

Simpson's $1/3^{\text{rd}}$ Rule of Integration

Major: All Engineering Majors

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Simpson's $1/3^{\text{rd}}$ Rule of Integration

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What is Integration?

Integration

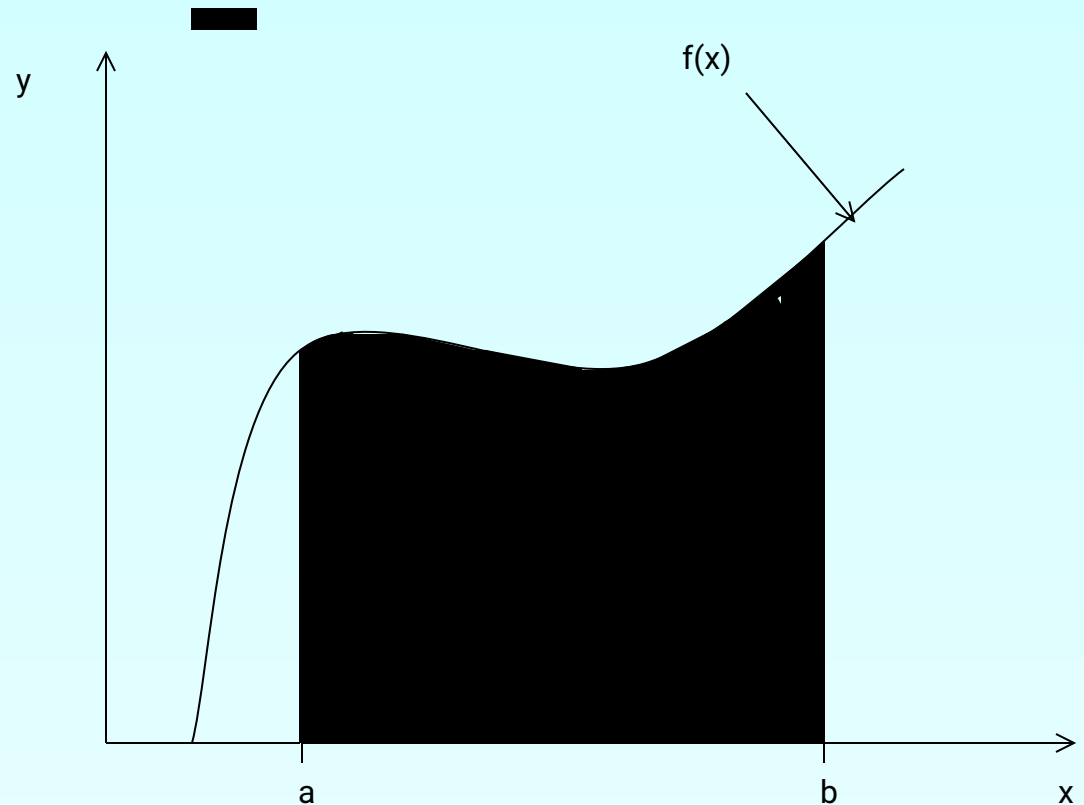
The process of measuring the area under a curve.

Where:

$f(x)$ is the integrand

a = lower limit of integration

b = upper limit of integration



Simpson's $1/3^{\text{rd}}$ Rule

Basis of Simpson's 1/3rd Rule

Trapezoidal rule was based on approximating the integrand by a first order polynomial, and then integrating the polynomial in the interval of integration. Simpson's 1/3rd rule is an extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.

Hence

Where $p(x)$ is a second order polynomial.

Basis of Simpson's 1/3rd Rule

Choose

and

as the three points of the function to evaluate a_0 , a_1 and a_2 .

Basis of Simpson's $1/3^{\text{rd}}$ Rule

Solving the previous equations for a_0 , a_1 and a_2 give

Basis of Simpson's $1/3^{\text{rd}}$ Rule

Then

Basis of Simpson's 1/3rd Rule

Substituting values of a_0 , a_1 , a_2 give

Since for Simpson's 1/3rd Rule, the interval $[a, b]$ is broken into 2 segments, the segment width

Basis of Simpson's $1/3^{\text{rd}}$ Rule

Hence

Because the above form has $1/3$ in its formula, it is called Simpson's $1/3^{\text{rd}}$ Rule.

Example 1

The distance covered by a rocket from $t=8$ to $t=30$ is given by

- a) Use Simpson's 1/3rd Rule to find the approximate value of x
- b) Find the true error,
- c) Find the absolute relative true error,

Solution

a)

Solution (cont)

b) The exact value of the above integral is

True Error

Solution (cont)

- c) Absolute relative true error,

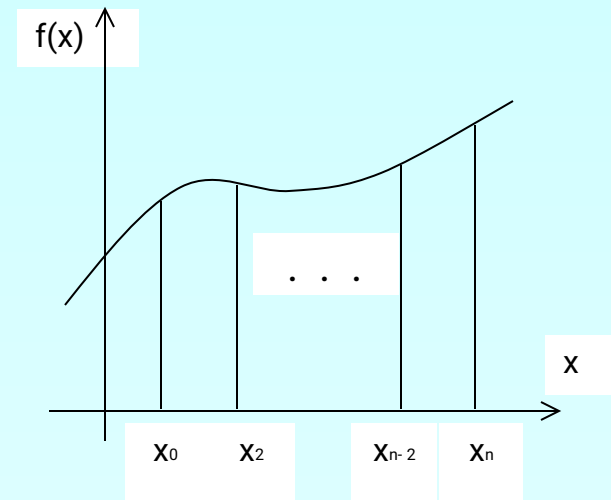
Multiple Segment Simpson's 1/3rd Rule

Multiple Segment Simpson's 1/3rd Rule

Just like in multiple segment Trapezoidal Rule, one can subdivide the interval $[a, b]$ into n segments and apply Simpson's 1/3rd Rule repeatedly over every two segments. Note that n needs to be even. Divide interval $[a, b]$ into equal segments, hence the segment width

where

Multiple Segment Simpson's 1/3rd Rule



Apply Simpson's 1/3rd Rule over each interval,

Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule

Since

Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule

Then

Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule

Example 2

Use 4- segment Simpson's 1/3rd Rule to approximate the distance covered by a rocket from $t= 8$ to $t=30$ as given by

- Use four segment Simpson's 1/3rd Rule to find the approximate value of x .
- Find the true error, for part (a).
- Find the absolute relative true error, for part (a).

Solution

a) Using n segment Simpson's 1/3rd Rule,

So

Solution (cont.)

Solution (cont.)

cont.

Solution (cont.)

- b) In this case, the true error is

- c) The absolute relative true error

Solution (cont.)

Table 1: Values of Simpson's 1/3rd Rule for Example 2 with multiple segments

n	Approximate Value	E_t	$ \epsilon_t $
2	11065.72	4.38	0.0396%
4	11061.64	0.30	0.0027%
6	11061.40	0.06	0.0005%
8	11061.35	0.01	0.0001%
10	11061.34	0.00	0.0000%

Error in the Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule

The true error in a single application of Simpson's $1/3^{\text{rd}}$ Rule is given as

In Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule, the error is the sum of the errors in each application of Simpson's $1/3^{\text{rd}}$ Rule. The error in n segment Simpson's $1/3^{\text{rd}}$ Rule is given by

Error in the Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule

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Error in the Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule

Hence, the total error in Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule is

Error in the Multiple Segment Simpson's $1/3^{\text{rd}}$ Rule

The term $\frac{1}{2}(f_0 + f_n)$ is an approximate average value of

Hence

where

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/simpsons_13rd_rule.html

THE END

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