Direct Method of Interpolation

Major: All Engineering Majors

Authors: Autar Kaw, Jai Paul

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What is Interpolation ?

Given (x_0,y_0), (x_1,y_1), …… (x_n,y_n), find the value of 'y' at a value of 'x' that is not given.
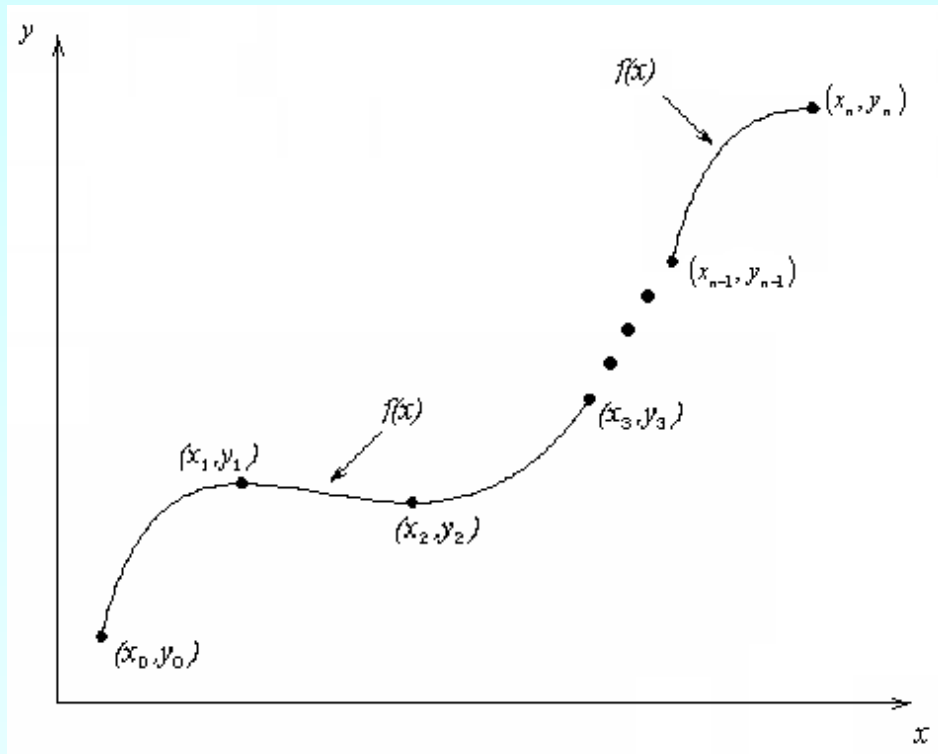

Figure 1 Interpolation of discrete.

Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate

Direct Method

Given 'n+1' data points (x_0,y_0), (x_1,y_1),………….. (x_n,y_n), pass a polynomial of order 'n' through the data as given below:

where a_0, a_1,………………. a_n are real constants.
- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.

Example 1

The upward velocity of a rocket is given as a function of time in Table 1.

Find the velocity at t=16 seconds using the direct method for linear interpolation.

Table 1 Velocity as a function of time.

0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67
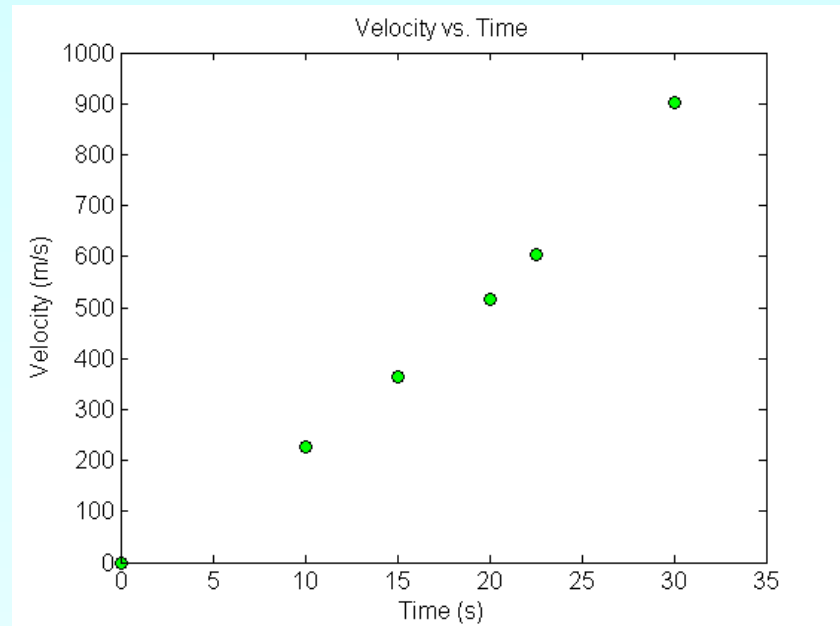
Figure 2 Velocity vs. time data for the rocket example

Linear Interpolation
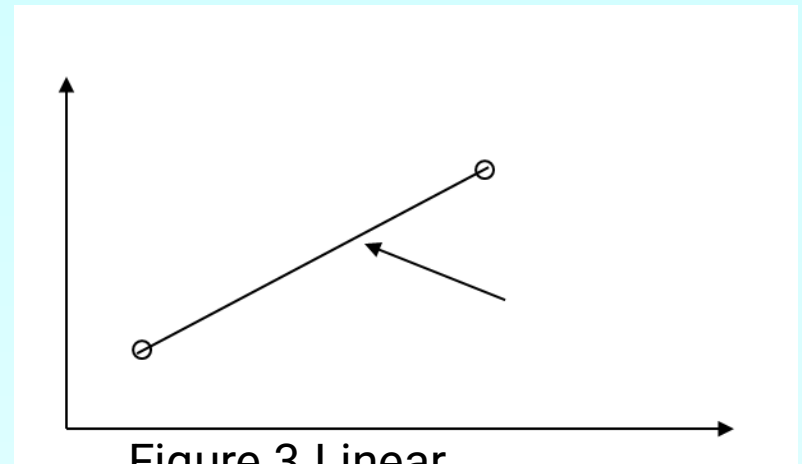

Figure 3 Linear interpolation.

Solving the above two equations gives,

    Hence

Example 2

The upward velocity of a rocket is given as a function of time in Table 2.

Find the velocity at t=16 seconds using the direct method for quadratic interpolation.

Table 2 Velocity as a function of time.

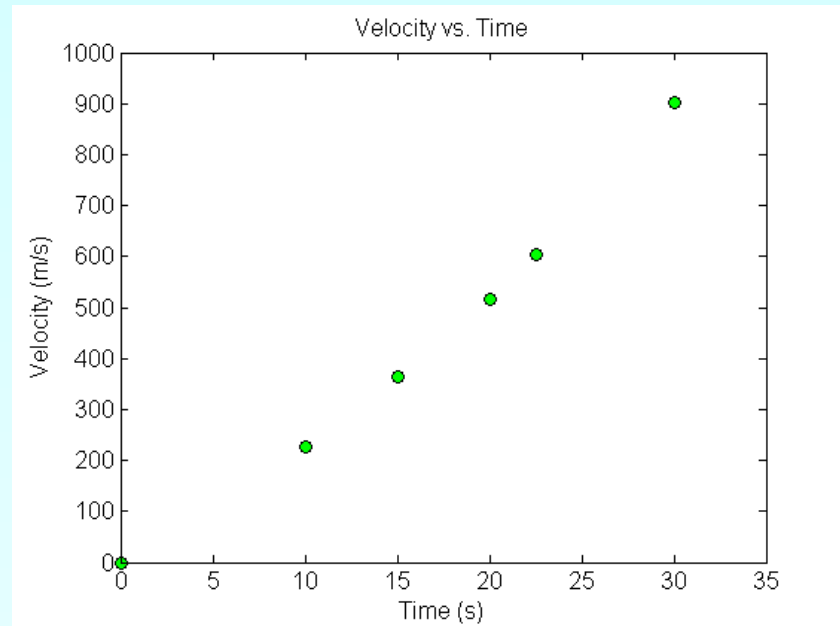|	
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Figure 5 Velocity vs. time data for the rocket example
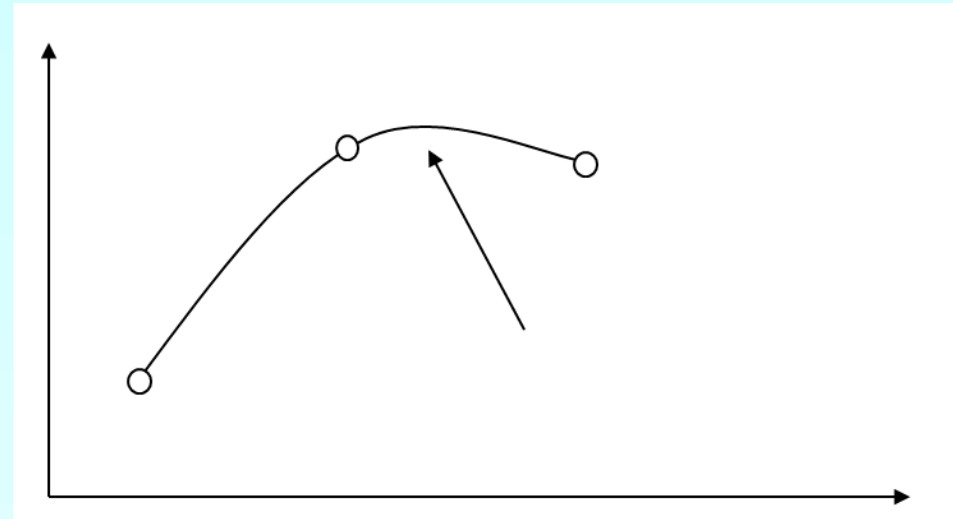
Quadratic Interpolation

Figure 6 Quadratic interpolation.

Solving the above three equations gives

Quadratic Interpolation (cont.)

The absolute relative approximate error   obtained between the results from the first and second order polynomial is

Example 3

The upward velocity of a rocket is given as a function of time in Table 3.

Find the velocity at t= 16 seconds using the direct method for cubic interpolation.

Table 3 Velocity as a function of time.

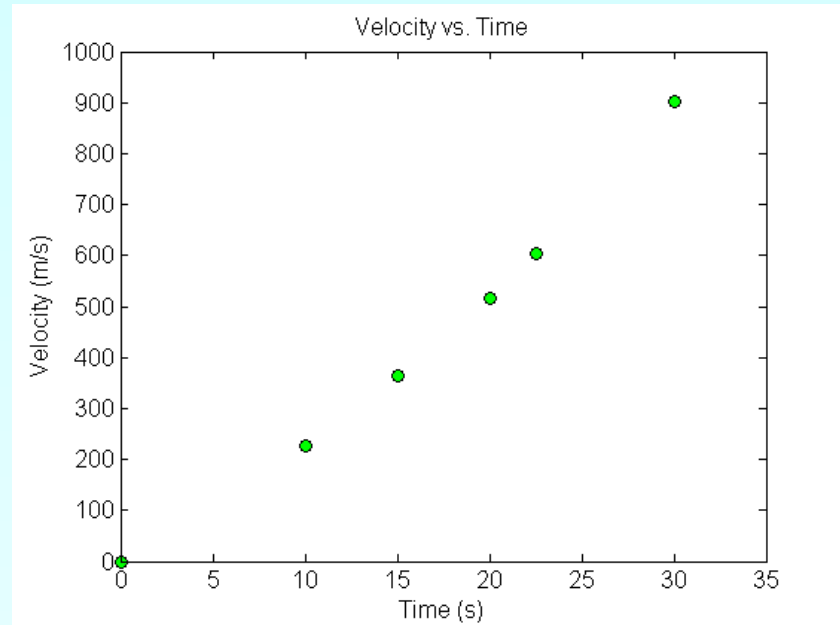|	
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Figure 6 Velocity vs. time data for the rocket example
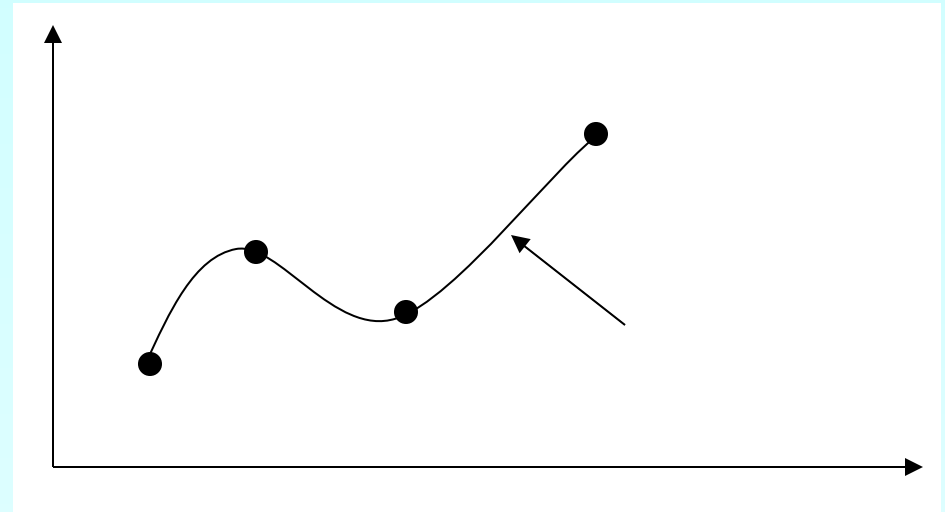
Cubic Interpolation

Figure 7 Cubic interpolation.

Cubic Interpolation (contd)

The absolute percentage relative approximate error   between second and third order polynomial is

Comparison Table

Table 4 Comparison of different orders of the polynomial.

t(s)	v (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Distance from Velocity Profile

Find the distance covered by the rocket from t=11s to t=16s ?

Acceleration from Velocity Profile

Find the acceleration of the rocket at t=16s given that

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple- choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/direct_method.html

THE END