Runge 4th Order Method

Major: All Engineering Majors

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Transforming Numerical Methods Education for STEM Undergraduates

Runge- Kutta 4th Order Method

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Runge- Kutta 4th Order Method

For

Runge Kutta 4th order method is given by

where

How to write Ordinary Differential Equation

How does one write a first order differential equation in the form of

Example

is rewritten as

In this case

Example

A ball at 1200K is allowed to cool down in air at an ambient temperature of 300K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by

Find the temperature at seconds using Runge- Kutta 4th order method.

Assume a step size of seconds.

Solution

Step 1:

Solution Cont

is the approximate temperature at

Solution Cont

Step 2:

Solution Cont

q_2 is the approximate temperature at

Solution Cont

The exact solution of the ordinary differential equation is given by the solution of a non- linear equation as

The solution to this nonlinear equation at t= 480 seconds is

Comparison with exact results

Figure 1. Comparison of Runge- Kutta 4th order method with exact solution

Effect of step size

Table 1. Temperature at 480 seconds as a function of step size, h

Step size, h	q (480)	E_t	$\lvert\epsilon_t\rvert\%$
480	−90.278	737.85	113.94
240	594.91	52.660	8.1319
120	646.16	1.4122	0.21807
60	647.54	0.033626	0.0051926
30	647.57	0.0008690	0.0001341
		0	9

(exact)

Effects of step size on Runge-Kutta 4th Order Method

Figure 2. Effect of step size in Runge- Kutta 4th order method

Comparison of Euler and Runge-Kutta Methods

Figure 3. Comparison of Runge- Kutta methods of 1st, 2nd, and 4th order.

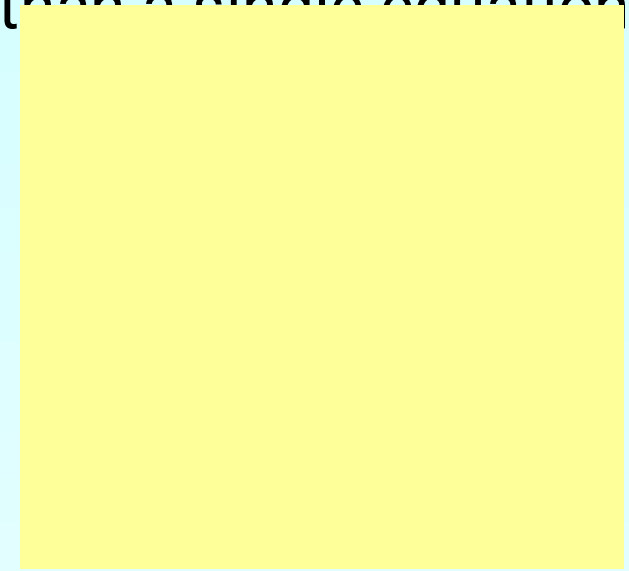
Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple- choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/runge_kutta_4th_method.html

Systems of Equations

- Many practical problems in engineering and science require the solution of a system of simultaneous ordinary differential equations rather than a single equation:



- Solution requires that n initial conditions be known at the starting value of x.

THE END

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