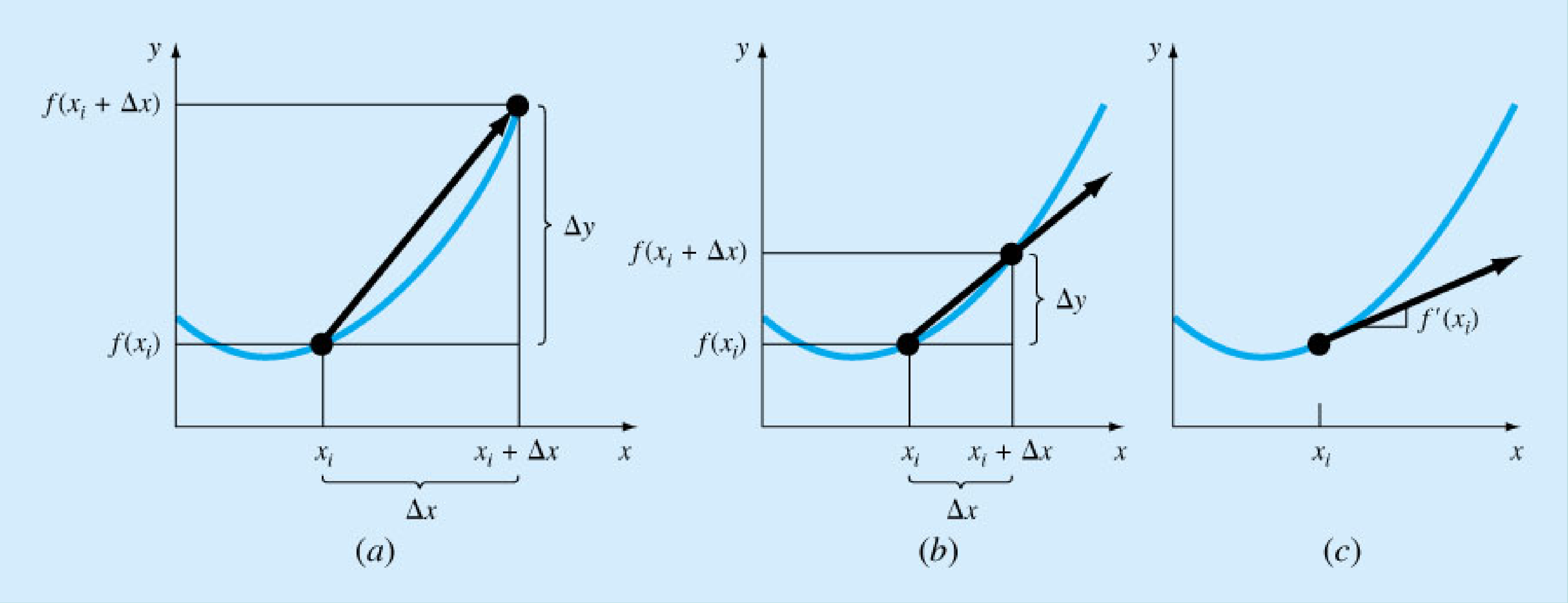
Numerical Differentiation and Integration

Part 6

- Calculus is the mathematics of change. Because engineers must continuously deal with systems and processes that change, calculus is an essential tool of engineering.

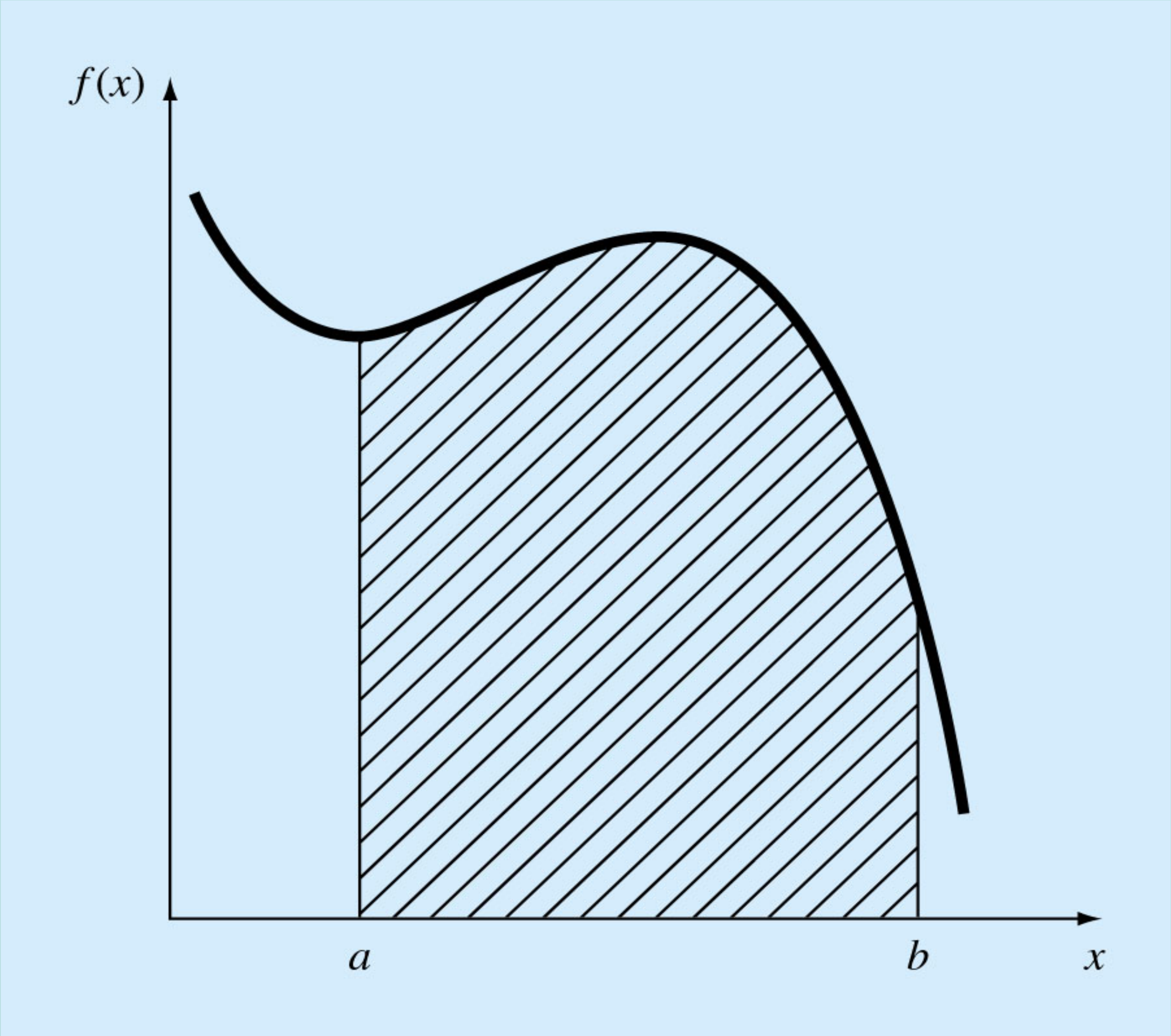
- Standing in the heart of calculus are the mathematical concepts of differentiation and integration:

Figure PT6.1



y
$f(x_i + \Delta x)$
Δy
$f(x_i)$
x_i
$x_i + \Delta x$
x
Δx
(a)
y
$f(x_i + \Delta x)$
Δy
$f(x_i)$
x_i
$x_i + \Delta x$
x
Δx
(b)
y
$f'(x_i)$
x_i
x
(c)

Figure PT6.2



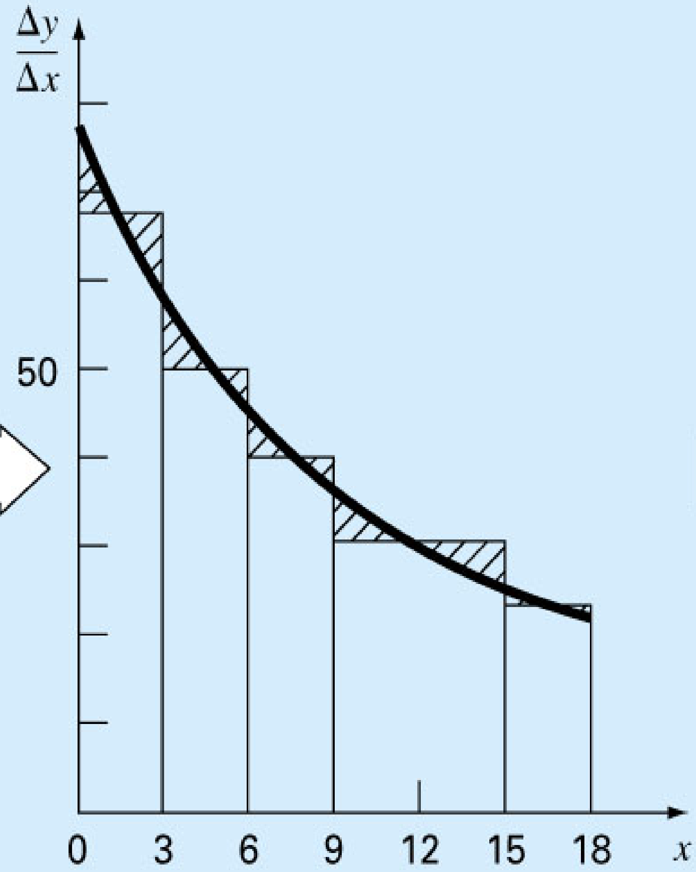
Noncomputer Methods for Differentiation and Integration

- The function to be differentiated or integrated will typically be in one of the following three forms:
  - A simple continuous function such as polynomial, an exponential, or a trigonometric function.
  - A complicated continuous function that is difficult or impossible to differentiate or integrate directly.
  - A tabulated function where values of x and f(x) are given at a number of discrete points, as is often the case with experimental or field data.

Figure PT6.4

x	y	$\Delta y/\Delta x$
0	0	
		66.7
3	200	
		50
6	350	
		40
9	470	
		30
15	650	
		23.3
18	720	

(*a*)



(*b*)

x	dy/dx
0	76.50
3	57.50
6	45.00
9	36.25
15	25.00
18	21.50

(*c*)

Figure PT6.7

(a)

$$\int_0^2 \frac{2 + \cos(1 + x^{3/2})}{\sqrt{1 + 0.5 \sin x}} e^{0.5x} \, dx$$

(b)

x	$f(x)$
0.25	2.599
0.75	2.414
1.25	1.945
1.75	1.993

(c)

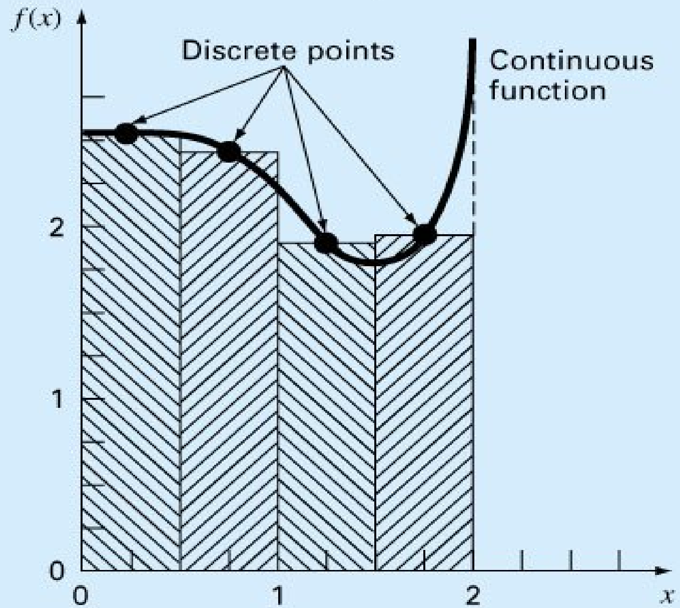
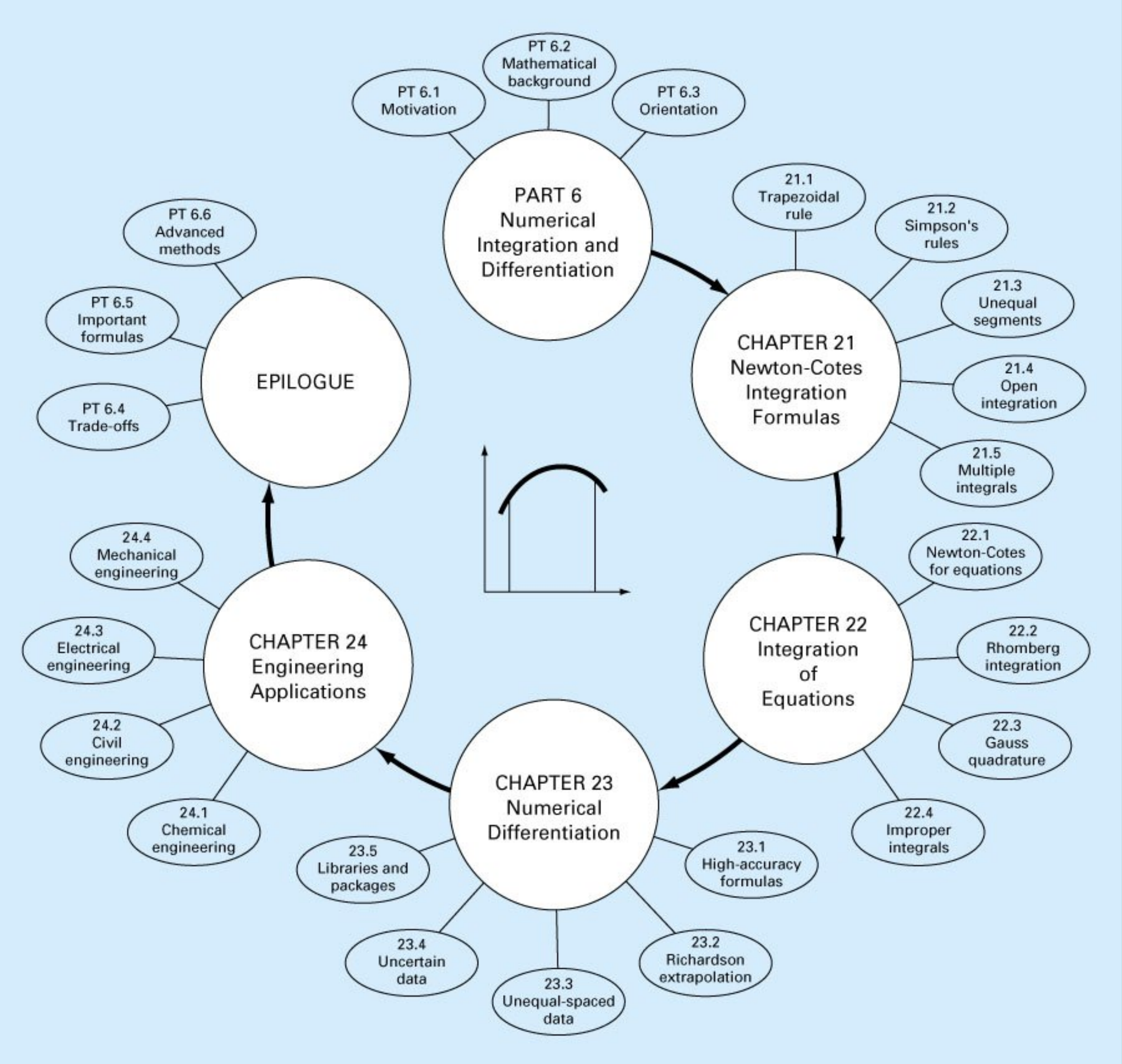


Figure PT6.10



Newton- Cotes Integration Formulas
Chapter 21

- The Newton- Cotes formulas are the most common numerical integration schemes.
- They are based on the strategy of replacing a complicated function or tabulated data with an approximating function that is easy to integrate:

Figure 21.1

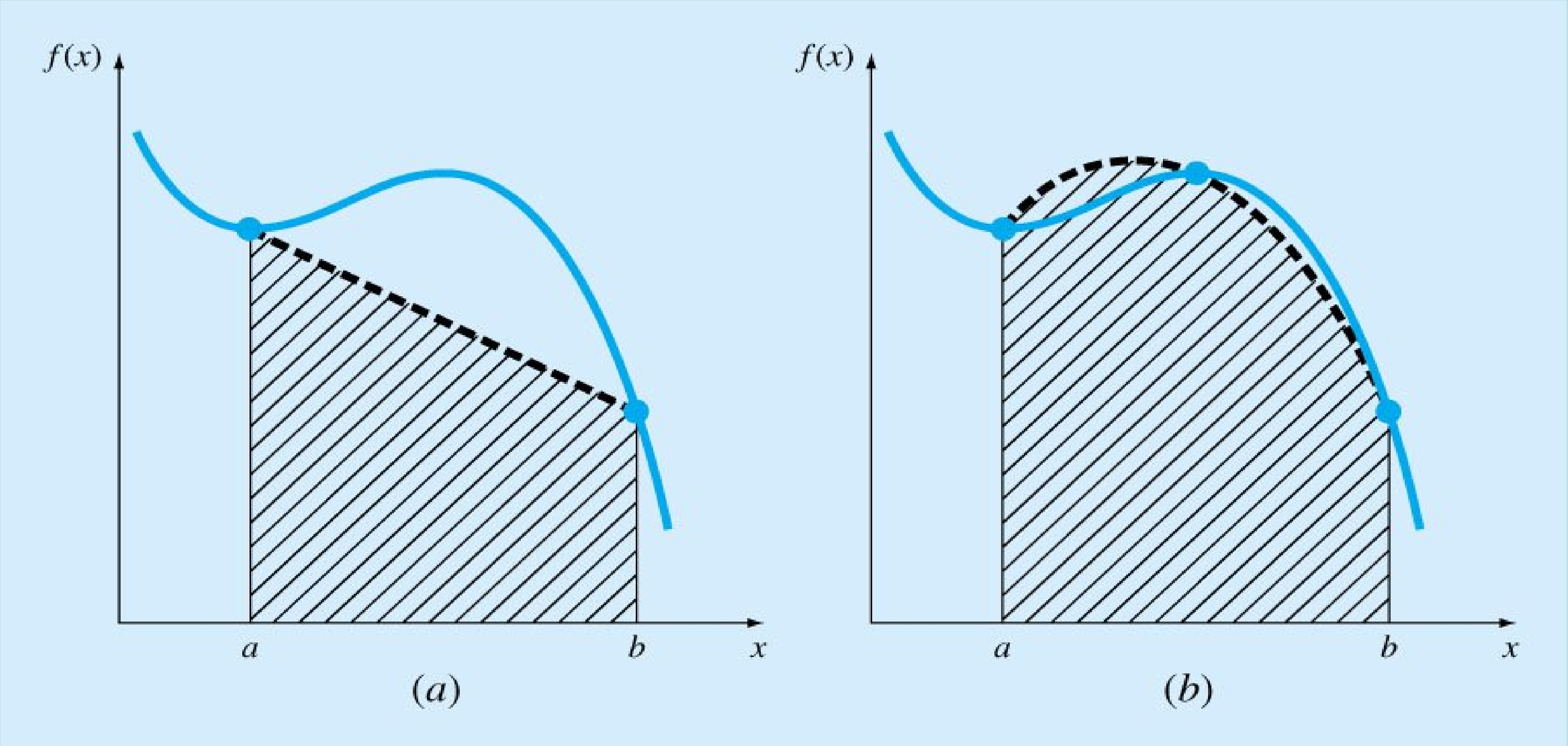
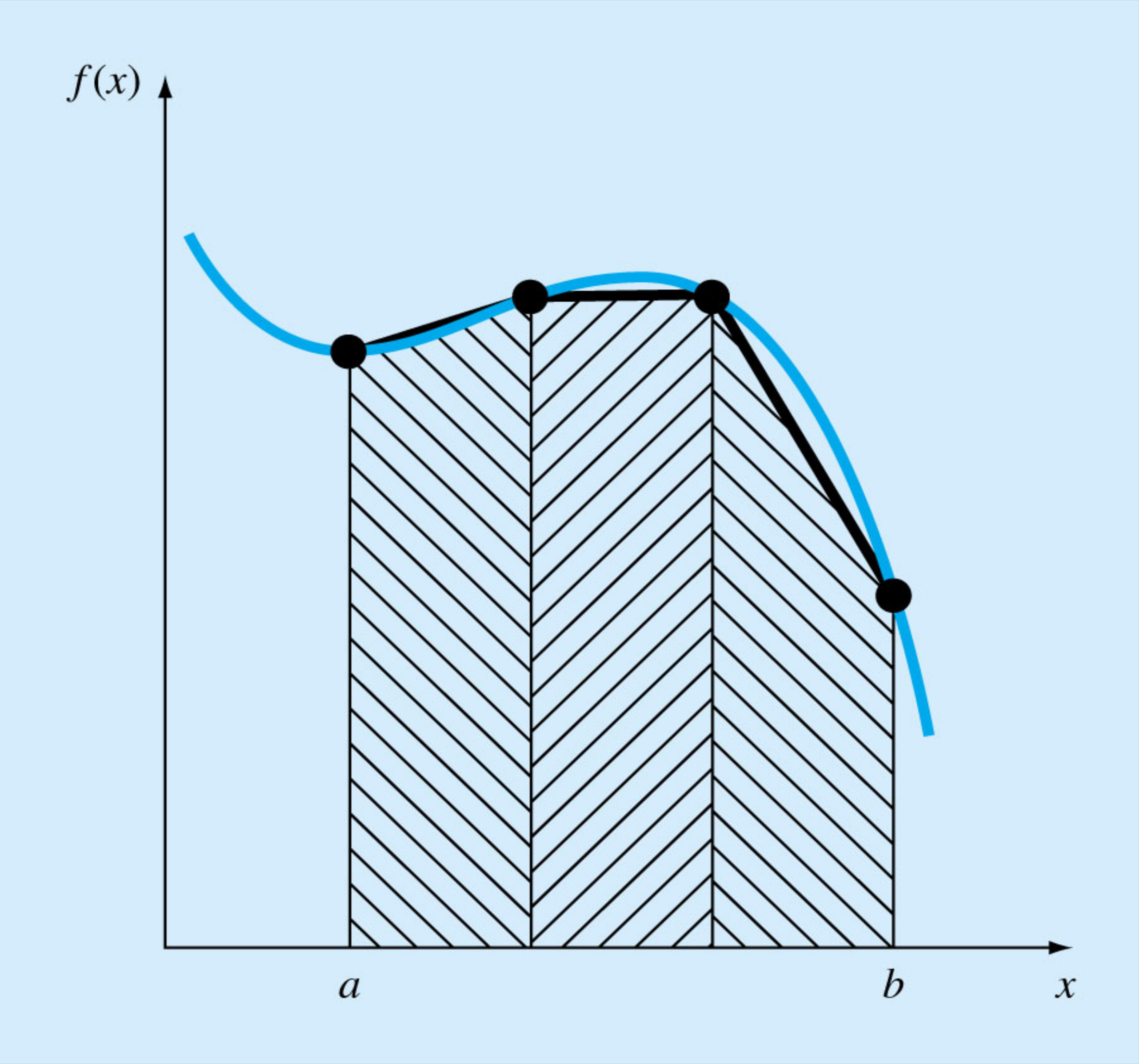


Figure 21.2



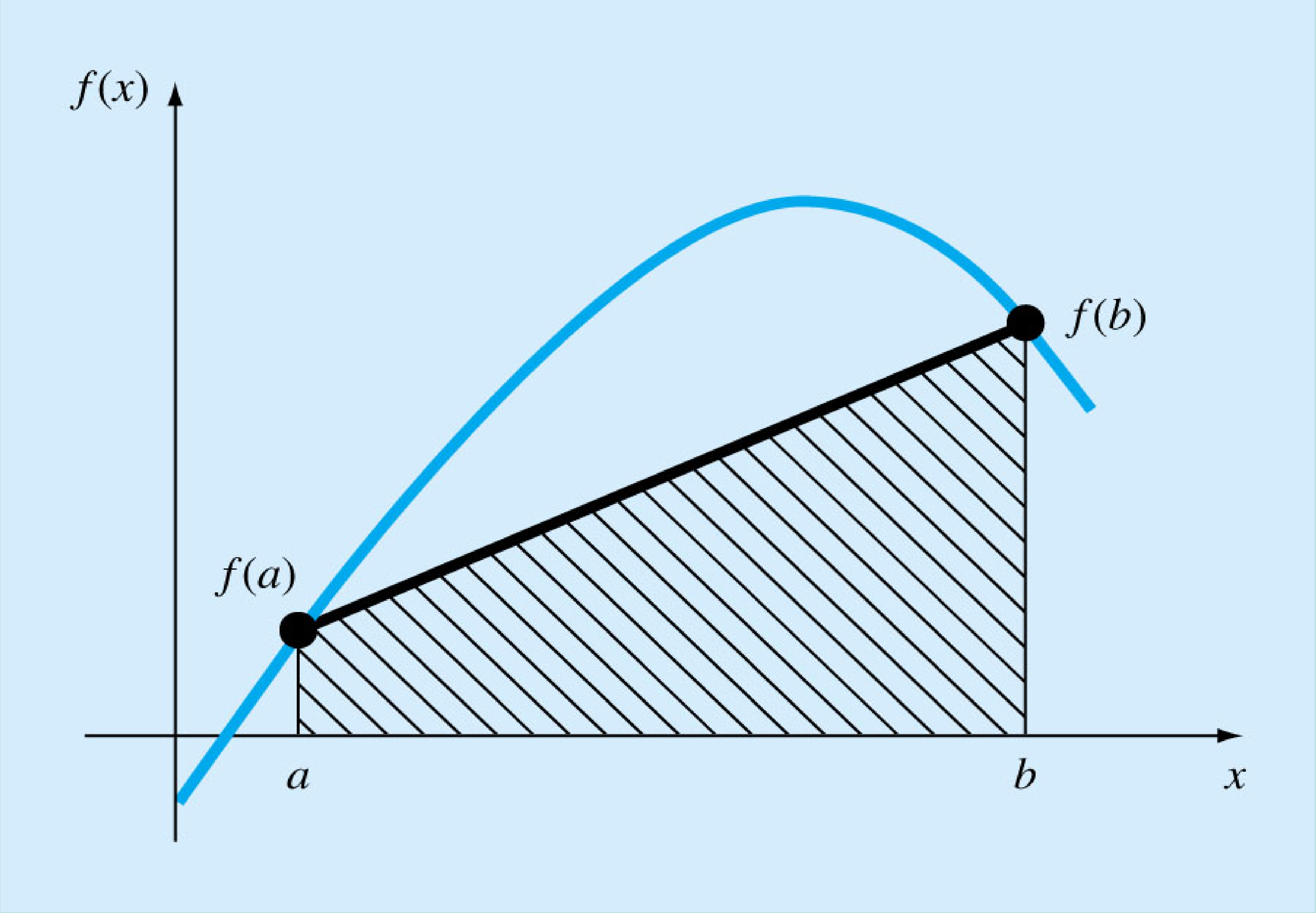
The Trapezoidal Rule

- The Trapezoidal rule is the first of the Newton-Cotes closed integration formulas, corresponding to the case where the polynomial is first order:

- The area under this first order polynomial is an estimate of the integral of f(x) between the limits of a and b:

Trapezoidal rule

Figure 21.4

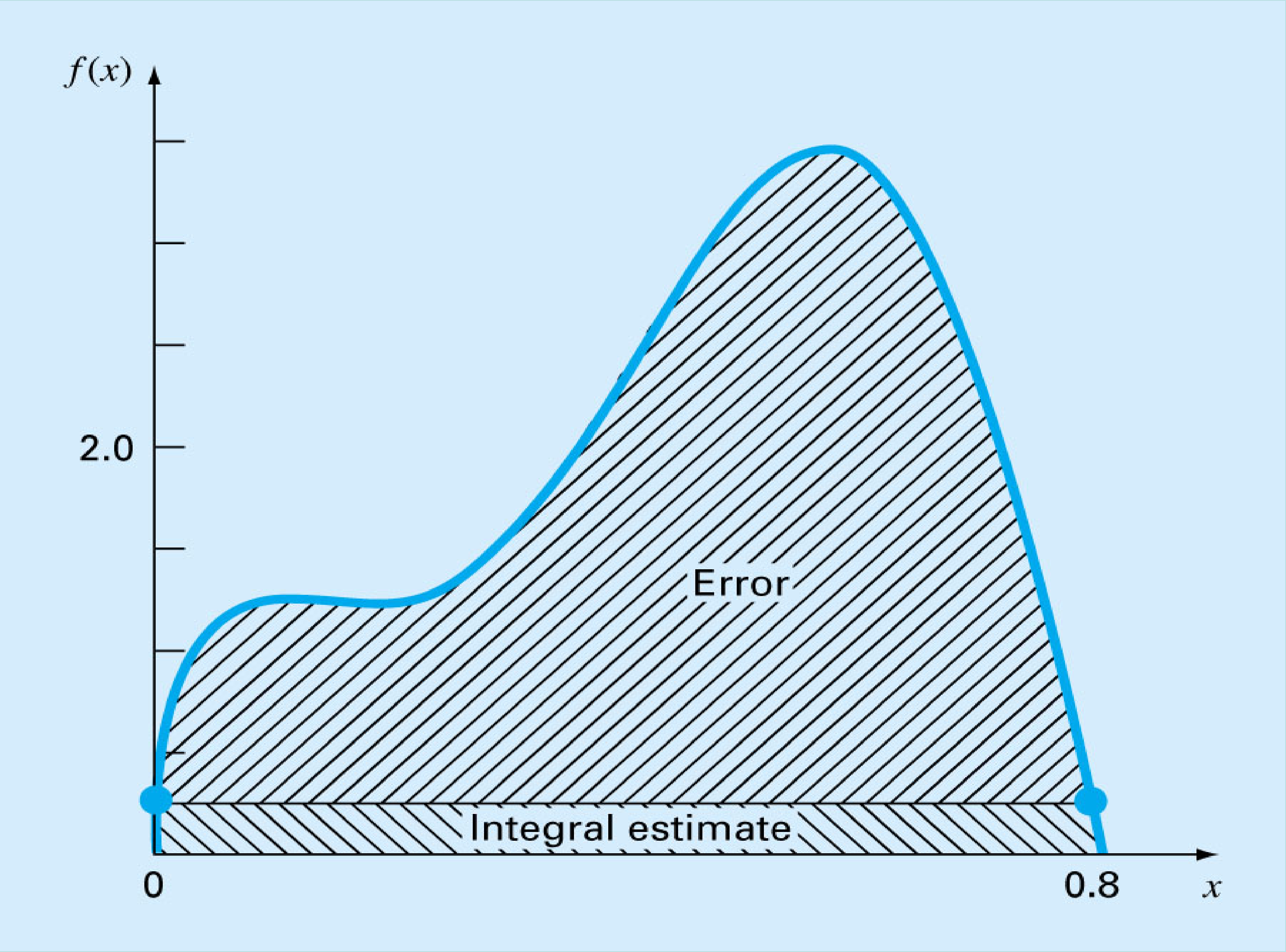


Error of the Trapezoidal Rule/

- When we employ the integral under a straight line segment to approximate the integral under a curve, error may be substantial:

where x lies somewhere in the interval from a to b.

Figure 21.6

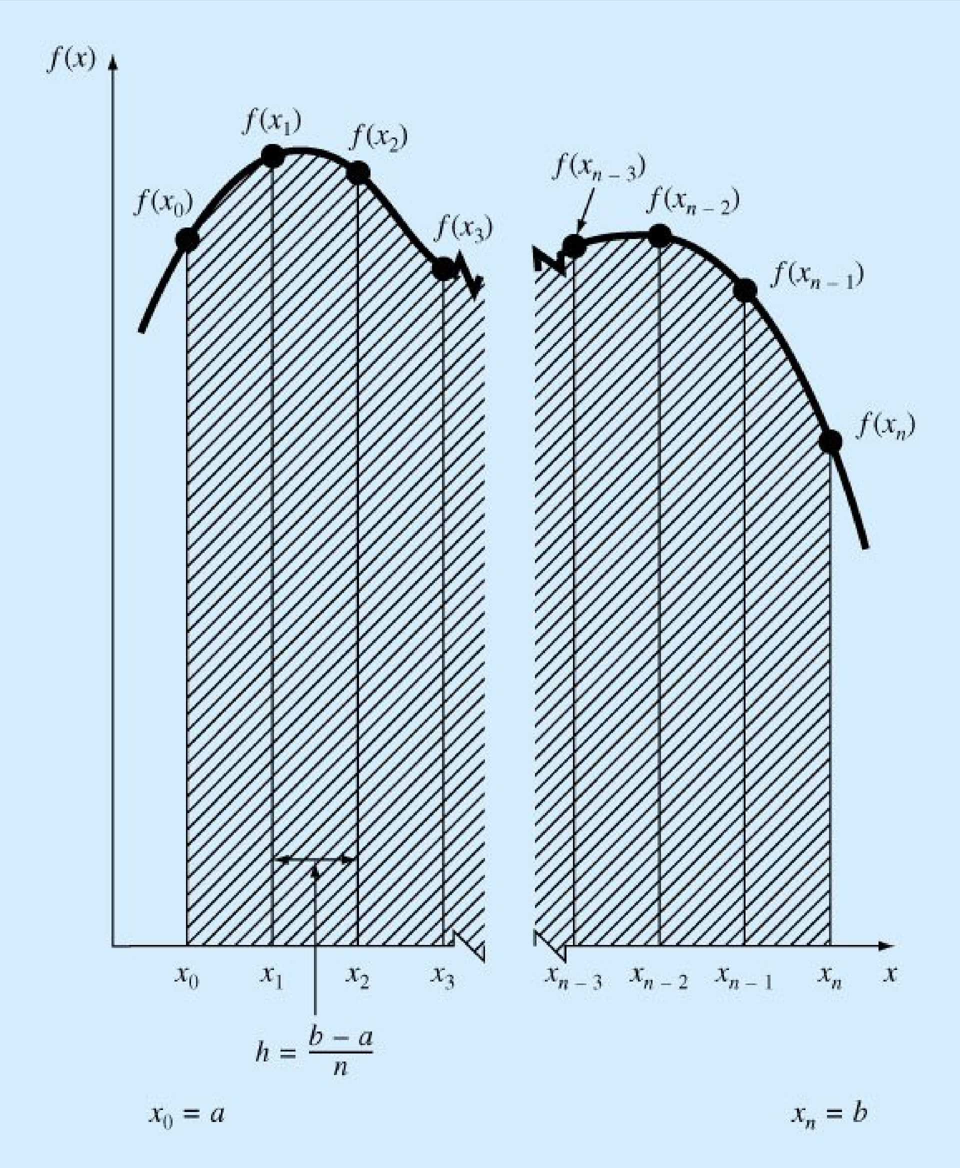


The Multiple Application Trapezoidal Rule/

- One way to improve the accuracy of the trapezoidal rule is to divide the integration interval from a to b into a number of segments and apply the method to each segment.
- The areas of individual segments can then be added to yield the integral for the entire interval.

Substituting the trapezoidal rule for each integral yields:

Figure 21.8



$f(x)$
$f(x_1)$
$f(x_2)$
$f(x_0)$
$f(x_3)$
$f(x_{n-3})$
$f(x_{n-2})$
$f(x_{n-1})$
$f(x_n)$
x_0
x_1
x_2
x_3
x_{n-3}
x_{n-2}
x_{n-1}
x_n
x
$h = \frac{b-a}{n}$
$x_0 = a$
$x_n = b$

- An error for multiple- application trapezoidal rule can be obtained by summing the individual errors for each segment:

Thus, if the number of segments is doubled, the truncation error will be quartered.

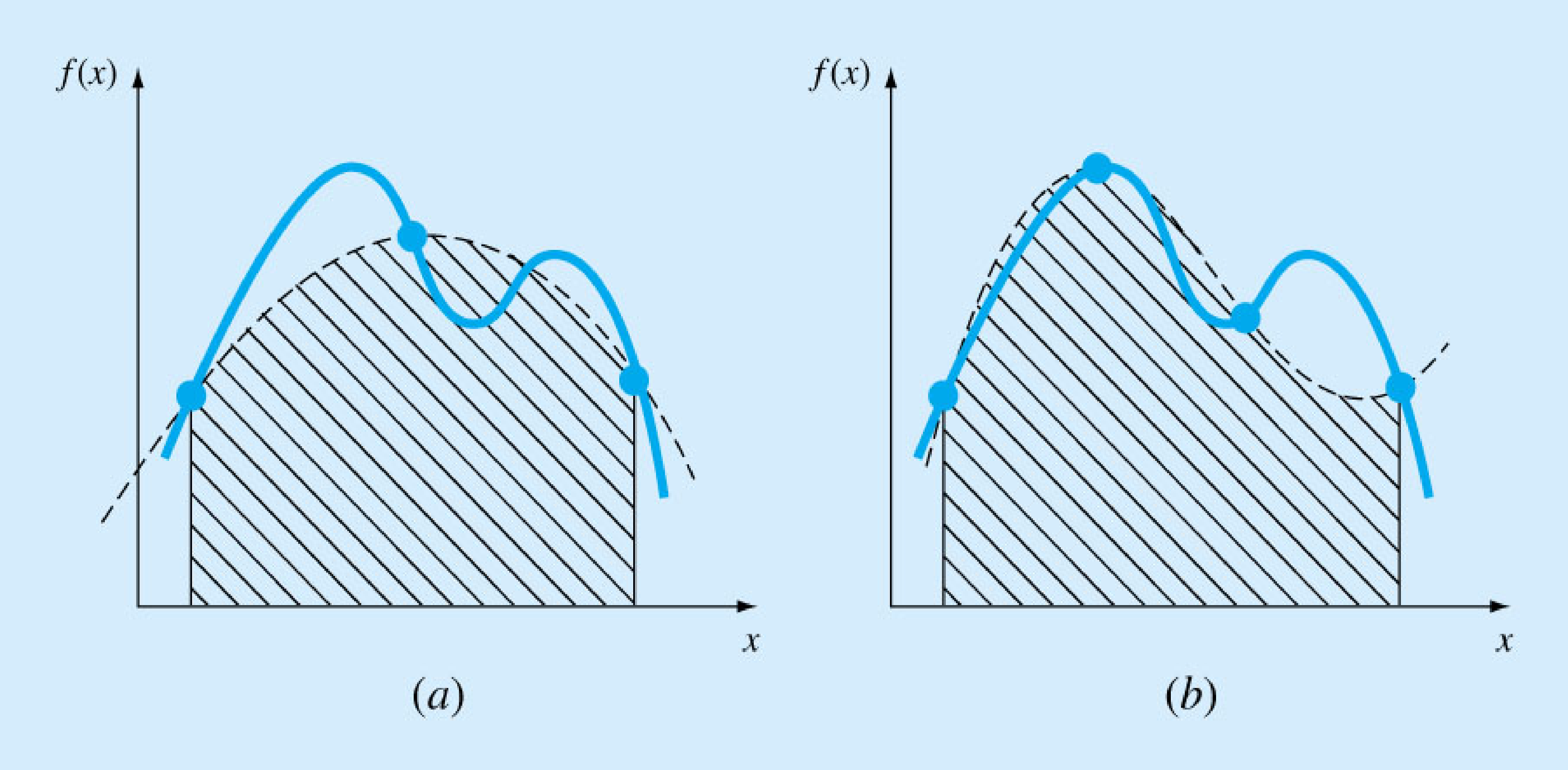
Simpson's Rules

- More accurate estimate of an integral is obtained if a high- order polynomial is used to connect the points. The formulas that result from taking the integrals under such polynomials are called Simpson's rules.

Simpson's 1/3 Rule/

- Results when a second- order interpolating polynomial is used.

Figure 21.10



Simpson's 1/3 Rule

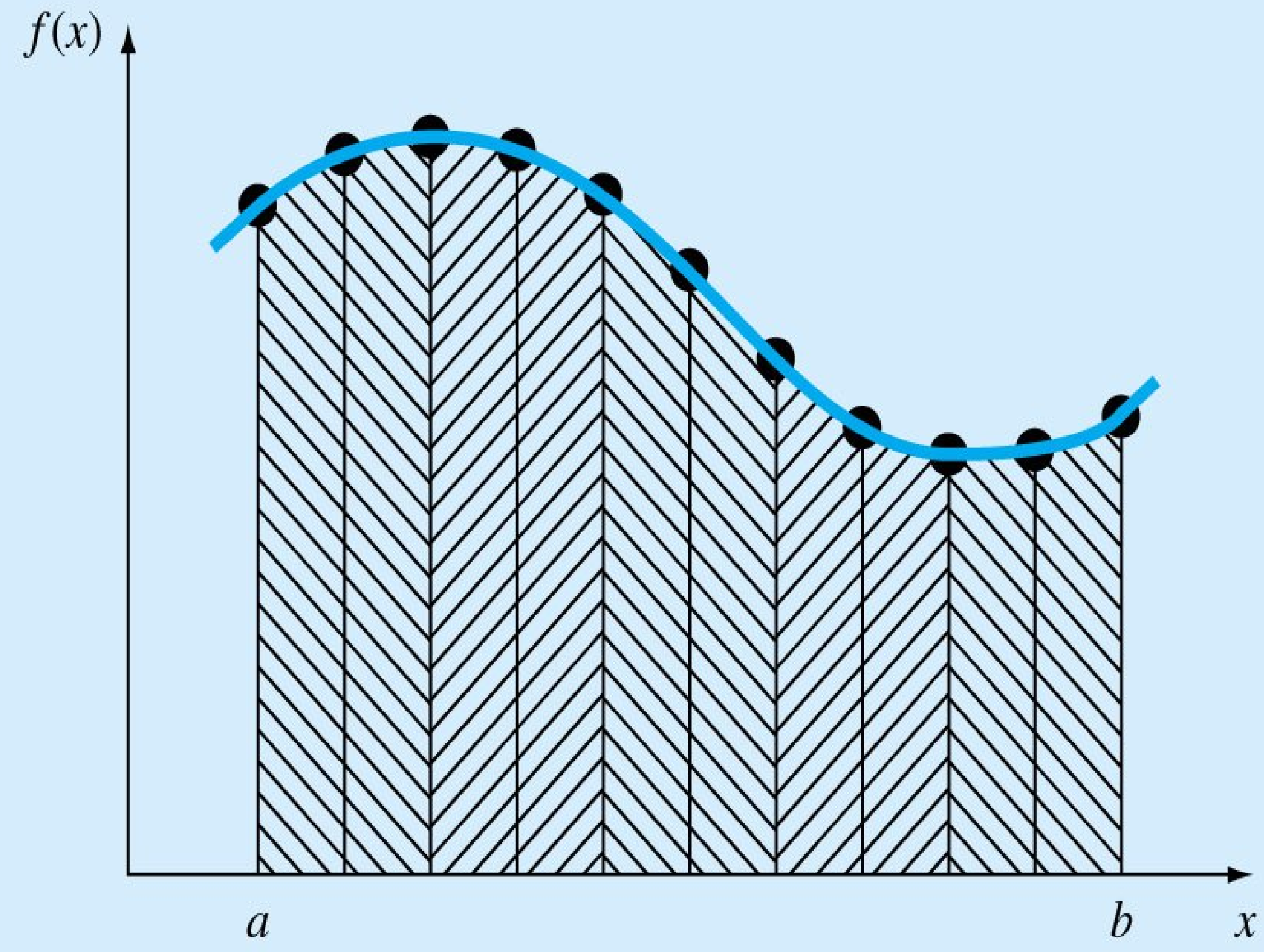
Single segment application of Simpson's 1/3 rule has a truncation error of:

Simpson's 1/3 rule is more accurate than trapezoidal rule.

The Multiple- Application Simpson's 1/3 Rule/

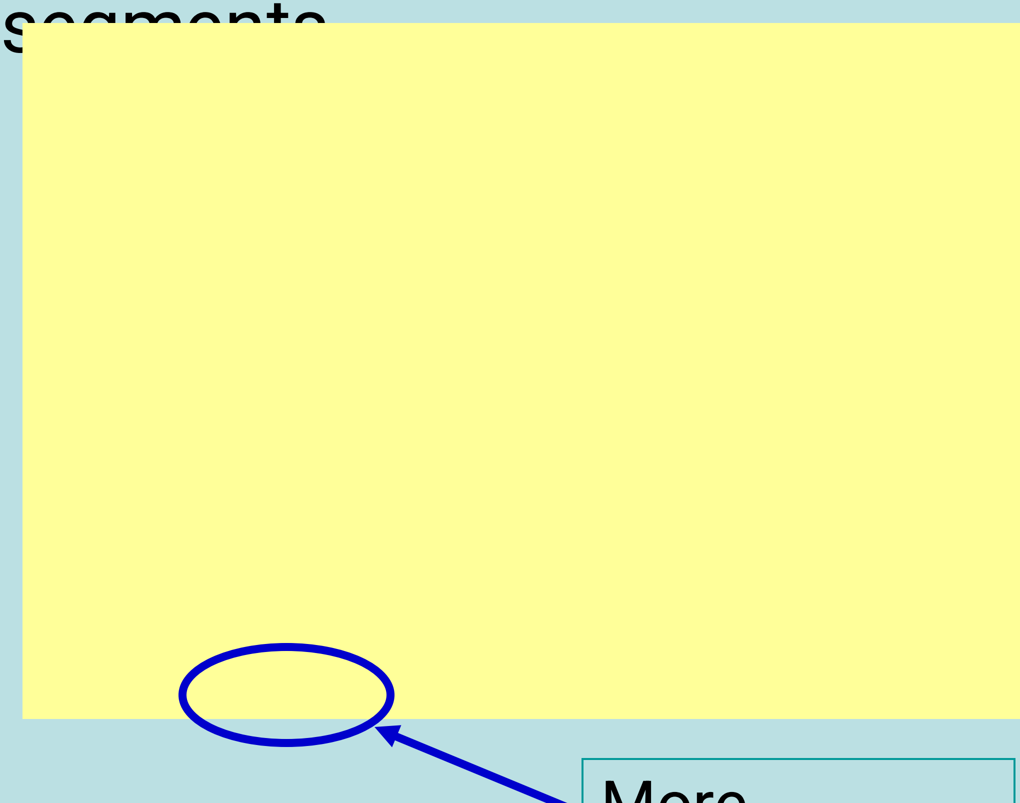
- Just as the trapezoidal rule, Simpson's rule can be improved by dividing the integration interval into a number of segments of equal width.
- Yields accurate results and considered superior to trapezoidal rule for most applications.
- However, it is limited to cases where values are equispaced.
- Further, it is limited to situations where there are an even number of segments and odd number of points.

Figure 21.11



Simpson's 3/8 Rule/

- An odd- segment- even- point formula used in conjunction with the 1/3 rule to permit evaluation of both even and odd numbers of segments.



More accurate

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Figure 21.12