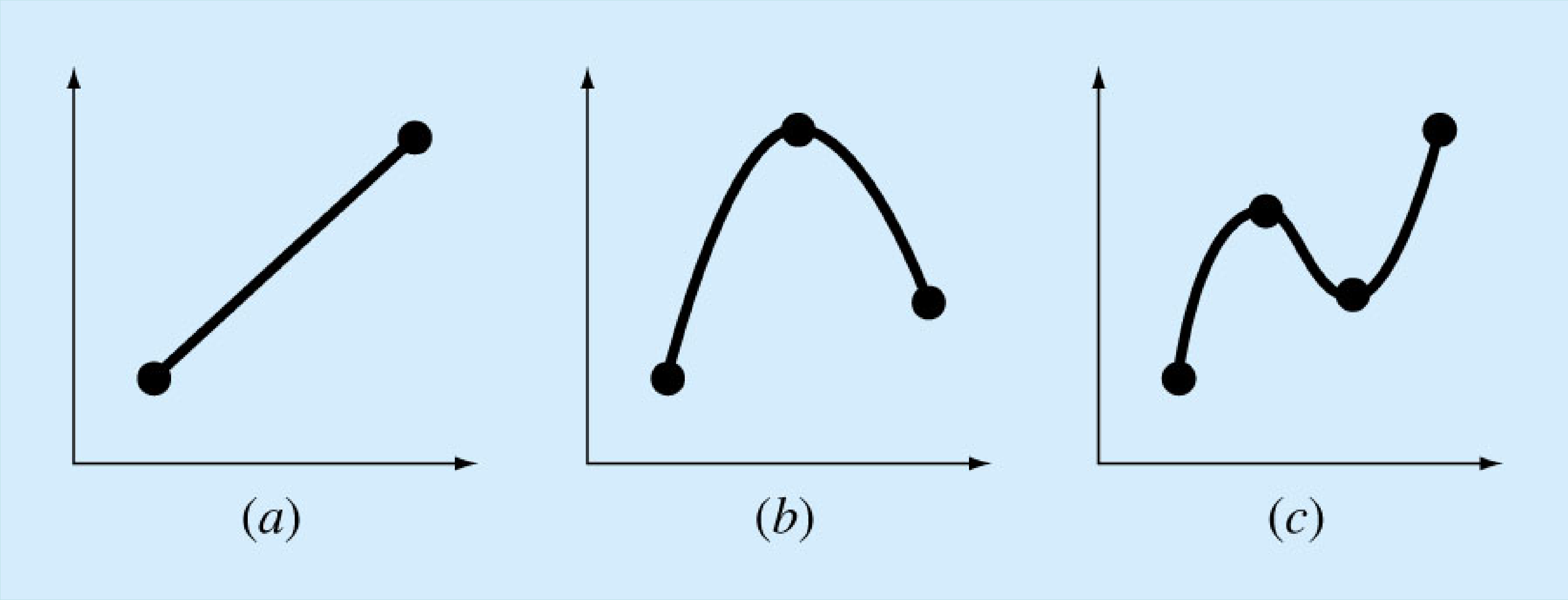
Interpolation
Chapter 18

- Estimation of intermediate values between precise data points. The most common method is:

- Although there is one and only one nth- order polynomial that fits n+1 points, there are a variety of mathematical formats in which this polynomial can be expressed:
  - The Newton polynomial
  - The Lagrange polynomial

Figure 18.1



Newton's Divided- Difference Interpolating Polynomials

Linear Interpolation/

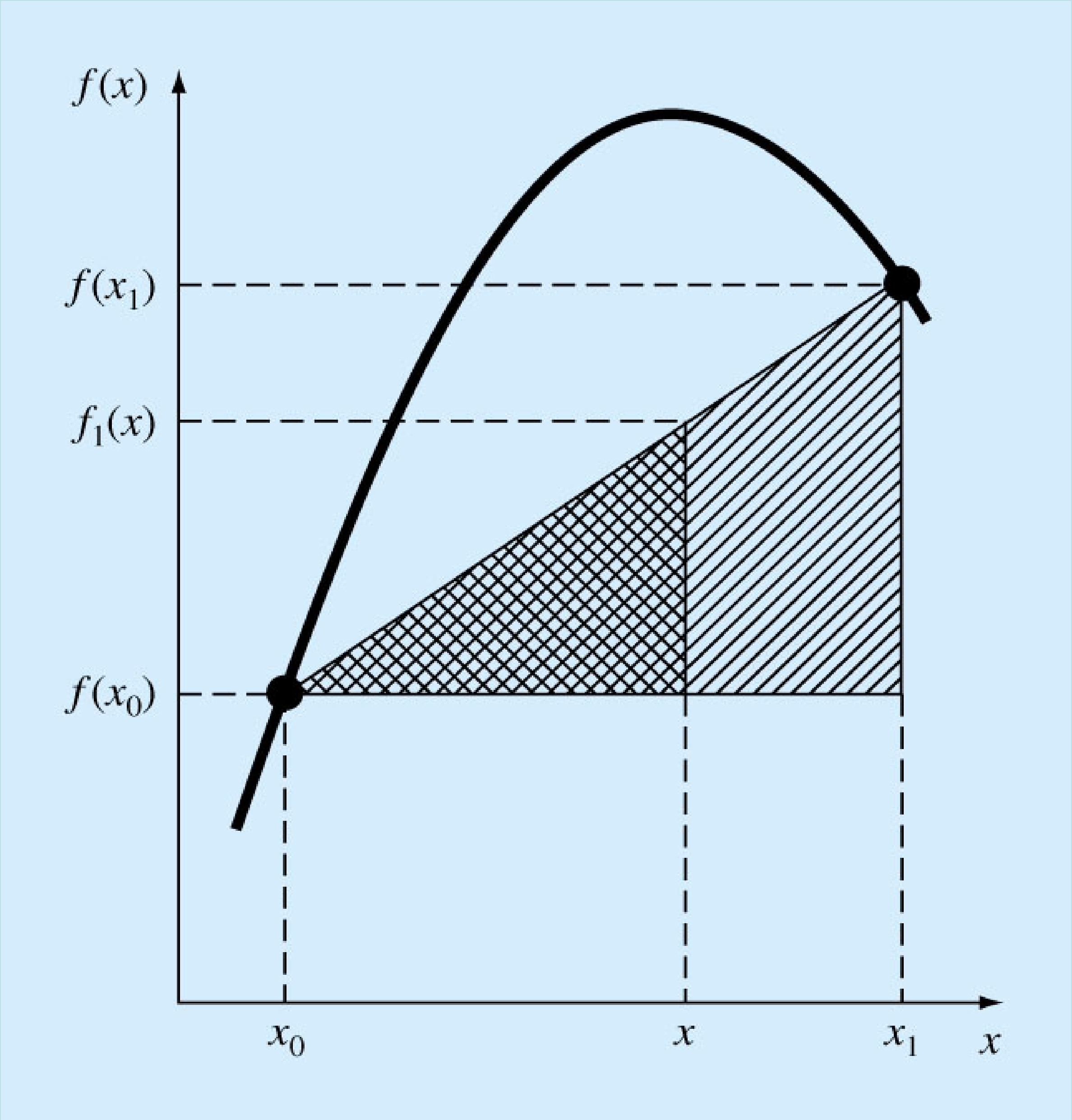
- Is the simplest form of interpolation, connecting two data points with a straight line.

Slope and a finite divided difference approximation to 1st derivative

Linear-interpolation formula

- $f_1(x)$ designates that this is a first- order interpolating polynomial.

Figure 18.2



Quadratic Interpolation/

- If three data points are available, the estimate is improved by introducing some curvature into the line connecting the points.
- A simple procedure can be used to determine the values of the coefficients.

General Form of Newton's Interpolating Polynomials/

Bracketed function evaluations are finite divided differences

Errors of Newton's Interpolating Polynomials/

- Structure of interpolating polynomials is similar to the Taylor series expansion in the sense that finite divided differences are added sequentially to capture the higher order derivatives.
- For an n^{th}- order interpolating polynomial, an analogous

x Is somewhere containing the unknown and he data

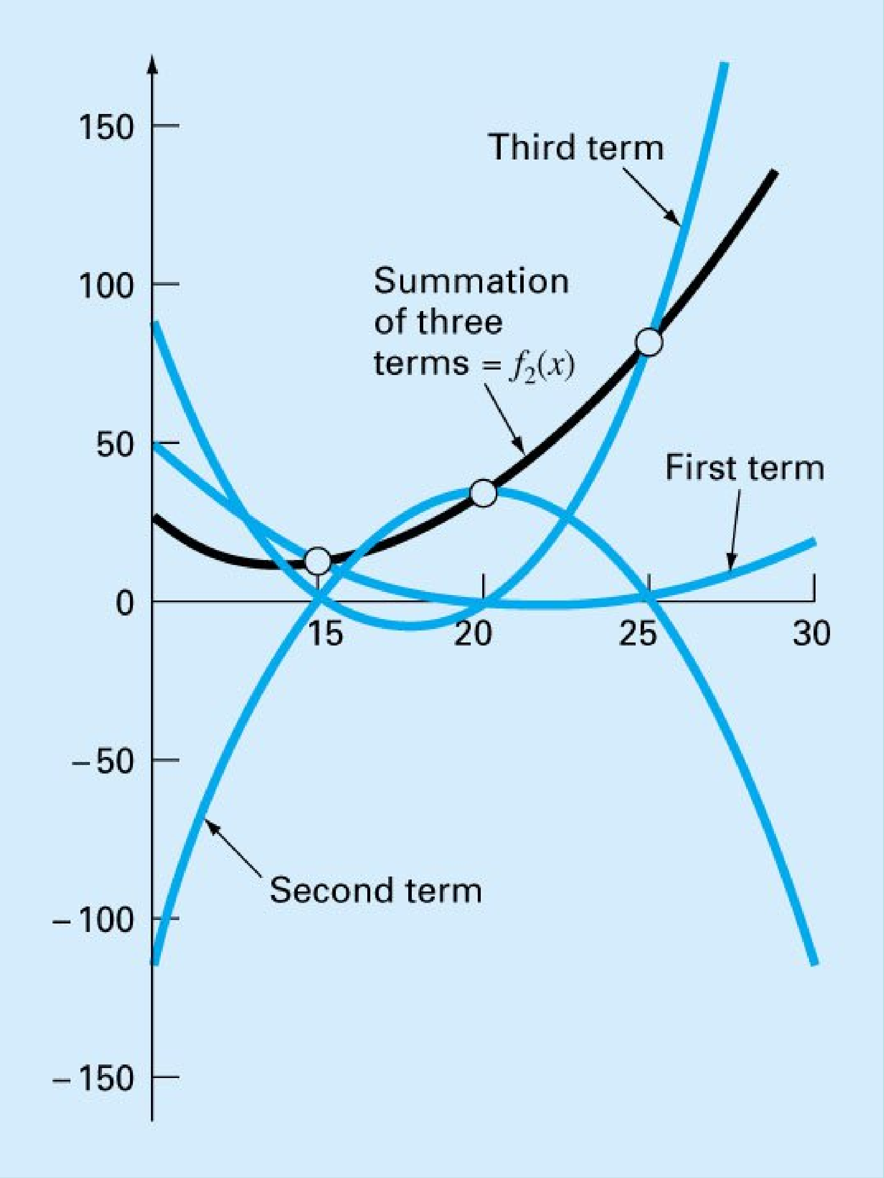
- For non differentiable functions, if an additional point $f(x_{n+1})$ is available, an alternative formula can be used that does not require prior knowledge of the function:

Lagrange Interpolating Polynomials

- The Lagrange interpolating polynomial is simply a reformulation of the Newton's polynomial that avoids the computation of divided differences:

- As with Newton's method, the Lagrange version has an estimated error of:

Figure 18.10



150
100
50
0
−50
−100
−150
15
20
25
30
Third term
Summation
of three
terms = $f_2(x)$
First term
Second term

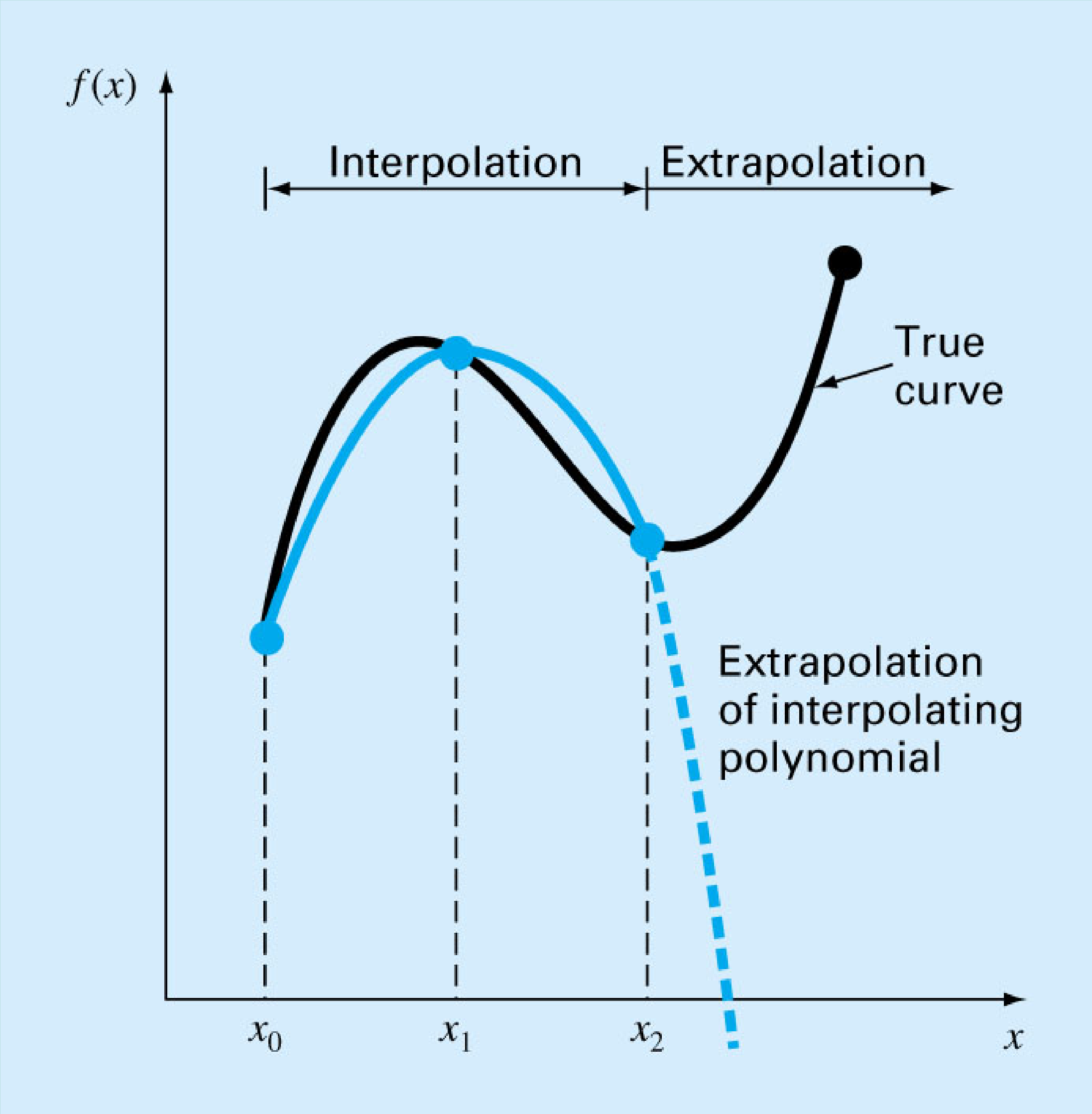
Coefficients of an Interpolating Polynomial

- Although both the Newton and Lagrange polynomials are well suited for determining intermediate values between points, they do not provide a polynomial in conventional form:

- Since n+1 data points are required to determine n+1 coefficients, simultaneous linear systems of equations can be used to calculate “a”s.

Where “x”s are the knowns and “a”s are the unknowns.

Figure 18.13



f(x)
Interpolation
Extrapolation
True
curve
Extrapolation
of interpolating
polynomial
x0
x1
x2
x

Spline Interpolation

- There are cases where polynomials can lead to erroneous results because of round off error and overshoot.
- Alternative approach is to apply lower-order polynomials to subsets of data points. Such connecting polynomials are called spline functions.

Figure 18.14

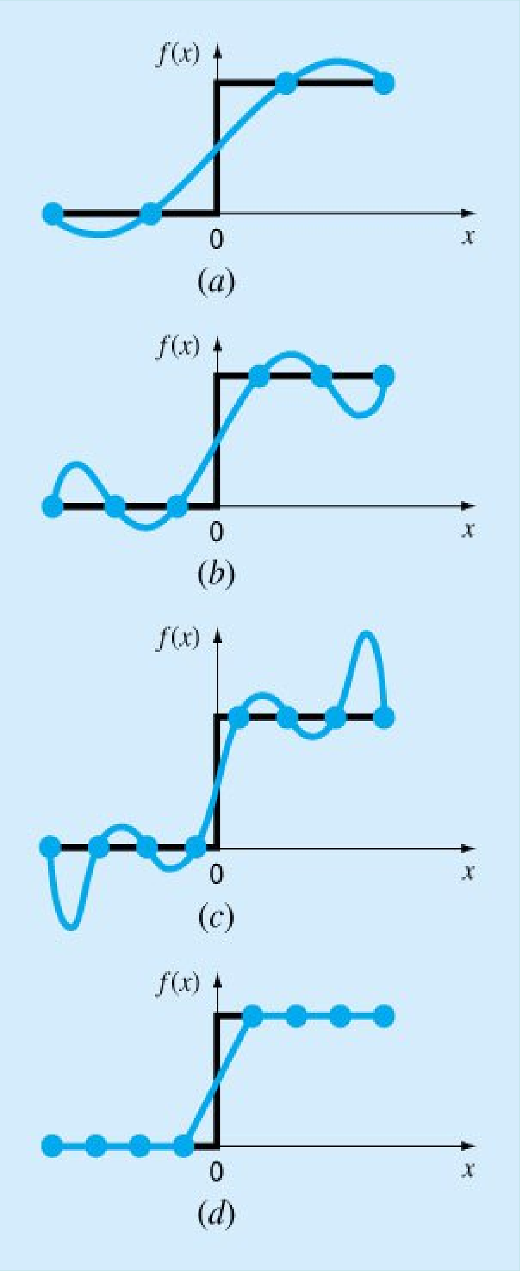


Figure 18.15

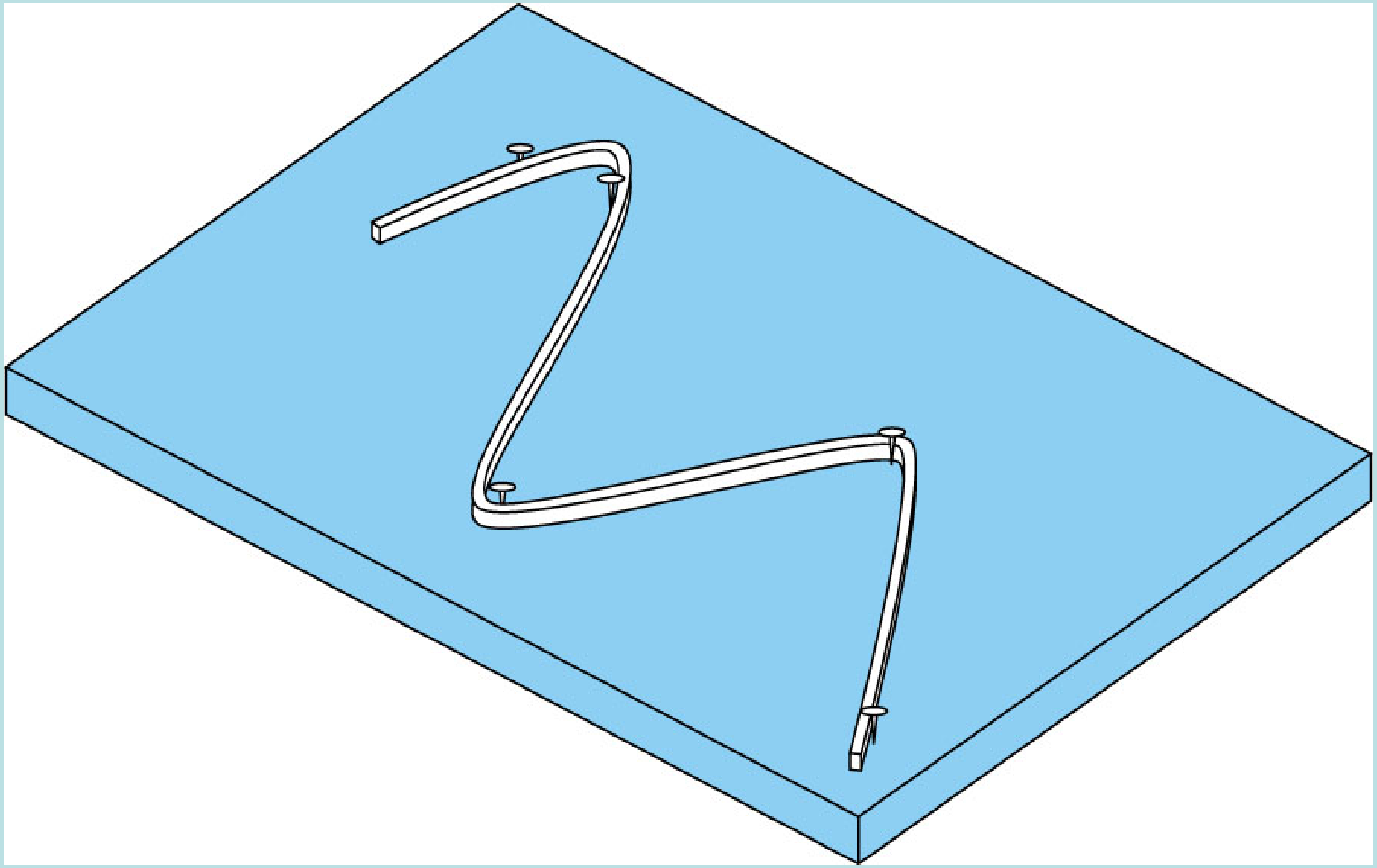
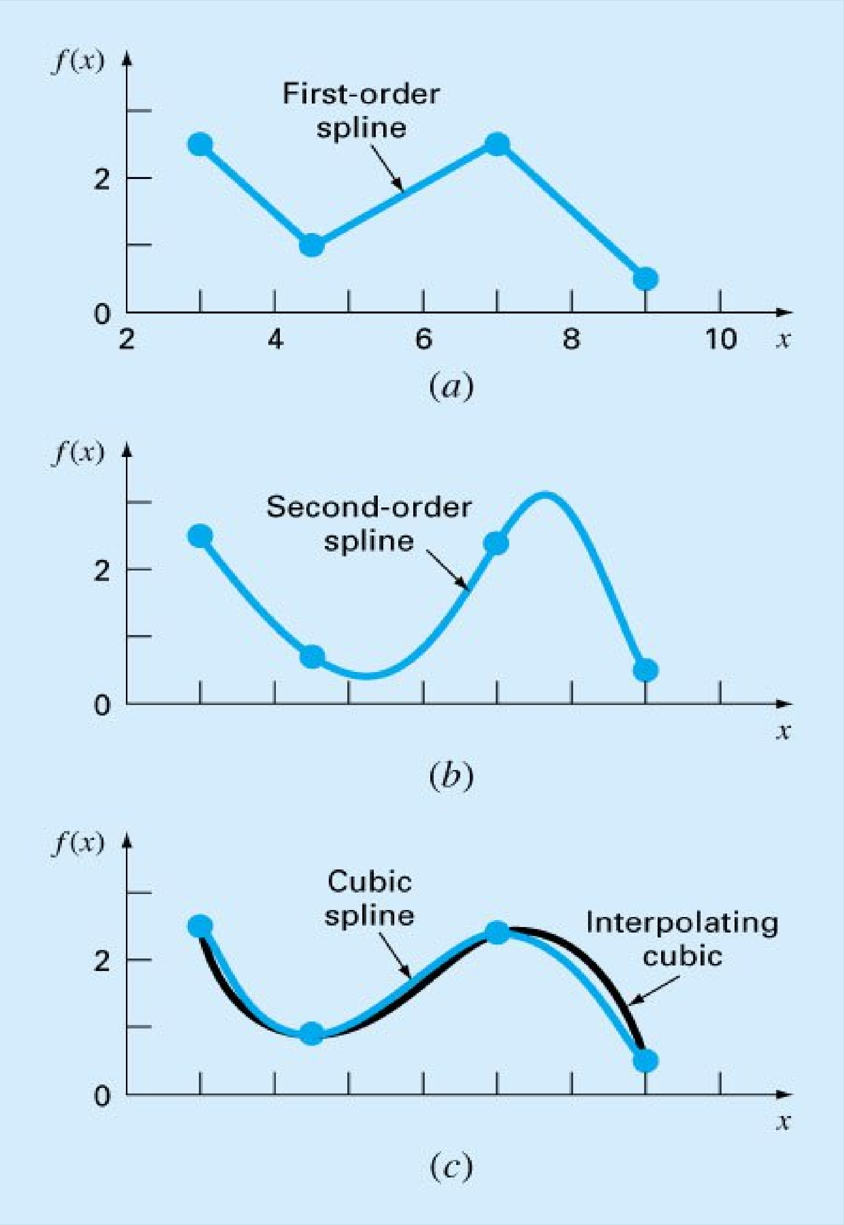
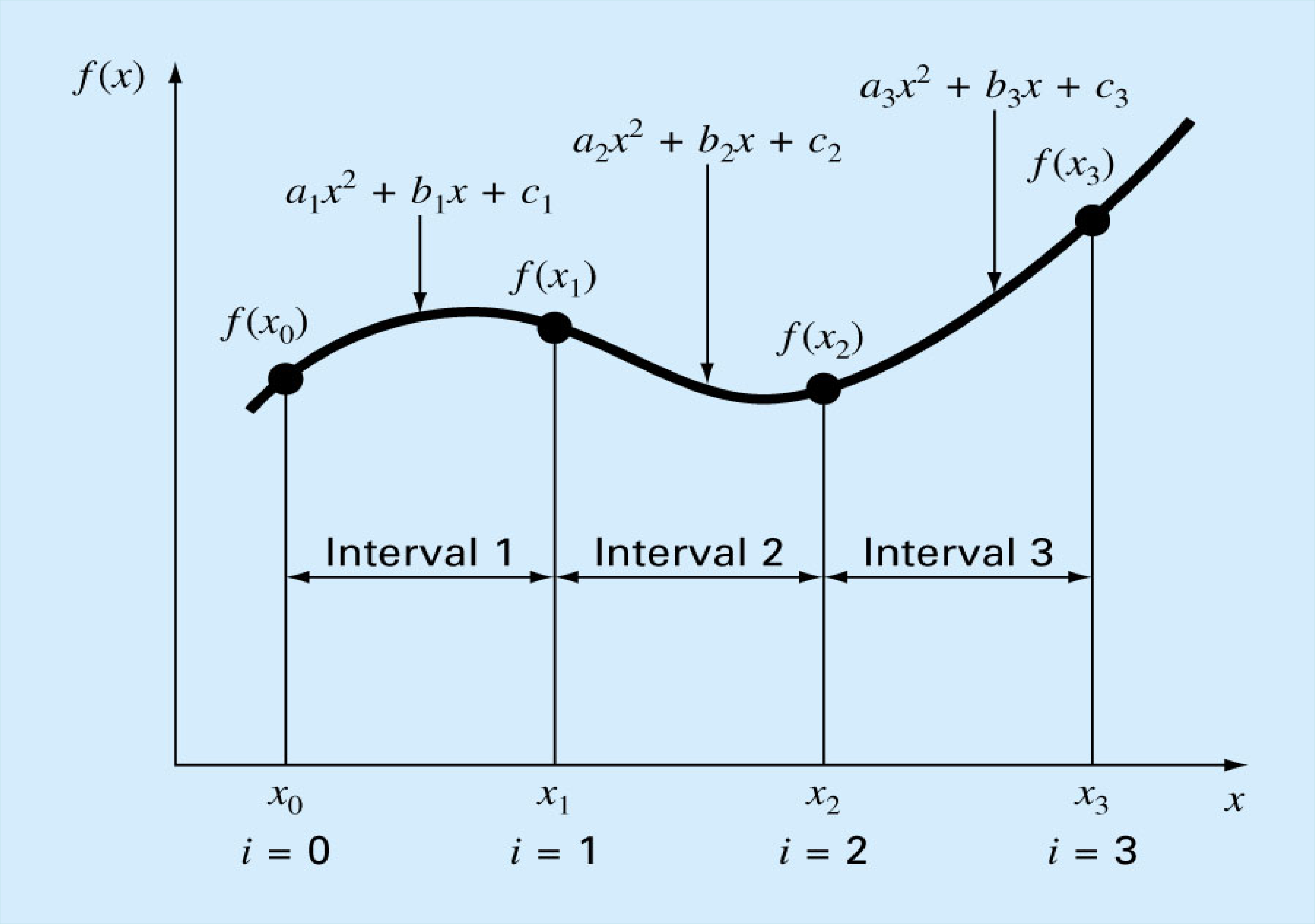


Figure 18.16



f(x)
First-order spline
2
0
2
4
6
8
10
x
(a)
f(x)
Second-order spline
2
0
x
(b)
f(x)
Cubic spline
Interpolating cubic
2
0
x
(c)

Figure 18.17



$f(x)$
$a_3x^2 + b_3x + c_3$
$a_2x^2 + b_2x + c_2$
$f(x_3)$
$a_1x^2 + b_1x + c_1$
$f(x_1)$
$f(x_0)$
$f(x_2)$
Interval 1
Interval 2
Interval 3
x_0
x_1
x_2
x_3
x
$i = 0$
$i = 1$
$i = 2$
$i = 3$