# Euler Method

Major: All Engineering Majors

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Transforming Numerical Methods Education for STEM Undergraduates

# Euler Method

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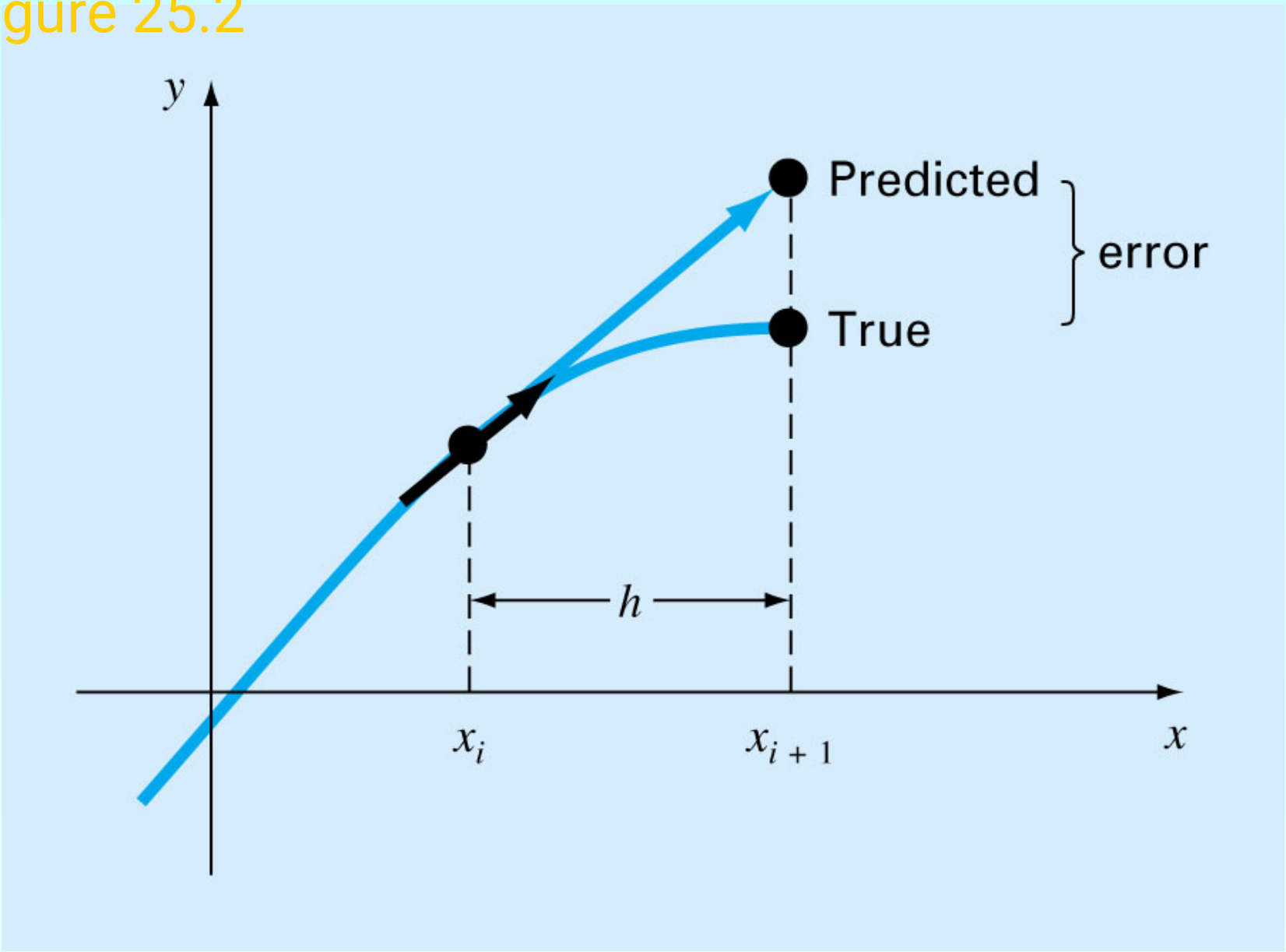
# Figure 25.2



# How to write Ordinary Differential Equation

How does one write a first order differential equation in the form of

Example

is rewritten as

In this case

# Example

A ball at 1200K is allowed to cool down in air at an ambient temperature of 300K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by

Find the temperature at seconds using Euler's method. Assume a step size of seconds.

# Solution

Step 1:

is the approximate temperature at

# Solution Cont

Step 2: For

is the approximate temperature at

# Solution Cont

The exact solution of the ordinary differential equation is given by the solution of a non- linear equation as

The solution to this nonlinear equation at t= 480 seconds is

# Comparison of Exact and Numerical Solutions

Figure 3. Comparing exact and Euler's method

# Effect of step size

Table 1. Temperature at 480 seconds as a function of step size, h

| Step, h | q(480) | $E_t$ | $\|\epsilon_t\|$% |
|---|---|---|---|
| 480 | −987.8 | 1635.4 | 252.54 |
| 240 | 1 | 537.26 | 82.964 |
| 120 | 110.32 | 100.80 | 15.566 |
| 60 | 546.77 | 32.607 | 5.0352 |
| 30 | 614.97 | 14.806 | 2.2864 |
| | 632.77 (exact) | | |

# Comparison with exact results

Figure 4. Comparison of Euler's method with exact solution for different step sizes

# Effects of step size on Euler's Method

Figure 5. Effect of step size in Euler's method.

# Errors in Euler's Method

It can be seen that Euler's method has large errors. This can be illustrated using Taylor series.

As you can see the first two terms of the Taylor series

are the Euler's method.

The true error in the approximation is given by

# Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple- choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/euler_method.html

# THE END

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