The physical significance of the view factor:

  The view factor between two surfaces represents the fraction of the radiative energy leaving one surface that strikes the other surface directly.

View factor definition



View Factor Derivation

$$dq_{1-2} = I \cos\theta_1 \, dA_1 \, d\omega_{2-1} \qquad \text{..........(1)}$$

$$d\omega_{2-1} = \frac{\cos\theta_2 \, dA_2}{L^2} \qquad \text{..........(2)}$$

But for black surface

$$I = \frac{E}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad \text{...........(3)}$$

Substituting eq. 2 and 3 in 1, we obtain

$$dq_{1-2} = \frac{\sigma T_1^4}{\pi} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{L^2} \qquad \text{..........(4)}$$

- Following the same procedure, we can write

$$dq_{2-1} = \frac{\sigma T_2^4}{\pi} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{L^2} \quad \text{.............(5)}$$

- The net of radiation between dA_1 and dA_2 is

$$dq_{1\to 2} = dq_{1-2} - dq_{2-1} \quad \text{.............(6)}$$

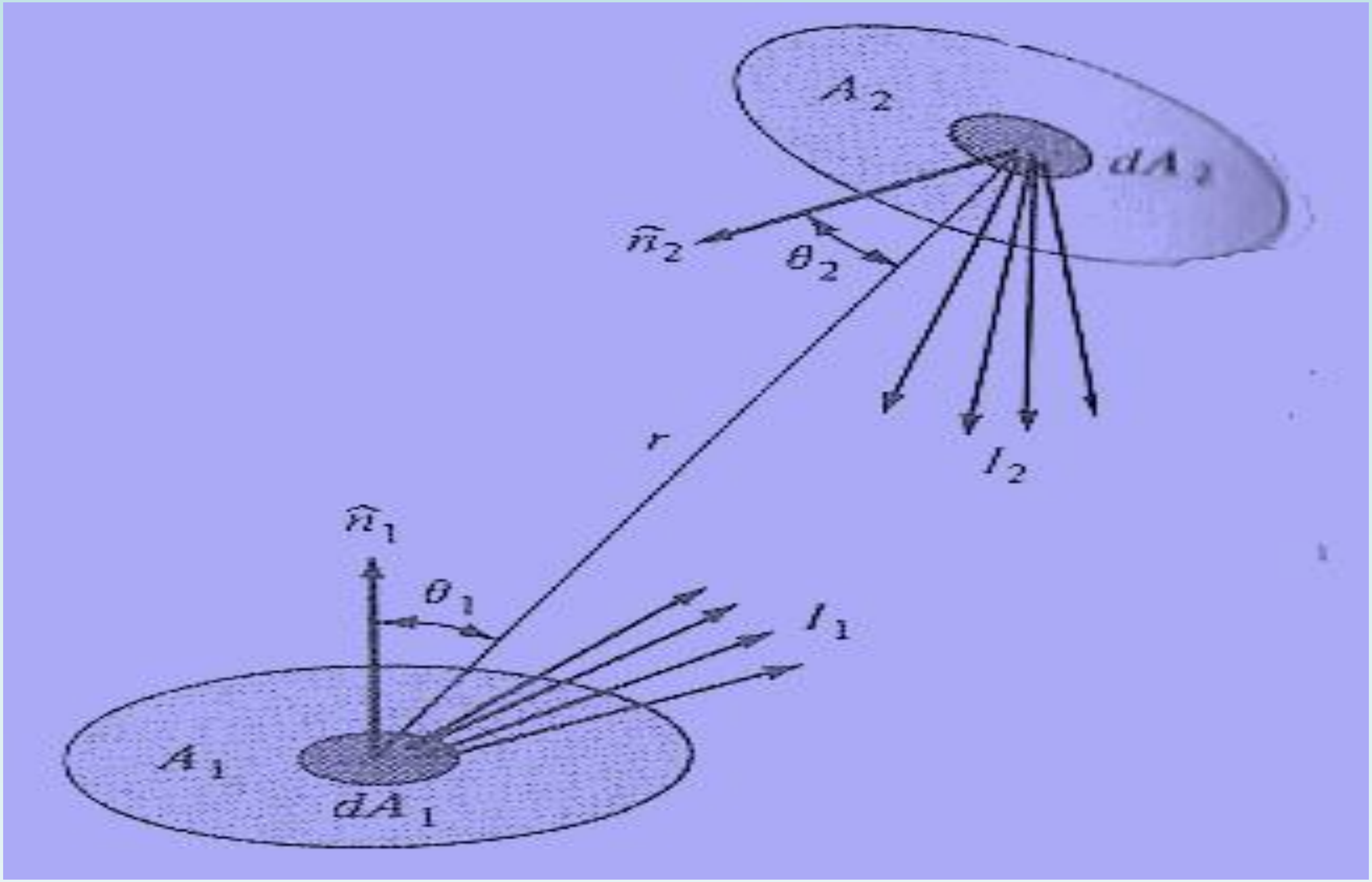
$$= \sigma(T_1^4 - T_2^4) \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi L^2} \quad \text{.............(7)}$$

On a per unit area basis

View factor, F_A

$$\frac{dq_{1\to 2}}{dA_1} = \sigma(T_1^4 - T_2^4) \frac{\cos\theta_1 \cos\theta_2 \, dA_2}{\pi L^2} \quad \text{.............(8)}$$

The elemental View Factor



Definition: The elemental View Factor

The *elemental view factor* $dF_{dA_1-dA_2}$, by definition, is the ratio of the radiative energy leaving dA_1 that strikes dA_2 directly to the radiative energy leaving dA_1 in all directions into the hemispherical space. It is given by

$$dF_{dA_1-dA_2} = \frac{\cos\theta_1 \cos\theta_2 \, dA_2}{\pi r^2}$$

The elemental view factor $dF_{dA_2-dA_1}$ from dA_2 to dA_1 is now immediately obtained from previous eq. by interchanging subscripts 1 and 2. We find

$$dF_{dA_2-dA_1} = \frac{\cos\theta_1 \cos\theta_2 \, dA_1}{\pi r^2}$$

Reciprocity Relation

- The reciprocity relation between the view factors $dF_{dA1-dA2}$ and $dF_{dA2-dA1}$ of the previous two Eqs. Is :

$$\mathbf{dA_1 dF_{dA1-dA2} = dA_2\ dF_{dA2-dA1}}$$

View Factor between two Finite Surfaces

$$F_{A_1-A_2} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, dA_2\, dA_1$$

Then the view factor $F_{A_2-A_1}$ from area A_2 to A_1 is immediately obtained by interchanging the subscripts 1 and 2.

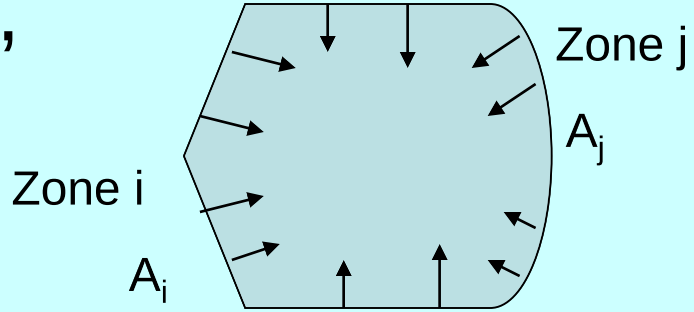
$$F_{A_2-A_1} = \frac{1}{A_2}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, dA_1\, dA_2$$

From the previous two equations, the reciprocity relation between the view factors F_{A1-A2} and F_{A2-A1} is given as

$$A_1 F_{A_1-A_2} = A_2 F_{A_2-A_1}$$

Properties of View Factors

- Assume an enclosure consisting of N zones, each of surface area A_i (i=1,2,…, N) as shown in figure
- Each zone is isothermal, a diffuse emitter and a diffuse reflector
- The surface of each zone may be plane, convex, or concave.



An N zone enclosure

- The view factors between surfaces A_i and A_j of the enclosure obey the following relation:

$$A_i F_{Ai\text{-}Aj} = A_j F_{Aj\text{-}Ai}$$

- Summation relation

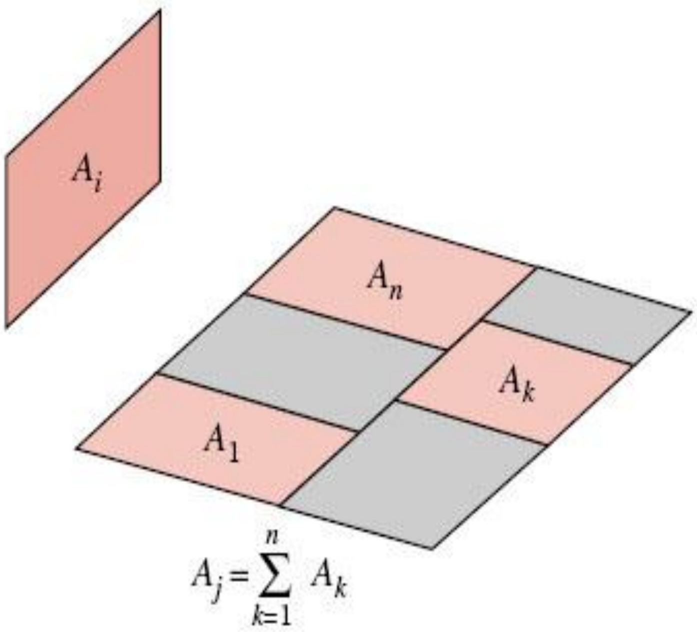
$$\sum_{k=1}^{N} F_{A_i - A_k} = 1$$

Note

$$F_{A_i - A_i} = 0 \quad \text{if } A_i \text{ plane or convex}$$
$$F_{A_i - A_i} \neq 0 \quad \text{if } A_i \text{ concave}$$

View-Factor Determination

- Equations were derived for simple shapes. These expressions given in tables.
- Graphs for simple configurations.
- Note: View factor Algebra



$F_{i\text{-}j} = F_{i\text{-}1} + F_{i\text{-}2} \ldots F_{i\text{-}n}$

$A_i F_{i\text{-}j} = A_i F_{i\text{-}1} + A_i F_{i\text{-}2} \ldots\ A_i F_{i\text{-}n}$

Use reciprocity relation

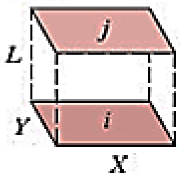
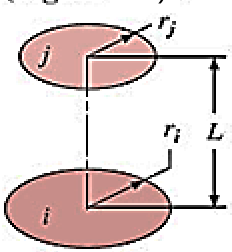
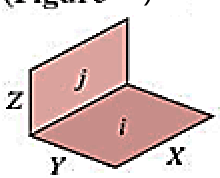
$A_j F_{j\text{-}i} = A_1 F_{1\text{-}i} + A_2 F_{2\text{-}i} + \ldots + A_n F_{n\text{-}i}$

Fj-i = [A1 F1-i+ A2 F2-i+…+ An Fn-i]/A_j

Fj-i = [A1F1-i+ A2 F2-i+…+ An Fn-i]/[A1+A2+…+An]

Areas used to illustrate view factor relations.

View Factors for Three-Dimensional Geometries

Geometry	Relation
Aligned Parallel Rectangles (Figure A) 	$\bar{X} = X/L, \bar{Y} = Y/L$ $F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}} \Big\{ \ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y} \Big\}$
Coaxial Parallel Disks (Figure B) 	$R_i = r_i/L, R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$
Perpendicular Rectangles with a Common Edge (Figure C) 	$H = Z/X, W = Y/X$ $F_{ij} = \frac{1}{\pi W}\Big(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\Big\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\Big\}\Big)$

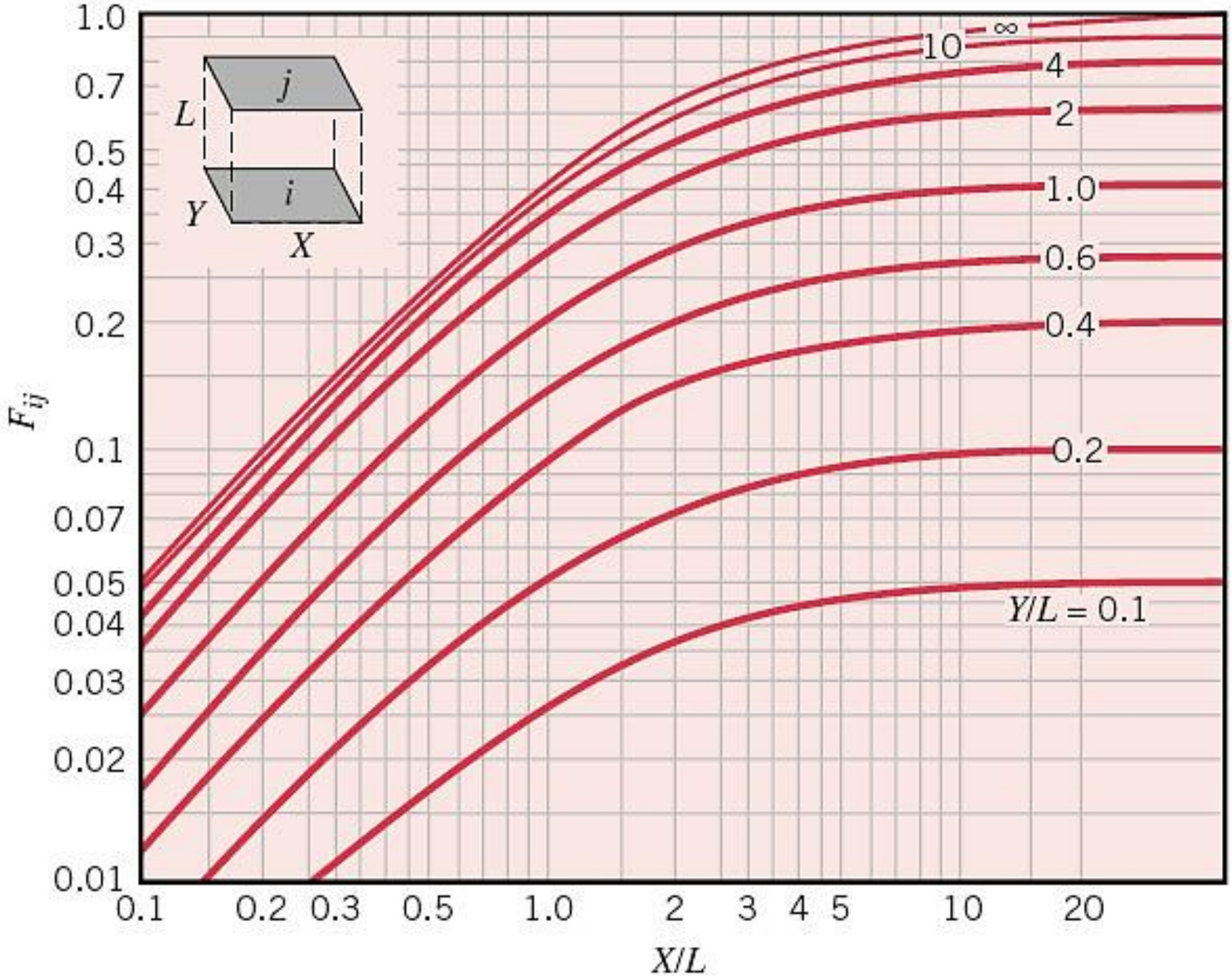


FIGURE A View factor for aligned parallel rectangles.

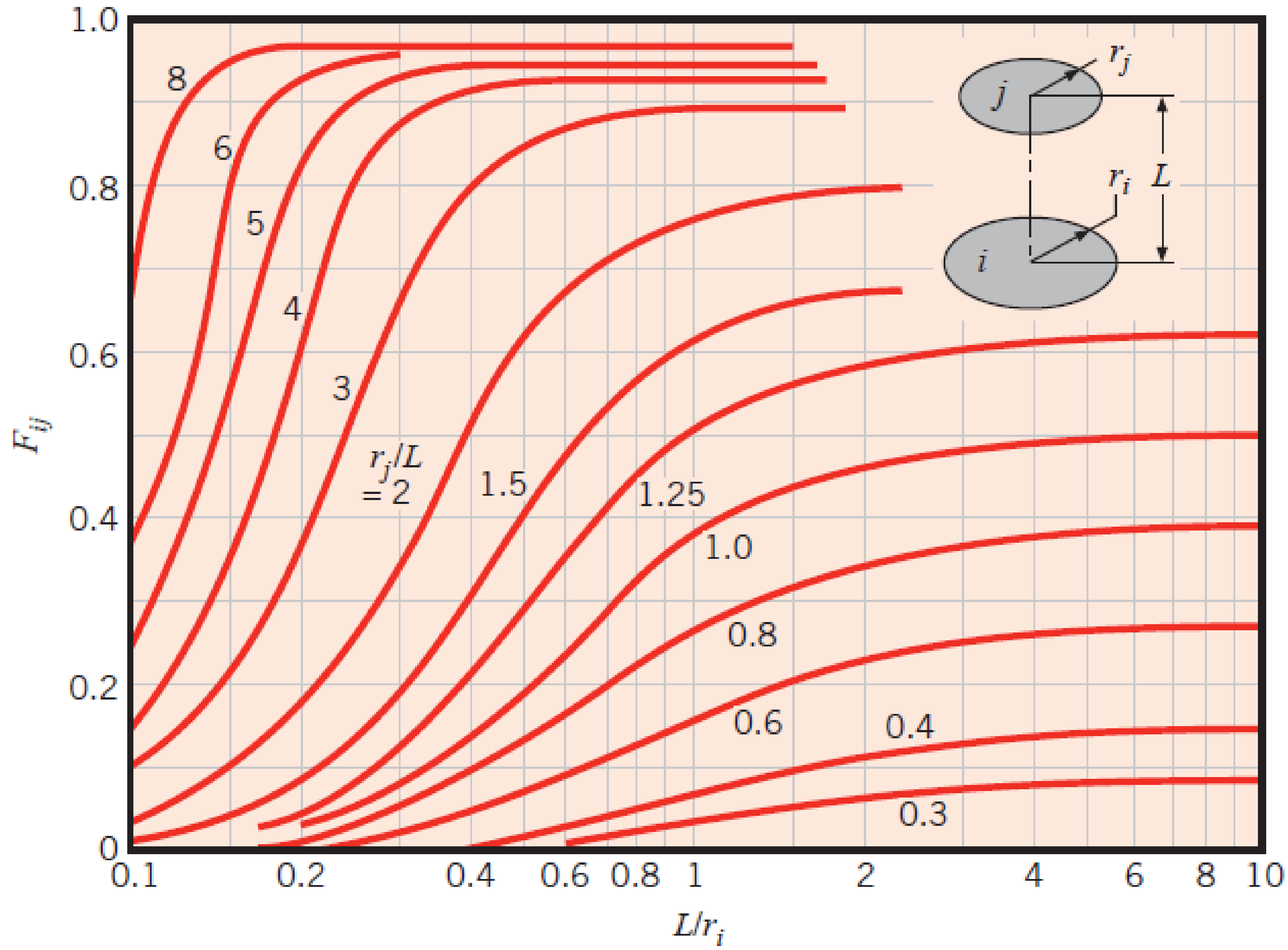


FIGURE B View factor for coaxial parallel disks.

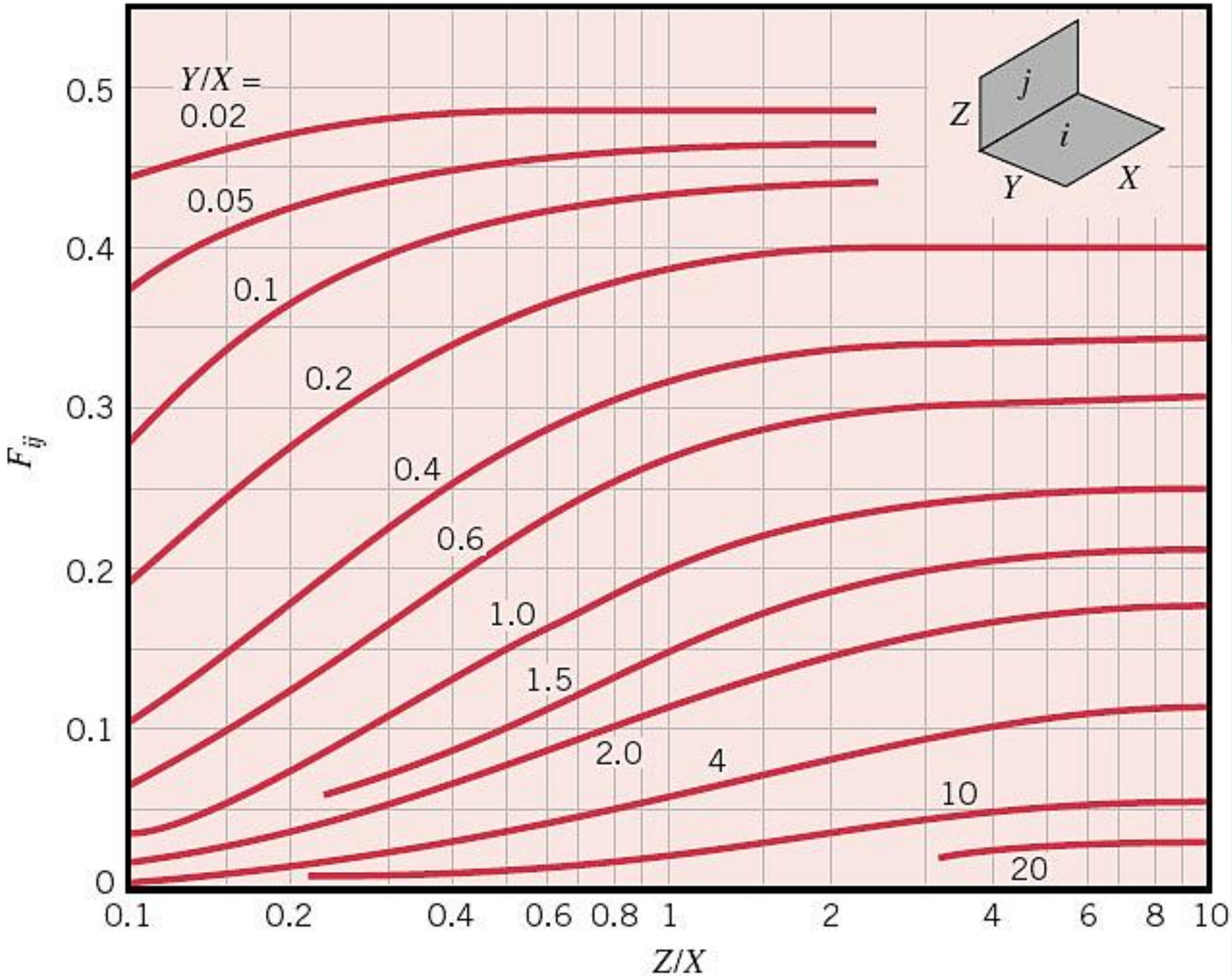
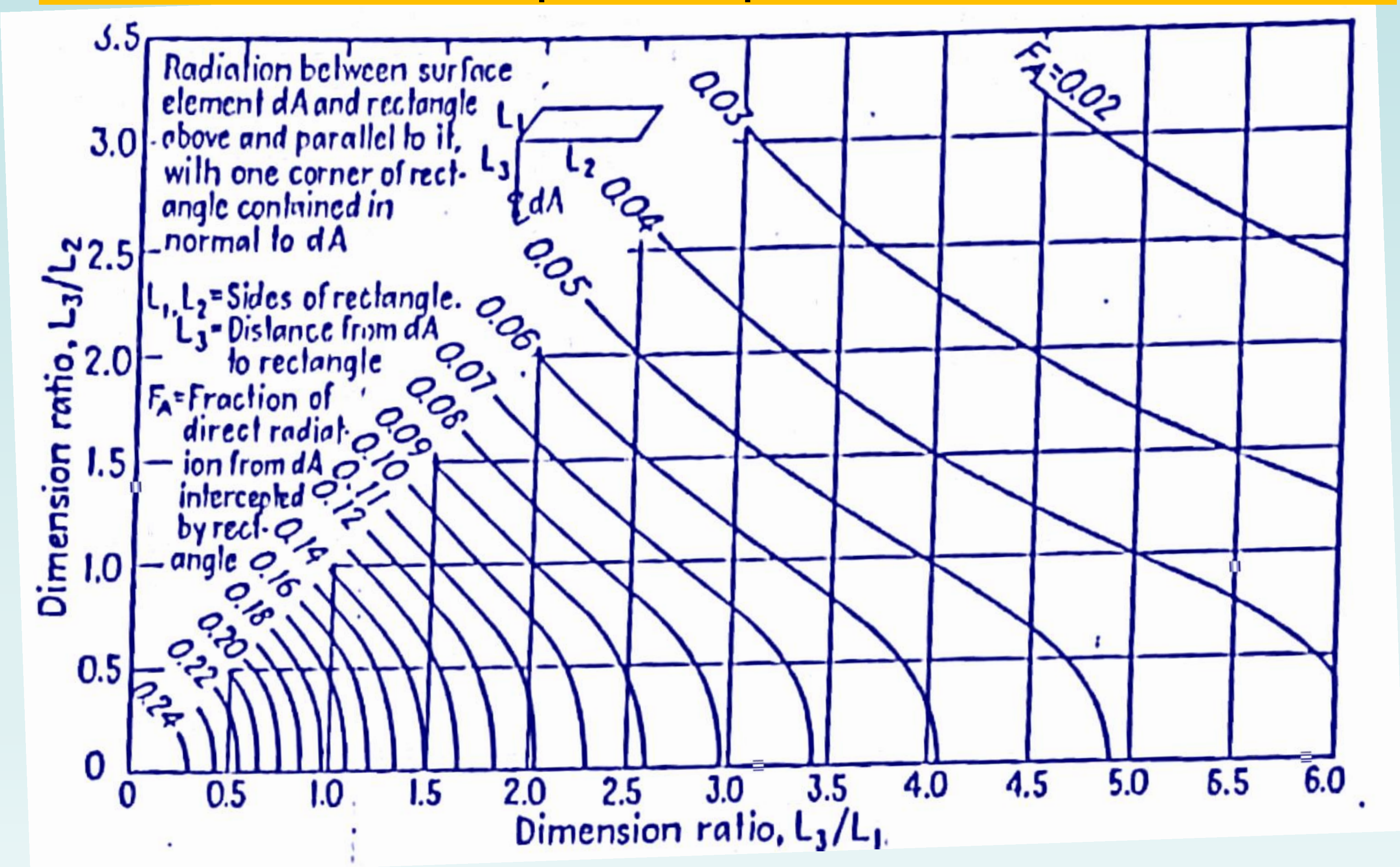


FIGURE C View factor for perpendicular rectangles with a common edge.

FIGURE D Radiation between an element and a parallel plane



Note

- The heat flux by radiation is given by:

$$q'' = F_{\varepsilon} F_{A} (T_1^4 - T_2^4)$$

where F_{ε} : Emissivity factor.

F_A : View factor.

Values of F_A and F_ε

	F_A	F_ϵ
(a) Surface A_1 small compared with the totally enclosing surface A_2	1	ϵ_1
(b) Surfaces A_1 and A_2 of parallel discs squares, 2:1 rectangles, long rectangles	Fig. B	$\epsilon_1 \epsilon_2$
(c) Surface dA_1 and parallel rectangular surface A_2 with one corner of rectangle above dA_1	Fig. D	$\epsilon_1 \epsilon_2$
(d) Surfaces A_1 or A_2 of perpendicular rectangles having a common side	Fig. C	$\epsilon_1 \epsilon_2$
(e) Surfaces A_1 and A_2 of infinite parallel planes or surface A_1 of a completely enclosed body is small compared with A_2	1	$\dfrac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2}\right) - 1}$
(f) Concentric spheres or infinite concentric cylinders with surfaces A_1 and A_2	1	$\dfrac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$