Trapezoidal Rule of Integration

Major: All Engineering Majors

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Transforming Numerical Methods Education for STEM Undergraduates

Trapezoidal Rule of Integration

What is Integration

Integration:

The process of measuring the area under a function plotted on a graph.

Where:

f(x) is the integrand

a= lower limit of integration

b= upper limit of integration

Basis of Trapezoidal Rule

Trapezoidal Rule is based on the Newton- Cotes Formula that states if one can approximate the integrand as an nth order polynomial…

where

and

Basis of Trapezoidal Rule

Then the integral of that function is approximated by the integral of that n^{th} order polynomial.

Trapezoidal Rule assumes n=1, that is, the area   under the linear polynomial,

Derivation of the Trapezoidal Rule

Method Derived From Geometry

The area under the
curve is a trapezoid.
The integral

Example 1

The vertical distance covered by a rocket from t=8 to t=30 seconds is given by:

- Use single segment Trapezoidal rule to find the distance covered.
- Find the true error,   for part (a).
- Find the absolute relative true error,   for part (a).

Solution

a)

Solution (cont)

a)

b)   The exact value of the above integral is

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Solution (cont)

b)

c)   The absolute relative true error,   , would be

Multiple Segment Trapezoidal Rule

In Example 1, the true error using single segment trapezoidal rule was large. We can divide the interval [8,30] into [8,19] and [19,30] intervals and apply Trapezoidal rule over each segment.

Multiple Segment Trapezoidal Rule

With

Hence:

Multiple Segment Trapezoidal Rule

The true error is:

The true error now is reduced from - 807 m to - 205 m.

Extending this procedure to divide the interval into equal segments to apply the Trapezoidal rule; the sum of the results obtained for each segment is the approximate value of the integral.

Multiple Segment Trapezoidal Rule

Divide into equal segments as
shown in Figure 4. Then the
width of each segment is:

The integral I is:

Figure 4: Multiple (n=4) Segment Trapezoidal Rule

Multiple Segment Trapezoidal Rule

The integral **I** can be broken into h integrals as:

Applying Trapezoidal rule on each segment gives:

Example 2

The vertical distance covered by a rocket from to seconds is given by:

a) Use two- segment Trapezoidal rule to find the distance covered.
b) Find the true error,   for part (a).
c) Find the absolute relative true error,   for part (a).

Solution

a) The solution using 2- segment Trapezoidal rule is

Solution (cont)

Then:

Solution (cont)

b) The exact value of the above integral is

so the true error is

Solution (cont)

The absolute relative true error,   , would be

Solution (cont)

Table 1 gives the values obtained using multiple segment Trapezoidal rule for

Exact Value= 11061 m

n	Value	E_t		
1	11868	- 807	7.296	- - -
2	11266	- 205	1.853	5.343
3	11153	- 91.4	0.8265	1.019
4	11113	- 51.5	0.4655	0.3594
5	11094	- 33.0	0.2981	0.1669
6	11084	- 22.9	0.2070	0.09082
7	11078	- 16.8	0.1521	0.05482
8	11074	- 12.9	0.1165	0.03560

Table 1: Multiple Segment Trapezoidal Rule Values

Example 3

Use Multiple Segment Trapezoidal Rule to find the area under the curve

from   to

Using two segments, we get   and

Solution

Then:

Solution (cont)

So what is the true value of this integral?

Making the absolute relative true error:

Solution (cont)

Table 2: Values obtained using Multiple Segment Trapezoidal Rule for:

n	Approximate Value		
1	0.681	245.91	99.724%
2	50.535	196.05	79.505%
4	170.61	75.978	30.812%
8	227.04	19.546	7.927%
16	241.70	4.887	1.982%
32	245.37	1.222	0.495%
64	246.28	0.305	0.124%

Error in Multiple Segment Trapezoidal Rule

The true error for a single segment Trapezoidal rule is given by:

where   is some point in

What is the error, then in the multiple segment Trapezoidal rule? It will be simply the sum of the errors from each segment, where the error in each segment is that of the single segment Trapezoidal rule.

The error in each segment is

Error in Multiple Segment Trapezoidal Rule

Similarly:

It then follows that:

Error in Multiple Segment Trapezoidal Rule

Hence the total error in multiple segment Trapezoidal rule is

The term   is an approximate average value of the

Hence:

Error in Multiple Segment Trapezoidal Rule

Below is the table for the integral

as a function of the number of segments. You can visualize that as the number of segments are doubled, the true error gets approximately quartered.

n	Value			
2	11266	- 205	1.854	5.343
4	11113	- 51.5	0.4655	0.3594
8	11074	- 12.9	0.1165	0.03560
16	11065	- 3.22	0.02913	0.00401

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple- choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/trapezoidal_rule.html

THE END