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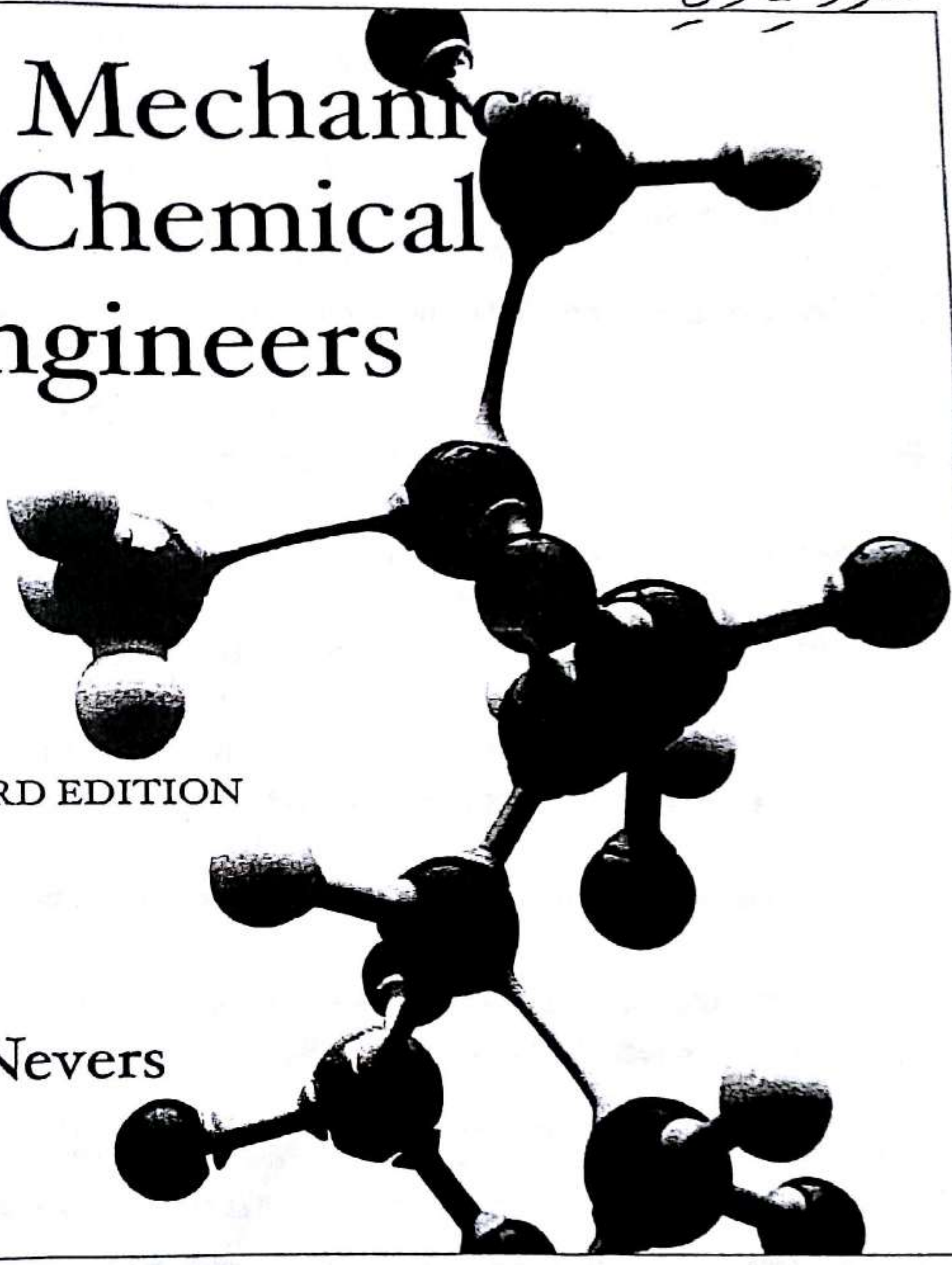
Fluid Mechanics for Chemical Engineers



THIRD EDITION

Noel de Nevers

McGRAW-HILL INTERNATIONAL EDITION

CONVERSION FACTORS*

Length:

1 ft = 0.3048 m = 12 in = mile / 5280 = nautical mile / 6076 = km / 3281

1 m = 3.281 ft = 39.37 in = 100 cm = 1000 mm = 10^6 micron = 10^{10} Å

= km / 1000

Mass:

1 lbm = 0.45359 kg = short ton / 2000 = long ton / 2240 = 16 oz (av.)

= 14.58 oz (troy) = metric ton (tonne) / 2204.63 = 7000 grains

= slug / 32.2

1 kg = 2.2046 lbm = 1000 g = (metric ton or tonne or Mg) / 1000

Force:

1 lbf = 4.4482 N = 32.2 lbm · ft / s^2 = 32.2 poundal = 0.4536 kgf

1 N = 0.2248 lbf = kg · m / s^2 = 10^5 dyne = kgf / 9.81

Volume:

1 ft^3 = 0.02831 m^3 = 28.31 L = 7.48 U.S. gal = 6.23 Imperial gal

= acre-ft / 43,560

1 U.S. gallon = 231 in^3 = barrel (petroleum) / 42 = barrel (beer, U.S.A.) / 31

= 4 U.S. quarts = 8 U.S. pints

= 3.785 L = 0.003785 m^3

1 m^3 = 35.29 ft^3 = 1000 L

Energy:

1 Btu = 1055 J = 1.055 kW · s = $2.93 \cdot 10^{-4}$ kWh = 252 cal = 777.97 ft · lbf

= $3.93 \cdot 10^{-4}$ hp · h

1 J = 1 N · m = 1 W · s = 1 V · C = $9.48 \cdot 10^{-4}$ Btu = 0.239 cal = 10^7 erg

= $6.24 \cdot 10^{18}$ eV

*These values are mostly rounded. There are several definitions for some of these quantities, e.g., the Btu and the calorie; these differ from each other by up to 0.2 percent. For the most accurate values see the *ASTM Metric Practice Guide*, ASTM Publication No. E 380-97, Philadelphia, 1997.

Power:

$$1\ \text{hp} = 0.746\ \text{kW} = 550\ \text{ft} \cdot \text{lbf}/\text{s} = 33{,}000\ \text{ft} \cdot \text{lbf}/\text{min} = 2545\ \text{Btu}/\text{h}$$

$$1\ \text{W} = 1.34 \cdot 10^{-3}\ \text{hp} = \text{J}/\text{s} = \text{N} \cdot \text{m}/\text{s} = \text{V} \cdot \text{A} = 0.239\ \text{cal}/\text{s}$$

$$= 9.49 \cdot 10^{-4}\ \text{Btu}/\text{s}$$

Pressure:

$$1\ \text{atm} = 101.3\ \text{kPa} = 1.013\ \text{bar} = 14.696\ \text{lbf}/\text{in}^2 = 33.89\ \text{ft of water}$$

$$= 29.92\ \text{in of mercury} = 1.033\ \text{kgf}/\text{cm}^2 = 10.33\ \text{m of water}$$

$$= 760\ \text{mm of mercury} = 760\ \text{torr}$$

$$1\ \text{psi} = \text{atm}/14.696 = 6.89\ \text{kPa} = 27.7\ \text{in}\ H_2O = 51.7\ \text{torr}$$

$$1\ \text{Pa} = \text{N}/\text{m}^2 = \text{kg}/\text{m} \cdot \text{s}^2 = 10^{-5}\ \text{bar} = 1.450 \cdot 10^{-4}\ \text{lbf}/\text{in}^2$$

$$= 0.0075\ \text{torr} = 0.0040\ \text{in}\ H_2O = 10\ \text{dyne}/\text{cm}$$

Viscosity:

$$1\ \text{cP} = 0.01\ \text{poise} = 0.01\ \text{g}/\text{cm} \cdot \text{s} = 0.001\ \text{kg}/\text{m} \cdot \text{s} = 0.001\ \text{N} \cdot \text{s}/\text{m}^2$$

$$= 0.001\ \text{Pa} \cdot \text{s} = 0.01\ \text{dyne} \cdot \text{s}/\text{cm}^2$$

$$= 6.72 \cdot 10^{-4}\ \text{lbm}/\text{ft} \cdot \text{s} = 2.42\ \text{lbm}/\text{ft} \cdot \text{h} = 2.09 \cdot 10^{-5}\ \text{lbf} \cdot \text{s}/\text{ft}^2$$

Kinematic viscosity:

$$1\ \text{cSt} = 0.01\ \text{Stoke} = 0.01\ \text{cm}^2/\text{s} = 10^{-6}\ \text{m}^2/\text{s} = 1\ \text{cP}/(\text{g}/\text{cm}^3)$$

$$= 1.08 \cdot 10^{-5}\ \text{ft}^2/\text{s} = \text{cP}/(62.4\ \text{lbm}/\text{ft}^3)$$

Temperature:

$$\text{K} = {}^\circ\text{C} + 273.15 = {}^\circ\text{R}/1.8 \approx {}^\circ\text{C} + 273 \qquad {}^\circ\text{C} = ({}^\circ\text{F} - 32)/1.8$$

$${}^\circ\text{R} = {}^\circ\text{F} + 459.67 \approx {}^\circ\text{F} + 460 = 1.8\ \text{K} \qquad {}^\circ\text{F} = 1.8{}^\circ\text{C} + 32$$

Psia, psig:

Psia means pounds per square inch, absolute. Psig means pounds per square inch, gauge, i.e., above or below the local atmospheric pressure.

Force-mass conversion factor, g_c

This factor is equal to dimensionless 1.00. Any dimensioned quantity may be multiplied or divided by g_c without changing the value of that quantity.

$$g_c = 1.0 = 32.2\ \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} = 1\ \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} = 1\ \frac{\text{lbm} \cdot \text{ft}}{\text{poundal} \cdot \text{s}^2} = 1\ \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2}$$

$$= 9.81\ \frac{\text{kgmass} \cdot \text{m}}{\text{kgforce} \cdot \text{s}^2}$$

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CHAPTER 1

INTRODUCTION

1.1 WHAT IS FLUID MECHANICS?

Mechanics is the study of forces and motions. Therefore, fluid mechanics is the study of forces and motions in fluids. But what is a fluid? We all can think of some things that obviously are fluids: air, water, gasoline, lubricating oil, and milk. We also can think of some things that obviously are not fluids: steel, diamonds, rubber bands, and paper. These we call solids. But there are some very interesting intermediate types of matter: gelatin, peanut butter, cold cream, mayonnaise, toothpaste, roofing tar, library paste, bread dough, and auto grease.

To decide what we mean by the word "fluid," we first have to consider the idea of shear stress. It is easiest to discuss shear stress in comparison with tensile stress and compressive stress; see Fig. 1.1.

In Fig. 1.1(*a*) a rope is holding up a weight. The weight exerts a force that tends to pull the rope apart. A stress is the ratio of the applied force to the area over which it is exerted (force/area). Thus, the stress in the rope is the force exerted by the weight divided by the cross-sectional area of the rope. The force that tries to pull things apart is called a *tensile force,* and the stress it causes is called a *tensile stress.*

In Fig. 1.1(*b*) a steel column is holding up a weight. The weight exerts a force that tends to crush the column. This kind of force is called a *compressive force,* and the stress in the column, the force divided by the cross-sectional area of the column, is called a *compressive stress.*

In Fig. 1.1(*c*) some glue is holding up a weight. The weight exerts a force that tends to pull the weight down the walls and thus to *shear* the glue. This force, which tends to make one surface *slide* parallel to an adjacent surface, is called a *shear force,* and the stress in the glue, the force divided by the area of the glue joint, is called a *shear stress.*

A more detailed examination of these examples would show that all three kinds of stress are present in each case, but those we have identified are the main ones. (For more information on this topic, see any text on strength of materials.)

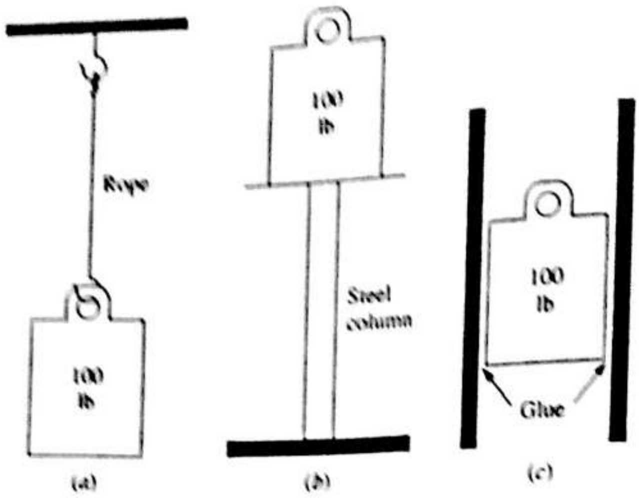

FIGURE 1.1
Comparison of tensile, compressive and shear stresses. (*a*) The rope is in tensile stess; (*b*) the column is in compressive stress; (*c*) the glue is in shear stress.

Solids are substances that can permanently resist very large shear forces. When subject to a shear force, they move a short distance (elastic deformation), thereby setting up internal shear stresses that resist the external force, *and then they stop moving*. Materials that obviously are fluids cannot permanently resist a shear force, no matter how small. When subject to a shear force, they start to move and *keep on moving as long as the force is applied.*

Substances intermediate between solids and fluids can permanently resist a small shear force but cannot permanently resist a large one. For example, if we put a "blob" of any obvious liquid on a vertical wall, gravity will make it run down the wall. If we attach a piece of steel or diamond securely to a wall, it will remain there, no matter how long we wait. If we attach some peanut butter to a wall, it will probably stay, but if we increase the shear stress on the peanut butter by spreading it with a knife, it will flow like a fluid. We cannot spread steel with a knife as we spread peanut butter.

If, as shown above, the relevant difference between peanut butter and steel is the magnitude of the shear stress that the material can resist, then the difference is one of degree, not of kind. At extreme shear stresses steel can be made to "flow like a fluid." In the remainder of this book we will be talking mostly about materials such as air and water, which cannot permanently resist any shear force. However, it is well to keep our minds open to other possibilities of "fluid" behavior [1]. (Numbers in brackets refer to items listed in the References at the end of the chapter.)

1.2 WHAT GOOD IS FLUID MECHANICS?

The problems in fluid mechanics are basically no different from those in "ordinary" mechanics (the mechanics of solids) or in thermodynamics. Therefore, in principle one can solve problems in fluid mechanics with the same methods used to solve problems in mechanics or thermodynamics. However, for many of the problems involving the flow of fluids (or the movement of bodies through fluids), we use a combination of the problem-solving methods of mechanics and thermodynamics. Furthermore, the methods that work for hydraulics problems (dams, canals, locks, river flow, etc.) are applicable, with slight modifications, to aerodynamics problems (airplanes, rockets, wind forces on bridges, etc.) and to problems of special interest to chemical engineers such as the flow in chemical reactors, in distillation columns, or in polymer extrusion dies. Therefore, it makes sense to combine the study of this class of similar problems into one discipline, which we call fluid mechanics.

Consider the important fluids in our lives: the air we breathe, the water we drink, many of the foods we consume, most of the fuels for heating our houses or propelling our vehicles, and the various fluids in our bodies that make up our internal environment. Without some idea of the behavior of fluids, we can have only a very limited understanding of how the world works.

Some of the subdivisions and applications of fluid mechanics are:

1. Hydraulics: the flow of water in rivers, pipes, canals, pumps, turbines.
2. Aerodynamics: the flow of air around airplanes, rockets, projectiles, structures.
3. Meteorology: the flow of the atmosphere.
4. Particle dynamics: the flow of fluids around particles, the interaction of particles and fluids (i.e., dust settling, slurries, pneumatic transport, fluidized beds, air pollutant particles, corpuscles in our blood).
5. Hydrology: the flow of water and water-borne pollutants in the ground.
6. Reservoir mechanics: the flow of oil, gas, and water in petroleum reservoirs.
7. Multiphase flow: coffee percolators, oil wells, carburetors, fuel injectors, combustion chambers, sprays.
8. Combinations of fluid flow: with chemical reactions in combustion, with electromagnetic phenomena in magnetohydrodynamics, with mass transport in distillation or drying.
9. Viscosity-dominated flows: lubrication, injection molding, wire coating, lava, and continental drift.

1.3 BASIC IDEAS IN FLUID MECHANICS

Fluid mechanics is based largely on working out the detailed consequences of four basic ideas:

1. The principle of the conservation of mass.
2. The first law of thermodynamics (the principle of the conservation of energy).
3. The second law of thermodynamics.
4. Newton's second law of motion, which may be summarized in the form $F = ma$.

Each of these four ideas is a generalization of experimental data. None of them can be deduced from the others or from any other prior principle. None of them can be "proven" mathematically. Rather, they stand on their ability to predict correctly the results of any experiment ever run to test them.

Sometimes in fluid mechanics we may start with these four ideas and the measured physical properties of the fluid(s) and proceed directly to solve mathematically for the desired forces, velocities, etc. This is generally possible only in the case of very simple flows. The observed behavior of a great many fluid flows is too complex to be solved directly from these four principles, so we must resort to experimental tests. Through the use of techniques called dimensional analysis (Chap. 9), we often can use the results of one experiment to predict the results of a much different experiment. Thus, careful experimental work is very important in fluid mechanics. With modern computers we can find useful numerical solutions to problems which would previously have required experimental tests. The methods for doing that are outlined in part IV of this book. As computers become faster and cheaper, we will see additional complex fluid mechanics problems solved on computers. Ultimately, though, the computer solutions must be tested experimentally.

These four ideas are applied to fluid mechanical problems as follows. This introductory chapter launches our study and defines some important terms. Then Part I of the book, Chaps. 2–4, deals with preliminaries. We will need these in our study of moving fluids, and they provide direct solutions and/or insight into many practical problems. Parts II and III, Chaps. 5–14, deal with the flow of fluids that are one-dimensional or can be treated as if they were. Part IV, Chaps. 15–20, deals with two- and three-dimensional fluid mechanics. Each of these sections will be described as we begin them.

Students using this book should have previously completed a course in elementary thermodynamics. Chapters 3 and 4 should serve as a review of matter previously covered; they are included because the principles involved are central to fluid mechanics. It is assumed that the student is familiar with the second law of thermodynamics, which is used occasionally. Remember that this entire book is devoted to the application of the four basic ideas and the results of experimental tests to fluid-flow problems. Although the details can become quite involved, the basic ideas are few.

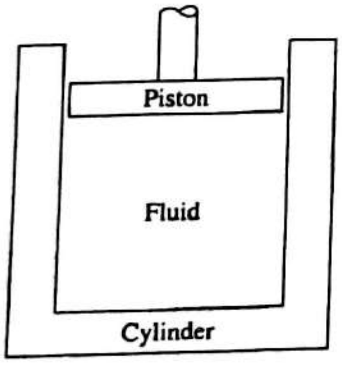

FIGURE 1.2
Piston and cylinder. If the fluid is a gas, we can move the piston up and down as much as we like, and the gas will expand or contract to fill the volume available. If the fluid is a liquid, we can move the piston down very little without producing extreme pressures; if we move it up, the liquid must partly evaporate to produce a gas to fill the space.

1.4 LIQUIDS AND GASES

Fluids are of two types, liquids and gases. On the molecular level these are quite different. In liquids the molecules are close together and are held together by significant forces of attraction; in gases the molecules are relatively far apart and have very weak forces of attraction. As a rule, the specific volumes of gases are $\approx$1000 times those of liquids, which means that the average intermolecular distance (center to center of the molecules) is roughly 10 times as far in a typical gas as in a typical liquid. As temperature and pressure increase, these differences become less and less, until the liquid and gas become identical at the critical temperature and pressure. The difference between the behavior of liquids and gases is most marked when these fluids are expanded. Suppose that some fluid completely fills the space below the piston in Fig. 1.2. When we raise the piston, the volume occupied by the fluid is increased.

If the fluid is a gas, it will expand readily, filling all the space vacated by the piston; gases can expand without limit to occupy space made available to them. But if the fluid is a liquid, then as the piston is raised, the liquid can expand only a small amount, and then it can expand no more. What fills the space between the piston and the liquid? Part of the liquid must turn into a gas by boiling, and this gas expands to fill the vacant space. This can be explained on the molecular level by saying that there is a maximum distance between molecules over which the attractive forces hold them together to form a liquid and that, when the molecules separate more than this distance, they cease behaving as a liquid and behave as a gas.

Because of their closer molecular spacing, liquids normally have higher densities, viscosities, refractive indices, etc., than gases (see Prob. 1.2). In engineering this frequently leads to quite different behaviors of liquids and gases, as we will see.

1.5 PROPERTIES OF FLUIDS

The physical properties of fluids that will enter our calculations most often are density, viscosity, and surface tension.

1.5.1 Density

The *density* ρ is defined the mass per unit volume:

$$\rho = \frac{m}{V} \tag{1.1}$$

We are all aware of the differences in density between various materials, such as that between lead and wood. How can we measure the density of a material? If we want to know the density of a liquid, we can weigh a bottle of known volume (determine its mass), fill it with the liquid, weigh it again, and compute the density with the aid of Eq. 1.1. (This is one of the standard laboratory methods of determining liquid density; the special weighing bottles designed for this purpose are called *pyncnometers*, Prob. 1.5) If we want to know the density of a cubical solid block, we can measure the length of its sides, compute its volume, weigh it, and apply these results to Eq. 1.1.

Now suppose we are asked to determine the density of a piece of Swiss cheese. If we have a large block of the cheese, we can cut off a cube, measure its sides, compute its volume, weigh it, and then calculate its density. This is an average density, one that includes the density of the air in the holes in the cheese. As long as we are dealing with large pieces of cheese, it is a satisfactory density. Suppose, however, we are asked to find the density at some point inside a large block of the cheese. If we can cut the cheese open, and if we find that the point in question is in the solid cheese and not in one of its holes, we can find the density easily enough or, if the point in question is in a hole, we find the density of the air in the hole. But if the point is on the surface of a hole, the problem is more difficult. Then the density is discontinuous; see Fig. 1.3. There is no meaningful single value of the density at x.

Why this long discussion about the density of Swiss cheese? Because the world is full of holes! Atomic physics tells us that even in a solid bar of steel the space occupied by the electrons, protons, and neutrons is a very small fraction of the total space; the rest presumably is empty. Furthermore, even at the molecular level there

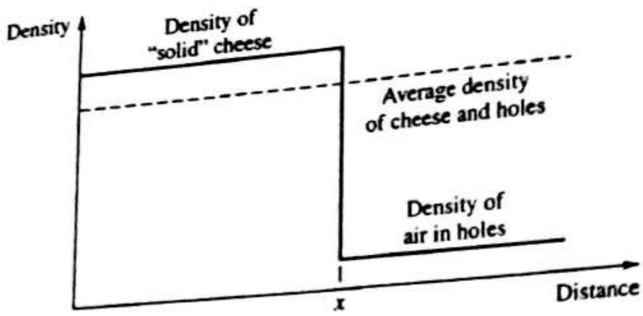

FIGURE 1.3
The density of Swiss cheese is not uniform from point to point, but has local point densities and an average density.

are holes; in a typical gas the space actually occupied by the individual gas molecules at any instant is a small fraction of the total space. Thus, in any attempt to speak of density at a given point we are in the same trouble as with the Swiss cheese. Therefore, we must restrict the definition of density to samples large enough to average out the holes. This causes no problem in fluid mechanics, because of the size of the samples normally used, but it indicates that the concept of density does not readily apply to samples of molecular and subatomic sizes.

In addition, we must be careful in defining the densities of composite materials. For example, a piece of reinforced concrete consists of several parts with different densities. In discussing such materials we must distinguish between the *particle densities* of the individual pebbles or steel-reinforcing bars and the *bulk density* of the mixed mass. When we refer to bulk density, our sample must be large compared with the dimensions of one particle. Some examples of composite solid materials are cast iron, fiberglass-reinforced plastics, and wood. Some examples of composite liquids are slurries, such as muds, milkshakes, and toothpaste, and emulsions, such as homogenized milk, mayonnaise, and cold cream. Smokes and clouds behave as composite gases.

Example 1.1. A typical mud is 70 wt. % sand and 30 wt. % water. What is its density? The sand is practically pure quartz (SiO_2), for which ρ_{sand} = 165 lbm / ft^3 (2.65 g / cm^3). See the inside back cover for the properties of water used in all examples and problems.

Here we assume that there is no volume change on mixing sand and water. There are volume changes on mixing for some substances like ethanol and water, but they are small enough to ignore for most problems, including this one. Then

$$\rho = \frac{m}{V} = \frac{m_{sand} + m_{water}}{V_{sand} + V_{water}} = \frac{m_{sand} + m_{water}}{(m/\rho)_{sand} + (m/\rho)_{water}} \tag{1.A}$$

[Every equation in this book has a number. Those, like this one, that are parts of examples or in other ways specific to some situation are identified with number-letter combinations, such as (1.A). General equations have number-number combinations, such as (1.1).]

We could simplify Eq. 1.A algebraically, but a more intuitive approach is to choose as our *basis* 100 lbm of mud, and substitute into Eq. 1.A, finding

$$\rho = \frac{m_{\text{sand}} + m_{\text{water}}}{\left(\frac{m}{\rho}\right)_{\text{sand}} + \left(\frac{m}{\rho}\right)_{\text{water}}} = \frac{70 \text{ lbm} + 30 \text{ lbm}}{\left(\frac{70 \text{ lbm}}{165 \text{ lbm / ft}^3}\right)_{\text{sand}} + \left(\frac{30 \text{ lbm}}{62.3 \text{ lbm / ft}^3}\right)_{\text{water}}}$$

$$= 110.4 \frac{\text{lbm}}{\text{ft}^3} = 1769 \frac{\text{kg}}{\text{m}^3} \tag{1.B}$$

■

The ■ indicates the end of an example.

1.5.2 Specific Gravity

Specific gravity of liquids and solids (SG) is defined as

$$\text{SG} = \frac{\text{density}}{\text{density of water at some specified temperature and pressure}} \tag{1.2}$$

This definition has the merit of being a ratio and, hence, a pure number, which is independent of the system of units chosen. Occasionally it leads to confusion, because some specific gravities are referred to water at 60°F, some to water at 70°F, and some to water at 39°F = 4°C (all at a pressure of 1 atm). The differences are small but great enough to cause trouble.

If the temperature of the water is specified as 39°F = 4°C, then the density of water is 1.000 g / cm^3. (The gram was defined to make this number come out 1.000). Thus, if this basis of measurement is chosen, then specific gravities become numerically identical with densities expressed in g / cm^3 or kg / L or metric tons / m^3. The mud in Example 1.1 has SG = 1.769.

Many process industries use special scales of fluid density, which are usually referred to as *gravities*. Some of them are the API gravity (American Petroleum Institute) for oil and petroleum products (Prob. 1.6), Brix gravity for the sugar industry, and Baumé gravity for sulfuric acid. Each scale is directly convertible to density; conversion tables and formulae are available in handbooks.

Specific gravities of gases are normally defined as

$$\begin{pmatrix}\text{SG of}\\ \text{a gas}\end{pmatrix} = \left(\frac{\text{density of the gas}}{\text{density of air}}\right)_{\text{Both at the same temperature and pressure}} \tag{1.3}$$

For ideal gases the specific gravity of any gas = $(M_{\text{gas}} / M_{\text{air}})$.

Throughout this text we use liquid and solid specific gravities referred to water at 4°C. Thus a liquid with a specific gravity of 0.8 is a liquid with a density of 0.8 g / cm^3.

1.5.3 Viscosity

Viscosity is a measure of internal, frictional resistance to flow. If we tip over a glass of water on the dinner table, the water will spill out before we can stop it. If we tip over a jar of honey, we probably can set it upright again before much honey flows out; this is possible because the honey has much more resistance to flow, more viscosity, than water. A more precise definition of viscosity is possible in terms of the following conceptual experiment.

Consider two long, solid plates separated by a thin film of fluid (see Fig. 1.4). This apparatus is easy to grasp conceptually and mathematically but difficult to use, because the fluid leaks out at the edges and gravity pulls the two plates together. Other devices that are more complex mathematically but easier to use are actually used to measure viscosities (see Example 1.2 and Chaps. 6 and 13). If we slide the upper plate steadily in the x direction with velocity V_0, a force will be required to overcome the internal friction in the fluid between the plates. This force will be different for different velocities, different plate sizes, different fluids, and different distances between the plates. We can eliminate the effect of different plate sizes, however, by measuring the force per unit area of the plate, which we define as the *shear stress* τ.

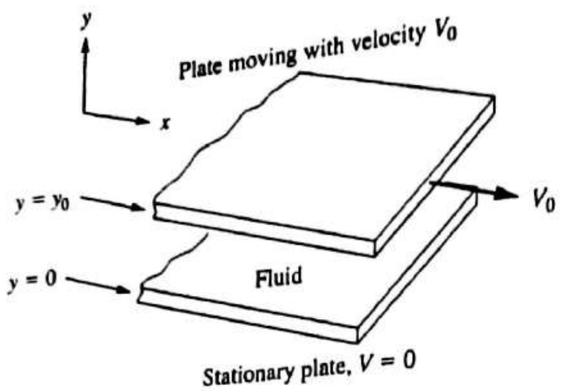

FIGURE 1.4
The sliding-plate experiment.

It has been demonstrated experimentally that at low values of V_0 the velocity profile in the fluid between the plates is linear, i.e.,

$$V = \frac{V_0 y}{y_0} \tag{1.C}$$

so that

$$\sigma = \begin{pmatrix}\text{shear rate, rectangular} \\ \text{coordinates}\end{pmatrix} = \frac{dV}{dy} = \frac{V_0}{y_0} \tag{1.D}$$

It also has been demonstrated experimentally that for most fluids the results of this experiment can be shown most conveniently on a plot of τ versus dV/dy (see Fig. 1.6). As shown here, dV/dy is simply a velocity divided by a distance. In more complex geometries it is the limiting value of such a ratio at a point. It is commonly called *shear rate, the rate of strain*, and *rate of shear deformation*, all of which mean exactly the same thing.

Example 1.2. Figure 1.5 shows a cutaway photograph of a concentric-cylinder ("cup and bob") viscometer also called a *Couette viscometer*. An inner cylinder (the bob) rotates inside a stationary outer cylinder (the cup). The shaft that drives the bob is instrumented to record both the angular velocity and the applied torque. The solid bob has $D_1 = 25.15$ mm and $L = 92.27$ cm. The surrounding cup has $D_2 = 27.62$ mm and is longer than the bob. When the bob is driven at 10 rpm, the observed torque is $\Gamma = 0.005$ Nm. What are τ and dV/dy?

This viscometer is simply the device in Fig. 1.4, wrapped around a cylinder. In this form, the leakage-at-the-edges problem and the difficulty of

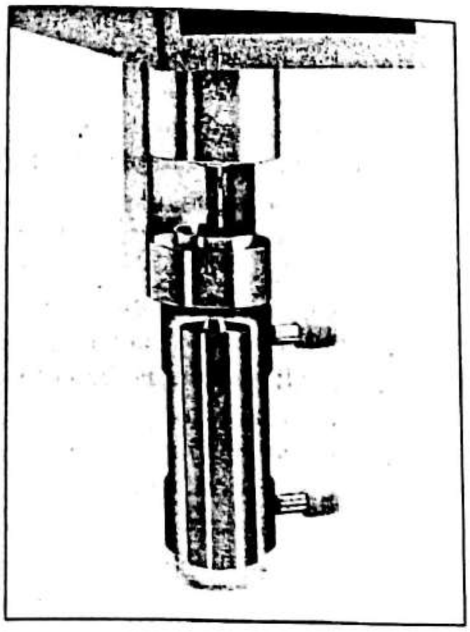

FIGURE 1.5
Cutaway photograph of a concentric-cylinder viscometer. This is simply the sliding-plate arrangement in Fig. 1.4, wrapped around a cylinder, thus eliminating the leaky edges in Fig. 1.4. The drive mechanism at the top holds the outer cylinder fixed and rotates the inner closed cylindrical bob. It provides a measured, controllable rotation rate and simultaneously measures the torque required to produce that rotation. The two flexible hoses circulate constant-temperature water or other fluid, to hold the whole apparatus at a constant temperature. Example 1.2 shows the dimensions of this device. (Courtesy of Brookfield Engineering Company.)

keeping the distance between the two surfaces constant are solved. (Fluid forces hold the rotating inner cylinder properly centered inside the outer cylinder.) Here we must replace the ys in Eq. 1.5 with rs, because the velocity is changing in the radial direction. $\Delta y = y_0$ is replaced by

$$\Delta r = 0.5(D_2 - D_1) = 0.5(27.62 - 25.15) = 1.235 \text{ mm} \tag{1.E}$$

and

$$V_0 = \pi D_1 \cdot \text{rpm} = \pi \cdot 25.15 \text{ mm} \cdot \frac{10}{\text{min}} = 790.1 \frac{\text{mm}}{\text{min}} = 13.17 \frac{\text{mm}}{\text{s}} \tag{1.F}$$

Thus,

$$\frac{dV}{dr} = \frac{V_0}{\Delta r} = \frac{13.17 \text{ mm / s}}{1.235 \text{ mm}} = 10.66 \frac{1}{\text{s}} \tag{1.G}$$

This is a linearized approximation of a cylindrical problem that understates the correct value, which is 12.26 (1 / s), (see Prob. 1.10), a difference of 15%. We will use the correct (cylindrical) value in the rest of this chapter.

The shear stress at the surface of the inner cylinder is

$$\tau = \frac{F}{A} = \frac{\Gamma / r_1}{\pi D_1 L} = \frac{0.005 \text{ Nm} / (0.5 \cdot 25.15 \text{ mm})}{\pi \cdot 25.15 \text{ mm} \cdot 92.37 \text{ mm}} = 5.45 \times 10^{-8} \frac{\text{N}}{\text{mm}^2} = 0.0545 \frac{\text{N}}{\text{m}^2} \tag{1.H}$$

■

This example ignores the stress on the bottom surface of the bob, a small effect, for which a correction is made in real viscosity measurements. The whole device is shown immersed in a constant-temperature bath, because the results are very temperature dependent.

The experiment in Example 1.2 can be repeated at different rotational speeds and the results plotted as shown in Fig. 1.6. Four different kinds of curve are shown as experimental results in the figure. All four of these results are observed in nature. The most common behavior is that represented by the straight line through the origin in the figure. This line is called Newtonian because it is described by Newton's law of viscosity:

$$\tau = \mu \frac{dV}{dy} \qquad \text{[Newtonian fluids]} \tag{1.4}$$

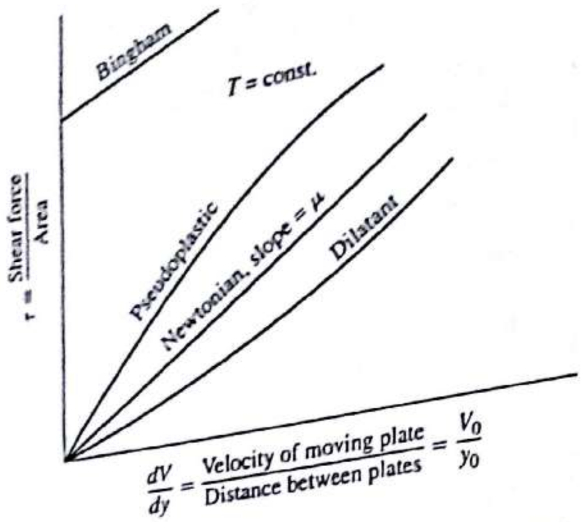

FIGURE 1.6
Possible outcomes of the sliding-plate experiment at constant temperature and pressure.

This equation says that the shear stress τ is linearly proportional to the velocity gradient dV/dy. It is also the definition of viscosity, because we can rearrange it to

$$\mu = \frac{\tau}{dV/dy} \tag{1.5}$$

Here μ is called the *viscosity* or the *coefficient of viscosity*. [We occasionally see this equation written with a minus sign in front of the τ. This is done so that the equation will have the same form as the heat-conduction and mass-diffusion equations ([2], p. 12). Since the shear stress acts in one direction on the rotating cylinder and in the opposite direction on the fluid adjacent to it, we can introduce this minus sign and reverse our idea of the direction of τ so that the result is always the same as in Eq. (1.5).] For Example 1.2, we would calculate

$$\mu = \frac{\tau}{dV/dy} = \frac{0.0.0545\ \text{N}/\text{m}^2}{12.26/\text{s}} = 0.0044\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} \tag{1.I}$$

For fluids such as air the value of μ is very low; therefore, their observed behavior is represented in Fig. 1.6 by a straight line through the origin, very close to the dV/dy axis. For fluids such as corn syrup the value of μ is very large, and the straight line through the origin is close to the τ axis.

Fluids that exhibit this behavior in the sliding-plate experiment or its cylindrical equivalent (i.e., fluids that obey Newton's law of viscosity) are called *Newtonian fluids*. All the others are called *non-Newtonian fluids*. Which fluids are Newtonian? All gases are Newtonian. All liquids for which we can write a simple chemical formula are Newtonian, such as water, benzene, ethyl alcohol, carbon tetrachloride, and hexane. Most dilute solutions of simple molecules in water or organic solvents are Newtonian, such as solutions of inorganic salts, or sugar in water, or benzene. Which fluids are non-Newtonian? Generally, non-Newtonian fluids are complex mixtures: slurries, pastes, gels, polymer solutions, etc. (some authors refer to them as *complex fluids*). Most non-Newtonian fluids are mixtures with constituents of very different sizes. For example, toothpaste consists of solid particles suspended in an aqueous solution of various polymers. The solid particles are much, much bigger than water molecules, and the polymer molecules are much bigger than water molecules.

In discussing non-Newtonian fluids we must agree on what we mean by viscosity. If we retain the definition given by Eq. 1.5, then the viscosity can no longer

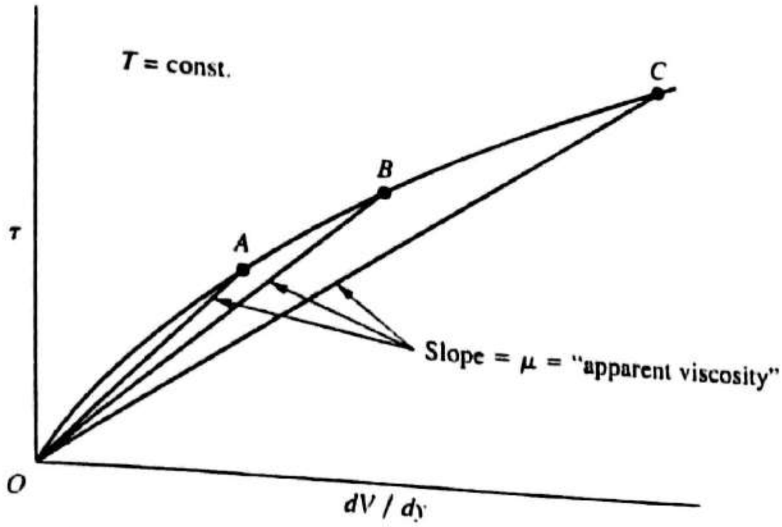

FIGURE 1.7
The "apparent viscosity" of a pseudoplastic fluid decreases as the shear rate increases.

be considered a constant independent of dV/dy for a given temperature, but must be considered a function of dV/dy. This is shown in Fig. 1.7. Here each of the lines OA, OB, and OC have slope μ, so the viscosity is decreasing with increasing dV/dy. (Viscosities defined as the slopes in Fig. 1.7 are often called *apparent viscosities.*) Using this definition, we can observe that there are three common types of non-Newtonian fluid (Fig. 1.6):

1. *Pseudoplastic fluids* show an apparent viscosity that decreases with increasing velocity gradient. Examples are most slurries, muds, polymer solutions, solutions of natural gums, and blood. These fluids are referred to as *shear thinning* fluids. This is the most common type of non-Newtonian behavior.
2. *Bingham fluids,* sometimes called *Bingham plastics,* resist a small shear stress indefinitely but flow easily under larger shear stresses. One may say that at low stresses the viscosity is infinite and at higher stresses the viscosity decreases with increasing velocity gradient. Examples are bread dough, toothpaste, applesauce, some paints, jellies, and some slurries.
3. *Dilatant fluids* show a viscosity that increases with increasing velocity gradient. This behavior is called *shear thickening;* it is uncommon, but starch suspensions and some muds behave this way. For these materials the liquid lubricates the passage of one solid particle over another; at high shear rate the lubrication breaks down, and the particles have more resistance to slipping past each other.

So far, we have assumed that the curve of τ versus dV/dy is not a function of time; i.e., if we move the sliding plate at a constant speed, we will always require the same force. This is true of most fluids, but not of all. A more complete picture

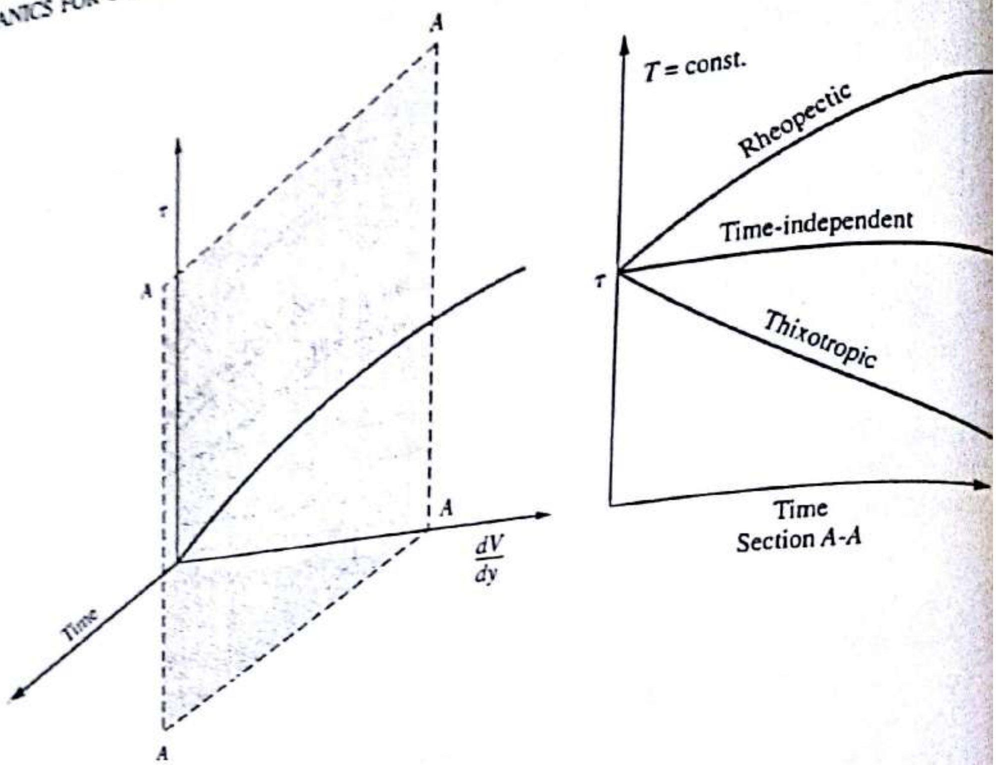

FIGURE 1.8
The viscosity of fluids can be independent of time of shearing or can increase or decrease with time as the fluid is sheared.

is given in Fig. 1.8. In Fig. 1.8 we see a constant dV/dy slice out of the solid constructed of τ versus dV/dy versus time. We see three possibilities:

1. The viscosity can remain constant with time, in which case the fluid is called *time independent.*
2. The viscosity can decrease with time, in which case the fluid is called *thixotropic.*
3. The viscosity can increase with time, in which case the fluid is called *rheopectic.*

All Newtonian fluids are time independent, as are most non-Newtonian fluids. Many thixotropic fluids are known, almost all of which are slurries or solutions of polymers, and a few examples of rheopectic fluids are known.

In addition, some fluids, called *viscoelastic fluids,* can show not only the kinds of behavior represented in Figs. 1.6 and 1.8 but also elastic properties, which allow them to "spring back" when a shear force is released. The most common examples of viscoelastic fluids are egg whites, cookie dough, and the rubber cement sold at stationery stores. Rubber cement's viscoelastic properties can be demonstrated most easily by starting to pour a little out of the bottle and then snapping it back into the bottle with a quick jerk of the hand. The same can be done with egg white. This is quite impossible with any ordinary fluid such as water: try it!

These strange types of fluid behavior are of considerable practical use. A good toothpaste should be a Bingham fluid, so that it can easily be squeezed out of the tube but will not drip off the toothbrush the way water or honey would. A good paint should be a thixotropic Bingham fluid, so that in the can it will be very viscous and the pigment will not settle to the bottom, but when it is stirred, it will become less viscous and can easily be brushed onto a surface. In addition, the brushing should temporarily reduce the viscosity so that the paint will flow sideways (under the influence of surface tension; see below) and fill in the brush marks (called *leveling* in the paint industry); then, as it stands, its viscosity should increase, so that it will not form drops and run down the wall.

Most engineering applications of fluid flow involve water, air, gases, and simple fluids. Therefore, most fluid-flow problems have to do with Newtonian fluids, as do most of the problems in this book. Non-Newtonian fluids are important, however, precisely because of their non-Newtonian behavior; they are discussed in Chap. 13.

The viscosity of simple gases, such as helium, can be calculated for all temperatures and pressures from the kinetic theory of gases using only one experimental measurement for each gas [2]. For the viscosities of most gases and all liquids several experimental data points are required, although ways of predicting viscosity change with changing temperature and pressure are available [3]. As a general rule, the viscosity of gases increases slowly with increasing temperature, and the viscosity of liquids decreases rapidly with increasing temperature. The viscosity of both gases and liquids is practically independent of pressure at low and moderate pressures.

The basic unit of viscosity is the *poise,* where $P = 1\ g/(cm \cdot s) = 0.1\ Pa \cdot s = 6.72 \times 10^{-2}\ lbm/(ft \cdot s)$ [See the inside front cover for conversion factors.] The poise is widely used for materials like high-polymer solutions and molten polymers. However, it too large a unit for most common fluids. By sheer coincidence the viscosity of pure water at about 68°F = 20°C is 0.01 poise; for that reason the common unit of viscosity in the United States is the centipoise, $cP = 0.01\ P = 0.01\ g/(cm \cdot s) = 0.001\ N \cdot s/m^2 = 0.001\ Pa \cdot s = 6.72 \times 10^{-4}\ lbm/(ft \cdot s)$. Hence, the viscosity of a fluid expressed in centipoise is the same as the ratio of its viscosity to that of water at room temperature. The viscosities of some common liquids and gases are shown in App. A.1. The computed viscosity of the fluid in Example 1.2 is 4.4 cP.

1.5.4 Kinematic Viscosity

In many engineering problems, viscosity appears only in the relation (viscosity/density). Therefore, to save writing we define

$$\text{Kinematic viscosity} = \nu = \mu / \rho \tag{1.6}$$

The most common unit of kinematic viscosity is the centistoke (cSt):

$$1\ \text{cSt} = \frac{1\ \text{cP}}{1\ \text{g}/\text{cm}^3} = 10^{-6}\,\frac{\text{m}^2}{\text{s}} = 1.08 \times 10^{-5}\,\frac{\text{ft}^2}{\text{s}} \tag{1.J}$$

at 68°F = 20°C, water has a kinematic viscosity of $1.004 \approx 1$ cSt. To avoid confusion over which viscosity is being used, some writers refer to the viscosity μ as the *absolute viscosity.* The kinematic viscosity has the same dimension ($\text{length}^2/\text{time}$) as the thermal diffusivity and the molecular diffusivity; in many problems it acts the same way as them. In Chap. 6 we will see some examples of the practical convenience of the kinematic viscosity.

1.5.5 Surface Tension

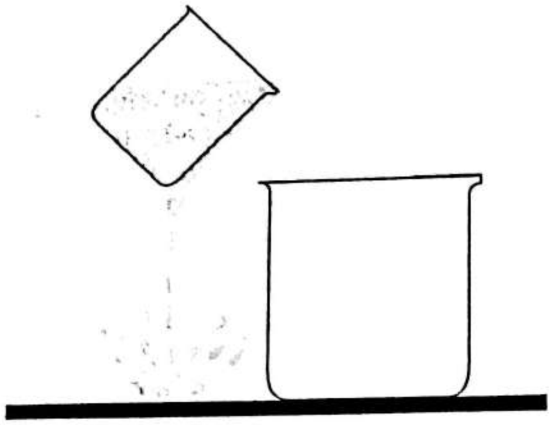

FIGURE 1.9
Disheartening effect of surface tension. The water dribbles down the surface of the container.

Liquids behave as if they were surrounded by a skin that tends to shrink, or contract, like a sheet of stretched rubber, a phenomenon known as *surface tension*. It is seen in many everyday events, the most disheartening of which is the tendency of water, when poured slowly from a glass, to dribble down the edge of the glass (see Fig. 1.9).

Surface tension is caused by the attractive forces in liquids. All of the molecules attract each other; those in the center are attracted equally in all directions, but those at the surface are drawn toward the center because there are no liquid molecules in the other direction to pull them outward (see Fig. 1.10). The "effort" of each molecule to reach the center causes the fluid to try to take a shape that will have the greatest number of molecules nearest the center, a sphere (Prob. 1.11). Any other shape has more surface per unit volume; therefore, regardless of the shape of a liquid the attractive forces tend to pull the liquid into a sphere. Other forces, such as gravity often oppose surface tension forces, so the spherical shape is only seen for small systems, such as small water drops on a water-repellent surface. The fluid thus tries to decrease its surface area to a minimum. (An analogous situation in two dimensions is observable in the behavior of some army ants. They travel in large groups, and, viewed from above, the swarm often looks like a circle. The reason appears to be that the ants are attracted by the scent of other ants and, hence all try to get to the place where the scent is strongest, the center. The ants all stay in one plane, so the result is the plane figure with the smallest possible ratio of perimeter to area—a circle [4].)

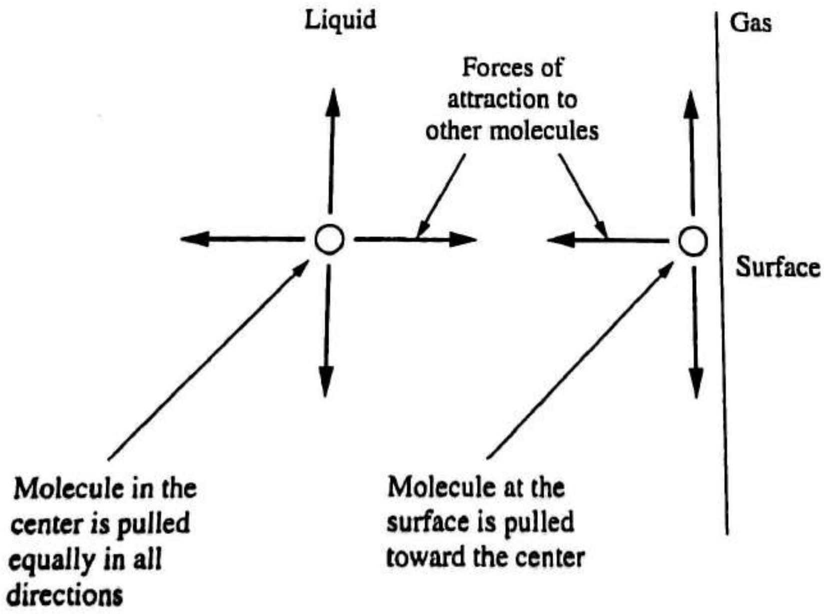

FIGURE 1.10
Surface tension is caused by the attractive forces between molecules.

The tendency of a surface to contract can be measured with the device

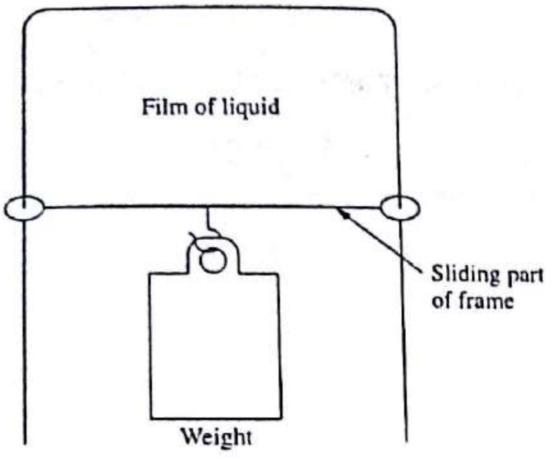

FIGURE 1.11
A very simple way to measure surface tension; see Example 1.3.

shown in Fig. 1.11. A wire frame with one movable side is dipped into a liquid and carefully removed with a film of liquid in the space formed by the frame. The film tries to assume a spherical shape, but since it adheres to the wire, it draws the movable part of the frame inward. The force necessary to resist this motion is measured by a weight. It is found experimentally that the ratio of the force to the length of the sliding part of the wire is always the same for a given liquid at a given temperature, regardless of the size of the apparatus. The liquid film in the frame has two surfaces (front and back), so the force-to-length ratio of *one* of the surfaces is exactly one-half of the total measurement. The surface tension of the liquid is then defined as

$$\text{Surface tension} = \frac{\text{force of one film}}{\text{length}} \qquad \text{or} \qquad \sigma = \frac{F}{l} \tag{1.7}$$

Example 1.3. The device in Fig. 1.10 has a sliding part 10 cm long. The mass needed to resist the inward pull of the fluid is 0.6 g, which exerts a force of 0.00589 N. What is the surface tension of the fluid?

From Eq. 1.7,

$$\sigma = \frac{F(\text{one film})}{l} = \frac{0.00589 \text{ N} / 2}{0.1 \text{ m}} = 0.0294 \frac{\text{N}}{\text{m}} = 0.000168 \frac{\text{lbf}}{\text{in}} \tag{1.K}$$

■

The device shown in Fig. 1.11 is easy to understand but not very practical as a measuring device; more practical ones are discussed in Chap. 14.

Surface tension is very slightly influenced by what the surrounding gas is—air or water vapor or some other gas. Typical values of the surface tension of liquids exposed to air are shown in Table 1.1. The traditional unit of surface tension is the dyne / cm = 0.001 N / m. At 68°F = 20°C, most organic liquids have about the same surface tension (≈25 dyne / cm) whereas that of water is about 3 times higher, and that of mercury is 20 times higher.

We indicated that the liquid adheres to the solid in the apparatus shown in Fig. 1.11. Liquids adhere strongly to some solids and not to others. For example, water adheres strongly to glass but very weakly to polyethylene. This greatly complicates the whole subject of surface tension; the phenomenon shown in Fig. 1.9 occurs much more often with glass, ceramic, or metal cups than with polyethylene or Teflon cups.

TABLE 1.1
Surface tensions of pure fluids exposed to air at 68°F = 20°C

Fluid	Surface tension, dyne / cm = 0.001 N / m	Surface tension, lbf / in
Acetic acid	27.8	0.000159
Acetone	23.7	0.000135
Benzene	28.25	0.000161
Carbon tetrachloride	26.95	0.000154
Ethyl alcohol	22.75	0.000130
n-Octane	21.8	0.000124
Toluene	28.5	0.000163
Water	72.74	0.000415
Mercury	484	0.002763

Extensive tables are available in the *Handbook of Chemistry and Physics*, annual editions, published by CRC Press, Boca Raton, Florida, and various other handbooks.

Two important effects attributable to surface tension are the capillary rise of liquids in small tubes and porous wicks (without which candles, kerosene lanterns or copper sweat-solder fittings would not work at all) and the tendency of jets of liquid to break up into drops (as from a garden hose or gasoline or diesel fuel injector or in an ink-jet printer). Surface tension effects are very important in systems involving large surface areas, such as emulsions (mayonnaise, cold cream, water-based paints) and multiphase flow through porous media (oil fields). We will discuss the effects in Chap. 14; see also references [5, 6].

1.6 PRESSURE

Pressure is defined as a compressive stress, or compressive force per unit area. In a stationary fluid (liquid or gas) the compressive force per unit area is the same in all directions. In a solid or in a moving fluid, the compressive force per unit area at some point is not necessarily the same in all directions. We can visualize why by squeezing a rubber eraser between our fingers; see Fig. 1.12. As we squeeze the eraser, it becomes thinner and longer, as shown. If we analyze the stresses in the eraser, we find that in the y direction the eraser is in compression, whereas in the x direction it is in tension. (This seems strange, but the eraser has been stretched in the x direction, and its elastic forces will pull it back when we let go; hence the tension.) The contraction in one direction and expansion in another in an elastic solid is described in terms of Poisson's ratio, discussed in any text on strength of materials. Because the tensile and compressive forces are at right angles to each other, there is also a strong shear stress at 45° to the x axis.

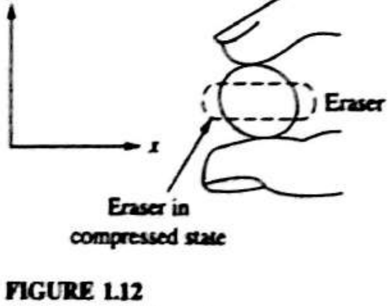

FIGURE 1.12
The response of an elastic solid to compression in one direction.

What would happen if we held our fingers in a cup of water and tried to squeeze the water between our fingers? Obviously, the water would run out from between our fingers, and our fingers would come together. Why? When we start to squeeze the water, it behaves like the eraser, setting up internal shear and tensile forces in the same directions as the eraser. However, ordinary fluids cannot permanently resist shear forces, so the water begins to flow and finally flows away. The eraser also flowed, until it had taken up a new shape, in which its internal tensile and shear resistance were enough to hold our fingers apart. Water cannot set up such resistance and so it simply flows away.

If we really wanted to squeeze the water, we would put it in some container that would prevent its flowing out to the side. If we did this with the eraser, then as we compressed it from the top, it would press out on the sides of the container. So also does water.

The foregoing is a description of why the pressure at a point in a fluid at rest is the same in all directions. It is not a proof of that fact; for a proof see App. B.1.

What we mean by pressure is not so clear for a solid as it is for a liquid or a gas. The compressive stresses at a given point in a solid are not the same in all directions. The usual definition of pressure in a solid is as follows: Pressure at a point is the average of the compressive stresses measured in three perpendicular directions. Since, as we have seen, these three stresses are all the same in a fluid at rest, the two definitions are the same. For a fluid in motion, the three perpendicular compressive stresses may not be the same. However, for this difference to be significant, the shear stresses must be very large, well outside the range of normal problems in fluid mechanics. Therefore, we normally extend the notion that pressure in a fluid at rest is the same in all directions to fluids in motion, with the reservation that at very high shear stresses (such as in the flow of metals or polymer melts through forming dies) this is not necessarily true. For polymer solutions and polymer melts the differences between the compressive stresses in directions at right angles to one another can be very significant and can lead to behavior quite different from the behavior of simple fluids; see [7].

In the solution of many problems, particularly those involving gases, it is most convenient to deal with pressures in an absolute sense, i.e., pressures relative to a compressive stress of zero; these are called *absolute pressures*. In the solution of many other problems, particularly those involving liquids with free surfaces, such as are encountered in rivers, lakes, and open or vented tanks, it is more convenient to deal with pressures above an arbitrary datum, the local atmospheric pressure. Pressures relative to the local atmospheric pressure are called *gauge pressures*.

Because both systems of measurement are in common use, it is necessary to make clear which kind of pressure we mean when we write "a pressure of 15 lb/in^2" [This unit is also called psi (pounds per square inch)]. It is usual to say "15 psi absolute" or "15 psia" for absolute pressure and "15 psi gauge" or "15 psig" for gauge pressure. The SI unit of pressure is the pascal, $Pa = N/m^2$. There does not seem to be a common set of abbreviations for Pascal absolute and Pascal gauge, so these must be written out.

Another two-datum situation familiar to the reader is found in the measurement of elevation. Mountain tops, road routes, and rivers are normally surveyed relative to

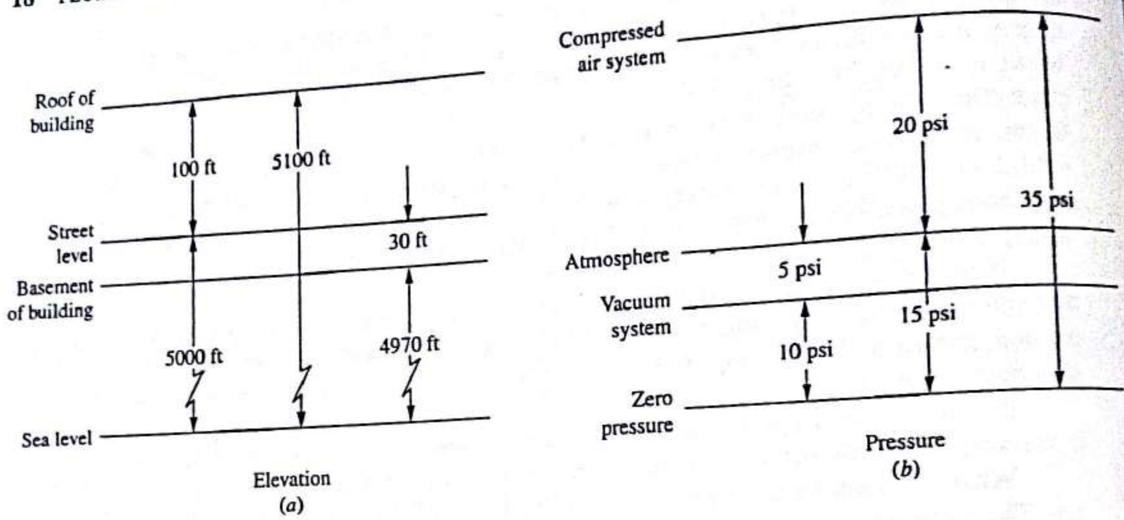

FIGURE 1.13
The relation between gauge and absolute pressure, and a comparison with elevation measurements.

mean sea level, which serves as an "absolute" datum, but most buildings are designed and constructed relative to some local elevation (usually a marker in the street); see Fig. 1.13. In both cases the most common measuring method gives answers in terms of the local datum. Most pressure gauges read the difference between the measured pressure and the local atmospheric pressure. For instance, the pressure gauge on the compressed air system in the figure would read 20 psig = 137.9 kPa gauge; the building height (by tape measure or transits) might be given as 100 ft = 30.5 m elevation. Both such measurements usually involve negative values, based on the local datum; the basement has a negative elevation relative to the street, −30 ft = −9.15 m, and the vacuum system has a negative pressure relative to the atmosphere, −5 psig = −34.5 kPa gauge.

Negative elevations relative to sea level can exist; the Dead Sea, for instance, is about 1200 feet (366 m) below sea level. Can negative absolute pressures exist? Certainly; a negative absolute pressure is a negative compressive stress, i.e., a tensile stress. These occur often in solids, very rarely in liquids, never in gases. They are rare in liquids because all liquids possess a finite vapor pressure. If the pressure of a liquid is reduced below its vapor pressure, the liquid boils and thus replaces the low pressure with the equilibrium vapor pressure of the liquid. However, this boiling never takes place spontaneously in an absolutely pure liquid [8], but rather occurs around small particles of impurities or at the wall of the container. (Most people have observed this phenomenon when they pour a cold carbonated drink into a glass; the bubbles form mostly at the edge of the glass, not in the bulk of the liquid. It can be shown dramatically by dropping some sugar into a cold, fresh glass of soft drink; do this over a sink!) Thus, if a liquid is very pure and the surfaces of its container are very smooth, the liquid can exist in tension at a negative absolute pressure. This situation is unstable, and a slight disturbance can cause the liquid to boil [9].

1.7 FORCE, MASS, AND WEIGHT

In fluid mechanics we are often concerned with forces, masses, and weights. The problem of units of force and mass is discussed in the next section. An unbalanced force makes things change speed or direction. Most forces in the world are balanced by opposite forces (a building exerts a force on the ground; the ground exerts an equal and opposite force on the building; neither moves). To make anything start moving or stop moving, we must exert an unbalanced force.

Mass is an indication of how much matter is present. The more matter, the more mass. (We may think of matter in any size, as bricks, molecules, atoms, nucleons, quarks, etc.). Mass is also an indicator of how hard it is to get some amount of matter moving or how hard it is to stop it once it is moving. We can all stop a baseball moving 50 ft / s (15.2 m / s) with little more damage than a possible sore hand. If we step in front of an automobile moving at the same speed, we will certainly be killed. The auto has much more mass; it is much harder to stop.

Weight is a force—the force that a body exerts due to the acceleration of gravity. When there is no gravity, there is no weight (e.g., in earth satellites there is no apparent gravity; this state is referred to as *weightlessness*).

1.8 UNITS AND CONVERSION FACTORS

Engineering is about real physical things, which can be measured and described in terms of those units of measure. Most engineering calculations involve these units of measure. It would be simple if there were only one set of such units that the whole world agreed upon and used; but that is not the case today. In the United States most measurements use the English system of units, based on the foot, the pound and the °F, but most of the world uses the metric (or SI) system of units based on the meter, the kg and the °C. The metric system has been legally accepted in the United States since 1866, and it has been the declared policy of the U.S. government to convert to metric since 1975 [10]. Progress has been disappointingly slow.

The situation is similar with languages; it would be easier if we all spoke one language. But we do not; the world has many languages. Educated Europeans all speak at least two languages well and generally can read one or two more. Similarly, U.S. engineers must be fluent in English and in metric units, be able to understand older literature written in the centimeter-gram-second (cgs) system, and in variant English systems that use the poundal or the slug and in specialized industrial units, like the 42-gal barrel for petroleum products or pressure differences expressed in inches of water. U.S. engineers must even deal with mixed systems, like automotive air pollutant emissions expressed in grams per mile. Furthermore, they must understand the differences between the common-use version of metric and SI, discussed below; they will be better able to deal with those differences if they understand why the differences arise.

In fluid mechanics we most often deal with dimensioned quantities, such as 12 ft / s (= 3.66 m / s), rather than with pure numbers such as 12 or 3.66. We often drop the units, for example, "I was driving 60," which in the United States normally means 60 mi / h, but in the rest of the world means 60 km / h. This is poor practice.

but common. In 1999 [11] a $125 million NASA Mars probe was destroyed because someone failed to check their units. In technical work we always make clear the units in which any value is expressed! To become competent at solving fluid mechanics problems we must become virtually infallible in the handling of such units and their conversion factors. For most engineers the major sources of difficulties with units and conversion factors are carelessness and the simultaneous appearance of force and mass in the same equation.

A useful "system" for avoiding carelessness and consistently converting the dimensions of engineering quantities from one set of units to another has two rules:

1. *Always* (repeat, *always*) include the dimensions with any engineering quantity you write down.
2. Convert the dimensions you have written down to the dimensions you want in your answer by multiplying or dividing by 1.

Example 1.4. We are required to convert a speed of 327 mi / h to a speed in ft / s. The first step is to write the equation

$$\text{Speed} = 327\ \text{mi / h} \tag{1.L}$$

This is not the same as 327 km / h or 327. If we omit the dimensions, our equation is meaningless. We now write, as an equation, the definition of a mile:

$$1\ \text{mi} = 5280\ \text{ft} \tag{1.M}$$

Dividing both sides of this equation by 1 mi, we find

$$\frac{1\ \text{mi}}{1\ \text{mi}} = 1 = \frac{5280\ \text{ft}}{\text{mi}} \tag{1.N}$$

You may not be used to thinking of 5280 ft / mi as being the same thing as 1, but Eq. 1.N shows that they are the same. Similarly, we write the definition of an hour as an equation,

$$1\ \text{h} = 3600\ \text{s} \tag{1.O}$$

and divide both sides by 3600 s to find

$$\frac{3600\ \text{s}}{3600\ \text{s}} = 1 = \frac{\text{h}}{3600\ \text{s}} \tag{1.P}$$

Again, you may not be used to thinking of 1 h / 3600 s as the same thing as 1, but it is. Now let us return to Eq. 1.L and multiply both sides by 1 twice, choosing our equivalents of 1 from Eqs. 1.N and 1.P:

$$\text{Speed} \cdot 1 \cdot 1 = \frac{327\ \text{mi}}{\text{h}} \cdot \frac{5280\ \text{ft}}{\text{mi}} \cdot \frac{\text{h}}{3600\ \text{s}} \tag{1.Q}$$

We can now cancel the two 1's on the left side, because they do not change the value of "Speed," and we can cancel the units that appear both above and below the line on the right side to find

$$\text{Speed} = \frac{327\ \cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\ \text{ft}}{\cancel{\text{mi}}} \cdot \frac{\cancel{\text{h}}}{3600\ \text{s}} = \frac{327 \cdot 5280\ \text{ft}}{3600\ \text{s}} = 480\ \frac{\text{ft}}{\text{s}} = 146\ \frac{\text{m}}{\text{s}} \tag{1.R}$$

■

This was an easy example, one you could certainly solve without going into as much detail as shown here, but it illustrates the procedure to be used in more complicated problems.

Example 1.5. Suppose Time equals 2.6 h. How many seconds is this? Again we begin by writing Time with its dimension as an equation:

$$\text{Time} = 2.6\ \text{h} \tag{1.S}$$

We want to know its value in seconds, so we divide by 1,

$$\text{Time} = 2.6\ \text{h} \cdot \frac{3600\ \text{s}}{\text{h}} = 2.6 \cdot 3600\ \text{s} = 9380\ \text{s} \tag{1.T}$$

■

How did we know to multiply by 1 h / 3600 s in Example 1.4 and to divide by 1 h / 3600 s in Example 1.5? In each case we chose the value of 1 that allowed us to cancel the unwanted dimension. Three ideas are involved here:

1. Dimensions are treated as algebraic quantities and multiplied or divided accordingly.
2. Multiplying or dividing any quantity by 1 does not change its value.
3. Any dimensioned equation can be converted to 1 = 1 by dividing through by either side.

Using the last procedure, we can write

$$1 = \frac{60\ \text{s}}{\text{min}} = \frac{12\ \text{in}}{\text{ft}} = \frac{7000\ \text{gr}}{\text{lbm}} = \frac{\text{mi}^2}{640\ \text{acres}} = \frac{\text{Btu}}{252\ \text{cal}} = \frac{\text{W}}{\text{VA}} = \text{etc.} \tag{1.8}$$

and as many other values of 1 as we like.

The previous examples did not involve the unit conversions that cause difficulties, the ones involving force and mass or thermal and mechanical energies. If everyone always used SI, we would never have those difficulties. In SI there is no difficulty with the units of force and mass; force is measured in newtons (N) and mass in kilograms (kg), and the only unit of energy is the mechanical-energy unit, the joule, where $J = N \cdot m$.

Unfortunately, in the English system (and in the traditional metric system as it is used by the public in Europe) there is difficulty with force-mass unit conversion. If we ask a typical European male what he weighs, he might well respond "80 kilos," meaning 80 kg. If he were speaking in SI he would not use kg as a unit of weight, because weight is a force and the SI unit of force is the newton. He should respond, "784.6 newtons" because that is the weight of an 80 kg mass in a standard gravitational field of $9.807\ \text{m/s}^2 = 32.17\ \text{ft/s}^2$. It is hard enough to teach novice engineers the difference between weight and mass; it is probably impossible to get the general public to take the view that a mass of 80 kg does not exert a force of 80 kg. To make this come out right, we need to decide that there are really *two* kilogram units, the kilogram-mass (kgm) and the kilogram-force (kgf). We can define these so that one kgm exerts a force of one kgf at standard gravity. That is what most of the people in the world actually do. Similarly, in the English system of units we need two kinds of pounds; pound-mass (lbm) and pound-force (lbf). Again we have defined these so that one lbm has a weight of (exerts a force of) one lbf at standard gravity.

Why does this cause problems? Because the kgm and kgf look like the same thing, so we are tempted to believe they are the same thing, and the lbm and the lbf look like the same thing, so we are tempted to believe they are the same thing. That is wrong. It is a trap for the unwary. They are not the same. This leads to serious errors in engineering calculations.

Newton's second law of motion is

$$F = ma \tag{1.9}$$

where F is force, m is mass, and a is acceleration. The pound-force (lbf) is defined as that force which, acting on a mass of 1 lbm, produces an acceleration of 32.2 ft / s^2. Substituting this definition into the last equation, we find

$$1 \text{ lbf} = \text{lbm} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \tag{1.U}$$

Dividing both sides of this by 1 lbf, we find

$$1 = \frac{\text{lbf}}{\text{lbf}} = 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \tag{1.V}$$

If we then make the mistake of canceling the lbm on the top and the lbf on the bottom right-hand side, we will conclude that $1 = 32.2$ ft / s^2. This is clearly wrong, and if we do it in a problem we will find that the dimensions do not check and the numerical value of the answer will be wrong by a factor of 32.2 (if we use English units) or 9.8 (if we use metric units). Similarly, in the traditional metric system we have

$$1 \text{ kfg} = \text{kgm} \cdot 9.8 \text{ m / s}^2 \tag{1.W}$$

and if we divide both sides by kgf, we find

$$1 = \frac{\text{kgf}}{\text{kgf}} = 9.8 \frac{\text{kgm} \cdot \text{m}}{\text{kgf} \cdot \text{s}^2} \tag{1.X}$$

If we then cancel kgm and kgf on the right side we will conclude that $1 = 9.8$ m / s^2, which is equally absurd.

How can we get out of this difficulty? One way is to always work exclusively in SI. In that case kg will always mean kgm, and kgf will never appear. Instead the unit of force will always be the N = (1 / 9.8) kgf. However, then we will be unable to deal with the public, who speak (unintentionally) in kgf and lbf, or to deal with those parts of the engineering literature that use kgf and lbf. The other way is to decide we must live with the kgf and lbf, and so we will regularly have to use the force-mass conversion factor whenever units of force and of mass occur in the same equation. This conversion factor has the following values:

$$1 = 9.8 \frac{\text{kgm} \cdot \text{m}}{\text{kgf} \cdot \text{s}^2} = 1.0 \frac{\text{kgm} \cdot \text{m}}{\text{N} \cdot \text{s}^2} = 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \tag{1.10}$$

Furthermore, we must know some history to understand the older literature. First, we must know that many older textbooks and articles used the symbol g_c to stand for this force-mass conversion factor. So whenever we see a g_c written into an equation, we must recognize it as a reminder that we must use the force-mass conversion factor. We must not confuse g_c, the force-mass conversion factor, with g, the acceleration of gravity; they are *not* the same.

Second, we should recognize that engineers using English units have tried to evade this difficulty by inventing two new units, the slug (1 slug = 32.2 lbm = 14.6 kg) and the poundal (pdl) (1 pdl = lbf / 32.2 = 0.138 N = 0.014 kgf). Using these, we have the following force-mass conversion factors:

$$1 = 9.8\,\frac{\text{kgm}\cdot\text{m}}{\text{kgf}\cdot\text{s}^2} = 1.0\,\frac{\text{kgm}\cdot\text{m}}{\text{N}\cdot\text{s}^2} = 32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 1.0\,\frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 1.0\,\frac{\text{lbm}\cdot\text{ft}}{\text{pdl}\cdot\text{s}^2} \tag{1.11}$$

The poundal finds little current use, but aeronautical engineers use the slug.

The kgf and the lbf have been around a long time, in spite of the efforts of scientists and engineers to replace them with the newton or the poundal. They survive because they seem natural to nonscientific users. Probably they will continue to be widely used, in spite of the efforts of the scientific community to replace them. Prudent engineers will learn to live with this fact, to use them when it seems appropriate, and to understand why they came about.

The second difficulty with units concerns mechanical and thermal units of energy. In SI the only unit of energy is the joule, 1 J = 1 N · m. This is clearly a mechanical unit, the product of a force and a distance. If we are transferring thermal energy (e.g., heating our houses or our soup), it seems natural to base the measurements on the quantity of thermal energy required to raise the temperature of some reference substance by some finite temperature interval. In the English system this quantity is the British thermal unit (Btu), which is the quantity of thermal energy required to raise the temperature of 1 lbm of water by 1°F. In the metric system the unit is the calorie (cal), which is the quantity of thermal energy required to raise the temperature of 1 g of water 1°C, or the kcal (kcal = 1000 cal; this is the "calorie" used in describing the energy content of foods). If we want to use the calorie or the Btu, then we need to convert from joules to calories or ft-lbf to Btu:

$$1 = \frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}} = \frac{\text{Btu}}{1055\ \text{J}} = \frac{\text{cal}}{4.18\ \text{J}} = \frac{\text{kcal}}{4180\ \text{J}} = \frac{\text{kcal}}{4.18\ \text{kJ}} \tag{1.12}$$

The Btu and the cal (or kcal) seem likely to continue in common usage; the Btu appears on almost all U.S. heating appliance and fuel bills (sometimes natural gas bills use the *therm* = 10^5 Btu), and kcal appears on numerous food products.

In summary, if we can do all our work in SI, we need never be concerned about force-mass conversions (N = kg · m / s) or energy conversions (J = N · m = W · s). If we are confronted with problems (or literature, or current U.S. legal definitions) involving the kgf, lbf, cal, kcal, or Btu, we must follow the rules outlined above. Always write down the dimensions, treat the dimensions as algebraic quantities, and multiply by 1 as often as needed to get the quantities into the desired set of units, using the appropriate values of the force-mass conversion factor and the thermal-mechanical energy conversion factor. Even in SI, if we stray from the basic units (m, kg, s, A, K, mol, and cd), we will need conversion factors such as

$$1 = \frac{1000\ \text{g}}{\text{kg}} = \frac{100\ \text{cm}}{\text{m}} = \frac{1000\ \text{mV}}{\text{V}} \tag{1.13}$$

Example 1.6. A mass of 10 lbm (4.54 kgm) is acted on by a force of 3.5 lbf (15.56 N or 1.59 kgf). What is the acceleration in ft / min²?

Rearranging Eq. 1.9, we find

$$a = F / m \tag{1.14}$$

Substituting, we find

$$a = \frac{3.5}{10} \frac{\text{lbf}}{\text{lbm}} \tag{1.Y}$$

Here we want the acceleration in ft / min², so we must multiply or divide by those equivalents of 1 that will convert the units:

$$a = \frac{3.5\ \text{lbf}}{10\ \text{lbm}} \cdot \frac{32.2\ \text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \cdot \left(\frac{60\ \text{s}}{\text{min}}\right)^2 = \frac{3.5 \cdot 32.2 \cdot 60^2}{10} \frac{\text{ft}}{\text{min}^2} = 40{,}570 \frac{\text{ft}}{\text{min}^2} \tag{1.Z}$$

or

$$a = \frac{15.56\ \text{N}}{4.54\ \text{kg}} \cdot \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} \cdot \left(\frac{60\ \text{s}}{\text{min}}\right)^2 = 123.4 \frac{\text{m}}{\text{min}^2} = 40{,}480 \frac{\text{ft}}{\text{min}^2} \tag{1.AA}$$

or

$$a = \frac{1.59\ \text{kgf}}{4.54\ \text{kgm}} \cdot \frac{9.8\ \text{kgm} \cdot \text{m}}{\text{kfg} \cdot \text{s}^2} \cdot \left(\frac{60\ \text{s}}{\text{min}}\right)^2 = 123.6 \frac{\text{m}}{\text{min}^2} = 40{,}540 \frac{\text{ft}}{\text{min}^2} \tag{1.AB}$$

The difference between these three answers is due to round-off error in the conversion factors used. If more figures had been carried (e.g., kgf = 9.80650 N), the answers would have agreed exactly, but since we know the input data to only two significant figures, our best answer, in all three cases, should be 40,500 ft / min². ■

Example 1.6 will be the last example in this book to use the kgf. Clearly the method of dealing with kgm and kgf is just the same as the method of dealing with lbm and lbf. For the rest of this book, we will use either lbm and lbf, or SI.

Example 1.7. An aluminum cell (Hall-Héroult process) has a current of 50,000 amp. If we assume it is 100% efficient, how much metallic aluminum does it produce per hour?

We first convert the current to gram equivalents per hour, using the necessary values of 1, one of which we take out of Prob. 1.16:

$$I = 50{,}000\ \text{A} \cdot \frac{\text{C}}{\text{A} \cdot \text{s}} \cdot \frac{3600\ \text{s}}{\text{h}} \cdot \frac{\text{g equiv}}{96{,}500\ \text{C}} = 1870 \frac{\text{g equiv}}{\text{h}} \tag{1.AC}$$

For aluminum,

$$27\ \text{g} = 1\ \text{mol} \tag{1.AD}$$

and

$$1\ \text{mol} = 3\ \text{g equiv} \tag{1.AE}$$

therefore,

$$I = 1870\,\frac{\text{g equiv}}{\text{h}} \cdot \frac{\text{mol}}{3\ \text{g equiv}} \cdot \frac{27\ \text{g}}{\text{mol}} \cdot \frac{\text{lbm}}{454\ \text{g}} = 37.1\,\frac{\text{lbm}}{\text{h}} = 16.8\,\frac{\text{kg}}{\text{h}} \tag{1.AF}$$

■

In solving Example 1.7 we multiplied by 1 six times. Nonetheless, the procedure is simple and straightforward. Each multiplication by 1 gets rid of an undesired dimension and brings us closer to an answer in the desired units. We saw that an apparently complex problem was really a simple conversion-of-units problem. In the course of our studies and our professional careers we will have to convert units as quickly and as easily as we now add and subtract. It will be easiest if we develop the habit of following the two rules given at the start of Sec. 1.8, namely:

1. Always include the dimensions with any engineering quantity you write.
2. Convert the dimensions you have written to the dimensions you want in your answer by multiplying or dividing by 1.

A short table of these conversion factors can be found inside the front cover of this text. The American Society for Testing and Materials (ASTM) [12] has prepared a much longer and more complete table, which reveals some additional complexity. For example, there are five different calorie definitions in common usage. The largest is 1.002 times the smallest. Only in the most careful work is this small a difference relevant. But if we are doing that kind of work, it is worthwhile to find, study, and use the ASTM tables.

1.9 PRINCIPLES AND TECHNIQUES

As discussed in Sec. 1.3, there are very few underlying ideas in fluid mechanics. With these few ideas we can solve a great variety of problems. In so doing, we can focus our attention either on the application of principles or on the techniques of solving problems. The author recommends attention to the principles. In the 10 years following his graduation from college, the engineering business was revolutionized by the digital computer, the transistor, and the space industry, among other things. None of these amounted to much in 1954, and they were not part of undergraduate courses.

All these technologies rigidly obey Newton's laws and the laws of thermodynamics. Students who learned "cookbook" techniques for solving problems on 1954 were not well prepared for the technologies that appeared during the next 10 years, but those who learned the basic principles and how to apply them could adapt to any one of them. There seems to be little reason to believe that the pace of technological change will become slower in the future. If we concentrate on learning techniques, energy conversion factor. Even in SI, if we stray from the basic units (m, kg, s, A, K, mol, and cd), we will need conversion factors such as

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1.10 ENGINEERING PROBLEMS

Although this book may fall into the hands of a practicing engineer, most of its readers will be college juniors; the following is addressed to them.

Engineering students start out in their freshman and sophomore years by doing “plug-in” problems. Given a problem statement, they select the appropriate formula either from the textbook or from their memory, and “plug in” the data in the problem to find the final answer. In their junior year they begin to find problems that can be readily reduced to plug-ins or to problems involving two or more equations that require some manipulations to be put in plug-in form. Furthermore, they may be exposed to problems that cannot be reduced to plug-ins and must be solved by trial and error. It is assumed that they can do simple plug-ins (such as gas-law calculations) without hesitation.

Instructors of third-year students would like to assign more complicated or difficult problems but generally cannot because:

1. The time required for them is too great—they cannot be done in the time that most students will devote to one homework problem.
2. The students would probably get intellectual indigestion on them. Therefore, at the third-year level most of the problems and examples in texts like this one are plug-ins or can be readily reduced to plug-ins.

When students start a senior laboratory or design course, they find their first real engineering problems. One of these may require 10 or 20 h of work and consist of 15 or 20 parts, each comparable to the problems and examples in this book. To deal with these problems, students break them into pieces small enough to handle as plug-ins. The interesting and exciting part of engineering is often the task of deciding how to divide a problem into reasonable pieces and how then to reassemble these pieces into a recognizable whole so that they fit together properly.

In the examples and problems in this book there are numerous simple plug-in problems. They are included because their solutions give the reader some feel for the numerical values involved in fluid mechanics. There are also more complex problems, in which two or more basic principles are involved (such as the mass balance and the energy balance). In these some manipulation is required to get the equations into plug-in form. The recommended procedure for solving such problems is this:

1. Make sure you understand precisely what the problem is; in particular, make sure you know precisely what is being asked for.
2. Decide which physical laws relate what you know to what you want to find.
3. Write the working form of these laws (as discussed later), and rearrange them to get the symbol for the quantity you seek standing alone to the left of the equal

sign. In so doing you will probably have to discard several terms in the physical-law equations. Discarding a term corresponds to making an assumption about the physical nature of the system (e.g., that a certain velocity is negligible). Thus, a list of such terms dropped is a list of assumptions made in solving the problem.

4. When step 3 is finished, the problem is reduced to a plug-in. Insert the given data, check the units, and find the numerical value of the answer.
5. Check the answer for plausibility: Does it indicate negative masses, velocities greater than the speed of light, or efficiencies greater than 100%? Does it pass the test of common sense, that is, do the results match your intuitive idea of what they should be? If not, is the difficulty with the calculations? or with your intuition? If neither is incorrect, perhaps you have made a new technical discovery! Also, re-examine the assumptions listed in step 3 to see whether they are consistent with the answer. If these checks are met, the answer probably is satisfactory.
6. If the problem is one that you may have to repeat with different data (such as the calculation of a fluid-flow rate from a measured pressure difference), then it might be worthwhile to see whether the answer can be put in a more convenient form, for example, some general plot or diagram. Perhaps the problem will occur often enough to justify programming its solution on a personal computer or entering it in a spreadsheet program.

In all engineering we must consider the degree of precision needed. Voltaire's famous dictum "The perfect is the enemy of the good!" describes the situation of the engineer. We could always spend more engineering effort, and do more testing, and thereby refine our design or our calculation a little more. But in any real problem the engineer's time is one of the limiting resources. We would all like the conditions that the famous architect Kobori Enshu demanded and received from the Japanese dictator Hideyoshi for the Katsura Villa: no limit on expense, no limit on time, and no client visits until the job is done. Many believe the result to be the greatest achievement of Japanese architecture and garden planning [13]. (If you are ever in Kyoto, visit it and decide for yourself.) But most engineers (and other professionals) are always working with limited time and limited budgets as well as clients who want intermediate progress reports. For us the goal is always to do the best possible, within the time, budget, and other constraints imposed by the client (or codes and regulations). So engineers must allocate their time well, handling routine things swiftly, and concentrating on those that are not routine and that may be a source of trouble. Much of what you learn in this book is routine to practicing engineers. The goal of this book is that students not only learn to do those routine things but also learn the scientific basis of the solution of those routine problems. In so doing, you will learn how engineers and scientists have turned yesterday's difficult problems into today's routine ones. That will help you to develop the habits of mind that will turn today's difficult problems into tomorrow's routine problems.

You should consider your degree of confidence in the answer to a problem. If the calculation used physical property data that is accurate to no more than ±5%, then it makes no sense to report the answer to 3 or more significant figures. If the solution presented required really speculative calculating approaches, or questionable input data, the reader should be alerted to that fact.

In the problems at the end of each chapter, one or two need to be broken down into simpler ones before they can be solved. The practice gained in doing these is well worth the effort.

1.11 WHY THIS BOOK IS DIFFERENT FROM OTHER FLUID MECHANICS BOOKS

Most undergraduate fluid mechanics books are written by mechanical or civil engineers. Please look at one; your impression will be that those books and this one are about totally different subjects. The reasons they look so different are these:

1. The fluid mechanics problems of greatest interest to mechanical and civil engineers (aerodynamics, flow around structures) are inherently two- or three-dimensional. They cannot be understood as or easily reduced to one-dimensional form. Most of the fluid mechanics problems of greatest interest to chemical engineers are inherently one-dimensional or can be understood and easily reduced to one-dimensional form. For this reason, civil and mechanical engineers start fluid mechanics as a three-dimensional study, and then derive the one-dimensional forms of greatest interest to chemical engineers from those three-dimensional forms.
2. Mechanical and civil engineers base most of their work on force and momentum. Those are the basic tools of the mechanical and civil engineer. Chemical engineers base most of their work on the conservation of mass and energy; the first course in chemical engineering is about mass and energy balances. Chemical engineers learn about force and momentum in physics but use them much less in their professional careers than they use mass and energy. The single most useful equation in fluid mechanics, Bernoulli's equation, can be found by starting with force and momentum, or with energy. Mechanical and civil engineers start with momentum. This book starts with energy. The energy approach makes much more sense to chemical engineers than does the momentum approach.
3. Momentum and force are vectors. For mechanical and civil engineers, fluid mechanics is inherently an exercise in vector calculus. Their books are full of vector equations. Many take the view that one of the main purposes of a fluid mechanics course is to immerse their students in the vector calculus, and make them exercise it. Mass and energy are scalars. Most of the quantities in chemical engineering are also scalars. Thus, chemical engineers have much less use of the vector calculus than do mechanical and civil engineers. Our graduate students are normally expected to become good at the vector calculus, but our undergraduates rarely use it.

For these reasons, this book uses scalars as much as possible and vectors only when necessary. It begins with the conservation of mass and energy and shows the vast range of practical fluid mechanical problems that can be solved with them, before it shows the momentum balance (which is inherently a vector balance) and shows the problems for which we need it. As a consequence, this book has far simpler mathematics than other fluids books. That does not mean that it sacrifices rigor: complexity is not rigor, or simplicity carelessness. In many cases the complete derivations are shown in appendices, with only the practical result shown in the main text.

In Parts II and III of this book, we cover the wide range of fluid mechanical problems of interest to chemical engineers that are best approached in a one-dimensional, energy-first approach. Then in Part IV we introduce the two- or three-dimensional, momentum-first approach, and discuss some of the chemical engineering problems that are best approached that way.

Figure 1.14 shows a chemical processing plant, in which lower-price chemicals are converted to higher-price (more useful) chemicals for profit and social benefit (and jobs for chemical engineers!). Many readers of this book will participate in the design, construction, and/or operation of similar plants. In such a plant the fluid flows are almost entirely inside pipes, pumps, vessels, fractionators, reactors, etc. We keep them inside because they are too valuable to waste and/or because their release would be dangerous or polluting. Almost all the flows in such a plant are most easily studied, predicted, and managed by the one-dimensional, mass-and-energy balance approach that forms Parts II and III of this book.

Figure 1.15 shows a schematic of a "cabin-type" industrial furnace. These are widely used for pyrolysis and reforming reactions in chemical engineering. Fifty years ago these were designed by hand using the one-dimensional methods presented in Parts II and III. With the recent spectacular advances in computer power, such furnaces are now designed using the two- and three-dimensional fluid mechanics methods presented in Part IV. Those methods and their computer implementation were largely developed by aeronautical engineers, to deal with the inherently

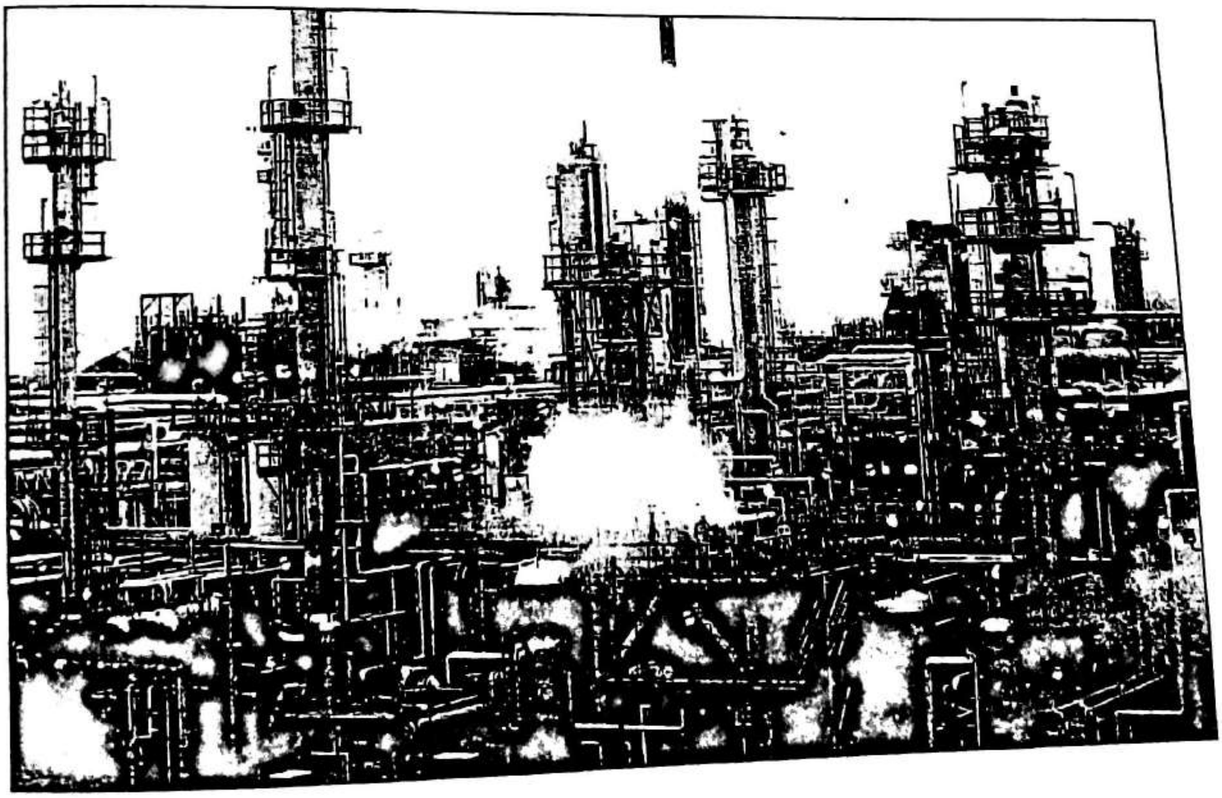

FIGURE 1.14
The mercaptan manufacturing unit at the Borger, Texas, complex of the Chevron Phillips Chemical Company. This plant is full of fluid flows, almost all of which are inside pipes, pumps, distillation columns, and associated vessels. (Courtesy of the Phillips Petroleum Company.)

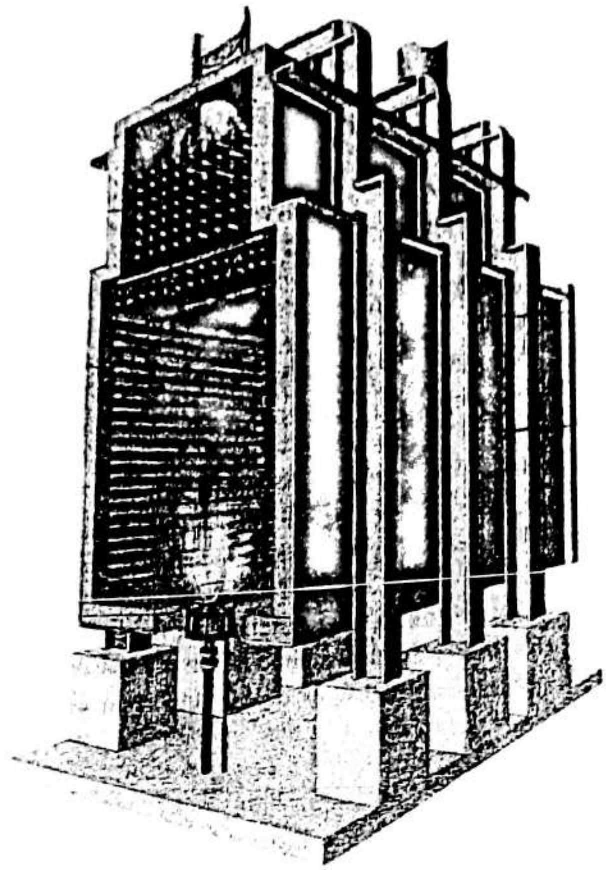

FIGURE 1.15
Cutaway drawing of a modern industrial furnace. The external steel frame supports the high-temperature refractory ceramic walls. There are multiple burners at the bottom, of which only one is shown. The flame heats the walls and the pipes through which the fluid being heated flows. Above the combustion chamber the hot gases pass over another bank of tubes, in which cooler fluid is warmed by the hot gases before they pass up the exhaust stacks seen at the top. (Courtesy of John Zinc Co. LLC.)

three-dimensional flow around airplanes. Furnace designers and other chemical engineers now use large computer codes to model the simultaneous three-dimensional fluid flow, heat transfer, and chemical reactions in such furnaces. The improvement in computational accuracy more than repays the additional cost and complexity. Part IV only introduces the basic ideas underlying such computations, and gives a bit of their history.

1.12 SUMMARY

1. Fluid mechanics is the study of forces and motions in fluids.
2. Fluids are substances that move continually when subjected to a shear force as long as the force is applied. Solids are substances that deform slightly when subjected to a shear force and then stop moving and permanently resist the force. There are, however, intermediate types of substance; the distinction between solid and liquid is one of degree rather than of kind.
3. Fluid mechanics is based on the principle of the conservation of matter, the first two laws of thermodynamics, Newton's laws of motion, and careful experiments.
4. Gases have weak intermolecular attractions and expand without limit. Liquids have much stronger intermolecular attractions and can expand very little. With increasing temperature and pressure, the differences between liquids and gases gradually disappear.
5. Density is mass per unit volume. Specific gravity of liquids is density / (density of water at 4°C). Specific gravity of gases is density / (density of air at the same T and P).

6. Viscosity is a measure of a fluid's resistance to flow. Most simple fluids are represented well by Newton's law of viscosity. The exceptions (non-Newtonian fluids) are generally complex mixtures, some of which are of great practical significance. Kinematic viscosity is viscosity divided by density.
7. Surface tension is a measure of a liquid's tendency to take a spherical shape, caused by the mutual attraction of the liquid's molecules.
8. Pressure is compressive force divided by area. It is the same in all directions for a fluid at rest and practically the same in all directions for most moving fluids.
9. In handling the units (dimensions) in this text, one should always write down the units of any dimensioned quantity and then multiply or divide by 1 to find the desired units in the answer.
10. Much of fluid mechanics can be based either on force and momentum, or on energy. This book, for chemical engineers, bases most of fluid mechanics on energy, thus dealing mostly with scalars instead of vectors. Momentum and vectors are used where they are needed.

PROBLEMS

See the Common Units and Values for Problems and Examples inside the back cover of this text. An asterisk (*) on the problem number indicates that the answer is in App. D.

1.1. In Sec. 1.3 the basic laws on which fluid mechanics rests are listed. How many of the basic laws of nature are not included in the list? To answer this question, make a list of what you consider to be the basic laws of nature. By basic laws, we mean laws that cannot be derived from other more basic ones; for example, Galileo's "laws of falling bodies" can be derived form Newton's laws and are not basic.

1.2. At low pressures there is a significant difference between the densities of liquids and of gases. For example, at 1 atm the densest gas known to the author is uranium hexafluoride, which has $M = 352$ g / mol; its normal boiling point is 56.2°C. Calculate its density in the gas phase at 1 atm and 56.2°C, assuming that it obeys the ideal gas law. The least dense liquid known to the author is liquid hydrogen, which at its normal boiling point, 20 K, has a density of 0.071 g / cm^3. Liquid helium also has a very low density, about 0.125 g / cm^3 (at 4 K). Excluding these remarkable materials, make a list of liquids which at 1 atm can exist at densities of less than 0.5 g / cm^3. A good source of data is *The Handbook of Chemistry and Physics*, CRC Press, Boca Raton, Florida, annual editions.

1.3.*For some oil and gas drilling operations we need a high-density drilling fluid (called "drilling mud"). Repeat Example 1.1 for a mud that is 50 wt. % water, 50 wt. % $BaSO_4$ (barite), $SG_{barite} = 4.49$.

1.4. Why are specific gravities most often referred to the density of water at 4°C instead of 0°C?

1.5.*A special-purpose piece of laboratory glassware, called a *pyncnometer*, is used to measure liquid densities. It has a volume of 25 cc, and a mass of 17.24 g when it is full of air. When filled with a liquid of unknown density, its mass = 45.00 g. What is the density of this liquid? How large an error do we make if we ignore the mass of the air that was in it when we weighed it and found m = 17.24 g?

1.6. The American Petroleum Institute (API) gravity (used extensively in the petroleum industry) is defined, in "degrees," by

$$\text{Deg API} = \frac{141.5}{\text{specific gravity}} - 131.5 \tag{1.AG}$$

Here the specific gravity is the ratio of the density of the liquid to that of water, both at 60°F. Sketch the relation between density in g/cm^3 and degrees API. What advantages of this scale might have led the petroleum industry to invent and adopt it?

1.7. Estimate the specific gravities (gas) for methane and propane. Their molecular weights are shown inside the back cover. (Commercial natural gas and commercial propane are mostly methane and propane, with small amounts of other substances, which may be ignored for this problem.) Which is more dangerous, a natural gas leak or a propane leak? Why?

1.8. What are the dimensions of dV/dy? What are the dimensions of shear stress? Shear stress in liquids is often called "momentum flux" [2]. Show that shear stress has the same dimensions as momentum / (area · time). What are the dimensions of viscosity?

1.9. List as many applications as you can of industrial, domestic, or other materials in which non-Newtonian viscosity behavior is desirable. In each case specify why this behavior is desirable.

1.10. In Example 1.2 we replaced a cylindrical problem with a linear approximation. The velocity distribution for this flow, taking the cylindrical character into account (see Prob. 15.22 and also [2], p. 91) is

$$V_\theta = \omega\left(\frac{k^2}{1-k^2}\right)\cdot\left(\frac{R^2}{r} - r\right) \tag{1.AH}$$

where R is the radius of the outer cylinder, r is the local radius, $k = r_{\text{inner cylinder}}/R$, and ω is the angular velocity of the inner cylinder.

(*a*) Verify that this distribution shows a zero velocity at the radius of the outer, nonmoving cylinder and shows $V_\theta = \omega kR$ at the surface of the inner, rotating cylinder.

(*b*) The shear rate in cylindrical coordinates, for a fluid whose velocity depends only on r (equivalent to dV/dy in rectangular coordinates) is given by

$$\sigma = \begin{pmatrix}\text{shear rate cylindrical}\\ \text{coordinates}\end{pmatrix} = r\frac{d}{dr}\left(\frac{V_\theta}{r}\right) \tag{1.AI}$$

Show that for the above velocity distribution, the shear rate at the surface of the inner cylinder is given by

$$\sigma = \omega\left(\frac{2}{1-k^2}\right) \tag{1.AJ}$$

(*c*) Show that the shear rate computed by Eq. 1.AJ using the values in Example 1.2 is 12.26 / s, which is 1.15 times the value for the flat approximation in Example 1.2. The manual for the viscometer shown in Fig. 1.5 provides formulae equivalent to those in this problem.

1.11.*Calculate the surface / volume of a sphere, a cube, and a right cylinder of height equal to diameter. Which has the least surface / volume?

1.12. A liquid under tensile stress is unstable [9]; a small disturbance can cause it to boil and thereby change to a stable state. Make a list of other unstable situations demonstrable in a chemistry or physics laboratory. The working criterion of instability is that a very small disturbance can cause a large effect.

1.13. Earth may be considered a sphere with a diameter of ≈8000 mi and an average SG of ≈5.5. What is its mass? What is its weight? Explain your answer.

1.14. A cubic foot of water at 68°F = 20°C weighs 62.3 lbf on earth.
(*a*) What is its density?
(*b*) What does it weigh on the moon ($g \approx 6\ \text{ft/s}^2$)?
(*c*) What is its density on the moon?

1.15.*How many U.S. gallons are there in a cubic mile? The total proven oil reserves of the U.S. are roughly 30×10^9 bbl. How many cubic miles is this?

1.16. In electrochemical equations it is common to write in the symbol $\mathcal{F}$ (called *Faraday's constant*) to remind the user to convert from moles of electrons to coulombs. This is just like the force-mass and thermal energy-mechanical energy conversion factor, namely,

$$\mathcal{F} = 1 = \frac{96{,}500\ \text{C}}{\text{g equiv of electrons}} \tag{1.AK}$$

1 g equiv of electrons = 6.02×10^{23} electrons. How many electrons are there in 1 C?

1.17. Older thermodynamics and fluids textbooks not only put the symbol g_c into equations to remind us to make the force-mass conversion but also put a J in equations to remind us to make the conversion from mechanical units of energy (e.g., ft · lbf) to thermal units of energy (e.g., Btu). Equation 1.11 shows the values of $g_c = 1$ for a variety of systems of units. Show the corresponding equation for J. (The use of the symbol g_c caused confusion because it is similar to g. Is there a symbol with which the J discussed in this problem can be confused?)

1.18.*As discussed in the text, the slug and the poundal were invented to make the conversion factor (mass length) / (force time^2) have a coefficient of 1. A new unit of length or a new unit of time could just as logically have been invented for this. Let us name those units the *toof* and the *dnoces*. What are the values of the toof and the dnoces in terms of the foot and the second?

1.19. In U.S. irrigation practice water is measured in acre-feet, which is the volume of water that covers an acre of land, one foot deep. What is the mass of an acre-foot of water (1 mi^2 = 640 acres)? What is the mass of a hectare-meter (ha · m) of water (km^2 = 100 ha)? Why would the acre-foot be a practical measure of irrigation water?

1.20. Einstein's equation $E = mc^2$ indicates that the speed of light squared must be expressible in units of energy per unit mass. What is the value of the square of the speed of light in Btu / lbm? In J / kg? The speed of light $c \approx 186{,}000$ mi / s = $2.998 \cdot 10^8$ m / s.

1.21.*A common basis for comparing rocket fuel systems is the *specific impulse*, defined as lbf of thrust produced divided by lbm / s of fuel and oxidizer consumed (see Chap. 7). The common values are 250 to 400 lbf · s / lbm. We frequently see the specific impulse referred to simply as "300 s." Is 300 s the same thing as 300 lbf s / lbm? European engineers regularly express the same quantity in terms of the equivalent exhaust velocity of the rocket. If a rocket has a specific impulse of 300 lbf s / lbm, what is its equivalent exhaust velocity?

1.22. Most U.S. engineers work with heat fluxes with the unit Btu / (h ft^2). In the rocket business the common unit is cal / (s cm^2). How many Btu / (h ft^2) is 1 cal / (s cm^2)? The proper SI unit is J / (m^2 s). How many Btu / (h ft^2) = 1 J / (m^2 s)?

1.23.*The Reynolds number, discussed in Chap. 6, is defined for a pipe as (velocity · diameter · density) / viscosity. What is the Reynolds number for water flowing at 10 ft / s in a pipe with a diameter of 6 in? What are its dimensions?

1.24. The flow of fluids through porous media (such as oil sands) is often described by *Darcy's equation* (see Chap. 11):

$$\frac{\text{Flow}}{\text{Area}} = \frac{\text{permeability}}{\text{viscosity}} \cdot \text{pressure gradient} \tag{1.AL}$$

The unit of permeability is the *darcy,* which is defined as that permeability for which a pressure gradient of 1 atm / cm for a fluid of 1 cP viscosity produces a flow of 1 cm^3/s through an area of 1 cm^2. What are the dimensions of the darcy? What is its numerical value in the dimension? Give the answer both in English units and in SI units.

1.25.*What mass (weight?) would be needed in Example 1.3 if the liquid had been water?

1.26. Determine the value of X in the equation,

$$1.0 \frac{\text{Btu}}{\text{lbm} \cdot {}^\circ\text{F}} = X \frac{\text{cal}}{\text{g} \cdot {}^\circ\text{C}} \tag{1.AM}$$

1.27. In strict SI, the only unit of pressure is the Pascal (Pa). The most widely used derived unit is the bar (bar = 10^5 Pa = 0.1 MPa). What is the relation between the bar and the pressure of the atmosphere at sea level? Why is the bar a popular choice for a working SI derived unit?

1.28.*Air pollutant emissions from autos and trucks in the United States are reported in a mixed metric-English unit, g / mi. Suggest reasons why this might be a practical unit.

1.29. Many European pressure gauges give the pressure in kg / cm^2. Is this kgm or kgf? Why would this be a convenient unit of pressure?

1.30. In the third part of Example 1.6, what would have happened if we had taken the force-mass conversion factor as 32.2 lbm · ft / (lbf · s^2) instead of 9.8 kgm · m / (kgf · s^2)?

REFERENCES FOR CHAPTER 1

1. Reiner, M. "The Flow of Matter." *Scientific American 201(6),* (1959), pp. 122–138.
2. Bird, R. B., E. N. Stewart, and W. E. Lightfoot, *Transport Phenomena,* 2nd ed. New York: Wiley, 2002.
3. Poling, B. E., J. M. Prausnitz, and J. P. O'Connell, *The Properties of Gases and Liquids,* 5th ed., New York: McGraw-Hill, 2001.
4. Schneirla, T. C., and G. Piel, "The Army Ant." *Scientific American 178(6),* (1948), pp. 16–23.
5. Boys, C. V. *Soap Bubbles and the Forces Which Mould Them,* paperback ed., Garden City, New York: Doubleday, 1959. First published 1902. This interesting, informative, semitechnical book is highly recommended. Reading time, about 3 h.
6. Davies, J. T., and E. K. Rideal, *Interfacial Phenomena,* 2nd ed. New York: Academic Press, 1963.
7. Lodge, A. S. *Elastic Liquids; An Introductory Vector Treatment of Finite-Strain Polymer Rheology.* London: Academic Press, 1964.
8. Kaschiev, D. *Nucleation; Basic Theory with Applications.* Oxford: Butterworth Heinemann, 2000.
9. Zimmerman, M. H. "How Sap Moves in Trees." *Scientific American 208(3),* (1963), pp. 133–142.
10. de Nevers, N. "The Poundal per Square Foot, the Pascal and SI Units." *Engineering Education 78(2),* (1987), p. 137.
11. Pollack, A. "Missing What Didn't Add Up, NASA Subtracted an Orbiter." *New York Times* (Oct. 1, 1999), Section A, p. 1.
12. ASTM, *Metric Practice Guide,* ASTM Publication # E 380-97$^{\epsilon}$, Philadelphia, PA: ASTM, 1997.
13. Leavitt, R. "Kyoto" in *Fodor's Japan and Korea, 1982,* edited by A. Tucker. New York: Fodor's Modern Guides, Inc., 1982.

CHAPTER 2

FLUID STATICS

In this chapter we apply Newton's law of motion, $F = ma$, to fluids at rest. We will see that this leads to a remarkably simple equation:

$$\frac{dP}{dz} = -\rho g \tag{2.1}$$

This equation and its applications are almost the whole of fluid statics.

In Chap. 7 we will apply Newton's law of motion to moving fluids. This chapter is really only part of the more general application made in Chap. 7. In Chaps. 5, and 6, however, we will need some of the results from this chapter, and the kinds of problem we deal with here are different from (and simpler than) those in Chap. 7; for these reasons a separate chapter on fluid statics is practical at this point. Remember that all we do in this chapter is apply $F = ma$ to a static fluid; the more general applications, covering both moving and static fluids, are discussed in Chap. 7.

In the most careful work, we would write Eq. 2.1 as a vector equation, because the acceleration of gravity has both magnitude and direction, as does the *gradient* of the pressure. We will see that this chapter can be developed, to give all the correct and useful results, without using vector calculus. But remember than any application of Newton's law of motion is a vector application. We will say more about that in the introduction to Chap. 7, where we consider the classes of problems in which we must use the vector nature of forces and of momentum, and again in Chap. 15, where we reintroduce Newton's law in three-dimensional vector calculus form.

2.1 THE BASIC EQUATION OF FLUID STATICS

For a simple fluid at rest the pressure is the same in all directions. This idea seems hard for students. Suppose you compress a small coil spring between your thumb and forefinger. The spring exerts the same force on thumb and forefinger, but in opposite directions. If you rotate your hand any way, it still exerts equal forces in two opposite directions. So also with pressures in fluids at rest. There are no shear stresses in a fluid at rest. These facts lead to the basic equation of fluid statics. Consider a small block of fluid that is part of a large mass of fluid at rest in a gravity field; see Fig. 2.1. Since the fluid is at rest, there are no accelerations, and the sum of the forces on any part of the fluid in any direction is zero. Let us consider the z direction, opposite to the direction of gravity. The forces that act on the small block of fluid in the z direction are the pressure forces on the top and bottom and the force of gravity acting on the mass of the element. Their sum (positive upward) is

$$(P_{z=0})\,\Delta x\,\Delta y - (P_{z=\Delta z})\,\Delta x\,\Delta y - \rho g\,\Delta x\,\Delta y\,\Delta z = 0 \tag{2.2}$$

Dividing by $\Delta x\,\Delta y\,\Delta z$ and rearranging, we find

$$\frac{P_{z=\Delta z} - P_{z=0}}{\Delta z} = -\rho g \tag{2.3}$$

If we now let Δz approach zero, then

$$\lim_{\Delta z \to 0} \frac{\Delta P}{\Delta z} = \frac{dP}{dz} = -\rho g \tag{2.1}$$

This is the basic equation of fluid statics, also called the *barometric equation.* It is correct only if there are no shear stresses on the vertical faces of the cube in Fig. 2.1. If there are such shear stresses, then they may have a component in the vertical direction, which must be added into the sum of forces in Eq. 2.2. For simple Newtonian fluids, shear stresses in the vertical directions can exist only if the fluid has a different vertical velocity on one side of the cube from that on the other (see Eq. 1.4). Thus, Eq. 2.1 is correct if the fluid is not moving at all, which is the case in fluid statics; or if it is moving but only in the x and y directions; or if it has a uniform velocity in the z direction. In this chapter we will apply it only when a fluid has no motion relative to its container or to some set of fixed coordinates. In later chapters we will

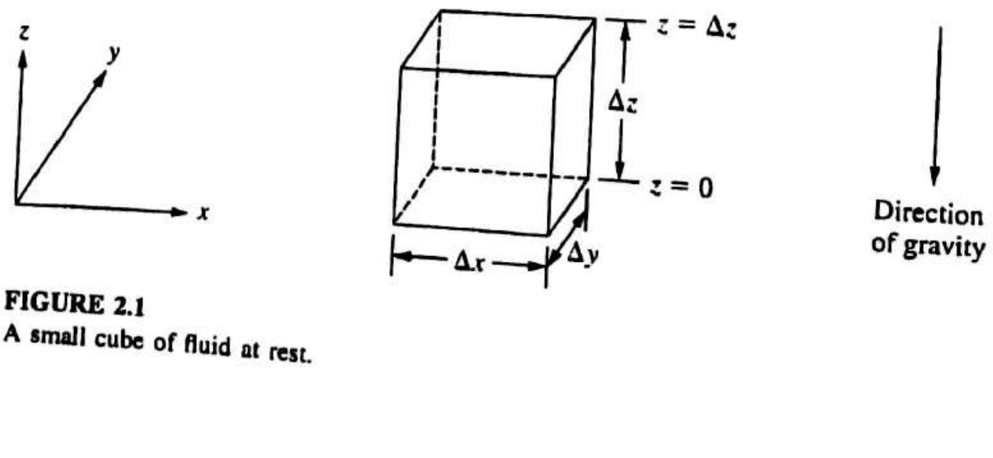

FIGURE 2.1
A small cube of fluid at rest.

apply it to flows in which there is no motion in the z direction or there is a motion with a uniform z component. We will also describe nonmoving fluids in accelerated motion in this chapter.

For complicated fluids, such as toothpaste, paints, and jellies, Eq. 2.1 is not correct, because these fluids can sustain small but finite shear stresses without any motion. The equation simply is not applicable. To find its equivalent, it is necessary to make up a sum of forces that includes shear forces on the vertical sides of the cube.

The barometric equation describes the change in pressure with distance upward, where "upward" is opposite to the direction of gravity, called z. (The minus sign appears in Eq. 2.1 because gravity points in the minus z direction.) If we want to know the change of pressure with distance in some other, nonvertical direction, call it direction a, then we can write

$$\frac{dP}{da} = \frac{dz}{da} \cdot \frac{dP}{dz} = -\rho g \frac{dz}{da} \tag{2.4}$$

But, as shown in Fig. 2.2,

$$\frac{dz}{da} = \frac{\Delta z}{\Delta a} = \cos\theta \tag{2.5}$$

where θ is the angle between the direction a and the z axis. Substituting this equation into Eq. 2.4, we have

$$\frac{dz}{da} = \frac{\Delta z}{\Delta a} = \cos\theta \qquad \text{or} \qquad \frac{dP}{da} = -\rho g \cos\theta \tag{2.6}$$

A particularly interesting direction a is the one at right angles to z, that is, any direction parallel to the x-y plane. For that direction θ is 90°, $\cos\theta$ is 0, and the pressure does not change with distance. Thus, from Eq. 2.6 we see that for a fluid at rest any surface that is perpendicular to the direction of gravity is a surface of constant pressure. The most interesting constant-pressure surface of a body of fluid at rest is the one with zero gauge pressure, that is, the surface in contact with the atmosphere. Since this is a constant-pressure surface, it must be everywhere perpendicular to the direction of gravity. On a global scale this makes the free surface of the oceans practically a sphere. (The earth is not quite spherical, being slightly flattened at the poles.) In typical engineering operations it means that the free surface of a liquid exposed to the atmosphere is practically a horizontal plane (Prob. 2.1).

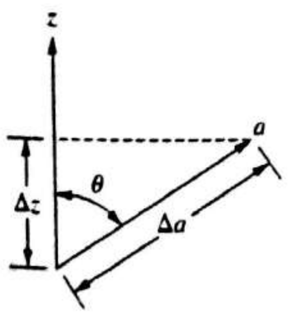

FIGURE 2.2
Relations between the a and z directions.

The product of density and gravity, which appears in Eq. 2.1, is often called the *specific weight*, and is given the symbol γ:

$$\rho g = \gamma = \text{specific weight} \tag{2.7}$$

At any place where the acceleration of gravity is equal to $32.2\ \text{ft/s}^2 = 9.81\ \text{m/s}^2$ (practically any place on the surface of the earth), the specific weight expressed in lbf/ft^3 (or kgf/m^3) is numerically equal to the density expressed in lbf/ft^3 (or kgf/m^3). The value in N/m^3 is numerically different.

Example 2.1. Calculate the specific weight of water at a place where the acceleration of gravity is 32.2 ft / s.

$$\gamma = \rho g = 62.3 \frac{\text{lbm}}{\text{ft}^3} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} = 62.3 \frac{\text{lbf}}{\text{ft}^3}$$

$$= 998.2 \frac{\text{kgf}}{\text{m}^3} = 9792 \frac{\text{N}}{\text{m}^3} \tag{2.A}$$

■

If one deals principally with fluid flows in which the forces of gravity are dominant, then one often can simplify the calculations by replacing the density in all equations by γ / g. In civil engineering hydraulics this is normally the case, and this is common practice. On the other hand, if one deals mostly with flows whose gravity terms are small compared with the other terms, then it is more convenient to work with ρ than with γ / g. In chemical engineering problems the gravity terms are normally small, so the specific weight is seldom used.

2.2 PRESSURE-DEPTH RELATIONSHIPS

Equation 2.1 is a separable, first-order differential equation that can be separated and integrated as follows:

$$\int dP = -\int \rho g \, dz \tag{2.8}$$

However, to perform the integration, it is necessary to have some relation between ρ, g, and z. In situations on the surface of the earth g is practically constant (see Sec. 2.8), so we may take it outside the integral sign. Several possible relations between ρ and z lead to simple integrations of the equation, as shown in the following material.

2.2.1 Constant-Density Fluids

No real substances have constant density; the density of all substances increases as the pressure increases. However, for most liquids at temperatures far below their critical temperatures, the effect of pressure on density is very small. For example, raising the pressure of water at 100°F from 1 to 1000 psia while holding the temperature constant causes the density to increase by 0.3%. In most engineering calculations we can neglect such small changes in density. Then we can take ρ outside the integral sign in Eq. 2.8 and find that the pressure change is

$$P_2 - P_1 = -\rho g(z_2 - z_1) \qquad \text{[constant density]} \tag{2.9}$$

Example 2.2. When the submarine *Thresher* sank in the Atlantic in 1963, it was estimated in the newspapers that the accident had occurred at a depth of 1000 ft (304.9 m). What is the pressure of the sea at that depth?

Seawater may be considered incompressible, with density 63.9 lbm / ft³ (1024 kg / m³). Thus

$$P_{1000 \text{ ft}} = 14.7 \frac{\text{lbf}}{\text{in}^2} + 63.9 \frac{\text{lbm}}{\text{ft}^3} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 1000 \text{ ft} \cdot \frac{\text{ft}^2}{144 \text{ in}^2} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}}$$

$$= 14.7 \frac{\text{lbf}}{\text{in}^2} + 444 \frac{\text{lbf}}{\text{in}^2} = 459 \frac{\text{lbf}}{\text{in}^2} \tag{2.B}$$

or

$$P_{304.9\text{ m}} = 101.3\text{ kPa} + 1024\,\frac{\text{kg}}{\text{m}^3}\cdot 9.81\,\frac{\text{m}}{\text{s}^2}\cdot 304.9\text{ m}\cdot\frac{\text{Pa}}{\text{N/m}^2}\cdot\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$= (101.3 + 3062.8)\text{ kPa} = 3.164\text{ MPa} \qquad (2.C)$$

■

In hydraulics problems and in all problems involving a free surface exposed to the atmosphere, we can further simplify Eq. 2.9 by working in gauge pressure. The gauge pressure is zero at the free surface: $P_{1\text{ gauge}} = 0$. We now define the depth as the distance measured downward from the free surface and give it the symbol h,

$$h = z_{\text{free surface}} - z \qquad (2.10)$$

in which case Eq. 2.9 simplifies to

$$P = \rho g h \qquad \text{[gauge pressure, constant density]} \qquad (2.11)$$

In Example 2.2, at $h = 1000$ ft the gauge pressure is $P = 444$ psig = 3062.8 kPa, gauge.

Example 2.3. A cylindrical oil-storage tank is 60 ft deep and contains an oil of density 55 lbm / ft^3. Its top is open to the atmosphere. What is the gauge-pressure-depth relation in this tank?

The gauge pressure is zero at the free surface. At the bottom it is

$$P_{\text{bottom}} = 55\,\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\,\frac{\text{ft}}{\text{s}^2}\cdot 60\text{ ft}\cdot\frac{\text{ft}^2}{144\text{ in}^2}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\text{ lbm}\cdot\text{ft}} = 22.9\,\frac{\text{lbf}}{\text{in}^2} = 158\text{ kPa} \quad (2.D)$$

From Eq. 2.11 we know that the pressure-depth relation is linear; see Fig. 2.3.

■

2.2.2 Ideal Gases

The density of gases changes significantly with pressure changes, so we must be cautious about taking the density outside the integral sign in Eq. 2.8. At low pressure the densities of most gases are well approximated by the ideal gas law,

$$\boxed{\rho = \frac{PM}{RT}} \qquad \text{[ideal gas]} \qquad (2.12)$$

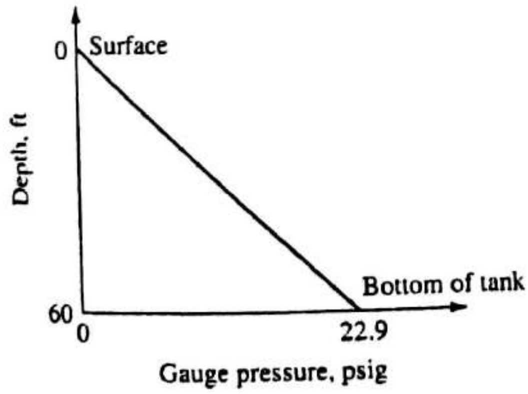

FIGURE 2.3
Pressure-depth relation in Example 2.3.

Here T is the absolute temperature, in ° Rankine or in Kelvins (T °R = T °F + 459.69, or T K = T °C + 273.15); R is the universal gas constant, whose value in various systems of units is shown on the inside back cover; M is the molecular weight, normally expressed in g / mol or lbm / lbmol. [This formulation of the ideal gas law gives the density in units of lbm / ft^3. In chemistry one often sees the ideal gas law written as $\rho = P / RT$, which gives the density in

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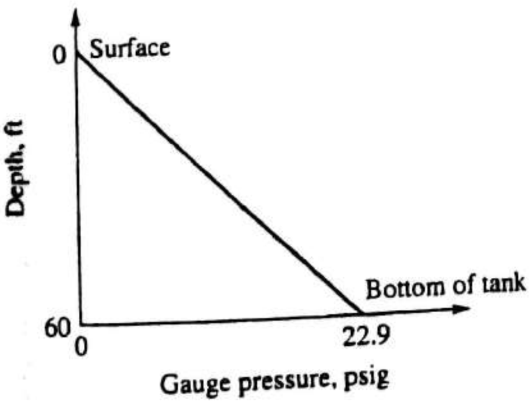


FIGURE 2.3
Pressure-depth relation in Example 2.3.

Here T is the absolute temperature, in ° Rankine or in Kelvins ($T\,°\text{R} = T\,°\text{F} + 459.69$, or $T\,\text{K} = T\,°\text{C} + 273.15$); R is the universal gas constant, whose value in various systems of units is shown on the inside back cover; M is the molecular weight, normally expressed in g / mol or lbm / lbmol. [This formulation of the ideal gas law gives the density in units of lbm / ft^3. In chemistry one often sees the ideal gas law written as $\rho = P / RT$, which gives the density in lbmol / ft^3 or mol / m^3. Multiplying the latter density by the molecular weight (in lbm / lbmol or g / mol) gives the density, shown here.]

Substituting Eq. 2.12 for the density in Eq. 2.1, we find

$$\frac{dP}{dz} = -\frac{PM}{RT}g \qquad \text{[ideal gas]} \tag{2.13}$$

If the temperature is constant, this can be separated and integrated as follows:

$$\int_1^2 \frac{dP}{P} = \frac{-gM}{RT}\int_1^2 dz \tag{2.14}$$

$$\ln\frac{P_2}{P_1} = \frac{-gM}{RT}(z_2 - z_1) \tag{2.15}$$

$$P_2 = P_1 \exp\left(\frac{-gM\Delta z}{RT}\right) \qquad \text{[isothermal, ideal gas]} \tag{2.16}$$

Example 2.4. At sea level the atmospheric pressure is 14.7 psia and the temperature is 59°F = 15°C = 519°R. Assuming that the temperature does not change with elevation (a poor assumption, but one that simplifies the mathematics and that will be reexamined in a few pages), calculate the pressure at 1000, 10,000, and 100,000 ft. For z = 1000 ft, we find

$$P_2 = P_1 \exp\left(\frac{-32.2\ \text{ft}/\text{s}^2 \cdot 29\ \text{lbm}/\text{lbmol} \cdot 1000\ \text{ft}}{(10.73\ \text{lbf}/\text{in}^2 \cdot \text{ft}^3/\text{lbmol}\cdot{}^\circ\text{R}) \cdot 519^\circ\text{R}} \cdot \frac{\text{ft}^2}{144\ \text{in}^2} \cdot \frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}\right)$$

$$= P_1 \exp(-0.03616) = \frac{P_1}{\exp 0.03616} = \frac{P_1}{1.0368} = 0.965\ \text{atm} \tag{2.E}$$

We can calculate the pressures at the other two elevations and show them, along with the results from the next example, in Table 2.1. ∎

How much error would we have made if we had used the constant-density formulae instead of taking the change in density into account?

Example 2.5. Rework Example 2.4, assuming that air is a constant-density fluid, which has the same density at all elevations as it has at 14.7 psia and 59°F.

Here we use Eq. 2.9:

$$P_2 - P_1 = -\rho_1 g(z_2 - z_1) = (-P_1 M / RT)g(z_2 - z_1) \tag{2.F}$$

$$P_2 = P_1[1 - (gM/RT)(z_2 - z_1)] \tag{2.G}$$

TABLE 2.1
Calculated atmospheric pressures for Examples 2.4 and 2.5

Elevation, z, ft	Elevation, z, m	$gM\,\Delta z / RT$	P_2, atm, Example 2.4	P_2, atm, Example 2.5
1,000	304.8	0.03616	0.965	0.964
10,000	3,048	0.3616	0.697	0.638
100,000	30,480	3.616	0.0269	−2.61

For 1000 feet we find

$$P_2 = P_1 \cdot \left(1 - \frac{32.2\ \text{ft}/\text{s}^2 \cdot 29\ \text{lbm}/\text{lbmol} \cdot 1000\ \text{ft}}{(10.73\ \text{lbf}/\text{in}^2 \cdot \text{ft}^3/\text{lbmol} \cdot {}^\circ R) \cdot 519^\circ\text{R}} \cdot \frac{\text{ft}^2}{144\ \text{in}^2} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2\ \text{lbm} \cdot \text{ft}}\right)$$

$$= P_1 \cdot (1 - 0.03616) = 0.964\ \text{atm} \tag{2.H}$$

This value, plus the corresponding ones for 10,000 and 100,000 ft, are shown in Table 2.1. ■

From Table 2.1 we see that up to 1000 ft the assumption of constant density creates a negligible error, at 10,000 ft it makes a 9% error, and at 100,000 ft it gives absurd results (negative absolute pressure in a gas??). Thus, for ordinary industrial-sized equipment (generally less than 1000 ft high) one can accurately calculate changes in gas pressure with elevation as if the gas had a constant density. On the other hand, in aeronautics and meteorological problems, in which the elevations are often from 10,000 to 100,000 ft, this simplification leads to disastrous errors.

In Examples 2.4 and 2.5 we made the simplifying assumption that the atmosphere was isothermal. Anyone who has gone to the mountains in the summer to get out of the heat did so because the atmosphere is not isothermal. To understand why the air temperature decreases with elevation, consider a mass of air being lifted from one elevation to a higher one (by a wind, for example, blowing it over a mountain range). The air mass expands because the pressure of the surrounding air decreases as it rises. The air mass is cooled because as it expands it does expansion work on the surrounding air. Air is a fairly poor conductor of heat, so during this process the rising air undergoes an expansion that is close to adiabatic and close to reversible. If it were exactly reversible and adiabatic, then the temperature-pressure-elevation relation would be exactly the isentropic one. For an isentropic atmosphere one can work out the following elevation-temperature and elevation-pressure relationships (Prob. 2.16):

$$P_2 = P_1\left(1 - \frac{k-1}{k} \cdot \frac{gM\Delta z}{RT_1}\right)^{k/(k-1)} \qquad \text{[isentropic, ideal gas]} \tag{2.17}$$

$$T_2 = T_1\left(1 - \frac{k-1}{k} \cdot \frac{gM\Delta z}{RT}\right) \qquad \text{[isentropic, ideal gas]} \tag{2.18}$$

Here k is the ratio of specific heats (discussed in Chap. 8); for air its value is practically constant at 1.4.

The isothermal atmosphere in Examples 2.4 and 2.5 would be observed if air were a perfect conductor of heat, evening out all temperature differences instantly. The isentropic atmosphere in Eqs. 2.17 and 2.18 would be observed if air were a perfect insulator against heat conduction, transferring no heat at all. Experimental measurements show that the real behavior of the atmosphere is intermediate between these two extremes. Heat is conducted outward from the earth not only by simple conduction in the air (which is fairly slow) but also by winds, which mix cold and warm air layers, and by condensation of water vapor and by infrared radiation. For calculation purposes meteorologists and aeronautical engineers have defined a "standard atmosphere," which agrees well with the *average* of many observations over the whole planet and all seasons of the year. As shown in Fig. 2.4, this standard atmosphere is

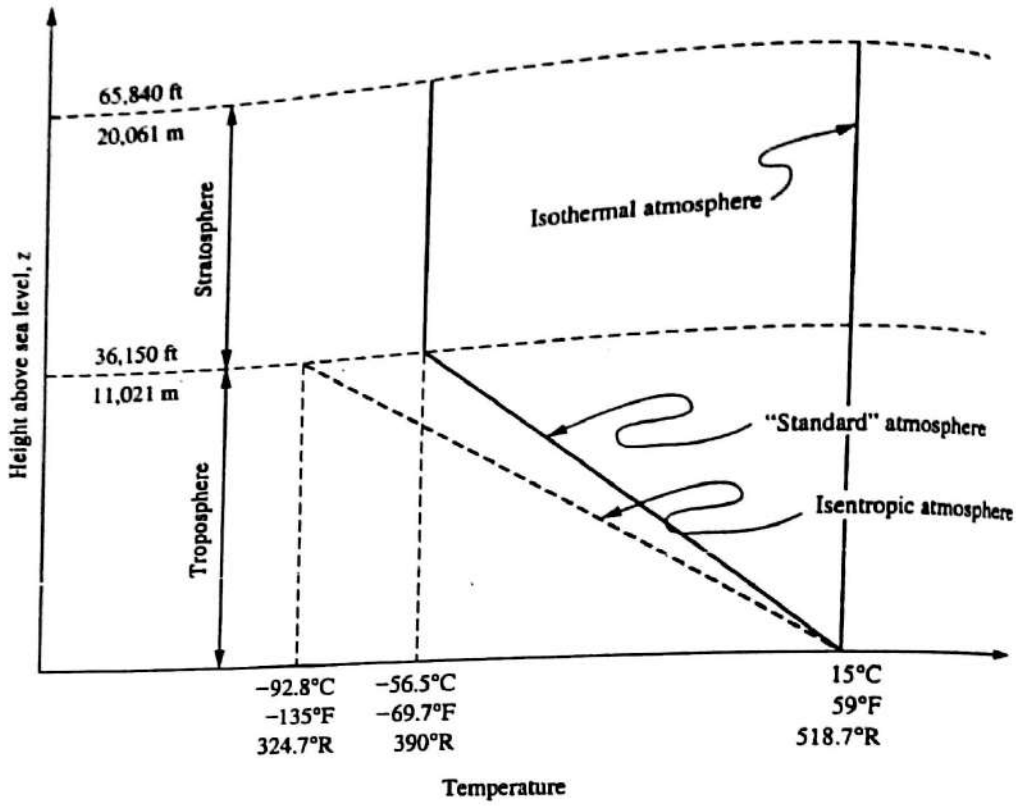


FIGURE 2.4
Comparison of standard atmosphere, isentropic atmosphere, and isothermal atmosphere.

indeed intermediate between the isothermal and isentropic atmospheres. It is an average; most interesting weather phenomena are caused by deviations from it. For a simple discussion of this see ([1], Chap. 5). From the standard-atmosphere temperature one may calculate a "standard" pressure-height curve (Prob. 2.17). Tables showing all the properties of the standard atmosphere are found in handbooks [2].

2.3 PRESSURE FORCES ON SURFACES

Static, simple fluids can exert only pressure forces on surfaces adjacent them. Since pressure is the normal (perpendicular) force per unit area, the pressure forces must act normal to the surface. Moving fluids can exert not only pressure forces but also shear forces, so the combined force exerted by a moving fluid on a surface is generally not normal to the surface. However, in problems involving moving fluids it is often convenient to treat the pressure and shear forces as separate and thus calculate the pressure force exactly as we do here, but use the pressure distribution on the surface corresponding to the flow situation rather than to the static-fluid one discussed in this chapter.

For an infinitesimal surface area the force exerted is

$$dF = P\,dA \tag{2.19}$$

This dF is a vector quantity; it has direction (perpendicular to the surface) and magnitude. For a plane surface all the differential dF vectors point in the same direction, so that we can find the total force simply by integrating this equation:

$$F = \int P\, dA \tag{2.20}$$

To calculate pressure forces on curved surfaces, we normally resolve the infinitesimal dF in Eq. 2.19 into its x and y components and integrate those. That calculation is shown in most civil and mechanical engineering fluid mechanics textbooks, for example, Reference 3 (Chap. 2).

If the pressure over an entire plane surface is constant, then Eq. 2.20 becomes

$$F = PA \qquad \text{[constant pressure, plane surface]} \tag{2.21}$$

Because the static pressure in gases changes very slowly with elevation, this is practically true for all moderate-sized flat surfaces exposed to gases, independent of the orientation of the surface. The same is not true for liquids.

For *horizontal* plane surfaces exposed to static liquids the pressure is constant over the entire surface, so Eq. 2.21 gives the required force.

Example 2.6. An oil-storage tank has a flat, horizontal, circular roof 120 ft in diameter. What force does the atmosphere exert on the roof?

$$F = PA = 14.7\,\frac{\text{lbf}}{\text{in}^2}\cdot\frac{\pi}{4}(120\text{ ft})^2\cdot 144\,\frac{\text{in}^2}{\text{ft}^2} = 2.39\cdot 10^7\text{ lbf} = 106.5\text{ MN} \tag{2.I}$$

■

The roof of the storage tank can withstand this startlingly large downward force because the gas or air inside it exerts an equal upward force, so the *net* force due to the pressure of the atmosphere and the pressure of the gas inside the tank is zero. Since these forces ordinarily cancel out of force calculations, it is customary to make such calculation in gauge pressure, whenever both sides of the surface are subjected to the pressure of the atmosphere in addition to the gauge pressure of the liquid. Such tanks normally are vented to the atmosphere, to prevent having a gauge pressure or a vacuum in the tank (which it is not designed to withstand). Often the vent will have a *vapor conservation valve*, which prevents the tank from "breathing" in and out with small changes of atmospheric pressure or of the temperature of the tank contents; such valves are normally set to open for an internal pressure of vacuum of less than ±0.1 psig. If for some reason that vent is closed (e.g., blocked by ice in a storm), then pumping liquid into the tank can bulge the tank outward, and pumping liquid out can collapse the tank inward. For fluids like gasoline this *tank breathing* can be a significant air pollution emission [1], which must be controlled.

Example 2.7. A layer of rain water 8 in deep collects on the roof of the oil-storage tank of Example 2.6. What net pressure force does it exert on the roof of the tank?

Here

$$P_{\text{gauge}} = \rho g h = 62.3\,\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\,\frac{\text{ft}}{\text{s}^2}\cdot\frac{8}{12}\,\text{ft}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}$$

$$= 41.5\,\frac{\text{lbf}}{\text{ft}^2} = 0.288\,\frac{\text{lbf}}{\text{in}^2} = 1.989\ \text{kPa} \tag{2.J}$$

$$F = PA = 41.5\,\frac{\text{lbf}}{\text{ft}^2}\cdot\frac{\pi}{4}(120\ \text{ft})^2 = 4.70\cdot 10^5\ \text{lbf} = 3.24\ \text{MN} \tag{2.K}$$

∎

We could have found exactly the same answer by asking what the weight W of the liquid on the roof was that is,

$$W = mg = V\rho g = \frac{8}{12}\,\text{ft}\cdot\frac{\pi}{4}(120\ \text{ft})^2\cdot 62.3\,\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\,\frac{\text{ft}}{\text{s}^2}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}$$

$$= 4.70\cdot 10^5\ \text{lbf} = 3.24\ \text{MN} \tag{2.L}$$

where V is volume of the liquid. This is typical of fluid-statics problems involving horizontal surfaces. Since we found the basic equation of fluid statics by considering the weight of the fluid, we could work this kind of problem just as well by simply considering the weight of the fluid involved. This large a weight would collapse the roof of an ordinary tank and of some other light-duty structures; proper rainfall drainage is important.

For *vertical* plane surfaces the pressure is not constant over the whole surface. Therefore, Eq. 2.20 must be used to find the force, and in general we cannot take the pressure outside the integral sign.

Example 2.8. The lock gate of a canal (Fig. 2.5) is rectangular, 20 m wide and 10 m high. One side is exposed to the atmosphere, the other side to water whose top surface is level with the top of the lock gate. What is the net force on the lock gate?

The net force is the force exerted by the water on the front of the gate minus the force exerted by the atmosphere on the back of the gate. Over the short vertical distance involved, the pressure of the atmosphere may be considered constant $= P_{\text{atm}}$. Thus, the force exerted on the back of the gate by the atmosphere is

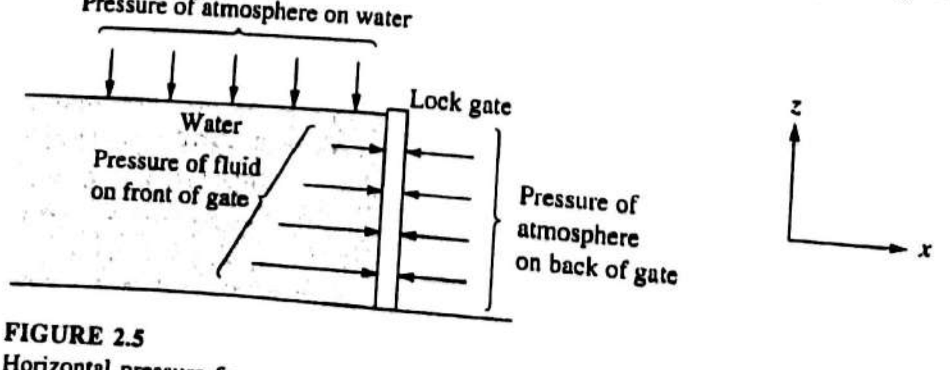


FIGURE 2.5
Horizontal pressure forces on a vertical surface.

$P_{atm}A$, where A is the area of the gate. The pressure at any point in the water is given by Eq. 2.11. Here we define W as the width of the gate and h as the depth below the free surface. Then, substituting Eq. 2.11 in Eq. 2.20, we find

$$F_{water} = \int P\,dA = \int (P_{atm} + \rho g h)\,dA = P_{atm}A + \rho g \int hW\,dh$$

$$= P_{atm}A + \rho g W \frac{h^2}{2}\Big]_0^{10\text{ m}} \tag{2.M}$$

The net force in the x direction is

$$F_{net} = F_{water} - F_{air} = P_{atm}A + \rho g W \frac{h^2}{2}\Big]_0^{10\text{ m}} - P_{atm}A \tag{2.N}$$

The two atmospheric pressure terms cancel each other, and

$$F_{net} = 998.2\,\frac{\text{kg}}{\text{m}^3}\cdot 9.81\,\frac{\text{m}}{\text{s}^2}\cdot 20\text{ m}\cdot \frac{h^2}{2}\Big]_{h=0}^{h=10\text{ m}} \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$= 9.80\text{ MN} = 2.20\cdot 10^6\text{ lbf} \tag{2.O}$$

■

In this problem—and in all others in which a liquid, open to the atmosphere, acts on one side of a surface and the atmosphere acts on the opposite side—the effect of the atmospheric pressure cancels. Thus, such problems can be worked most easily by using gauge pressure. If it had been used in this problem, it would have given exactly the result just shown.

$P = \rho g h$
$w = mg$

In the next section (and some other problems of practical interest) we want to know the x or y components of the pressure force on some surface that is curved, or that is flat but not perpendicular to the x or y axis. The basic procedure is to write

$$dF_x = \begin{pmatrix} x\text{-component} \\ \text{of } dF \end{pmatrix} = \sin\theta\cdot dF = \sin\theta\cdot P\,dA \tag{2.22}$$

and

$$dF_y = \begin{pmatrix} y\text{-component} \\ \text{of } dF \end{pmatrix} = -\cos\theta\cdot dF = -\cos\theta\cdot P\,dA \tag{2.23}$$

where θ is the angle between the normal to the surface and the vertical. If P is constant, then the x or y components of the pressure force are equal to P times the projected area of the surface in the x or y direction.

2.4 PRESSURE VESSELS AND PIPING

Figure 2.6 shows part of an oil refinery "tank farm." Three different types of storage vessels are shown. The largest are cylindrical with vertical axes and flat bottoms; these are used for liquids stored at approximately atmospheric pressure. The spherical tanks are used to store liquids (and rarely gases) at pressures substantially above atmospheric. The sausage-shaped tanks (horizontal cylinders with hemispherical ends) are also used to store liquids (and rarely gases) at high pressures. The choice between

FIGURE 2.6
Part of an oil refinery "tank farm" showing 24 flat-bottomed atmospheric pressure storage tanks, 2 high-pressure spherical storage tanks, and 24 high-pressure sausage-shaped storage tanks. [Courtesy of Chicago Bridge and Iron Company (CB&I).]

these three types of tank is based on economics, which is mostly driven by the necessity to make them strong enough to resist the pressure of the fluid they contain.

Returning to the oil-storage tank in Example 2.3 and Fig. 2.3, we can ask, how thick do the walls of the tank have to be to contain the fluid inside? Figure 2.7 shows an atmospheric-pressure, flat-bottomed tank like those shown in Fig. 2.6. Part (*a*) shows the whole tank, which is a cylindrical shell with a flat bottom and which rests on a concrete or gravel foundation. The tank has a lightweight roof (either flat or domed in Fig. 2.6).

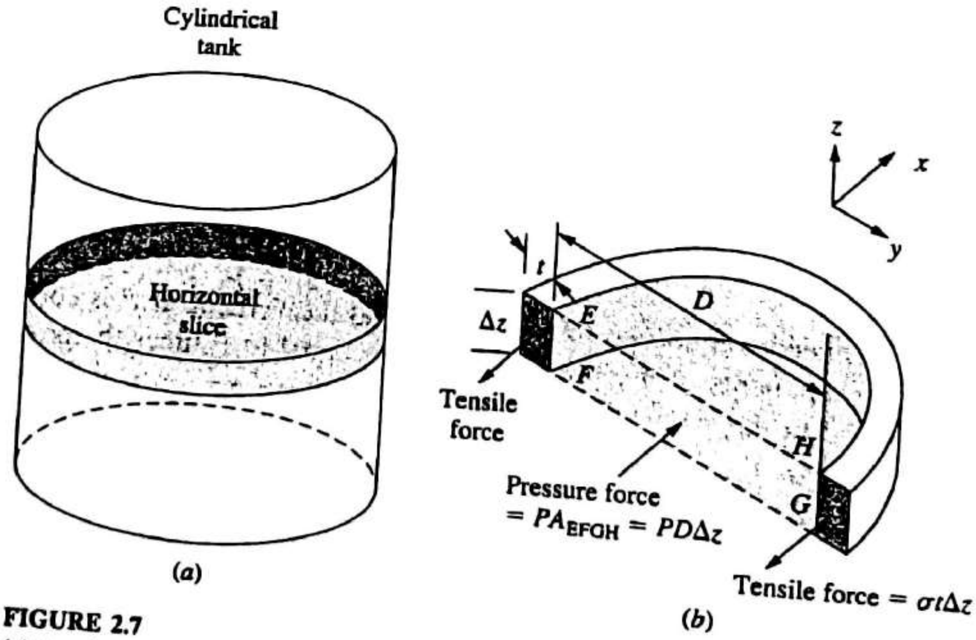


FIGURE 2.7
(*a*) A cylindrical flat-bottomed tank, showing a horizontal slice used in part (*b*). (*b*) A force balance on half of the horizontal slice, showing the pressure force in the *x* direction, and the tensile force in the two pieces of the tank's shell, which resists this pressure force.

Part (*a*) shows a horizontal slice (like one pancake in a stack) and part (*b*) shows that slice cut vertically (like a half pancake). Part (*b*) shows both the steel shell of the tank, whose thickness is exaggerated for clarity, and the liquid that is within the "half pancake."

Making a force balance in the x direction on the piece shown on Fig. 2.7(*b*), we see that the pressure force on the liquid surface E-F-G-H acts in the positive x direction, while the tensile forces in the two cut pieces of the shell of the tank act in the negative x direction. The liquid shown in the tank section in part (*b*) exerts a force radially outward over the part of the tank it contacts, but we are only interested in the x component of that force, which is equal to the x component of the force that the rest of the fluid exerts on this segment of fluid. We do not bother with forces in the y or z directions, because they do not concern us here. If the tank is not in the act of rupturing, then the sum of the forces in the x direction (or any other) must be zero, so we may write

$$P \cdot D \cdot \Delta z = 2\sigma_{\text{tensile}} \cdot \Delta z \cdot t \qquad (2.24)$$

where σ_{tensile} is the tensile stress in the shell, P is the gauge pressure, also assumed to be uniform, and t is the thickness of the metal shell. Now we make the *thin-walled assumption* that σ_{tensile} is uniform over the wall thickness (see below). Solving for the required thickness of the shell, we find

$$t = \frac{PD}{2\sigma_{\text{tensile}}} \qquad \text{[cylindrical, thin-walled assumption]} \qquad (2.25)$$

The tensile stress in Eq. 2.25 is resisted by the external metal hoops in barrels and in wooden water tanks; it is normally called the *hoop stress.*

Example 2.9. If the design tensile stress (normally $\frac{1}{4}$ of the stress at rupture) of the tank wall is 20,000 psia, how thick must the shell of the tank in Example 2.3 be at the bottom of the tank? The diameter of the tank is 120 ft.

Substituting directly into Eq. 2.25, we write

$$t = \frac{(22.9\ \text{lbf/in}^2) \cdot 120\ \text{ft}}{2 \cdot 20{,}000\ \text{lbf/in}^2} = 0.0688\ \text{ft} = 0.825\ \text{in} = 2.10\ \text{cm} \qquad (2.P)$$

■

From this example, we see that this large a tank requires a fairly thick wall. We also see from Fig. 2.3 that the gauge pressure falls to zero at the top, so the thickness required to resist the internal pressure has a triangular shape, thick at the bottom, zero at the top. As a practical matter we cannot have a zero thickness at the top—that would be impossible to build, would not support the roof of the tank, and would not resist wind and seismic forces. But such tanks are actually made up by welding (or bolting) together prefabricated curved plates, the thickness of which decreases from bottom to top (Prob. 2.28).

The actual design of this type of tank [4] adds a corrosion allowance to the thickness calculated in Eq. 2.25 and uses practically the thickness computed in Example 2.9 (adequate to resist pressure forces) at the bottom, but a thickness based on wind and seismic forces at the top. Equation 2.25 (and the corresponding equation for spherical containers shown below) makes the *thin-walled vessel* assumption. For pressures above about 3000 psia the required wall thicknesses become large enough that this uniform-stress assumption becomes inaccurate, and one must use *thick-walled vessel* equations, not shown here [5]. If $t/D > 0.25$ one should use such formulae. The barrels of firearms are normally thick-walled vessels, as are extremely high-pressure chemical reactors (see Prob. 2.33 and Table 2.2).

The other two types of tank shown in Fig. 2.6 (spherical and sausage-shaped) are *pressure vessels* designed to contain fluids with pressures much higher than that due to gravity (Examples 2.3 and 2.9). They are not vented to the atmosphere. The sausage-shaped tanks to the right in Fig. 2.6 are the standard storage tanks for propane, with a design pressure of 250 psig. The analysis of the required shell thickness for them is the same as in Fig. 2.7. In this case the cylinder is horizontal, so the pressure at the bottom is greater than the pressure at the top. However, these tanks are typically about 10 ft in diameter, and liquid propane has a SG of ≈0.5, so the difference in pressure from the top to the bottom of the liquid in the tank is

$$\Delta P = \rho g h = 0.5 \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 10 \text{ ft} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 2.16 \text{ psi} = 14.9 \text{ kPa} \quad (2.Q)$$

which is less than 1% of the 250 psig design pressure, and is normally ignored. Thus, we may design such vessels by using Eq. 2.25.

Example 2.10. Estimate the necessary wall thickness for a horizontal cylindrical pressure vessel with a diameter of 10 ft, a working pressure of 250 psig, and a design tensile stress of 20,000 psig. This is similar to Example 2.9, in which the pressure was due to gravity,

$$t = \frac{(250 \text{ lbf/in}^2) \cdot 10 \text{ ft}}{2 \cdot (20{,}000 \text{ lbf/in}^2)} = 0.0625 \text{ ft} = 0.75 \text{ in} = 1.90 \text{ cm} \quad (2.R)$$

■

For spherical pressure vessels, the same calculation procedure leads to a different equation. Figure 2.8 shows a spherical pressure vessel cut in half, with the wall

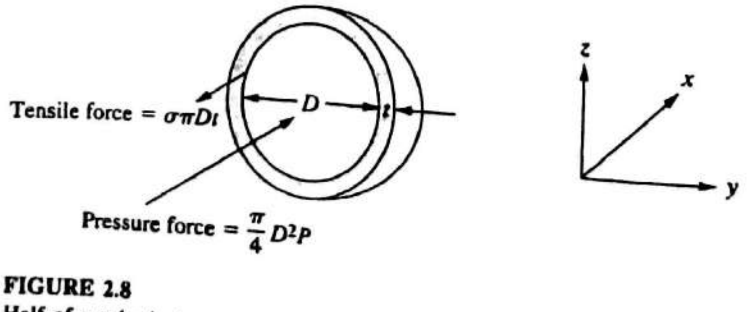


FIGURE 2.8
Half of a spherical pressure vessel, cut along its mid-plane, showing the pressure force in the x direction and the tensile force in the shell, which resists this pressure force.

thickness and diameter shown. The internal pressure force, acting in the plus x direction along the plane of the cut, is equal and opposite to the tensile stress in the wall in the plane of the cut, which acts in the minus x direction. Setting these equal and opposite, we find

$$P \cdot \frac{\pi}{4} D^2 = \sigma_{\text{tensile}} \cdot \pi D \cdot t \tag{2.26}$$

or

$$t = \frac{PD}{4\sigma_{\text{tensile}}} \qquad \text{[spherical vessel, thin-walled assumption]} \tag{2.27}$$

This shows that for equal pressure and diameter, the required wall thickness for a spherical pressure vessel is exactly one-half that for a cylindrical pressure vessel. This suggests that one can store a given volume of high-pressure fluid in a container with much less metal in the walls if the container is spherical than if it is cylindrical. Figure 2.6 shows two such spherical containers in an oil refinery, holding high-pressure fluids. For space travel applications, where weight is critical, high-pressure fluids are always stored in spherical containers, to minimize container weight. The hemispherical ends of the sausage containers shown in Fig. 2.6 normally have thicknesses about one-half the thickness of the cylindrical section as suggested by Eqs. 2.25 and 2.27.

However, economics often dictate the use of the sausage-shaped containers of which 24 are shown in Fig. 2.6. These can be mass-produced in factories and shipped complete, whereas the spherical containers shown in Fig. 2.6 are too large to ship, so they are prefabricated in factories and then assembled in place. The supports for the spherical containers are more complex than those for the sausages, and the number of pieces is greater (look at the number of pieces on a soccer ball!). If a special steel with a high price per pound is needed (e.g., liquid natural gas shipping or storage), then the spherical container is often more economical. For most high-pressure liquid storage applications, the sausage container, in spite of its extra weight of metal, is often the most economical (see Prob. 2.36).

Ordinary pipes and tubes are thin-walled pressure vessels. The relation between their dimensions and their safe working pressure is given by Eq. 2.25 (with an added thickness as a corrosion allowance, and a joint efficiency term for welded pipe). The distillation towers, reflux drums, and other vessels in Fig. 1.14 are also pressure vessels, whose external shells are designed by the same formulae as are the sausage-shaped storage tanks in Fig. 2.6. For significant internal pressures (like the 250 psig in the previous examples) the required wall thickness to contain the pressure is enough that the resulting vessels are self-supporting and need no external structural support (other than foundations). Vessels with lower working pressures often have walls thin enough that they need external or internal bracing to resist gravity, wind forces, or seismic forces, [4]. The large trucks that deliver gasoline to local service stations have a standard truck chassis, on which the gasoline tank is mounted; the low-pressure gasoline tank is not very strong. The large trucks that deliver propane to regional distribution plants do not have a truck chassis; the high-pressure propane tank is strong enough that the wheels and axles are attached directly to it.

2.5 BUOYANCY

We can calculate the force exerted by static fluids on floating and immersed bodies by integrating the vertical component of the pressure force over the entire surface of the body. This leads to a very simple generalization, called *Archimedes' principle*, which is much easier to apply than the integration over the whole surface.

Consider the floating block of wood shown in Fig. 2.9. The block is at rest, so the sum of forces in any direction on it is zero. The only forces acting on it are the gravity force and the total pressure force around its entire surface; these must be equal and opposite. The vertical component of the pressure force itegrated around the entire surface of a floating or submerged body is called a *buoyant force*. The buoyant force over the entire surface is then given by

$$F_{\text{vertical}} = F_z = \int -P \cos\theta \, dA \tag{2.28}$$

The $\cos\theta$ appears in this equation because the pressure forces act directly inward normal to all the surfaces they contact, whereas the vertical component is that pressure force times the cosine of the angle between the direction of the pressure force and the vertical direction, θ. For the block shown in Fig. 2.9, $\cos\theta$ is zero for the sides, -1 for the bottom, and $+1$ for the top; so

$$F_z = (P_{\text{bottom}} - P_{\text{top}})\,\Delta x\,\Delta y \tag{2.29}$$

Here

$$(P_{\text{bottom}} - P_{\text{top}}) = \rho_{\text{liquid}} g h + \rho_{\text{air}} g (l - h) \tag{2.30}$$

Multiplying by $\Delta x\,\Delta y$ gives

$$F_z = \rho_{\text{liq}} g V_{\text{liq}} + \rho_{\text{air}} g V_{\text{air}} \tag{2.31}$$

where V_{liq} is the volume of liquid displaced, and V_{air} is the volume of air displaced. Thus the buoyant force is exactly equal to the weight of both fluids displaced. This is *Archimedes' principle*. In most cases the term for the weight of air in Eq. 2.31 is negligible compared with that for the weight of the water involved (density of water ≈800 times that of air!). For floating bodies Archimedes' principle is often restated: "A floating body displaces a volume of fluid whose weight is exactly equal to its own." If a body is completely immersed in a fluid, then there is only one term on the right in Eq. 2.31 and the statement becomes, "The buoyant force on a completely submerged body is equal to the weight of fluid displaced."

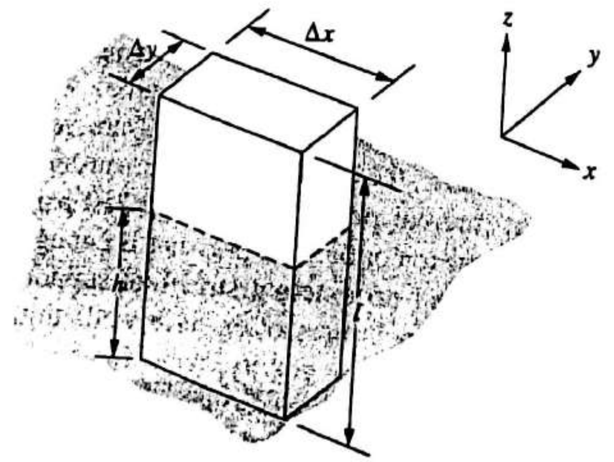


FIGURE 2.9
A floating block of wood, used to show Archimedes' principle.

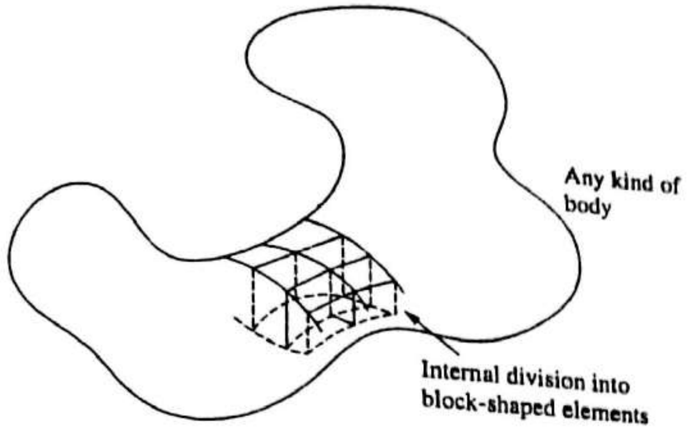


FIGURE 2.10
Any arbitrary-shaped object can be thought of as being made up of many blocks with vertical sides, like the one in Fig. 2.9.

The preceding statements were worked out for a block with the axis vertical. This was convenient, because the pressure on the vertical sides did not contribute to the buoyant force. However, the result is true for any kind of body because, as shown in Fig. 2.10, any shape at all can be visualized as made up of such blocks.

If the blocks are large, as shown in the figure, then their combined volume will be a rough approximation to the volume of the body. However, as the x and y dimensions of the blocks decrease, the blocks form a steadily improving approximation to the body, becoming identical with it as the x and y dimensions approach zero. Nonetheless, for each block, no matter how small, the foregoing argument holds, and therefore Archimedes' principle holds for any shape of body. Thus, although it would be very difficult to perform the indicated pressure integration over a body shaped like an octopus, if we know its volume (and hence the volume of fluid it displaces), then we can easily calculate the buoyant force by means of Archimedes' principle.

Example 2.11. A helium balloon is at the same pressure and temperature as the surrounding air (1 atm, 20°C) and has a diameter of 3 m. The weight of the plastic skin of the balloon is negligible. How much payload can the balloon lift?

The buoyant force is the weight of air displaced:

$$F_{\text{buoyant}} = \rho_{\text{air}} g V_{\text{balloon}} \tag{2.S}$$

The weight of helium is

$$W_{\text{helium}} = \rho_{\text{helium}} g V_{\text{balloon}} \tag{2.T}$$

Therefore, the payload is

$$\begin{aligned}
\text{Payload} &= F_{\text{buoyant}} - W_{\text{helium}} = V_{\text{balloon}} g(\rho_{\text{air}} - \rho_{\text{helium}}) \\
&= Vg\frac{P}{RT}(M_{\text{air}} - M_{\text{helium}}) \\
&= \frac{\pi}{6}\cdot(3\text{m})^3\cdot\frac{9.81\text{ m}}{\text{s}^2}\cdot\frac{1\text{ atm}}{[8.2\cdot10^{-5}\text{ m}^3\cdot\text{atm}/(\text{mol}\cdot\text{K})]\cdot293.15\text{K}} \\
&\quad\cdot\left(29\frac{\text{g}}{\text{mole}} - 4\frac{\text{g}}{\text{mole}}\right)\cdot\frac{\text{kg}}{1000\text{ g}}\cdot\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 144.2\text{ N} = 32.4\text{ lbf}
\end{aligned} \tag{2.U}$$

■

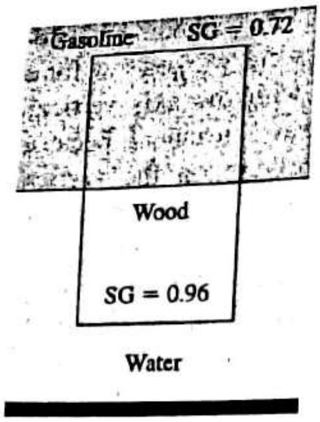


FIGURE 2.11
A block of wood floating at a gasoline-water interface.

Example 2.12. A block of wood is floating at the interface between a layer of gasoline and a layer of water, see Fig. 2.11. What fraction of the wood is below the interface?

Here the weight of the wood is equal to the buoyant force, which is in turn equal to the weight of the two fluids displaced:

$$V_{\text{wood}}\rho_{\text{wood}}g = V_{\text{water}}\rho_{\text{water}}g + V_{\text{gasoline}}\rho_{\text{gasoline}}g \qquad (2.V)$$

where V_{wood} is the volume of the block, and V_{water} and V_{gasoline} are the volumes of water and gasoline displaced. Dividing by $g\rho_{\text{water}}$, we find

$$V_{\text{wood}}\text{SG}_{\text{wood}} = V_{\text{water}} + V_{\text{gasoline}}\text{SG}_{\text{gasoline}} \qquad (2.W)$$

where SG is the specific gravity. But since

$$V_{\text{wood}} = V_{\text{water}} + V_{\text{gasoline}} \qquad (2.X)$$

we may eliminate V_{gasoline}

$$V_{\text{wood}}\text{SG}_{\text{wood}} = V_{\text{water}} + (V_{\text{wood}} - V_{\text{water}})\,\text{SG}_{\text{gasoline}} \qquad (2.Y)$$

and we then find

$$\frac{V_{\text{water}}}{V_{\text{wood}}} = \frac{\text{SG}_{\text{wood}} - \text{SG}_{\text{gasoline}}}{1 - \text{SG}_{\text{gasoline}}} = \frac{0.96 - 0.72}{1 - 0.72} = 0.857 \qquad (2.Z)$$

■

This result appears paradoxical. The gasoline pushes down on the top of the block, not up on it at any point, yet the volume of gasoline displaced enters the buoyant force calculation. However, if we examine the pressure integral around the surface, we see that the pressure difference from top to bottom of the block does indeed involve the gasoline in the way shown.

Remember that the basic operation is the integration of the vertical components of the pressure force over the entire surface of the body. The convenient result of this integration is Archimedes' principle: the buoyant force is equal to the weight of the fluid displaced.

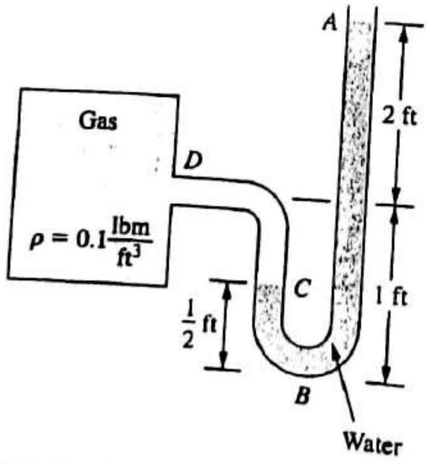


FIGURE 2.12
A simple manometer, filled with colored water.

2.6 PRESSURE MEASUREMENT

Pressures usually are measured by letting them act across some area and opposing them with either a gravity force or the force of a compressed spring. The gravity-force method uses a device called a *manometer,* described in the following example,

Example 2.13. Figure 2.12 shows a tank of gas connected to a manometer. The manometer is a U-shaped glass or transparent plastic tube open to the atmosphere at one end and containing colored water. From the elevations shown calculate the gauge pressure in the vessel.

We want to know the pressure at D. The simple way to work all manometer problems is to start with some pressure we know and work step by step to the pressure we want to find. In this case, we know the gauge pressure at A is zero, because the manometer is open to the atmosphere at A. The water is practically a constant-density fluid; therefore, we can use Eq. 2.11 to find the pressure at B:

$$P_B = P_A + \rho_{\text{water}} g h_B = 0 + \rho_{\text{water}} g \cdot 3\text{ ft} \tag{2.AA}$$

To find the pressure at C we need Eq. 2.9:

$$P_C = P_B - (\rho_{\text{water}} g \cdot \tfrac{1}{2}\text{ ft}) \tag{2.AB}$$

To find the pressure at D we use the same equation (i.e., we assume that the density *change* of the gas is negligible):

$$P_D = P_C - (\rho_{\text{gas}} g \cdot \tfrac{1}{2}\text{ ft}) \tag{2.AC}$$

Adding these three equations and canceling like terms, we find

$$\begin{aligned} P_D &= (\rho_{\text{water}} g) \cdot \left(3\text{ ft} - \frac{1}{2}\text{ ft}\right) - \left(\rho_{\text{gas}} g \cdot \frac{1}{2}\text{ ft}\right) \\ &= 32.2\,\frac{\text{ft}}{\text{s}^2}\left[\left(62.3\,\frac{\text{lbm}}{\text{ft}^3}\cdot 2.5\text{ ft}\right) - \left(0.075\,\frac{\text{lbm}}{\text{ft}^3}\cdot 0.5\text{ ft}\right)\right]\cdot\frac{\text{lbm}\cdot\text{s}^2}{32.2\text{ lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144\text{ in}^2} \\ &= 1.08\,\frac{\text{lbf}}{\text{in}^2} - 0.0003\,\frac{\text{lbf}}{\text{in}^2} = 1.08\,\frac{\text{lbf}}{\text{in}^2}\text{gauge} = 7.46\text{ kPa} \end{aligned} \tag{2.AD}$$

■

The example illustrates several points.

1. The contribution of the section of the manometer full of gas is only 0.03% of the answer. It is generally neglected in manometer problems.
2. Manometers that are open to the atmosphere are gauge-pressure devices and should be calculated in gauge pressure.
3. In reading such a device, we normally read an elevation; the actual operational reading in Example 2.13 was 2.5 ft. For many purposes it is convenient to think of pressures and to report them in terms of such manometer readings as heights; the U.S. air conditioning industry, for example, commonly refers to all pressure in air conditioning ducts as "inches of water," and most U.S. vacuum-equipment manufacturers refer to vacuums as "inches of mercury."
4. At no place in our last calculation did the cross-sectional area of the manometer tube enter. Therefore, this tube can be of any convenient size and need not be made of constant-diameter tubing. The only measurements necessary are the fluid densities, which can be looked up in handbooks, and the differences in elevation, which can be read directly with tape measures and rulers. Thus, manometers require neither calibration nor testing with standards; one simply connects them and takes the reading. There is no requirement that the tubes be vertical, only that we can read the vertical distance between the horizontal liquid surfaces.

5. It may seem that we went to a lot of trouble for such a simple problem. This is true; working engineers use shorter calculation methods than the one shown here. However, with complicated systems, such as two-fluid manometers, the shortcuts are confusing, and the step-by-step method just shown is always reliable.

Example 2.14. A two-fluid manometer is often used to make it unnecessary to read small differences in liquid level. The one shown in Fig. 2.13 is measuring the pressure difference between two tanks. What is that pressure difference?

We want to know $P_A - P_E$. All the fluids have practically constant density, so we can use Eq. 2.9. We begin by calling P_E known. Then

$$P_D = P_E + (\rho_{\text{water}} g \cdot 1 \text{ ft}) \tag{2.AE}$$

$$P_C = P_D + (\rho_{\text{oil}} g \cdot 2 \text{ ft}) \tag{2.AF}$$

$$P_B = P_C - (\rho_{\text{oil}} g \cdot 1 \text{ ft}) \tag{2.AG}$$

$$P_A = P_B - (\rho_{\text{water}} g \cdot 2 \text{ ft}) \tag{2.AH}$$

Adding these and canceling like terms, we find

$$P_A - P_E = \rho_{\text{water}} g(1 \text{ ft} - 2 \text{ ft}) + \rho_{\text{oil}} g(2 \text{ ft} - 1 \text{ ft}) = 1 \text{ ft} \cdot g(\rho_{\text{oil}} - \rho_{\text{water}})$$

$$= 1 \text{ ft} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot (1.1 - 1.0) \cdot 62.3 \frac{\text{lbm}}{\text{ft}^2} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 0.043 \frac{\text{lbf}}{\text{in}^2} = 298 \text{ Pa} \tag{2.AI}$$

■

This reading corresponds to a pressure difference of 0.1 ft of water. The actual reading of this two-fluid manometer is 1 ft. If we assume that we can read liquid level differences with an accuracy of ±0.005 ft (= ±0.06 in), then a simple water manometer would have an uncertainty of 5% for this difference; the two-fluid manometer shown has an uncertainty of 0.5%.

Because a manometer is a device for measuring pressure differences, to use one to measure absolute pressure we must measure the difference between the pressure in question and a perfect vacuum. In principle this is impossible, because there is no such thing as a perfect vacuum, but in practice we may produce vacuums of sufficient quality that the error introduced by calling them perfect is negligible. This idea is used in the mercury barometer shown in Fig. 2.14. This common device for measuring the pressure of the atmosphere is found in most laboratories.

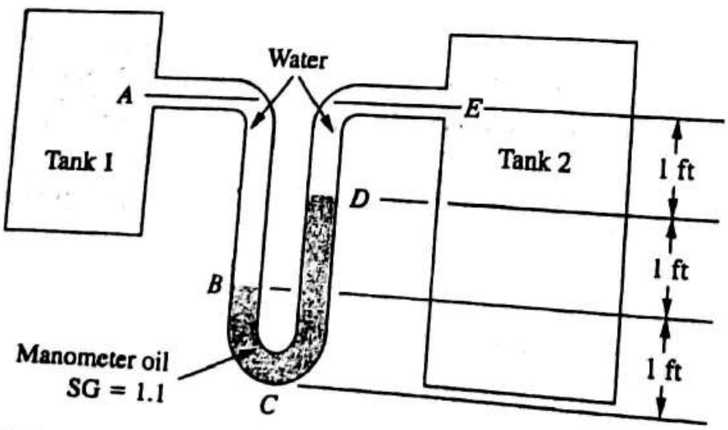


FIGURE 2.13
A two-fluid manometer, with water and a manometer oil.

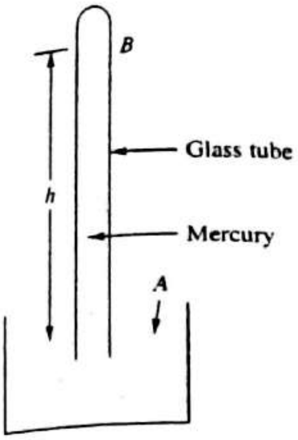


FIGURE 2.14
Mercury barometer.

The atmosphere acts on the mercury in the cup at the bottom, and the weight of the column of mercury opposes it. Calculating this, we find

$$P_A - P_B = \rho_{Hg} g h \tag{2.AJ}$$

where P_B is the pressure in the vapor space above the liquid mercury. In well-built manometers this will be simply the vapor pressure of mercury, which at 68°F = 20°C is about 10^{-6} atm; this is so small compared with 1 atm that it can be neglected. Thus, although the barometer, like all manometers, measures pressure differences, it can be used with satisfactory accuracy as an absolute-pressure device in this case; see Prob. 2.56.

The second way to measure pressure is to let the pressure act on some piston, which compresses a spring, and to measure the displacement. Figure 2.15 shows an *impractical but illustrative* way of doing this. The fluid whose pressure is to be measured presses on the piston, compressing the spring and moving the pointer along the scale. If we know the area of the piston, the spring constant, and the pointer reading for zero pressure, we can calculate the pressure on the piston from the pointer position.

Example 2.15. The piston in Fig. 2.15 has an area of 100 cm², and the spring constant k is 100 N / cm. We set the pointer so that there is a zero reading when both sides of the piston are exposed to the atmosphere. Now we attach the gauge to a tank with an unknown pressure, and the pointer moves to 2.5 cm. What is the pressure in the tank?

Here the net force acting on the piston is

$$F_{net} = (P_{tank} - P_{atm}) \cdot A = A P_{tank,\ gauge} \tag{2.AK}$$

This must be equal to the force on the spring, which is $k\,\Delta x$, and therefore

$$P_{tank,\ gauge} = \frac{k\,\Delta x}{A} = \frac{(100\ \text{N / cm}) \cdot 2.5\ \text{cm}}{100\ \text{cm}^2} \cdot \left(\frac{100\ \text{cm}}{\text{m}}\right)^2 = 25\ \text{kPa} = 3.62\,\frac{\text{lbf}}{\text{in}^2} \tag{2.AL}$$

■

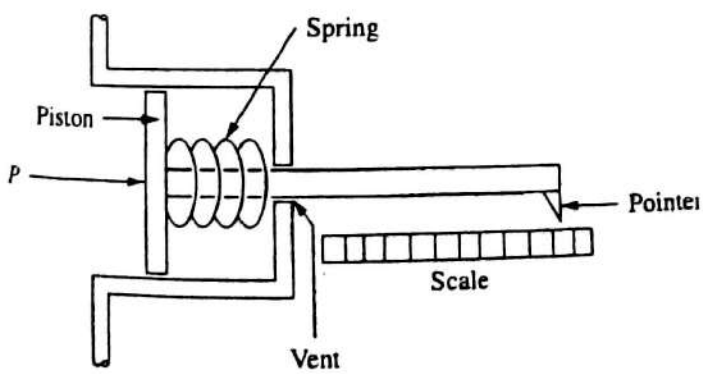


FIGURE 2.15
Piston-and-spring pressure gauge.

From this example we observe the following:

1. This device, like the manometer, measures pressure differences. To use it as an absolute-pressure device, we must make it compare pressure with a vacuum. We can do this by placing it in an evacuated chamber.

2. This device, unlike the manometer, requires a precise measurement of its dimensions or a calibration. Spring-type pressure gauges usually are calibrated by comparing their reading with those of manometers like the one shown in Fig. 2.16, or other equivalent devices (see Prob. 2.65).

The gauge shown in Fig. 2.15 is impractical because of the problem of leakage around the piston. The most widely used type of spring pressure gauge uses a *bourdon tube*, as shown in Fig. 2.16. A bourdon tube is a stiff, flattened metal tube bent into a circular shape; the fluid whose pressure is to be measured is inside the tube. One end of the tube is fixed, and the other is free to move inward or outward. The inward or outward movement of the free end moves a pointer through a linkage-and-gear arrangement. As Fig. 2.16 shows, the tube cross section is a flattened circle. Internal pressure makes its cross section become closer to circular, like blowing up a balloon, which stresses the outer surface, thus tending to straighten the curved tube. With a high enough pressure the tube would become straight with a circular cross section [6]. The tube itself serves as the spring; it is made of metal, which is stiff and has a reasonable spring constant. The internal pressures are low enough that the tube returns to the cross section shown in Fig. 2.16 when the pressure is removed. With such a tube the calculation of the movement as a function of the inside and outside pressures is more difficult than with the linear piston-and-spring gauge of Fig. 2.15. However, the bourdon tube is a very convenient shape and causes no

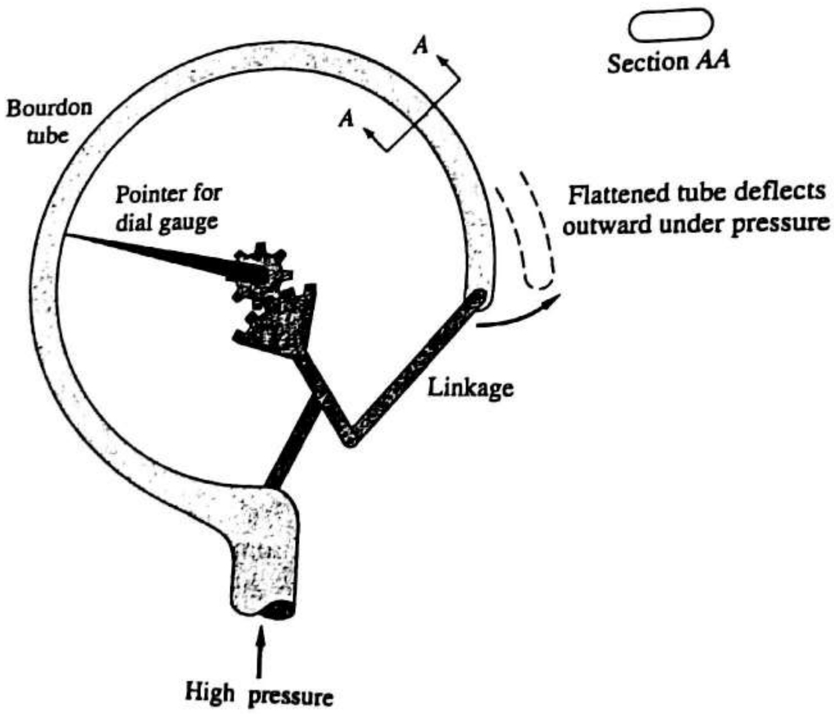


FIGURE 2.16
Bourdon-tube pressure gauge. The whole assembly is in a shallow, cylindrical container. The tube and linkage are at the back, a sheet with numbers comes next, the pointer is in front of that, and a glass cover plate protects the whole assembly.

leakage problems, as does the piston-and-spring gauge. Since both are calibrated devices, the difficulty in calculating the performance of the bourdon tube is not a real disadvantage. Bourdon-tube pressure gauges are simple, rugged, leak-free, reasonably reliable, and cheap; they are the most widely used type of industrial pressure gauge.

Neither the manometer nor the bourdon-tube gauge is suited to measuring rapidly changing pressures. Both are unsatisfactory for this purpose because of their high inertial mass; this mass makes them move slowly to accommodate a change in pressure, and so their readings lag behind a rapidly changing pressure. For rapidly changing pressures (such as pressure fluctuations in rocket motors, or the rapid oscillations in air pressure that we call sound, which are measured with a microphone), two other types of pressure gauges respond much more quickly. One is the diaphragm gauge, which is similar to that of Fig. 2.15 but has, instead of the piston and spring, a thin metal diaphragm, which acts as both. When the pressure increases, the diaphragm stretches very slightly; the stretch is detected by an electric strain gauge (or other electronic means) and recorded electrically. The advantage of the diaphragm over the bourdon tube is its very low mass, which allows it to move quickly in response to a change in pressure. The other type of rapid-response pressure gauge is the quartz-crystal piezometer, which uses the change in electrical properties of quartz crystals with change in pressure. Other electronic pressure gauges are available that take advantage of the response of fixed or oscillating microstructures to changes in external pressure or other electronic phenomena.

2.7 MANOMETER-LIKE SITUATIONS

In Sec. 2.6 we discussed manometers as pressure-measuring devices. There are many other fluid mechanical situations that are most easily understood if we analyze them just as we analyze manometers. Several examples are shown here.

Figure 2.17 shows a schematic cross section of a percolator-type coffee maker. In it, the pot is filled to a height z_1 with water. The basket above the water is filled with ground coffee. The whole assembly is placed on a stove and heated from below. When the water has been warmed, it begins to flow in irregular spurts up the central tube; it is diverted by the cap on the top, falls on the coffee grounds, and percolates through them, extracting the water-soluble constituents of the ground coffee, to make the hot drink many of us enjoy.

How can the fluid do this? Here we have a fluid flowing from a low elevation to a high one, with no mechanical device lifting it. How can that be? To answer the question, we compute the pressures at B and C. It will be easiest if we do this all in gauge pressure. In that case, the pressure at B will be

$$P_B = \rho g z_1 \tag{2.AM}$$

and if the fluid in the tube is up to the level where it spills out at D, then the pressure at C will be

$$P_C = \rho g z_2 \tag{2.AN}$$

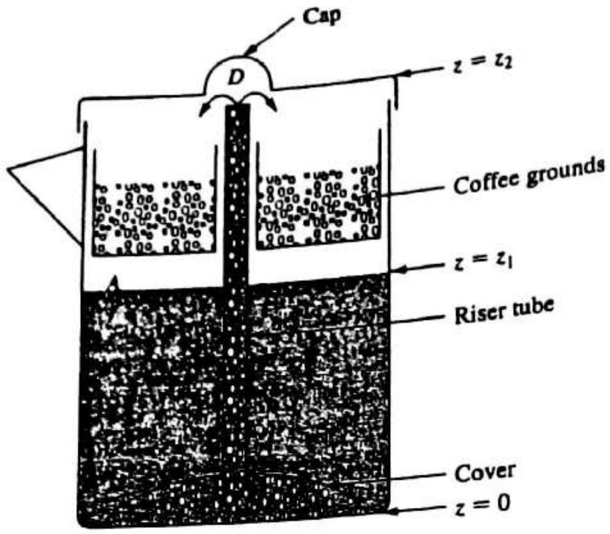


FIGURE 2.17
Coffee percolator, showing fluid flow driven by boiling

and

$$P_B - P_C = g[(\rho z)_1 - (\rho z)_2] \quad (2.AO)$$

When we put the pot on the stove, the density inside and outside the riser tube will be the same (that of water), and the liquid in the tube will stand at the same level as the liquid outside, z_1. There will be no flow.

As the water at the bottom is heated by the stove, the loose-fitting cover prevents it from mixing with the rest of the fluid in the pot, so that a small amount of liquid is heated to its boiling point. When it boils, the bubbles of steam flow by buoyancy up through the riser tube. While they do so, the average density of the gas-liquid mixture in the riser tube decreases. If there is no net flow under the loose-fitting cover, then the pressure difference from one side of it to the other, $(P_B - P_C)$, must be zero, and the level in the riser tube, z_2, must increase to keep this pressure difference equal to zero.

When the generation rate of bubbles becomes high enough that z_2 becomes greater than the height of the top of the riser tube, then a mixture of steam and water will flow out the top of the tube. If the rate of generation of steam bubbles increases even more, then the average density of the steam-water mixture in the riser tube will fall low enough that $(P_B - P_C)$ can no longer be zero but must become a positive number. Then the pressure force due to gravity will force water from the pot under the loose-fitting cover, and the circulation will be established, with flow downward under the cover, up the riser tube, and down through the coffee grounds. For that flow, we can no longer use the simple equations of fluid statics, which we have used so far in this discussion; the methods of Chaps. 5, 6, 7, and 12 must be used. But this simple discussion shows how the pressure forces that move the fluids in coffee percolators arise. The exact same discussion applies to geysers (where the flow is intermittent instead of the steady flow in the boiling coffee pot) and to the circulation system in most steam boilers, in gas- and propane-fired refrigerators, and in the reboilers of many distillation columns. In all of these, the formation of bubbles of steam (or the vapor of some other liquid being boiled) lowers the average density in one leg of the "equivalent manometer," producing the pressure difference that drives the flow.

Such pressure differences can also arise in systems that do not involve boiling liquids, as is illustrated in Example 2.16

Example 2.16. Figure 2.18 shows a schematic of a home fireplace, with part of the house that surrounds it. The burning logs in the fireplace heat the gases

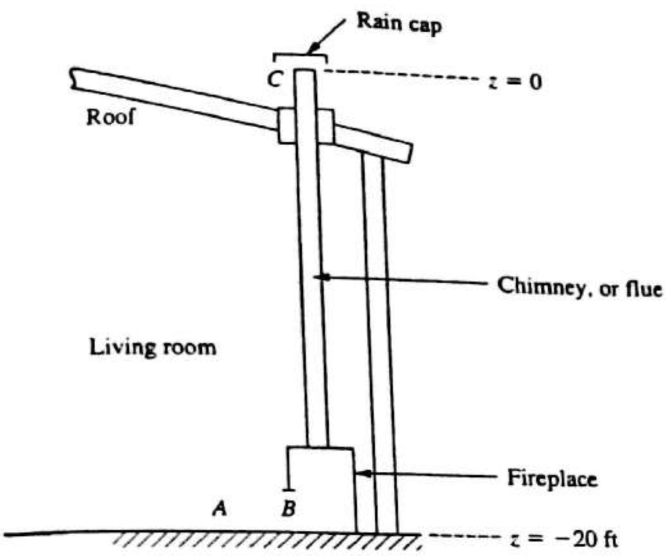


FIGURE 2.18
Home fireplace, showing fluid flow driven by temperature differences.

in the chimney to 300°F. If we treat this as a static situation, what will be the difference in pressure between the air in the room adjacent to the fireplace and the air inside the fireplace at the same level?

Here we assume that the house is leaky enough, or has an open window, so that the pressure inside the house is the same as the pressure in the atmosphere outside. (This is true for older houses, but not necessarily true for modern "tight" energy-conserving houses, which have much less air exchange with the surroundings!) Here, as shown in the figure, we have taken the elevation datum, $z = 0$, at the top of the chimney; this choice makes the solution simple. Taking the pressure at $z = 0$ to be atmospheric pressure, and working in gauge pressures, we can compute that

$$P_A = \rho_{\text{air}} g z_1 \tag{2.AP}$$

and that

$$P_B = \rho_{\text{flue gas}} g z_1 \tag{2.AQ}$$

The chimney is also called a *flue*, and the gas in it is normally called *flue gas*. Then

$$P_A - P_B = (\rho_{\text{air}} - \rho_{\text{flue gas}})\, g z_1 \tag{2.AR}$$

Assuming that both the air and the flue gas are ideal gases, we can express the density of each in terms of the ideal gas law and write

$$P_A - P_B = g z_1 \frac{P}{R}\left[\left(\frac{M}{T}\right)_{\text{air}} - \left(\frac{M}{T}\right)_{\text{flue gas}}\right] \tag{2.AS}$$

In the most careful work we need to take into account the small difference in molecular weights of air and flue gas, but here we can assume that the molecular weights are practically equal. Then we multiply and divide by $(M/T)_{\text{air}}$ and substitute ρ_{air} for its ideal gas equivalent:

$$P_A - P_B = gz_1 \frac{PM_{\text{air}}}{RT_{\text{air}}}\left[1 - \frac{T_{\text{air}}}{T_{\text{flue gas}}}\right] = gz_1\rho_{\text{air}}\left[1 - \frac{T_{\text{air}}}{T_{\text{flue gas}}}\right]$$

$$= 32.2\,\frac{\text{ft}}{\text{s}^2}\cdot 20\text{ ft}\cdot 0.075\,\frac{\text{lbm}}{\text{ft}^3}\cdot\left[1 - \frac{528°\text{R}}{760°\text{R}}\right]\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\text{ lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144\text{ in}^2}$$

$$= 0.0032\,\frac{\text{lbf}}{\text{in}^2} = 22\text{ Pa} \tag{2.AT}$$

■

This is a small pressure difference. But as we will see in Chap. 5, very small pressure differences can produce significant velocities in gas flows; this small pressure difference would produce a velocity, in frictionless air flow, of about 20 ft / s ≈ 6 m / s.

In this example the calculation was made for a static fluid. In the real situation, the fluid would be set in motion by the pressure difference calculated above, and we would need the methods developed in later chapters to compute the velocity. But this calculation shows how pressure differences can arise not only in boiling liquids but also in gases if one side of an "equivalent manometer" is heated to a temperature higher than the other. This explains how chimneys work. For all but the largest furnaces the air flow is driven through the furnace by the pressure difference computed here (called "natural draft"). That explains why large furnaces have tall stacks; the available pressure difference as just shown is proportional to the height of the stack. Many large furnaces now use powered fans to drive the gases through them (called "forced draft"). The choice between natural and forced draft is based on economics; the tall stacks are expensive, but once installed they require no power to run the forced-draft fans.

This calculation also explains many meteorological phenomena. Oceans and lakes are heated and cooled slowly by the sun because currents and waves mix their upper layers; the ground surface on the shore heats more rapidly in the daytime and cools more rapidly at night because it can transfer heat up and down only by conduction, which is slow compared to the convective mixing in bodies of water. Thus, during the day the hot ground surface heats the air above it, and hot air above the ground plays the same role as the hot flue gas in Example 2.16. The winds blow from the ocean or lake onto the shore. At night the ground cools, and cools the air above it, and the direction of the pressure gradient reverses, causing the wind to blow from the shore out over the body of water. The monsoon rains of India and parts of tropical Africa (and in weaker form, the southwestern United States) are the same phenomenon on a much larger scale. In the summer the heated air over the continents rises, and the cooler moist air from the adjacent oceans flows in and brings rains. The same description applies to firestorms, in which the rising, heated air above a large forest or urban fire induces strong winds blowing inward toward its center. Volcanoes are also manometer-like situations, but more complex than these meteorological examples.

2.8 VARIABLE GRAVITY

So far we have assumed that the acceleration of gravity is constant, $32.2 \text{ ft}/\text{s}^2 = 9.81 \text{ m}/\text{s}^2$ in the minus z direction. This is not exactly true in any problem involving two different elevations. However, the change in gravity with change in elevation is quite small. Near the surface of the earth, the acceleration of gravity is proportional to the reciprocal of the square of the distance from the center of earth. The radius of the earth is about 4000 mi = 6440 km, so the acceleration of gravity 1 mi = 1.609 km above the surface is $4000^2 / 4001^2 = 0.9995$ times the acceleration of gravity at the surface. Few engineering problems include data precise enough to justify making such corrections. (Deposits of metal ores perturb the acceleration of gravity above them; sensitive gravitometers flown in airplanes can detect the small gravity perbations caused by such deposits. Such gravitometers are widely used in mineral exploration.)

In two types of problem, however, nonconstant gravity is important:

1. Space travel and rocket problems; in these the distances from the earth become significant compared with 4000 mi, so the changing value of gravity must be taken into account.
2. Acceleration and centrifugal force problems.

Since this is a chapter on fluid statics, it seems a strange place to consider acceleration or centrifugal force problems, in which the fluid is certainly moving. We do so because, in these problems the fluid is not moving relative to its container or relative to other parts of the fluid. Really, all problems in terrestrial fluid statics involve moving fluids, because the fluids are on the earth, and the earth is rotating about its axis and revolving around the sun, and the sun is moving through space. As long as the individual particles of fluid are not moving relative to each other, we can treat such moving problems by the methods of fluid statics. Such motions of fluids are called *rigid-body motions*.

2.9 PRESSURE IN ACCELERATED RIGID-BODY MOTIONS

We now repeat the derivation of Eq. 2.1 for the case in which an entire mass of fluid is in some kind of accelerated rigid-body motion. Again we will use the small, cubical element of fluid shown in Fig. 2.1 and consider it to be part of a larger mass of fluid. In Sec. 2.1 we showed that if the fluid was not being accelerated, then the sum of the forces on it must be zero. If the fluid was being accelerated, then the sum of the forces acting on it, in the direction of the acceleration, must equal the mass times the acceleration. For the cubical element of fluid being accelerated in the vertical $(+z)$ direction, we rewrite Eq. 2.2 as

$$(P_{z=0})\,\Delta x\,\Delta y - (P_{z=\Delta z})\,\Delta x\,\Delta y - \rho g\,\Delta x\,\Delta y\,\Delta z = \rho\,\Delta x\,\Delta y\,\Delta z\,\frac{d^2 z}{dt^2} \qquad \text{(2.AU)}$$

Dividing by $\Delta x \, \Delta y \, \Delta z$ and taking the limit as Δz approaches zero, we find

$$\frac{dP}{dz} = -\rho\left(g + \frac{d^2z}{dt^2}\right) \tag{2.32}$$

which for constant-density fluids can be integrated to

$$P_2 - P_1 = -\rho\left(g + \frac{d^2z}{dt^2}\right)(z_2 - z_1) \qquad \text{[constant density]} \tag{2.33}$$

and which for gauge pressure simplifies further to

$$P = -\rho h\left(g + \frac{d^2z}{dt^2}\right) \qquad \text{[constant density, gauge pressure]} \tag{2.34}$$

Example 2.17. An open tank containing water 5 m deep is sitting on an elevator. Calculate the gauge pressure at the bottom of the tank

(*a*) when the elevator is standing still,
(*b*) when the elevator is accelerating upward at the rate of 5 m/s^2, and
(*c*) when the elevator is accelerating downward at the rate of 5 m/s^2.

From Eq. 2.11, part (*a*) is simply

$$P_{\text{bottom}} = \rho g h = 998.2\,\frac{\text{kg}}{\text{m}^3} \cdot 9.81\,\frac{\text{m}}{\text{s}^2} \cdot 5\text{ m} \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \cdot \frac{\text{Pa}}{\text{N/m}^2}$$
$$= 49.0\text{ kPa} = 7.11\,\frac{\text{lbf}}{\text{in}^2} \tag{2.AV}$$

For parts (*b*) and (*c*) we use Eq. 2.34:

$$P_{\text{bottom}} = \rho h\left(g + \frac{d^2z}{dt^2}\right) = 998.2\,\frac{\text{kg}}{\text{m}^3} \cdot 5\text{ m} \cdot \left(9.81\,\frac{\text{m}}{\text{s}^2} + 5\,\frac{\text{m}}{\text{s}^2}\right) \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \cdot \frac{\text{Pa}}{\text{N/m}^2}$$
$$= 74.0\text{ kPa} = 10.73\,\frac{\text{lbf}}{\text{in}^2} \tag{2.AW}$$

and

$$P_{\text{bottom}} = \rho h\left(g + \frac{d^2z}{dt^2}\right) = 998.2\,\frac{\text{kg}}{\text{m}^3} \cdot 5\text{ m} \cdot \left(9.81\,\frac{\text{m}}{\text{s}^2} - 5\,\frac{\text{m}}{\text{s}^2}\right) \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \cdot \frac{\text{Pa}}{\text{N/m}^2}$$
$$= 24.0\text{ kPa} = 3.48\,\frac{\text{lbf}}{\text{in}^2} \tag{2.AX}$$

■

Most of us can tell which way an elevator starts to move, by sensing these small changes in our weight.

If the acceleration is not in the same direction as gravity or in the direction opposite to it, it must be in some direction *a*, as in Fig. 2.2. Then we can take the summation of forces in the a direction and substitute $g\cos\theta$ for g in Eq. 2.34:

$$\frac{dP}{da} = -\rho\left(g\cos\theta + \frac{d^2a}{dt^2}\right) \tag{2.35}$$

Example 2.18. A rectangular tank of orange juice on a cart is moving in the x direction with a steady acceleration of 1 ft / s^2; see Fig. 2.19. What angle does its free surface make with the horizontal?

Here we assume that the tank has been under acceleration so long that the initial sloshing back and forth of the liquid at the start of acceleration has died out and that the fluid is truly in rigid-body motion. In the figure the points A and B are both on the free surface; neglecting the very slight change in atmospheric pressure over this change in elevation, we may say that the gauge pressure is zero at both points. Then we can calculate the pressure at C from the pressure at A by using Eq. 2.35. Here we are applying it in the y direction ($a = y$), so we have $\cos\theta = 1$ and $d^2y / dt^2 = 0$. Hence, the result is the same as Eq. 2.11:

$$P_C = -\rho g\,\Delta y \tag{2.AY}$$

(Here Δx and Δy are both negative moving from the free surface, so P_C is positive.) We may also calculate the gauge pressure at C by using Eq. 2.35 for the horizontal direction, in which case we have $a = x$, $\cos\theta = 0$, and

$$P_C = -\rho\,\Delta x\,\frac{d^2x}{dt^2} \tag{2.AZ}$$

But the pressure at C is the same no matter how we calculate it, so we may eliminate P_C between these two equations and rearrange to find

$$\frac{\Delta y}{\Delta x} = \frac{d^2x / dt^2}{g} = \tan\theta \tag{2.BA}$$

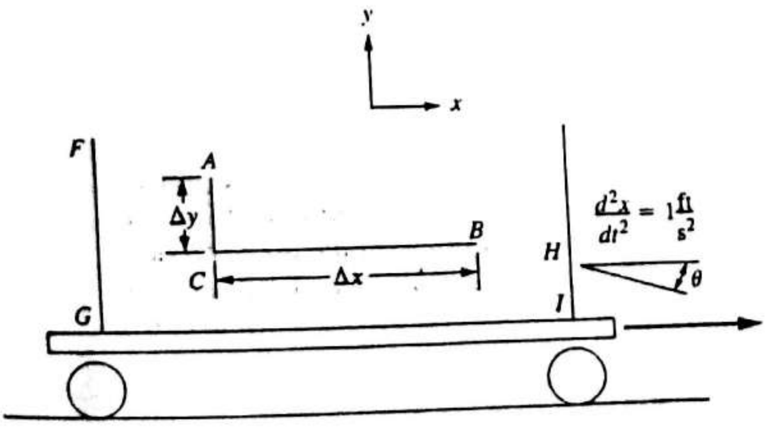


FIGURE 2.19
A system in linear acceleration.

where θ is the angle shown in Fig. 2.19. For this problem

$$\theta = \text{arc tan}\,\frac{d^2x/dt^2}{g} = \text{arc tan}\,\frac{1\ \text{ft}/\text{s}^2}{32.2\ \text{ft}/\text{s}^2} = 1.76° \tag{2.BB}$$

■

To calculate the pressure at any point in the tank, we may now use Eq. 2.11, being careful to measure the depth from the free surface vertically above the point in question. The force on the wall *FG*, for example, is exactly the same at every point as it would be if the cart were standing still and filled with liquid up to point *F*. The force on wall *HI* is exactly the same as it would be if the cart were standing still and filled with liquid up to the level of *H*.

Example 2.18, a case of uniform, rectilinear acceleration holds little practical interest, because such an acceleration acting for a reasonable period of time (e.g., long enough for the sloshing to die out) would produce enormous velocities. However, it serves as an introduction to the more interesting case of ***rigid-body rotation***. Consider an open-topped cylindrical tank of water with a vertical axis. The system is initially at rest; then the tank is set in steady motion, rotating about its vertical axis. At first the fluid in the center will not be affected by the rotation of the walls but will stand still, and only the fluid near the walls will rotate. This sets up motions of parts of the fluid relative to each other, so that this is not a fluid-statics problem. Eventually, however, the shear forces due to this relative motion will bring the fluid at the center to the same angular velocity as the tank, and thereafter there is no relative motion within the fluid. Once the fluid in the center reaches the same angular velocity as the wall of the container, the whole of the fluid moves as if it were a rigid body; hence the name, "rigid-body rotation." Pressures in rigid-body rotation can be calculated by the method of fluid statics.

Example 2.19. An open-topped can of water 30 cm in inside diameter is rotating at 78 rpm. It has been rotating a long time and is in rigid-body rotation. What is the shape of the free surface?

A cross section of this system is sketched in Fig. 2.20. Here we use the same procedure as we did for Fig. 2.19, calculating the pressure at C in two directions. To simplify the calculation, we choose C to be at exactly the same elevation as the lowest point on the free surface. As in Example 2.18, we assume that the pressures at A and B are the same, the local atmospheric pressure. Then, from Eq. 2.35 applied for the z direction, we can write

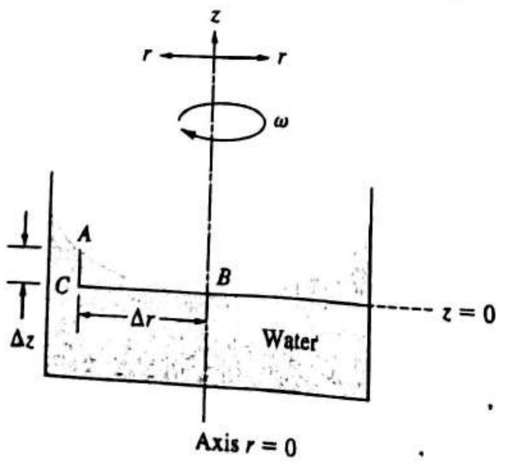


FIGURE 2.20
A system with rotation, leading to centrifugal acceleration.

$$P_{C,\,\text{gauge}} = -\rho g\,\Delta z \tag{2.BC}$$

because the rotational acceleration is perpendicular to the z axis. In the radial direction, which is the r direction in Fig. 2.20, the only forces acting on the element of fluid are the pressure forces and the centripetal acceleration, whose magnitude is given by

$$\text{Centripetal acceleration} = -(\text{angular velocity})^2 \cdot \text{radius}$$

$$a_c = -\omega^2 r \tag{2.BD}$$

Substituting this for d^2a/dt^2 in Eq. 2.35 and noting that $\cos\theta$ is zero for the radial direction, we find

$$\frac{dP}{dr} = \rho\omega^2 r \tag{2.36}$$

(No minus sign appears here because the centrifugal force points in the $+r$ direction, whereas the gravity force points in the $-z$ direction.) We then find the gauge pressure at C:

$$P_C = \int_{r=0}^{r=\Delta r} \rho\omega^2 r\, dr = \rho\omega^2 \frac{r^2}{2}\bigg]_0^{\Delta r} = \rho\omega^2 \frac{(\Delta r)^2}{2} \tag{2.37}$$

The pressure at C is the same no matter how we calculate it, so we may eliminate P_C between these two equations and divide ρg to find

$$-\Delta z = \frac{\omega^2}{2g}(\Delta r)^2 \tag{2.BE}$$

If we now let the elevation of point B (the lowest point on the free surface) be $z = 0$, then the length Δz is minus the value of z at point A, and Δr is the value of r at point A; so those points on the free surface are described by

$$z = \frac{\omega^2}{2g} r^2 \tag{2.38}$$

The free surface is a parabola with vertex at the center of the can. The height of the free surface at the wall of the can is

$$z = \frac{(2\pi \cdot 78\text{ rpm})^2 \cdot (15\text{ cm})^2}{2 \cdot 9.81\text{ m/s}^2} \cdot \left(\frac{\text{min}}{60\text{ s}}\right)^2 \cdot \frac{\text{m}}{100\text{ cm}} = 7.65\text{ cm} = 3.01\text{ in} \tag{2.BF}$$

■

To find the pressure at any point in the rotating system (with the axis of rotation vertical) we use Eq. 2.11 and measure the distance down from the free surface directly upward from the point in question. The pressure at any point on the wall of the can in Fig. 2.20 is exactly the same as if the can were not rotating and were filled to the level of the rotating free surface at the edge of the can.

Example 2.20. An industrial centrifuge has a basket with a 30 in diameter and is 20 in high. Its speed is 1000 rpm; see Fig. 2.21. If the liquid layer against the wall of the centrifuge is 1 in thick at the top, how thick is it at the bottom?

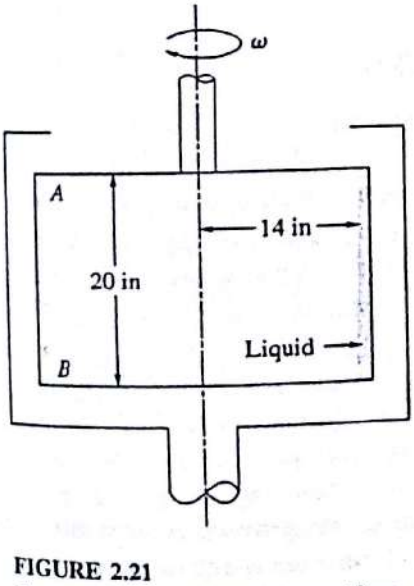


FIGURE 2.21
The basket of an industrial centrifuge.

This is really the same problem as Ex. 2.19, except that only part of the parabolic free surface is present. To solve it, we write Eq. 2.38 twice, for points A and B on the figure, and we then subtract one from the other;

$$z_A - z_B = \frac{\omega^2}{2g}(r_A^2 - r_B^2) \tag{2.39}$$

The only unknown here is r_B. Solving for it, we find

$$\begin{aligned} r_B &= \left[r_A^2 - (z_A - z_B)\frac{2g}{\omega^2} \right]^{1/2} \\ &= \left[(14\text{ in})^2 - \frac{20\text{ in} \cdot 2 \cdot 32.2\text{ ft}/\text{s}^2}{(2\pi \cdot 1000/\text{min})^2} \cdot \left(\frac{60\text{ s}}{\text{min}}\right)^2 \cdot \left(\frac{12\text{ in}}{\text{ft}}\right) \right]^{1/2} \\ &= (196\text{ in}^2 - 1.4\text{ in}^2)^{1/2} = 13.95\text{ in} = 0.354\text{ m} \end{aligned} \tag{2.BG}$$

■

Thus, the liquid film is 1.05 in (2.67 cm) thick at the bottom. One may readily calculate that at the outer side of the centrifuge, the ratio of the centrifugal acceleration to that of gravity is

$$\left(\frac{\text{Centrifugal acceleration}}{\text{Gravity acceleration}}\right) = \frac{\omega^2 r}{g} = \frac{(2\pi \cdot 1000/60\text{ s})^2 \cdot 1.25\text{ ft}}{32.2\text{ ft}/\text{s}^2} = 426 \tag{2.BH}$$

This ratio explains why liquids can be separated from solids much more effectively in such a centrifuge than by simple gravity draining, both in industrial usage and in home clothes washing machines. We will see in Chap. 10 that a centrifugal pump is a modified centrifuge; and in studying air pollution control, the widely used cyclone separators that collect particles are also modified centrifuges.

2.10 MORE PROBLEMS IN FLUID STATICS

Having worked out the basic equation and its simplifications for constant density, gauge pressure, isothermal and isentropic ideal gas, centrifugal-force fields, etc., we can attack a wide range of problems. In this text we will pass over some types of problems that have been widely treated elsewhere. Forces, distribution of forces, overturning moments, etc., on dams, retaining walls, flood gates, etc., are treated in all texts on civil or mechanical engineering fluid mechanics, such as that by White [3]. The subject of the buoyancy and stability of ships (why some turn over and others do not) is treated in the same texts. That analysis shows that the parallelepiped blocks floating with their axes vertical in Figs. 2.11 and 2.13 are unstable; they will instantly turn over to have their long axes horizontal. One may test that with a short piece of lumber in a pond; it never floats vertically. Figures 2.11 and 2.13 are convenient for showing the derivations, but physically unstable. The behavior of lighter-than-air craft is covered in books on aeronautics, for example, Prandtl and Tietjens [7].

2.11 SUMMARY

1. For simple fluids at rest, the pressure-depth relationship is given by the basic equation of fluid statics, $dP / dz = -\rho g$, found by considering the weight of a small element of fluid and the pressure change with depth necessary to support that weight.
2. For constant-density fluids the basic equation can be integrated to $P_2 - P_1 = -\rho g(z_2 - z_1)$. This equation is an excellent approximation for liquids and a good approximation for gases when the change in elevation is small.
3. For changes in elevation measured in thousands of feet, gases cannot be treated as constant-density fluids. For isothermal, isentropic, or constant-temperature-gradient behavior, the basic equation can be easily integrated for ideal gases.
4. Problems involving liquids with free surfaces are generally easiest to work in gauge pressure, in which case the basic equation simplifies further to $P_{\text{gauge}} = \rho g h$.
5. The force exerted by a static fluid on any infinitesimal surface is given by $dF = P\, dA$ and points normal to the surface. This equation can be integrated to find total forces and the x and y components of pressure forces for a wide variety of situations.
6. The necessary wall thickness of pressure vessels and pipes for modest internal pressures are calculated using the thin-walled approximation.
7. The buoyant force exerted by a fluid on a floating or submerged body is equal to the weight of the fluid displaced.
8. Most pressure-measuring devices either balance the pressure against the weight of a column of fluid, in which case the height of the fluid column is the reading, or let the pressure act on some area, compressing a spring, in which case the deflection of the spring is the reading.
9. The manometer, used for measuring fluid pressure differences, is a logical model for *manometer-like* flows, including most chimneys and flues, and circulating fluid flows, driven by heating one side of the *equivalent manometer.*
10. Problems involving accelerated motion can be handled by the methods of fluid statics if the particles of fluid do not move relative to each other, for example, in rigid-body rotation.

PROBLEMS

See the Common Units and Values for Problems and Examples inside the back cover of this text. An asterisk (*) on the problem number indicates that the answer is in App. D.

2.1.* A large petroleum storage tank is 100 ft in diameter. The free surface is really a very small part of a sphere with radius ≈4000 mi (the radius of the earth). If one drew an absolutely straight line from the liquid surface at one side of the tank to the liquid surface directly across the diameter on the other side, how deep into the fluid would that line go? In most fluid mechanics problems we ignore the curvature of the earth. Does this calculation support that simplification?

2.2. Calculate the specific weight of water at a place where the acceleration of gravity is $32.2\ \text{ft/s}^2 = 9.81\ \text{m/s}^2$. Express your answer in lbf/ft^3 and in kgf/m^3. Calculate its specific weight on the moon, where $g \approx 6\ \text{ft/s}^2 \approx 2\ \text{m/s}^2$.

2.3.* Calculate the specific weight of water in the SI system of units.

2.4. Calculate the pressure gradient due to gravity in water, in psi / ft. Most experienced engineers round this to 0.5 psi / ft, and use it for making routine calculations in their heads.

2.5. Most swimmers find the pressure at a depth of about 10 ft painful to ears. What is the gauge pressure at this depth?

2.6. A new submarine can safely resist an external pressure of 1000 psig. How deep in the ocean can it safely dive?

2.7. The tallest buildings in the world (excluding TV towers, which are not buildings in the common sense) are the Petronas Twin Towers in Kuala Lumpur, 1483 ft tall. If the pressure in the supply line to the drinking fountain on the top floor (perhaps 1450 ft high) is 15 psig, what is the required pressure in the supply line at street level? Assume zero flow in the water line.

2.8.* The deepest point in the oceans of the world is believed to be in the Marianas Trench, southeast of Japan; there the depth is about 11,000 m. What is the pressure at that point?

2.9. In the deep oil fields of Louisiana one occasionally encounters a fluid pressure of 10,000 psig at a depth of 15,000 ft. If this pressure is greater than the hydrostatic pressure of the drilling fluid in the well from the surface, the result may be a blowout, which is dangerous to life and property. Assuming that you are responsible for selecting the drilling fluid for an area where such pressures are expected, what is the minimum density drilling fluid you can use, assuming a surface pressure 0 psig for the drilling fluid?

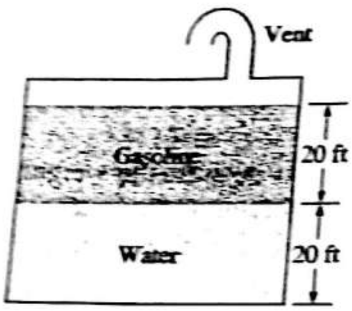


FIGURE 2.22
A storage tank holding two fluids.

2.10. The tank in Fig. 2.22 contains gasoline and water. What is the absolute pressure at the bottom? Sketch the curve of gauge pressure versus depth for this tank.

2.11. Large hydrocarbon storage tanks normally have a valve on their vents that allows free flow of air in or out when liquid is being pumped in or out but that prevents air flow for small pressure differences caused by wind, solar heating of the tank, and changes in atmospheric pressure. These reduce the *breathing losses* of valuable hydrocarbons as well as the amount of atmospheric pollution. These valves are typically set to open at an internal pressure of 4 in of water and an external pressure of 2 in of water.

(*a*) Estimate the force on the roof of the tank in Example 2.6 at the opening pressures for internal pressure and for vacuum.

(*b*) Why is the setting larger for internal pressure than external pressure?

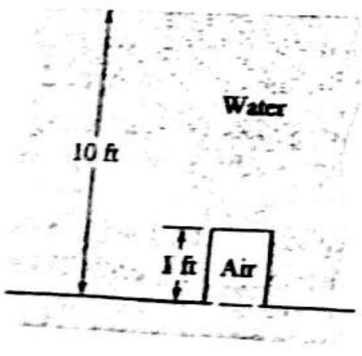


FIGURE 2.23
Figure for Prob. 2.12.

2.12.* An open-ended can 1 ft long is originally full of air at 70°F. The can is now immersed in water, as shown in Fig. 2.23. Assuming that the air stays at 70°F and behaves as a ideal gas, how high will the water rise in the can?

2.13. Normally we assume that liquids are constant-density fluids. To find out how large an error we make that way, compute the pressure at the deepest point in the oceans (about 11,000 m) two ways;

(*a*) Assume seawater is a constant-density fluid with properties shown inside the back cover of this book.

(*b*) Assume that the density of water is given by $\rho = \rho_0[1 + \beta(P - P_0)]$. The definitions of the symbols in this equation and the value of β for water are given in App. A.8.

2.14. On a very cold day in Antarctica the temperature of the air is −60°F. Assuming that the air remains isothermal up to a 10,000-ft elevation and that the pressure at sea level is 1 atm, estimate the pressure at 10,000 ft.

2.15. An airplane takes off from sea level and is climbing at 2000 ft / min. The plane is not pressurized, so that the pressure of the cabin is falling as the plane rises. At sea level (just after takeoff), how fast is the pressure falling (psi / min or kPa / min)?

2.16. Derive Eqs. 2.17 and 2.18, starting with P / ρ^k = constant and $\rho = PM / (RT)$.

2.17. For the "standard atmosphere" shown in Fig. 2.4,
(*a*) derive the pressure-height relation for the troposphere,
(*b*) calculate the pressure at the troposphere-stratosphere interface, and
(*c*) derive the pressure-height relation for the stratosphere.

2.18.*(*a*) At what height does the equation for an isentropic atmosphere, Eq. 2.17, indicate that the temperature of the air is 0 K? Assume that the surface temperature is 59°F = 15°C.
(*b*) What is the physical significance of this prediction?
(*c*) What is the predicted pressure (Eq. 2.16) for this elevation?

2.19. For most problems we assume that P_{atm} = 14.7 psia. This is a reasonable approximation for sea level but not for other elevations. What is the average atmospheric pressure at
(*a*) Salt Lake City, whose elevation is 4300 ft;
(*b*) 10,000 ft, the elevation to which the cabins on commercial airliners are pressurized;
(*c*) on the top of Mt. Everest (29,028 ft)?
(For simplicity, use the isothermal atmosphere; but see also Prob. 2.21.)

2.20.*What is the sea-level temperature gradient in °F / ft in
(*a*) the standard atmosphere (Fig. 2.4) and
(*b*) the isentropic atmosphere (Eq. 2.17) with a surface temperature of 59°F = 15°C?
(The negative of this gradient, called the *lapse rate*, is widely used in meteorology.)

2.21. The conditions at sea level are 14.7 psia and 59°F = 15°C. Calculate the pressure and temperature at 10,000 ft according to
(*a*) the isothermal atmosphere,
(*b*) the isentropic atmosphere, and
(*c*) the standard atmosphere.

2.22.*What is the mass of the entire atmosphere of the earth? The earth may be considered a sphere of radius ≈4000 mi. All of the atmosphere is so close to the surface of the earth that all of it may be considered to be subjected to the same acceleration due to gravity.

2.23. The oil-storage tank in Examples 2.6 and 2.7 has a vent to the atmosphere to allow air to move in or out as the tank is filled or emptied. This vent is plugged by snow in a blizzard while the oil is being pumped out of the tank, and the gauge pressure in the tank falls to −1 psig. What is the *net* force on the roof of the tank?

P = F/A

2.24.*In the hydraulic lift in Fig. 2.24 the total mass of car, rack, and piston is 1800 kg. The piston has a cross-sectional area of 0.2 m^2. What is the pressure in the hydraulic fluid in the cylinder if the car is not moving?

2.25. Hoover Dam is approximately 230 m high and 76 m wide at the top. Consider it to be a rectangle (only approximately true). When the water is up to the top, what is the pressure at the bottom? What is the net force tending to move the dam?

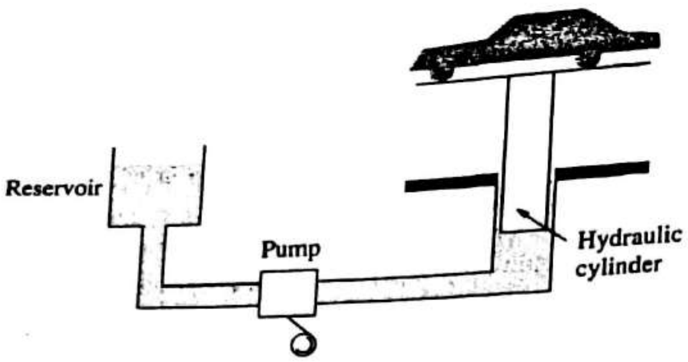


FIGURE 2.24
Hydraulic lift.

2.26. Example 2.8, leading to Eq. 2.M, was simple because the width of the surface was constant. Suppose that instead of being a rectangle, the lock was an isosceles triangle, apex down, 20 m wide at the top and 10 m deep.

(*a*) Show that the width, instead of being constant is given by

$$W = 20\text{ m} \cdot \left(1 - \frac{h}{10\text{ m}}\right) \quad (2.BI)$$

(*b*) Show that replacing the W in Eq. 2.M with this value of W and carrying out the integration leads to

$$F = \rho g \cdot 20\text{ m} \int h\left(1 - \frac{h}{10\text{ m}}\right) dh = \rho g \cdot 20\text{ m}\left[\frac{h^2}{2} - \frac{h^3}{3 \cdot 10\text{ m}}\right]_0^{10\text{ m}} \quad (2.BJ)$$

(*c*) Calculate the net force on the triangular lock gate, and compare it to that on the rectangular lock gate in Example 2.8.

(*d*) Show that the right-hand integral in Eq. 2.M can be written as

$$F = \rho g A h_c \quad (2.BK)$$

where h_c is the depth of the centroid of the surface exposed to the fluid, defined as

$$\begin{pmatrix}\text{Depth of}\\ \text{centroid}\end{pmatrix} = h_c = \frac{\int h dA}{A} \quad (2.BL)$$

for any shape.

(*e*) For a triangle, $h_c = h_{\text{maximum}} / 3$ measured from the base toward the apex. Repeat part (*c*) using this simplification. Are the answers the same?

2.27.* A dam has an upstream face that is vertical and has the shape of a semicircle with a diameter of 100 m at the top. Water is up to the top of the dam. The atmosphere presses on the rear of the dam. What is the net horizontal force on the dam? Work this problem two ways:

(*a*) by direct integration of the pressure force as shown in part (*b*) of the preceding problem.

(*b*) by using the centroid of depth as in parts (*d* and *e* of the preceding problem). The centroid of a semicircle about its diameter is $2D / 3\pi$. Centroids of all common geometric figures are shown in books on strength of materials.

2.28. Example 2.9 shows the calculated thickness at the bottom needed for steel plate making up the shell of a vertical-axis, flat-bottomed, atmospheric-pressure storage tank. In the United States such tanks are normally made from steel sheets either 8 or 10 ft wide, so their heights are normally multiples of 8 or 10 ft. The 60-ft-high tank in that example would be made of 6 bands, one atop the other, each made of steel sheets 10 ft wide, each with its own uniform thickness. The lowest band would have the thickness calculated in Example 2.9.

(*a*) What would the required thickness be for the remaining bands above the bottom band?
(*b*) If the calculation in part (*a*) leads to a thickness of less than about 0.25 in, then a plate with 0.25 in thickness will be used. Is that the case here? Suggest reasons why a plate with less than 0.25 in thickness will not be used.
(*c*) The thicknesses calculated above are wasteful, because they use a uniform thickness, instead of one that tapers from bottom to top, which would keep the stress constant in the whole wall of the tank. If we could buy such tapered plates, in which the thickness at the top of one band exactly matched the thickness at the bottom of the next upper band, how many pounds of steel (SG = 7.9) would we save on that tank? For really big tanks, the steel mills will roll tapered such plates for you, to make this savings, but that is uncommon.

2.29. The largest vertical-axis, flat-bottomed, atmospheric-pressure storage tanks have heights of about 70 ft and diameters of about 400 ft. The largest spherical storage tanks are about 80 ft in diameter. The largest sausage-shaped tanks are about 11 ft in diameter and 90 ft long. Suggest reasons for these maximum dimensions for each shape of container.

2.30. *We want to select a pipe with an inside diameter of 1 ft that will withstand an internal pressure of 1000 psig. The steel to be used has a maximum allowable tensile stress of 40,000 psi but, to allow for a safety factor of 4, we design for a maximum stress of 10,000 psi. How thick must the pipe walls be?

2.31. From the data given in App. A.2 on the diameter and wall-thickness of schedule 40 pipes, sizes 2.5 in and larger, show that these correspond almost exactly to the formula

$$t = A + BD \qquad (2.BM)$$

where t is the wall thickness, D is the diameter, A is an arbitrary constant known as the "corrosion allowance," and B is the value one would compute from Eq. 2.25. Calculate the values of A and B in this equation from the best straight line through a plot of thickness versus diameter data for schedule 40 pipes.

2.32. The thin-walled formulae are based on the assumption that the stress is practically uniform across the cross section of the vessel wall. It is generally used when $D_o / D_i < 1.5$. Sketch what the stress distribution would be for a vessel if internal pressure caused both the inside and outside diameter to increase by the same amount. Estimate how big the difference in pressure from inside to outside would be for a pipe or vessel with $D_o / D_i < 1.5$.

2.33. The thin-walled formulae in the text, Eqs. 2.25 and 2.27, are simplifications of the formulae in the piping codes [5], which are reproduced in Table 2.2.
(*a*) Repeat Example 2.10, using the thin-walled formula for a cylinder from Table 2.2, with $E_J = 1.00$ and $C_C = 0.0$. How much difference does it make?
(*b*) Same as part (*a*), but use the thick-walled formula from Table 2.2.
(*c*) For a cylindrical vessel with $r_i = 5.00$ ft, $S = 20{,}000$ psi, $E_J = 1.00$, and $C_C = 0.0$, prepare a plot of t versus P showing the calculated t from Eq. 2.25 and from the thin- and thick-walled equations from Table 2.2. Cover the pressure range from 0 to 10,000 psi.

2.34. An ordinary rifle has a maximum pressure of about 50,000 psig during firing. This peak value occurs a short way down the barrel from the chamber and lasts about 0.001 s. Farther down the barrel, the peak pressure is substantially less than this [8].
(*a*) If the inside diameter of the barrel is 0.22 in, estimate the required thickness of the barrel wall and the barrel diameter by the thin-walled and thick-walled formulae

TABLE 2.2
Vessel wall thickness formulae, Ref. [5]

Thickness equations	Limiting conditions		
Cylindrical shells			
Thin-walled: $t = \dfrac{P \cdot r_i}{SE_J - 0.6P} + C_C$	$t \le r_i/2$	or	$P \le 0.385SE_J$
Thick-walled: $t = r_i\left(\dfrac{SE_J + P}{SE_J - P}\right)^{1/2} - r_i + C_C$	$t > r_i/2$	or	$P > 0.385SE_J$
Spherical shells			
Thin-walled: $t = \dfrac{P \cdot r_i}{2SE_J - 0.2P} + C_C$	$t \le 0.356r_i$	or	$P \le 0.665SE_J$
Thick-walled: $t = r_i\left(\dfrac{2SE_J + 2P}{2SE_J - P}\right)^{1/3} - r_i + C_C$	$t > 0.356r_i$	or	$P > 0.665SE_J$

Here t = wall thickness, r_i = internal radius, S = design stress, E_J = joint efficiency, and C_C = corrosion allowance.

in Table 2.2 and by Eq. 2.25. Use $S \approx 80{,}000$ psig. Rifles are seamless, so that they have no joint, making $E_J = 1.00$. Their owners take good care of them, so that $C_C = 0.0$.

(*b*) The common practice in design of ordinary pressure vessels is to use a value of S (in Table 2.2) of 20,000 psig instead of 80,000 psig, thus providing a safety factor of 4. Does this work for the formulae in Table 2.2? For Eq. 2.27? Could one design rifles using $S = 80{,}000$ psi and then apply a suitable safety factor to the calculated wall thickness? Would that work in the formulae in Table 2.2? For Eq. 2.31?

(*c*) A simple, single-shot target rifle has an inside barrel diameter of 0.22 in. The barrel tapers from the chamber to the outlet, have a diameter of 0.74 in near the chamber and 0.605 in at the muzzle. Using all three of the values calculated in part (*a*), estimate the safety factor in the diameter at the thick end of the barrel. (Much thinner-walled barrels can be used on guns. They work, but they have very small safety factors and are risky [8]. They are also inaccurate, because the thin-walled barrel is flexible, and flexes during firing. Target rifles often have barrels thicker than the values shown here, not for safety but for stiffness and accuracy.)

2.35. Estimate the required wall thickness for the lowest ring of a flat-bottomed atmospheric pressure tank that will store water, with height 64 ft, diameter 200 ft, and $\sigma = 30{,}000$ psig, using Eq. 2.25. The American Petroleum Institute (API) Standard [9] uses a calculation method somewhat different from the equations in Table 2.2 and Eq. 2.25. Using it, they compute a wall thickness (page K-7) of 1.092 in. How does that compare with the value you compute in this example?

2.36. We wish to purchase a steel tank to store 20,000 gallons of propane; design pressure is 250 psig, and design stress is 20,000 psi. What will be the weight of metal in the shell (excluding foundations, valves, manholes, etc.) if the tank is

(*a*) cylindrical with hemispherical ends, like those in Fig. 2.6. The cylindrical section will have length = 6 times diameter.

(*b*) spherical.

Assume that the simple thin-walled formulae (Eqs. 2.25 and 2.27) may be used. For steel, SG = 7.9.

2.37. Assuming that steel pipes have an allowable wall stress of 10,000 psi, calculate the maximum internal pressure allowable for a 5 in, schedule 40 pipe that has an outside diameter of 5.563 in and an inside diameter of 5.047 in.

2.38. For a cylindrical vessel with spherical ends, what is the relation between the circumferential (hoop) stress in the cylindrical section and the axial stress in the same section?

2.39.*Archimedes is said to have discovered the buoyancy rules, which are called Archimedes' principle, when he was asked by the King of Syracuse in Sicily to determine whether a crown was pure gold, as the goldsmith said, or was an alloy. At that time no chemical means were known for settling the question without destroying the crown. Archimedes was struck with the idea of how to do so while taking a bath, and he jumped out of his tub and ran through the streets yelling "Eureka" ("I have found it"). The story goes that he was so excited that he did not bother to get dressed before doing this.

Suppose that in testing the crown Archimedes found that in air it had a weight of 5.0 N and a weight of 4.725 N in water. Assuming that the crown was made of gold or of silver or of an alloy of both, what percentage by volume was the gold? Assume that the density of gold-silver alloys is $\rho_{\text{alloy}} = \rho_{\text{silver}} + (\text{vol.\% gold}) \cdot (\rho_{\text{gold}} - \rho_{\text{silver}}) / 100$. The densities of gold and silver are 19.3 and 10.5 g / cm^3, respectively.

2.40.*A helium balloon has a flexible skin of negligible weight and infinite capacity for expansion, so that the helium is always at the same pressure as the surrounding air. If the balloon moves up and down slowly, then the temperature of the gas in the balloon will be practically the same as that of the surrounding air. If the mass of helium in the balloon is 10 lbm, how much payload can it lift under the following conditions:
(*a*) 1 atm and 70°F,
(*b*) 0.01 atm and 0°F, and
(*c*) 0.001 atm and −100°F? Assume that helium behaves as an ideal gas.

2.41. Helium is preferred to hydrogen in balloons because it is nonflammable. However, hydrogen has only half the weight of helium. By how much would the payload of the balloon in Example 2.11 have been increased if hydrogen had been used to fill it instead of helium?

2.42. Currently, recreational balloons are not filled with hydrogen or helium but with hot air; the pilot has a small propane burner to heat the air in the balloon. If the balloon is a sphere 20 m in diameter, and if the total weight of balloon, pilot, passenger compartment, propane burner, propane tank, ropes, etc. is 200 kg, what average temperature must the air in the balloon have to just barely lift the balloon? Assume that the air inside and outside the balloon both have atmospheric pressure and have equal molecular weights, 29 gm / mol. (The latter is slightly inaccurate because of the products of combustion inside the balloon; this inaccuracy is small.)

2.43.*A sample of lead is weighed on a pan balance by means of brass weights. It weighs 2.500 lbf.
(*a*) With the same set of brass weights, what would the lead weigh if the entire scale with weights and lead were at the bottom of a tank of water?
(*b*) If they were in a vacuum chamber? Here $SG_{\text{brass}} = 8.5$ and $SG_{\text{lead}} = 11.3$.

2.44. Rework Example 2.12, not by Archimedes' principle, but by assuming the block has only vertical and horizontal faces and calculating the difference in pressure between the top and bottom faces.

2.45. A 150-lb drunkard falls in a vat of whiskey. Whiskey has SG = 0.92, whereas the drunkard has SG = 0.99. The drunkard, who wants to stay alive long enough to drink his fill of the whiskey, "treads water," keeping his head above the whiskey. If his head up to his mouth is 15% of his body volume, how much upward force must he exert by "treading water" to keep his head out of the whiskey?

2.46. *It is proposed to build a raft of pine logs for carrying a cargo on a river. The cargo will weigh 500 kg, and it must be kept entirely above the water level. How many kilograms of pine logs must we use to make the raft, if the logs may be entirely submerged, and they have SG = 0.80?

2.47. A sunken battleship weighs 40,000 tons. It may be considered to be all steel, SG = 7.9.

(*a*) We now propose to raise the battleship by sinking steel tanks adjacent to it, attaching them to the battleship, and then blowing the water out of them with compressed air, making them buoyant. Assuming that the compressed-air tanks will have negligible mass, what volume must they have to raise the battleship? Assume that the battleship is in seawater and that the insides of the battleship are completely filled with water.

(*b*) It has been suggested that we could raise the ship by attaching a cable to it and hauling it up. If the cable has a working tensile stress of 20,000 psi, how thick would it have to be?

(*c*) If the battleship is 1000 ft deep and the cable is of uniform thickness, what is the stress in the cable at the top due to the weight of the cable alone?

2.48. A swimming pool is emptied for cleaning. The dimensions of the pool are 20 ft · 30 ft · 6 ft (average depth). A rainstorm causes the water table in the ground around the pool to rise so that the water level in the ground is up to 1 ft below the surface of the ground and thus up to within one foot of the top of the pool. The liquid pressure exerted by this groundwater on the pool is the same as if the pool were immersed to a depth of 5 ft in pure water. What is the upward (buoyant) force exerted by the groundwater on the pool?

2.49. *On July 2, 1982, Larry Walters attached 42 helium-filled weather balloons to a lawn chair, sat in it, and took a balloon ride high over Los Angeles [10]. People who saw him were amazed; air traffic controllers were dumbfounded. Estimate the diameter of the individual weather balloons, which are assumed to be spherical and to all have the same diameter. Assume that Mr. Walters, plus the lawn chair, plus the ropes, the empty balloons, and the miscellaneous things he took along had a weight of 200 lbf.

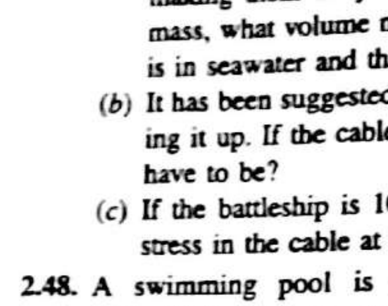


FIGURE 2.25
Simple manometer.

2.50. A rowboat is in a circular swimming pool with diameter 10 ft. The person in the rowboat throws overboard a 100 lbm block of steel, SG = 7.9, which sinks to the bottom. Does this action cause the level in the swimming pool to rise, stay the same, or decline? How much?

2.51. The fluid shown shaded in the manometer of Fig. 2.25 is ethyl iodide, SG = 1.93. The heights are h_1 = 44 in and h_2 = 8 in

(*a*) What is the gauge pressure in the tank?

(*b*) What is the absolute pressure in the tank?

2.52. *The two tanks in Fig. 2.26 are connected through a mercury manometer. What is the relation between Δz and Δh?

2.53. Figure 2.27 is a schematic diagram of a general two-fluid manometer. What is $P_A - P_B$ in terms of h, g, ρ_1, and ρ_2? If we want maximal sensitivity—that is, $\Delta h / (P_A - P_B)$ as large as possible—what relation of ρ_1 to ρ_2 should we choose?

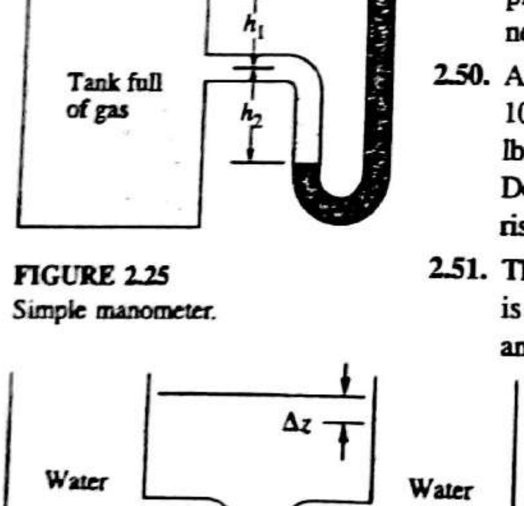


FIGURE 2.26
Mercury-water two-fluid manometer.

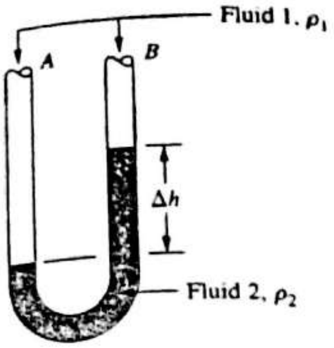


FIGURE 2.27
Two-fluid manometer.

2.54.* For low pressure differences the inclined manometer shown in Fig. 2.28 is often used (this device is so often used to measure the "draft" of a furnace that its common name is a *draft tube*).
(*a*) If the scale is set to read zero length at $P_A = P_B$ and the manometer fluid is colored water, what will the reading be at $P_A - P_B = 0.1$ lbf / in^2?
(*b*) What would the reading of an ordinary manometer with vertical legs be for this pressure difference?

2.55. The manometer in Prob. 2.54 has as its reservoir a cylinder with a diameter of 2 in. The tube has a diameter of $\frac{1}{8}$ in. The scale is set to read zero at $P_A = P_B$. When the level is at the 10 in mark, how much has the level in the reservoir fallen?

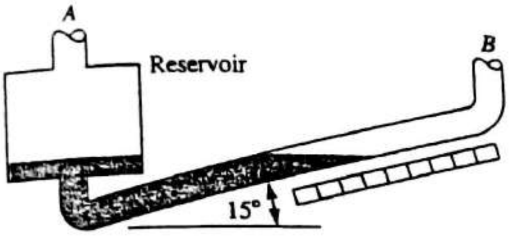


FIGURE 2.28
Draft tube.

2.56. The conventional barometer shown in Fig. 2.14 is filled with mercury.
(*a*) How high must it be to record a pressure of 1 atm?
(*b*) How high would it have to be if we used water instead of mercury as the barometer fluid.
(*c*) How large an error in pressure would we make with a water barometer by ignoring the pressure of the water in space above the liquid?

2.57. Television and newspaper meteorologists regularly show atmospheric high and low pressure regions. Typically, a high will have a sea level pressure of about 1025 millibar and a low will have a sea level pressure of 995 millibar. (Engineers would state these as 1.025 and 0.995 bars, but meteorologists always use the millibar.) Assuming a static atmosphere (impossible, but useful for this problem), estimate the average temperature difference between ground level and the top of the troposphere between the high and the low needed to cause this pressure difference. Could these pressure difference be caused by differences in moisture content?

2.58.* A common scheme for measuring the liquid depth in tanks is shown in Fig. 2.29. Compressed air or nitrogen flows slowly through a "dip tube" into the liquid. The gas-flow rate is so low that the gas may be considered a static fluid. The pressure gauge is 6 ft above the end of the dip tube.
(*a*) If the pressure gauge reads 2 psig and the dip tube is 6 in from the bottom of the tank, what is the depth of the liquid in the tank? Here $\rho_{\text{liquid}} = 60$ lbm / ft^3 and $\rho_{\text{gas}} = 0.075$ lbm / ft^3.
(*b*) Customarily, engineers read these gauges as if ρ_{gas} were zero. How much error is made by such a simplification?

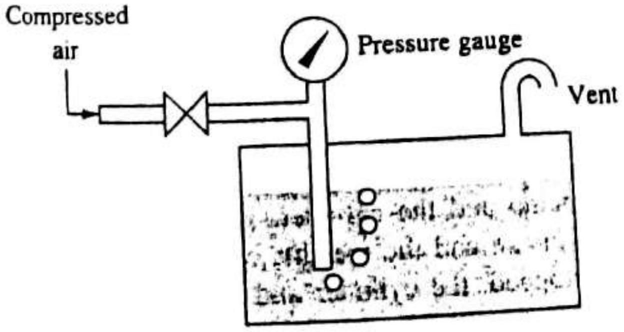


FIGURE 2.29
Dip-tube depth gauge.

2.59. The system shown in Fig. 2.30 is used to measure the density of a liquid in a tank. Compressed air or

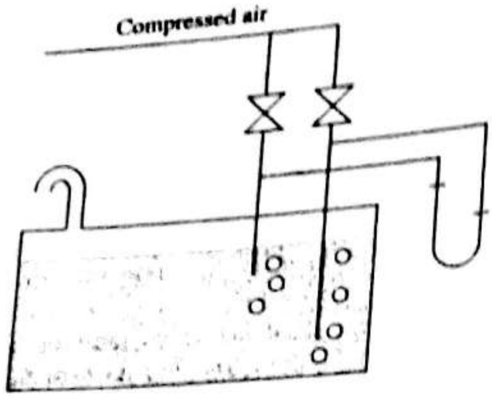


FIGURE 2.30
Two dip-tube density gauge.

nitrogen flows at a very low rate through two dip tubes, whose ends are vertically 1.00 m apart. The difference in pressure between the two dip tubes is measured by a water manometer, which reads 1.5 m of water. The gas-flow rate is so slow that the gas in the dip tubes may be considered a static fluid. The density of the gas is 1.21 kg / m³. What is the density of the fluid in the tank?

2.60.* A furnace has a stack 100 ft high. The gases in the stack have $M = 28$ g / mol and $T = 300°F$. If the pressures of the air and the gas in the stack are equal at the top of the stack, what is the pressure difference at the bottom of the stack?

2.61. An oil well is 10,000 ft deep. The pressure of the oil at the bottom is equal to the pressure of a column of seawater 10,000 ft deep. (This is typical of oil fields; most of them, at the time of discovery, have about the pressure of a hydrostatic column of seawater of equal depth; there are exceptions.) The density of the oil is 55 lbm / ft³. What is the gauge pressure of the oil at the top of the well (at the surface)?

2.62.* A natural-gas well contains methane, which is practically an ideal gas. The pressure at the surface is 1000 psig.

(*a*) What is the pressure at a depth of 10,000 ft?

(*b*) How much error would be made by assuming that methane was a constant-density fluid? Assume the temperature is constant at 70°F.

2.63. An oil pipeline was constructed to transport an oil with SG = 0.8 for a distance of 10 mi. The country was hilly, so that the line made many ups and downs. These may be considered equivalent to 10 rises of 200 ft, followed by descents of 200 ft. When the pipe was completed it was tested by pumping water through it. The water flowed satisfactorily with an inlet pressure of 150 psi. Then the oil was slowly fed into the pipe. As the oil flowed the pressure at the inlet end began to rise, and the flow rate began to fall. Finally, the flow stopped altogether, while the pressure at the inlet side remained at 150 psi. Explain what caused this. (*Hint:* This is a manometer problem.)

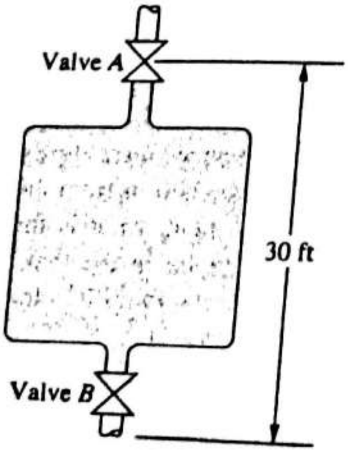


FIGURE 2.31
A tank that can be collapsed by draining.

2.64. The tank in Fig. 2.31 is completely full of water; there is no air. Both valves are closed; now we open valve *B* and allow the water to drain out, without opening valve *A*. What is the minimum pressure that will be reached in the tank?

2.65.* Bourdon tube pressure gauges and some electronic ones are inherently calibrated devices. The standard device for calibrating them is the *dead-weight tester.* This is a laboratory-sized equivalent of the hydraulic lift in Fig. 2.24. Precision weights are placed on the lift, instead of the automobile. The connection for the pressure gauge to be tested is placed between the pump and the cylinder. When the pump has just lifted the piston and the weights on it off the bottom, the pump is stopped, the cylinder and weights are rotated by hand to make sure they are not sticking, and the reading of the pressure gauge is recorded. More weights are

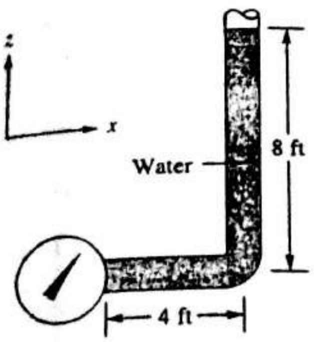


FIGURE 2.32
A very simple accelerometer.

added and the process repeated. If the diameter of the piston is 0.500 in, and the weight of the piston plus weights is 25.00 lbf, what is the pressure exerted on the pressure gauge? Manufacturers claim that these testers are accurate to ±0.5% of the pressures calculated this way.

2.66. A popular, low cost way of marking the tops of fence posts level uses a transparent plastic garden hose. One partly fills the hose with water, holds the two ends to two different posts, and adjusts the liquid level in one end of the hose (by raising or lowering it) until it is level with the top of the post that is being used for reference. Then one marks the level of the fluid in the hose on the next post; and so on. Will this system work if there are trapped air bubbles in the hose?

2.67. Rework Example 2.17 for the elevator falling freely, i.e., for downward acceleration of 9.81 m / s^2.

2.68.*The device in Fig. 2.32 consists of two pieces of pipe of 1 in inside diameter that is connected to a pressure gauge. The whole apparatus is on a elevator, which moves in the z direction. The pressure gauge reads 5 psig.

(*a*) How fast is the elevator accelerating?
(*b*) Which way?

2.69. The rectangular tank in Fig. 2.33 is sitting on a cart. We now slowly accelerate the cart. What is the maximum acceleration we can give the cart without having the fluid spill over the edge of the tank?

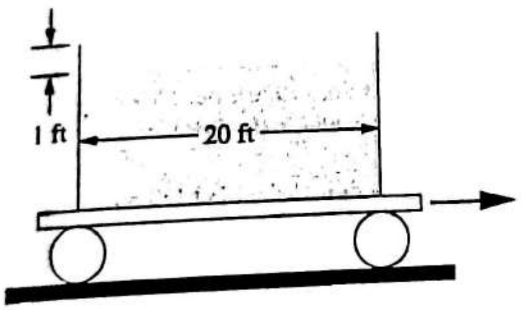


FIGURE 2.33
Figure for Prob. 2.69.

2.70.*A closed tank contains water and heating oil, SG = 0.96, and is completely full of the two liquids, with no air space at the top. The tank is being steadily accelerated in the *x* direction at 1 ft / s^2. What angle does the water-oil interface make with the vertical?

2.71. If the fluid in the centrifuge in Example 2.20 is water, what is the gauge pressure at the outer wall of the centrifuge (under the layer of water 1 in thick)?

2.72.*In the centrifuge in Example 2.20 a solid particle of volume 0.01 in^3 is settling through the fluid.

(*a*) When it is almost at the wall, where the radius is 15 in, what is the buoyant force acting on it?
(*b*) Which way does the buoyant force act?

2.73. The tank and manometer shown in Fig. 2.34 are mounted on a merry-go-round that is revolving at 10 rpm. The vessel is filled with a gas of negligible density; the manometer fluid is water. What is the pressure in the vessel?

2.74. A cylindrical, open-topped can contains a layer of gasoline 4 in deep on top of a layer of water 4 in deep. The can is now set on

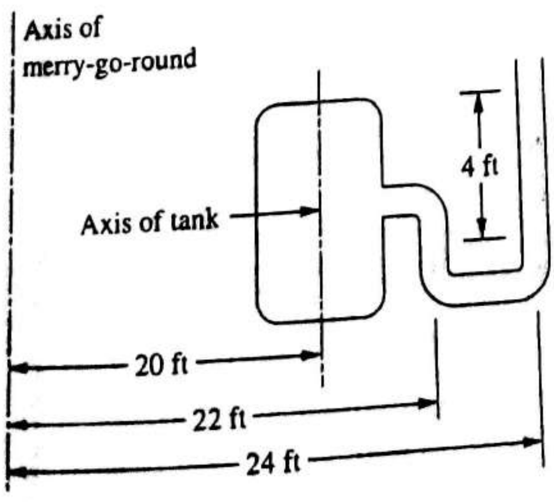


FIGURE 2.34
Manometer on merry-go-round.

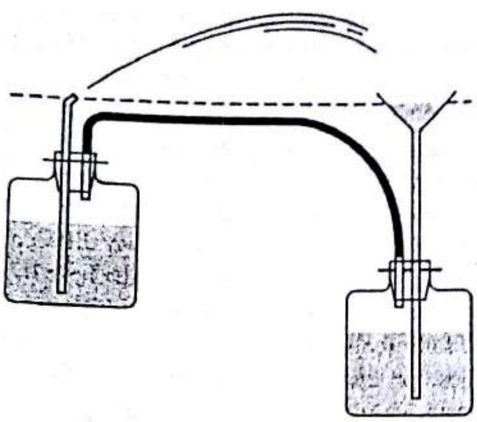


FIGURE 2.35
Gravity fountain.

a phonograph turntable and rotated about its vertical axis at 78 rpm. Describe mathematically the shape of the gasoline-air and gasoline-water interfaces.

2.75. Figure 2.35 is a sketch of a fountain arrangement made of two glass jars with rubber stoppers, several lengths of glass tubing, a funnel, and a piece of rubber tubing. The level of the jet and the level of the water in the funnel are exactly the same. The space above the water in each bottle is full of air, as is the rubber tube connecting the two bottles. An inventor has come to us, telling us that with this arrangement the water will squirt high in the air, much higher than the water level in the funnel. Is she right? Explain your answer.

REFERENCES FOR CHAPTER 2

1. de Nevers, N. *Air Pollution Control Engineering,* 2nd ed. New York: McGraw-Hill, 2000, p. 120.
2. *Handbook of Chemistry and Physics,* Boca Raton, Florida: CRC Press, annual editions.
3. White, F. M. *Fluid Mechanics,* 4th ed. New York: McGraw-Hill, 1999.
4. Meyers, P. E. *Aboveground Storage Tanks.* New York: McGraw-Hill, 1997.
5. "Rules for Construction of Pressure Vessels." *ASME Boiler and Pressure Vessel Code, Vol. III, Div. 1.* New York: ASME, 1998 (updated regularly).
6. Jennings, F. B. "Theories of the Bourdon Tube." *ASME Transactions 78,* (1956), pp. 55–64.
7. Prandtl, L., and O. G. Tietjens. *Fundamentals of Hydro- and Aeromechanics* (L. Rosenhead, transl.). New York: Dover, 1957.
8. Rinker, R. A. *Understanding Firearm Ballistics.* 4th ed. Apache Junction, Arizona: Mulberry House Publishers, 2000.
9. American Petroleum Institute (API). *Standard 650, Welded Steel Tanks for Oil Storage.* Washington, D.C.: API, 1997.
10. Plimpton, G. "The Man in the Flying Lawn Chair." *New Yorker* (June 1, 1998), pp. 62–67.

CHAPTER

3

THE BALANCE EQUATION AND THE MASS BALANCE

Much of engineering is simply careful accounting of things other than money. The accountings are called mass balances, energy balances, component balances, momentum balances, etc. In this chapter we examine the basic idea of a balance and then apply it to mass. The result is the *mass balance,* one of the four basic ideas listed in Sec. 1.3. Chemical engineers use some form of the balance equation in almost every problem they encounter.

3.1 THE GENERAL BALANCE EQUATION

Let us illustrate the general balance idea by making a *population balance* around the State of Utah. The population of Utah can change by:

1. Births
2. Deaths
3. Immigration
4. Emigration

Adding these with the correct algebraic signs and equating them to the increase in population, we get

$$\text{Increase in population} = \text{births} - \text{deaths} + \text{immigration} - \text{emigration} \quad (3.1)$$

This equation is a special case of the general balance equation:

$$\text{Accumulation} = \text{creation} - \text{destruction} + \text{flow in} - \text{flow out} \quad (3.2)$$

We can now make four comments:

1. These equations must apply to some period of time. If we simultaneously talk about the births in one year and the deaths in one month, the balance will be queer indeed. If in the population balance we are talking about one year, then we can divide Eq. 3.1 by one year to find

$$\begin{pmatrix}\text{Annual increase}\\ \text{in population}\end{pmatrix} = \begin{pmatrix}\text{annual}\\ \text{birth rate}\end{pmatrix} - \begin{pmatrix}\text{annual}\\ \text{death rate}\end{pmatrix} + \begin{pmatrix}\text{annual}\\ \text{immigration rate}\end{pmatrix} - \begin{pmatrix}\text{annual}\\ \text{emigration rate}\end{pmatrix} \quad (3.3)$$

   This is a *rate equation*. If someone promises you a million dollars, you will be happy. If he pays you at the rate of $0.01 per year, you will be unhappy; we are all normally interested in rates.

2. If we apply the population balance to the State of Utah for a one-day period, we will find misleading rates. The number of births per day fluctuates; the annual rate is practically constant. To get meaningful rates, the period over which measurements are made must be long enough to average out fluctuations. (There are some situations in which we want to study the short-time fluctuations, e.g., the statistical study of turbulence. For such studies it is worthwhile to make balances over time periods short enough for these fluctuations not to "average out.")

3. In the example above, the balance was made over an identifiable set of boundaries (the legal boundaries of the state of Utah; see Prob. 3.1). A general principle of engineering balances is that there can be no meaningful balance without a carefully *defined and stated set of boundaries*. The set of boundaries need not be fixed, but they must be identifiable. Suppose a group of people was shipwrecked in the Antarctic and took refuge on a floating iceberg. We could make a population balance around the iceberg, and it would have the same terms in it as did our population balance around the state of Utah. The boundaries of the iceberg are perfectly well-defined, but the iceberg is not fixed in place, and its size is constantly changing.

   Whatever is inside a set of boundaries is often called *system*. Everything that is outside the boundaries we will call the *surroundings*. Thus, the boundaries divide the whole universe into two parts, the system and the surroundings.

   For some problems it is convenient to choose as our system the contents of some closed container, which does not allow flow into or out of it. For such a system the balance equation reduces to

$$\text{Accumulation} = \text{creation} - \text{destruction} \qquad [\text{closed system}] \quad (3.4)$$

Such a system is called a *closed system*. An example might be the population of a sealed space capsule traveling through space, for which the population balance equation would be

$$\text{Increase in population} = \text{births} - \text{deaths} \qquad (3.A)$$

The closed system is widely used in chemistry. It is very convenient when a chemical reaction is taking place in a closed container, in which new species may be created by chemical reaction and old ones destroyed but none flow into or out of the container.

An *open system* is usually some kind of container or vessel that has flow in and out across its boundaries at some small number of places. This is used much more commonly in engineering than is the closed system and will be used extensively in this book.

We consider flows in and out of most open systems only at some small number of places: for example, a household water heater that has one cold water inlet pipe, one hot water outlet pipe, one drain pipe, and one connection for a pressure relief valve. If we choose as a system some arbitrary region of space that can have flow in or out over its entire boundary, then this system is called a *control volume*. In this book we will treat any control volume as a special kind of open system.

4. The balance equation deals only with *changes* in the thing being accounted for, not with the total amount present. The population balance given above tells the change in the population of Utah but not the numerical value of the population. If we want to know the numerical value of the population of that state, we may conduct a census. Alternatively, if we could find birth, death, immigration, and emigration data from the time that the first person entered the state to the present, we could compute the change in population starting with population zero. Mathematically, this is

$$\text{Current population} = \int_{\text{time at population}=0}^{\text{present}} (\text{rate of change of population})\, d(\text{time}) \qquad (3.5)$$

Beginners are often tempted to find a place in their balances for the total amount contained, such as a numerical value of population; resist this temptation!

To what can the balance equation be applied? It can be applied to any countable set of units or to any *extensive property*. An extensive property is one that doubles when the amount of matter present doubles. Some examples are mass, energy, entropy, mass of any chemical species, momentum, and electric charge. Some examples of countable units are people, apples, pennies, molecules, home runs, electrons, and bacteria.

The balance equation cannot be applied to uncountable individuals (units) or to *intensive properties*. Intensive properties are independent of the amount of matter present. Some examples are temperature, pressure, viscosity, hardness, color, honesty, electric voltage, beauty, and density. An example of uncountable individuals is all the decimal fractions between 0 and 1.

Because the balance equation is so important, we consider one more nonengineering illustration.

Example 3.1. Write out the appropriate terms to apply Eq. 3.2 to your bank account.

By inspection we can write that

$$\begin{pmatrix}\text{Increase in bank}\\ \text{balance}\end{pmatrix} = \begin{pmatrix}\text{interest}\\ \text{payments}\end{pmatrix} - \text{charges} + \text{deposits} - \text{withdrawals} \qquad (3.B)$$

■

As in the population example, the value of the current bank balance does not enter into this equation. (It is unfortunate that the current value of your account is called a *bank balance,* which conflicts with our use of the term *balance.*) In both the population example and this example, there are terms that are roughly proportional to the current value of what is accounted for. The birth and death rates in any state are roughly proportional to the total population, and the interest payment in your bank account is proportional to the current amount in the account. Thus the current values do often enter such balance equations indirectly. But they have no direct entry into the accountings.

3.2 THE MASS BALANCE

Our example of a balance equation in the preceding section would be of interest to demographers but not necessarily to engineers. The most important chemical engineering balance is the mass balance. Mass obeys the general balance equation: creation and destruction terms are zero. Thus, the mass balance is

$$\begin{pmatrix}\text{Increase in mass within}\\ \text{the chosen boundaries}\end{pmatrix} = \begin{pmatrix}\text{flow of}\\ \text{mass in}\end{pmatrix} - \begin{pmatrix}\text{flow of}\\ \text{mass out}\end{pmatrix} \qquad (3.6)$$

The careful application of this equation is necessary to most fluid-mechanics problems. We can divide by time and find

$$\begin{pmatrix}\text{Rate of increase of mass}\\ \text{within the chosen boundaries}\end{pmatrix} = \begin{pmatrix}\text{flow rate}\\ \text{of mass in}\end{pmatrix} - \begin{pmatrix}\text{flow rate}\\ \text{of mass out}\end{pmatrix} \qquad (3.7)$$

The mass balance cannot be derived from any prior principle. Like all the other basic "laws of nature," it rests on its ability to explain observed facts. Every careful experimental test indicates that it is correct. Mass can exist in a variety of forms, for example, solid, liquid, gas, and some other bizarre forms, and can convert from one to the other. When liquid water evaporates we see the liquid disappear, but we have no visual evidence that the mass of the surrounding air increased by exactly the mass of the water vapor thus produced. Lavoisier made the first clear statement of the law [1] and demonstrated that if processes similar to the evaporation of water were carried out in a closed glass jar resting on a balance, there was no loss of mass; the visible water had changed to invisible water vapor, but the mass of the contents of the jar did not change. The idea that mass is conserved seems quite obvious to us now, but it was not known nor believed by the human race before about 1780. The key discovery was that gases had mass, which was not intuitively obvious to scientists or the public before then.

We will see in Chap. 4 that mass and energy can be converted from one to the other. In most engineering problems we can neglect this fact and use the simple formulation in Eq. 3.5 (but we cannot neglect it in dealing with atomic bombs or the energy source of the sun). There is no experimental evidence on earth that matter is created except by conversion of energy to mass, as just described. A more interesting prospect is the idea of the "steady-state universe," put forward by the British astronomer Fred Hoyle. According to his theories, matter is being created all the time, everywhere; however, the rate is very slow, about one hydrogen atom per hour per cubic mile of space [2]. No instruments now exist that could detect such an event, so a confirmation of this theory on an earthbound scale seems impossible at present. Hoyle claimed that the experimental observations of the behavior of the farthest galaxies support his theories; most other astronomers disagree. Although these theories have no foreseeable application to engineering problems, it is well to keep an open mind on the subject of the *absolute* nature of the mass balance or the other laws of nature as we currently understand them.

Example 3.2. Consider the simple pot-bellied stove, burning natural gas, shown in Fig. 3.1. Applying Eq. 3.7 to this stove, we choose as our system boundaries the walls of the stove. Then Eq. 3.7 becomes

$$\begin{pmatrix}\text{Rate of increase of}\\ \text{mass within the chosen}\\ \text{boundaries}\end{pmatrix} = \begin{pmatrix}\text{mass flow rate}\\ \text{of gas in}\end{pmatrix} + \begin{pmatrix}\text{mass flow}\\ \text{rate of air in}\end{pmatrix} - \begin{pmatrix}\text{mass flow rate}\\ \text{of exhaust}\\ \text{gas out}\end{pmatrix} \qquad (3.C)$$

■

Here we have two mass-flow-in terms. There is no limit to the number of such terms. Recall our population balance around the state of Utah; we could have immigration by airplane, car, boat, train, etc. We would have a term for each and add the terms to get the total immigration term. Similarly here we add the individual mass-flow-in terms to get the total mass-flow-in term.

The mass balance has several other names that are in wide use. These are the *principle of conservation of mass*, the *continuity equation*, the *continuity principle*, and the *material balance*. They all mean exactly the same thing as mass balance, namely, that mass obeys the general balance equation, with no creation or destruction.

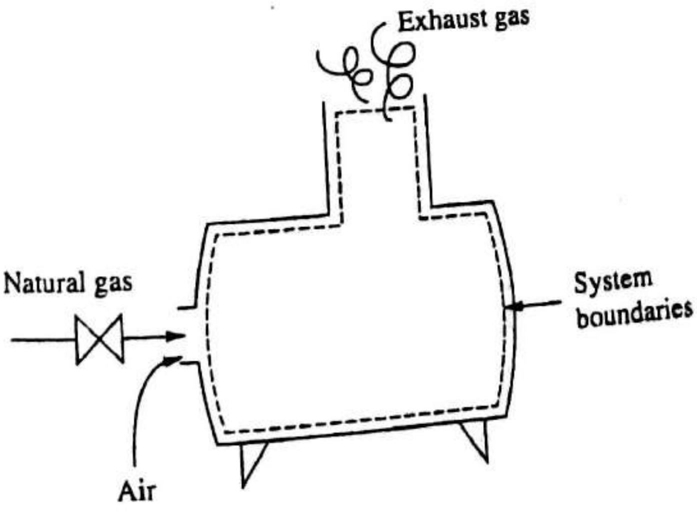


FIGURE 3.1
Pot-bellied stove.

3.3 STEADY-STATE BALANCES

When the pot-bellied stove in Fig. 3.1 is first lighted after being turned off for a long time, the temperature of its various parts will change rapidly. After a certain time it will be warmed up, and thereafter the temperature of the various parts will not change with time. During the warm-up period, the velocities and temperatures of the gases passing through it at some fixed point will be changing with time. A thermometer at some fixed point in the flue will register a continually increasing temperature. After the stove has warmed up, this thermometer will register a constant temperature. When the stove has warmed up and is running steadily we speak of it as being at steady state.

A steady state does not mean that nothing is changing; it means that nothing is changing with respect to time. Consider a waterfall with a steady flow over it. From the viewpoint of a particle of water there is a rapid increase in velocity as it falls and a sudden decrease in velocity at the bottom. From the viewpoint of an observer watching one specific point in space the waterfall is always the same: there is always water going by at a fixed velocity. Mathematically, if the velocity V is some function of time and position,

$$V = f(t, x, y, z) \tag{3.8}$$

then at steady state

$$\left(\frac{\partial V}{\partial t}\right)_{x,\,y,\,z} = 0 \qquad \text{[steady state]} \tag{3.9}$$

We may similarly write for steady state that $(\partial / \partial t)_{x,\,y,\,z}$ of *any* measurable property of the system at *any* point is zero. Thus, if we write the balance equation for some measurable quantity such as mass and divide by dt to find the rate form, then we see that the left-hand side (the time rate of mass increase within the system) must be zero, because at every point in the system the mass contained is not changing with time. Entirely analogous arguments indicate that at steady state the accumulation term must be zero for all possible balances, including the energy and momentum balances, which we will discuss in Chaps. 4 and 7.

Returning now to the pot-bellied stove of Example 3.2, we see that, at steady state, the mass balance simplifies to

$$0 = \begin{pmatrix}\text{Mass flow rate}\\ \text{of gas in}\end{pmatrix} + \begin{pmatrix}\text{mass flow rate}\\ \text{of air in}\end{pmatrix} - \begin{pmatrix}\text{flow rate of}\\ \text{exhaust gas out}\end{pmatrix} \tag{3.D}$$

This is the familiar "flow in equals flow out" idea, which is true *only* for steady state, with no creation or destruction.

Example 3.3. For the pot-bellied stove of Example 3.2 we now make a steady-state carbon dioxide balance. By chemical analysis we find that the amount of carbon dioxide in the natural gas and in the air is small enough to ignore; so, omitting the unnecessary terms from Eq. 3.2, we find

$$0 = \begin{pmatrix}\text{Creation rate}\\ \text{of carbon}\\ \text{dioxide}\end{pmatrix} + \begin{pmatrix}\text{destruction}\\ \text{rate of}\\ \text{carbon dioxide}\end{pmatrix} - \begin{pmatrix}\text{mass flow rate}\\ \text{of carbon dioxide}\\ \text{out in exhaust gas}\end{pmatrix} \tag{3.E}$$

Chemical analysis of the exhaust gas indicates that it contains 8% to 12% carbon dioxide, so the mass flow rate out is not negligible. Thus, for this equation to be satisfied, there must be significant creation minus destruction of carbon dioxide in the stove; i.e., carbon dioxide is formed by combustion in the stove. In this case, the destruction term is negligible. ■

If we made a similar balance for natural gas we would see that the destruction term would be approximately equal to the mass-flow-in term. In the field of chemical reactions, the creation and destruction terms are very important and cannot be ignored. The momentum balance (Chap. 7) includes creation and destruction terms, as does the entropy balance or the second law of thermodynamics (see any elementary textbook on thermodynamics). Thus, although the two most common balances, the mass and energy balances, have no creation or destruction terms, one should remember that these terms are very important in some other balances.

3.4 THE STEADY-STATE FLOW, ONE-DIMENSIONAL MASS BALANCE

Consider the steady-state flow of some fluid in a pipe of varying cross section, Fig. 3.2. If we apply the steady-state mass balance equation to the system shown, we find

$$\text{Mass flow rate in at point 1} = \text{mass flow rate out at point 2} \tag{3.F}$$

In general, velocity is not the same at every point in a cross section of a pipe; it is faster near the center than at the walls. (One may verify this for the analogous open-channel flow by dropping bits of wood or leaves on a flow of water in a ditch or gutter and noting that those in the center go faster than those at the side.) Therefore, to calculate the total flow rate in across the system boundaries at point 1, we should break the area across which the flow is coming in into small subareas (A), over each of which the flow is practically uniform:

$$\text{Mass flow rate in at point 1} = \sum_{\text{many subareas}} \rho A V \tag{3.10}$$

Here the individual elements of area must be taken perpendicular to the local flow velocity. For flow in a straight pipe or channel this is no problem, because the flow is all in one direction, and the area we normally consider is one perpendicular to the flow. If we then take the limit, as each subarea becomes infinitely small, the term on the right becomes the integral, over the entire system boundary at point 1, of $\rho V\, dA$. Therefore, the steady-state mass balance for the system shown in Fig. 3.2 is

$$0 = \int_{\text{area 1}} \rho V\, dA - \int_{\text{area 2}} \rho V\, dA \tag{3.G}$$

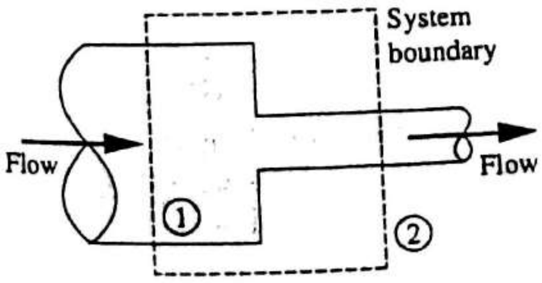


FIGURE 3.2
A system with one flow in and one flow out.

But we could choose points 1 and 2 to be any locations in the pipe, so for steady-state flow in a pipe or channel this equation becomes

$$\int_{\substack{\text{area at any boundary} \\ \text{perpendicular to the flow}}} \rho V \, dA = \text{constant} \qquad \text{[steady flow in a pipe or channel]} \tag{3.11}$$

3.4.1 Average Velocity

No real flow has a completely uniform velocity over the whole cross section. But for many problems we use an appropriate average velocity as if it were uniform across the whole cross section. The constant in Eq. 3.11 is the total mass per unit time passing down the pipe or channel, called the *mass flow rate*. It is normally measured in kg / s or lbm / s and given the symbol $\dot{m}$. If the density is uniform across the cross section of the pipe or channel (almost always practically true) then we may further define

$$\begin{pmatrix}\text{Volumetric} \\ \text{flow rate}\end{pmatrix} = Q = \frac{\text{mass flow rate}}{\text{density}} = \frac{\dot{m}}{\rho} \tag{3.12}$$

(In civil engineering books this quantity is called the *discharge*.) If we now divide the volumetric flow rate by the cross-sectional area of the pipe or channel, we find

$$\begin{pmatrix}\text{Average} \\ \text{velocity}\end{pmatrix} = V_{\text{average}} = \frac{Q}{A} \tag{3.13}$$

Example 3.4. A typical self-service gasoline pump puts 15 gal of fuel into your tank in 2 min. The inside diameter of the nozzle is 1.0 in. What are the volumetric flow rate, mass flow rate, and average velocity?

The volumetric flow rate is

$$Q = \frac{V}{t} = \frac{15 \text{ gal}}{2 \text{ min}} = 7.5 \frac{\text{gal}}{\text{min}} = 0.0167 \frac{\text{ft}^3}{\text{s}} = 0.00047 \frac{\text{m}^3}{\text{s}} \tag{3.H}$$

The density of gasoline varies from refiner to refiner and with time of the year. On average, it has an SG of about 0.72, so that

$$\dot{m} = Q\rho = 0.0167 \frac{\text{ft}^3}{\text{s}} \cdot 0.72 \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3} = 0.75 \frac{\text{lbm}}{\text{s}} = 0.34 \frac{\text{kg}}{\text{s}} \tag{3.I}$$

and

$$V_{\text{average}} = \frac{Q}{A} = \frac{0.0167 \text{ ft}^3 / \text{s}}{(\pi / 4) \cdot (1 \text{ in})^2} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} = 3.06 \frac{\text{ft}}{\text{s}} = 0.93 \frac{\text{m}}{\text{s}} \tag{3.J}$$

■

3.4.2 Velocity Distributions

For most of the rest of Parts I, II, and III of this book we will characterize a flow in a pipe or channel as having one velocity (the *block flow* or *plug flow* assumption), the average velocity calculated above. How good an approximation is that? How big a price in accuracy do we pay for the huge calculational simplification we get that way?

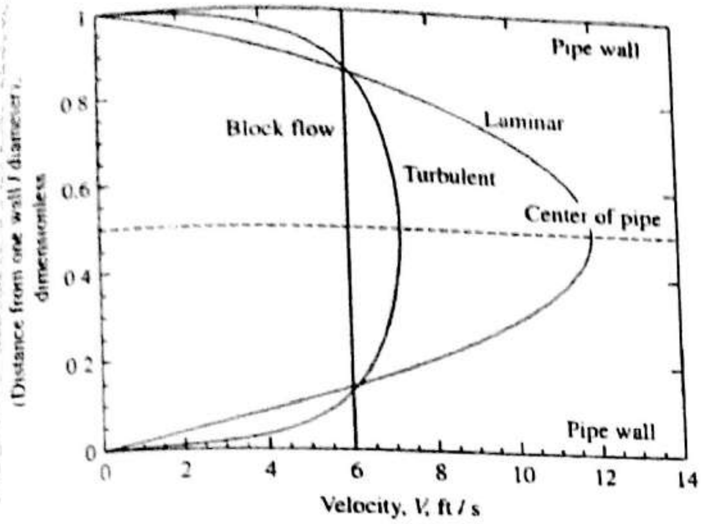


FIGURE 3.3
Velocity distributions in a circular pipe with an average velocity of 6 ft / s. The Block flow curve (vertical line) corresponds to the simplification that the whole flow may be represented by its average velocity. The Laminar flow curve (discussed in detail in Chap. 6) shows that for this type of flow the velocity at the center of the pipe is twice the average velocity, and the velocity distribution is parabolic. The Turbulent curve is an approximation of experimental measurements, represented by "Prandtl's 1 / 7 power rule" (see Prob. 3.10 and Chap. 17).

Figure 3.3 compares the velocity distributions in a pipe computed by various assumptions. In all three cases the average velocity, $V_{\text{average}} = 6$ ft / s, a common velocity in industrial pipe flow. Figure 3.3 shows that for the *block flow* assumption, the velocity is constant at 6 ft / s over the entire cross section of the pipe. *Turbulent flow,* discussed in Chap. 6, is the most common type of flow in industrial pipes, tubes, and channels. The curve shown for turbulent flow is an approximation; see Prob. 3.8. It shows that the velocity goes to zero at each of the pipe walls, as you can observe in flows in a river or rain gutter. For the average velocity to be 6 ft / s, the maximum velocity at the center of the channel must be 7.35 ft / s. *Laminar flow,* also discussed in Chap. 6, occurs in very small pipes and channels (e.g., almost all the blood flows in your body) and for high-viscosity fluids (pouring syrup on your pancakes) but not very frequently in common industrial flows. We will speak more about it in subsequent chapters. For laminar flow, as for turbulent flow, the velocity at the pipe walls is zero. To have the average velocity be 6 ft / s, the maximum velocity at the center must be 12 ft / s.

The average kinetic energy (KE) and average momentum in a pipe flow (discussed in Chap. 7) are always somewhat larger than those corresponding to the average velocity, because in calculating them (Probs. 3.10 and 3.11) we see that the velocity appears to the second or third power. Table 3.1 shows how much difference it makes if we assume either of these three distributions. For example, laminar flow is quite different from block or turbulent flows; we will speak more about that in Chap. 6. But turbulent flow is not much different from block flow. If we estimate the kinetic energy in a turbulent flow by the block flow (average velocity) assumption we

TABLE 3.1
Comparisons of block flow, turbulent and laminar velocity distributions

	Block flow	Turbulent flow	Laminar flow
Maximum velocity	$V_{average}$	$1.22\ V_{average}$	$2.00\ V_{average}$
Minimum velocity	$V_{average}$	0	0
Kinetic energy per unit mass	$\dfrac{V^2_{average}}{2}$	$1.06 \cdot \dfrac{V^2_{average}}{2}$	$2.00 \cdot \dfrac{V^2_{average}}{2}$
Total momentum in the flow	$V^2_{average}\, \rho A$	$1.014 \cdot V^2_{average}\, \rho A$	$1.333 \cdot V^2_{average}\, \rho A$

will make an error of only about 6%. If we estimate the momentum in the flow by the same simplification, we will make an error of about 1.4%. We rarely have input data (in industrial situations) accurate enough to worry about errors this small, so we will normally ignore these differences. For the rest of Parts I, II, and III of this book we will make the block-flow assumption, that the velocity of flow in a duct, pipe, or channel is adequately represented by its average value. Where we do not make that assumption, we will make that clear in the text. For the most careful work, reconsider that assumption, looking again at Table 3.1. Table 3.1 applies only to flow in a circular pipe or duct; for other geometries (the atmosphere, the oceans, all two- and three-dimensional flows), the results are more complex.

With this simplification, and the additional (very good) assumption that the density of the fluid is constant across the cross section, the integration in Eq. 3.11 can be easily performed, giving

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m} = \text{constant} \qquad \text{[steady flow in a pipe or channel]} \tag{3.14}$$

Example 3.5. In a natural-gas pipeline at station 1 the pipe diameter is 2 ft and the flow conditions are 800 psia, 60°F, and 50 ft / s velocity. At station 2 the pipe diameter is 3 ft and the flow conditions are 500 psia, 60°F. What is the velocity at station 2? What is the mass flow rate?

Solving Eq. 3.14 for V_2, we find

$$V_2 = V_1 \frac{\rho_1}{\rho_2}\frac{A_1}{A_2} = 50\,\frac{\text{ft}}{\text{s}} \cdot \frac{\rho_1}{\rho_2} \cdot \frac{\left(\dfrac{\pi}{4}\right)(2\text{ ft})^2}{\left(\dfrac{\pi}{4}\right)(3\text{ ft})^2} \tag{3.K}$$

The density of natural gas (principally methane) at 800 psia and 60°F is approximately 2.58 lbm / ft^3, and at 500 psia and 60°F it is approximately 1.54 lbm / ft^3 [3]. Therefore,

$$V_2 = V_1 \frac{\rho_1}{\rho_2}\frac{A_1}{A_2} = 50\,\frac{\text{ft}}{\text{s}} \cdot \frac{2.58(\text{lbm / ft}^3)}{1.54(\text{lbm / ft}^3)} \cdot \frac{\left(\dfrac{\pi}{4}\right)(2\text{ ft})^2}{\left(\dfrac{\pi}{4}\right)(3\text{ ft})^2} = 37.2\,\frac{\text{ft}}{\text{s}} = 11.3\,\frac{\text{m}}{\text{s}} \tag{3.L}$$

$$\dot{m} = \rho_1 V_1 A_1 = 2.58\,\frac{\text{lbm}}{\text{ft}^3} \cdot 50\,\frac{\text{ft}}{\text{s}} \cdot \frac{\pi}{4}(2\text{ ft})^2 = 405\,\frac{\text{lbm}}{\text{s}} = 184\,\frac{\text{kg}}{\text{s}} \tag{3.M}$$

■

For liquids at temperatures well below their critical temperature the changes in density with moderate temperature and pressure changes are small. Therefore, for liquids we can divide the density out of Eq. 3.14, finding

$$A_1V_1 = A_2V_2 = \frac{\dot{m}}{\rho} = \text{constant} \quad \begin{bmatrix}\text{constant density}\\ \text{steady flow in a}\\ \text{pipe or channel}\end{bmatrix} \tag{3.15}$$

Mass divided by density equals volume; therefore, the constant in this equation (the mass flow rate divided by the density) is the *volumetric flow rate*, Q, discussed above.

Example 3.6. Water is flowing in a pipe. At point 1 the inside diameter is 0.25 m and the velocity is 2 m / s. What are the mass flow rate and the volumetric flow rate? What is the velocity at point 2 where the inside diameter is 0.125 m?

$$\dot{m} = \rho_1 V_1 A_1 = 998.2\,\frac{\text{kg}}{\text{m}^3}\cdot 2\,\frac{\text{m}}{\text{s}}\cdot\frac{\pi}{4}(0.25\text{ m})^2 = 98.0\,\frac{\text{kg}}{\text{s}} = 216\,\frac{\text{lbm}}{\text{s}} \tag{3.N}$$

$$Q = \frac{\dot{m}}{\rho} = V_1A_1 = 2\,\frac{\text{m}}{\text{s}}\cdot\frac{\pi}{4}(0.25\text{ m})^2 = 0.09817\,\frac{\text{m}^3}{\text{s}} = 3.46\,\frac{\text{ft}^3}{\text{s}} \tag{3.O}$$

$$V_2 = V_1\frac{A_1}{A_2} = 2\,\frac{\text{m}}{\text{s}}\,\frac{\left(\frac{\pi}{4}\right)(0.25\text{ m})^2}{\left(\frac{\pi}{4}\right)(0.125\text{ m})^2} = 8\,\frac{\text{m}}{\text{s}} = 24.2\,\frac{\text{ft}}{\text{s}} \tag{3.P}$$

■

3.5 UNSTEADY-STATE MASS BALANCES

The steady-state behavior of systems, shown in the preceding examples, is very important. Most of the examples and problems shown in elementary textbooks concern steady-state behavior. However, unsteady-state behavior is probably more important. The characteristics of the two are compared in Table 3.2.

A power plant burns fuel and produces electricity by means of a boiler, turbine, condenser, generator, etc.; its steady-state behavior is fairly easy to calculate. However, its behavior when the power demand on the generator is suddenly increased or decreased is much more difficult to calculate. The power company would prefer

TABLE 3.2
Comparison of steady-state and unsteady-state processes

Property	Steady state	Unsteady state
Calculations	Generally easy	More difficult
Normally requires calculus?	No	Yes
Setup in laboratory	Difficult	Easy
Large-scale industrial use	Desirable	Undesirable
Efficiency	Generally high	Generally lower
Capital cost per unit of production		
Large-volume product (e.g., gasoline)	Low	High
Small-volume product (e.g., pharmaceuticals)	High	Low

to have a steady load, because then they could always operate the plant at its maximum efficiency. Nonetheless, they must plan for and be equipped for sudden load disturbances (e.g., a lightning strike shuts down a major consumer, thus quickly reducing the power demand). It has also been observed that most industrial disasters, such as explosions and fires, do not occur during periods of steady-state operation but during startup or shutdown of some processing unit, for example, the Chernobyl nuclear disaster in 1986 near Kiev. It occurred during an unusual shutdown. Thus we see that the unsteady-state behavior is very important and worthy of our attention.

Unsteady-state mass balances do not introduce any new ideas beyond those seen so far. However, as shown by the following examples, they generally lead to more complicated mathematics.

Example 3.7. The microchip diffusion furnace in Fig. 3.4 contains air, which may be considered an ideal gas. The vacuum pump is pumping air out prior to beginning the thermal diffusion step. During the pumpout process the heating coils in the tank hold the temperature in the tank constant at 68°F. The volumetric flow rate at the inlet of the pump, independent of pressure, is 1.0 ft³/min. How long does it take the pressure to fall from 1 atm to 0.0001 atm?

We choose as our system the tank up to the pump inlet. For this system the mass balance gives

$$\left(\frac{dm}{dt}\right)_{\text{system}} = -\dot{m}_{\text{out}} \tag{3.Q}$$

But we know that

$$m_{\text{system}} = V_{\text{system}}\,\rho_{\text{system}} \tag{3.R}$$

where V_{system} is the volume of the system, which does not change. Thus,

$$\left(\frac{dm}{dt}\right)_{\text{system}} = V_{\text{system}}\frac{d\rho_{\text{system}}}{dt} \tag{3.S}$$

Furthermore,

$$\dot{m}_{\text{out}} = Q_{\text{out}}\,\rho_{\text{out}} \tag{3.T}$$

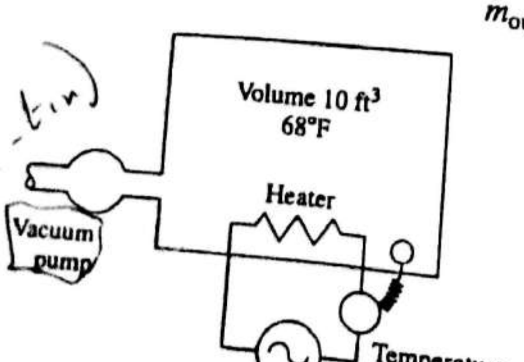


FIGURE 3.4
Evacuation of a microchip diffusion furnace.

But Q_{out} is constant and

$$\rho_{\text{out}} = \rho_{\text{system}} \tag{3.U}$$

so that

$$V_{\text{sys}}\frac{d\rho_{\text{sys}}}{dt} = -Q_{\text{out}}\,\rho_{\text{sys}} \tag{3.V}$$

This is separable, first-order differential equation, which can be rearranged to

$$\frac{d\rho_{\text{sys}}}{\rho_{\text{sys}}} = -\frac{Q_{\text{out}}}{V_{\text{sys}}}\,dt \tag{3.W}$$

and integrated from initial to final states, yielding

$$\ln \frac{\rho_{\text{sys, final}}}{\rho_{\text{sys, initial}}} = -\frac{Q_{\text{out}}}{V_{\text{sys}}}\Delta t \tag{3.16}$$

For low-pressure gases at constant temperature the densities are proportional to the pressures, so we can solve for the required time:

$$\Delta t = \frac{V_{\text{sys}}}{Q_{\text{out}}}\ln\frac{P_{\text{initial}}}{P_{\text{final}}} = \frac{10\,\text{ft}^3}{1\,\text{ft}^3/\text{min}}\ln\frac{1\text{ atm}}{0.0001\text{ atm}} = 92.1\text{ min} \tag{3.X}$$

■

Example 3.8. In Example 3.7 we considered a vacuum system with zero leaks. No real vacuum systems are totally leak-free; engineers work very hard to keep the leakage rate as low as possible. If the tank in Example 3.7 has a leak of 0.0001 lbm / min of air, what will the pressure-time plot look like, and what will be the final pressure?

Equation 3.Q becomes

$$\left(\frac{dm}{dt}\right)_{\text{system}} = -\dot{m}_{\text{out}} + \dot{m}_{\text{in}} \tag{3.Y}$$

we then follow the preceding problem, retaining $\dot{m}_{\text{in}}$ as a constant in our equations. We find

$$V_{\text{sys}}\frac{d\rho_{\text{sys}}}{dt} = -Q_{\text{out}}\,\rho_{\text{sys}} + \dot{m}_{\text{in}} \tag{3.Z}$$

$$\frac{d\rho_{\text{sys}}}{[\rho_{\text{sys}} - (\dot{m}_{\text{in}}/Q_{\text{out}})]} = -\frac{Q_{\text{out}}}{V_{\text{sys}}}dt \tag{3.AA}$$

$$\ln\frac{\rho_{\text{sys, final}} - (\dot{m}_{\text{in}}/Q_{\text{out}})}{\rho_{\text{sys, initial}} - (\dot{m}_{\text{in}}/Q_{\text{out}})} = -\frac{Q_{\text{out}}}{V_{\text{sys}}}\Delta t \tag{3.17}$$

If we ask how long it takes this system to reach 0.0001 atm, we will find that it can never get there. To see why, we ask what its steady state pressure is by setting $\Delta t = \infty$. That can only be possible if the numerator of the fraction on the left becomes zero, or

$$\rho_{\text{sys, final}} = \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}} = \frac{0.0001\text{ lbm/min}}{1\text{ ft}^3/\text{min}} = 0.0001\frac{\text{lbm}}{\text{ft}^3} \tag{3.AB}$$

At 68°F = 20°C the density of air is 0.075 lbm / ft^3, and for ideal gases densities are proportional to pressures, so

$$P_{\text{steady state}} = 1\text{ atm}\cdot\frac{0.0001\text{ lbm/ft}^3}{0.075\text{ lbm/ft}^3} = 0.00133\text{ atm} \tag{3.AC}$$

Solving Eq. 3.17 for the density at any time, we find

$$\rho_{\text{sys, any time}} = \left(\rho_{\text{sys, initial}} - \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}}\right)\exp\left(-\frac{Q_{\text{out}}}{V_{\text{sys}}}\Delta t\right) + \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}}$$

$$= (0.075 - 0.0001)\frac{\text{lbm}}{\text{ft}^3}\exp\left(-\frac{0.1}{\text{min}}\Delta t\right) + 0.0001\frac{\text{lbm}}{\text{ft}^3} \tag{3.AD}$$

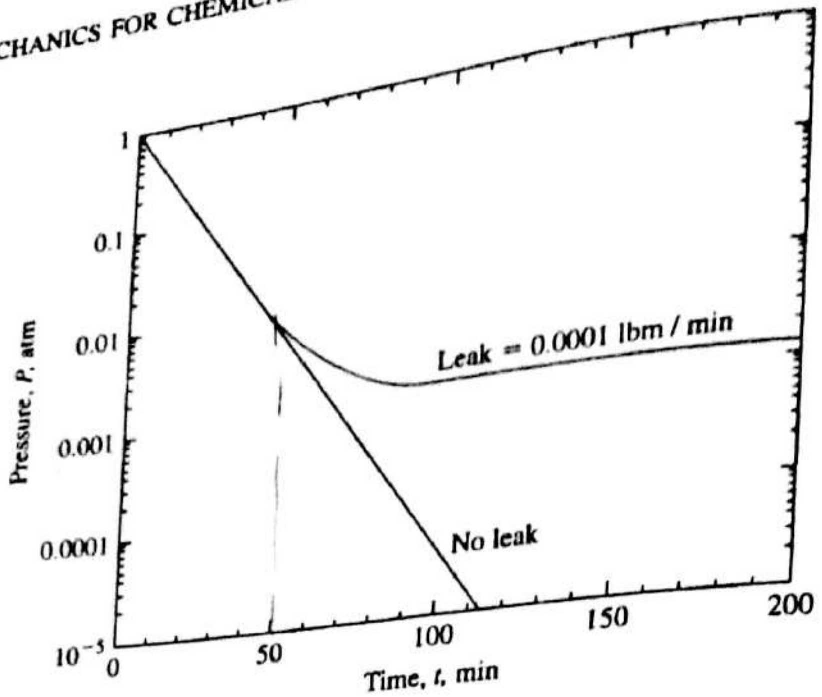


FIGURE 3.5
Calculated pressure-time behavior of the microchip diffusion furnace with zero leakage (Example 3.7) and with a constant 0.0001 lbm / min leakage (Example 3.8).

For $\Delta t = 50$ min, we have

$$\rho_{\text{sys, 50 min}} = 0.0749 \frac{\text{lbm}}{\text{ft}^3} \exp\left(-\frac{0.1}{\text{min}} \cdot 50 \text{ min}\right) + 0.0001 \frac{\text{lbm}}{\text{ft}^3}$$

$$= 0.000505 + 0.0001 = 0.000605 \frac{\text{lbm}}{\text{ft}^3} \tag{3.AE}$$

and

$$P_{50 \text{ min}} = 1 \text{ atm} \frac{0.000605 \text{ lbm / ft}^3}{0.075 \text{ lbm / ft}^3} = 0.00806 \text{ atm} \tag{3.AF}$$

The same calculation is repeated for other times, on a spreadsheet. The resulting pressure-time curves for this example and the previous one are shown in Fig. 3.5. ∎

In many unsteady-state mass-balance problems it is convenient to take as the system the fluid within some container. Thus, as the mass of fluid increases or decreases, the volume of the system will change.

Example 3.9. A cylindrical tank 3 m in diameter, with axis vertical, has an inflow line of 0.1 m inside diameter and an outflow line of 0.2 m inside diameter. Water is flowing in the inflow line at a velocity of 2 m / s and leaving by the outflow line at a velocity of 1 m / s. Is the level in the tank rising or falling? How fast?

Here we take as our system the instantaneous mass of water in the tank. For this system

$$\left(\frac{dm}{dt}\right)_{\text{system}} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}} \tag{3.AG}$$

For any fluid we have $m = \rho V$ and $\dot{m} = \rho Q$. Substituting these into the last equation and canceling the constant density, we find

$$\left(\frac{dV}{dt}\right)_{\text{system}} = Q_{\text{in}} - Q_{\text{out}} \tag{3.18}$$

The volumetric flow in or out is equal to VA, so

$$\left(\frac{dV}{dt}\right)_{\text{sys}} = 2\,\frac{\text{m}}{\text{s}}\cdot\frac{\pi}{4}(0.1\text{ m})^2 - 1\,\frac{\text{m}}{\text{s}}\cdot\frac{\pi}{4}(0.2\text{ m})^2$$

$$= 0.0157 - 0.0314 = \boxed{-0.0157\,\frac{\text{m}^3}{\text{s}}} \tag{3.AH}$$

The volume of liquid in the tank is decreasing, and the level is falling. The rate of decrease of volume is equal to the cross-sectional area times the rate of fall of the level:

$$\left(\frac{dV}{dt}\right)_{\text{system}} = A\,\frac{dz_{\text{surface}}}{dt}$$

$$\frac{dz_{\text{surf}}}{dt} = \frac{1}{A}\frac{dV}{dt} = \frac{1}{(\pi/4)(3\text{ m})^2}\left(-0.0157\,\frac{\text{m}^3}{\text{s}}\right)$$

$$= -0.0022\,\frac{\text{m}}{\text{s}} = -0.00673\,\frac{\text{ft}}{\text{s}} \tag{3.AI}$$

■

3.6 MASS BALANCES FOR MIXTURES

In the preceding examples, the flowing materials have been uniform single species, such as air or water. In most of the rest of this book we will deal with such uniform single species. However, there are many problems of great interest in which two or more components mix inside the system we are considering. If we make the simplest possible mixing assumption—perfect mixing of all components—then we can apply the simple balance equation as we have done before and find useful answers. The perfect mixing assumption is obviously a great simplification of what must occur in nature, but it is often used because the results are so simple and useful. Several examples illustrate the idea.

Example 3.10. Figure 3.6 is a sketch of a rectangular city with length L and width W. The wind blows over the city in the x direction with velocity V. Atmospheric turbulence mixes the air over the city up to the height H, so we may assume that the air in the "box" with dimensions L times W times H is well mixed and has the same pollutant concentration c everywhere. The air flowing into the upwind side of the city has pollutant concentration b (which stands for *background concentration*). The city emits pollutants into the atmosphere uniformly over its surface with an emission rate q. {Here q would have dimensions like $\text{kg}/(\text{m}^2\cdot\text{s})$. This uniform-emission assumption is a fair one for emissions from autos or small industry, which are more or less uniformly spread over the city, but a very poor one for emissions from a single large factory or power

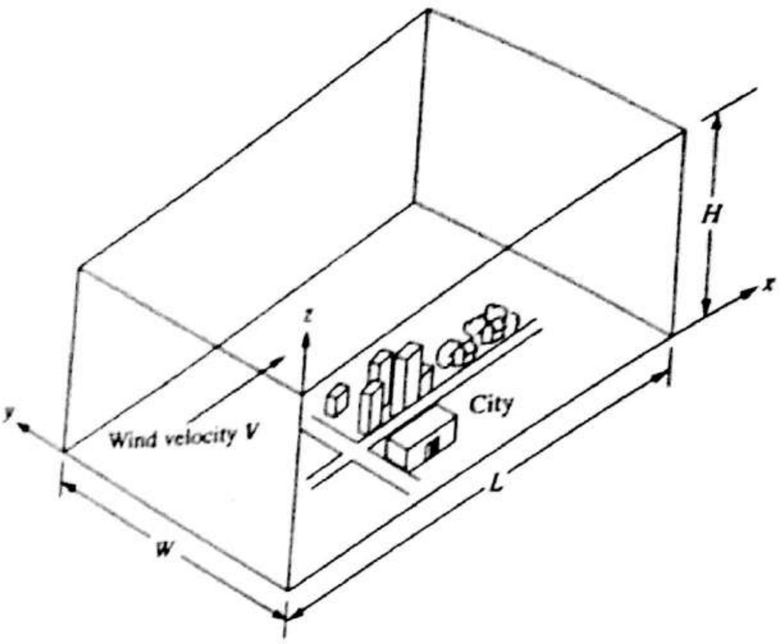


FIGURE 3.6
Idealized city used for Example 3.10.

plant; such emissions are treated a very different way in air pollutant modeling and regulation. [4], Chap. 6.) What is the concentration of pollutant in the air over the city in terms of *q*, *V*, *W*, *L*, and *H*?

Here we make the steady-state assumption, that the concentration is not changing with time, so that the algebraic sum of the flows of pollutant in and out must be zero. Writing that sum, we see

$$0 = \begin{pmatrix} \text{Flow rate of} \\ \text{pollutant into} \\ \text{city from upwind} \end{pmatrix} + \begin{pmatrix} \text{flow rate of} \\ \text{pollutant into} \\ \text{city air from city} \end{pmatrix} - \begin{pmatrix} \text{flow rate of pollutant} \\ \text{out of downwind} \\ \text{edge of city} \end{pmatrix} \tag{3.AJ}$$

The pollutant flow rates are expressed as concentrations (e.g., kg/m^3) times volumetric flow rates (e.g., m^3/s), so

$$0 = bVWH + qLW - cVWH$$

$$c = b + \frac{qL}{VH} \tag{3.19}$$

Equation 3.19 says that the pollutant concentration in the city is equal to that in the air entering the city (the background concentration) plus a term (qL/VH) that indicates how much the pollutant concentration has been increased by the emissions from the city itself. This is the "box model" or "proportional" or "roll-back" equation, which has played a very important role in the formulation of air pollution regulations in the USA [4]. ■

Example 3.11. Our paint shop will use a special paint that contains benzene as a solvent. In the course of an 8-h day the paint will evaporate 5 kg (22 lb) of benzene ($q = 5$ kg / 8 hr). The shop dimensions are 10 m × 4 m × 4 m. To protect the health of our workers, we must limit the concentration of benzene in the shop air to less than or equal to the industrial hygiene standard for benzene [5], which was 1.3 mg / m³ in 2003. If we want to keep the concentration c of benzene in the shop at or below this permitted concentration, how large a flow of ventilating air must we supply?

This problem is very similar to Example 3.10. Here we assume that the benzene is well mixed into the air in the shop and that the air leaving the shop will have the permitted benzene concentration. Making a steady-state benzene balance on the shop, taking the inlet air flow as Q, we write

$$0 = \dot{m}_{\substack{\text{benzene in}\\ \text{inlet air}}} + \dot{m}_{\substack{\text{benzene evaporated}\\ \text{from paint}}} - \dot{m}_{\substack{\text{benzene in}\\ \text{outlet air}}} = Qb + q - Qc \quad \text{(3.AK)}$$

Now we observe that there is negligible benzene in the incoming air ($b = 0$), so we can solve for Q, finding

$$Q = \frac{q}{c} = \frac{5 \text{ kg} / 8 \text{h}}{1.3 \text{ mg} / \text{m}^3} \cdot 10^6 \frac{\text{mg}}{\text{kg}} = 481{,}000 \frac{\text{m}^3}{\text{h}}$$
$$= 8000 \frac{\text{m}^3}{\text{min}} = 283{,}000 \frac{\text{ft}^3}{\text{min}} \quad \text{(3.AL)}$$

■

This example (called the well-mixed model in industrial hygiene [6]) shows that, for the assumption of perfect mixing of the benzene into the shop air, it is quite straightforward to compute the required dilution air to meet the industrial hygiene standard. We can also see that this is an impossibly large airflow rate. If we divide the above flow rate by the cross-sectional area of the shop (4 m × 4 m), we find

$$\text{Velocity} = \frac{Q}{A} = \frac{8000 \text{ m}^3 / \text{min}}{16 \text{ m}} = 500 \frac{\text{m}}{\text{min}} = 8.33 \frac{\text{m}}{\text{s}} = 27 \frac{\text{ft}}{\text{s}} = 18 \frac{\text{mi}}{\text{h}} \quad \text{(3.AM)}$$

This very high velocity could hardly be used inside a paint shop. Our practical alternatives are to choose a less toxic solvent, for which the permitted concentration is higher, or to devise some kind of ventilation system, like a laboratory fume hood, that will prevent the mixing of the benzene with the air the workers breathe or to provide the workers with personal protective devices. We also need to consider the air pollution consequences of emitting 5 kg / day of benzene to the atmosphere; in most U.S. cities that would require a permit and probably some form of capture or destruction of the benzene.

These two examples appear here because every chemical engineer is both an environmental engineer and a safety engineer. We are responsible for protecting the public and the workers under our supervision from harm due to our activities. The completely mixed model used here is also useful for more traditional chemical engineering problems.

TABLE 3.3
Comparison of Examples 3.7 and 3.12

Type of variable	Example 3.7, vacuum pump down	Example 3.12, tank washout
Capacity variable	Tank volume, V_{system}	Liquid volume, V_{liquid}
Flow variable	Pump-out rate, Q_{out}	Flow-through rate, Q_{liquid}
Concentration variable	Gas density, ρ_{system}	Salt concentration, c_{salt}
Starting variable	Initial gas density, $\rho_{sys,\ init}$	Initial concentration, $c_{salt,\ init}$
Resulting ratio	$\dfrac{\rho}{\rho_{init}} = \dfrac{P}{1\ \text{atm}}$	$\dfrac{c_{salt}}{c_{salt,\ initial}}$
Time variable	Time, t	Time, t

Example 3.12. A tank contains $1000\ m^3$ of salt solution, with salt concentration $= 10\ kg/m^3$. At time zero, salt-free water starts to flow into the tank at a rate of $10\ m^3/min$. Simultaneously, salt solution flows out of the tank at $10\ m^3/min$, so that the volume of solution in the tank is always $1000\ m^3$. A mixer in the tank keeps the concentration of salt in the entire tank uniform so that the concentration in the effluent is the same as the concentration in the tank. What is the concentration in the effluent as a function of time?

This example is exactly the same as Example 3.7, with the variables renamed, as shown in Table 3.3. The reader may make the substitutions shown there and have the resulting solution to this problem. See also Prob. 3.21. This same problem appears in heat transfer and mass transfer, with the variables renamed. ∎

3.7 SUMMARY

1. Balances are important in engineering.
2. All balances can be made from the general balance equation (accumulation = creation − destruction + flow in − flow out) by dropping the unnecessary terms.
3. All balances can be divided by time to make rate equations.
4. In any balance it is necessary to choose and state the boundaries over which the balance is made. Whatever is inside the boundaries over which the balance is made is called the "system." Whatever is outside is called the "surroundings."
5. The most important engineering balance is the mass balance, in which the creation and destruction terms are zero. This is also called "the continuity equation" or "the principle of conservation of mass."
6. In applying the mass balance to flowing fluids, we normally speak of the mass flow rate and the volumetric flow rate. We also normally assume that the average velocity adequately represents the fluid behavior, although we know that the most precise work must take into account the fact that the velocity is not uniform across the flow.
7. The completely mixed model, used in several examples in this chapter, is immensely useful.

PROBLEMS

See the Common Units and Values for Problems and Examples inside the back cover of this text. An asterisk (*) on the problem number indicates that the answer is in App. D.

3.1. In our balance equation for the population of the state of Utah, we must resolve several questions of definition. For example, are out-of-state students to be counted in the population of Utah? Are tourists driving through the state to be counted while they are here? List several other ambiguous groups for which we must make a definition.

3.2. Write out the balance for the number of one-dollar bills in circulation in the United States.

3.3. Write out the balance for the mass of refined sugar in the state of Idaho (which is a sugar-producing state).

3.4. A child's toy balloon has just had its neck released. It is zipping through the air and shooting air out its neck. Write out the balance for the mass of air involved. Are your system boundaries fixed in space? Are they fixed in size? Are they identifiable?

3.5. Write a carbon-atom balance for an automobile that is driving at a constant speed; include the carbon atoms that are bound in chemical compounds as well as the free carbon atoms. Consider other flows of carbon atoms than those in the exhaust gas. There are more terms in this balance than most students expect.

3.6. Write a mass balance for an exploding firecracker.

3.7.* A river has a cross section that is approximately a rectangle 10 ft deep and 50 ft wide. The average velocity is 1 ft / s. How many gallons per minute pass a given point? What is the average velocity (assuming steady flow) at a point downstream, where the channel shape has changed to 7 ft in depth and 150 ft in width?

3.8. The annual flow of the Colorado River below Glen Canyon Dam is approximately 10^7 acre-ft / yr, where an acre-ft is the volume needed to cover one acre, one foot deep $\approx 4.35 \cdot 10^4$ ft^3. This flow is not steady over the year, but varies from season to season. At a point where the river is 200 ft wide and 10 ft deep (assume for this problem that the river has a rectangular cross section), what is the velocity of the river, averaged over the whole year?

3.9. In March 1996 a special release of water, $Q = 45{,}000$ ft^3 / s, was made from the Glen Canyon Dam, to create an "artificial flood" in the Grand Canyon.

(*a*) The flow was through 8 pipes, each with an internal diameter of 8 ft. Estimate the velocity through those pipes.

(*b*) Estimate the average velocity of the river at some point downstream of the dam, where the width of the river was 200 ft and its average depth was 10 ft.

3.10. There is steady flow in a circular pipe. The average velocity is given by

$$V_{\text{average}} = \frac{Q}{A} = \int_{r=0}^{r=r_{\text{wall}}} V \cdot 2\pi r \, dr \,/\, \pi r_{\text{wall}}^2 \qquad (3.20)$$

Calculate the ratio of the average velocity to the maximum velocity for each of the following cases. Compare your results to those shown in Table 3.1.

(*a*) The flow is laminar (to be discussed in Chap. 6), and the local velocity at any point in the pipe is given by

$$V = V_{\max}\left(\frac{r_{\text{wall}}^2 - r^2}{r_{\text{wall}}^2}\right) \qquad (3.21)$$

where r is the radial distance from the center of the pipe and r_{wall} is the radius at the wall of the pipe.

(*b*) The flow is turbulent (to be discussed in Chap. 6), and the local velocity is given by

$$V = V_{max}\left(\frac{r_{wall} - r}{r_{wall}}\right)^{1/7} \tag{3.22}$$

(*c*) This is *Prandtl's 1/7 power rule,* which is a good but not outstanding approximation of the velocity distribution in turbulent flow in a circular pipe. It is the best *simple* mathematical description of that distribution; see Table 17.1 and Fig. 17.7. At higher average velocities the 1/7 is replaced by 1/10. Repeat part (*b*) using 1/10 instead of 1/7.

3.11. See the preceding problem.

(*a*) The average kinetic energy per unit mass in any flow in any circular conduit is given by

$$\begin{pmatrix}\text{Average kinetic}\\ \text{energy, per}\\ \text{unit mass}\end{pmatrix} = \frac{\int_{r=0}^{r=r_{wall}} (V^2/2)\cdot V \cdot 2\pi r\, dr}{\int_{r=0}^{r=r_{wall}} V \cdot 2\pi r\, dr} \tag{3.23}$$

but the denominator of this fraction equals

$$\int_{r=0}^{r=r_{wall}} V\cdot 2\pi r\, dr = Q = V_{average}\, A = \pi r_{wall}^2\, V_{average} \tag{3.24}$$

so that Eq. 3.23 simplifies to

$$\begin{pmatrix}\text{Average kinetic}\\ \text{energy, per}\\ \text{unit mass}\end{pmatrix} = \frac{\int_{r=0}^{r=r_{wall}} V^3 \cdot r\, dr}{r_{wall}^2\, V_{average}} \tag{3.25}$$

Show that substituting Eqs. 3.21 and 3.22 for V in this equation, integrating, and simplifying leads to the values shown in Table 3.1.

(*b*) The total momentum flow in a pipe is given by

$$\begin{pmatrix}\text{Total momentum}\\ \text{flow}\end{pmatrix} = \int_{\text{whole flow area}} V\, dm = \int_{r=0}^{r=r_{wall}} V\cdot \rho V \cdot 2\pi r\, dr \tag{3.26}$$

For block flow this simplifies to

$$\begin{pmatrix}\text{Total momentum}\\ \text{flow}\end{pmatrix} = V_{average}^2\, \rho A \qquad \text{[block flow]} \tag{3.27}$$

Show that substituting Eqs. 3.21 and 3.22 for V in Eq. 3.26, integrating, and simplifying leads to the values shown in Table 3.1. Show the corresponding values for 1/10 instead of 1/7 in Eq. 3.22.

3.12. An ideal gas is flowing in a constant-diameter pipe at a constant temperature. What is the relation of average velocity to pressure?

3.13.* A column of soldiers is marching 12 abreast at a speed of 4 mi/h. To get through a narrow pass they must crowd in to form a column 10 men abreast. Assuming steady flow, how fast are the soldiers moving when they are 10 abreast?

3.14.* A water tank has an inflow line 1 ft in diameter and two outflow lines of 0.5 ft diameter. The velocity in the inflow line is 5 ft/s. The velocity out one of the outflow lines is 7 ft/s. The mass of water in the tank is not changing with time. What are the volumetric flow rate, mass flow rate, and velocity in the other outflow line?

$D_{in} = 1\ ft \Rightarrow V = 5\ ft/s$

$D_{out} = 0.5\ ft \Rightarrow V_1 = 7$

3.15. A compressed-air vessel has a volume of 10 ft^3. Cooling coils hold its temperature constant at 68°F. The pressure now in the vessel is 100 psia. Air is flowing in at the rate of 10 lbm / h. How fast is the pressure increasing?

3.16.* Repeat Example 3.8 for a leak rate of 0.001 lbm / min.

3.17. The tank in Example 3.8 has a leak that admits air at an *unknown but constant* rate. We find that it takes 72 min to reach a pressure of 0.001 atm.
(*a*) What is the leakage rate, in lbm / min or some equivalent units?
(*b*) What will the steady-state pressure be?

3.18. The tank in Example 3.8 has a leak that admits air at the rate

$$\dot{m} = \frac{0.0005 \text{ lbm}}{\text{min} \cdot \text{atm}} (P_{\text{outside}} - P_{\text{tank}}) \tag{3.AN}$$

How long does it take the pump to reduce the pressure in the tank from 1 atm to 0.01 atm? What is the steady-state pressure in the tank?

3.19.* A lake has a surface area of 100 km^2. One river is bringing water into the lake at the rate of 10,000 m^3 / s, while another is taking water out at 8000 m^3 / s. Evaporation and seepage are negligible. How fast is the level of the lake rising or falling?

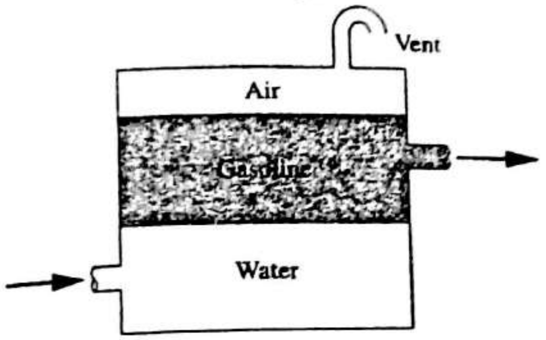


FIGURE 3.7
Tank with two fluids for Prob. 3.20.

3.20. The tank in Fig. 3.7 has an inflow line with a cross-sectional area of 0.5 ft^2 and an outflow line with a cross-sectional area of 0.3 ft^2. Water is flowing in the inflow line at a velocity of 12 ft / s, and gasoline is flowing out the outflow line at a velocity of 16 ft / s. How many lbm / s of air are flowing through the vent? Which way?

3.21.* A vacuum chamber has a volume of 10 ft^3. When the vacuum pump is running, the steady-state pressure in the chamber is 0.1 psia. The pump is shut off, and the following pressure-time data are observed:

Time after shutoff, min	Pressure, psia
0	0.1
10	1.1
20	2.1
30	3.1

Calculate the rate of air leakage into the vacuum chamber when the pump is running. Air may be assumed to be an ideal gas. The air temperature may be assumed constant at 68°F. Can you use Fig. 3.5 in this problem?

3.22. Finish Example 3.12, showing the resulting numerical values and their dimensions.

3.23. Repeat Prob. 3.22, except now there is a layer of solid salt on the bottom of the tank, which is steadily dissolving into the solution at a rate of 5 kg / min, and the inflowing water contains no salt.

3.24. Repeat Prob. 3.22, except that the outflow is only 9 m^3 / min, so that the total volume of liquid contained in the tank is increasing by 1 m^3 / min.

3.25. Rework Example 3.7 with the following change. The tank is now somewhat flexible, so that it is being slowly crushed by the surrounding pressure. If it is crushed at such a rate that its volume decreases steadily by 0.1 ft^3 / min, and this rate of volume decrease begins as soon as the vacuum pump starts, how long does it take the pressure to fall from 1 atm to 0.0001 atm?

3.26.*While Moses was crossing the Red Sea, he took up a liter of water, examined it, and then threw it back. The tides, currents, evaporation, and rainfall, have been steadily mixing the waters of the world's oceans since, so we may assume (for this problem only!) that the molecules in that liter of water have now been uniformly distributed over the waters of all the oceans of the world. If you pick up a liter of water from the ocean and examine it, how many molecules will it contain that were in the liter which Moses examined? State clearly your assumptions and simplifications.

3.27. The typical human being breathes about 10 times / min and takes in about 1 liter per breath. Assuming that the atmosphere has been perfectly mixed since Julius Ceasar's time, estimate the number of air molecules that you take in with a single breath that at some time were breathed in and out by Julius Caesar, who lived 56 yr.

REFERENCES FOR CHAPTER 3

1. Poirier, J. P. *Lavoisier, Chemist, Biologist, Economist.* Philadelphia: University of Pennsylvania Press, 1996.
2. Gamow, G., "Modern Cosmology." *The New Astronomy,* New York: Simon and Schuster, 1959, p. 14.
3. Starling, K. E. *Fluid Thermodynamic Properties for Light Petroleum Systems,* Houston: Gulf Publishing, 1973, p. 12.
4. de Nevers, N. *Air Pollution Control Engineering.* 2nd ed. New York: McGraw-Hill, 2000, p. 120.
5. *Threshold Limit Values for Chemical Substances and Physical Agents.* Cincinnati: American Conference of Governmental Industrial Hygienists. Annual editions. The value shown is from the 2001 edition.
6. Keil, C. B. *Mathematical Models for Estimating Occupational Exposure to Chemicals.* Fairfax, Virginia: American Industrial Hygienists Association, 2000.

CHAPTER
5
BERNOULLI'S EQUATION

The energy balance for steady, incompressible flow, called Bernoulli's equation, is probably the most useful single equation in fluid mechanics.

5.1 THE ENERGY BALANCE FOR A STEADY, INCOMPRESSIBLE FLOW

We begin with Eq. 4.17,

$$du + d\left(\frac{P}{\rho}\right) + g\,dz + d\left(\frac{V^2}{2}\right) = \frac{dQ}{dm} + \frac{dW_{\text{n.f.}}}{dm} \qquad \text{[steady flow, open system]} \quad (4.17)$$

which applies to the changes from one point to the next along the direction of flow in any steady flow of a homogeneous fluid. Electrostatic, magnetic, and surface energies are assumed to be negligible.

Multiplying by minus 1 and regrouping produce

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = \frac{dW_{\text{n.f.}}}{dm} - \left(\Delta u - \frac{dQ}{dm}\right) \tag{5.1}$$

Here ΔP stands for $P_{\text{out}} - P_{\text{in}}$, etc. This equation is the preliminary form of Bernoulli's equation. To save paper, in the rest of this chapter we will speak of Bernoulli's equation as B.E. The original form of B.E. was developed by Daniel Bernoulli (1700–1782) in an entirely different way. By considering momentum balance (Chap. 7) for a frictionless fluid he found $\Delta(P/\rho + gz + V^2/2) = 0$, the same as Eq. 5.1 but without the two terms to the right of the equal sign. The original equation was not applicable to flows containing pumps or turbines or to flows in which fluid friction was important. Equation 5.1, based on the energy balance, is applicable to all the flows to which the original, momentum-based, frictionless B.E. applies as well as to those that have significant friction and / or pumps. Some writers refer to Eq. 5.1 as the *extended form of B.E.* or the *engineering form of B.E.*

Before converting it to the final form, let us see what each of the terms represents physically. The P/ρ terms are injection-work terms, representing the work required to inject a unit mass of fluid into or out of the system, or both. The gz terms are potential-energy terms, representing the potential energy of a unit mass of fluid above some arbitrary datum plane. Since they appear only as Δgz, it is unnecessary in most problems to know or to state what that datum is. The $V^2/2$ terms show the kinetic energy per unit mass of fluid. The $dW_{n.f.}/dm$ term represents the amount of work done on the fluid per unit mass of fluid passing through the system (this does not include injection work, which was specifically excluded). Most often this represents work input from a pump or compressor, or work output in a turbine or expansion engine.

5.2 THE FRICTION-HEATING TERM

We are all familiar with friction heating, as seen in the smoking brakes and tires of an auto that has stopped suddenly and in the high temperature of a saw that is cutting wood. We are less familiar with the idea of friction heating in fluids, because the temperature increases produced by friction heating in fluids are generally much less than those produced by rubbing two solids together. These temperature increases are less for the following reasons:

$u = -g\Delta Z$
$= \frac{\Delta u}{C_v}$

1. The amount of frictional work per unit mass in typical fluid-flow problems is generally less than in the examples cited above. In these examples the friction-heating energy is concentrated in a small volume; in fluid flows it is spread over a larger volume of fluid.
2. The heat capacity of liquids is generally greater than that of solids. For example, the amount of heat required to raise the temperature of 1 lbm of water by 1°F will raise the temperature of 1 lbm of steel by about 8°F.

Example 5.1. One kilogram of water falls over a 100 m waterfall and lands in the pool at the bottom. This converts the potential energy it had at the top of the fall to internal energy. How much does the temperature of the water increase?

In real waterfalls we must consider evaporation of part of the falling water, which cools the remaining water. But ignoring that for this example, we solve Eq. 5.1 for the change in internal energy,

$$\Delta u = -g(\Delta z) = -9.81\,\frac{\text{m}}{\text{s}^2}(-100\text{ m})\cdot\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\cdot\frac{\text{J}}{\text{N}\cdot\text{m}}$$
$$= 981\,\frac{\text{J}}{\text{kg}} = 328.1\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}} \tag{5.A}$$

and the temperature increase is

$$\Delta T = \frac{\Delta u}{C_V} = \frac{981 \text{ J/kg}}{4184 \text{ J/(kg}\cdot{}^\circ\text{C)}} = 0.23^\circ\text{C} = 0.42^\circ\text{F} \tag{5.B}$$

■

This example shows why we rarely think about friction heating in liquids; the calculated temperature increase, even for this large change in potential energy, is below our ability to sense by sticking our finger in the water.

Friction heating involves the conversion of some other kind of energy (kinetic or potential) or of external work (injection, shaft, or expansion) into internal energy. For constant-density materials (gas, liquid, or solid) the only other way (excluding magnetic, electrostatic, etc.) the internal energy per unit mass can change is through external heating or cooling. Thus,

$$\Delta u = \frac{d(\text{friction heating})}{dm} + \frac{dQ}{dm} \qquad \begin{bmatrix}\text{constant-density} \\ \text{materials only}\end{bmatrix} \tag{5.2}$$

Solving this equation for the friction heating per unit mass, we see that it is given by the $\Delta u - dQ/dm$ term on the right of Eq. 5.1.

This friction heating is not connected with any heating or cooling of the fluid through heat transfer with the surroundings and has the same meaning whether the fluid is being heated or cooled. This may be seen by considering the simple, frictionless heater for a constant-density fluid shown in Fig. 5.1. For such a heater there is no change in elevation or velocity and, because there is no friction, there is no change in pressure. Similarly, there is no pump or compressor work, so B.E. simplifies to

$$0 = -\left(\Delta u - \frac{dQ}{dm}\right) \qquad \text{[frictionless heater]} \tag{5.3}$$

If, however, there were friction in the heater, then $\Delta u - dQ/dm$ would be a positive number, whose value would be exactly equal to the amount of friction heating per unit mass.

The increased internal energy produced by friction heating is generally useless for industrial purposes, so friction heating is often referred to as *friction loss*. Energy does not disappear in this case. Rather, energy of a valuable form is converted to energy of a normally useless form; hence the "loss" of energy (really, of useful energy).

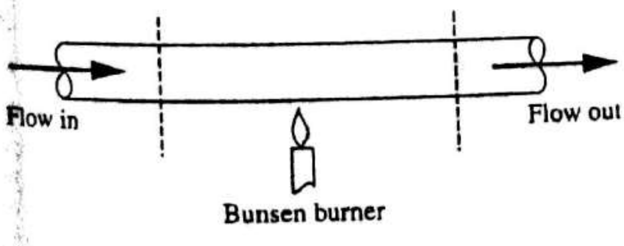


FIGURE 5.1
A simple frictionless heater.

As discussed in Sec. 2.2, there is no such thing as an absolutely incompressible fluid. Furthermore, there are some situations in which even a fluid with a very small compressibility, such as water, behaves in a compressible way. Thus, we speak of an *incompressible flow*, by which we mean a flow in which the changes in density are unimportant, rather than of an incompressible fluid. As a general rule, all steady flows of liquids and most steady flows of gases at low velocities (see Sec. 5.6) may be considered incompressible, whereas some unsteady flows of liquids (see Sec. 7.4) and all steady flows of gases at high velocities may not be considered incompressible. We will consider the flow of gases at high velocities in Chap. 8, where we will see that the same terms that appear in B.E. will reappear in different combinations. Therefore, we will apply B.E. only to incompressible flows and use only the incompressible-flow meaning of $\Delta u - dQ/dm$, that is, friction heating per unit mass.

To save writing, we now introduce a new symbol for the friction heating per unit mass,

$$\Delta u - \frac{dQ}{dm} = \mathscr{F} = \begin{pmatrix}\text{friction heating}\\ \text{per unit mass}\end{pmatrix} \qquad \begin{bmatrix}\text{constant-density}\\ \text{flow}\end{bmatrix} \tag{5.4}$$

Here we use $\mathscr{F}$ to avoid confusion with F for force. Most civil engineering texts call this quantity gh_f or gh_L, where g is the acceleration of gravity and h_f or h_L stands for *friction head loss* (Sec. 5.4). Some thermodynamics textbooks introduce the idea of the lost work in explaining the second law of thermodynamics. It can be shown that for a constant-density fluid at the heat reservoir temperature the friction heating per unit mass is exactly equal to the lost work per unit mass, so some texts call this term *LW*. Other texts call it $(-\Delta P/\rho)_{\text{friction}}$, since for the most common pipe friction problem, steady flow in horizontal, constant-area pipes, $\mathscr{F} = (-\Delta P/\rho)_{\text{friction}}$.

Substituting the definition of $\mathscr{F}$ into Eq. 5.1 changes it to the final working form of B.E.,

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = \frac{dW_{\text{n.f.}}}{dm} - \mathscr{F} \tag{5.5}$$

One may show as a consequence of the second law of thermodynamics that $\mathscr{F}$ is zero for frictionless flows and positive for all real flows. One sometimes calculates flows in which $\mathscr{F}$ is negative. This indicates that the assumed direction of the flow is incorrect; for the assumed conditions at the inlet and outlet locations the flow is thermodynamically possible only in the opposite direction. On the other hand, frictionless flows are reversible; any flow described by B.E. in which $\mathscr{F}$ is zero could be reversed in direction without any change in magnitude of the velocities, pressures, elevations, etc.

Since for all real flows $\mathscr{F}$ is positive, the effect in Eq. 5.5 with a minus sign before $\mathscr{F}$ is to indicate that friction causes a decrease in pressure or a decrease in elevation or a decrease in velocity or a decrease in the work that can be extracted by a turbine or an increase in the work that must be put in by a pump or some combination of these effects.

In Eq. 5.5 we now have only terms that can be measured mechanically; we have eliminated the Q and u terms, which require thermal measurements. Therefore, this equation, the working form of B.E., is often referred to as the *mechanical-energy balance*. Mechanical energy is conserved only if we include an "energy destruction" term $\mathscr{F}$. This equation has the same restrictions as Eq. 5.1 and, in addition, the restriction that the effects of changes in density are negligible.

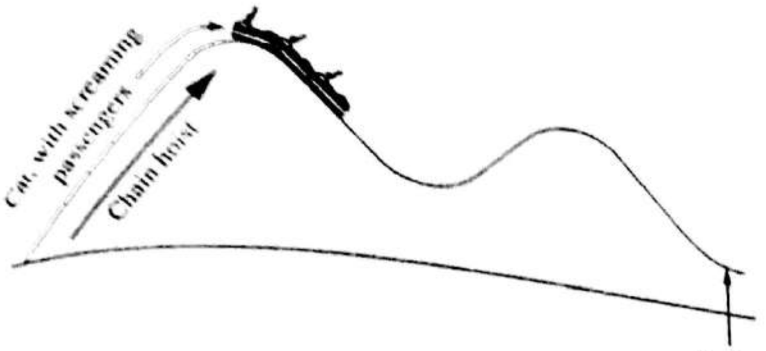


FIGURE 5.2
A simple roller coaster, which illustrates four of the five terms in Bernoulli's equation.

In most applications we will be dealing with flow in a pipe or channel and will assume that the fluid velocity is constant across a given cross section perpendicular to the flow. This approximation is excellent for most engineering problems (see Table 3.1); one interesting exception is discussed in Sec. 5.11.

B.E. deals with the conversion of one kind of energy to another. These changes are illustrated in a common roller coaster, Fig. 5.2. At the left, the car with passengers is lifted from ground level to the top of the first hill by a chain hoist driven by an electric motor, which engages teeth on the bottom of the car. For this part of the trip the change in potential energy, Δgz, is equal to the work input ($\Delta W_{\text{n.f.}} / m$). At the top of the first hill, the car disengages from the chain, pauses a moment for the passengers to anticipate what comes next, and then descends to the first valley. In this part of the trip the decrease in potential energy is practically equal to the increase in kinetic energy; at the first valley the car is going very fast. From the first valley to the top of the second hill, the car's kinetic energy decreases as its potential energy increases. The top of the second hill is always somewhat lower than the top of the first hill, because there has been some friction slowing the car, both due to air resistance and due to rolling friction on the track. If there were no friction, the car could go up and down to the same original height forever; with friction, the top of each succeeding high point must be lower than the preceding one. At the end of the ride (which has more than the two hills shown here), brakes on the track slow the car (convert kinetic energy to friction heating), bringing it to a safe stop at the end of the ride. Four of the five terms in B.E. appear in this description of a roller coaster. The fifth, involving pressure, is discussed in Sec. 5.5.

5.3 ZERO FLOW

The basic equation of fluid statics is a limited form of Eq. 5.5. If we apply Eq. 5.1 between any two points in a fluid flow in which the velocities are slowly becoming zero, then there will be no work or friction and the kinetic energy terms will approach zero so that

$$\Delta\left(\frac{P}{\rho} + gz\right) = 0 \qquad \text{[zero flow]} \tag{5.C}$$

Rearranging, we find

$$\frac{1}{\rho}\Delta P = -g\,\Delta z \tag{5.D}$$

or

$$\lim_{\Delta z \to 0} \frac{\Delta P}{\Delta z} = \frac{dP}{dz} = -\rho g \tag{2.1}$$

which is the basic equation of fluid statics. It is found in Chap. 2 by making a force balance around an elemental particle of fluid. The derivation shown here points out only that Eq. 5.5 is general enough to include cases of zero flow.

5.4 THE HEAD FORM OF BERNOULLI'S EQUATION

In many problems, particularly those involving flow of water in dams, canals, and open channels, it is convenient to divide both sides of Eq. 5.5 by g to find

$$\Delta\left(\frac{P}{\rho g} + z + \frac{V^2}{2g}\right) = \frac{dW_{\text{n.f.}}}{g\,dm} - \frac{\mathscr{F}}{g} \tag{5.6}$$

which is called the *head form* of B.E.

Every term in Eq. 5.6 has the dimension of length. The lengths are at least conceptually convertible into elevation Δz above some datum plane. These elevations are commonly referred to as "heads." ("Head" is apparently a variant spelling and pronunciation of "height.") Thus, we would refer to the various terms in Eq. 5.6 as the *pressure head, gravity head, velocity head, pump or turbine head,* and *friction head loss.* One occasionally sees the terms *static head,* which is the sum of the pressure and gravity heads, and *dynamic head,* which is the sum of the static head and the velocity head.

There is no simple, universal rule for deciding when to use the head form of B.E. and when to use the energy form, Eq. 5.5; if correctly applied, both give the same result. Through practice engineers learn which is the most convenient for a given problem. Civil engineers use the head form much more than do chemical engineers; but the terms velocity head and pump head occur often in chemical engineering.

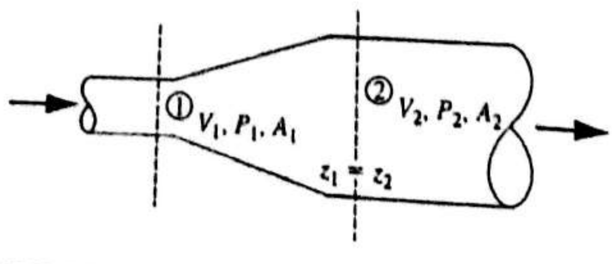


FIGURE 5.3
A simple diffuser in which a fluid flow is slowed in an orderly fashion.

5.5 DIFFUSERS AND SUDDEN EXPANSIONS

In the following sections we will see several examples of flow in which a moving fluid is slowed to a stop. Here we consider two ways of slowing down a fluid: a diffuser and a sudden expansion. A diffuser is a gradually expanding pipe or duct, as sketched in Fig. 5.3.

Writing B.E. for the pipe between locations 1 and 2, both at the same elevation, we find

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} = -\mathcal{F} \tag{5.E}$$

From the mass balance for a constant-density fluid we have

$$V_2 = \frac{V_1 A_1}{A_2} \tag{5.F}$$

$V_2 A_2 = V_1 A_1$

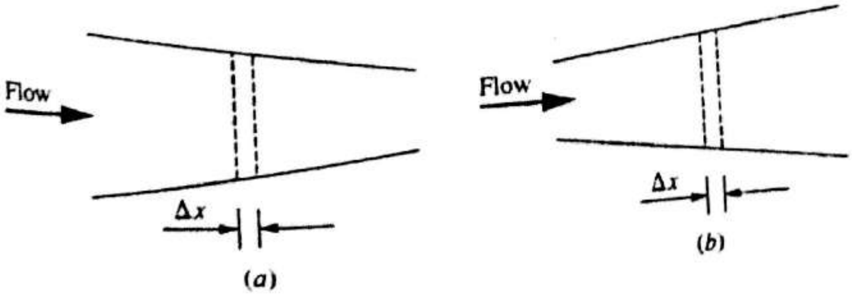
and, substituting for V_2 in Eq. 5.E, we find

$$P_2 - P_1 = \rho \frac{V_1^2}{2}\left(1 - \frac{A_1^2}{A_2^2}\right) - \rho\mathcal{F} \tag{5.7}$$

This increase in pressure that accompanies the decrease in velocity is often called *pressure recovery*. In such a device kinetic energy is converted partly into injection work (shown by an increase in pressure) and partly into friction heating.

Students find it hard to visualize why the pressure increases as the fluid slows down in steady flow. First, observe that in a constant-density steady flow the velocity can only change from one point to another if the cross-sectional area of the flow changes. In a constant-cross-section area pipe or duct, in steady flow, the velocity is the same at each downstream location. Figure 5.4 shows two types of flow channel, one of which contracts in the flow direction, the other of which expands in the flow direction.

The left part of Fig. 5.4 is the common garden-hose nozzle with which the reader is familiar. In it the cross-sectional area decreases in the flow direction, and the velocity increases. Most students have observed that behavior; the slow-moving flow in the garden hose is converted to the much-faster moving jet of water by the nozzle. If we consider the small section marked Δx, we see that the fluid in it must be accelerating, From Newton's second law we know that $F = ma$; and if the acceleration is in the flow direction, then there must be a net force acting on this slice of fluid, in the flow direction. The only forces acting are the pressure forces, which act on the slice from behind and from in front (we ignore the small shear forces at the walls of the duct). For the algebraic sum of these forces to point in the flow direction (which must occur if the flow is accelerating), the downstream pressure must be less than the upstream pressure. The right part of Fig. 5.4 is obviously the mirror



1- Diffusers
2- Pitot Tube
3- Pitot static tube
4- Venturi meter
5- orifice meter
6- Rotameter

FIGURE 5.4
Two flow channels, one of which, (*a*), contracts in the flow direction, the other of which, (*b*), expands in the flow direction.

image of the left. Students have seldom experienced this flow—it has no common example like a garden hose nozzle. It is called a *diffuser* and appears in various industrial devices. In it the flow is slowing down, because the cross-sectional area perpendicular to the flow is increasing. If we apply the $F = ma$ discussion to it, we see that it is the same, with the words upstream and downstream and increasing and decreasing interchanged. The pressure must *increase* in the flow direction across the section marked Δx for there to be a net force that decelerates the fluid.

Students have experienced the increase in velocity in rolling down a hill on bicycles or skates and the decrease in velocity in rolling up a hill. The gz and $V^2/2$ terms in B.E. show that interaction. The preceding paragraph shows that there is an analogous behavior as fluid flows up or down a *pressure hill;* down the pressure hill, the fluid speeds up (in frictionless flow) and up the hill, the fluid slows down. The P/ρ and $V^2/2$ terms in B.E. show that behavior.

It is possible to build diffusers in which friction heating is only about one-tenth of the decrease in kinetic energy; or, as is commonly stated, the *pressure recovery* is about 90 percent of the maximum possible from a frictionless diffuser.

Now consider a fluid flowing through a duct into a large tank of fluid with no net velocity, as shown in Fig. 5.5. This is called a *sudden expansion.* Here point 2 is chosen far away from the fluid inlet, so that the velocity at point 2 is negligible. Writing B.E. between points 1 and 2, we find

$$P_2 - P_1 = \frac{\rho V_1^2}{2} - \rho \mathcal{F} \tag{5.G}$$

which is quite similar to Eq. 5.7. Here, however, the friction term is much larger than that for the diffuser, because instead of the fluid being brought to rest in an orderly fashion it is stopped by a chaotic mass of eddies, which convert all its kinetic energy into internal energy. Thus, it is an experimental observation that for such sudden expansions the friction heating per unit mass is almost exactly equal to the decease in kinetic energy per unit mass, and there is no pressure recovery at all. Therefore, the pressure of a fluid flowing into such a sudden expansion is the same as the pressure of the fluid into which it flows. This conclusion is limited to flows with velocities less than the speed of sound; it does not apply to sonic or supersonic flows, which we will discuss in Chap. 8.

These two ways of stopping a fluid are analogous to stopping a fast-moving auto by letting it run up a hill and thereby converting its kinetic energy into useful potential energy and to stopping it with its brakes and thereby converting its kinetic energy into useless internal energy in the brakes. Most students have ridden on roller coasters and hence are comfortable with the idea of converting from potential energy (at the top of the roller coaster) to kinetic energy (at the first valley) and then back to potential energy again at the top of the next rise. They are less used to the idea of a "pressure hill," but from B.E. we see that a rapidly moving fluid stream can convert its kinetic energy to potential energy by climbing a gravity hill, or into injection work by climbing a "pressure hill," or into internal energy by friction heating.

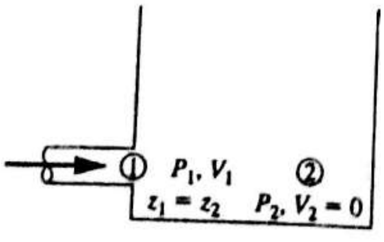


FIGURE 5.5
A sudden expansion in which a fluid flow is slowed in a chaotic fashion.

5.6 B.E. FOR GASES

B.E., as we have written it, is exactly correct for constant-density fluids and practically correct for all flows in which the density changes are unimportant. For liquids this includes almost all steady flows. We show here that it also is practically correct for low-velocity gas flows.

Example 5.2. The tank in Fig. 5.6 is full of air at 68°F = 20°C. The air is flowing out at a steady rate through a smooth, frictionless nozzle to the atmosphere. What is the flow velocity for various tank pressures?

At point 1 the velocity is negligible and, as discussed in Sec. 5.5, the pressure at point 2 is equal to the local atmospheric pressure, if the flow is subsonic. Making these insertions in B.E., without friction, taking 1 to be in the tank away from the nozzle, and 2 to be in the jet, just outside the nozzle, we find

$$V_2 = \left[\frac{2(P_1 - P_{\text{atm}})}{\rho}\right]^{1/2} \tag{5.8}$$

Which value of the density should we use here? It is obviously different at the two states, because the pressure is not the same at the two states. However, if the pressure change is small, the two densities will be practically the same. Let us use the upstream density (but see Prob. 5.5). This will be given by substituting the ideal gas law, $\rho = MP_1 / RT_1$ in Eq. 5.8.

$$V_2 = \left[\frac{2RT_1}{P_1 M}(P_1 - P_{\text{atm}})\right]^{1/2} \tag{5.9}$$

Using this equation, we can calculate V_2 for various values of P_1. For example, if P_1 is (P_{atm} + 0.01 psig), then

$$V_2 = \left[\frac{2 \cdot (10.73 \text{ psi} \cdot \text{ft}^3 / {}^\circ\text{R} \cdot \text{lbmol}) \cdot 528^\circ\text{R}}{(14.71 \text{ psi})(29 \text{ lbm / lbmol})} \cdot 0.01 \frac{\text{lbf}}{\text{in}^2} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} \cdot \frac{32.2 \text{ lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right]^{1/2}$$

$$= \left[1231 \frac{\text{ft}^2}{\text{s}^2}\right]^{1/2} = 35 \frac{\text{ft}}{\text{s}} = 10.7 \frac{\text{m}}{\text{s}} \tag{5.H}$$

Equation 5.9 is based on the assumption of a constant-density fluid, which is not exactly correct here; the exactly correct result for this system, taking gas expansion into account, is developed in Chap. 8. The velocities calculated from Eq. 5.9 and the correct solution from Chap. 8 are compared in Table 5.1. ■

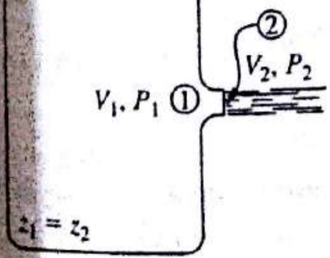


FIGURE 5.6
A gas flowing through a nozzle driven by a modest pressure difference.

From the values in Table 5.1, it is clear that to assume that gas flows are incompressible and are described by B.E. causes a very small error at gas velocities below about 200 ft / s. Even at a velocity of 700 ft / s (213 m / s) the error caused by assuming incompressible flow is only about 5%. The rightmost column in Table 5.1 shows why the answers from simple B.E. and the high-velocity calculations in Chap. 8 differ. Using the methods in Chap. 8, we

TABLE 5.1
Comparison of Bernoulli's equation and high-velocity gas flow equations for the flow in Fig. 5.6

$(P_1 - P_2)$, psia	V_2 from Eq. 5.9, ft / s	V_2 from Chap. 8, ft / s	T_2 from Chap. 8, °F
0.01	35	35	67.9
0.1	110	111	67.0
0.3	190	191	65.6
0.6	267	268	62.0
1	340	343	58.2
2	466	476	49.1
3	554	572	40.7
5	678	713	25.6

see that as the velocity increases, the gas temperature falls; for the lowest row in the table the temperature is 42°F less than the starting temperature. In a high-velocity gas flow the gas can convert some of its internal energy to kinetic energy, so the velocity will be higher and the temperature lower than those we calculate using the constant density assumption in B.E.

Most air-conditioning and low-speed aircraft problems involve velocities below 200 ft / s (61 m / s), so these problems can be solved with engineering accuracy by B.E. On the other hand, where there are significant pressure changes for gases in flow, which lead to high velocities, the density changes must be taken into account, as shown in Chap. 8. Observe also the very high velocities caused by very small pressure differences acting on gases. The inverse of this observation is that for ordinary flow velocities the pressure differences in gases are at least an order of magnitude smaller than for the corresponding flow velocities in liquids. The reason is that the pressure appears in this type of calculation only as $(\Delta P / \rho)$ and for gases ρ is typically about 1 / 800 the value for liquids.

Application of B.E. to a simple, horizontal pump or compressor with equal-sized inlet and outlet pipes (so that there is no velocity change) leads to

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{\Delta P}{\rho} + \mathscr{F} \tag{5.10}$$

If we ignore friction, this equation becomes

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{\Delta P}{\rho} \qquad \begin{bmatrix}\text{frictionless pump or compressor,}\\ \text{constant-density fluid}\end{bmatrix} \tag{5.11}$$

Real pumps and compressors are never frictionless. We normally define the efficiency of a pump or compressor as the ratio of this frictionless work requirement to the work actually needed to drive the pump or compressor. Equation 5.11 is the exactly correct frictionless work requirement for constant-density fluids and is practically correct for most pumps that are pumping real liquids. For gases, whose density will change in a compressor, it is not exactly correct. The correct result, taking the change of gas density into account, is developed in Chap. 10. However, if the pressure change ΔP is small compared with the inlet pressure P_{in}, then Eq. 5.11 gives a very good estimate of the required frictionless work. For example, if $\Delta P / P$ is 0.1 or less, then the result from Eq. 5.11 is certain to be within 10 percent of the calculated result which takes density changes into account. This pressure range includes most fans, blowers, air-conditioning systems, vacuum cleaners, etc., but does not include air compressors that inflate the tires of our vehicles or that drive paint sprayers or pneumatic tools (see Prob. 5.58).

5.7 TORRICELLI'S EQUATION AND ITS VARIANTS

The most interesting applications of B.E. include the effects of friction. Before we can solve these, we must learn how to evaluate the $\mathcal{F}$ term, which we will do in Chap. 6. However, there are many flow problems in which the friction heating terms are small compared with the other terms and can be neglected. We can solve these by means of B.E. without the friction heating term. A good example of this type of problem is the tank-draining problem, which leads to Torricelli's equation.

Example 5.3. The tank in Fig. 5.7 is full of water and open at the top. There is a frictionless nozzle near the bottom, the diameter of which is small compared with the diameter of the tank. What is the velocity of the flow out of the nozzle?

To solve this problem, we apply Eq. 5.5 between the free surface at the top of the tank, location 1, and the jet of fluid as it leaves the tank, location 2. In addition to the assumptions built into B.E. we make the following:

1. The diameter of the tank is so large that the velocity at the free surface is practically zero, $V_1 \approx 0$.
2. The pressures at locations 1 and 2 are the local atmospheric pressures. The pressure of the atmosphere is not exactly the same at both points, but it is practically the same; so we assume $\Delta P = 0$.
3. There is no friction or external work.
4. Flow is steady; that is, the level at the top of the tank is not falling. This means that fluid must be flowing into the tank somewhere exactly as fast as it flows out at location 2.

Subject to these restrictions, we may write

$$g(z_2 - z_1) + \frac{V_2^2}{2} = 0 \qquad (5.I)$$

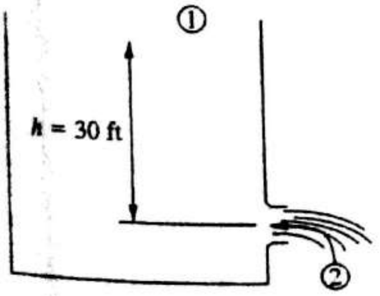


FIGURE 5.7
The flow described by Torricelli's equation.

Here $z_2 - z_1 = -h$, so

$$V_2 = (2gh)^{1/2} \quad \text{[Torricelli's equation]} \qquad (5.12)$$

Torricelli's equation says that the fluid velocity is exactly the same as the velocity the fluid would attain by falling freely from rest a distance h. Substituting the numerical values, we find

$$V_2 = \left(2 \cdot 32.2\,\frac{\text{ft}}{\text{s}^2} \cdot 30\ \text{ft}\right)^{1/2} = 43.9\,\frac{\text{ft}}{\text{s}} = 13.4\,\frac{\text{m}}{\text{s}} \qquad (5.J)$$

■

This is the classic tank-draining solution. It is correct only for situations in which the assumptions made in finding Torricelli's equation apply; in Examples 5.4 and 5.5 we examine some situations in which they may not apply.

Example 5.4. Repeat Example 5.3, making the area of the outlet nozzle 1 ft^2 and the cross-sectional area of the tank 4 ft^2.

In this case we cannot assume that the velocity at the free surface is zero, as we did in Example 5.3, so

$$g(-h) + \frac{V_2^2 - V_1^2}{2} = 0 \tag{5.K}$$

Using the mass balance for a constant-density fluid, we can solve for V_1 in terms of V_2, A_1, and A_2 and substitute for V_1, finding

$$g(-h) + \frac{1}{2}\left[V_2^2 - \left(\frac{V_2 A_2}{A_1}\right)^2\right] = 0 \qquad -gh + \frac{V_2^2}{2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = 0 \tag{5.L}$$

$$V_2 = \left[\frac{2gh}{1 - (A_2 / A_1)^2}\right]^{1/2} \tag{5.13}$$

Inserting the numerical values, we find

$$V_2 = \left(\frac{2 \cdot 32.2\ \text{ft}/\text{s}^2 \cdot 30\ \text{ft}}{1 - (1/4)^2}\right)^{1/2} \tag{5.M}$$

This is the answer from Example 5.3, divided by $(15/16)^{1/2}$:

$$V_2 = \frac{43.9\ \text{ft}/\text{s}}{(15/16)^{1/2}} = 45.3\ \frac{\text{ft}}{\text{s}} = 13.8\ \frac{\text{m}}{\text{s}} \tag{5.N}$$

■

Why does the water flow faster in this case? All the water in the tank has measurable kinetic energy; it is flowing down at a velocity of 11.4 ft / s. In Example 5.3 the water in the tank has immeasurably small kinetic energy.

What happens in Example 5.4 if the cross-sectional area of the tank is equal to the cross-sectional area of the outlet, that is, if $A_2 = A_1$? If we substitute this in Eq. 5.13, it predicts an infinite velocity! Therefore, Eq. 5.13 does not describe this situation. Recall the assumptions that went into that equation. First, there is the B.E. assumption of steady flow. Second, there is the assumption in Eqs. 5.12 and 5.13 that friction is negligible. Suppose we have a vertical pipe of constant cross-sectional area and steady flow downward. Suppose also that the pressure gauges at two different elevations read the same value. Then this situation is analogous to that in Example 5.4, with $A_1 = A_2$. Returning to Eq. 5.5, we see that the only terms that can be significant are

$$g(z_2 - z_1) = -\mathcal{F} \tag{5.O}$$

This situation, in which the friction forces are dominant, is quite different from the situation shown in Fig. 5.7, from which we found Torricelli's equation, and is not covered by the frictionless assumption of Torricelli's equation.

Example 5.5. Repeat Example 5.3, making the tank contain carbon dioxide gas at the same temperature and pressure as the surrounding atmosphere.

This looks strange: an open tank full of gas! However, it is easy to demonstrate in the laboratory or kitchen by mixing a little bicarbonate of soda and vinegar in a cup. When the bubbling has stopped, the cup will be filled with carbon dioxide gas. This gas is heavier than air and can be poured from cup to cup, visibly. However, it mixes slowly with the air and ultimately will disperse by diffusion.

Returning to the problem, it appear at first to be the same as Example 5.3. However, there is a big difference, namely, we cannot ignore the difference in atmospheric pressure between locations 1 and 2. The other assumptions for Example 5.3 appear sound, so B.E. becomes

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^2}{2} = 0 \tag{5.P}$$

From the basic equation of fluid statics we can calculate

$$P_2 - P_1 = -\rho_{\text{air}} g (z_2 - z_1) \tag{5.Q}$$

Now we must be careful, because there are two densities in our problem: the ρ_{air} shown here and the ρ in B.E. If we follow the derivation of B.E. back to its source, we see that the ρ in it is the ρ of the fluid that is flowing; we label it ρ_{CO_2}. Combining these two equations, we find

$$\frac{-\rho_{\text{air}} g (z_2 - z_1)}{\rho_{CO_2}} + g(z_2 - z_1) + \frac{V_2^2}{2} = 0;$$

$$0 = \frac{V_2^2}{2} + g(z_2 - z_1)\left(1 - \frac{\rho_{\text{air}}}{\rho_{CO_2}}\right) \tag{5.R}$$

Solving for V_2, we find

$$V_2 = \left[2gh\left(1 - \frac{\rho_{\text{air}}}{\rho_{CO_2}}\right)\right]^{1/2} \tag{5.14}$$

If we assume the air and carbon dioxide behave as ideal gases and are at the same temperature and pressure, their densities are proportional to their molecular weights, 29 and 44 g / mol, respectively, so

$$V_2 = \left(2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 30 \text{ ft}\right)^{1/2} \cdot \left(1 - \frac{29}{44}\right)^{1/2} = 43.9 \frac{\text{ft}}{\text{s}} \cdot 0.34^{1/2}$$

$$= 25.6 \frac{\text{ft}}{\text{s}} = 7.8 \frac{\text{m}}{\text{s}} \tag{5.S}$$

■

If the difference in atmospheric pressure is important in Example 5.5, is it also important in Example 5.3? Equation 5.14 applies as well to Example 5.3 as it does to Example 5.5. Therefore, if we want to take the effect of the difference in atmospheric pressure into account in Example 5.3, we should use Eq. 5.14. This is equivalent to multiplying the answer in Example 5.3 by $(1 - \rho_{\text{air}} / \rho_{\text{water}})^{1/2}$. For water and air at normal temperature and pressure, this is about

$$\left(1 - \frac{\rho_{\text{air}}}{\rho_{\text{water}}}\right)^{1/2} = \left(1 - \frac{0.075 \text{ lbm / ft}^3}{62.3 \text{ lbm / ft}^3}\right)^{1/2} = (0.9988)^{1/2} = 0.9994 \tag{5.T}$$

Ignoring the change in atmospheric pressure in Torricelli's equation for air and water makes an error of ≈0.06 percent (much less than the error introduced by some of the other assumptions). We are justified in leaving this term out if the ratio of the density of the surrounding fluid to that of the flowing fluid, $\rho_{\text{surrounding fluid}} / \rho_{\text{moving fluid}}$, is much less than 1. This is true in most hydraulics problems but not in two-liquids problems (Probs. 5.15 and 5.16).

We will discuss one more variant of Torricelli's equation in Sec. 5.10.

5.8 B.E. FOR FLUID-FLOW MEASUREMENT

Several important types of fluid-flow measuring devices are based on the frictionless form of B.E. Where the friction effects in these devices become significant, they are normally accounted for by introducing empirical coefficients and retaining the frictionless form of B.E., rather than by introducing the friction term into B.E. Thus, we consider these devices before we discuss the friction term in B.E., even though these devices obviously involve some friction. These devices have been in common use for at least 100 years. Modern electronics and computers have made possible other types of flow-measuring devices not based on B.E. The devices described here are still more widely used than the computer-electronic ones, because the B.E. devices are simple, reliable, and cheap.

5.8.1 Pitot Tube

The simplest *pitot tube* (H. Pitot, 1695–1771) is sketched in Fig. 5.8. This is sometimes called an *impact tube* or *stagnation tube.* It consists of a bent, transparent tube with one vertical leg projecting out of the flow and another leg pointing directly upstream in the flow.

At location 1 the flow is practically undisturbed by the presence of the tube and hence has the velocity that would exist at location 2 if the tube were not present. At location 2 the flow has been completely stopped by the tube that has been inserted, so $V_2 = 0$. Writing B.E. between locations 1 and 2, we find

$$\frac{P_2 - P_1}{\rho} - \frac{V_1^2}{2} = -\mathcal{F} \tag{5.U}$$

But inside the pitot tube the fluid is not moving, so the pressure at location 2 is given by

$$P_2 = P_{\text{atm}} + \rho g(h_1 + h_2) \tag{5.V}$$

If all the fluid flow is in the horizontal direction, then the basic equation of fluid statics can be used to find the vertical change in pressure with depth inside the flow, so that

$$P_1 = P_{\text{atm}} + \rho g h_2 \tag{5.W}$$

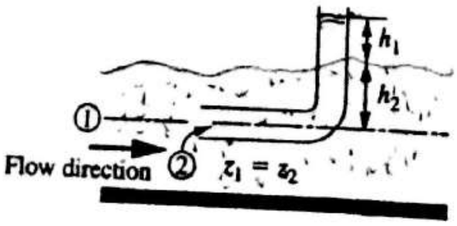


FIGURE 5.8
Pitot tube for fluid velocity measurement.

Substituting Eqs. 5.V and 5.W in Eq. 5.U and rearranging, we find

$$V_1 = (2gh_1 + 2\mathcal{F})^{1/2} \tag{5.X}$$

It has been found experimentally that the friction heating term in Eq. 5.X is normally less tha

1 percent of the total; it may be ignored, giving

$$V_1 = (2gh_1)^{1/2} \quad \text{[pitot tube]} \tag{5.15}$$

The pitot tube allows one to measure a liquid height (a very easy thing to measure) and calculate a velocity from it by means of B.E. The device, exactly as shown in Fig. 5.8, is used for finding velocities at various points in open-channel flow and for determining the velocities of boats.

Example 5.6. A pitot tube exactly as shown in Fig. 5.8 is used for measuring the velocity of a sailboat. When the water level in the tube is 1 m above the water surface, how fast is the boat going?

$$V_1 = \left(2 \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot 1 \text{ m}\right)^{1/2} = \left(19.62 \frac{\text{m}^2}{\text{s}^2}\right)^{1/2} = 4.43 \frac{\text{m}}{\text{s}} = 14.5 \frac{\text{ft}}{\text{s}} \tag{5.Y}$$

■

5.8.2 Pitot-Static Tube

The pitot tube shown in Fig. 5.8 is suitable for liquid open-channel flow but not for flow of the atmosphere or flow in pipes. For the latter two uses, it is combined with a second tube, called a *static tube,* shown in Fig. 5.9. This is the most common type, with the pitot (or impact) tube inside the surrounding static tube. This combination is often simply called a pitot tube.

As the figure shows, the tube that faces the flow is the high-pressure side, whereas the surrounding tube that has openings perpendicular to the flow is the low-pressure side. These two are connected to opposite sides of some appropriate pressure-difference measuring device. Experimental tests have shown that for a well-designed

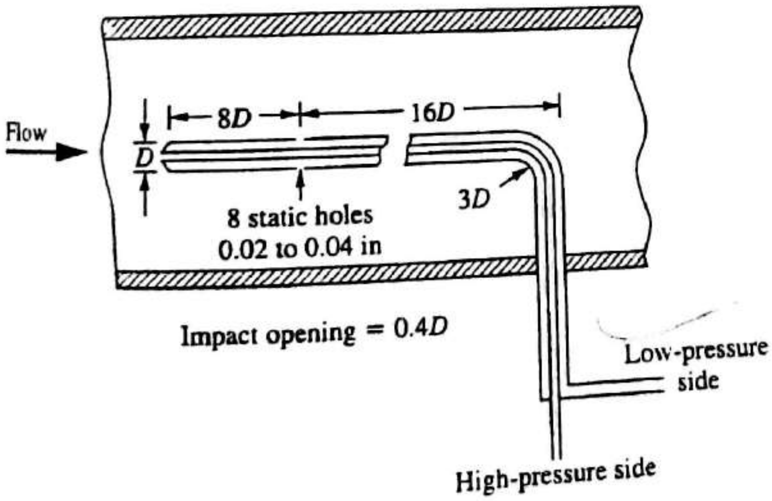


FIGURE 5.9
Pitot-static tube for fluid velocity measurement. The "low pressure side" and "high pressure side" are connected to some appropriate pressure-difference measuring device. The dimensions shown are typical of those on the devices used to measure stack velocities in air pollution sampling. The pitot-static tubes used in aircraft are similar in concept but somewhat different in dimensions. They are often heated to prevent ice-plugging.

pitot-static tube the friction effect is negligible, so we may read the pressure difference from the meter and calculate the velocity from Eq. 5.U rearranged:

$$V_1 = \left(\frac{2\Delta P}{\rho}\right)^{1/2} \quad \text{[pitot-static tube]} \tag{5.16}$$

Example 5.7. Air is flowing in the duct in Fig. 5.9. The pressure-difference gauge attached to the pitot-static tube indicates a difference of 0.05 psi. What is the air velocity?

$$V = \left(\frac{2 \cdot 0.05 \text{ lbf / in}^2}{0.075 \text{ lbm / ft}^3} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} \cdot \frac{32.2 \text{ lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2} = 78.6\,\frac{\text{ft}}{\text{s}} = 23.9\,\frac{\text{m}}{\text{s}} \tag{5.Z}$$

■

The pitot-static tube is the standard device for measuring the air speed of airplanes and is often used for measuring the local velocity in pipes or ducts, particularly in air pollution sampling procedures. One can easily identify the pitot-static probes on airplanes. Multiengine planes have them near the nose, at the side below the pilot's window. Single-engine propeller planes place the probe below the wing, far enough out from the center not to be influenced by the propeller. Look for these the next time you are at the airport! For measuring flow in enclosed ducts or channels, the venturi meter and orifice meters discussed below are more convenient and more frequently used.

5.8.3 Venturi Meter

Figure 5.10 shows a horizontal *venturi meter* (G. Venturi, 1746–1822). It consists of a truncated cone in which the cross-sectional area perpendicular to flow decreases, a short cylindrical section, and a truncated cone in which the cross-sectional area increases to its original value. There are pressure taps both upstream and in the short cylindrical section (the "throat"); they are connected to some pressure-difference-measuring device, usually a manometer. Applying B.E. between locations 1 and 2, we find

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} = -\mathcal{F} \tag{5.AA}$$

The friction in these devices is normally small, so the $\mathcal{F}$ term is dropped. Using the mass balance for a constant-density fluid, we can write V_1 in terms of V_2, A_2, and A_1.

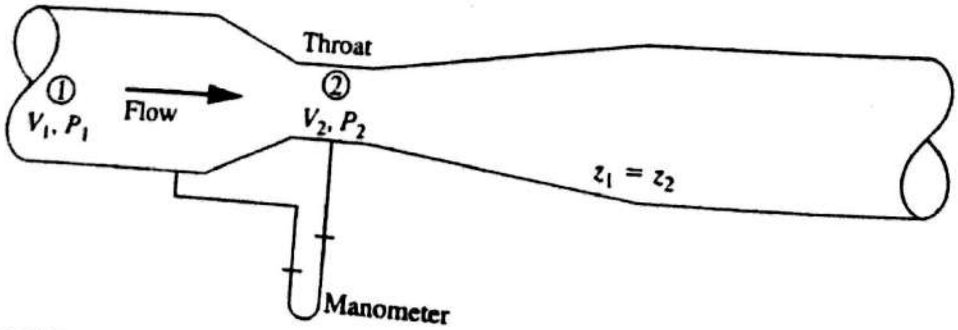


FIGURE 5.10
Venturi meter for fluid velocity measurement.

Substituting in Eq. 5.AA and rearranging, we find

$$V_2 = \left[\frac{2(P_1 - P_2)/\rho}{1 - (A_2^2/A_1^2)}\right]^{1/2} \qquad \text{[venturi meter]} \qquad (5.17)$$

Example 5.8. The venturi meter in Fig. 5.10 has water flowing through it. The pressure difference $P_1 - P_2$ is 1 psi. The diameter at point 1 is 1 ft, and that at point 2 is 0.5 ft. What is the volumetric flow rate through this meter?

From Eq. 5.17,

$$V_2 = \frac{\left(\dfrac{2 \cdot 1\ \text{lbf}/\text{in}^2}{62.3\ \text{lbm}/\text{ft}^3} \cdot \dfrac{144\ \text{in}^2}{\text{ft}^2} \cdot \dfrac{32.2\ \text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2}}{\{1 - [(\pi/4)(0.5\ \text{ft})^2]^2/[(\pi/4)(1\ \text{ft})^2]^2\}^{1/2}} = 12.7\ \frac{\text{ft}}{\text{s}} = 3.9\ \frac{\text{m}}{\text{s}} \qquad (5.\text{AB})$$

The volumetric flow rate is

$$Q = V_2 A_2 = 12.7\ \frac{\text{ft}}{\text{s}} \cdot \frac{\pi}{4}(0.5\ \text{ft})^2 = 2.49\ \frac{\text{ft}^3}{\text{s}} = 0.070\ \frac{\text{m}^3}{\text{s}} \qquad (5.\text{AC})$$

■

It is found experimentally that the flow rate calculated from Eq. 5.17 is slightly higher than that actually observed. This is due partly to friction heating in the meter, which we have assumed to be zero, partly to the fact that the flow is not entirely uniform across any cross section of the pipe, and partly to the fact that the flow is not perfectly one-dimensional, as we have also tacitly assumed. One could attempt to account for these differences by using a more complicated formula than Eq. 5.17; however, the more common approach is to introduce an empirical coefficient into Eq. 5.17, called *the coefficient of discharge*, C_v:

$$V_2 = C_v\left[\frac{2(P_1 - P_2)/\rho}{1 - (A_2^2/A_1^2)}\right]^{1/2} \qquad (5.18)$$

A large number of experimental tests have shown that C_v depends only on the Reynolds number, a dimensionless group whose significance will be discussed in Chaps. 6 and 9; these results are summarized in Fig. 5.11.

Example 5.9. Rework Example 5.8, taking into account the experimental results summarized in Fig. 5.11.

This requires a trial-and-error solution because, to calculate V, we need to know C_v, which is a function of V. The procedure is as follows.

1. Assume $V = V_{\text{Ex. 5.8}} = 12.7$ ft / s.
2. Compute the Reynolds number, $\mathscr{R}$ at point 1.

$$\mathscr{R}_1 = \frac{V_1 D_1 \rho}{\mu} = \frac{V_2(A_2/A_1)D_1\rho}{\mu}$$
$$= \frac{(12.7\ \text{ft}/\text{s}/4) \cdot 1\ \text{ft} \cdot 62.3\ \text{lbm}/\text{ft}^3}{1\ \text{cP} \cdot 6.72 \cdot 10^{-4}\ \text{lbm}/\text{ft} \cdot \text{s} \cdot \text{cP}} = 2.9 \cdot 10^5 \qquad (5.\text{AD})$$

3. On Fig. 5.11 we read $C_v = 0.984$.

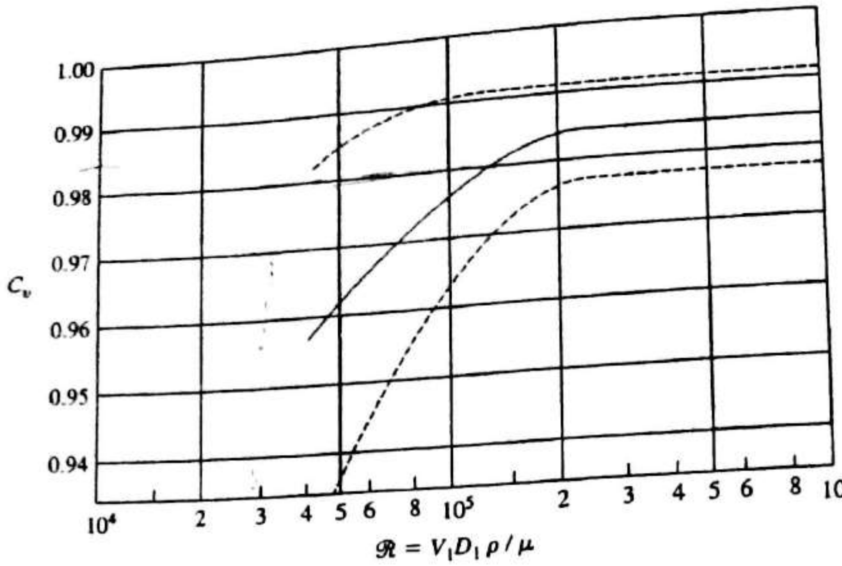


FIGURE 5.11
Discharge coefficients for venturi meters. Here velocities and diameters, V_1 and D_1 are measured at point 1 in Fig. 5.10. The solid line represents the best average of the available data; the dotted lines represent the range of the scatter in the experimental data. (From *Fluid Meters, Their Theory and Practice*, 5th ed., ASME, New York, 1959. Reproduced with permission of the publisher.)

4. $V_{\text{revised}} = 0.984 \cdot 12.7\,\frac{\text{ft}}{\text{s}} = 12.5\,\frac{\text{ft}}{\text{s}}.$ (5.AE)

5. We should now repeat steps 2 and 3, using this revised value of V. However, in comparing these, we ask, "How much would C_v be changed by using $V_{\text{revised}} = 12.5$ ft / s in calculating the Reynolds number (step 2) and then using a new value of C_v?" Clearly, because of the shape of Fig. 5.11 this would cause a negligible change; so a revised C_v would be the same, and we accept $V = 12.5$ ft / s as a satisfactory estimate of the velocity. Then

$$Q = 12.5\,\frac{\text{ft}}{\text{s}} \cdot \frac{\pi}{4}(0.5\text{ ft})^2 = 2.45\,\frac{\text{ft}^3}{\text{s}} = 0.069\,\frac{\text{m}^3}{\text{s}} \qquad (5.\text{AF})$$

If the velocity had been much lower, not corresponding to the horizontal part of the curve in Fig. 5.11, this trial-and-error solution probably would have taken several steps; normally these meters are designed to operate at high velocities, on the right-hand side of Fig. 5.11, so that this trial and error is very simple. ■

The foregoing is all based on a horizontal venturi meter. If we use the setup shown in Fig. 5.12 and take the manometer reading as a pressure difference to get our value of $(P_1 - P_2)$ in Eq. 5.18, then the result is quite independent of the angle to the vertical of the venturi meter. The reason is that the elevation change in the meter is compensated by the elevation change in the manometer legs. Consider the venturi meter in Fig. 5.12.

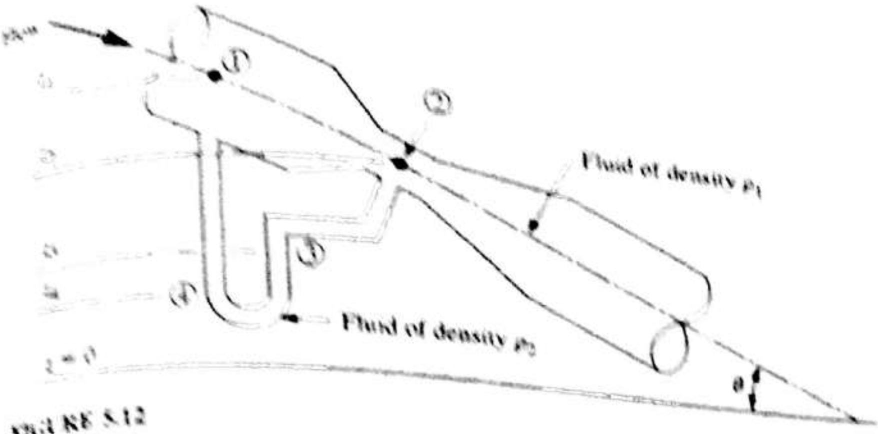


FIGURE 5.12
An inclined venturi meter, with the pressure difference measured by a manometer.

Applying B.E. between points 1 and 2 on this figure and solving for $V_2(1 - A_2^2/A_1^2)^{1/2}$ gives

$$V_2\left(1 - \frac{A_2^2}{A_1^2}\right)^{1/2} = \left[\frac{2(P_1 - P_2)}{\rho} + 2g(z_1 - z_2)\right]^{1/2} \tag{5.AG}$$

To solve for $(P_1 - P_2)$, let us call P_2 known and work our way through the manometer step by step:

$$P_3 = P_2 + \rho_1 g(z_2 - z_3) \tag{5.AH}$$

$$P_4 = P_3 + \rho_2 g(z_3 - z_4) \tag{5.AI}$$

$$P_1 = P_4 - \rho_1 g(z_1 - z_4) \tag{5.AJ}$$

Adding these equations and canceling like terms, we find

$$P_1 = P_2 + \rho_1 g[(z_2 - z_1) - (z_3 - z_4)] + \rho_2 g(z_3 - z_4) \tag{5.AK}$$

$$P_1 - P_2 = -\rho_1 g(z_1 - z_2) + g(z_3 - z_4)(\rho_2 - \rho_1) \tag{5.AL}$$

Substituting this in Eq. 5.18, we see that the elevation $(z_1 - z_2)$ does indeed cancel, and we find

$$V_2 = \left[\frac{2g(z_3 - z_4)(\rho_2 - \rho_1)}{\rho_1(1 - A_2^2/A_1^2)}\right]^{1/2} \quad \begin{bmatrix}\text{inclined venturi}\\ \text{meter with manometer}\end{bmatrix} \tag{5.19}$$

But $g(z_3 - z_4)(\rho_2 - \rho_1)$ is precisely the pressure difference we would have calculated for the manometer reading if we had not taken the difference in length of the manometer legs into account. The result found above is true for any angle θ; so we conclude that, if the venturi meter is connected as shown in Fig. 5.12, we can neglect the angle to the vertical and simply use Eq. 5.19 (but see Prob. 5.34).

5.8.4 Orifice Meter

The venturi meter described above is a reliable flow-measuring device. Furthermore, it causes little pressure loss (that is, the actual value of $\mathcal{F}$ is small). For these reasons it is widely used, particularly for large-volume liquid and gas flows. However, the meter

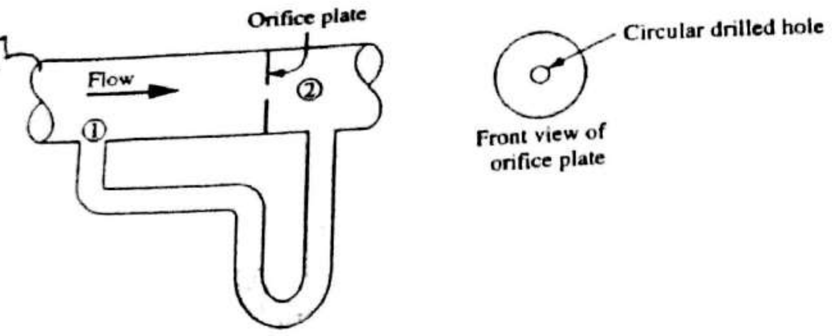


FIGURE 5.13
Orifice meter for fluid velocity measurement.

is relatively complex to construct and hence expensive. For small pipelines, its cost seems prohibitive, so simpler devices have been invented, such as the *orifice meter*.

As shown in Fig. 5.13, the orifice meter consists of a flat orifice plate with a circular hole drilled in it. There is a pressure tap upstream from the orifice plate and another just downstream. If the flow direction is horizontal and we apply B.E., ignoring friction, from point 1 to point 2 in the figure, we find Eq. 5.17, exactly the same equation we found for a venturi meter. However, in this case we cannot so easily assume frictionless flow and uniform flow across any cross section of the pipe as we can in the case of the venturi meter.

As in the case of the venturi meter, experiments indicate that, if we introduce a discharge coefficient and thus form Eq. 5.18, then that coefficient is a fairly simple function of the ratio of the diameter of the orifice hole to the diameter of the pipe, D_2 / D_1, and the Reynolds number; the relation is shown in Fig. 5.14.

Example 5.10. Water is flowing at a velocity of 1 m/s in a pipe 0.4 m in diameter. In the pipe is an orifice with a hole diameter of 0.2 m. What is the measured pressure drop across the orifice?

Rearranging Eq. 5.18, we find

$$\Delta P = \frac{\rho V_2^2}{2C_v^2} \cdot \left(1 - \frac{A_2^2}{A_1^2}\right) = \frac{\rho V_2^2}{2C_v^2}\left(1 - \frac{D_2^4}{D_1^4}\right) \tag{5.AM}$$

From the mass balance for steady flow, we know that

$$V_2 = V_1 \frac{A_1}{A_2} = 1\,\frac{\text{m}}{\text{s}} \cdot \frac{(\pi/4)\cdot(0.4\ \text{m})^2}{(\pi/4)\cdot(0.2\ \text{m})^2} = 4\,\frac{\text{m}}{\text{s}} = 13.1\,\frac{\text{ft}}{\text{s}} \tag{5.AN}$$

The Reynolds number $\mathscr{R}_2$ based on D_2 is calculable and will be found to be about $1.6 \cdot 10^6$; so, from Fig. 5.14, we have $C_v = 0.62$. Hence

$$P_1 - P_2 = \frac{(998.2\ \text{kg/m}^3)\cdot(4\ \text{m/s})^2}{2 \cdot 0.62^2} \cdot (1 - 0.5^4) \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \cdot \frac{\text{Pa}}{\text{N/m}^2}$$
$$= 19.5\ \text{kPa} = 2.83\ \text{psi} \tag{5.AO}$$

■

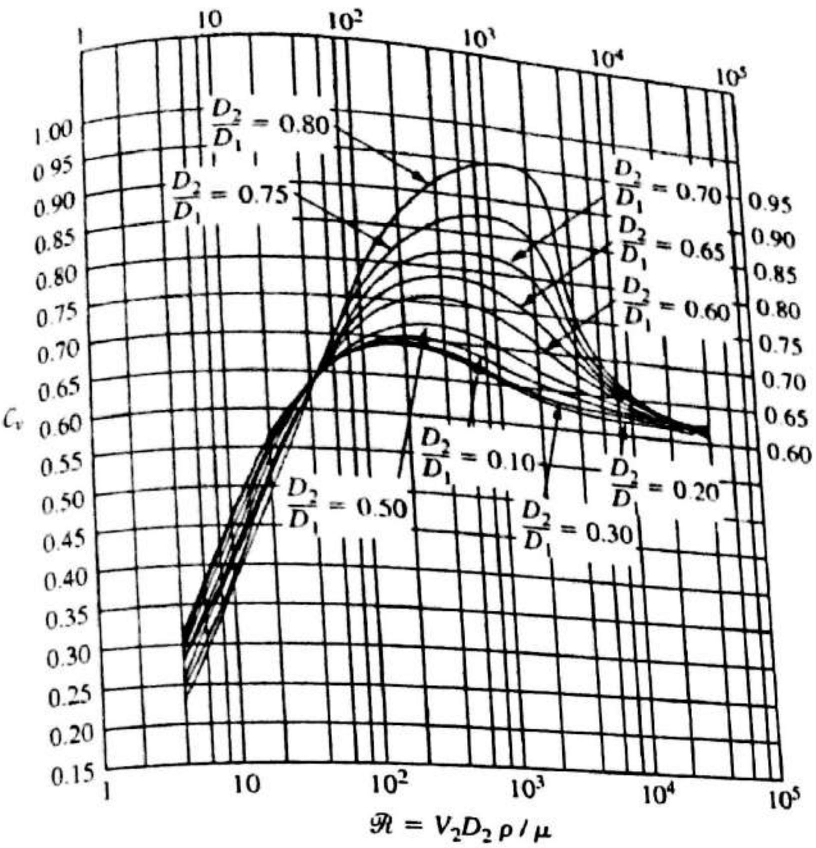


FIGURE 5.14
Discharge coefficients for drilled-plate orifices. (From G. L. Tuve and R. E. Sprenkle, "Orifice discharge coefficients for viscous liquids," *Instruments* 6:201 (1933). Reproduced by permission of the publisher.)

From Fig. 5.14 we see that for small orifice holes ($D_2 / D_1 \leq 0.4$) and high flow rates ($\mathscr{R}_2 > \sim 1000$), C_v is approximately equal to 0.6. These conditions occur in most typical industrial orifice applications, so many practicing engineers automatically write down $C_v = 0.6$ for orifice meters, or for the flow through any simple orifice. In new applications it is best to check Fig. 5.14 to see whether this simplification applies. By the mathematical methods of potential flow (Chap. 16), one may show that an ideal orifice should have $C_v = \pi / (\pi + 2) = 0.611$ [1].

Figure 5.14 is based on a standard location of the upstream and downstream pressure taps. When the taps are in some other location, the value of C_v will be different [2]. In comparison with venturi meters, orifice meters have high pressure losses—high $\mathscr{F}$—and correspondingly high pumping costs, but because they are mechanically simple they are cheap and easy to install. For flows in small-sized pipes, orifice meters are much more common than venturi meters.

The values of C_v in Fig. 5.14 are applicable only to drilled-plate orifices (sometimes called *square-edge orifices*, because the edges of the hole are not rounded). Some other standard types also are used, and sets of C_v curves for these have been published [3].

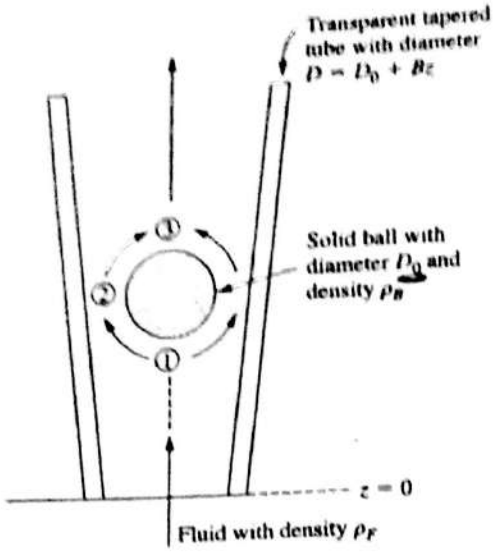


FIGURE 5.15
Rotameter for fluid velocity measurement. (The taper of the tube is exaggerated; real rotameter tubes have a much smaller taper.)

5.8.5 Rotameters

The four previously discussed devices use a fixed geometry and read a pressure difference that is proportional to the square of the volumetric flow rate. A rotameter uses a fixed pressure difference, and a variable geometry, which is a simple function of the volumetric flow rate. Figure 5.15 shows a schematic view of a simple *rotameter*. It consists of a tapered transparent (glass or plastic) tube, in which the fluid whose flow is to be measured flows upward, and an interior float, which may have several shapes, and is shown in Figure 5.15 as a spherical ball.

Suppose the upward flow shown in Fig. 5.15 is steady, so that the ball is not moving, and is fast enough to hold the ball steadily suspended in the flow. If we make a force balance around the ball (positive downward), we find

$$0 = F_{\text{gravity}} + F_{\text{pressure from above}} - F_{\text{buoyancy}} - F_{\text{pressure from below}} \tag{5.AP}$$

If we assume that the pressure below the ball is practically uniform across the ball's lower surface, and similarly for the pressure across the ball's upper surface, and remember from Chap. 2 that the z component of that pressure force will be simply the pressure times the projected area of the ball, we find

$$0 = \frac{\pi}{6} D_0^3 \rho_{\text{ball}} g + P_3 \frac{\pi}{4} D_0^2 - \frac{\pi}{6} D_0^3 \rho_{\text{fluid}} g - P_1 \frac{\pi}{4} D_0^2 \tag{5.AQ}$$

$$\frac{\pi}{6} D_0^3 (\rho_{\text{ball}} - \rho_{\text{fluid}}) g = \frac{\pi}{4} D_0^2 (P_1 - P_3) \tag{5.AR}$$

From B.E., we can find that

$$P_1 - P_2 = \rho_{\text{fluid}} \cdot \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) = \rho_{\text{fluid}} \cdot \frac{V_2^2}{2} \cdot \left(1 - \frac{A_2^2}{A_1^2} \right) \tag{5.AS}$$

But $(A_2 / A_1)^2$ is generally much less than 1, so we can drop it in the last term above. And as discussed previously, the flow from 2 to 3 is a sudden expansion, so that P_3 is very nearly the same as P_2. Making these substitutions in Eq. 5.AS and solving for V_2, we find

$$V_2 = \left[\frac{4 D_0 g}{3} \cdot \frac{\rho_{\text{ball}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}} \right]^{1/2} \quad \text{[rotameter]} \tag{5.20}$$

Thus, applying B.E. (and some judicious assumptions), we find that for a given diameter of the ball and of the densities of ball and fluid, there is only one possible value of V_2 that will keep the ball steadily suspended! That means that for any flow rate, Q

the ball must move to that elevation in the tapered tube where $V_2 = Q / A_2$. But

$$A_2 = \frac{\pi}{4}[(D_0 + Bz)^2 - D_0^2] = \frac{\pi}{4}[2Bz + (Bz)^2] \tag{5.AT}$$

The taper of the tube, B, is generally small enough that the $(Bz)^2$ term in Eq. 5.AT is small compared to $2Bz$ and can be dropped. Then it follows that the height, z, at which the ball stands is linearly proportional to the volumetric flow rate Q.

This treatment is simple; more complex treatments [4] lead to similar conclusions. Because of some of the assumptions that went into finding Eq. 5.20, we should not assume that we can compute the true velocities from it; an empirical coefficient like the orifice coefficient would enter. However, most rotameters are treated as calibrated devices; for a given tube, float, and fluid, the Q-z curve is measured and thereafter one simply reads the float position and looks up the flow rate from that calibration curve.

Example 5.11. Our rotameter has been calibrated for nitrogen at room temperature and atmospheric pressure; the calibration shows that for a reading (float position) of 50 percent of the height of the rotameter tube the volumetric flow rate is 100 cm^3 / min. We now need to measure the flow of helium at room temperature and atmospheric pressure, using the same rotameter. When the reading is 50 percent of full scale, estimate the helium volumetric flow rate.

From Eq. 5.20 we know that the velocity V, and hence the volumetric flow rate, for a given float position

$$V \propto \left(\frac{\rho_{\text{ball}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}}\right)^{1/2} \tag{5.AU}$$

Here the density of the ball, if it is made of almost any solid material, is at least 1000 times the density of nitrogen at atmospheric pressure, so we can safely drop the ρ_{fluid} in the numerator, from which it follows that the velocity is proportional to $1 / (\text{fluid density})^{1/2}$. Thus

$$Q_{\text{helium}} = Q_{\text{nitrogen}}\left(\frac{\rho_{\text{nitrogen}}}{\rho_{\text{helium}}}\right)^{1/2} = Q_{\text{nitrogen}}\left(\frac{M_{\text{nitrogen}}}{M_{\text{helium}}}\right)^{1/2}$$

$$= 100\,\frac{\text{cm}^3}{\text{min}}\left(\frac{28}{4}\right)^{1/2} = 265\,\frac{\text{cm}^3}{\text{min}} \tag{5.AV}$$

■

Rotameters are very widely used for measuring low flow rates. The simple spherical ball float is used for the smallest flows, and more complex float designs are used for larger flow rates.

5.9 NEGATIVE ABSOLUTE PRESSURES: CAVITATION

In certain flows B.E. can predict negative absolute pressures, as shown by the following two examples. In gases, negative absolute pressures have no physical meaning at all. When B.E. predicts a negative absolute pressure for a gas flow, then the flow probably contains velocities much too high for the assumptions of B.E.; the equations developed in Chap. 8 must then be used.

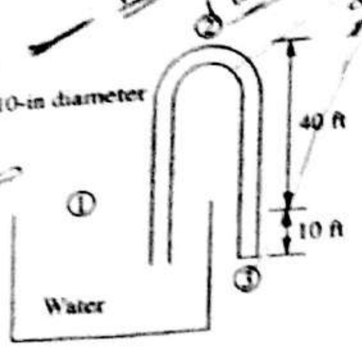


FIGURE 5.16
A siphon that will not work; see Example 5.12.

In liquids, negative absolute pressures can exist under very rare conditions, but they are unstable. Normally, when the absolute pressure on a liquid is reduced to the vapor pressure of the liquid, the liquid boils. This converts the flow to a two-phase flow, which has a much higher value of $\mathcal{F}$ than does the corresponding one-phase flow. Thus, when B.E. predicts a pressure less than the vapor pressure of the liquid, the flow as calculated is physically impossible; the actual flow will have a much higher friction effect, and the flow velocity will be less than that assumed in the original calculation.

Example 5.12. Figure 5.16 shows a siphon that is draining a tank of water. What is the absolute pressure at point 2?

Applying B.E. without friction from the free surface, point 1, to the outlet, point 3, we find

$$V_3 = [2g(h_1 - h_3)]^{1/2} = [2 \cdot (32.2 \text{ ft/s}) \cdot 10 \text{ ft}]^{1/2} = 25.3 \text{ ft/s} = 7.71 \text{ m/s} \tag{5.AW}$$

Then applying B.E. between points 1 and 2, we find

$$P_2 = P_1 - \rho\left[\frac{V_2^2}{2} + g(z_2 - z_1)\right]$$

$$= 14.7\frac{\text{lbf}}{\text{in}^2} - 62.3\frac{\text{lbm}}{\text{ft}^3}\left[\frac{(25.3 \text{ ft/s})^2}{2} + 32.2\frac{\text{ft}}{\text{s}^2}\cdot 40 \text{ ft}\right]\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2 \text{ lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 14.7\frac{\text{lbf}}{\text{in}^2} - 21.6\frac{\text{lbf}}{\text{in}^2} = -6.91\frac{\text{lbf}}{\text{in}^2} = -47.6 \text{ kPa} \quad ??? \tag{5.AX}$$

■

This flow is physically impossible. One may show that, when water is open to the atmosphere, such siphons can never lift water more than about 34 ft (10.4 m) above the water surface, even with zero velocity; the siphon shown in Fig. 5.16 will not flow at all. In this example the physically unreal, negative pressure was mostly a result of the gravity term in B.E. Negative absolute pressures can also be predicted by B.E. for horizontal flows in which gravity plays no role.

Example 5.13. Water flows from a pressure vessel through a venturi meter to the atmosphere; see Fig. 5.17. $P_1 = 10$ psig and $A_2/A_3 = 0.50$. What is the pressure at location 2?

Applying B.E. without friction between locations 1 and 3, we find

$$V_3 = \left[2\frac{(P_1 - P_3)}{\rho}\right]^{1/2} = \left[2\cdot\frac{10 \text{ lbf/in}^2}{62.3 \text{ lbm/ft}^3}\cdot\frac{32.2 \text{ lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\cdot\frac{144 \text{ in}^2}{\text{ft}^2}\right]^{1/2}$$

$$= 38.6\frac{\text{ft}}{\text{s}} = 11.8\frac{\text{m}}{\text{s}} \tag{5.AY}$$

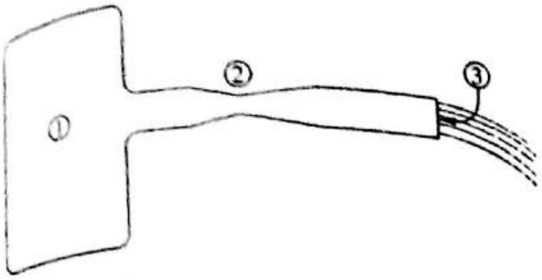


FIGURE 5.17
A horizontal venturi; see Example 5.13.

The mass balance gives us

$$V_2 = V_3\frac{A_3}{A_2} = 38.6\,\frac{\text{ft}}{\text{s}}\cdot\frac{1}{0.50}$$

$$= 77.2\,\frac{\text{ft}}{\text{s}} = 23.5\,\frac{\text{m}}{\text{s}} \qquad \text{(5.AZ)}$$

Applying B.E. without friction between locations 1 and 2, we find

$$P_2 = P_1 - \rho\frac{V_2^2}{2} = 24.7 \text{ psia} - \frac{62.3 \text{ lbm / ft}^3(77.2 \text{ ft / s})^2}{2}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2 \text{ lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 24.7 \text{ psia} - 40.1 \text{ psi} = -15.4 \text{ psia} = -106 \text{ kPa, abs} \quad ??? \qquad \text{(5.BA)}$$

■

This flow also is physically unreal. At such high velocities the frictional effects become large; so the frictionless assumption above is a poor one. If the frictional effect were negligible, then the fluid would boil in the venturi and thus convert to a two-phase flow with a much lower flow rate. The device sketched in Fig. 5.17 is widely used as a vacuum pump. An opening in the side of the tube at point 2 will suck in air; such devices, attached to faucets, are widely used as a laboratory source of modest vacuum. As the above example shows, with a high flow one produces a negative absolute pressure. But with modest flows one does not produce an impossible flow, and does produce a useful vacuum.

Example 5.13 shows that as the velocity increases in horizontal flow, the pressure falls. The pressure decrease can cause boiling of the liquid. Dramatic examples of this phenomenon occur in pumps, turbines, and ship's propellers, as shown in Fig. 5.18. In these devices the fluid is often speeded up to a velocity at which it forms a vapor bubble. Then the bubble flows to a region of higher pressure and collapses. The collapse can cause a sudden pressure pulse, and the pulses, occurring at high frequencies, can damage the pump, turbine, etc. The phenomenon of local boiling due to velocity increase is called *cavitation*, the study of which is an important part of modern research in fluid machines [5]. We will say a little more about this in Chap. 10.

5.10 B.E. FOR UNSTEADY FLOWS

B.E. is steady-flow equation; however, it can be successfully applied to some unsteady flows if the changes in flow rate are slow enough to be ignored. To decide how slow the change must be to be ignored, we reason as follows. For a steady flow $(\partial V / \partial t)_{x,y,z}$ is zero. This means that, although an observer riding with the fluid would observe a changing velocity, an observer watching a specific point in the system would observe no change in velocity with respect to time. We are safe in ignoring unsteady-flow effects if $(\partial V / \partial t)_{x,y,z}$ for all points in the system is small compared with the acceleration we are considering, that is, the acceleration of gravity or the acceleration due to pressure forces, $(dP / dL) / \rho$. If, on the other hand, $(\partial V / \partial t)_{x,y,z}$ at any point in the

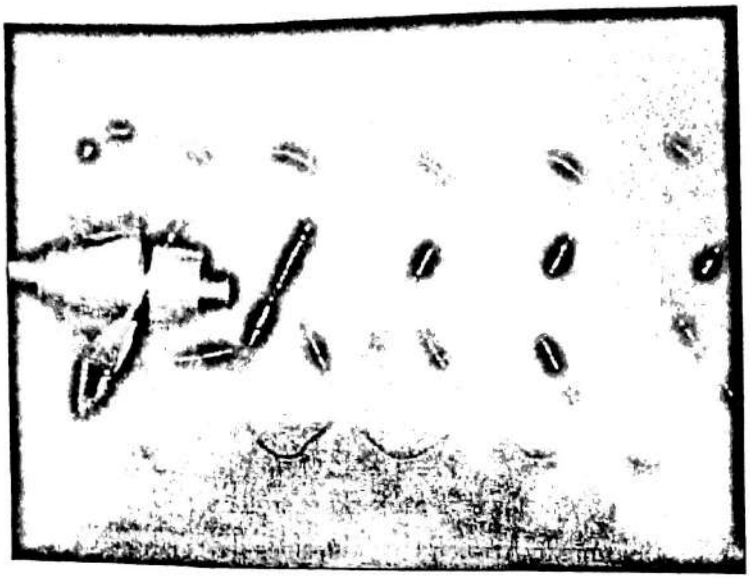


FIGURE 5.18
Cavitation bubbles formed at the tips of a propeller. The propeller is rotating in a channel with flow from left to right. At the tips of the blades the pressure is low enough to cause the water to boil. The bubbles have short lifetimes; the collapse of the unstable bubbles produces shock waves in the water, which can be destructive. (Photograph from the Garfield Thomas Water Tunnel Building, Applied Research Laboratory, The Pennsylvania State University.)

system is comparable to the largest of the other acceleration terms in the system, then we cannot safely apply B.E. to the system.

Example 5.14. If the tank in Fig. 5.7 is cylindrical with a diameter of 10 m, and if the outlet nozzle is 1 m in diameter, how long does it take the fluid level to drop from 30 m above the tank outlet to 1 m above the tank outlet?

Here we assume that the $(\partial V / \partial t)_{x,y,z}$ is small; we will check that assumption later. The instantaneous flow rate is assumed to be given by B.E., which here takes the form of Torricelli's equation:

$$V_2 = (2gh)^{1/2} \tag{5.12}$$

But, by the mass balance for an incompressible fluid,

$$V_2 = V_1 \frac{A_1}{A_2} \tag{5.BB}$$

where V_1 is the rate at which the free surface of the tank is moving downward, which is equal to $-dh/dt$; so

$$V_2 = \frac{-dh}{dt} \cdot \frac{A_1}{A_2} = (2gh)^{1/2}; \qquad \frac{-dh}{h^{1/2}} = \frac{A_2}{A_1}(2g)^{1/2}\, dt \tag{5.BC}$$

$$-\int_{h_1}^{h_2} \frac{dh}{h^{1/2}} = -2h^{1/2}\Big]_{h_1}^{h_2} = \frac{A_2}{A_1}(2g)^{1/2} \int_{t_1}^{t_2} dt = \frac{A_2}{A_1}(2g)^{1/2}\, t\Big]_{t_1}^{t_2} \tag{5.BD}$$

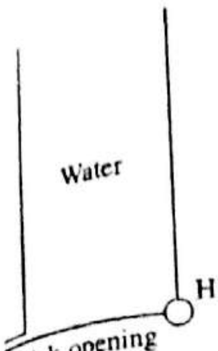


FIGURE 5.19
Opening the valve produces a flow not described by Bernoulli's equation.

Therefore,

$$\Delta t = t_2 - t_1 = \frac{-2(h_2^{1/2} - h_1^{1/2})}{(A_2 / A_1)(2g)^{1/2}} \quad \begin{bmatrix}\text{frictionless} \\ \text{tank draining}\end{bmatrix} \tag{5.21}$$

Inserting numbers, we find

$$\Delta t = \frac{-2[(1\text{ m})^{1/2} - (30\text{ m})^{1/2}]}{[(\pi / 4)\cdot(1\text{ m})^2 / (\pi / 4)\cdot(10\text{ m})^2]\cdot(2\cdot 9.81\text{ m / s}^2)^{1/2}}$$
$$= 2.02 \cdot 10^2 \text{ s} = 3.37 \text{ min} \tag{5.BE}$$

The maximum velocity in this tank is at the outlet, and all other velocities are proportional to it; therefore, the maximum value of $(\partial V / \partial t)_{x,y,z}$ must occur at the outlet. Differentiating Torricelli's equation with respect to time, we find

$$\frac{\partial V_2}{\partial t} = \frac{(2g)^{1/2}}{2h^{1/2}}\frac{dh}{dt} \tag{5.BF}$$

Substituting for dh / dt, we find

$$\frac{\partial V_2}{\partial t} = \left(\frac{g}{2h}\right)^{1/2} \cdot V_2 \frac{A_2}{A_1} = \left(\frac{g}{2h}\right)^{1/2} (2gh)^{1/2} \frac{A_2}{A_1} = g\frac{A_2}{A_1} \tag{5.BG}$$

Thus, in this example the maximum value of $(\partial V / \partial t)_{x,y,z}$ is $\frac{1}{100}$ the acceleration of gravity, and the unsteady-flow aspect of the problem can safely be neglected. ■

Most of the flow problems in which the unsteady flow cannot be neglected and hence in which B.E. cannot be applied involve starting the flow from rest or sudden stopping of the flow. Consider the pipe and valve shown in Fig. 5.19. Initially the pipe is practically full of fluid and the valve is closed. Then the valve is suddenly opened. If the friction effect is negligible, then the fluid will fall freely, maintaining its cylindrical shape, just as a solid rod would. In this case the entire outflow process takes place during the flow-starting period; the whole fluid is still accelerating when the last particle of fluid leaves the pipe. Friction and surface tension complicate the picture, but for low-viscosity fluids in short, large-diameter pipes the result described above is experimentally observed.

In this case all the fluid has the same velocity, and thus $(\partial V / \partial t)_{x,y,z}$ is the same at all points where there is fluid. Here it is equal to g, so it is of the same size as the largest accelerations in B.E., and the test indicates that we cannot safely apply B.E. to this problem.

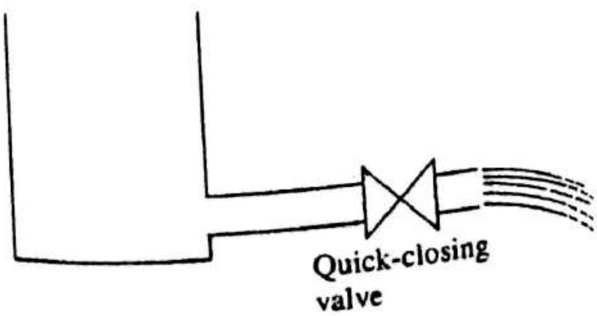


FIGURE 5.20
Closing the value quickly produces a flow not described by Bernoulli's equation.

The other general type of unsteady-flow problem that cannot be solved by B.E. is the problem with sudden valve closing, which leads to a phenomenon called *water hammer*.

Figure 5.20 shows a tank from which a liquid flows through a pipe, at the end of which is a quick-closing valve. If the liquid is flowing steadily and the valve is suddenly closed, the flow during the closing process cannot be described by B.E. B.E. would indicate that, once the valve closed, the pressure throughout the system would be the pressure given by the basic equation of fluid statics. Actually, at the time the valve is being closed, the fluid in the pipe has significant kinetic energy, and the sudden shutting of the valve requires that kinetic energy be converted either to internal energy, with a rise in temperature, or to injection work, with a rise in pressure.

This chapter has concentrated on problems most easily solved by the energy balance (of which B.E. is a restricted form). The problem of suddenly stopping the fluid, in Fig. 5.20, and the problem of starting it from rest are both more easily solved by the momentum balance (Chap. 7). We will return to this problem and the problem of what happens when the valve in Fig. 5.20 is suddenly opened in Chap. 7. For now we simply note that although B.E. is immensely useful, there are some problems for which it is not useful; the starting and stopping of the flow in Fig. 5.20 is one of those problems.

5.11 NONUNIFORM FLOWS

So far in this chapter, and in the vast majority of problems in pipes, channels, ducts, etc., we assume that the velocity is practically uniform across the pipe, duct, or channel, so that we may associate one velocity with the entire flow at any one downstream location perpendicular to the flow. In most flows of practical interest to chemical engineers this simplification introduces negligible errors (see Table 3.1). However there are some very simple and common flows for which this is not the case. The simplest and most illustrative example of this type is the flow over a sharp-edged weir.

Figure 5.21 shows schematically the flow in an open channel that passes over a sharp-edged weir. One may study a very similar flow in a kitchen sink by pouring water out of a rectangular baking dish, at a high enough velocity that the flow does not dribble down the side of the dish but rather flows freely away from the edge, as shown in Fig. 5.21. The flow over the weir in Fig. 5.21 is simpler than the flow out of baking dish, because the weir is assumed to extend a long way into and out of the page, so that the complications where it meets the walls of the channel, equivalent to the effects of the corners of the dish, can be ignored.

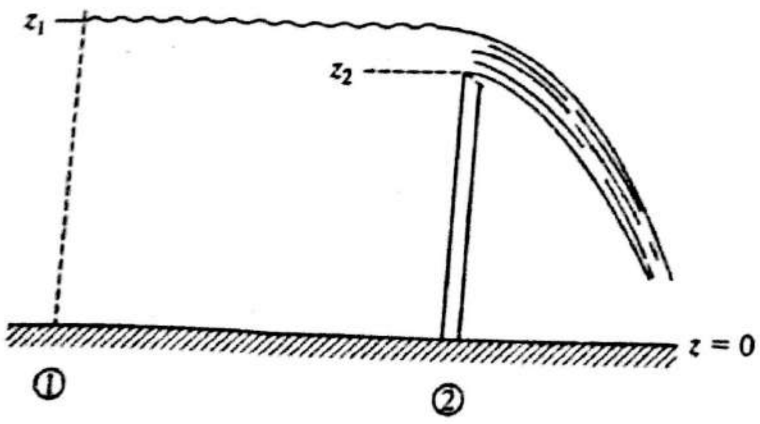


FIGURE 5.21
Flow over a weir.

Here at 1, far upstream from the weir, the velocity is presumably uniform (ignoring the effects of friction), equal to V_1. Clearly at 1 we do not have a single value of z; instead, we have elevations ranging from $z = 0$ to $z = z_1$. Similarly, we do not have a single pressure, but we have gauge pressures ranging from zero at the free surface to $P = \rho g z_1$ at the bottom.

Fortunately, this causes little difficulty because the sum $P/\rho + gz + V^2/2$ is constant at point 1, independent of z, because as z declines the pressure rises, according to the basic equation of fluid statics, to keep the sum of the $P/\rho + gz$ terms constant.

The same is not true at 2. There the flow is open to the air at both sides, so that the gauge pressure at 2 must be zero, at all elevations above the weir. We can thus write B.E. between an arbitrary upstream point at 1 and some elevation z (above z_2) at point 2. Because the sum $P/\rho + gz$ is independent of z at 1, we choose $z = z_1$, for which $P_1 = 0$, and write

$$gz_1 + \frac{V_1^2}{2} = gz + \frac{V_2^2}{2} \qquad V_2 = \left[2g(z_1 - z) + \frac{V_1^2}{2}\right]^{1/2} \tag{5.BH}$$

This says that at the free surface the velocity should be the same as the velocity at 1, which in most cases is negligibly small, and that at the bottom of the overflowing stream, the velocity should be a maximum, with the value given by the above equation, with $z = z_2$. One can try this by pouring water from a baking dish in the sink and seeing that this is not exactly the case. The internal friction in the flow does not allow the fluid at the surface to go that slowly, adjacent to the much faster-flowing fluid just below it. Instead the faster-flowing fluid drags the surface fluid along, faster than Eq. 5.BH predicts. One may also see that as the surface fluid speeds up, its elevation falls, so that instead of being completely level up to the weir, as Fig. 5.21 shows, the free surface actually falls slightly just before the weir, to satisfy the energy balance.

Ignoring this disagreement between Eq. 5.BH and what we can see in the sink, we can compute the expected flow rate by considering a length W into the page in Fig. 5.21. To simplify the integration, we now measure elevations downward from the surface, defining $h = z_1 - z$, and write

$$Q = \int V\,dA = W\int_0^{h_1}\left(2gh + \frac{V_1^2}{2}\right)^{1/2} dh \tag{5.BI}$$

Normally V_1 is small enough that we can drop it from the right side of the above equation, and integrate to find

$$Q = W\sqrt{2g}\,\frac{h_1^{3/2}}{3/2} \tag{5.22}$$

Experimental results show that the $3/2$ power dependence of Q on h_1 is correct but that the flow rate is less than predicted by Eq. 5.22, typically about 67 percent of what that equation predicts [6].

The same differences in velocity from top to bottom of the flow that we calculate here are certainly present in all the horizontal-flow examples in this chapter. However, in a flow like that shown in Fig. 5.7, the difference in elevation from the top to the bottom of the exit flow is so small compared to the elevation change from the free surface in the tank to the centerline of the exit that we make a negligible error in ignoring the minor differences in velocity from top to bottom of the exit flow. The same is true of most of the flows of practical interest to chemical engineers. But for shallow gravity-driven flows, for example, the flow over weirs in distillation columns, clarifiers, etc., one must take them into account.

Figure 5.21 shows the jet that flows over the weir becoming thinner as it flows. This is also explained by B.E., as shown in the next example.

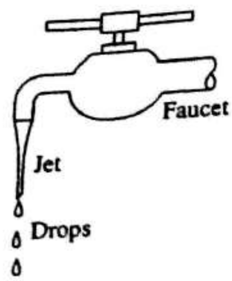


FIGURE 5.22
Slow flow from a partly opened faucet, producing a jet that contracts as it falls and finally breaks up into drops.

Example 5.15. A faucet in an ordinary sink with a partially opened valve is shown in Fig. 5.22. The flow tapers from the faucet, forming a thinner and thinner stream as it flows downward. Eventually it breaks up into drops, due to surface tension, discussed in Chap. 14. If the diameter of the falling column of fluid is 2 cm and its velocity 0.5 m/s where it leaves the faucet, what is the expected diameter at 0.5 m below the faucet?

Taking 1 to be the exit of the faucet and 2 to be 0.5 m lower, we find from B.E. that

$$V_2^2 - V_1^2 = 2gh \qquad V_2 = (V_1^2 + 2gh)^{1/2} \tag{5.BJ}$$

$$V_2 = \left[\left(0.5\,\frac{\text{m}}{\text{s}}\right)^2 + 2\cdot 9.81\,\frac{\text{m}}{\text{s}^2}\cdot 0.5\text{ m}\right]^{1/2}$$

$$= \left(10.06\,\frac{\text{m}^2}{\text{s}^2}\right)^{1/2} = 3.17\,\frac{\text{m}}{\text{s}} = 10.4\,\frac{\text{ft}}{\text{s}} \tag{5.BK}$$

By material balance for a constant-density fluid,

$$A_2 = A_1\,\frac{V_1}{V_2} = \frac{0.5\text{ m/s}}{3.17\text{ m/s}} = 0.158 \tag{5.BL}$$

and correspondingly

$$D_2 = D_1\left(\frac{A_2}{A_1}\right)^{1/2} = 2\text{ cm}\cdot 0.158^{1/2} = 0.79\text{ cm} = 0.31\text{ in} \tag{5.BM}$$

As the velocity of the fluid increases according to B.E., the column shrinks laterally according to the material balance. ∎

For inherently two- or three-dimensional flows, like the flow around an airplane, the simple application of B.E. from one point to another in the flow, which we have used here, is only applicable if the two points chosen are on a single streamline, as discussed in Chap. 16.

5.12 SUMMARY

1. B.E. is the energy-balance equation for steady flow of constant-density fluids.
2. For constant-density fluids the term $(\Delta u - dQ/dm)$ in B.E. represents the friction heating per unit mass, $\mathcal{F}$.
3. Although no fluid has exactly constant density, B.E. can be applied with negligible error to almost all steady flows of liquids and to steady flow of gases at low velocities, because in those flows the effect of changes in density is negligible.

4. A large number of fluid-measuring devices are based on the frictionless form of B.E. Friction is present in these devices. The commonly used equations for these devices retain the frictionless B.E. form and add empirical correction factors to deal with the effects of friction.
5. B.E. can predict negative absolute pressures for some impossible flows. For gas flows, this prediction normally means that the velocities are too high for B.E. to apply. For liquids, it normally means that the fluid will boil, leading to a two-phase flow and a much lower velocity than predicted.
6. Although B.E. is a steady-flow equation, it can be used for unsteady flows if the time rate of change of velocity at every point in the system is small compared with the accelerating forces (e.g., the acceleration of gravity). It is not useful for problems involving sudden starting and stopping of flows; those are best solved with the momentum balance.
7. Normally we ignore the differences in velocity perpendicular to the flow in applying B.E. to the flow in pipes and channels. This causes negligible errors, except in shallow, gravity-driven flows, like the flows over weirs.

PROBLEMS

See the Common Units and Values for Problems and Examples inside the back cover. An asterisk (*) on the problem number indicates that the answer is in App. D.

5.1.* If a body falls 1000 ft in free fall and then is stopped by friction in such a way that all its kinetic energy is converted into internal energy, how much will the temperature of the body increase if

(*a*) It is steel, $C_V = du / dT = 0.12$ Btu / lbm · °F.

(*b*) If it is water, $C_V = du / dT = 1.0$ Btu / lbm · °F? Here C_V is the heat capacity at constant volume.

5.2. Show that in the head form of B.E. each term has the dimension of a length.

5.3.* Water is flowing in a pipe at a velocity of 8 m / s. Calculate the pressure increase and the increase in internal energy per unit mass for each of the following ways of bringing it to rest:

(*a*) A completely frictionless diffuser with infinitely large A_2.

(*b*) A diffuser that has 90 percent of the pressure recovery of a frictionless diffuser, with infinitely large A_2.

(*c*) A sudden expansion.

5.4. A fluid is flowing in a frictionless diffuser in which $A_2 / A_1 = 3$ and $V_1 = 10$ ft / s. Calculate the pressure recovery $(P_2 - P_1)$

(*a*) For the fluid being water.

(*b*) For the fluid being air.

5.5. Rework Example 5.2, calculating the density from the formula $\rho = MP_{avg} / RT$ where P_{avg} is $0.5(P_1 + P_{atm})$. Compare the results with those shown in Table 5.1.

5.6. Torricelli's equation can be reduced to a simple plot of V as a function of h. Prepare such a plot for heights up to 1000 ft.

5.7.* The tank shown in Fig. 5.7 is modified to have an outflow area of 2 ft^2. The diameter of the tank is so large that it may be considered infinite. The height h is now 12 ft. How many cubic feet per second are flowing out? Assume frictionless flow.

5.8. Repeat Example 5.3 when the fluid is gasoline.

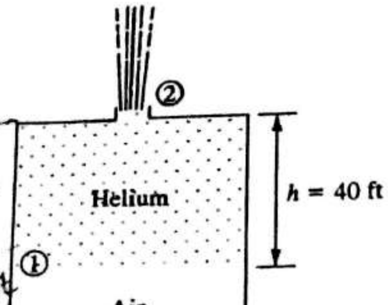


FIGURE 5.23
Buoyancy-driven gas flow.

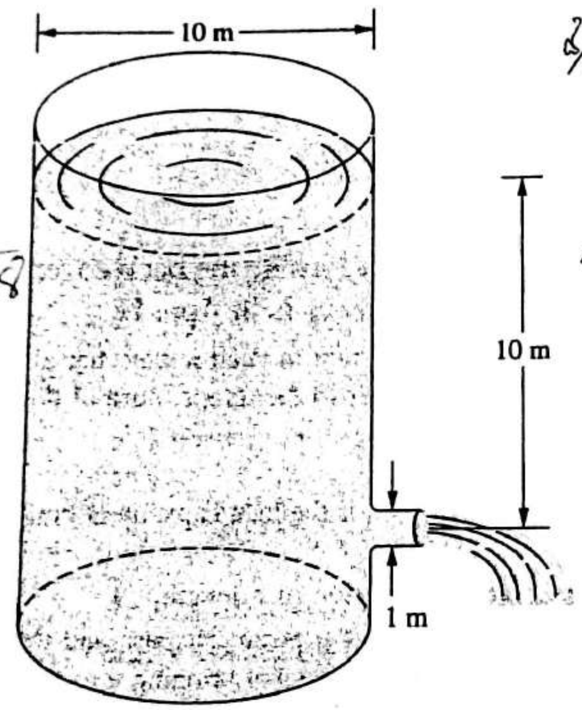


FIGURE 5.24
Tank-draining flow.

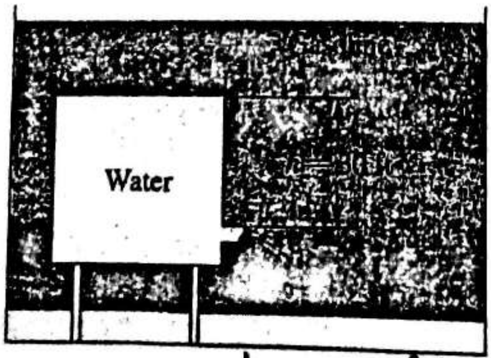


FIGURE 5.25
Two-fluid, gravity-driven flow.

5.9.*Repeat Prob. 5.7, except that now the horizontal cross-sectional area of the tank is 5 ft^2.

5.10. Hoover Dam has a height of 726 ft. Assume the water is up to the top of the dam on the upstream side. If one were to drill a hole through its base and let the water squirt out, and if friction were negligible, what velocity of water jet would we expect?

5.11. An ocean liner strikes an iceberg, which tears a 5 m^2 hole in its side. The center of the hole is 10 meters below the ocean surface. Estimate the volumetric flow rate of water into the ship.

5.12. Rework Example 5.5, making the fluid in the tank air be at the same temperature and pressure as the air of the atmosphere. Is the answer from Eq. 5.12 plausible? Would the answer from Eq. 5.14 be plausible?

5.13.*The tank in Fig. 5.23 is full of helium, at the same temperature and pressure as the surrounding atmosphere. Assuming steady, frictionless flow, what is the velocity of helium through the hole?

5.14. The tank in Figure 5.24 is cylindrical with a diameter of 10 m. The outlet is a cylindrical frictionless nozzle, with diameter 1 m. The top of the tank is open to the atmosphere. When the level in the tank is 10 m above the centerline of the outlet, how fast is the level in the tank falling?

5.15. In Fig. 5.25 a tank of water is immersed in a larger tank of gasoline, and the water is flowing out through a hole in the bottom. What is the velocity of this flow?

5.16.*In the vessel in Fig. 5.26 water is flowing steadily in frictionless flow under the barrier. What is the velocity of the water flow under the barrier?

5.17. In the tank and standpipe in Fig. 5.27, which way is the fluid flowing? *Hint:* Write B.E., taking the two free surfaces as points 1 and 2. Compute the magnitude and sign of $\mathcal{F}$ for flow in each direction.

5.18.*In the tank in Fig. 5.28, water is under a layer of compressed air that is at a pressure of 20 psig. The water is flowing out through a frictionless nozzle that is 5 ft below the water surface. What is the velocity of the water?

5.19. In the preceding problem, if the liquid level remains constant, and we slowly lower the air pressure, at some air pressure the velocity will be 10 ft / s. What pressure will that be?

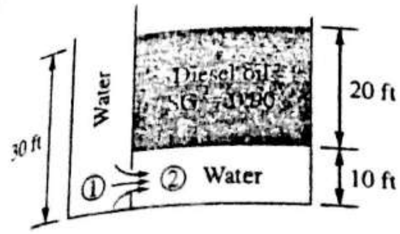


FIGURE 5.26
Two-fluid, gravity-driven flow.

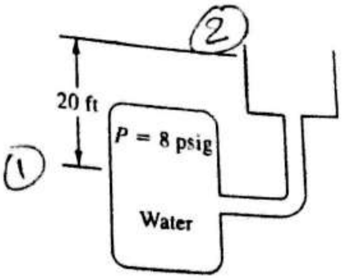


FIGURE 5.27
Which way does this system flow?

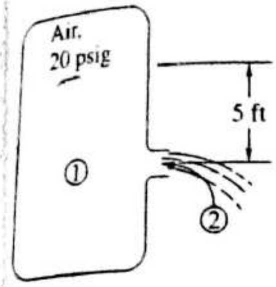


FIGURE 5.28
Flow driven by gravity and pressure.

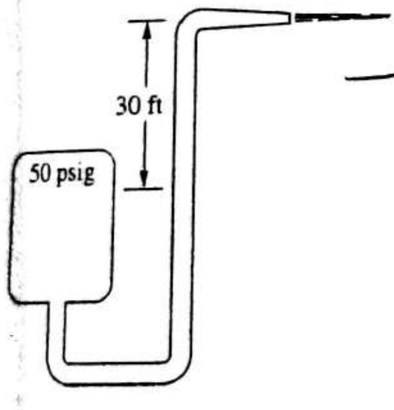


FIGURE 5.29
Flow driven by pressure against gravity.

5.20. The system in Fig. 5.29 consists of a water reservoir with a layer of compressed air above the water and a large pipe and nozzle. The pressure of the air is 50 psig, and the effects of friction can be neglected. What is the velocity of the water flowing out through the nozzle?

5.21.*The tank in Fig. 5.30 has a layer of mercury under a layer of water. The mercury is flowing out through a frictionless nozzle. What is the velocity of the fluid leaving the nozzle?

5.22. The compressed-air-driven water rocket shown in Fig. 5.31 is ejecting water vertically downward through a frictionless nozzle. When the pressure and elevation are as shown, what is the velocity of the fluid leaving the nozzle?

5.23.*An industrial centrifuge is sketched in Fig. 5.32. The fluid in the basket is water. The radii are $r_1 = 21$ in and $r_2 = 20$ in. The basket is revolving at 2000 rpm. There is a small hole in the outer wall of the centrifuge, through which the fluid is flowing in frictionless flow. What is the velocity of flow through this hole?

5.24. Flow-recorder charts frequently have a square scale rather than a linear one. Why?

5.25.*A pitot tube is being designed for use as a speedometer on power boats. For ease of construction the

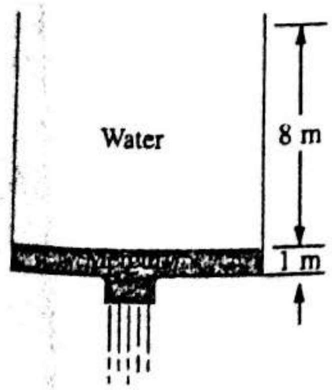


FIGURE 5.30
Two-fluid, unsteady tank draining.

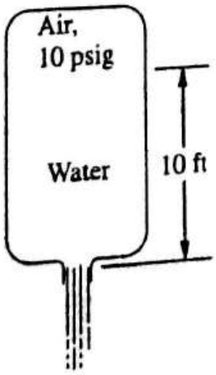


FIGURE 5.31
Compressed air-water rocket.

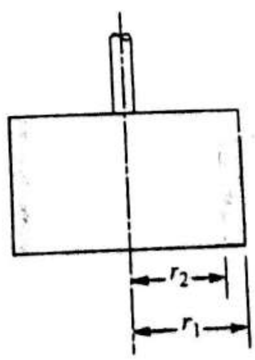


FIGURE 5.32
Centrifuge basket with leak.

tubes will not extend more than 10 ft above the water. What is the maximum speed at which they can be used?

5.26. A pitot-static tube is to be used to measure a flow of air. The manometer fluid is water. We will not use the tube for flows so slow that the elevation difference in the manometer is less than 0.5 in because smaller differences are hard to read. What is the smallest air velocity at which we can use this pitot-static tube?

5.27.*Repeat Prob. 5.26 except that now we measure the flow of gasoline, and the manometer fluid is water.

5.28. A pitot-static tube is used to measure an airplane's air speed. When the pressure-difference gauge reads 0.3 psig, how fast is the plane going?

(*a*) At sea level where the air density is about 0.075 lbm / ft^3?

(*b*) At an altitude of 10,000 ft, where the air density is about 0.057 lbm / ft^3?

(*c*) Does this difference cause problems for the pilot? (You may have to ask your pilot or aeronautical engineering friends for help on part *c*.)

5.29. A pitot tube, connected to a bourdon-tube pressure gauge, is used to measure the speed of a boat. The tube is just below the waterline and faces directly forward. When the boat is going 60 km / h, what is the reading of the pressure gauge?

5.30. In this book and most textbooks, equations are correct for any set of units. In "applied" or "practical" publications, one regularly sees equations that are unit specific. For example, Jack Caravanos (*Quantitative Industrial Hygiene; A Formula Workbook*, ACGIH, Cincinnati, 1991, page 66) gives the following equation for the air-flow velocity in a duct, based on measurements with a pitot-static tube:

$$V = 4005\sqrt{VP} \tag{5.BN}$$

where V is the velocity in ft / min, and VP is the "velocity pressure" in inches of water. Is this consistent with Eq. 5.16?

5.31. If the venturi meter in Example 5.10 is to be used on a day-to-day basis, then it will be useful to have a plot of volumetric flow rate versus pressure drop, so that one can read the pressure drop and simply look up the volumetric flow rate. Sketch such a plot for flow rates of 1 to 10 ft^3 / s.

5.32. In the venturi meter shown in Fig. 5.10, the flowing fluid is air, the manometer fluid is water, $(D_2 / D_1) = 0.5$, and the manometer reading is 1 ft. Estimate the velocity at point 2.

5.33. The venturi meter in Example 5.8 is now set at 30° to the horizontal, as in Fig. 5.12. The flowing fluid is gasoline. The fluid in the bottom of the manometer is colored water. The reading of the manometer is $z_3 - z_4 = 1$ ft. What is the volumetric flow rate of the gasoline?

5.34.*Repeat Prob. 5.33, except that the two pressure taps have been replaced with pressure gauges. These are placed on the side of the pipe, so that they indicate pressures on the pipe centerline. The gauge at point 1 reads 7 psig, and the gauge at point 2 reads 5 psig. The difference in elevation between the gauges, $(z_1 - z_2)$, is 2 ft. What is the volumetric flow rate of the gasoline?

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5.35. In the apparatus in Fig. 5.33 what is the volumetric flow rate?

5.36.*The venturi meter in Fig. 5.34 has air flowing through it. The manometer, as shown, contains both mercury and water. The cross-sectional areas at the upstream location and at the throat are 10 ft^2 and 1 ft^2. What is the volumetric flow rate of the air? The discharge coefficient C_v equals 1.0.

5.37. Modern autos in the United States, Europe, and Japan have mostly replaced the carburetors of older autos with fuel injectors. But autos produced for developing countries still have carburetors, as do power tools such as lawnmowers. The carburetors in automobiles and power tools are much more complicated versions of the carburetor shown in

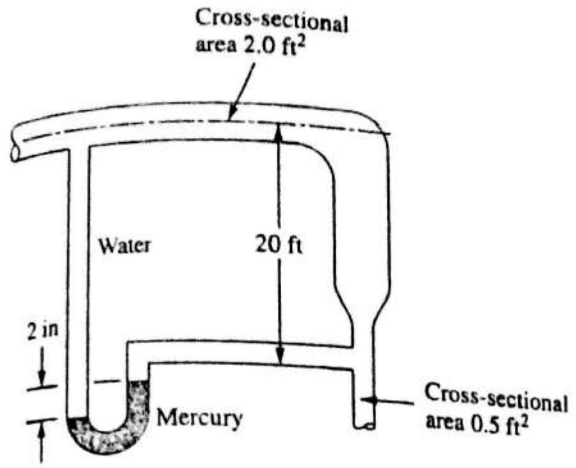


FIGURE 5.33
Device for Prob. 5.35.

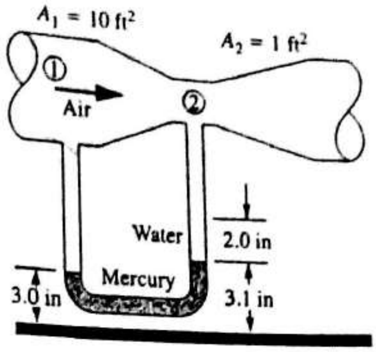


FIGURE 5.34
Venturi meter with two-fluid manometer.

Fig. 5.35, but they operate the same way as that simple one. The cross-sectional areas at points 1 and 3 are large enough that the velocities there can be considered negligible compared to the velocity at point 2, and the pressures at points 1 and 3 are both approximately equal to atmospheric pressure. The gasoline enters from a constant-liquid-level, atmospheric-pressure reservoir through a large-diameter tube and a small jet, which may be considered a frictionless nozzle with diameter D_j. The diameter at the throat of the venturi, point 2, is D_2.

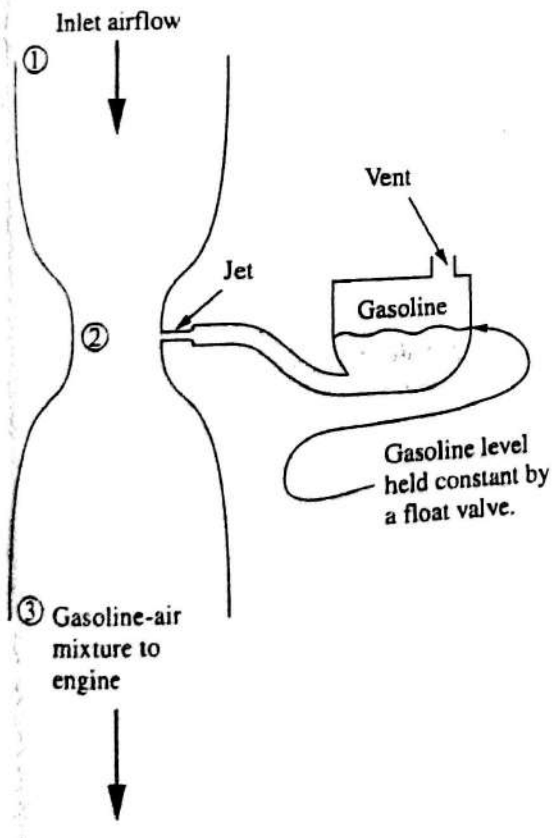


FIGURE 5.35
Elementary carburetor. The liquid level in the reservoir is held constant by a float valve.

(*a*) Write the equation for the air-fuel ratio, which is the (mass flow rate of air) / (mass flow rate of fuel), in terms of the diameters of the throat, the jet, etc.

(*b*) How does this air-fuel ratio change with changes in air flow rate to the engine? (The air flow rate to the engine is governed by the setting of the throttle plate, which is connected to the driver's accelerator pedal, and located between the part of the carburetor shown here and the engine.)

(*c*) If we want an air-fuel ratio of 15 lbm / lbm (typical of gasoline engines), what ratio of D_1 / D_2 should we choose?

(*d*) If the carburetor shown here gives an air-fuel ratio of 15 at sea level, will it give the same, a higher, or a lower fuel-air ratio in Denver, elevation 5280 ft above sea level?

5.38. In the United States natural gas is normally piped inside buildings at a pressure of 4 in of water, whereas propane is piped inside buildings at a pressure of 11 in of water. Why?

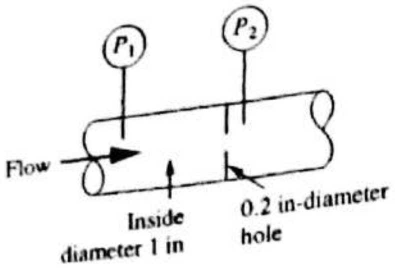


FIGURE 5.36
Orifice meter.

5.39. An oil (density 55 lbm / ft³) is flowing through the orifice in Fig. 5.36. The oil velocity is 1 ft / s in the pipe. $C_v = 0.6$. What is the indicated value of $P_1 - P_2$?

5.40. One occasionally sees Eq. 5.18 written

$$V_2 = \frac{C_v}{(1 - A_2^2 / A_1^2)^{1/2}} \cdot \left[\frac{2(P_1 - P_2)}{\rho} \right]^{1/2} \quad (5.BO)$$

One then defines a new coefficient, $C = C_v / (1 - A_2^2 / A_1^2)^{1/2}$, which is called the *coefficient of discharge, approach velocity corrected*. Sketch this coefficient for $D_2 / D_1 = 0.8$ and for $D_2 / D_1 = 0.2$ on a graph like Fig. 5.14.

5.41. Mercury is flowing at 1 ft / s in a 1-in diameter pipe. We want to select a drilled plate to insert in the pipe so that the pressure-drop signal across it will be 3 psig. What diameter should we select for the orifice hole?

5.42.*A venturi meter, Fig. 5.10, has $A_2 / A_1 = 0.5$. The fluid flowing is water. The pressure at point 1 is 20 psia.

(*a*) What is the velocity at point 2 that corresponds to a pressure at point 2 of 0.0 psia?

(*b*) If the water is at 200°F, its vapor pressure is 11.5 psia. What is the highest velocity possible at point 2 at which water at 200°F will not boil?

5.43. For a siphon similar to that sketched in Fig. 5.16, we want the fluid to have a velocity of 10 ft / s in the siphon pipe. If we assume that flow is frictionless and that the minimum pressure allowable is 1 psia, what is the maximum height that the top of the siphon may have above the liquid surface level?

5.44. In Example 5.12, what is the highest possible value of $(z_2 - z_1)$ for which cavitation will not occur? The vapor pressure of water at 20°C = 68°F is 0.34 psia.

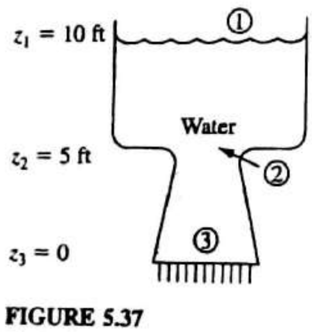


FIGURE 5.37
Vertical, gravity-driven venturi.

5.45. The tank in Fig. 5.37 is open to the atmosphere at the top and discharges to the atmosphere at the bottom. The cross-sectional areas are; 1, very large; 2, 1.00 ft²; 3, 1.50 ft². The flow is steady and frictionless. What is the pressure at 2?

5.46.*A ship's propeller has an outside diameter of 15 ft. When the ship is loaded, the uppermost part of the propeller is submerged 4 ft. If the water is at 60°F (vapor pressure, 0.26 psia), what is the maximum speed of the propeller, in revolutions per minute, at which cavitation cannot be expected to occur at the tip of the propeller?

5.47. Is cavitation likely to be as severe a problem with the propellers of submarines as with the propellers of surface ships? Why?

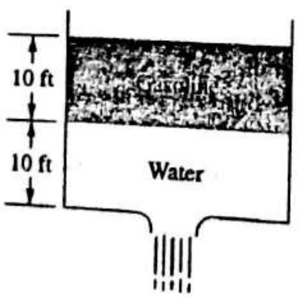


FIGURE 5.38
Two-fluid tank draining.

5.48. In Example 5.14, instead of the tank being initially filled to a depth of 30 m above the outlet with water, it is filled to a depth of 10 m with water and then has a layer of 20 m of gasoline on top of the water. How long does it take the level of the top of the gasoline to fall from 30 m above the outlet to 1 m above the outlet?

5.49.*The tank in Fig. 5.38 is cylindrical and has a vertical axis. Its horizontal cross-sectional area is 100 ft². The hole in the bottom has a cross-sectional area of 1 ft². The interface between the gasoline and the water remains perfectly horizontal at all times. That interface is now 10 ft above

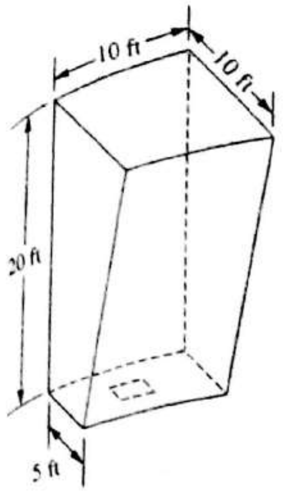


FIGURE 5.39
Tank draining with non-constant cross section.

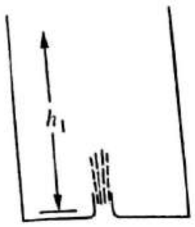


FIGURE 5.40
Gravity inflow.

the bottom. How soon will gasoline start to flow out the bottom? Assume frictionless flow.

5.50. (*a*) In Figure 5.28, if the tank diameter is 10 ft and a pressure regulator supplies compressed air as needed to keep the pressure in the gas space constant at 20 psig, how long will it take for the water level to fall from 5 ft to 1 ft?

(*b*) Repeat part (*a*) except that the compressed air system is turned off so that the pressure above the water falls as the liquid flows out and the gas expands. Assume that the gas is an ideal gas and that its temperature remains constant at 20°C = 68°F. Initially the gas space is 1 ft high, and at $P = 20$ psig.

5.51. The open-topped tank shown in Fig. 5.39 is full to the top with water. The bottom opening is uncovered, so that the water runs out into the air. The cross-sectional area of the bottom opening is 1 ft². How long does it take the tank to empty? Assume frictionless flow.

5.52. An open-ended tin can, Fig. 5.40, has a hole punched in its bottom. The can is empty and is suddenly immersed in water to the depth h_1 shown and then held steady. The area of the hole is 0.5 in², and the horizontal cross-sectional area of the can is 20 in². If we assume that the flow through the hole in the bottom of the can is frictionless, how long does it take the can to fill up to the level of the surrounding water?

5.53. A fluid mechanics demonstration device has the same flow diagram as Fig. 5.6. The tank is rectangular, 6 in by 5.5 in. The outlet opening is circular, $D = 0.30$ in. The tank is filled with water and allowed to drain. How long will it take the level to fall from 11 in above the centerline of the opening to 1 in above it?

5.54. A 1-gal paint can, diameter 6.5 in and height 7.5 in, is filled with methane. A 0.25 in. diameter hole in the top is covered with masking tape, as is a 0.5 in. diameter hole at the bottom; see Fig. 5.41. At time zero, the two masking tapes are removed, and a stopwatch is started. The methane flows upward out the hole in the lid by gravity and is lighted, producing a yellowish flame. For all the calculations below, assume that the flow resistance through the hole in the bottom is negligible.

(*a*) What is the initial velocity of the methane through the hole in the top?

(*b*) As the methane in the can is replaced by the inflowing air, that velocity falls and the flame becomes smaller and bluer. What is the relation between that velocity and elapsed time? Here use the two classic mixing models of chemical engineering, totally unmixed flow, in which the air and methane form separate layers, one above the other, and totally mixed flow, in which the concentration of methane in the mixture inside the container is always uniform throughout the container.

(*c*) When the flow rate through the opening at the top becomes less than the laminar flame speed for methane-air mixtures, 1.1 ft / s, the flame burns back into the container, where the velocity is less than 1.1 ft / s and spreads rapidly, producing a bang and a flash, and propelling the container's lid into the air. The internal safety chain prevents the lid from hurting anyone, and pulls the can up off its bottom. How long does it take for this to occur, according to the totally unmixed and totally mixed models in part (*b*)? The observed time is about 325 s. This demonstration is described in detail in [7].

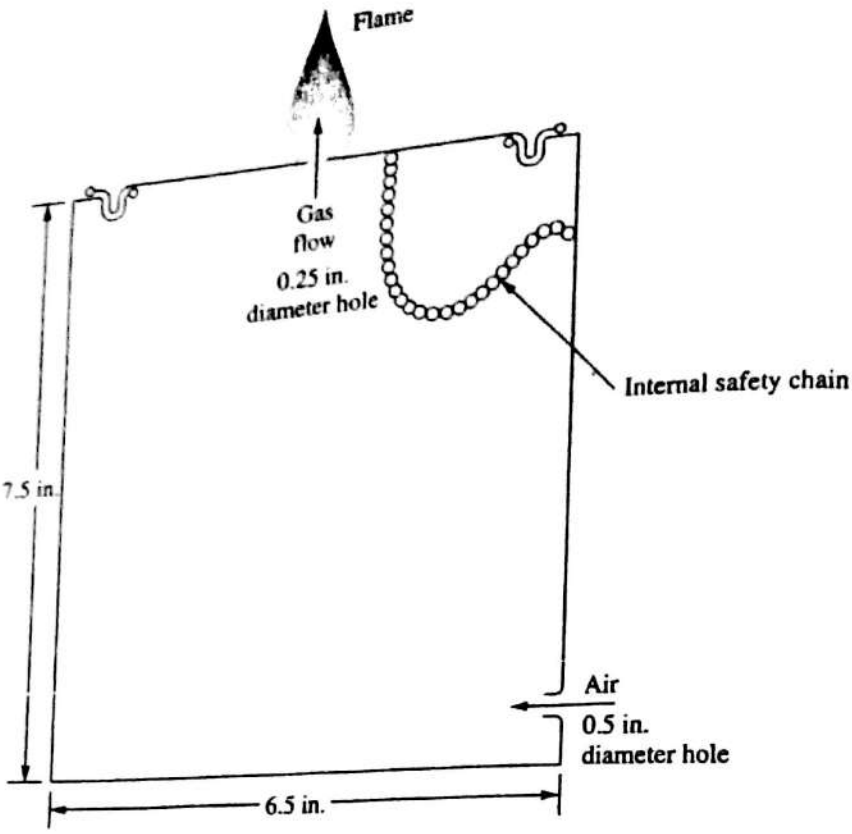


FIGURE 5.41
Simple time bomb; see Prob. 5.54.

5.55. Repeat Example 5.14 with the water in the tank being replaced with propane. Assume zero mixing between the propane and the air above it.

5.56. Figure 5.42 shows a toy fluid-mechanics demonstrator, which consists of a wooden (or plastic) spool, a piece of cardboard, and a thumbtack. When one blows hard enough downward into the spool, the cardboard is held firmly against the spool; when one stops blowing, the cardboard falls away by gravity. Sketch a pressure-radius plot for pressure along line *A–A* in the sketch while air is flowing. Use the axes shown in the lower part of the figure.

The function of the thumbtack is to prevent the cardboard from moving sideways; otherwise it plays no role in the device. Assume that the cardboard is stiff enough that the distance between the cardboard and the spool is constant, independent of radius. The device works with a piece of ordinary flexible paper, but the mathematics are more complex because the distance between paper and spool is not constant. A piece of adhesive tape holding the thumbtack in place helps.

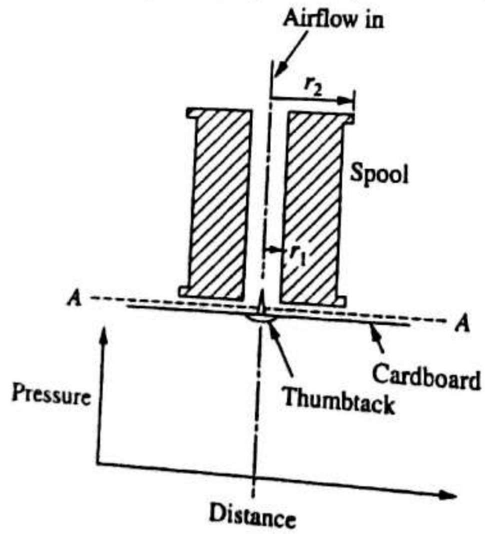


FIGURE 5.42
Spool and cardboard fluid mechanics demonstrator.

5.57. In the spool-and-cardboard demonstrator in the preceding problem, the hole in the spool has a diameter of 7.1 mm, the outside diameter of the spool is 35 mm, and the space between the spool and the cardboard disk is estimated to be 0.2 mm. One lung full of air is about 1 L and is blown out in about 2 s. Based on these values, estimate the lowest pressure likely to occur in the space between the spool and the cardboard.

5.58. For frictionless pumps and compressors pumping constant-density fluids, the required work is given by Eq. 5.11. If the fluid is an ideal gas, then that equation becomes

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{RT}{M}\int\frac{dP}{P} \tag{5.BP}$$

For very small pressure changes this is practically

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{RT}{M}\frac{\Delta P}{P_1} \qquad [\Delta P << P_1] \tag{5.BQ}$$

Almost all real compressors are intermediate between adiabatic (no heat transfer to the surroundings) and isothermal (complete thermal equilibrium with the surroundings). For those two cases the required work for ideal gases is shown in Chap. 10 to be

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{RT}{M}\ln\frac{P_2}{P_1} \qquad \text{[isothermal, frictionless]} \tag{5.BR}$$

and

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{RT_1}{M}\cdot\frac{k}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right] \qquad \text{[adiabatic, frictionless]} \tag{5.BS}$$

Here T_1 is the inlet temperature, k is the ratio of specific heats (to be discussed in Chap. 8), which is practically constant for any gas ($\approx$1.40 for air), and P_1 and P_2 are the inlet and outlet pressures, respectively. To show how these formulae compare, prepare a plot of $[M / RT_1]\cdot(dW_{\text{n.f.}} / dm)$ versus P_2 / P_1 for air, showing curves for each of the three equations, for the range $1.0 < P_2 / P_1 < 1.3$. Here the calculated work is that done to drive the pump or compressor, which is work done on the system and has a positive sign.

5.59. Figure 5.43 shows an air-cushion car, of the type widely used to slide heavy loads over relatively smooth surfaces. In it, a fan or blower forces air under pressure into the confined space under the car. This air supports the car and its load. Some of the air continually leaks out through the gap between the skirt of the car and the ground; the fan must supply enough air to make up for this leakage. Assuming that the car and its payload have a total mass of 5000 lbm, that the car is circular with a diameter of 10 ft, and that the clearance between the skirt of the car and the floor is 0.01 in, calculate the air flow rate. Then, assuming that the blower is 100 percent efficient and isothermal (Prob. 5.58), calculate the required blower horsepower.

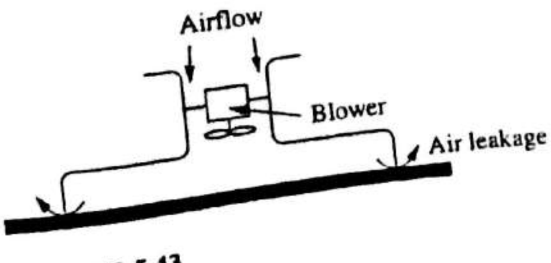


FIGURE 5.43
Air-cushion car.

5.60.* An air-inflated "bubble" structure has a skin that weighs 1.0 lbf / ft^2. Real ones are cylindrical domes, but for this problem consider it to be a flat roof, held up by the pressure inside it. The floor area is 20,000 ft^2. All such structures have some leakage, which must be supplied by a fan that runs constantly. The true leakage area consists of many small pinholes in the

fabric, leaky seams, etc. For this problem assume that the leakage is equivalent to the frictionless flow through 5 ft^2 of opening.
(*a*) Estimate the gauge pressure inside the structure.
(*b*) Estimate the leakage flow rate that must be made up for by the fan.
(*c*) Estimate the power requirement for the fan, which is assumed to be 100 percent efficient.

5.61. Water flows (100 m^3/s) in a channel 50 m wide and spills over a sharp-edged weir.
(*a*) Estimate the difference in elevation between the upstream flow and the top of the weir.
(*b*) If the upstream channel is 5 m deep, what is the upstream velocity?
(*c*) How large a percentage error are we likely to have made in neglecting this velocity in formulating Eq. 5.22?

5.62. In Example 5.3 we computed the exit velocity by Torricelli's equation, which does not take into account the fact that at the bottom of the jet the velocity will be higher than at the top, as discussed in Sec. 5.11. How large an error are we likely to have made? If the jet is passing thorough a perfectly rounded entrance with an outlet diameter of 0.5 ft, and the centerline of the jet is 30 ft below the fluid surface, how much difference should there be between the velocities at the top and the bottom of the jet?

5.63. A slow-moving stream of water flows from a faucet into a sink. It is observed that the width of the stream decreases with distance from the faucet. If the flow leaves the faucet, vertically downward, in the form of a cylindrical jet with diameter 0.25 in. and a velocity of 1 ft / s, what will be its diameter one ft below the faucet?

5.64. In Example 5.15, if the column is expected to break into drops when its diameter is 0.1 in, how far below the faucet should this occur?

5.65. Equation 5.BJ shows V as $f(h)$ for flow from a faucet. Show the corresponding equation for D as $f(h)$.

5.66. A meteorologist, discussing a record-breaking hurricane said, "It had a pressure of 850 millibars in the center, so it had winds of 250 miles an hour!" Explain this statement in terms of B.E.

REFERENCES FOR CHAPTER 5

1. Lamb, H. *Hydrodynamics*, New York: Dover, 1945, p. 99.
2. Boyce, M. P. "Transport and Storage of Fluids." In *Perry's Chemical Engineers' Handbook, 7*, ed. R. H. Perry, D. W. Green, and J. O. Maloney. New York: McGraw-Hill, 1997, pp. 10–14.
3. "Flows of Fluids Through Valves, Fittings and Pipes," *Technical Paper No. 410,* Crane Company, 475 N. Gary Ave., Carol Stream, IL, 1957, p. A-19.
4. Fischer, K. "How to Predict Calibration of Variable-Area Flow Meters." *Chemical Engineering 59,* no. 6 (1952), pp. 180–184. One must read this paper carefully to see that it is discussing corrections of only a few percent to the simple equations presented in this text.
5. Li, S. C., ed. *Cavitation of Hydraulic Machinery,* London: Imperial College Press, 2000.
6. Kindsvater, C. E. and R. W. Carter. "Discharge Characteristics of Rectangular Thin-plate Weirs." *Transactions of the American Society of Civil Engineers 124* (1959), pp. 772–822. This long paper, with comments by others, shows that using a coefficient of 0.67 in Eq. 5.22 will reproduce most of the published test data over a wide range of conditions to ±5%.
7. de Nevers, N. "An Inexpensive Time Bomb" *Chem. Eng. Ed. 8,* 1974, pp. 98–101.

CHAPTER 6

FLUID FRICTION IN STEADY, ONE-DIMENSIONAL FLOW

In Chap. 5 we found the working form of Bernoulli's equation (B.E.)

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = \frac{dW_{\text{n.f.}}}{dm} - \mathcal{F} \tag{5.5}$$

and applied it to problems in which we could set the friction term, $\mathcal{F}$, equal to zero. In this chapter we show how to evaluate the $\mathcal{F}$ term for the very important and practical case of steady flow in one dimension, as in a pipe, duct, or channel. Using the $\mathcal{F}$ terms we evaluate here, we can use Eq. 5.5 for a much wider range of problems than those we have considered so far, including many problems of great practical interest to chemical engineers. Keep in mind that our main reason for evaluating $\mathcal{F}$ is to put the proper relation for $\mathcal{F}$ into Eq. 5.5, and then solve the resulting equation for the appropriate pressures, velocities, elevations, pipe diameters, etc.

The form of the friction-loss term is strongly dependent on the geometry of the system. The problem is much simpler if the flow is all in one direction, as in a pipe, rather than in two or three dimensions, as around an airplane. Therefore, we will first consider fluid friction in long, constant-diameter pipes in steady flow. This case is of great practical significance and is the easiest case to treat mathematically. Starting and stopping of flow in pipes are discussed in Sec. 7.4. In Sec. 6.13 we will consider the frictional drag on particles in steady, rectilinear motion, which, although it is two-dimensional, gives results quite similar to those found in long, straight pipes.

In Part IV we will investigate two- and three-dimensional flows by using some of the ideas from this chapter and introducing several others.

6.1 THE PRESSURE-DROP EXPERIMENT

The classic pressure-drop experiment to determine $\mathcal{F}$ is performed on an apparatus like that shown in Fig. 6.1. In this experiment we set the volumetric flow rate of the fluid with the flow-regulating valve. We measure the volumetric flow rate with the tank or bucket on the scale and a stop watch. At steady state we read pressure gauges P_1 and P_2 and record their difference. Usually we are interested in pressure drop per unit length, so we divide the pressure drop by distance Δx (the length of the test section) and plot $[(P_1 - P_2) / \Delta x]$ against volumetric flow rate Q.

Regardless of what Newtonian liquid is flowing or what kind of pipe we use, the result is always of the form shown in Fig. 6.2, and for all gases *at low velocities* the result is the same as that shown.

The salient features of Fig. 6.2 are that *for one specific fluid flowing in one specific pipe:*

1. At low flow rates the pressure drop per unit length is proportional to the volumetric flow rate to the 1.0 power.
2. At high flow rates the pressure drop per unit length is proportional to the volumetric flow rate raised to a power that varies from 1.8 (for very smooth pipe) to 2.0 (for very rough pipes).
3. At intermediate flow rates there is a region where the experimental results are not easily reproduced. The two curves for the other two regions are shown dotted, extrapolated into this region. The flow can oscillate back and forth between these two curves, and take up values between them. If the experimental apparatus is like that in Fig. 6.1, with a more or less constant value of dP / dx, then the volumetric flow rate will oscillate horizontally between the two curves, producing an irregular pulsing flow.

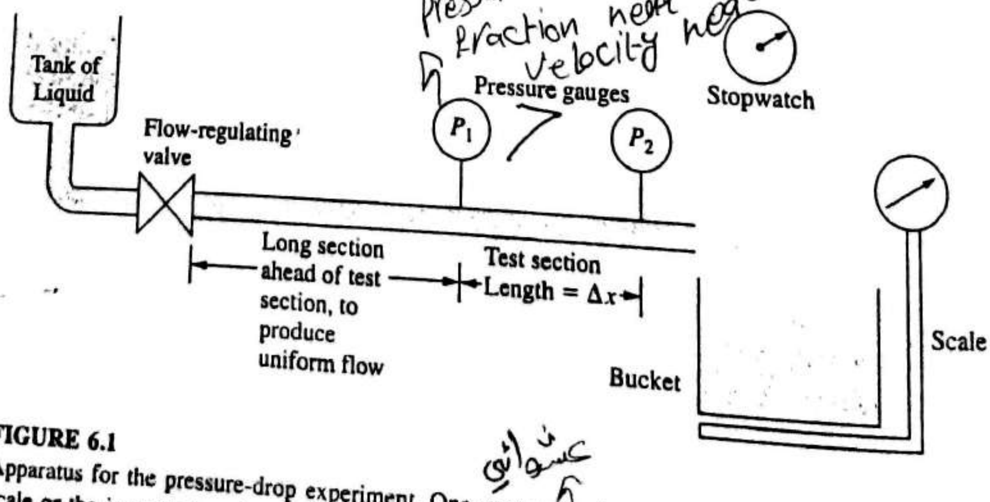


FIGURE 6.1
Apparatus for the pressure-drop experiment. One may read either the increase in weight on the scale or the increase in volume in the bucket based on calibration marks on its sides. The flow-rate-measuring method here, called "bucket and stopwatch," is the most accurate method known and is used to calibrate other methods, such as those shown in Chap. 5.

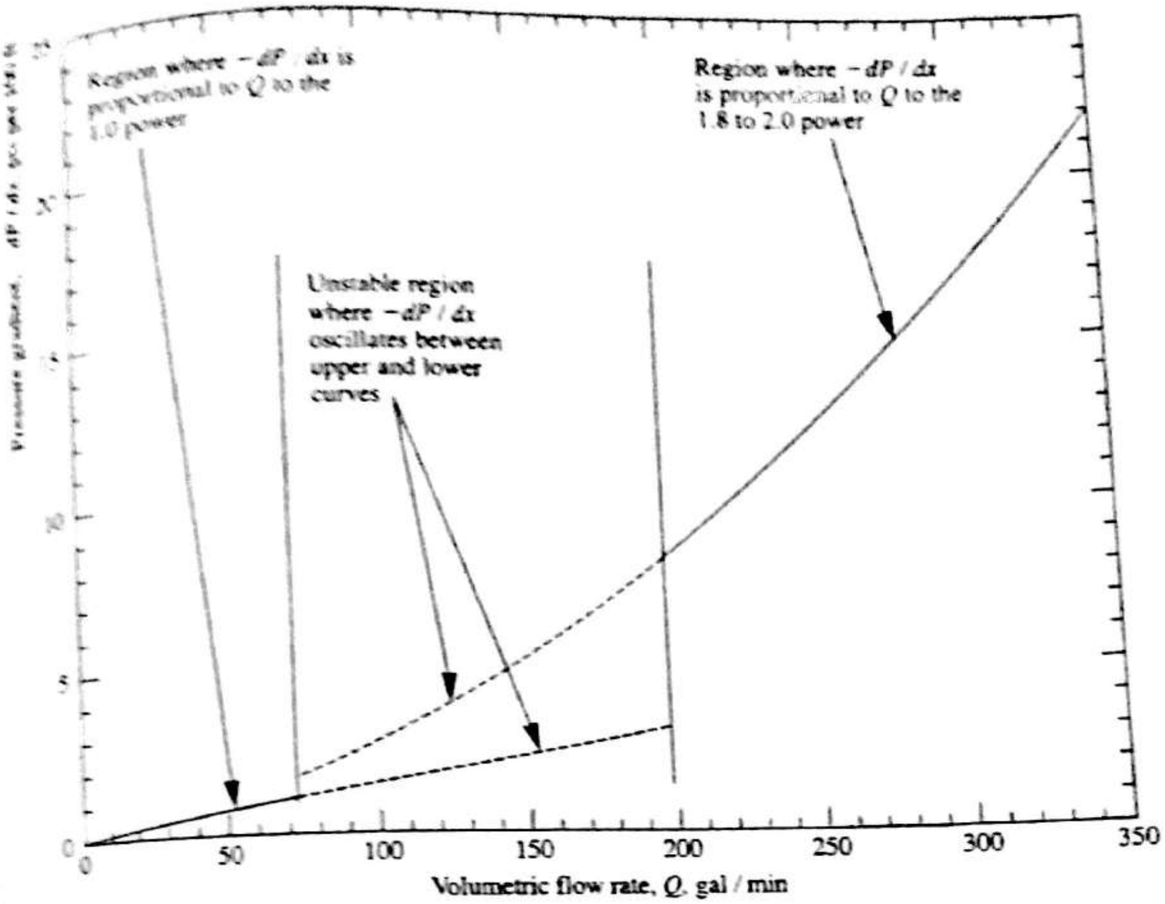


FIGURE 6.2
Typical pressure-drop curve for a specific fluid in a specific pipe. These are calculated values for an oil with SG = 1.0 and μ = 50 cP, flowing in a 3 in schedule 40 pipe. For other fluids and other pipes the plot looks the same, but the numerical values are different. If the volumetric flow rate, Q, is constant, then in the unstable region the flow will oscillate vertically between the two curves of $-dP/dx$. If, instead, $-dP/dx$ is fixed (e.g., a flow by gravity from a reservoir), then in the unstable region the flow will oscillate horizontally between two values of Q.

This experiment is relatively easy to run, and the curves have been found for many combinations of pipe and fluid. However, since all possible combinations have not been tested, it would be convenient to have some way of calculating the results of a new combination without having to test it. Furthermore, no inquisitive mind will be satisfied with Fig. 6.2 without asking why it has three regions so different from each other.

6.2 REYNOLDS' EXPERIMENT

Osborne Reynolds [1] explained the strange shape of Fig. 6.2. In an apparatus similar to that of Fig. 6.1 but made of glass, he arranged to introduce a liquid dye into the flowing stream at various points. He found that, in the low flow rate region (in which $-dP/dx$ is proportional to the flow rate), the dye he introduced formed a smooth, thin, straight streak down the pipe: there was no mixing perpendicular to the axis of the pipe. This type of flow, in which all the motion is in the axial direction, is now called *laminar flow* (the fluid appears to move in thin shells or layers, or *laminae*).

FIGURE 6.3
Laminar and turbulent flows for a thin stream of smoke rising in a room with very weak air currents.

He also found that in the high flow rate region, where $-dP/dx$ is proportional to the volumetric flow rate to the 1.8 to 2.0 power, no matter where he introduced the dye it rapidly dispersed throughout the entire pipe. A rapid, chaotic motion in all directions in the pipe was superimposed on the overall axial motion and caused the rapid, crosswise mixing of the dye. This type of flow is now called *turbulent flow.*

The two types of curve (linear and approximately parabolic) in Fig. 6.2 thus were shown to represent two radically different kinds of flow. The distinction is very important, as we will see; students should observe both types in the world about them. Perhaps the easiest example to see is the smoke from a cigarette rising in a still room shown in Fig. 6.3. The smoke rises in a smooth, laminar flow for about a foot, and then the flow converts to turbulent flow, with random chaotic motion perpendicular to the major, upward flow direction. This case, although easy to demonstrate in the laboratory or in the living room, is much harder to analyze mathematically than Reynolds' pipe-flow experiment, so we return to the latter.

Reynolds showed further that the region of unreproducible results between the regions of laminar and of turbulent flow is the region of transition from the one type of flow to the other, called the *transition region.* The reason for the poor reproducibility here is that laminar flow can exist in conditions in which it is not the stable flow form, but it fails to switch to turbulent flow unless some outside disturbance such as microscopic roughness on the pipe wall or very small vibrations in the equipment triggers the transition. Thus, in the transition region the flow can be laminar or turbulent, and the pressure drop or flow rate can suddenly change by a factor of 2. Under some circumstances the flow can alternate back and forth between being laminar and turbulent, causing the pressure drop to oscillate between a higher and a lower value; or for a constant pressure drop, as in Fig. 6.1, the velocity can oscillate between a higher and a lower value.

Besides clarifying the strange shape of Fig. 6.2, Reynolds made the most celebrated application of dimensional analysis (Chap. 9) in the history of fluid mechanics. He showed that for smooth, circular pipes, for all Newtonian fluids, and for all pipe diameters the transition from laminar to turbulent flow occurs when the dimensionless group $DV\rho/\mu$ has a value of about 2000. Here D is the pipe diameter, V is the average fluid velocity in the pipe, ρ is the fluid density, and μ is the fluid viscosity. This dimensionless group is now called the Reynolds number, $\mathcal{R}$:

$$\begin{pmatrix}\text{Reynolds number for}\\ \text{flow in a circular pipe}\end{pmatrix} \quad \mathcal{R} = \frac{DV\rho}{\mu} = \frac{DV}{\nu} = \frac{4Q}{\pi D \nu} \tag{6.1}$$

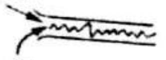
Re < 2100 Liminar

Re > 4000 turbulent

< Re < 4000

Re = ρ V D → Diamiter

TABLE 6.1
Comparison of laminar, transition, and turbulent flows

	Type of flow		
	Laminar	**Transition**	**Turbulent**
Behavior of dye streak	Dye in  Flow	Oscillates between laminar and turbulent	Dye in  Flow
Pressure drop proportional to	$Q^{1.0}$	Oscillates from one value to another; very difficult to measure	$Q^{1.8}$ (very smooth pipes) to $Q^{2.0}$ (very rough pipes)
Reynolds number	<2000	≈2000 to 4000	>4000

The transition region on Fig. 6.2 corresponds to Reynolds numbers between about 2000 and 4000. For Reynolds numbers above about 4000, the flow is stably turbulent. For flows other than pipe flow, some other appropriate length is substituted for the pipe diameter in the Reynolds number, producing a different Reynolds number, as will be discussed later. All Reynolds numbers are (some length · velocity · density / viscosity).

The difference between laminar and turbulent flows is one of the most important differences in fluid mechanics. The equations in this book for laminar flow do not describe turbulent flow, nor do the turbulent flow equations describe laminar flow. If you learn nothing else in this chapter, learn that. In pipe flow, the boundary between laminar and turbulent flow is the region from Reynolds number ≈2000 to ≈4000. This means that almost all flows of gases and liquids like water in ordinary-sized pipes are turbulent. The only exceptions to that statement are flows of fluids much more viscous than water, such as asphalt, maple syrup, or polymer solutions. (The fluid used as an example to make up Fig. 6.2 is 50 times as viscous as water; if that figure had been made for water, the laminar region would have practically disappeared into the left axis!) However, in very small tubes or other flow passages the flow is normally laminar. The flow in the heart and the major arteries near it in our bodies and those of most animals our size are turbulent. The rest of the blood flow in our bodies is laminar, as is the flow of fluids in filters, in groundwater, and in oil fields. (These latter are not exactly pipe flow, but as shown in Chap. 11, the flow passages between the solid particles in filters and in the ground behave as irregular-shaped pipes.) River flows are mostly turbulent, and the main flows of the atmosphere are turbulent, but in low-wind situations and in the stratosphere the atmosphere can be laminar. Both laminar and turbulent flows are important; you could not read this statement without the turbulent flow near your heart or the laminar flow of blood to your brain and eyes.

The results of Reynolds' experiments are summarized on Table 6.1.

6.3 LAMINAR FLOW

Laminar flow is the simplest flow, so we discuss it first. Consider a steady laminar flow of an incompressible Newtonian fluid in a horizontal circular tube or pipe. A section of the tube Δx long with inside radius r_0 is shown in Fig. 6.4. We arbitrarily select a rod-shaped

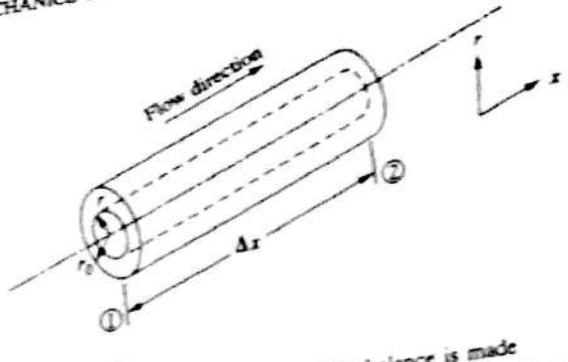


FIGURE 6.4
Force-balance system in pipe flow. The balance is made around the cylindrical rod-shaped volume, symmetrical about the centerline.

element of the fluid, symmetrical about the center, with radius r, and compute the forces acting on it. Here it is assumed that location 1 is well downstream from the place where the fluid enters the tube. This analysis is not correct for the tube entrance (see Part IV). The flow is steady and all in the axial direction. There is no acceleration in the x direction, so the sum of the forces acting in the x direction on the rod-shaped element we have chosen must be zero. There is a pressure force acting on each end, equal to the pressure times the cross-sectional area of the end. These act in opposite directions; their sum *in the positive x direction* is

$$\text{Pressure force} = P_1(\pi r^2) - P_2(\pi r^2) = \pi r^2(P_1 - P_2) \tag{6.2}$$

Along the cylindrical surface of our rod-like element the pressure forces have no component in the x direction and can be ignored, but there is a shear force resisting the flow. The shear force acts in the direction opposite to the pressure gradient, which is in the flow direction, and its magnitude is

$$\text{Shear force} = 2\pi r\,\Delta x \cdot (\text{shear stress at } r) = 2\pi r\,\Delta x \cdot \tau \tag{6.3}$$

Since the pressure force and shear force are the only forces acting in the x direction, and since the sum of the forces is zero, these must be equal and opposite. Equating their sum to zero and solving for the shear stress at r, we find

$$\tau = \begin{pmatrix}\text{Shear stress acting} \\ \text{on the central rod} \\ \text{at radius } r\end{pmatrix} = \frac{-r(P_1 - P_2)}{2\Delta x} \tag{6.4}$$

The minus sign shows that our intuition is correct, and τ acts in the minus x direction. (See the discussion of the sign of the shear stress in Sec. 1.5). Equation 6.4 applies to steady laminar or turbulent flow of any kind of fluid in any circular pipe or tube.

Here we have applied Newton's law, $F = ma$, to the particularly simple case in which there is no acceleration and sum of the forces is therefore zero; in Chap. 7 and Part IV we will see how to apply it to more complicated cases.

We saw in Chap. 1 that for Newtonian fluids in laminar motion the shear stress is equal to the product of the viscosity and the velocity gradient. Substituting in Eq. 6.4, we find

$$\mu \frac{dV}{dr} = -r\frac{P_1 - P_2}{2\Delta x} \tag{6.5}$$

For steady laminar flow the pressure gradient $(P_1 - P_2)/\Delta x$ does not depend on radial position in the pipe, so we may integrate this to

$$V = \frac{-r^2}{4\mu} \cdot \frac{P_1 - P_2}{\Delta x} + \text{constant} \tag{6.6}$$

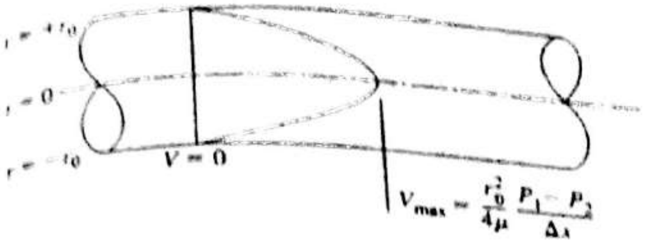


FIGURE 6.5
Velocity distribution in steady, laminar flow of a Newtonian fluid in a circular pipe.

To solve for the value of the constant, we need one observational fact: that for the flow of everything except rarefied gases, the fluid at the solid surface clings to the surface. This is not an intuitively obvious fact, and we cannot derive it from some prior principle. The behavior is quite different from that of solids, whose sliding surfaces do slip over one another so that there is a sharp discontinuity in velocity at the sliding boundary. However, one may observe that it is so by watching the behavior of bits of wood or leaves on the surface of a stream: those at the center move rapidly, those near the bank slowly, and those right at the bank not at all (this condition is often referred to as the *no-slip condition;* one kind of rarefied gas flow is called, logically enough, *slip flow*). From this observational fact it follows that at $r = r_0$ (at the pipe wall), $V = 0$; so

$$0 = \frac{-r_0^2}{4\mu}\cdot\frac{P_1 - P_2}{\Delta x} + \text{constant} \tag{6.7}$$

Substituting this value of the constant in Eq. 6.6 and factoring, we find

$$V = \frac{r_0^2 - r^2}{4\mu}\cdot\frac{P_1 - P_2}{\Delta x} \tag{6.8}$$

This equation says that for steady, laminar flow of Newtonian fluids in circular pipes:

1. The velocity is zero at the tube wall ($r = r_0$).
2. The velocity is a maximum at the center of the pipe ($r = 0$).
3. The magnitude of this maximum velocity is

$$V_{max} = \frac{r_0^2}{4\mu}\cdot\frac{P_1 - P_2}{\Delta x} \tag{6.A}$$

4. The pressure drop per unit length is independent of fluid density and is proportional to the first power of the local velocity and the first power of the viscosity.
5. The velocity-radius plot is a parabola; see Fig. 6.5.

In engineering we are generally more interested in the volumetric flow rate Q than in the local velocity V. To find the Q of a uniform-velocity flow we multiply the velocity by the cross-sectional area perpendicular to flow (see Table 3.1). The veloc-ity of the laminar flow described above is not uniform, so we must integrate velocity times area over the whole pipe cross section.

$$Q = \int_{\text{tube}} V\,dA = \int_{r=0}^{r=r_0} \frac{r_0^2 - r^2}{4\mu}\cdot\frac{P_1 - P_2}{\Delta x}\cdot 2\pi r\,dr$$

$$= \frac{P_1 - P_2}{\Delta x}\cdot\frac{\pi}{2\mu}\left[\frac{r_0^2 r^2}{2} - \frac{r^4}{4}\right]_{r=0}^{r=r_0} = \frac{P_1 - P_2}{\Delta x}\cdot\frac{\pi}{\mu}\cdot\frac{r_0^4}{8} = \frac{P_1 - P_2}{\Delta x}\cdot\frac{\pi}{\mu}\cdot\frac{D_0^4}{128} \tag{6.9}$$

This equation was developed by Hagen and also, independently, by Poiseuille [2a]. In the United States it is most commonly called the Poiseuille equation (pronounced "pwah-zoo-y"). It shows that the pressure drop $(P_1 - P_2) / \Delta x$ is proportional to the first power of the volumetric flow rate Q, as shown in Fig. 6.2. The solution is immensely satisfying; using only very simple mathematics, we find a complete description of the flow. The description has been experimentally verified so well that, when laminar-flow experiments in circular pipes disagree with it, the experiments are in error.

From Eq. 6.9 it can also be shown (Prob. 6.4) that

$$V_{avg} = \frac{Q}{\pi r^2} = \frac{V_{max}}{2} \tag{6.10}$$

$$\text{Average ke} = \frac{\int (V^2/2)\, dQ}{\int dQ} = \frac{\int (V^2/2) V \cdot 2\pi r\, dr}{\int V \cdot 2\pi r\, dr} = (V_{avg})^2 \tag{6.11}$$

Both of these values are shown in Table 3.1.

To see how this fits in with B.E. (Eq. 5.5), we apply B.E. from point 1 to point 2 in Fig. 6.4 and substitute for $-\Delta P$ from Eq. 6.9 to find

$$\mathscr{F} = \frac{-\Delta P}{\rho} = Q\, \Delta x \frac{\mu}{\rho} \cdot \frac{128}{\pi D_0^4} \tag{6.12}$$

Equation 6.12 relates the $\mathscr{F}$ in B.E. to the flow rate, diameter, length, and (viscosity / density) of a horizontal flow in which gravity plays no role. If we repeat the entire derivation for a vertical flow in a pipe in which the pressure is constant throughout (Prob. 6.1), we find that the ΔP term is replaced with a $\rho g\, \Delta z$ term. When we substitute this in Eq. 6.12 and then calculate $\mathscr{F}$ from B.E., we find

$$\mathscr{F} = -g\Delta z = Q\, \Delta x \frac{\mu}{\rho} \cdot \frac{128}{\pi D_0^4} \tag{6.13}$$

Thus, for either horizontal or vertical laminar flow we find

$$\boxed{\mathscr{F} = Q\, \Delta x \frac{\mu}{\rho} \cdot \frac{128}{\pi D_0^4}} \quad \begin{bmatrix}\text{laminar flow} \\ \text{only!}\end{bmatrix} \tag{6.14}$$

We may readily extend the argument to show that this equation applies also to flows at any angle to the vertical, so that Eq. 6.14 is the general description of friction heating in laminar flow of Newtonian fluids in circular pipes.

Example 6.1. Oil at a rate of 50 gal / min is flowing steadily from tank A to tank B through 3000 ft of 3-in schedule 40 pipe; see Fig. 6.6. (Appendix A.2 shows the dimensions of standard U.S. schedule 40 pipe sizes. The inside diameter (ID) of 3-in schedule 40 pipe is 3.068 in.) The oil has a density of 62.3 lbm / ft³ and a viscosity of 50 cP. The levels of the free surfaces are the same in both tanks. Tank B is vented to the atmosphere. What is the gauge

Q = 50 gal/min ρ = 62.3 lbm/ft³ ΔZ = 0

Δx = 3000 ft D₀ = 3.068

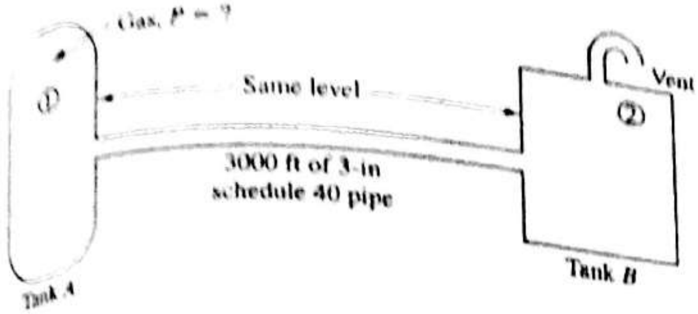


FIGURE 6.6
A pressure-driven fluid transfer from one tank to another, used in Examples 6.1 and 6.4.

pressure in tank A required to produce this flow rate? (Pushing liquids from one container to another with gas pressure is common in industry, particularly when a leak of the fluid would be dangerous.)

Applying B.E. between the free surface in tank A (point 1) and the free surface in tank B (point 2), we see that the velocities are negligible. Since there is no change in elevation or pump or compressor work, we have

$$\Delta \frac{P}{\rho} = -\mathscr{F} \tag{6.B}$$

The density is constant, so in tank A the gauge pressure $(P_1 - P_2)$ is $-(-\rho\mathscr{F})$. *If the flow in the pipe is laminar*, then we can solve for this pressure from Eq. 6.14. The average velocity is

$$V_{\text{avg}} = \frac{Q}{A} = \frac{50 \text{ gal / min}}{(\pi / 4)(3.068 \text{ in})^2} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} \cdot \frac{\text{min}}{60 \text{ s}} \cdot \frac{\text{ft}^3}{7.48 \text{ gal}}$$
$$= 2.17 \frac{\text{ft}}{\text{s}} = 0.66 \frac{\text{m}}{\text{s}} \tag{6.C}$$

Therefore, the Reynolds number is

$$\mathscr{R} = \frac{(3.068 / 12) \text{ ft} \cdot 2.17 \text{ ft / s} \cdot 62.3 \text{ lbm / ft}^3}{50 \text{ cP} \cdot 6.72 \cdot 10^{-4} \text{ lbm / (ft} \cdot \text{s} \cdot \text{cP)}} = 1028 \tag{6.D}$$

As shown before, steady pipe flow is laminar if $\mathscr{R} < 2000$; so we have laminar flow here and are safe in substituting for $\mathscr{F}$ from Eq. 6.14. Multiplying through by $-\rho$, we find

$$-\Delta P = P_1 - P_2 = Q \frac{128}{\pi} \cdot \frac{\mu}{D_0^4} \Delta x$$
$$= 50 \frac{\text{gal}}{\text{min}} \cdot \frac{128}{\pi} \cdot \frac{50 \text{ cP}}{(3.068 \text{ in})^4} \cdot 3000 \text{ ft} \cdot 231 \frac{\text{in}^3}{\text{gal}}$$
$$\cdot 2.09 \cdot 10^{-5} \frac{\text{lbf} \cdot \text{s}}{\text{cP} \cdot \text{ft}^2} \cdot \frac{\text{min}}{60 \text{ s}} \cdot \frac{\text{ft}}{12 \text{ in}} = 23.1 \frac{\text{lbf}}{\text{in}^2} = 159 \text{ kPa} \tag{6.E}$$

This is the gauge pressure in tank A required to produce a flow of 50 gpm. ■

Readers may check that this corresponds to

$$-\frac{\Delta P}{\Delta x} = \frac{23.1 \text{ psi}}{3000 \text{ ft}} = 0.77 \frac{\text{psi}}{100 \text{ ft}} \tag{6.F}$$

$Q = \rho A$

$R = \frac{D \rho V}{\mu}$

which is the value plotted for 50 gpm in Fig. 6.2. The laminar part of that figure was made by repeating this example for a variety of flow rates (on a spreadsheet).

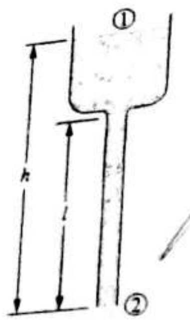


FIGURE 6.7
Typical capillary viscometer; see Example 6.2.

Example 6.2. A typical capillary *viscometer* (a device for measuring viscosity) has the flow diagram shown in Fig. 6.7. It consists of a large-diameter reservoir and a long, small-diameter, vertical tube. The sample is placed in the reservoir, and the flow rate due to gravity is measured. The tube is 0.1 m long and has a 1 mm ID. The height of the fluid in the reservoir above the inlet to the tube is 0.02 m. The fluid being tested has a density of 1050 kg / m^3. The flow rate is 10^{-8} m^3 / s. What is the viscosity of the fluid?

Applying B.E. between the free surface in the reservoir (point 1) and the fluid leaving the bottom of the viscometer (point 2), we see that the pressure at each point is atmospheric and that there is no pump or compressor work. We can neglect the velocity in the reservoir, so B.E. becomes

$$g(z_2 - z_1) + \frac{V_2^2}{2} = -\mathscr{F} \tag{6.G}$$

The kinetic-energy term here is negligible compared with the other two terms. This is found in most laminar-flow problems, so we drop the kinetic-energy term and find

$$\mathscr{F} = -g\,\Delta z \tag{6.H}$$

Substituting for $\mathscr{F}$ from Eq. 6.14 and solving for μ, we find

$$\mu = \frac{\rho g(-\Delta z)\,\pi D_0^4}{128 Q\,\Delta x}$$

$$= \frac{1050\ \text{kg/m}^3 \cdot 9.81\ \text{m/s}^2 \cdot 0.12\ \text{m} \cdot \pi \cdot (0.001\ \text{m})^4}{128 \cdot (10^{-8}\ \text{m}^3/\text{s}) \cdot 0.1\ \text{m}} \cdot \frac{10^3\ \text{cP} \cdot \text{m} \cdot \text{s}}{\text{kg}}$$

$$= 30.3\ \text{cP} = 0.0303\ \text{Pa} \cdot \text{s} \tag{6.I}$$

■

Viscometers of the same type, but slightly more complicated than the one described above, are very widely used. In using them we recognize that:

1. They must not be used at flow rates so high that flow is turbulent (Prob. 6.8).
2. They must be long enough that the error introduced by applying the Poiseuille equation (which only applies well downstream from the entrance) to the whole tube is small. For a brief introduction to the problem of *entrance flow*, to which Poiseuille's equation does not apply, see Part IV.
3. The Poiseuille equation applies only to Newtonian fluids, so this type of apparatus can be used simply only for such fluids; see Chap. 13.

4. Because the viscosity measured by such a device is proportional to D^4, a small error in the diameter measurement leads to a large error in the viscosity measurement. For this reason these devices ordinarily are calibrated by using a fluid of known viscosity and by determining the appropriate average diameter from the calibration.

5. As discussed in Chap. 5, the $\mathscr{F}$ term represents the conversion of mechanical energy into internal energy. Normally this conversion results in an increase of temperature. It can be shown (Prob. 6.6) that in this case the temperature change is negligible. However, in more viscous liquids, which are pumped through capillary viscometers, there can be a significant temperature rise. In most fluids a small temperature rise can cause a large viscosity change, so the temperature rise must be minimized.

6. The commercial versions of this device normally require the user to use a stopwatch and measure the time for the liquid level to pass from one mark on the glass tube to another. The resulting measurement is a time. For that reason viscosities are often reported as times, e.g., *Saybolt Seconds Universal* (SSU), the standard viscosity measurement for fuel oils in the United States. Formulae for converting from SSU or other time-for-the-interface-to-pass-the-marks in various standard capillary-tube-gravity viscometers are given in handbooks.

6.4 TURBULENT FLOW

Why does the preceding analysis not work for turbulent flow? Equation 6.4 is correct for steady laminar or turbulent flow of any kind of fluid, but the substitution of $\mu(dV/dy)$ for the shear stress is correct only for *laminar flow* of *Newtonian fluids*. In laminar flow in a tube there is no motion perpendicular to the tube axis. In turbulent flow there is no *net* motion perpendicular to the tube axis, but there does exist an intense, local, oscillating motion perpendicular to the tube axis. The transfer of fluid perpendicular to the net axial motion causes an increase in shear stress over the value given above for laminar flow of Newtonian fluids. This is most easily seen in an analogy. Consider two students playing catch with baseballs. One is standing on the ground, the other on a railroad car; see Fig. 6.8.

In Fig. 6.8(a) the railroad car is not moving, and both students throw the ball back and forth in the plus and minus y direction. Each time one catches the ball, the student experiences a force and, if the student throws it back at the same speed, the student exerts an equal force in the opposite direction. Therefore, the net effect of their throwing the ball back and forth is that a force is exerted on each one, tending to move them apart in the plus or minus y direction. There is no force in the x direction.

In Fig. 6.8(b) the train is moving at constant speed in the x direction. Each student still throws the ball in the plus or minus y direction. However, because of their relative motion, each one receives the ball moving, not in the y direction, but at an angle between the x and y directions; the directions of the balls (relative to the two students) are shown by the arrows. Since each one receives the balls in these directions, the force exerted by a student in stopping the ball consists, not only of the

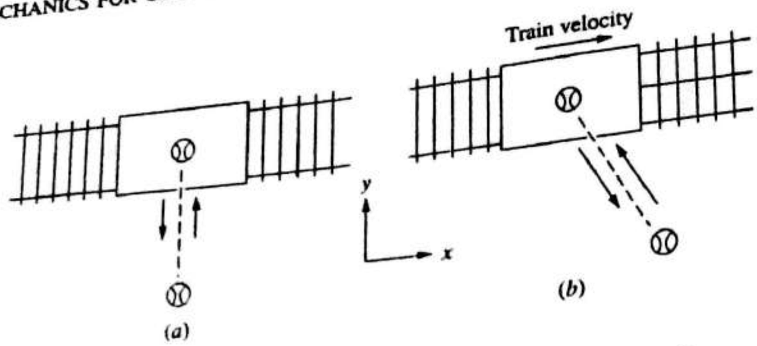


FIGURE 6.8
Illustration by analogy of shear forces due to turbulence: (*a*) top view of students playing catch, neither moving; (*b*) top view of students playing catch, one moving perpendicular to the direction of throwing the ball.

y component, which the other student put into the ball by throwing it, but also of the x component due to their relative motion. When the train is moving, in addition to the y-directed force there is a force tending to retard the train and to drag the stationary student along in the x direction.

Exactly the same thing happens in turbulent fluid flow. The exchange of fluid between the faster-moving fluid in the center of the tube and the slower-moving fluid near the wall increases the shear stress over that which would exist in laminar flow. This extra stress is called a *Reynolds stress* after Reynolds, who first explained it. Thus, the actual $\mathcal{F}$ in turbulent flow is greater than that predicted by Poiseuille's equation.

In the case of the students throwing the balls, the extra stress is proportional to the velocity of the train and the number of times per second which they throw the balls back and forth. In the case of the corresponding stress in a fluid in turbulent flow the stress is proportional to the velocity gradient dV/dy times the average mass of fluid passing back and forth across a surface of constant y (across which there is no *net* flow). Since the velocity goes from zero at the pipe wall to the average velocity near the center, the velocity gradient should be some function of V_{avg}/D. If we now assume that this is a linear proportion and that the magnitude of the flow of mass back and forth across a surface of constant y is proportional to the average velocity, then it follows that

$$\mathcal{F} \propto \frac{V_{\text{avg}}^2}{D} \tag{6.15}$$

Furthermore, the friction heating should be proportional to the length of the pipe. Including this idea, we find

$$\mathcal{F} \propto \frac{\Delta x V_{\text{avg}}^2}{D} \tag{6.16}$$

This equation rests on the plausible assumption that the friction heating is proportional to the length of the pipe and on the more questionable assumption that it is proportional to V_{avg}^2/D. Do these assumptions agree with the experimental data? The answer

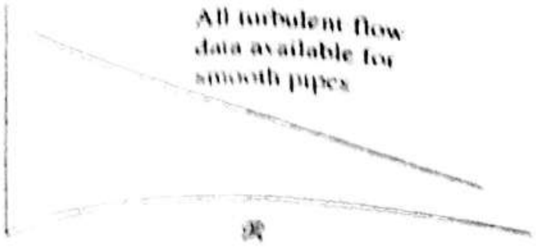


FIGURE 6.9
Blasius and Stanton's friction factor plot.

is, Yes and no. To save writing, we now define a new term, the *friction factor f*, which is equal to half the proportionality constant in Eq. 6.16, and drop the avg subscript on the velocity, so that

$$\mathcal{F} = 2f\frac{\Delta x V^2}{D} = 4f\frac{\Delta x}{D}\cdot\frac{V^2}{2} \tag{6.17}$$

$$f = \text{friction factor}$$

$$= \frac{\mathcal{F}}{4(\Delta x/D)(V^2/2)} \tag{6.18}$$

$F = f\,\dfrac{4\Delta x V^2}{2D}$

To test these assumptions, Blasius and Stanton [2*b*] calculated the friction factor for a large variety of pipe-flow experiments with smooth pipes. They found that the friction factor was not a constant, as predicted by the above simple theory, but decreased slowly with increasing Reynolds number. However, all the data for smooth pipes of various diameters at all velocities for a large range of fluids formed a single curve on a plot of friction factor versus Reynolds number; see Fig. 6.9.

Once plots like this came into common use, it became apparent that they were very good for smooth pipes, such as glass pipes or drawn metal tubing, but that the pressure drops they predicted were too low for rough pipes, such as those made from cast iron or concrete. It appeared that the roughness of the pipe surface influenced f. To resolve the question Nikuradse [3] measured the pressure drop in various smooth pipes to the inside of which he had glued sand grains. He found that for a given value of ε/D, where ε is the size of the sand particle and D is the pipe diameter, he could plot all his results on one curve on Fig. 6.9, but that there were different curves for different values of ε/D. This ratio, ε/D, is called the *relative roughness*. Figure 6.10 is currently the most commonly used friction factor plot (which chemical engineers normally call a *friction factor plot*, and some other disciplines refer to as a *Moody diagram*), prepared by Moody [4*a*], who based it on Nikuradse's data and on all the other available data on flow in pipes.* Moody also suggested the working values for the absolute roughness, shown in Table 6.2.

Figure 6.10 shows that, as the relative roughness becomes greater and greater, the assumptions that went into Eq. 6.16 become better and better; f becomes a constant that is independent of diameter, velocity, density, and fluid viscosity.

To make life hard for the working engineer, there are two values of the friction factor in common use The one shown in Eq. 6.18 appears in most chemical engineering books, but in mechanical engineering and civil engineering books there appears

$$f_{\text{civ, mech}} = \frac{\mathcal{F}}{(\Delta x/D)(V^2/2)} = 4f_{\text{chem}} \tag{6.19}$$

*The friction behavior of a pipe with sand grains glued to the wall is somewhat different from that of a commercial pipe. This is believed to be due to the wide range of sizes and shapes of the rough spots in a commercial pipe compared with the uniform size and shape of the sand grains used by Nikuradse. Moody [4] made Fig. 6.9 according to the Colebrook equation [5], which agrees with the data on commercial pipes. The differences between the two kinds of roughness have been discussed ([6], p. 529).

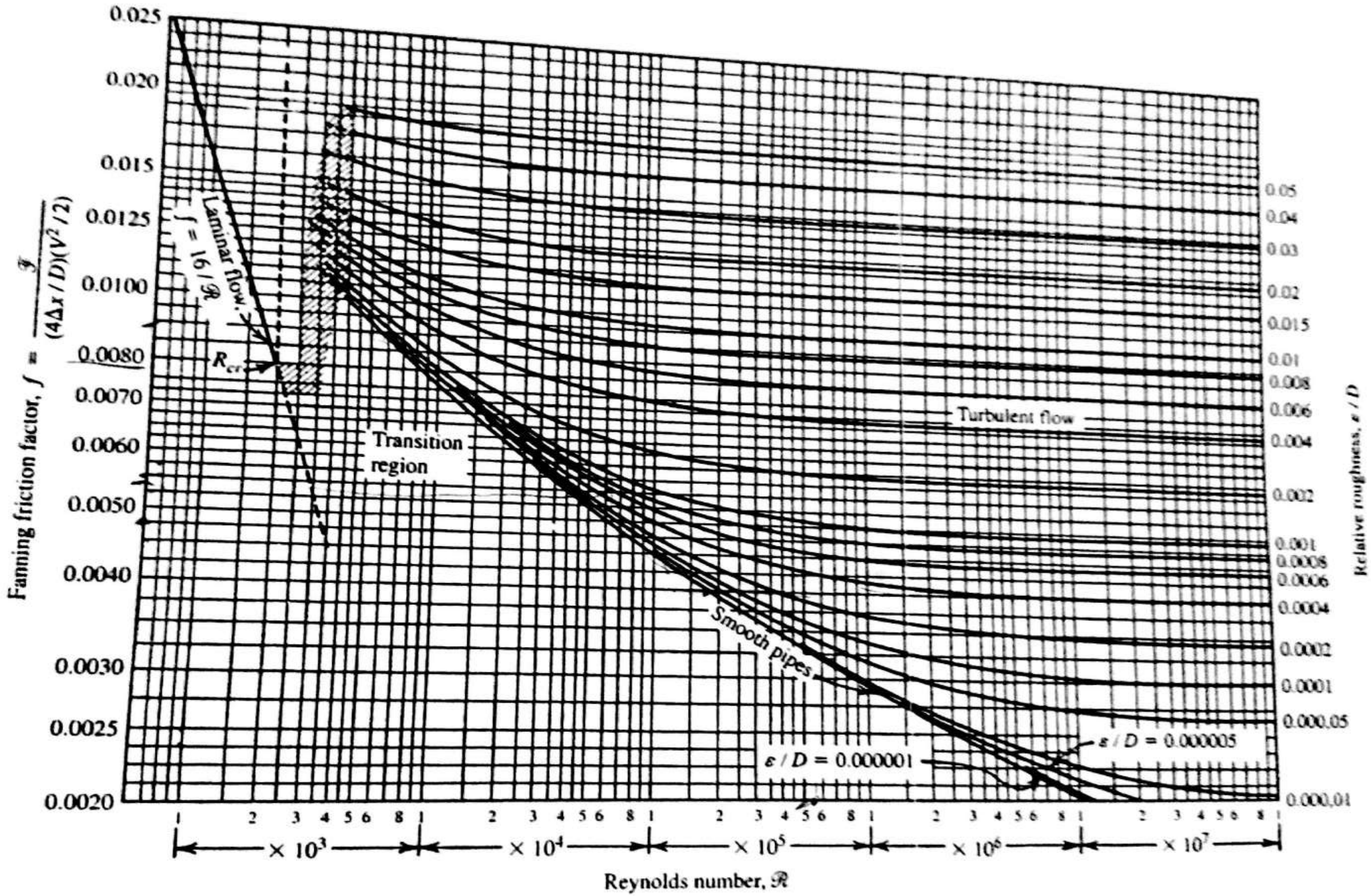


FIGURE 6.10
Friction factor plot for circular pipes. (From L. W. Moody, "Friction factors for pipe flow," *Trans. ASME* 66:672 (1944). Reproduced by permission of the publisher.)

TABLE 6.2
Values of surface roughnesses for various materials,* to be used with Fig. 6.10

	Surface roughness	
	ε, ft	ε, in
Drawn tubing (brass, lead, glass, etc.)	0.000005	0.00006
Commercial steel or wrought iron	0.00015	0.0018
Asphalted cast iron	0.0004	0.0048
Galvanized iron	0.0005	0.006
Cast iron	0.00085	0.010
Wood stave	0.0006–0.003	0.0072–0.036
Concrete	0.001–0.01	0.012–0.12
Riveted steel	0.003–0.03	0.036–0.36

*From Moody [4].

The existence of the two values means that, whenever engineers plan to use a chart like Fig. 6.10 or an equation with f in it, they must check to see on which of the two f values the chart or equation is based. Throughout this book we will use the value of f_{chem} defined by Eq. 6.18. It is often called the *Fanning friction factor*, while the one 4 times as large is called the *Darcy* or *Darcy-Weisbach friction factor*: $f_{\text{Fanning}} = \tau / (\rho V^2 / 2)$; $f_{\text{Darcy-Weisbach}} = 4\tau / (\rho V^2 / 2)$.

There is really not much point in having a curve for laminar flow on a friction factor plot, since laminar flow in a pipe can be solved analytically. Poiseuille's equation (Eq. 6.8) can be rewritten (Prob. 6.12) as

$$f = \frac{16}{\mathcal{R}} \tag{6.20}$$

Plotting any equation this simple is unnecessary. However, the laminar-flow line usually is included in friction factor plots, as it is in Fig. 6.10. Furthermore, the turbulent and transition region curves on Fig. 6.10 can be represented with very good accuracy by [7]

$$f = 0.001375 \cdot \left[1 + \left(20{,}000\,\frac{\varepsilon}{D} + \frac{10^6}{\mathcal{R}}\right)^{1/3}\right] \tag{6.21}$$

which has no theoretical basis, but reproduces the turbulent region in Fig. 6.10 well (see Prob. 6.39).

Example 6.3. Read the value of the friction factor from Fig. 6.10 for $\mathcal{R} = 10^5$ and $(\varepsilon / D) = 0.0002$, and compare that value to the value from Eq. 6.21.

From Fig. 6.10, as closely as I can read it, $f = 0.00475$. From Eq. 6.21,

$$f = 0.001375 \cdot \left[1 + \left(20{,}000 \cdot 0.0002 + \frac{10^6}{10^5}\right)^{1/3}\right] = 0.0047 \tag{6.J}$$

The difference between the two values is less than our ability to read Fig. 6.10. ■

As seen here, Fig. 6.10 can be reduced to two equations; so why bother with it? It has great historic significance and considerable intuitive content. Most modern engineers have quick computer programs that solve the type of problems presented in most of the rest of this chapter. So the hand solutions, using Fig. 6.10, are presented to help the student understand and develop an intuitive feel for what is going on in those programs. The student is advised to program Eq. 6.21 into a spreadsheet and, after making a few chart lookups on Fig. 6.10, only to glance at that figure and use the spreadsheet, which is its equivalent, to find working values for problems (and exams, if you can bring your spreadsheet with you!).

6.5 THE THREE FRICTION FACTOR PROBLEMS

The friction factor plot, Fig. 6.10, relates six parameter of the flow:

1. Pipe diameter, D.
2. Average velocity, V_{avg}.
3. Fluid density, ρ.
4. Fluid viscosity, μ.
5. Pipe roughness, ε.
6. The friction heating per unit mass, $\mathcal{F}$.

Therefore, given any five of these, we can use Fig. 6.10 to find the sixth.

Often, instead of being interested in the average velocity V_{avg}, we are interested in the volumetric flow rate,

$$Q = \frac{\pi}{4} D^2 V_{avg} \tag{6.K}$$

The three most common types of problem are shown in Table 6.3. For all of these problems the equations to be solved are shown in Table 6.4. This appears to be a formidable list of equations, but as the following examples show, their solution, while tedious by hand, is straightforward, and they are readily solved by computer. We will begin with a type 1 problem by hand and then do types 2 and 3 by spreadsheet.

Example 6.4. In Example 6.1, Fig. 6.6, we have decided that we wish to transport 300 gal / min, instead of the 50 gal / min in that example. Now what is the required pressure in Tank *A?*

This is 6 times the required volumetric flow rate in Example 6.1; the average velocity is $V = 2.17 \text{ ft/s} \cdot 6 = 13.0 \text{ ft/s}$. If the flow here were laminar, as it was in that example, then we could simply multiply the required pressure by 6. But in that example the Reynolds number $\mathcal{R} = 1028$. Here we have the same pipe diameter, viscosity, and density, but 6 times the velocity, so we must have $\mathcal{R} = 1032 \cdot 6 = 6164$. This is >4000, so the flow is sure to be turbulent.

TABLE 6.3
The three friction factor problems

Type	Given	To find
1	$D, \varepsilon, \rho, \mu, Q$	$\mathcal{F}$
2	$D, \varepsilon, \rho, \mu, \mathcal{F}$	Q
3	$\varepsilon, \rho, \mu, \mathcal{F}, Q$	D

TABLE 6.4
The equations to be solved in all pipe-flow-with-friction problems

Bernoulli's equation	$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = \frac{dW_{n.f.}}{dm} - \mathcal{F}$
Friction heating term in B.E.	$\mathcal{F} = 4f\frac{\Delta x}{D}\cdot\frac{V^2}{2}$
Reynolds number	$\mathcal{R} = \frac{DV\rho}{\mu} = \frac{DV}{\nu} = \frac{4Q}{\pi D\nu}$
Friction factor, laminar flow, if $\mathcal{R} < 2000$ or	$f = \frac{16}{\mathcal{R}}$
Friction factor, turbulent flow, if $\mathcal{R} > 4000$	$f = 0.001375\cdot\left[1 + \left(20{,}000\frac{\varepsilon}{D} + \frac{10^6}{\mathcal{R}}\right)^{1/3}\right]$
Volumetric flow rate as function of velocity, some problems	$Q = \frac{\pi}{4}D^2V_{avg}$

To use Fig. 6.10 or Eq. 6.21, we need a value of ε/D. Reading the value for commercial steel pipe from Table 6.2, we have

$$\frac{\varepsilon}{D} = \frac{0.0018\text{ in}}{3.068\text{ in}} = 0.0006 \tag{6.L}$$

Then we enter Fig. 6.10 at the right at $\varepsilon/D = 0.0006$ and follow that curve to the left to $\mathcal{R} = 6192$, finding (as best we can read that crowded part of the chart) $f = 0.009$. We may check that value from Eq. 6.21, finding 0.00905. We will discuss later the uncertainties in friction factor values, so for now we accept 0.0091 as a good estimate of f.

The B.E. analysis is the same as in Example 6.1, leading to Eq. 6.B. Combining that with Eqs. 6.17 and 6.21, we find

$$\Delta P = 4f\frac{\Delta x}{D}\rho\frac{V^2}{2}$$

$$= 4\cdot 0.0091\frac{3000\text{ ft}}{(3.068/12)\text{ ft}}\cdot 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{(13.0\text{ ft/s})^2}{2}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\text{ lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144\text{ in}^2} \tag{6.M}$$

$$= 484\text{ psi} = 3340\text{ kPa}$$

This corresponds to 16.1 psi / 100 ft, which is the value shown on the turbulent flow line in Fig. 6.2, which in turn was made by repeating this calculation in a spreadsheet for various values of Q. It is $(484/23.1) \approx 21$ times the value in Example 6.1. If the flow had remained laminar, it would be 6 times the value in Example 6.1. ■

In all such problems (and the ones that follow) it is necessary to convert the volumetric flow rate (gal / min or ft^3 / s or m^3 / s) into linear velocity (in this case, 300 gal / min in a 3-in pipe = 13.0 ft / s). This routine calculation can be simplified by the use of App. A.2, which shows the volumetric flow rate in gal / min corresponding to a velocity of 1 ft / s for all schedule 40 standard U.S. pipe sizes. In the foregoing example we could have looked up the value of 23.00 (gal / min) / (ft / s) for

3-in pipe and computed

$$V = \frac{300 \text{ gal / min}}{23.0 (\text{gal / min}) / (\text{ft / s})} = 13.0 \frac{\text{ft}}{\text{s}} \tag{6.N}$$

Like all Type 1 problems, this was quite straightforward. We used all the equations in Table 6.4 except the laminar friction factor equation. The sequence of operations was

$$\begin{pmatrix}\text{Volumetric} \\ \text{flow rate, } Q\end{pmatrix} \rightarrow V \rightarrow \mathcal{R} \rightarrow f \rightarrow \mathcal{F} \rightarrow \Delta P \tag{6.O}$$

Please review the calculation to see that all these steps were used. For Types 2 and 3, we cannot proceed as easily as this but must resort to a trial-and-error solution; that is easy with a spreadsheet.

Example 6.5. A gasoline storage tank drains by gravity to a tank truck; see Fig. 6.11. The pipeline between the tank and the truck is 100 m of 0.1 m diameter commercial steel pipe. The properties of gasoline are given in the Common Units and Values for Problems and Examples. Both tank and truck are open to the atmosphere, and the level in the tank is 10 m above the level in the truck. What is the volumetric flow rate of the gasoline?

Applying B.E. between the free surface in the tank, point 1, and the free surface in the truck, point 2, we see that all terms cancel except

$$\Delta(gz) = -\mathcal{F} = -4f\frac{\Delta x}{D}\cdot\frac{V^2}{2} \tag{6.22}$$

This is a type 2 problem. The equation contains two unknowns, V and f; therefore, to solve it we need an additional equation or relationship among the variables listed. The second relationship is provided by Fig. 6.10, which relates f and V. We could use Eq. 6.21 to replace either f or V in terms of the other; some computer programs do that. Others follow the trial-and-error procedure here, replacing the chart lookups with applications of Eq. 6.21.

Here we know the fluid properties and the pipe diameter (0.1 m ID). From Table 6.2 we have $\varepsilon = 0.0018$ in; so

$$\frac{\varepsilon}{D} = \frac{0.0018 \text{ in}}{0.1 \text{ m}}\cdot\frac{\text{m}}{39.37 \text{ in}} = 0.00046 \tag{6.P}$$

From Fig. 6.10 we see that for this value of the relative roughness the possible range of f for turbulent flow is 0.0042 to about 0.008. As our first guess, let us try $f_{\text{first guess}} = 0.005$. Then from Eq. 6.22, rearranged to solve for

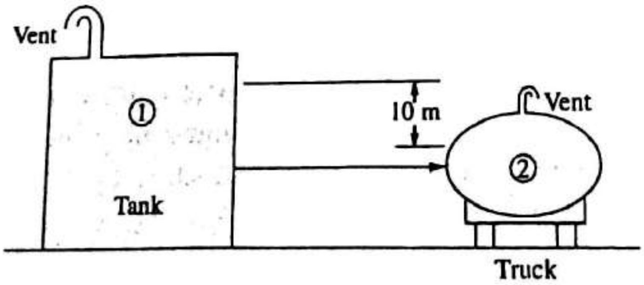


FIGURE 6.11
A gravity-driven fluid transfer from one tank to another, used in Example 6.5. The air from the headspace displaced by the liquid flow into the truck exits by the vent on the tank roof. In the United States such tank vent emissions are controlled to prevent air pollutant emissions.

$V_{\text{first guess}}$, we have

$$V_{\text{first guess}} = \left[\frac{2g(-\Delta z)}{4f}\cdot\frac{D}{\Delta x}\right]^{1/2} = \left(\frac{2\cdot 9.81\ \text{m/s}^2\cdot 10\ \text{m}}{2\cdot 0.005}\cdot\frac{0.1\ \text{m}}{100\ \text{m}}\right)^{1/2}$$
$$= 4.4\ \frac{\text{m}}{\text{s}} \tag{6.Q}$$

If this is a good guess, then the computed f should match our first $f_{\text{first guess}}$. Using $V_{\text{first guess}}$, we compute

$$\mathcal{R}_{\text{first guess}} = \frac{0.1\ \text{m}\cdot(4.4\ \text{m/s})\cdot 720(\text{kg/m}^3)}{0.0006\ \text{kg/m}\cdot\text{s}} = 5.3\cdot 10^5 \tag{6.R}$$

From Fig. 6.10 for this value of $\mathcal{R}$ and ε/D we read $f \approx 0.0045$, and from Eq. 6.21 we compute $f \approx 0.0044$. We could repeat the process by hand, using $f_{\text{second guess}} = 0.0044$, and continue until we had satisfactory agreement between the successive values of f. But computers do this very easily for us, so we proceed on a spreadsheet as shown in Table 6.5.

The first column of Table 6.5 shows the names of the variables, the second shows the nature of each variable, and the third shows the values shown above, based on $f_{\text{first guess}} = 0.005$. We see at the bottom of the second column that the ratio of $f_{\text{computed}}/f_{\text{guessed}} = 0.887$. We next ask the spreadsheet's numerical solution package ("goal seek" on Excel spreadsheets) to make the value of $f_{\text{computed}}/f_{\text{guessed}}$ become equal to 1.00 by changing the value of f_{guessed}. We see that for an f_{guessed} of 0.00442, that ratio becomes $1.0007 \approx 1.00$. We could get more significant figures of agreement, but the input data do not justify that so we accept the values in the column at the right as correct. Then

$$Q = \frac{\pi}{4}\cdot(0.1\ \text{m})^2\cdot 4.71\ \frac{\text{m}}{\text{s}} = 0.0370\ \frac{\text{m}^3}{\text{s}} = 1.306\ \frac{\text{ft}^3}{\text{s}} = 386\ \frac{\text{gal}}{\text{min}} \tag{6.S}$$

■

TABLE 6.5
Numerical solution to Example 6.5

Variable	Type	First guess	Solution
D, m	Given	0.1	0.1
L, m	Given	100	100
Δz, m	Given	−10	−10
ε, in	Given	0.0018	0.0018
ρ, kg/m³	Given	720	720
μ, cP	Given	0.6	0.6
f, guessed	**Guessed**	**0.005**	**0.00442**
V, m/s	Calculated	4.429	4.710
$\mathcal{R}$	Calculated	531,533	565,222
ε/D	Calculated	0.000457	0.000457
f_{computed}	Calculated	0.004435	0.004424
$f_{\text{computed}}/f_{\text{guessed}}$	**Check value**	0.88701	**1.0007**

Example 6.6. We want to transport 500 ft^3 / min of air horizontally from our air conditioner to an outbuilding 800 ft away. The air is at 40°F and a pressure of 0.1 psig. At the outbuilding the pressure is to be 0.0 psig. We will use a circular sheet metal duct, which has a roughness of 0.00006 in. Find the required duct diameter.

Here we are applying B.E. to a compressible fluid. However, as discussed in Sec. 5.6, for low fluid velocities B.E. gives the same result as the analysis that takes compressibility into account. Applying B.E. from the inlet of the duct, point 1, to its outlet, point 2, we find

$$\frac{\Delta P}{\rho} = -\mathscr{F} = -4f\frac{\Delta x}{D}\cdot\frac{V^2}{2} \tag{6.23}$$

This is the Type 3 problem, in which we know everything but the pipe diameter. The equation contains the three unknowns f, D, and V; therefore, we need two additional relations. One is supplied by Fig. 6.10; the other, by the continuity equation, which shows that Q (which is given in the problem statement) is equal to $V(\pi/4)D^2$. We could use this relation to eliminate V or D from Eq. 6.23, but this is not particularly convenient. Rather, we proceed by trial and error. First we guess a pipe diameter and calculate the pressure drop from Eq. 6.23. Then we compare the calculated pressure drop with the known value, 0.1 psi, and readjust the guessed pipe diameter until we find the diameter for which the pressure drop is 0.1 psi.

For the density of air we use the average pressure between inlet and outlet and the ideal gas law:

$$\rho_{\text{air}} = \frac{PM}{RT} = \frac{14.75\ \text{lbf/in}^2 \cdot 29\ \text{lbm/lbmol}}{10.73[\text{lbf}\cdot\text{ft}^3/(\text{in}^2\cdot\text{lbmol}\cdot{}^\circ\text{R})]\cdot 500^\circ\text{R}} = 0.080\,\frac{\text{lbm}}{\text{ft}^3} \tag{6.T}$$

For air at 40°F we have $\mu = 0.017$ cP (see App. A.1). For our first trial we select $D_{\text{first guess}} = 1$ ft. Then

$$\left(\frac{\varepsilon}{D}\right)_{\text{first guess}} = \frac{0.00006\ \text{in}}{12\ \text{in}} = 0.000005 \tag{6.U}$$

$$V_{\text{first guess}} = \frac{(500\ \text{ft}^3/\text{min})\cdot(\text{min}/60\ \text{s})}{(\pi/4)(1\ \text{ft})^2} = 10.6\,\frac{\text{ft}}{\text{s}} = 3.23\,\frac{\text{m}}{\text{s}} \tag{6.V}$$

$$\mathscr{R}_{\text{first guess}} = \frac{1\ \text{ft}\cdot 10.6\ \text{ft/s}\cdot 0.080\ \text{lbm/ft}^3}{0.017\ \text{cP}\cdot 6.72\cdot 10^{-4}\ \text{lbm/ft}\cdot\text{s}\cdot\text{cP}} = 7.43\cdot 10^4 \tag{6.W}$$

From Fig. 6.10 we read $f \approx 0.0049$ and, from Eq. 6.21, $f = 0.00465$. Then

$$\Delta P = \rho_{\text{air}}\left(-4f\frac{\Delta x}{D}\cdot\frac{V^2}{2}\right)$$

$$= -0.080\,\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{4}{2}\cdot 0.00465\cdot\frac{800\ \text{ft}}{1\ \text{ft}}\cdot\left(10.6\,\frac{\text{ft}}{\text{s}}\right)^2\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2}$$

$$= -0.0146\,\frac{\text{lbf}}{\text{in}^2} = -101\ \text{Pa} \tag{6.X}$$

TABLE 6.6
Numerical solution to Example 6.6

Variable	Type	First guess	Solution
Q, cfm	Given	500	500
D, ft	Guessed	1	0.667
L, ft	Given	800	800
ε, in	Given	0.00006	0.00006
ρ, lbm / ft^3	Given	0.08	0.08
μ, cP	Given	0.017	0.017
V, ft / s	Calculated	10.6	23.8
$\mathcal{R}$	Calculated	74301	111396
ε / D	Calculated	0.000005	7.496E-06
$f_{computed}$	Calculated	0.00465	0.00425
$\Delta P_{computed}$, psi	Calculated	0.01446	0.1000
Allowed ΔP, psi	Given	0.1	0.1
$\Delta P_{computed} / \Delta P_{allowed}$	Check value	0.1446	1.000

Our first guess of the diameter is too large, because it would result in a pressure drop due to friction that is only about one-seventh of that available; i.e., a duct 1 ft in diameter will do, but we can use a smaller one and still get the required flow with the available pressure difference. We could make a second guess of the diameter and repeat the calculation, but it is much easier to let our computer do this. Table 6.6 shows the spreadsheet solution. As in Table 6.5, the first column lists the variables, the second describes the variables, and the third corresponds to $D_{first\ guess} = 1$ ft, with the values shown above in this example. To find the solution in the fourth row we let the spreadsheet's numerical solution routine vary the value of D to make the check value in the lower right corner become 1.0000. This shows that the required diameter is 0.667 ft = 8.00 in = 0.203 m. ■

6.6 SOME COMMENTS ABOUT THE FRICTION FACTOR METHOD AND TURBULENT FLOW

1. The three preceding examples show how we use the friction factor plot, Fig. 6.10, or its numerical equivalent, Eqs. 6.20 and 6.21. However, the calculations for turbulent and transition flow are not reliable to better than ±10 percent, because the exact values of the roughnesses are seldom known to better than that accuracy. Furthermore roughnesses of pipes change over time as they corrode or collect deposits. It is common practice in the long-distance oil and gas pipeline industry to regularly force a scraper (called a "pig") through their pipelines. This cleans the inner pipe surface, thus greatly lowering the roughness and lowering the required pressure drop. The savings in pumping cost more than repay the cost of this regular cleaning.
2. The plot is made up for sections of pipe that contain no valves, elbows, sudden contractions, sudden expansions, etc. These are probably present in all the actual

situations described in Examples 6.3, 6.4, 6.5, and 6.6. We will discuss how to account for them in Secs. 6.8 and 6.9.

3. The data on which the plot is based are all taken well downstream of the entrance to the pipe. We will discuss the entrance region briefly in Part IV.
4. The friction factor plot is a generalization of experimental data. One should not attach much theoretical significance to it. So far no one has been able to calculate friction factors for turbulent flow without starting with experimental data.
5. It can be easily shown that for turbulent flows the heat-transfer and mass-transfer coefficients are related fairly simply to the friction factor f. This is so because the eddy that transports momentum (and thus increases the shear stress) also transports heat and mass and, thus, increases the heat and mass transfer. This subject is discussed in heat-transfer and mass-transfer texts as the *Reynolds analogy.*
6. In Part IV we will discuss briefly the measured velocity distributions in turbulent pipe flow. Now we simply note that the velocity profile of turbulent pipe is much flatter than that of laminar pipe flows (see Fig. 3.3). Most of the fluid flows in a central core, which moves almost as a unit at nearly the same velocity throughout. There is a thin layer near the pipe wall in which the velocity drops rapidly from the high velocity of the central core to the zero velocity at the wall. Thus, it is quite reasonable to treat the average velocity of a turbulent pipe flow as the velocity representing the entire flow.
7. Now that we all have computers, and many of us have access to programs that solve the above examples quickly and easily. The hand solution of these problems and the reading of friction factors from Fig. 6.10 are part of an chemical engineer's cultural background, but most likely not part of her day-to-day tool kit. However, knowing how these methods work helps the engineer know what those convenient computer programs are doing, and whether or not they are applicable in some unusual situation.

6.7 MORE CONVENIENT METHODS

The friction factor plot, Fig. 6.10, is a very great generalization; all the pressure-drop data for all Newtonian fluids, pipe diameters, and flow rates are put on a single graph. However, as shown in Examples 6.4, 6.5, and 6.6, the plot is tedious to use by hand. Therefore, before the computer age, working engineers rearranged the same experimental data in numerous forms that are more convenient. The resulting methods are more convenient but less compact than the friction factor methods; for instance, one might be using 20 charts instead of only one. These are now mostly of historical interest, because our computer programs are so good. However, the student is likely to encounter some of them and wonder how they are organized. Furthermore, many of them allow more intuitive insight into these flows than the computers programs do. Some of these methods are shown in this section.

Suppose we decide to build an oil refinery, a city water supply system, or an aircraft carrier. We will have to deal with a very large number of fluid flows. We could calculate the friction effect for each from Fig. 6.10. However, in any of these projects we would probably use U.S. standard pipe sizes for practically all of the flows. From App. A.2 we see that they constitute a fairly small number of sizes. For pipes of a given size and of the same material, the diameter and relative roughness are constant. Therefore, for a given size of pipe there are only four variables: $\mathscr{F}$ per foot, Q, ρ, and μ. These can be plotted (for one pipe size) in a way that makes calculations of friction loss very easy. Thus, if we spend the time to make about 10 such plots for the common U.S. pipe sizes, we can save considerable work in designing our refinery, water system, or aircraft carrier. Naturally, oil companies, water-supply companies, and the Navy have done just that.

In making such plots and tables it is customary to set them up for the most common problem, which is the long, horizontal, constant-diameter pipe. For such a pipe, B.E., rearranged, is

$$\left(\frac{-\Delta P}{\Delta x}\right)_{\text{friction}} = \frac{\rho}{\Delta x}\mathscr{F} = \rho\left(\frac{4f}{D}\cdot\frac{V^2}{2}\right) \quad \left(\begin{array}{l}\text{steady flow, horizontal}\\ \text{pipe, with no pumps or}\\ \text{compressors}\end{array}\right) \tag{6.24}$$

so the charts customarily can be read directly in $-\Delta P/\Delta x$, dropping the "friction" subscript. If we must use such a chart for some other type of problem, we may read the appropriate $-\Delta P/\Delta x$ and then use Eq. 6.24 to find $\mathscr{F}$.

Figure 6.12 is an example of such a plot. This figure shows, for a 3-in pipe, the pressure drop per 1000 ft as a function of volumetric flow rate, kinematic viscosity (viscosity / density), and specific gravity. The plot is logarithmic on both axes, but the log scale is different for each. A plot like this can be made directly from Fig. 6.10 (Prob. 6.29).

Example 6.7. Rework Example 6.1 by using Fig. 6.12.

Here the B.E. analysis is the same as in Example 6.1. We start on the chart at the right at 50 gal / min and read horizontally to the left to 50.0 cSt line, and then vertically downward. The bottom section allows for fluids of various specific gravities; here the specific gravity is ≈1.00, so we read to the bottom of the plot, finding 7.7 psi / 1000 ft. In this example the pipe is 3000 ft long, so the pressure drop is 3 times 7.7 = 23.1 psi. The perfect agreement with Example 6.1 should not surprise us; the laminar part of Fig. 6.12 was made up from the same equations we used there. ■

Figure 6.12 is a "convenience" chart made up from Fig. 6.10. It is well suited to the needs of an oil company, which spends large sums of money in pumping fluids with a wide range of viscosities, sometimes in laminar flow, sometimes in turbulent flow. But it is poorly suited to the needs of a city water-supply company, which deals almost exclusively with water. When Fig. 6.12 was made from Fig. 6.10, the pipe diameter and roughness were held constant. If we are dealing with water, we can assume that the temperature is constant (which is approximately true in city water systems) and that the absolute roughness of the pipe wall is constant (also approximately true in city water systems). Then the pressure drop as a function of pipe diameter and flow rate can be tabulated for all flows of water at the chosen temperature. Appendix A.3 is such a table, made up for the flow of water at

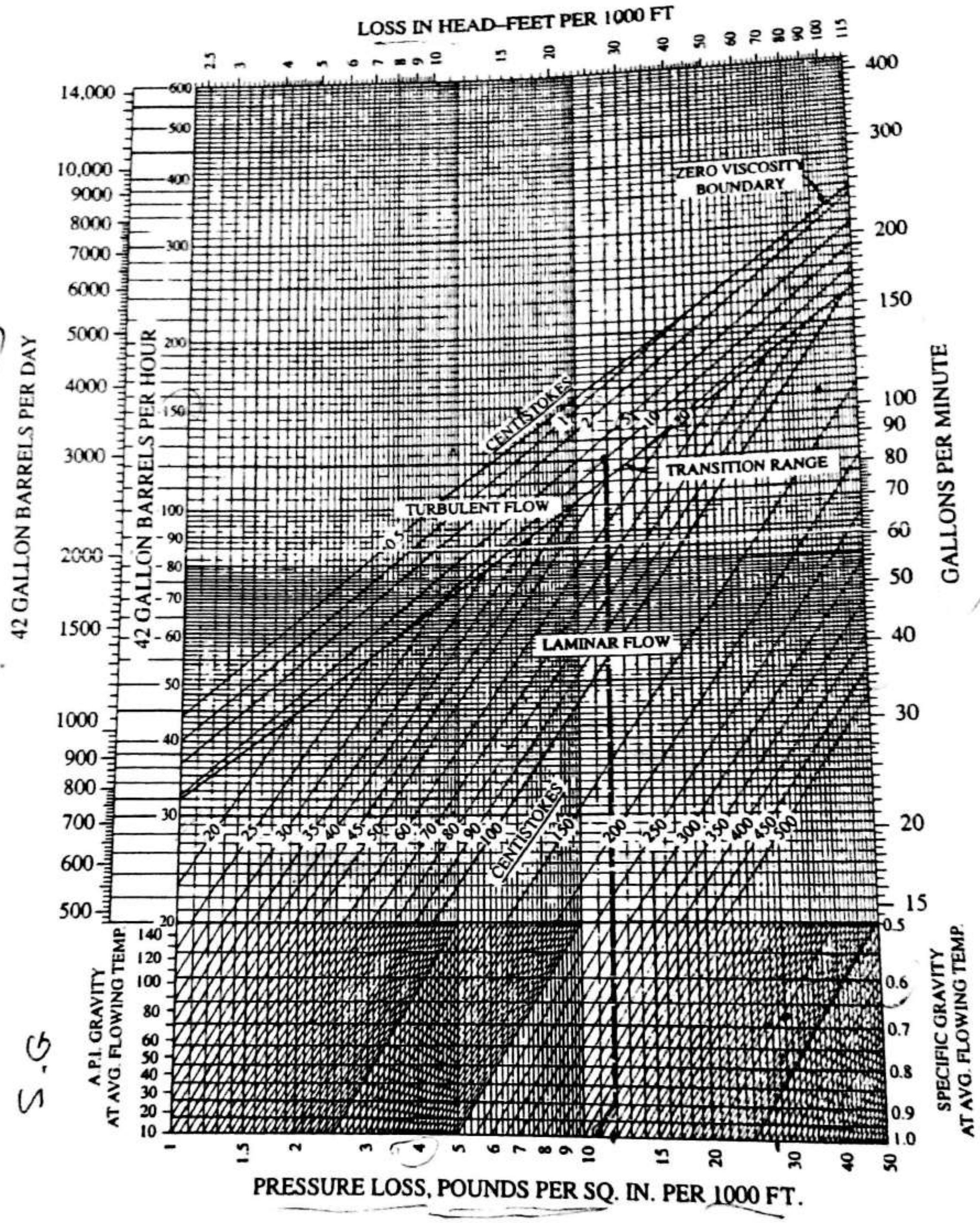


FIGURE 6.12

Pressure drop in a 3-in schedule 40 pipe, 3.068 in inside diameter. Example shown; flow rate = 120 barrels per hour (BPH); kinematic viscosity = 10 cSt; specific gravity = 0.9; pressure loss (follow dashed line) = 10.7 psi / 1000 ft. (Courtesy of the Board of Engineers, Standard Oil Company of California.)

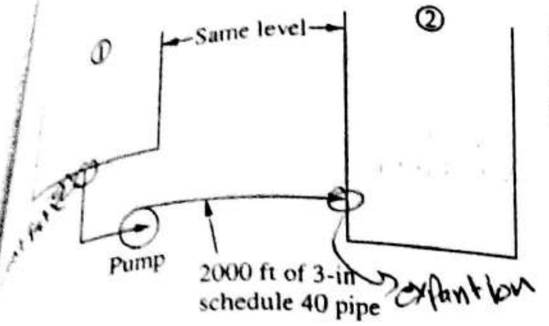


FIGURE 6.13
A pump-driven fluid transfer from one tank to another, used in Example 6.8.

60°F through schedule 40 pipe (the most common size in United States industrial practice).

Example 6.8. Two reservoirs are connected by 2000 ft of 3-in pipe. We want to pump 200 gal / min of water from one to the other. The levels in the reservoirs are the same, and both are open to the atmosphere; see Fig. 6.13. What are the pump work per unit mass, the required pump pressure rise, and the required pump power?

Applying B.E. from the free surface of the first reservoir, point 1, to the free surface of the second, point 2, we see that all terms are zero except

$$0 = \frac{dW_{\text{n.f.}}}{dm} - \mathcal{F} \tag{6.Y}$$

The pump work (positive because of the thermodynamic sign convention) is equal to the friction loss. We could solve this problem using Fig. 6.10 or the spreadsheets in Tables 6.5 and 6.6. But it is faster and easier using App. A.3.

We start at the left of App. A.3 at 200 gal / min and read horizontally to the column for 3-in pipe, where the pressure drop is indicated as (3.87 psi) / (100 ft). The pipe is 2000 ft long, so the pressure drop due to friction is

$$\mathcal{F} = \left(\frac{dW_{\text{n.f.}}}{dm}\right)_{\text{pump}} = \left(\frac{-\Delta P}{\rho}\right)_{\text{friction}}$$

$$= \frac{1}{\rho} \cdot 3.87 \frac{\text{psi}}{100\ \text{ft}} \cdot 2000\ \text{ft} = \frac{1}{\rho} \cdot 77.4 \frac{\text{lbf}}{\text{in}^2} \tag{6.Z}$$

The pump must increase the pressure of the fluid flowing through it by 77.4 psi to overcome the friction in the 2000 ft of pipe.

The pump power required is

$$\text{Po} = \frac{dW_{\text{n.f.}}}{dt} = \frac{dW_{\text{n.f.}}}{dm}\dot{m} = \dot{m}\frac{1}{\rho} \cdot 77.4 \frac{\text{lbf}}{\text{in}^2} \tag{6.AA}$$

Here

$$\dot{m} = 200 \frac{\text{gal}}{\text{min}} \cdot \frac{\text{min}}{60\ \text{s}} \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3} \cdot \frac{\text{ft}^3}{7.48\ \text{gal}} = 27.8 \frac{\text{lbm}}{\text{s}} \tag{6.AB}$$

$$\text{Po} = \frac{77.4\ \text{lbf}/\text{in}^2}{62.3\ \text{lbm}/\text{ft}^3} \cdot 27.8 \frac{\text{lbm}}{\text{s}} \cdot \frac{32.2\ \text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} \cdot \frac{\text{hp} \cdot \text{s}}{550\ \text{ft} \cdot \text{lbf}}$$

$$= 8.90\ \text{hp} = 6.63\ \text{kW} \tag{6.AC}$$

The pump power computed here is the amount of mechanical power delivered to the fluid. For a 100 percent efficient pump this would also be the power input to the pump. Real pumps are never 100 percent efficient (nor are the electric motors which drive most of them); their actual behavior is discussed in Chap. 10. ■

In using App. A.3 remember where it comes from; each entry represents a calculation like that in Example 6.4. One may solve by Fig. 6.10 or Eq. 6.21 for the value we read from the table here and see that the values agree. By making such calculations for a large number of flow rates and pipe diameters we can make App. A.3. Thus, that appendix is simply Fig. 6.10, rearranged for the special case of 60°F water flowing in schedule 40 pipes.

Just as oil refinery engineers have made charts convenient to their class of problems, so also have air-conditioning engineers prepared Fig. 6.14, on which they can quickly solve most pressure drop problems in common air-conditioning use [8]. Here they have recognized that almost all flow in air-conditioning ducts is of air at about 70°F and 14.7 psia, for which the viscosity and density are known, and that most of the ducts are made of galvanized steel or aluminum, for which ε is known. Thus, they have one fewer variable than the oil refinery engineers (who treat fluids with a variety of viscosities), so that instead of having to have a separate plot for each size pipe, like Fig. 6.13, they can have one plot that covers all sizes.

Example 6.9. Repeat Example 6.6 by using Fig. 6.14. Here the figure was made for air at 68°F, which we can tell from the assumed density of 0.075 lbm / ft^3, while our problem is for 40°F and a density of 0.080 lbm / ft^3. We know that the viscosities do not match perfectly either. However, we ignore these differences for the moment and simply use Fig. 6.14. We know the flow rate (500 ft^3 / min), the pressure drop (0.1 psi), and the pipe length (800 ft). We need to convert the pressure drop to pressure drop per unit length, which we can do either as

$$\frac{\Delta P}{\Delta x} = \frac{0.1 \text{ psi}}{800 \text{ ft}} \cdot \frac{27.69 \text{ in } H_2O}{\text{psi}} = 0.00346 \frac{\text{in } H_2O}{\text{ft}} = 0.346 \frac{\text{in } H_2O}{100 \text{ ft}} \quad (6.AD)$$

or as

$$\frac{\Delta P}{\Delta x} = \frac{0.1 \text{ psi}}{800 \text{ ft}} \cdot \frac{6.895 \cdot 10^3 \text{ Pa}}{\text{psi}} \cdot \frac{3.28 \text{ ft}}{\text{m}} = 28.3 \frac{\text{Pa}}{\text{m}} \quad (6.AE)$$

These give the same entry point on the abscissa. Reading at the intersection of this pressure gradient and 500 cfm, we find that the required pipe diameter is about 8.2 in and the velocity is about 1350 ft / min = 22.5 ft / s. The close agreement with Example 6.5 (8.0 in, 23.5 ft / s) simply shows that Fig. 6.14 was made using the standard friction factor plot. The small differences between the density and viscosity of air at 70°F and 40°F are the probable cause of the differences shown. ■

Such convenient charts as Figs. 6.13 and 6.14 and App. A.3 are widely used in industry for routine calculations, even in the computer age. When engineers leave the university and join industrial firms, they find that their colleagues have a large supply of them. It is worth the young engineer's while to trace them back to their sources. Not only will they discover how the convenient methods fit in with the ideas learned in the university, but also they will see more clearly the limitations of the convenient methods. Then they can use them for the routine parts of complex jobs, saving their creative efforts for the non-routine parts that will test their talents and education.

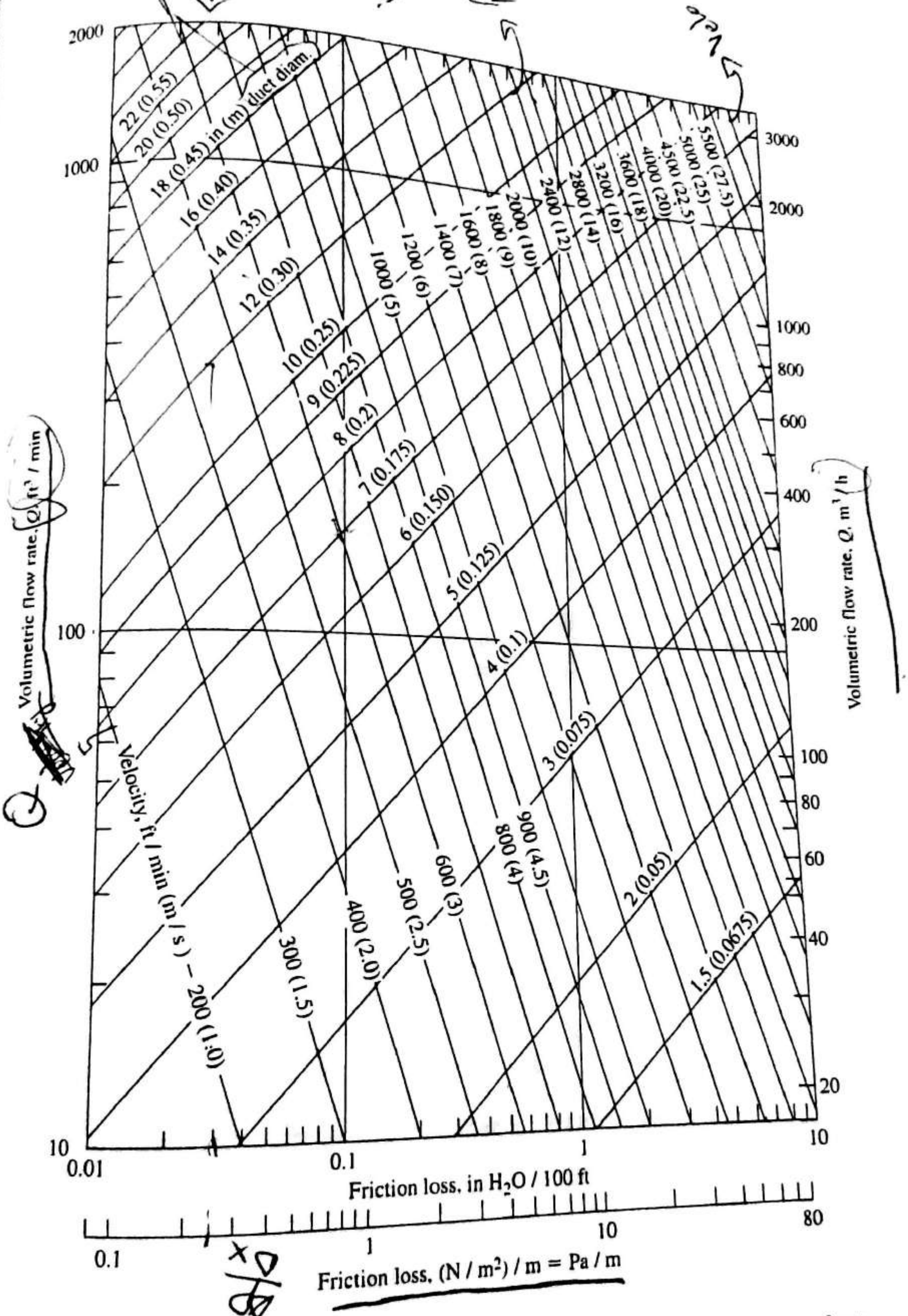


FIGURE 6.14
Friction of air in straight ducts for volumetric flow rates of 10 to 2000 ft^3 / min (20 to 3000 m^3 / h). Based on standard air of 0.075 lb / ft^3 (1.2 kg / m^3) density, flowing through average, clean, round, galvanized metal ducts having approximating 40 joints per 100 ft (30 m). Do not extrapolate below the chart. (Reprinted from the *1972 ASHRAE Handbook—Fundamentals*, with permission.)

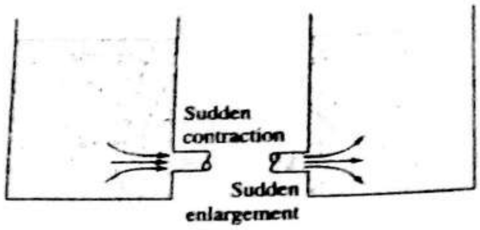


FIGURE 6.15
Flows in sudden contractions and enlargements.

6.8 ENLARGEMENTS AND CONTRACTIONS

The first part of this chapter was devoted to the steady flow of a fluid in a part of a circular pipe, well downstream from the pipe entrance. However, in each of the examples in this chapter there were places where the flow entered and left a pipe. In Examples 6.1, 6.3, and 6.4 the fluid flowed from the first reservoir into a pipe and left the pipe to enter a second reservoir. Friction losses are associated with these transitions; see Fig. 6.15.

For the enlargements we can calculate the friction effect on the basis of some simple assumptions; the calculation, made by means of the momentum balance, is given in Sec. 7.3. The results of that calculation can be put in the form

$$\mathcal{F} = K\frac{V^2}{2} \tag{6.25}$$

where K is an empirical constant, called the *resistance coefficient*, that is dependent on the ratio of the two pipe diameters involved, and V is the larger of the two velocities involved. The experimental data agree with it reasonably well [9]. No one has successfully calculated the friction effect of sudden contractions without having resort to experimental data. However, it can be shown [9] that the experimental data can also be represented by the same equation. The experimental values of K for contractions are shown in Fig. 6.16, as are the calculated values of K for sudden expansions.

From Fig. 6.16 it is clear that, the larger the change of diameter, the greater the pressure losses. The reasons for the losses are as follows.

1. In a sudden expansion the fluid is slowed down from relatively high velocity and high kinetic energy in the small pipe to relatively low velocity and low kinetic energy in the large pipe. If this process took place without friction, the kinetic energy would be converted to injection work with a resulting pressure increase. In a sudden expansion the process takes place as a fluid mixes and eddies around the enlargement. The kinetic energy of the fluid is converted into internal energy. Therefore, when the downstream velocity is zero, the friction loss is equal to the upstream kinetic energy. This is shown by Eq. 6.25 with $K = 1$, which is the value for zero downstream velocity in Fig. 6.16 (see the discussion in Sec. 5.5).
2. In a sudden contraction the flow does not come into the pipe entirely in the axial direction. Rather, it comes from all directions, as sketched in Fig. 6.15. (The flow is not completely one-dimensional, but rather two- or three-dimensional.) On entering the pipe the flow follows the pattern shown in Fig. 6.17.

The fluid forms a neck, called the *vena contracta*, just downstream of the tube entrance. The flow into the neck is caused by the radial inward velocity of the fluid

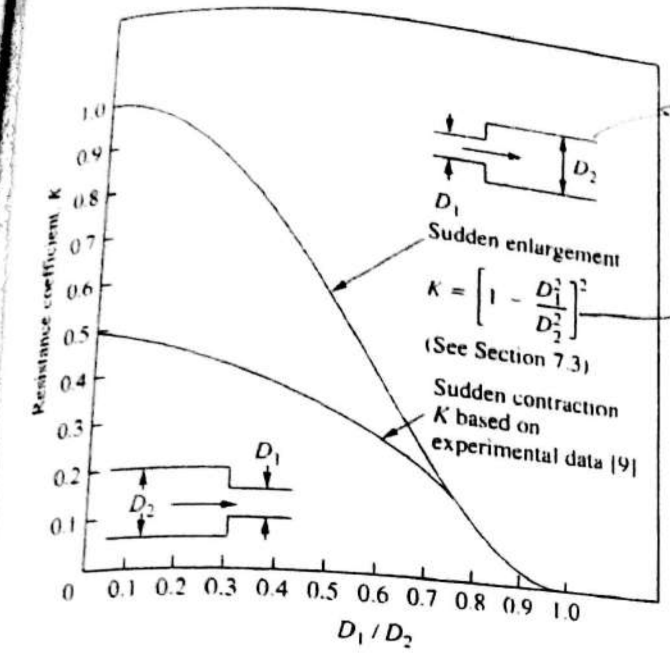


FIGURE 6.16
Resistance due to sudden enlargements and contractions. The resistance coefficient K is defined in Eq. 6.25. (From Crane Technical Paper No 410, reproduced by permission of the Crane Company.)

approaching the tube. Because it is coming radially inward, the fluid overshoots the tube wall and goes into the neck. This neck is surrounded by a collar of stagnant fluid. In the neck the velocity is greater than the velocity farther downstream. Thus, the kinetic energy decreases from the neck to some point downstream, where the velocity is practically uniform over the cross section of the pipe. This kinetic energy is not all recovered as increased pressure but leads to the friction loss shown in Eq. 6.25 with the values of K from Fig. 6.16.

Our discussion of entrance and exit losses has concerned turbulent flow only. In laminar flow these effects generally are negligible, because the kinetic energies generally are negligible compared with the viscous effects.

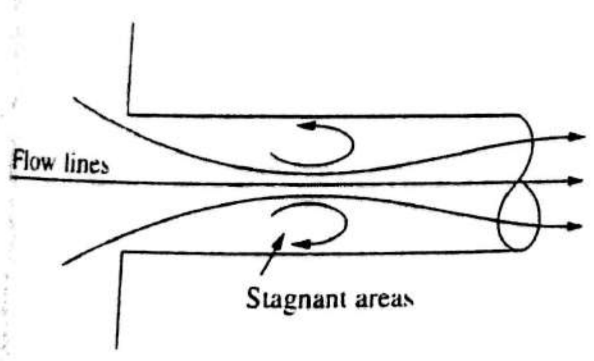


FIGURE 6.17
Flow pattern (turbulent flow) in a sudden contraction, showing that the flow is not truly one-dimensional, but comes in with a radial component, causing it to form a narrow neck before it straightens out into one-dimensional pipe flow.

Example 6.10. Calculate the error made in Example 6.4 by neglecting the expansion and contraction losses.

From Fig. 6.16 we see that for flow from a tank to a pipe the coefficient K is 0.5 and for flow from a pipe to a tank it is 1.0. Thus, the friction loss due to the expansion and contraction should be $1.0 + 0.5 = 1.5 \cdot$ (kinetic energy of the fluid in the pipe). In Example 6.4 the velocity was 13.0 ft / s; therefore, it can be shown that

$$
\begin{aligned}
-\Delta P_{\substack{\text{expansion}\\ \text{and contraction}}} &= \rho \mathcal{F} = \rho K \frac{V^2}{2} \\
&= 62.3\,\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{1.5}{2}\cdot\left(13.0\,\frac{\text{ft}}{\text{s}}\right)^2\cdot\frac{\text{ft}^2}{144\ \text{in}^2}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}} \\
&= 1.70\,\frac{\text{lbf}}{\text{in}^2} = 11.75\ \text{kPa}
\end{aligned}
\tag{6.AF}
$$

■

This is ≈0.4 percent of the 484 psi calculated in Example 6.4. In this example the pipe was long (3000 ft). If the pipe were short, the contraction and expansion losses would be just as large, but the percentage error in neglecting them would be much greater. We can also consider the role of valves by using Eq. 6.25. A completely closed valve is the same as that equation with $K = \infty$; substituting that into Eq. 6.AF and then into Eq. 5.5 leads to $V = 0$, which is the result a closed valve should produce. Flow-regulating valves (such as those in your kitchen and bathroom sinks) are, in effect variable Ks; wide open, they have a small value of K, fully closed they have an infinite value of K, and by handle adjustment they can take up any value in between, thus controlling the flow.

6.9 FITTING LOSSES

In addition to expansions and contractions, in most fluid systems we must take into account the effect of valves, elbows, etc. They are much more complex to analyze than the one-dimensional flows we have considered so far (take apart an ordinary household faucet and study its flow path; it is much more complicated than that of a straight pipe). Efforts at calculating the friction losses in such fittings have been made, and the results have been correlated in two convenient ways which allow us to treat them as if they were one-dimensional problems.

The first way to correlate these test results is to assume that for a given flow

$$
\begin{pmatrix}\mathcal{F}\text{ through}\\ \text{a valve or fitting}\end{pmatrix} = \begin{pmatrix}\text{a constant for}\\ \text{that kind of}\\ \text{valve or fitting}\end{pmatrix}\cdot\begin{pmatrix}\mathcal{F}\text{ through a length}\\ \text{of pipe} = \text{one pipe}\\ \text{diameter}\end{pmatrix} \tag{6.26}
$$

If for a given kind of fitting this constant turns out to be independent of the kind of flow, and independent of the pipe diameter, then this correlation will be very easy to use. In turbulent flow, the constant in this equation is practically independent of pipe size, flow rate, and nature of the fluid flowing. If we know the value of the constant for a particular kind of fitting, we can calculate an "equivalent length of pipe" that would have the same friction effect as the fitting, and we can add this length to the actual length of the pipe to find an *adjusted length*, which gives practically the same friction effect as does the actual pipe including fittings. The constant in the equation is dimensionless. Typical values are shown in Table 6.7; they are referred to as *equivalent lengths*.

TABLE 6.7
Equivalent lengths and K values for various kinds of fitting*

Type of fitting	Equivalent length, L/D, dimensionless	Constant, K, in Eq. 6.25, dimensionless
Globe valve, wide open	350	6.3
Angle valve, wide open	170	3.0
Gate valve, wide open	7	0.13
Check valve, swing type	110	2.0
90° standard elbow	32	0.74
45° standard elbow	15	0.3
90° long-radius elbow	20	0.46
Standard tee, flow-through run	20	0.4
Standard tee, flow-through branch	60	1.3
Coupling	2	0.04
Union	2	0.04

*Source: Reference 10.

Example 6.11. Rework Example 6.4 on the assumption that, in addition to the 3000 ft of 3-in pipe, the line contains two globe valves, a swing check valve, and nine 90° standard elbows.

Using the constants in Table 6.7, we can calculate the equivalent length of 3-in pipe that would have the same friction effect as these fittings. This is:

$$\sum L/D = 2 \cdot 350 + 1 \cdot 110 + 9 \cdot 32 = 1098 \qquad \text{(6.AG)}$$

From Eq. 6.26 we see that this is the number of pipe diameters needed to have the same friction loss as the fittings. Thus, the equivalent length is $1098 \cdot [(3.068/12)\text{ ft}] = 281$ ft. Therefore, the adjusted length of the pipe is

$$\begin{pmatrix}\text{Adjusted}\\ \text{length}\end{pmatrix} = \begin{pmatrix}\text{actual pipe}\\ \text{length}\end{pmatrix} + \begin{pmatrix}\text{equivalent length}\\ \text{for fittings}\end{pmatrix}$$
$$= 3000 + 281 = 3281 \text{ ft} \qquad \text{(6.AH)}$$

The total pressure drop is

$$-\Delta P_{\text{total}} = 484 \text{ psi} \cdot \frac{3281 \text{ ft}}{3000 \text{ ft}} = 529 \text{ psi} \qquad \text{(6.AI)}$$

and that due to the valves and fittings

$$-\Delta P_{\text{valves and fittings}} = 529 \text{ psi} - 484 \text{ psi} = 45 \text{ psi} = 310 \text{ kPa} \qquad \text{(6.AJ)}$$

■

The second way to represent the same experimental data for the friction losses in valves or fittings is to assign a value of K in Eq. 6.25 to each kind of fitting. Those values, based on friction-loss experiments, are also shown in Table 6.7.

Example 6.12. Repeat Example 6.11, using the K values in Table 6.7.

Using those values, we compute that

$$\sum K_{\text{valves and fittings}} = 3 \cdot 6.3 + 1 \cdot 2.0 + 9 \cdot 0.74 = 27.56 \qquad \text{(6.AK)}$$

and

$$-\Delta P_{\text{valves and fittings}} = 27.56 \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3} \cdot \frac{(13.0 \text{ ft/s})^2}{2} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 31 \text{ psi} = 216 \text{ kPa} \tag{6.AL}$$

■

The fact the second method of making this estimate gives an answer only 69% of the first reminds us that these procedures give only a fair estimate of the pressure drop, not as reliable an estimate as we can make for flow in a straight pipe. These two methods appear to be quite different, but are not. If we write the friction heating for each method,

$$\mathcal{F} = 4f\left(\frac{L}{D}\right)_{\text{equivalent}} \frac{V^2}{2} = K_{\text{fitting}} \frac{V^2}{2} \tag{6.27}$$

we see that they are the same if $4f(L/D)_{\text{equivalent}} = K_{\text{fitting}}$. The equivalent length method lets $\mathcal{F}$ of a fitting vary with the size of the pipe and the Reynolds number, whereas the K method makes it independent of those. Both seem to match the experimental data about as well as each other. Lapple [10] suggests that the equivalent length method matches experimental results better when $\mathcal{R} < 10^5$ and the K method matches experimental results better when $\mathcal{R} > 10^5$.

Laminar flow has yielded little experimental data on which to base pressure-drop correlations for valves and fittings. Generally, the adjusted length calculated by the method given above will be correct for turbulent flow but will be too large for laminar flow. Empirical guides to estimating the adjusted length for laminar flow have been published [11].

Do not attach theoretical significance to these empirical relations for fitting losses. They are simply the results of careful tests of specific cases, arranged in a way that is useful in predicting the behavior of new systems. Fitting losses and expansion and contraction losses are often lumped as *minor losses* even though for a short piping system they may be larger than the straight pipe loss.

6.10 FLUID FRICTION IN ONE-DIMENSIONAL FLOW IN NONCIRCULAR CHANNELS

6.10.1 Laminar Flow in Noncircular Channels

Steady laminar flow in a circular pipe is one of the simplest flow problems. A somewhat harder problem is steady flow of an incompressible Newtonian fluid in some constant-cross-section duct or pipe that is not circular, such as a rectangular duct or an open channel. For laminar flow of a Newtonian fluid the problem can be solved analytically for several shapes. Generally, the velocity depends on two dimensions. In several cases of interest the problems can be solved by the same method we used to find Eq. 6.9; i.e., setting up a force balance around some properly chosen section of the flow, solving for the shear stress, introducing the Newtonian law of viscosity for the shear stress, and integrating to find the velocity distribution. From the velocity distribution the volumetric flow rate-pressure drop relation is found.

That these are all similar to the solution for laminar flow in a horizontal circular tube may be seen by comparing the horizontal, steady-flow solutions with that for a circular tube. For a circular tube,

$$Q = \left(\frac{P_1 - P_2}{\Delta x} \cdot \frac{1}{\mu}\right) \cdot \frac{\pi}{128} D_0^4 \tag{6.9}$$

For a slit between two parallel plates (Prob. 6.48),

$$Q = \left(\frac{P_1 - P_2}{\Delta x} \cdot \frac{1}{\mu}\right) \cdot \frac{1}{12} l h^3 \tag{6.28}$$

where h is the distance between plates and l is the width of the slit. If both sides are divided by l, the left-hand side becomes the volumetric flow rate per unit width. For an annulus (Prob. 6.49),

$$Q = \left(\frac{P_1 - P_2}{\Delta x} \cdot \frac{1}{\mu}\right) \cdot \frac{\pi}{128} (D_o^2 - D_i^2)\left[D_o^2 + D_i^2 - \frac{D_o^2 - D_i^2}{\ln(D_o / D_i)}\right] \tag{6.29}$$

where D_o is the outer diameter and D_i is the inner diameter. These equations differ only by the terms at the far right, which account for the different geometries. Most of the cases that can be worked out by simple mathematics have been summarized by Bird [12, Chaps. 2–4] and Sakiadis [13]. One may show (Prob. 6.52) that as the spacing in the annulus become small ($D_i \rightarrow D_o$), Eq. 6.29 reduces to

$$Q = \left(\frac{P_1 - P_2}{\Delta x} \cdot \frac{1}{\mu}\right) \cdot \frac{1}{12} \cdot \pi D \left(\frac{D_o - D_i}{2}\right)^3 \tag{6.30}$$

which is the same as Eq. 6.28 with the slit length and slit width renamed. This is one of many circular or annular problems that can be simplified by converting them to equivalent straight or planar problems.

6.10.2 Seal Leaks

An extremely important chemical engineering application of Eq. 6.30 is the problem of seal leakage. Figure 6.18 shows three kinds of seals. Figure 6.18(a) shows a static seal, as exists between the bottle cap and the top of a soft-drink or beer bottle. A thin washer of elastomeric material is compressed between the metal cap and the glass bottle top. This compressed material forms a seal that prevents the escape of CO_2 (carbonation). The leakage rate is not exactly zero, but it is small enough to hold the carbonation for many years. Leaks through this kind of seal are generally unimportant. Sealing is more difficult when one of the sealed surfaces moves relative to the other.

Figure 6.18(b) shows a simple compression seal between a housing and a shaft. The example shown is a water faucet, in which a nut screws down over the body of

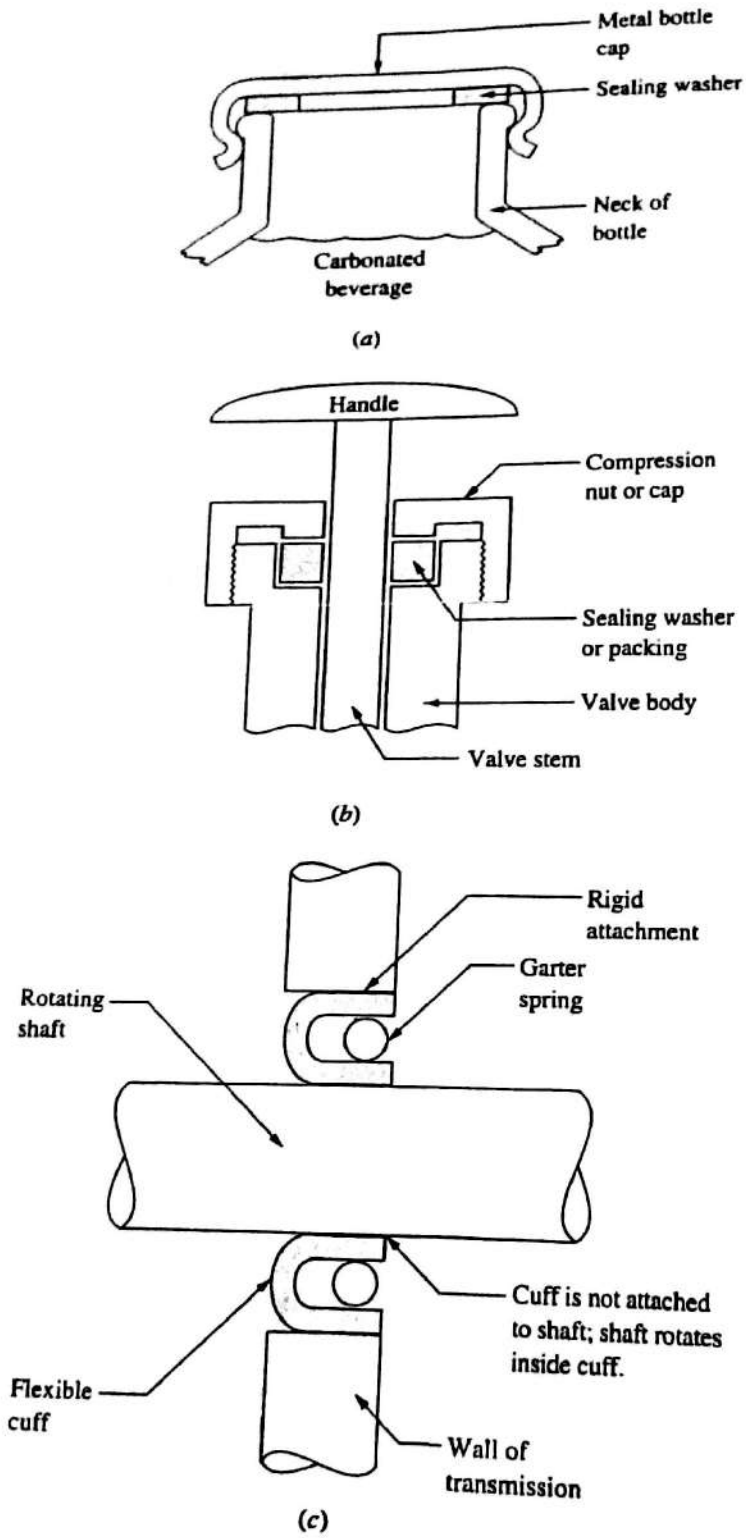


FIGURE 6.18
Three kinds of seals: (*a*) a static seal, as exists between a soft-drink or beer bottle and its bottle cap, (*b*) a packed seal, as exists between the valve stem and valve body of simple faucets, and as also exists on many simple pumps, (*c*) a rotary seal of the type common on the drive shafts of automobiles and some pumps.

the faucet to compress an elastomeric seal, which is trapped between the body of the faucet and the stem of the valve. The compressed seal must be tight enough to prevent leakage of the high-pressure water inside the valve out along the edge of the stem, but not so tight that the valve cannot be easily rotated by hand. Students are probably aware from personal experience that this type of seal often leaks. If the leak is a small amount of water into the bathroom sink, that causes little problem; tightening the nut normally reduces the leak to a rate low enough that it becomes invisible (but does not become zero!).

Example 6.13. A valve has a seal of the type shown in Fig. 6.18(*b*). Inside the valve is gasoline at a pressure of 100 psig. The space between the seal and the valve stem is assumed to have an average thickness of 0.0001 in. The length of the seal, in the direction of leakage, is 1 in. The diameter of the valve stem is 0.25 in. Estimate the gasoline leakage rate.

This is the case described by Eq. 6.30. Inserting values, we have

$$Q = \frac{100\ \text{lbf}/\text{in}^2}{1\ \text{in}} \cdot \frac{1}{12 \cdot 0.6\ \text{cP}} \cdot \pi \cdot 0.25\ \text{in} \cdot (10^{-4}\ \text{in})^3 \cdot \frac{\text{cP} \cdot \text{ft}^2}{2.09 \cdot 10^{-5}\ \text{lbf} \cdot \text{s}} \cdot \frac{144\ \text{in}^2}{\text{ft}^2}$$

$$= 7.5 \cdot 10^{-5}\ \frac{\text{in}^3}{\text{s}} = 0.27\ \frac{\text{in}^3}{\text{h}} = 1.3 \cdot 10^{-9}\ \frac{\text{m}}{\text{s}} \qquad \text{(6.AM)}$$

$$\dot{m} = Q\rho = 0.27\ \frac{\text{in}^3}{\text{h}} \cdot 0.026\ \frac{\text{lbm}}{\text{in}^3} = 0.007\ \frac{\text{lbm}}{\text{h}} = 0.0032\ \frac{\text{kg}}{\text{h}} \qquad \text{(6.AN)}$$

Tests indicate that the average leak rate from many oil refinery valves, processing this kind of liquid, is about 0.024 lbm / h, 3.5 times the value calculated here [13]; see Prob. 6.49. There are enough of these valves in a typical oil refinery or chemical plant that they contribute significantly to the overall emissions of hydrocarbons and chemicals to the atmosphere [14, p. 347]. ■

Figure 6.18(*c*) shows in greatly simplified form the seal that surrounds the drive shaft of an automobile, where that shaft exits from the transmission. The inside of the transmission is filled with oil. The flexible seal is like a shirt cuff turned back on itself, with the outside held solidly to the wall of the transmission and the inside held loosely against the rotating shaft by a "garter spring." If we set that spring loosely, then there will be a great deal of leakage. If we set it very tight, then the friction and wear between the cuff and the shaft that rotates inside it will be excessive. The setting of the tension on that spring is a compromise between the desire for low leakage and the desire for low friction and wear (buyers expect these seals to last as long as the auto!). That compromise normally leads to a low, but not a zero leakage rate; a small amount of oil is always dripping out and accumulating on the floor of our garages. Valves and pumps also have shafts that must rotate and hence have the same kind of leakage problem. Pumps and valves in chemical engineering all have the same kind of leakage problem shown Example 6.13. The seals regularly used are more complex versions of the ones shown there.

6.10.3 Turbulent Flow in Noncircular Channels

We are no more able to calculate the pressure drop in steady, *turbulent* flow in a noncircular channel than we are in a circular one. However, it seems reasonable to expect that we could use the friction-loss results for circular pipes to estimate the results for other shapes. Let us *assume* that for a given fluid the shear stress at the wall of any conduit is the same for a given V_{average}, independent of the shape of the conduit. Then, from a force balance on a horizontal section like that leading to Eq. 6.3, we conclude that in steady flow

$$\Delta P \cdot \begin{pmatrix}\text{area perpendicular}\\ \text{to the flow}\end{pmatrix} = \begin{pmatrix}\text{wall shear}\\ \text{stress}\end{pmatrix} \cdot \begin{pmatrix}\text{wetted}\\ \text{perimeter}\end{pmatrix} \cdot \Delta x \tag{6.31}$$

Rearranging this, we find

$$\frac{\Delta P}{\Delta x} = \tau \cdot \begin{pmatrix}\text{wetted}\\ \text{perimeter}\end{pmatrix} \Big/ \begin{pmatrix}\text{area perpendicular}\\ \text{to the flow}\end{pmatrix} \tag{6.32}$$

We now define a new term:

$$\begin{pmatrix}\text{Hydraulic}\\ \text{radius}\end{pmatrix} = \text{HR} = \begin{pmatrix}\text{area perpendicular}\\ \text{to the flow}\end{pmatrix} \Big/ \begin{pmatrix}\text{wetted}\\ \text{perimeter}\end{pmatrix} \tag{6.33}$$

For a circular pipe this is

$$\text{HR} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} = \frac{D}{4} \qquad [\text{circular pipe}] \tag{6.AO}$$

If the assumptions that went into Eq. 6.31 are correct, then we can construct the ratio of the pressure drop per unit length in a noncircular conduit to that in a circular one:

$$\frac{(\Delta P / \Delta x)_{\text{noncircular}}}{(\Delta P / \Delta x)_{\text{circular}}} = \frac{1 / \text{HR}}{4 / D} = \frac{D}{4\,\text{HR}} \tag{6.AP}$$

$$\left(\frac{\Delta P}{\Delta x}\right)_{\text{noncircular}} = \left(\frac{\Delta P}{\Delta x}\right)_{\text{circular}} \left(\frac{D}{4\,\text{HR}}\right) \tag{6.AQ}$$

But for turbulent flow $(\Delta P / \Delta x)_{\text{circular}}$ is given by the friction-factor equation, Eq. 6.23. Substituting, we get

$$\left(\frac{\Delta P}{\Delta x}\right)_{\text{noncircular}} = \frac{-4f\rho V^2}{2D} \cdot \frac{D}{4\,\text{HR}} = \frac{-f\rho V^2}{2\,\text{HR}} \tag{6.34}$$

Alternatively, we may write

$$\mathcal{F}_{\text{noncircular}} = \frac{f\,\Delta x}{\text{HR}} \cdot \frac{V^2}{2} \tag{6.35}$$

What value of f should we use in Eqs. 6.34 and 6.35? Experimental results indicate that those equations work *fairly well* if one uses the ordinary friction factor plot (Fig. 6.10) but replaces the diameter in $\mathcal{R}$ and in ε / D with 4 HR. The equations do not work well for shapes that depart radically from circles, such as long, narrow slits.

Example 6.14. Air at 1 atm and 68°F is flowing in a long, rectangular duct whose cross section is 1 ft by 0.5 ft, with V_{avg} = 40 ft / s. The roughness of the duct is 0.00006 in. What is the pressure drop per unit length?

First we calculate the hydraulic radius:

$$\text{HR} = \frac{0.5\ \text{ft}^2}{(2 \cdot 1\ \text{ft}) + (2 \cdot 0.5\ \text{ft})} = 0.167\ \text{ft} \tag{6.AR}$$

$$\mathcal{R} = \frac{V\rho(4\ \text{HR})}{\mu} = \frac{40\ \text{ft/s} \cdot 0.075\ \text{lbm/ft}^3 \cdot (4 \cdot 0.1667)\ \text{ft}}{0.018\ \text{cP} \cdot (6.72 \cdot 10^{-4}\ \text{lbm/ft} \cdot \text{s} \cdot \text{cP})} = 1.65 \cdot 10^5 \tag{6.AS}$$

$$\frac{\varepsilon}{D} = \frac{\varepsilon}{4\ \text{HR}} = \frac{0.00006\ \text{in}}{(4 \cdot 0.1667)\ \text{ft} \cdot (12\ \text{in/ft})} = 7.5 \cdot 10^{-6} \tag{6.AT}$$

From Fig. 6.10 (for this low an ε / D we use the "smooth-tubes" curve) we find $f = 0.0039$ or, using Eq. 6.21, we find $f = 0.00390$. Using the latter value in Eq. 6.34, we find

$$-\frac{\Delta P}{\Delta x} = \frac{0.00390 \cdot 0.075\ \text{lbm/ft}^3 \cdot (40\ \text{ft/s})^2}{2 \cdot 0.1667\ \text{ft}} \cdot \frac{\text{ft}^2}{144\ \text{in}^2} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2\ \text{lbm} \cdot \text{ft}}$$

$$= 3.0 \cdot 10^{-4}\ \frac{\text{lbf/in}^2}{\text{ft}} = 0.82\ \frac{\text{in H}_2\text{O}}{100\ \text{ft}} = 6.8\ \frac{\text{Pa}}{\text{m}} \tag{6.AU}$$

■

We may check this result by using Fig. 6.14. Here we assume that the pressure drop in this rectangular duct should be similar but not identical to that for the same volumetric flow rate in a circular duct with the same cross-sectional area. The diameter of such a duct would be

$$D = \sqrt{\frac{4}{\pi} A} = \sqrt{\frac{4}{\pi}\ 0.5\ \text{ft}^2} = 0.80\ \text{ft} = 9.6\ \text{in} \tag{6.AV}$$

Entering Fig. 6.14 at V = 2400 ft / min, and interpolating between the 9-in and 10-in diameter lines we find approximately the same pressure gradient values shown in Eq. 6.AU. Many air-conditioning ducts are rectangular or square, because for a given cross-sectional area they can fit in a smaller ceiling space than an equivalent circular duct. This comparison shows that the pressure gradient in them is very similar to that in a circular duct of equal cross-sectional area.

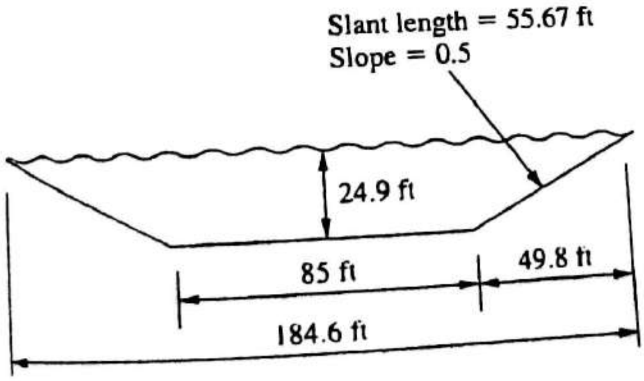


FIGURE 6.19
Cross-sectional view of one of the large irrigation water canals of California's Central Valley Project. The sloping sides have $dy / dx = 0.50$. The cross-sectional area (at the design depth of 24.9 ft) = 3356 ft³, the wetted perimeter = 196.4 ft, and HR = 17.09 ft. See Example 6.15.

Example 6.15. Figure 6.19 shows the cross section of one of the canals in the Central Valley Project irrigation system in California. The slope is 0.00004 ft / ft = 0.21 ft / mile. What are the velocity and volumetric flow rate in this canal?

First we apply B.E. from some upstream point in the canal to some downstream point in the canal. Since both points are open to the atmosphere, the pressures are the same. For steady flow of a constant-density fluid in a canal of constant cross-sectional area the velocities at the two points are the same. There is no pump or turbine work. Therefore, the remaining terms are

$$g\,\Delta z = -\mathcal{F} \tag{6.AW}$$

This says that the decrease in potential energy is exactly equal to the energy "loss" due to friction, i.e., the mechanical energy converted to internal energy. Substituting for $\mathcal{F}$ from Eq. 6.23, we find

$$-g\,\Delta z = \frac{f\,\Delta x\,V^2}{2\,\mathrm{HR}}; \qquad V = \left(\frac{2\cdot \mathrm{HR}\cdot g}{f}\cdot\frac{-\Delta z}{\Delta x}\right)^{1/2} \tag{6.AX}$$

In the previous example the wetted perimeter was the entire perimeter of the duct. Here we do not include the part of the perimeter facing the air, because the air exerts little resistance to the flow compared with the walls of the canal. The reader can verify this by watching the flow of leaves or bits of wood on any open stream or irrigation ditch; those at the center move much faster than those at the edges. If the air restrained the flow as much as the solid walls do, then the whole top surface of the flow would not move at all, just as the fluid right at the solid boundaries does not move. Therefore, the hydraulic radius is

$$\mathrm{HR} = \frac{\text{flow area}}{\text{wetted perimeter}} = \frac{3356\ \mathrm{ft}^2}{196.36\ \mathrm{ft}} = 17.09\ \mathrm{ft} \tag{6.AY}$$

The absolute roughness of a concrete-lined irrigation ditch is estimated from Table 6.2 at the low end of the range of values shown, 0.001 ft, so the estimated relative roughness is

$$\frac{\varepsilon}{4\,\mathrm{HR}} = \frac{0.001\ \mathrm{ft}}{4\cdot 17.09\ \mathrm{ft}} = 0.000015 \tag{6.AZ}$$

Here we do not know the velocity, so we cannot directly compute $\mathcal{R}$. However, we can guess a velocity, and proceed. We take $V_{\text{first guess}} = 1$ ft / s. Then

$$\mathcal{R}_{\text{first guess}} = \frac{4\cdot 17.09\ \mathrm{ft}\cdot 1.0\ \mathrm{ft/s}\cdot 62.3\ \mathrm{lbm/ft^3}}{1.0\ \mathrm{cP}\cdot(6.72\cdot 10^{-4}\ \mathrm{lbm/ft\cdot s\cdot cP})} = 6.3\cdot 10^6 \tag{6.BA}$$

Thus, from Fig. 6.10 or Eq. 6.21 we find $f_{\text{first guess}} = 0.0024$; therefore,

$$V_{\text{second guess}} = \left(\frac{2\cdot 17.09\ \mathrm{ft}\cdot 32.2\ \mathrm{ft/s^2}}{0.0024}\cdot 0.00004\right)^{1/2} = 4.28\ \frac{\mathrm{ft}}{\mathrm{s}} \tag{6.BB}$$

The design value is 3.89 ft / s, indicating that we have chosen a smaller value for the roughness of the concrete canal walls than did the designer (who had experimental data on such canals). We could compute a second guess of $\mathcal{R}$ and f, finding $f = 0.0023$, but we are not justified in doing this, because of our uncertainty of the roughness.

Using the design value of the velocity, we find

$$Q = VA = 3.89\,\frac{\text{ft}}{\text{s}} \cdot 3356\ \text{ft}^2 \approx 13{,}100\,\frac{\text{ft}^3}{\text{s}} \tag{6.BC}$$

which is the design volumetric flow rate of this section of the canal system. ■

6.11 MORE COMPLEX PROBLEMS INVOLVING B.E.

Now that we can evaluate all the terms in B.E., we may consider some of the more interesting types of problem that this equation can be used to solve.

Example 6.16. A large, high-pressure chemical reactor contains water at a pressure of 2000 psi. A 3-in schedule 80 line connecting to it ruptures at a point 10 ft from the reactor. What is the flow rate through this break?

This is an unsteady-state problem; the reactor pressure will fall during the outflow. However, if the reactor is large, the unsteady-state contribution can be neglected, and we will do so here. Applying B.E. from the free liquid surface in the reactor to the exit of the pipe, we neglect the potential-energy terms, which are negligible, and the small velocity at the free surface. The remaining terms are

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2}{2} = -\mathcal{F} \tag{6.36}$$

The flow rate in this case will be much higher than is used in common industrial practice, so App. A.3 will be of no use to us. Here the friction loss consists of two parts: the entrance loss into the pipe and the loss due to the flow through the 10 ft of pipe. Substituting from Eqs. 6.25 and 6.17, we find

$$\mathcal{F} = \underbrace{K\frac{V^2}{2}}_{\textit{entrance}} + \underbrace{4f\frac{\Delta x}{D}\cdot\frac{V^2}{2}}_{\textit{straight pipe}} \tag{6.BD}$$

Substituting Eq. 6.BD into Eq. 6.36, we find

$$\frac{P_2 - P_1}{\rho} = \frac{-V_2^2}{2} - \frac{KV_2^2}{2} - 4f\frac{\Delta x}{D}\cdot\frac{V_2^2}{2} = \frac{-V_2^2}{2}\left(1 + K + 4f\frac{\Delta x}{D}\right) \tag{6.BE}$$

$$V_2 = \left(\frac{2(P_1 - P_2)/\rho}{1 + K + 4f(\Delta x/D)}\right)^{1/2} \tag{6.BF}$$

From Fig. 6.16 we can read $K = 0.5$ (the diameter of the line is much smaller than the tank diameter). From App. A.2 we find that for 3-in schedule 80 pipe the inside diameter is 2.900 in. Then, from Table 6.2,

$$\frac{\varepsilon}{D} = \frac{0.0018 \text{ in}}{2.900 \text{ in}} = 0.00062 \tag{6.BG}$$

It is safe to assume that the Reynolds number here will be very high; so on Fig. 6.10 we select as our first guess a friction factor at the far right of the diagram, which for an ε / D of 0.00062 gives us $f = 0.0043$. Then

$$V_2 = \left[\frac{2(2000 - 15) \text{ lbf / in}^2}{62.3 \dfrac{\text{lbm}}{\text{ft}^3} \cdot \left(1 + 0.5 + \dfrac{4 \cdot 0.0043 \cdot 10 \text{ ft}}{(2.900 / 12) \text{ ft}}\right)} \cdot \frac{32.2 \text{ lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} \right]^{1/2} \tag{6.BH}$$

$$= \left[\frac{2 \cdot 1985 \cdot 32.2 \cdot 144}{62.3 \cdot (1 + 0.5 + 0.71)} \cdot \frac{\text{ft}^2}{\text{s}^2} \right]^{1/2} = 365 \frac{\text{ft}}{\text{s}} = 111 \frac{\text{m}}{\text{s}} \tag{6.BI}$$

We then check to see whether our assumed friction factor is correct:

$$\mathscr{R} = \frac{(2.9 / 12) \text{ ft} \cdot 62.3 \cdot \text{lbm / ft}^3 \cdot 365 \text{ ft / s}}{1.0 \text{ cP} \cdot (6.72 \cdot 10^{-4} \text{ lbm / ft} \cdot \text{s} \cdot \text{cP})} = 8.2 \cdot 10^6 \tag{6.BJ}$$

From Fig. 6.10 we see that our assumed f was correct. From App. A.2 we see that for 1 ft / s the flow rate is 20.55 gal / min; so the flow rate is

$$Q = 365 \frac{\text{ft}}{\text{s}} \cdot \frac{20.55 \text{ gal / min}}{\text{ft / s}} = 7500 \frac{\text{gal}}{\text{min}} = 0.47 \frac{\text{m}^3}{\text{s}} \tag{6.BK}$$

This is the instantaneous volumetric flow rate. As the flow continues, the pressure and flow rate will both decrease. ■

Appendix A.3 is of no use in this case, because the flow velocity is much larger than normal pipeline velocities. Ignoring the kinetic energy of the exit fluid, or the friction loss in the pipe, or the entrance loss would have given a significantly incorrect answer.

Example 6.17. A fire truck, Fig. 6.20, is sucking water from a river and delivering it through a long hose to a nozzle, from which it issues at a velocity of 100 ft / s. The total flow rate is 500 gal / min. The hoses have a diameter equivalent to that of a 4-in schedule 40 pipe and may be assumed to have the same relative roughness. The total length of hose, corrected for valves, fittings, entrance, etc., is 300 ft. What is the power required of the fire truck's pump?

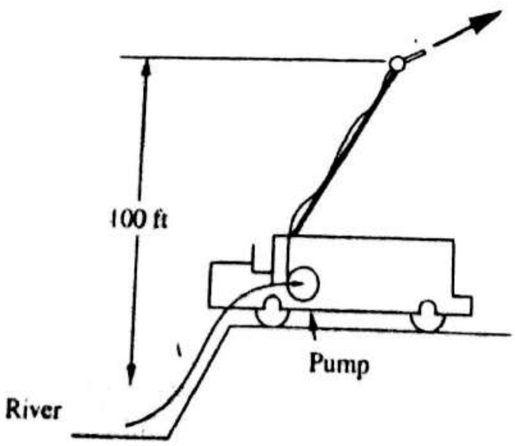


FIGURE 6.20
Fire truck pumping water from a river, used in Example 6.17.

Applying B.E. from the surface of the river, point 1, to the outlet of the nozzle, point 2, we find

$$g(z_2 - z_1) + \frac{V_2^2}{2} = \frac{dW_{\text{n.f.}}}{dm} - \mathscr{F} \tag{6.BL}$$

Table A.3

Here we may find the friction-loss term from App. A.3 and Eq. 6.24:

$$\mathcal{F} = \frac{-\Delta P}{\Delta x} \cdot \frac{\Delta x}{\rho} = \frac{5.65 \text{ lbf/in}^2}{100 \text{ ft}} \cdot \frac{300 \text{ ft}}{62.3 \text{ lbm/ft}^3} \cdot \frac{32.2 \text{ lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \cdot \frac{144 \text{ in}^2}{\text{ft}^2}$$
$$= 1260 \frac{\text{ft}^2}{\text{s}^2}$$

Then we have

$$\frac{dW_{\text{n.f.}}}{dm} = 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 100 \text{ ft} + \frac{(100 \text{ ft/s})^2}{2} + 1260 \frac{\text{ft}^2}{\text{s}^2} = 9480 \frac{\text{ft}^2}{\text{s}^2} \tag{6.BM}$$

$$\dot{m} = 500 \frac{\text{gal}}{\text{min}} \cdot 8.33 \frac{\text{lbm}}{\text{gal}} \cdot \frac{\text{min}}{60 \text{ s}} = 69.5 \frac{\text{lbm}}{\text{s}} = 31.6 \frac{\text{kg}}{\text{s}} \tag{6.BN}$$

Therefore,

$$\text{Po} = \frac{dW_{\text{n.f.}}}{dt} = \frac{dW_{\text{n.f.}}}{dm}\dot{m} \tag{6.BO}$$
$$= 9480 \frac{\text{ft}^2}{\text{s}^2} \cdot 69.5 \frac{\text{lbm}}{\text{s}} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \cdot \frac{\text{hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lbf}}$$
$$= 37 \text{ hp} = 27.6 \text{ kW} \tag{6.BP}$$

■

Figure 6.21 illustrates a class of problems that occurs very often in water supply networks in which multiple reservoirs are connected to multiple users. Water flows from one reservoir through a pipe to a division point (called a *node*), whence it flows to two other reservoirs via separate pipes. The elevations of the reservoirs are shown. The task is to compute the flow through each pipe branch, assuming steady flow. The three-reservoir example illustrates the idea, but not the complexity that can exist in municipal and industrial plant water supply networks that have grown and been modified over time.

For such long pipes the kinetic-energy terms in B.E. will be negligible, and the gauge pressures at the free surfaces are all zero. If these two simplifications are not appropriate, the problems can still be solved, but not as simply as is shown here. Writing B.E. for the three sections of the pipe in Fig. 6.21, we find

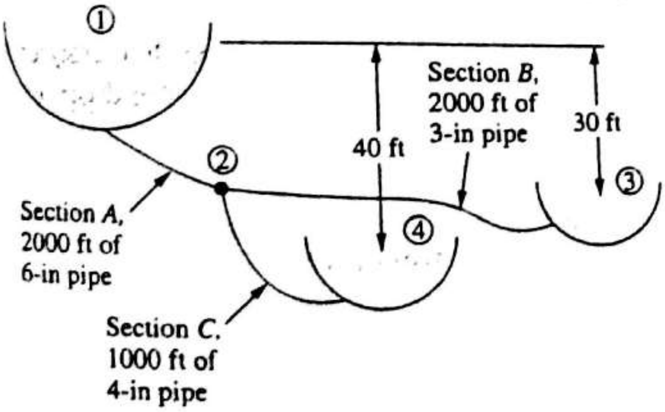


FIGURE 6.21
Multiple-reservoir system with branching pipes, typical of city water systems that have grown and been modified over time.

$$\frac{P_2}{\rho} + g(z_2 - z_1) = -\mathcal{F}_A = -\left(4f\frac{\Delta x}{D} \cdot \frac{V^2}{2}\right)_A \tag{6.BQ}$$

$$\frac{-P_2}{\rho} + g(z_3 - z_2) = -\mathcal{F}_B = -\left(4f\frac{\Delta x}{D} \cdot \frac{V^2}{2}\right)_B \tag{6.BR}$$

$$\frac{-P_2}{\rho} + g(z_4 - z_2) = -\mathcal{F}_C = -\left(4f\frac{\Delta x}{D} \cdot \frac{V^2}{2}\right)_C \tag{6.BS}$$

From the mass balance we find

$$Q_A = Q_B + Q_C; \qquad V_A = V_B \frac{A_B}{A_A} + V_C \frac{A_C}{A_A} \tag{6.BT}$$

Here we have four equations relating four unknowns (the three Vs and P_2). However, to solve the problem, we must use the correct values of the three fs, which are related to the pipe diameters and the Vs by the friction-factor chart, Fig. 6.10, or by Eq. 6.21. Thus, we could also think of this as a system with seven unknowns and seven equations (taking the friction-factor chart or Eq. 6.21 three times). Because of the forms of Eq. 6.21 there is no possibility that one can solve these equations analytically. The solution must be by trial and error.

In the problem statement the elevation at point 2 is not given, and the pressure at point 2 is unknown. In practice, we will know the elevation at point 2, and the flows will determine the pressure there, which will be different from the pressure which exists there when no fluid is flowing. The problem is inherently a trial and error, easy on computers. We begin by defining

$$\alpha = \left(\frac{P_2 - P_1}{\rho g} + z_2 - z_1 \right) \tag{6.BU}$$

and guessing a value of α. Using it we can solve each of Equations 6.BQ, 6.BR and 6.BS, for the velocities in each of the pipe sections. From those velocities we compute the three volumetric flow rates and check to see if the algebraic sum of the volumetric flow rates into point 2 is zero. If not, we make a new guess of α and repeat the flow calculations, to find the unique value of α which makes that sum = zero.

For this three-branch, one-node example the trial and error is quite easy (Prob. 6.68). For more complex examples it is not. A widely used systematic procedure for solving this type of system was developed by Cross [15]. Computer programs to carry out that solution are available [16].

6.12 ECONOMIC PIPE DIAMETER, ECONOMIC VELOCITY

From the foregoing we can easily calculate the flow rate, given the pipe diameter and pressure drop, or calculate the pipe diameter, given the flow rate and pressure drop, etc. A much more interesting question is, Given the design flow rate, what size pipe should we select? It is possible that the choice is dictated by aesthetics; e.g., the pipe goes through a lobby, and we want it to be the same size as other exposed pipes in the lobby. Or it may be dictated by the supply; e.g., we have on hand a large amount of surplus 4-in pipe that we want to use up. Most often the choice is based on economics; the engineer is asked to make the most economical selections, all things considered.

For economic analysis we must consider two possibilities:

1. The fluid is available at a high pressure and will eventually be throttled to a low pressure, so the energy needed to overcome friction losses may come from the available pressure drop.

2. The fluid is not available at a high pressure, so a pump or compressor is needed to overcome the effects of fluid friction.

The first is simple: we select the smallest size of pipe that will carry the required flow with the available pressure drop. Example 6.6 is that case.

If the effects of friction must be overcome by a pump or compressor, then the total annual costs of the pump-pipeline system are the following:

1. Power to run the pump.
2. Maintenance charges on pump and line.
3. Capital-cost charges for both line and pump.

How these change with increasing pipe diameter is sketched in Fig. 6.22. The figure indicates the following:

1. The larger the pipe diameter, the greater the capital charges. The cost of pipelines is roughly proportional to the pipe diameter; bigger pipes cost more to buy, require more expensive supports, take longer to install, etc. The cost of the pump is proportional to the cost of the pipe and is included in it.
2. The maintenance cost is practically independent of the pipe size.
3. The pumping cost goes down rapidly as the pipe size goes up. The pumping cost is proportional to the pressure drop (see Example 6.8), which for turbulent flow is proportional to the velocity to the 1.8 to 2.0 power divided by the diameter. The velocity (for constant flow rate) is proportional to the reciprocal of the square of the diameter, so the pumping cost is proportional to the reciprocal of the diameter to the 4.6 to 5 power.

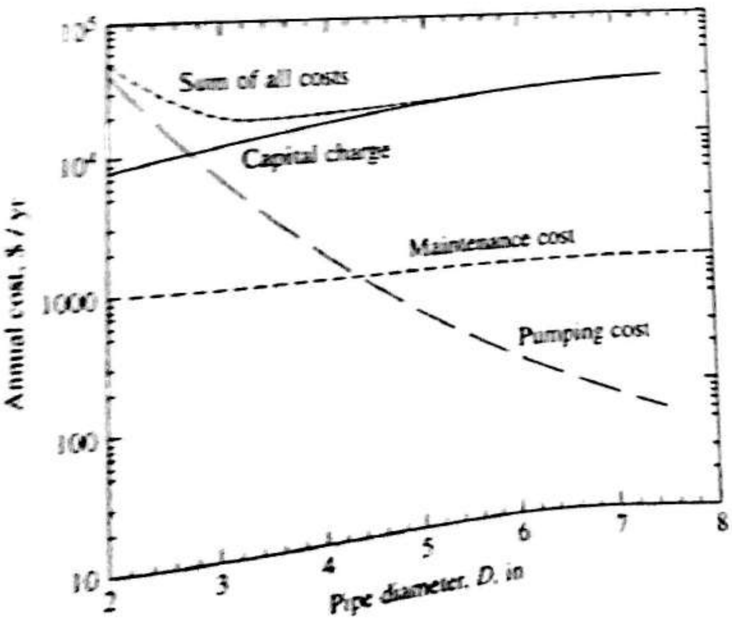


FIGURE 6.22
Relation between capital, operating, and maintenance costs for a pipeline with pumped flow. The numerical values are based on the economic data in Example 6.18.

As Fig. 6.22 shows, the sum of these has a rather broad minimum. This minimum occurs at the economic pipe diameter. Here we are taking the sum of a power cost during some finite period, e.g., a year, and the one-year charge for owning the pipeline and the pump, whose lifetime will be many years. Books on engineering economics or process design show various sophisticated ways to do that [17]. Here we use the simplest possible way, computing the annual capital charge, which is equivalent

This drag force is equal and opposite to the "thrust" force that must be supplied by the engine and propeller to keep the plane moving at this speed. The lift force is given by

$$F_{\text{lift}} = AC_l\rho\frac{V^2}{2} \tag{6.CJ}$$

which is the same as the drag force multiplied by C_l / C_d:

$$F_{\text{lift}} = 945 \text{ N} \cdot \frac{0.8}{0.04} = 18{,}900 \text{ N} = 4249 \text{ lbf} \tag{6.CK}$$

The lift is equal to the maximum gross, loaded weight of the aircraft. ■

The last example shows why the lift and drag coefficients are so useful to aeronautical engineers. Their ratios, C_l / C_d, are equal to the allowable ratio of total aircraft weight to thrust of the power plant. Normally both C_l and the C_l / C_d ratio are functions of aircraft speed and of the angle between the oncoming airstream and the wing surface [22]. This also shows why commercial aircraft fly as high as they can. To maintain level flight they need a lift equal to their weight and a thrust equal to their drag. The lift and drag are both proportional to ρV^2. For a given weight the required speed goes up as the square root of the air density goes down. The drag has the same relationship. So the higher they go, the lower the air density, and the faster they can go for a given hourly fuel input. Thus, their fuel cost per hour remains constant as they go up, but their fuel cost per mile goes down. (They also deliver the customers to their destination sooner, which the customers like, and pay for fewer hours of work to pilots and flight attendants.)

6.14 SUMMARY

1. The steady flow of fluids in constant-cross-section conduits can be of two radically different kinds: laminar, in which all the motion is all in the flow direction, and turbulent, in which there is a chaotic crosswise motion perpendicular to the net flow direction. The same is true for unconfined flows like the oceans or the atmosphere.
2. In laminar flow the pressure drop per unit length is proportional to the first power of the volumetric flow rate. The entire flow behavior can be calculated simply. The calculation requires the observational fact that fluid clings to solid surfaces, i.e., the velocity at the surface is zero.
3. In turbulent flow the pressure drop per unit length is proportional to the flow rate to the 1.8 to 2.0 power. The behavior cannot be calculated without experimental data.
4. All experimental data on the turbulent flow of Newtonian fluids in circular pipes can be represented on the friction factor plot.
5. The friction factor plot can be replaced by two fairly simple equations. The first, for laminar flow, is simply a rearrangement of Poiseuille's equation and is restricted to laminar flow in a circular tube, for which it is rigorous. The second is simply a satisfactory fit of the experimental data. With these two equations we can completely replace the friction factor plot. But the plot has considerable intuitive content and is still useful for hand calculations.

6. All data on turbulent flow through valves and fittings can be correlated by assuming that each kind of fitting contributes as much friction as a certain number of pipe diameters of straight pipe. That number is about the same for one kind of fitting, independent of pipe size, fluid properties, etc. An alternative approach assigns a resistance coefficient, K, to each kind of fitting.
7. Laminar flow in a few kinds of noncircular conduits can be analyzed by the same technique used for circular pipes.
8. Turbulent flow friction losses in many kinds of noncircular conduits can be estimated by substituting 4 times the hydraulic radius for the diameter in the Reynolds number, ε / D, the friction factor plot, and B.E.
9. The economic size for a pipe is the size with the lowest sum of annual charges for the purchased cost of the pipe and pump and the annual power cost of running the pump or compressor needed to overcome friction. For turbulent flow this results in an economic velocity which is practically independent of everything but fluid density; it is about 6 ft / s for most low viscosity liquids and about 40 ft / s for air and other gases under normal conditions.
10. The forces of fluids flowing over bodies are ordinarily correlated by the drag equation, in which the drag coefficient plays the same role as does the friction factor in pipe flow.

PROBLEMS

See the Common Units and Values for Problems and Examples inside the back cover. An asterisk (*) on a problem number indicates that its answer is shown in App. C. In all problems in this chapter, unless a statement is made to the contrary, assume that all pipes are schedule 40, commercial steel (see App. A.2).

6.1. Derive the equivalents of Eqs. 6.5 and 6.9 for fluid flow in the vertical direction, taking gravity into account. Then generalize them for fluid flow at any angle, taking gravity into account.

6.2. Air is flowing through a horizontal tube with a 1.00 in inside diameter. What is the maximum average velocity at which laminar flow will be the stable flow pattern? What is the pressure drop per unit length at this velocity?

6.3.*Repeat Prob. 6.2, for water.

6.4. Show the derivations of Eqs. 6.10 and 6.11.

6.5. Show the effect on the calculated viscosity, in the viscometer in Example 6.2, of the 10 percent error in the measurement of
(*a*) Flow rate
(*b*) Fluid density
(*c*) Tube diameter.

6.6. In Example 6.2 how much does the internal energy per unit mass of the fluid increase as it passes through the viscometer? Assume that there is no heat transfer from the fluid to the wall of the viscometer. If the heat capacity of the fluid is 0.5 Btu / lbm · °F = 2.14 kJ / kg · °C how much does the fluid's temperature rise?

6.7.*A circular, horizontal tube contains asphalt, with μ = 100,000 cP (= 1000 Poise) and ρ = 70 lbm / ft^3. The tube radius is 1.00 in. Asphalt may be considered a Newtonian fluid for the purposes of this problem, although it is not always one. We now apply a pressure gradient of 1.0 (psi) / ft. What is the steady state flow rate?

6.8. What is the Reynolds number in Example 6.2? What is the lowest fluid viscosity for which one should use this viscometer?

6.9. The simple capillary viscometer in Example 6.2 is not the same as that actually used. In the practical versions, there are two marks on the glass, and the user reads the time for the fluid level to pass between the two marks. In Example 6.2, if the upper reservoir has a diameter of 10 mm, how many seconds does it take the level in the reservoir to fall from 0.02 to 0.01 m? (In actual practice the marks are placed in narrow parts of a tube above and below the reservoir, so that reading the passage of the interface is easy. To see how these work look at the viscometers in any laboratory glassware catalog.)

6.10.*If the students in Fig. 6.8(*a*) throw a standard 0.31 lbm baseball back and forth, if its velocity in flight is 40 mi / h, and if each one throws it an average of once every 10 s, what is the average force in the y direction tending to separate them?

6.11. Show that in Fig. 6.8(*b*) the force in the x direction is independent of how fast the balls move in the y directions.

6.12. Show that, if we define the shear stress at the pipe wall as $\tau = f\rho V^2 / 2$ and then calculate the pressure gradient for horizontal flow, we find Eq. 6.18.

6.13. Show that Poiseuille's equation may be rewritten as $f = 16 / \mathcal{R}$.

6.14.*A fluid is flowing in a pipe. The pressure drop is 10 psi per 1000 ft. We now double the volumetric flow rate, holding the diameter and fluid properties constant. What is the pressure drop if the new Reynolds number

(*a*) Is 10?
(*b*) Is 10^8?

6.15.*As discussed in the text, there are two friction factors in common use, which means that there are two versions of Fig. 6.10 in common use. When one encounters a friction factor plot and wants to know on which of the definitions it is based, the easiest way is to look at the label on the laminar flow line. For the Fanning friction factor used in this book, that is labeled $f = 16 / \mathcal{R}$. For a chart based on the Darcy-Weisbach friction factor, what is the label on the laminar flow line?

6.16. Water is flowing at an average velocity of 7 ft / s in a 6-in pipe. What is the pressure drop per unit length?

6.17. An oil with a kinematic viscosity of 5 cSt and SG = 0.80 is flowing in a 3-in pipe. The pressure drop is 30 psi per 1000 ft. What is the flow rate in gallons per minute? Show the solution two ways:

(*a*) Using Fig. 6.12.
(*b*) Using Fig. 6.10 and/or Eqs. 6.20 and 6.21.

6.18. We want to transport 200 gal / min of fluid through a 3-in pipe. The available pressure drop is 28 psi per 1000 ft. The fluid properties are SG = 0.75 and μ = 0.1 cP. Is a 3-in pipe big enough?

6.19.*Oil is flowing at a rate of 150 gal / min, with μ = 1.5 cP, and SG = 0.87 in a 3-in pipe 1000 ft long. What is the pressure drop? Calculate it two ways:

(*a*) By Fig. 6.10 and/or Eqs. 6.20 and 6.21.
(*b*) By Fig. 6.12.

6.20. In Example 6.5, how much does the temperature of the gasoline rise as it flows through the pipe? Assume that there is no heat transfer from the gasoline and that its heat capacity is 0.6 Btu / lbm · °F.

6.21. (*a*) Set up the spreadsheet program shown in Table 6.5 and verify the values.
(*b*) Using that spreadsheet, find the corresponding values for $\Delta z = -30$ m.

6.22. (*a*) Set up the spreadsheet program shown in Table 6.6 and verify the values.
(*b*) Using that spreadsheet, find the corresponding values for Q = 2000 cfm.

6.23.*Two large water reservoirs are connected by 5000 ft of 8-in pipe. The level in one reservoir is 200 ft above the level in the other, and water is flowing steadily through the pipe from one reservoir to the other. Both reservoirs are open to the atmosphere. How many gallons per minute are flowing? Show both the solution based on Table A.3 and that based on Fig. 6.10/Eq.6.21.

6.24. In Prob. 6.23 we want to replace the existing pipe with a new one, which will transmit 10,000 gal / min under the same conditions. What size of pipe should we choose?

6.25.*Two tanks are connected by 500 ft of 3-in pipe. The tanks contain an oil with $\mu = 100$ cP and SG = 0.85. The level in the first tank is 20 ft above the level in the second, and the pressure in the second is 10 psi greater than the pressure in the first. How much oil is flowing through the pipe? Which way is it flowing?

6.26. We are offered some pipes made of a new kind of plastic. To test their roughness, we pump water through a 3-in pipe made of this material at an average velocity of 40 ft / s. The observed friction factor is 0.0032. Estimate the absolute roughness of this plastic.

6.27. As discussed in Sec. 6.5, the friction factor plot, Fig. 6.10, relates six variables and therefore can be used for finding any of the six if the other five are known. Examples 6.4, 6.5, and 6.6 show how to find three of these quantities, given all the others. Problem 6.26 shows how to find a fourth, given all the others. The remaining two are the density and viscosity of the flowing fluid. *Turbulent-flow* pressure drops are almost never used for determining fluid viscosities or densities. Discuss why this is so.

6.28. Equation 6.21 leads easily to quick trial-and-error solutions to all the problems in Sec. 6.5. One could also use it to eliminate a variable and thus reduce those trial-and-error solutions to a single equation. Show the algebra of that elimination and the resulting single-variable equations for Examples 6.5 and 6.6. Are those equations likely to be easier to solve analytically or numerically?

6.29. On a piece of log paper 2 cycles by 2 cycles, make up the equivalent of Fig. 6.12 for a 2-in pipe. Show the following:

(*a*) The zero-viscosity boundary.
(*b*) The laminar-flow region.
(*c*) The turbulent-flow region.
(*d*) The transition region.

6.30. Calculate the pressure drop per unit length for the flow of 100 gal / min of air in a 3-in pipe by using Fig. 6.12.

6.31.*Oil of a kinematic viscosity of 20 cSt is flowing in a 3-in pipe. According to Fig. 6.12,

(*a*) What is the highest volumetric flow rate at which the flow is certain to be laminar?
(*b*) What Reynolds number does this correspond to?
(*c*) What is the lowest volumetric flow rate at which the flow is certain to be turbulent?
(*d*) What Reynolds number does this correspond to?

6.32. Do any of the values in App. A.3 correspond to laminar flow?

6.33. Check the values in App. A.3 for 24-in pipe to see whether they correspond to a constant friction factor or whether the pipe size is so large that the friction factor corresponds to the "smooth tubes" curve in Fig. 6.10. The inside diameter of 24-in schedule 40 pipe is 22.624 in.

6.34. Rework Example 6.5 by using App. A.3. Show what corrections, if any, are needed in the solution if it is assumed that gasoline has the same flow properties as water. In that example the pipe ID was 0.1 m = 3.94 in. From App. A.2 we see that 4-in schedule 40 pipe has 4.03 in ID. Work the problem assuming that 3.94 ≈ 4.03, and then estimate how much difference this simplification makes.

6.35. Estimate the pressure loss for 1000 ft^3 / min of air flowing in a 12-in diameter air-conditioning duct 1000 ft long.

6.36. Estimate the required pipe diameter to transport 100 m^3 / h of air with a friction loss of 1 Pa / m.

6.37. Estimate the volumetric flow rate of air for a pressure drop of 5 Pa / m in a duct with diameter 0.125 m.

6.38. Estimate the pressure drop for 1000 ft^3 / min of hydrogen flowing in a 6-in diameter pipe 500 ft long in two ways:
(*a*) Using Fig. 6.10, or Eq. 6.21.
(*b*) Using Fig. 6.14, and suitable corrections for its much lower density than that of air.

6.39. The common friction factor plot (Fig. 6.10) is based on the Colebrook equation [5],

$$\frac{1}{\sqrt{f}} = -4 \log\left(\frac{\varepsilon / D}{3.7} + \frac{1.255}{\mathcal{R}\sqrt{f}}\right) \tag{6.59}$$

which itself is a data-fitting equation with no theoretical basis. It is difficult to use, because f appears on both sides, once as the argument of a logarithm. There are other data-fitting equations that attempt to reproduce Eq. 6.59 with a more easily used form, of which one of the most popular is that due to Haaland [23],

$$f = 0.25 \Big/ \left(-1.8 \log\left[\frac{6.9}{\mathcal{R}} + \left(\frac{\varepsilon / D}{3.7}\right)^{1.11}\right]\right)^2 \tag{6.60}$$

another equation, [24], is

$$f = 0.0624 \Big/ \left[\log\left(\frac{\varepsilon}{3.7 D_H} + \frac{5.74}{\mathcal{R}^{0.8}}\right)\right]^2 \tag{6.61}$$

where D_H is the hydraulic diameter, twice the hydraulic radius.
Find the friction factor for $\mathcal{R} = 2.00 \cdot 10^5$, and $\varepsilon / D = 0.0006$,
(*a*) From Fig. 6.10.
(*b*) From the Colebrook equation (Eq. 6.59).
(*c*) From the Haaland equation (Eq. 6.60).
(*d*) From Eq. 6.21.
(*e*) From Eq. 6.61.

6.40. Figure 6.14 is the "standard chart" for air-conditioning applications. It is based on Fig. 6.10 and the assumptions that the air flowing is at 1 atm and 68°F, i.e., the same assumption as given inside the back cover of the book. The plot is logarithmic on both axes, but the length of a decade on the vertical axis is greater than that on the horizontal axis.
(*a*) If a given duct diameter corresponded to a fixed value of f, what should the (line? curve?) for that diameter look like on this plot?
(*b*) Is that shape observed for the small ducts, i.e., those with $D < 5$ in?
(*c*) Is that shape observed for large ducts, i.e., those with $D > 10$ in?
(*d*) Why are these different?
(*e*) Figure 6.14 shows a pressure gradient of almost exactly 0.2 in of water / 100 ft for 1000 ft^3 / min in a 12-in duct. What value of the absolute roughness ε does that correspond to?
(*f*) How does the value just determined in part (*e*) compare to the value for steel in Table 6.2? Explain!

6.41. Examples 6.11 and 6.12 show significantly different values for the pressure drop due to the valves and fittings, calculated by the equivalent length method and the K method. Is that result a function of pipe diameter? To answer this question, prepare a plot of $\Delta P_{\text{valves and fittings}}$ versus pipe diameter for the same fluid as in Example 6.4 (SG = 1.00, μ = 50 cP) and the same average velocity, V_{avg} = 13.0 ft / s, covering the pipe diameter range of 3 in (that example) to 12 in, with the same fittings as in Examples 6.11 and 6.12, by the methods in Examples 6.11 and 6.17.

6.42. *Water is flowing at a rate of 1500 gal / min in a horizontal, 10-in pipe that is 50 ft long and contains two standard 90° elbows and a swing-type check valve. Estimate the pressure drop using both methods of accounting for the elbows and the valve.

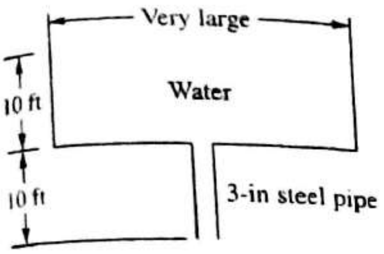


FIGURE 6.27
Tank draining by gravity, with pipe and entrance friction, Prob. 6.45.

6.43. A piping system consists of 100 ft of 2-in pipe, a sudden expansion to 3-in pipe, and then 50 ft of 3-in pipe. Water is flowing at 100 gal / min through the system. What is the pressure difference from one end of the pipe to the other?

6.44. *Two large water tanks are connected by a 10 ft piece of 3-in pipe. The levels in the tanks are equal. When the pressure difference between the tanks is 30 psi, what is the volumetric flow rate through the pipe?

6.45. The water in Fig. 6.27 is flowing steadily. What is the flow rate?

6.46. *We are going to lay a length of 6-in steel pipe for a long distance and allow water to flow through it by gravity. If we want a flow rate of 500 gal / min, how much must we slope the pipe (i.e., how many feet of drop per foot of pipe length or how many ft / mi)?

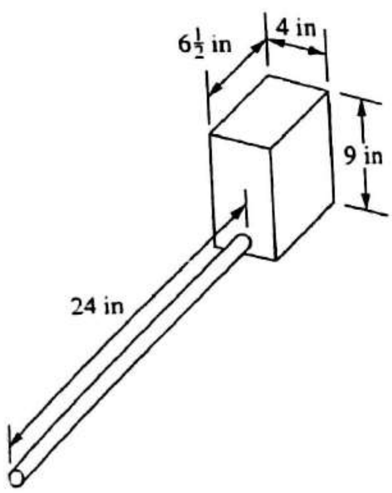


FIGURE 6.28
A portable demonstrator of tank draining with friction, Prob. 6.47.

6.47. A 1-gal can full of water has the dimensions shown in Fig. 6.28. There is a horizontal piece of $\frac{1}{4}$-in galvanized pipe inserted in the bottom. The end of the pipe is unplugged, and the water is allowed to flow out of the tank.

(*a*) How long will it take the level in the tank to fall from 7 in above the centerline of the pipe to 1 in above the centerline of the pipe? Make whatever assumptions seem plausible.

(*b*) As the level falls, the flow slows down, until it finally converts from turbulent to laminar. How far will the level be above the centerline of the pipe when this transition occurs?

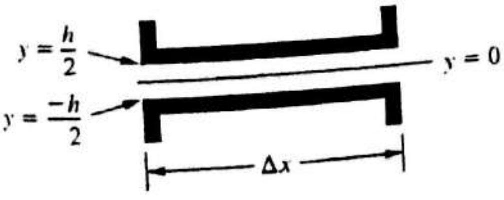


FIGURE 6.29
Suggested dimensions for Prob. 6.48.

6.48. Derive Eq. 6.28. It is suggested that you use the coordinates shown in Fig. 6.29. Here the flow is in the x direction from left to right, and the slit extends a distance l in the z direction. Choose as your element for the force balance a piece symmetrical about the y axis (other choices are possible but lead to more difficult mathematics). *Hint:* This is a repeat of the derivation of Eq. 6.8 in a different geometry. Simply follow that derivation, changing the geometry.

6.49. Derive Eq. 6.29. This equation is derived in detail in Bird et al. [12, p. 54].

6.50. Example 6.13 shows that the calculated leakage rate is less than the average observed rate for typical valves in oil refineries by a factor of 3.5. In that example we assumed that the average thickness of the leakage path was 0.0001 in. If we held all the other values in that example constant except this thickness, what value of the thickness corresponds to the observed leakage rate?

6.51. In Example 6.13 we replaced Eq. 6.29 (flow in an annulus) with Eq. 6.30 (the linear simplification of Eq. 6.29). How much difference does it make in our answer? Check by repeating Example 6.13, using Eq. 6.29.

6.52. (*a*) Show how one obtains Eq. 6.30 from Eq. 6.28.
(*b*) Show the ratio of $Q_{\text{Eq. 6.30}} / Q_{\text{Eq. 6.28}}$.
(*c*) Using a spreadsheet, show the value of this ratio for $D_o / D_i = 1.1, 1.01, 1.001$, and 1.0001.

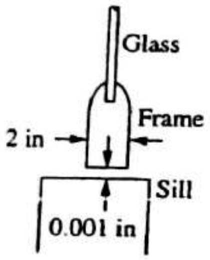


FIGURE 6.30
Leakage flow beneath a window, Prob. 6.53.

6.53.*The wooden frame of a window is 2 in thick; see Fig. 6.30. The bottom of the window closes against the sill with a space between frame and sill of 0.001 in. The width of the window (distance perpendicular to the paper in the figure) is 2 ft. When the wind is blowing toward the window and creating a pressure difference of 0.01 psi across the window, what is the volumetric flow rate of air through the space between frame and sill?

6.54. The cylindrical vessel in Fig. 6.31 is full of water at a pressure of 1000 psig. The top is held on by a flanged joint, which has been ground smooth and flat, with a clearance of 10^{-5} in, as shown. The diameter of the vessel is 10 ft. Estimate the leakage rate through this joint.

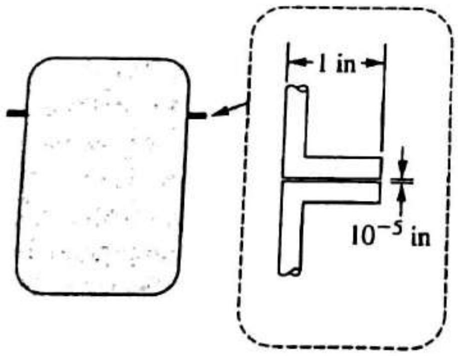


FIGURE 6.31
Leakage flow through a flanged joint, Prob. 6.54.

6.55. Calculate the hydraulic radius for each of the following shapes:
(*a*) A semicircle with the top closed.
(*b*) A semicircle with the top open.
(*c*) A closed square.
(*d*) An annulus.

6.56.*Rework Example 6.6, assuming that a square duct is to be used. For equal cross-sectional areas and equal wall thicknesses, what is the ratio of the weight per foot of a square duct to that of a circular one? Based on this, which would normally be chosen if there were no space constraints? Look around public buildings in which such ducts are visible, and examine where circular ducts are used and where square or rectangular ducts are used. Do your observations agree with your answer to this problem?

6.57.*In Example 6.15, what values of f and ε did the designer use to estimate $V = 3.97$ ft / s?

6.58. (*a*) In hydraulics books one regularly encounters the *Chézy formula* for open channel flow,

$$V = C\sqrt{\text{HR} \cdot \frac{-\Delta z}{\Delta x}} \tag{6.62}$$

in which we have changed from the notation normally shown in those books to the notation in this book. What value of C makes Eq. 6.62 the same as Eq. 6.AX?

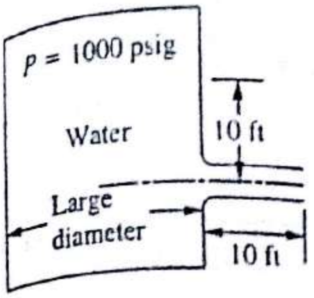


FIGURE 6.32
Pressure and gravity-driven flow with friction, Prob. 6.59.

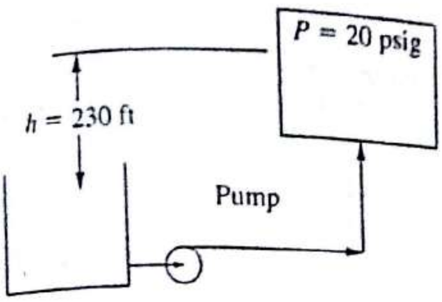


FIGURE 6.33
Pumped fluid transfer with both pressure and elevation change, Prob. 6.60.

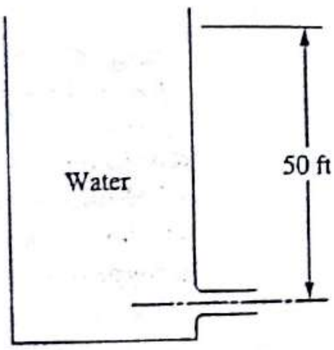


FIGURE 6.34
Gravity draining flow with friction, Prob. 6.61.

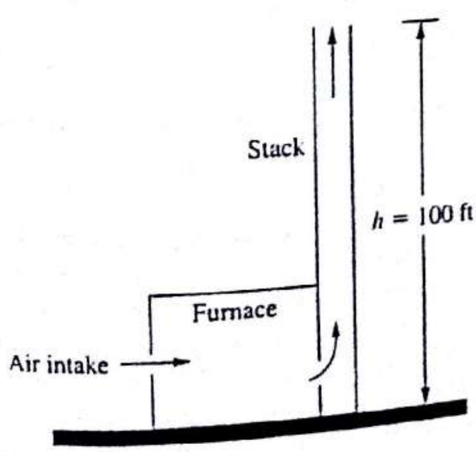


FIGURE 6.35
Flow in a furnace and chimney, Prob. 6.62.

(*b*) The same books often show the *Manning coefficients* to use in the *Chézy formula*, given by

$$C = \alpha \frac{\text{HR}^{1/6}}{n} \tag{6.63}$$

in which n is a roughness parameter, whose value is ≈ 0.012 for finished cement and α is a dimensional conversion factor. Determine the value α that corresponds to Example 6.15.

6.59.* In Fig. 6.32 the 3-in pipe is joined to the tank by a well-designed adapter, in which there is no entrance loss. What is the instantaneous velocity in the pipe?

6.60. In Fig. 6.33 water is being pumped through a 3-in pipe. The length of the pipe plus the equivalent length for fittings is 2300 ft. The design flow rate is 150 gal / min.

(*a*) At this flow rate, what pressure rise across the pump is required?

(*b*) If there are no losses in pump, motor, coupling, etc., how many horsepower must the pump's motor deliver?

6.61.* The tank in Fig. 6.34 is attached to 10 ft of 5-in pipe. The losses at the entrance from the reservoir to the pipe are negligible. What is the velocity at the exit of the pipe?

6.62. The flue gas in the stack in Fig. 6.35 is at 350°F and has $M = 28$ g / mol. The stack diameter is 5 ft, and the friction factor in the stack is 0.005. In passing through the furnace the air changes significantly in density, because it is heated by the combustion and then cooled in giving up heat to the working parts of the furnace. Thus, we cannot rigorously apply B.E. in the form we use in this problem (we could do so by integrating from point to point, over points so close together that the density change was negligible, but that would be very difficult in such a complex flow through a furnace). However, experimental data on the friction effects of furnaces indicate that if we treat them as constant-density devices with flowing fluids having the density and viscosity of the gas in the stack, then we can use Eq. 6.25. For this furnace, for those assumptions, $K \approx 3.0$. Thus, in applying B.E. to this furnace and stack, assume that the air changes at the inlet to the furnace to a gas with $M = 28$ g / mol and $T = 350$°F, and then maintains that M and T throughout the furnace and the stack. Estimate the velocity of the gases in the stack.

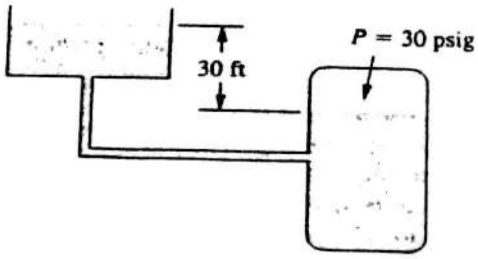


FIGURE 6.36
Flow driven one way by gravity and the opposite way by pressure difference, Prob. 6.63.

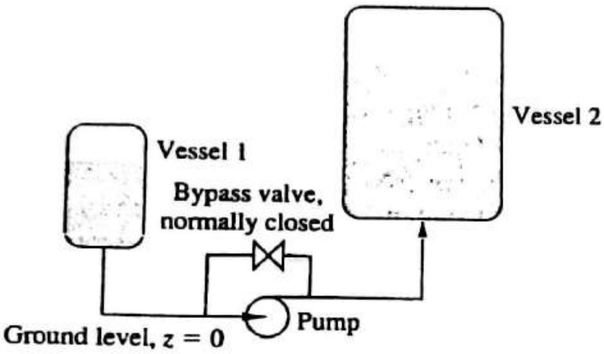


FIGURE 6.37
A somewhat more realistic pumping situation, Prob. 6.64.

TABLE 6.A
Values for Prob. 6.64

	Vessel 1	Vessel 2
P_{max}, psig	20	81
P_{min}, psig	8	47
Max liquid level, above $z = 0$, ft	43	127
Min liquid level, above $z = 0$, ft	21	100

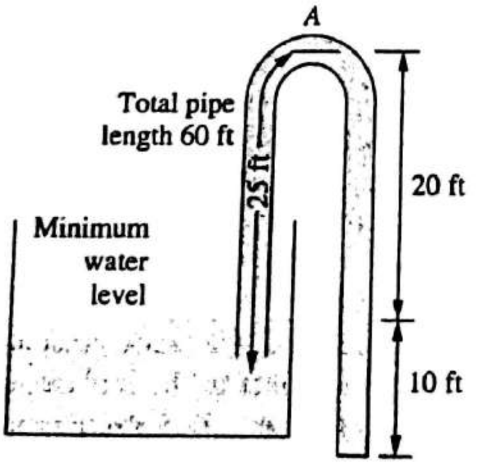


FIGURE 6.38
Siphon with friction, Prob. 6.66.

6.63.*The vessels in Fig. 6.36 are connected by 1000 ft of 3-in pipe (neglect fitting and entrance and exit losses). In each vessel the diameter is so large that V is negligible. The fluid is an oil with ν = 100 cSt, and ρ = 60 lbm / ft^3. How many gallons per minute are flowing? Which way?

6.64. The two vessels shown in Fig. 6.37 have the design conditions shown in Table 6.A. The connecting line between the vessels is 3-in pipe that is 627 ft long, containing six elbows, four gate valves, and one globe valve. The fluid to be pumped has a specific gravity range of 0.80 to 0.85 and a kinematic viscosity range of 2 to 5 cSt. The flow rate is 150 to 200 gal / min. We are ordering the pump. What values will we specify of

(*a*) the flow rate?

(*b*) the pump head, $\Delta P / \rho g$, in feet? For this problem the head form of B.E. is convenient.

6.65.*If we shut off the pump in the system in Prob. 6.64 and open the bypass around it, what are the maximum and minimum values of the volumetric flow rate? Which way does it go? Neglect the friction losses in the pump bypass line. Assume that the globe valve is always wide open.

6.66. Figure 6.38 shows a siphon, which will be used to empty water out of a tank. The siphon is made of 10-in pipe, 60 ft long. When the water is at its minimum level, as shown, what is the volumetric flow rate, and what is the pressure at the top (point A)? The bend at the top of the siphon is equivalent to two 90° long-radius elbows.

6.67. The National Park Service has recently decided to construct a pipeline to carry water across the Grand Canyon from the relatively water-rich North Rim to the arid South Rim. A cross section of the system is shown in Fig. 6.39. The length of pipeline between the springs and the river crossing is 10 mi, and between the river crossing and the pumping station it is 4 mi. The desired flow rate is 1000 gal / min. The pressure at the springs and at the pumping station may be assumed

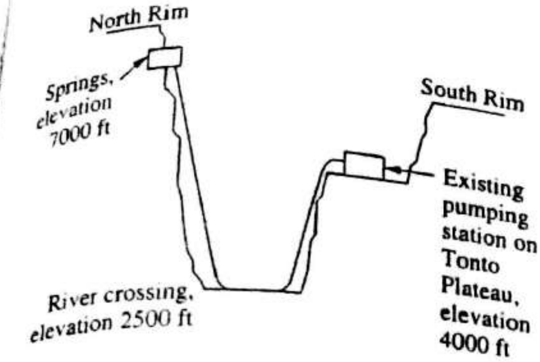


FIGURE 6.39
Elevations for the freshwater pipeline across the Grand Canyon, Prob. 6.67.

atmospheric. Because all the materials must be brought into place on muleback, which is costly, there is a considerable incentive to make the pipe as lightweight as possible. Recommend what material the pipe should be made of, what its inside diameter should be, and what its wall thickness should be.

6.68. Solve for the steady state flows in Fig. 6.21. As a first guess take $\alpha = 10$ ft, which is close to, but not equal the value of α which makes the sum of the volumetric flow rates into point 2 = 0.

6.69. Repeat Prob. 6.68 for the case where $(z_3 - z_1) = -5$ ft and $(z_4 - z_1) = -40$ ft. *Hint:* In this case, not all the flows will be in the same direction as in Prob. 6.68.

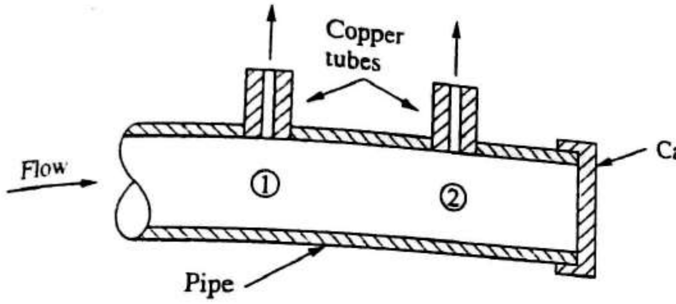


FIGURE 6.40
Part of a manifold, Prob. 6.70.

6.70.*Figure 6.40 shows the end of a *manifold* in which one major pipe feeds into a number of smaller pipes that branch from it. Many air-conditioning ducts are manifolds. This figure shows only the last two branches. The flow out through the branches depends almost entirely on the pressure in the main channel opposite them, and very little on the velocity in that channel. For the flow as shown, will the velocity out of tube 1 be greater or less than that out of tube 2;

(*a*) For zero friction in the pipe flow (because it is so much bigger than the tubes)?

(*b*) For substantial friction in the pipe flow (because a cylindrical rod has been inserted, thus making the cross-sectional area perpendicular to flow much less)?

(*c*) Dr. J. Q. Cope, vice president of Chevron Research when the author started there in 1958, used this device to teach humility to new Ph.D.s. He would describe the device, without mentioning the insertable rod. Then he would goad the new Ph.D. into betting him which jet would squirt higher when water was introduced. Then he would go and get the device, inserting or removing the rod as needed before returning to collect on his bet. What practical lessons might a new engineer learn from this story [25]?

6.71. Check the assumed friction factor in Example 6.18. For the value of relative roughness shown, the range of possible friction factors in turbulent flow is 0.004 to 0.01. How much would the economic diameter differ from Example 6.18 if $f = 0.01$?

6.72. The type of calculation of economic diameter of pipes shown in Sec. 6.12 was apparently first performed by Lord Kelvin [26] in connection with the problem of selecting the economic diameter for long-distance electric wires. To see how he obtained his result, derive the formula for the economic diameter of an electric conductor (analogous to Eq. 6.47) using the following information: The purchased cost of the whole transmission line (including poles, insulators, land, construction labor, etc.) is A times the mass of metal in the wire, where A has dimensions of \$/lbm. The annual cost of owning the transmission line (which includes interest on the capital investment in the line, payment on the principal of that investment, taxes, and maintenance) is B times the purchased cost of the whole transmission line, where B has dimensions of $1/y$. The electrical energy that is lost due to resistive heating in the wire costs C, where C has dimensions of \$/kWh. The resistive heating is given by $Q = I^2R$, where R is the resistance of the wire and I is the current. The resistance of the wire is given by $R = r\,\Delta x / [(\pi/4)D^2]$, where r is the resistivity (with dimension ohm-ft), Δx is the length of the wire, and D is the wire diameter.

Your formula for the economic diameter should be written in terms of the current to be carried (*not* in terms of the voltage), and in terms of the other variables listed above, plus any others that you consider necessary. Do not be concerned about numerical units conversions; your final equation should be like Eq. 6.47, showing D_{econ} as a function of the appropriate variables to the appropriate powers. (Kelvin's solution is still correct for low-voltage transmission lines, but not for the high-voltage lines now used, in which the major loss is not resistive heating but corona discharge from the wire's surface.)

6.73. (*a*) Work out the equation equivalent to Eq. 6.47 for laminar flow. Start with Eq. 6.45 and substitute $f = 16/\mathcal{R}$. Simplify the resulting equation, finding

$$D_{\text{econ, laminar}} = \left[\frac{32\pi \cdot \text{PC} \cdot (4/\pi)^2 \mu Q^2}{\text{CC} \cdot \text{PP}}\right]^{1/3} \tag{6.64}$$

(*b*) Check to see whether the laminar part of Fig. 6.23 is made up by this equation, by calculating the economic velocity for 200 gpm and 2000 cSt, and comparing your answer to the value you read from that figure. Use the economic values that are shown with that figure.

(*c*) The lines of constant viscosity on Fig 6.23 have slope ≈ 0.2. Does this agree with Eq. 6.64?

(*d*) Figure 6.23 indicates that the economic velocity is practically independent of fluid density. Does this agree with Eq. 6.64?

6.74. It has been proposed to solve Los Angeles' air pollution problem by pumping out the contaminated air mass every day. The area of the L.A. Basin is 4083 mi^2. The contaminated air layer is roughly 2000 ft thick. Suppose we plan to pump it out every day, a distance of 50 mi to Palm Springs. (It is assumed that the residents of Palm Springs will not object, which is not a very good assumption.)

(*a*) Estimate the economic velocity in the pipe.

(*b*) Estimate the required pipe diameter.

(*c*) Estimate the pressure drop.

(*d*) Estimate the pumping power requirement.

(*e*) Comment on the feasibility of this proposal.

6.75. It has been proposed to solve the water problem in Los Angeles by importing water from the mouth of the Columbia River, where vast amounts flow into the sea. One way to do this would be with a pipeline and pumping station. Both ends of the pipe are at sea level, so the only pumping cost would be the cost of overcoming the friction loss. The pipe length would be about 1000 mi. Assuming that we want to move 10^7 acre-ft a year (1 acre-ft = 43,560 ft^3), estimate the horsepower of pumps required. State your assumptions.

6.76. If in Example 6.18 the fluid were water contaminated with hydrofluoric acid, we would have to use a special corrosion-resistant pipe. Suppose that this pipe had a value of PP exactly 10 times that of carbon-steel pipe. What would D_{econ} be?

6.77. If Table 6.8 were based on Eq. 6.49 and the friction factor were held constant, then the product of the economic velocity and the cube root of the density would be a constant. How much does it vary from being a constant? What is the cause of this variation?

6.78. Does the result in Example 6.18 agree exactly with the data in Table 6.8 and Fig. 6.23? If not, how much does it disagree, and what is the most probable cause of the difference?

6.79. You are selected to design the fuel line for a Mars-landing rocket. Money is unimportant: low mass is the main goal. Decide what the significant "economic" factors in this problem are, and write in general form the equivalent of Eq. 6.47 for this problem.

6.80. A 1 μ diameter spherical particle with SG = 2.0 is ejected from a gun into air at a velocity of 10 m / s. How far does it travel before it is stopped by viscous friction? (This distance is the *Stokes' stopping distance*, which appears often in the fine particle literature.) Ignore the effect of gravity.

(*a*) Work out the equation in general terms, by writing $F = ma$. The drag force is the only force acting on the particle, after it leaves the gun, operating in the direction opposite the direction of motion, and is given by Eq. 6.54

$$F = -3\pi\mu DV = ma = \frac{\pi}{6}D^3\rho\frac{dV}{dt}; \qquad \frac{dV}{dt} = -\frac{18\mu V}{D^2\rho} \tag{6.65}$$

Substitute $dt = dx / V$, separate the variables, cancel the two V terms, and integrate from $V = V_0$ at $x = 0$ to $V = 0$ at $x_{\text{Stokes' stopping distance}}$ to find

$$x_{\text{Stokes' stopping}} = \frac{V_0 D^2 \rho}{18\,\mu} \tag{6.66}$$

One often sees this equation with a C, for the Cunningham correction factor in the numerator.

(*b*) Insert the numerical values and find the value of $x_{\text{Stokes' stopping distance}}$.

(*c*) On the basis of the logic of this calculation, how long does it take the particle to come to exactly zero velocity? How long does it take it to come to 1 percent of V_0?

(*d*) How far does the particle fall by gravity (which we ignored in this derivation) in the time it takes to come to 1 percent of V_0?

6.81.*Rework Example 6.19 for the particle settling in water at 68°F instead of in air.

6.82. Rework Example 6.20 for the ball falling in glycerin instead of in water. $\mu_{\text{glyc}} = 800$ cP, and $\rho_{\text{glyc}} = 78.5$ lbm / ft^3.

6.83.*A spherical balloon is 10 ft in diameter and has a buoyant force 0.1 lbf greater than its weight. What is its terminal velocity rising through air?

6.84. A standard baseball has a diameter of 2.9 in and a mass of 0.31 lbm. Good fast-ball pitchers can throw one at about 100 mi / h.

(*a*) Neglecting the effect of the stitching on the ball and the spin of the ball, estimate the drag force of the air on the ball.

(*b*) The distance from the pitcher's mound to home plate is 60 ft. If the ball left the pitcher's hand at 100 mi / h, how fast will it be going when it reaches home plate, subject to the simplifications in part (*a*)?

6.85. Probably the most-studied kick in soccer history was David Beckham's free-kick goal in the England-Greece World Cup Qualifiers in 2001, [27]. This kick even made a movie title ("Bend it like Beckham," 2002).

The kick left his foot at 36 m / s, 27 m from the goal. It was high enough to pass over the screen of defenders, and spinning enough on a vertical axis to curve toward the corner of the goal. It appeared to be aimed above the goal, but suddenly slowed down dramatically in flight, and fell into the upper corner of the goal. Explain in terms of Fig. 6.24 how this was possible. A standard soccer ball weighs ≈425 g, and has a diameter of ≈22.3 cm.

6.86. You have certainly observed that you can walk much faster in air than you can when you are up to your neck in a swimming pool. Why? Is that mostly because the water is more viscous than air? Or because the water is more dense than air? Or some other answer?

6.87.*A 120-lbm parachutist jumps from a plane and falls in free fall for a while before opening her chute.

(*a*) If she falls head first, then her projected area perpendicular to the direction of fall is ≈1 ft^2 and C_d ≈ 0.7. What is her terminal velocity? How many seconds must she fall to reach 99 percent of this terminal velocity, assuming that the drag coefficient is independent of velocity? How far does she fall in this number of seconds?

(*b*) If instead of falling head first, she spreads out her arms and legs and lies horizontally, then her projected area will be ≈6 ft^2 and C_d ≈ 1.5. Repeat part (*a*) for this condition.

6.88. We want to design a parachute. The requirement is that at terminal velocity the rider will have a velocity equal to the maximum velocity the rider would reach jumping to the ground from a 10-ft-high roof. The rider weighs 150 lbf. The parachute will be circular and its drag coefficient C_d = 1.5. What diameter must the parachute have?

6.89. Occasionally, car companies advertise that their sports-model automobiles have very low values of C_d, typically about 0.3 for teardrop-shaped cars. That drag coefficient is based on the frontal area. If a car has that C_d and a width of 6 ft and a height of 5 ft and is going 70 mi / h,

(*a*) What is the air resistance of the car?

(*b*) How much power must be expended to overcome this air resistance?

6.90. In Examples 6.19 and 6.20, we assumed Stokes' law applies, calculated *V*, and then checked $\mathcal{R}_p$ to see whether the assumption of Stokes' law was a good one. If our assumption was not a good one, then the *V* calculated in the first step was a wrong velocity, and the calculated $\mathcal{R}_p$ was wrong, too. Is there any chance that this procedure can lead to a combination of *V* and $\mathcal{R}_p$ that indicates that Stokes' law should be obeyed when actually the $\mathcal{R}_p$ based on the correct solution is outside the range of Stokes' law?

6.91. Check the results of Examples 6.19 and 6.20 on Fig. 6.26.

6.92. A spherical raindrop with a diameter of 0.001 in is falling at its terminal velocity in still air. How fast is it falling?

6.93. In James Bond movies the hero is often swimming and has to dive deep into the water to escape the bullets from the enemy helicopter flying above him. How deep should he dive? Assume the bullet is a sphere of diameter 0.5 in and mass 0.027 lbm. It hits the surface of the water vertically at a velocity of 1000 ft / s and will not inflict serious injury if it is slowed down to a velocity of 100 ft / s or less. For the purposes of this problem only, assume that the drag coefficient is constant, independent of velocity, and equal to 0.1.

6.94. A bullet was fired straight up at 2700 ft / s. The bullet had a mass of 150 grains (the standard mass unit in U.S. gun lore, 1 lbm = 7000 grains), a more-or-less cylindrical shape with a sharp point, and a diameter of 0.30 in.

(*a*) If there were zero air resistance, how high would it go? How long would it take to reach that altitude? How long would it take it to come back to ground? What would its velocity be when it came back to ground?

(*b*) It actually went up 9000 ft in 18 s and returned to earth in 31 more seconds for total time of 49 s and arrived with a velocity of 300 ft / s [28]. Based on these values, estimate the drag coefficient going up, and the drag coefficient coming down. Assume in both cases that the bullet retained its vertical orientation due to its axial rotation (i.e., did not tumble end-over-end) and that the drag coefficient was a constant (only a fair assumption).

6.95. Figure 6.26 (and the calculations on which it is based) are widely used in chemical engineering to describe separations processes based on gravity. Large particles can be separated into distinct sizes by screens, but smaller particles cannot. Instead they are separated into size fractions or different density fractions by processes that take advantage of their different settling velocities in air or water. Many mineral separations function by making a very uniform particle size sample with screens, and then separating by specific gravity in differential settlers, mostly in water. In these devices the settling velocity is less than that shown in Fig. 6.26, because that figure (and the examples in the chapter) assume that each particle is far from any other particle. In these devices the particles are close to one another.

(*a*) Sketch the flow around a particle which is settling in a closed container, by itself, and then in a mud or slurry in which there are many other nearby particles. Indicate why one would expect the other particles (and the flow of fluid which they induce) to cause the particle to settle more slowly.

(*b*) This slowing is called *hindered settling*. Its effect can be estimated [29] by

$$V_{\text{terminal, hindered}} = V_{\text{terminal, isolated}}(1 - c)^n \tag{6.67}$$

in which c is the volume fraction of solids and $n = 4.65$ in the Stokes' law region, with value decreasing to 2.33 for $\mathcal{R}_p > 1000$. Using this value, estimate the settling velocity of a simple spherical particle of SG = 3 and $D = 20\ \mu$ in water all by itself, and in a mud that has $c = 0.4$.

REFERENCES FOR CHAPTER 6

1. Reynolds, O. "An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous and of the Law of Resistance in Parallel Channels." *Philosophy Transactions of the Royal Society 174* (1883). The history of this work and its relation to others are discussed in Rouse and Ince [2, p. 206].
2. Rouse, H., S. Ince. *History of Hydraulics*. Iowa City, Iowa: Iowa Institute of Hydraulic Research, 1957, (*a*) p. 160, (*b*) p. 323.
3. Nikuradse, J. "Stroemungsgesetze in rauhen Rohren (Flow Laws in Rough Tubes)." *VDI-Forsch.-Arb. Ing.-Wes. Heft 361* (1933).
4. Moody, L. W. "Friction Factors for Pipe Flow." *Transactions of the American Society of Mechanical Engineers 66* (1944), (*a*) pp. 671–684, (*b*) Hunter Rouse's comments at the end of the article.
5. Colebrook, C. F. "Turbulent Flow in Pipes, with Particular Reference to the Transition Region Between Smooth and Rough Pipe Laws," *Journal of the Institute of Civil Engineering 11* (1938–1939), pp. 133–155.
6. Schlichting, H., K. Gersten. *Boundary-Layer Theory*. 8th ed. Berlin: Springer, 2000.
7. I do not know who is the author of this equation. If any of my readers can inform me, I will give proper credit in future printings.
8. *ASHRAE Handbook of Fundamentals*. Atlanta, Georgia: ASHRAE. New editions every few years.
9. O'Brian, M. P., G. H. Hickox. *Applied Fluid Mechanics*. New York: McGraw-Hill, 1937, p. 211.
10. Lapple, C. E. "Velocity Head Simplifies Flow Computation." *Chemical Engineering 56* (1949). *56(5)*, pp. 96–104.

CHAPTER

7

THE MOMENTUM BALANCE

Newton's second law of motion, often called Newton's *equation of motion,* is commonly written

$$F = ma \tag{7.1}$$

It is easily applied in this form to the motion of rigid bodies like the falling bodies of elementary physics. It also can be easily applied in this form to the motion of fluids that are moving in rigid-body motion, as discussed in Chap. 2. However, for fluids that are moving in more complicated motions, e.g., in pipes or around airplanes, it is difficult to use Eq. 7.1 in the form shown. Therefore, in this chapter we will rewrite the equation in the form of a momentum balance. The momentum balances given in this chapter are rearrangements of Eq. 7.1, and they, too, are often referred to in the engineering literature as *equations of motion.*

The momentum balance form will prove very convenient for solving fluid-flow problems. In particular, it will allow us to find out something about complicated flows through a system without having to know in detail what goes on inside the system. In this way the momentum balance is similar to the mass and energy balances. For example, by using the mass and energy balances we can find out some things about a turbine or compressor from the inlet and outlet streams only, without knowing in detail what goes on inside. We also will frequently apply the momentum balance "from the outside."

Remember that all of this chapter is simply the manipulation and application of Eq. 7.1. In this chapter the applications are one-dimensional; in Part IV we apply the same ideas to two- and three-dimensional flows.

7.1 MOMENTUM

Momentum, like energy, is an abstract quantity. Unlike energy, it is defined in terms of simpler quantities, mass and velocity. The definition of momentum is given in terms of the momentum of a body:

$$\begin{pmatrix}\text{Momentum}\\ \text{of a body}\end{pmatrix} = \begin{pmatrix}\text{mass of}\\ \text{the body}\end{pmatrix} \cdot \begin{pmatrix}\text{velocity of}\\ \text{the body}\end{pmatrix} = mV \tag{7.2}$$

It makes no sense to speak of momentum as separate from bodies because, as we see from this equation, if there is no mass, there is no momentum. Only bodies—of solid, liquid, or gas—have mass, so momentum can exist only in connection with some body.*

Furthermore, momentum is a vector. We have applied the balance equations to mass and energy, which are scalar, here we will apply it to a vector and get similar results. Most often in dealing algebraically with vectors, one uses the scalar components of the vector rather than the vector itself. For example, Eq. 7.1 may be written in vector form

$$\mathbf{F} = m\mathbf{a} \tag{7.3}$$

(**boldface** indicates a vector). However, any vector can be resolved into the vector sum of three scalar components multiplied by unit vectors, in three mutually perpendicular directions.** For example,

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \tag{7.4}$$

where F_x, F_y, and F_z are the scalar components of the vector **F** in the x, y, and z directions, and **i**, **j**, and **k** are unit vectors in the x, y, and z directions, respectively. Similarly, we can resolve the acceleration vector **a** and rewrite Eq. 7.3 in the following forms:

$$F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} = m(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}) \tag{7.5}$$

$$(F_x - ma_x)\mathbf{i} + (F_y - ma_y)\mathbf{j} + (F_z - ma_z)\mathbf{k} = 0 \tag{7.6}$$

But this equation is the equation of a new vector, the $(\mathbf{F} - m\mathbf{a})$ vector, which is seen to be zero. For a vector to be zero each one of its scalar components must be zero, so this equation is exactly equivalent to

$$F_x - ma_x = 0; \qquad F_y - ma_y = 0; \qquad F_z - ma_z = 0 \tag{7.7}$$

*Neutrinos and light quanta (photons) apparently possess momentum but not rest mass and thus might be considered exceptions to this statement. However, they are observable only when moving at high velocities, at which time they have considerable energy and, hence, relativistic mass, so this statement is correct even for them.

**As far as we know, we live in a three-dimensional universe, so we speak of three mutually perpendicular directions. If we lived in an n-dimensional universe there would be n mutually perpendicular directions. In some problems it is convenient to consider n-dimensional "spaces," in which a vector is resolved into n "perpendicular" components.

This shows us that we may consider any vector equation as a shorthand way of writing three scalar equations. Vector calculus is a powerful tool for deriving equations describing multidimensional problems. In electromagnetic problems and problems involving moving coordinate axes (e.g., gyroscopes) it is easiest to work directly with the vector quantities. However, for solving practical fluid-mechanics problems it is almost always more convenient to use the three scalar (component) equations, which are the exact equivalent of the vector equation. In this chapter we will show the momentum balance both as a vector equation and as its more useful scalar equivalents. We will show the application of the vector calculus approach to fluid mechanics problems in Part IV.

One distinct complication with the momentum balance as compared with the mass and energy balances concerns the algebraic signs of the momentum terms. In the energy and mass balances we have little trouble with signs, because we seldom consider negative energies and never consider negative masses.† On the other hand, if we wish to represent a velocity in the minus x direction, we write it as

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k} \tag{7.A}$$

where V_z and V_y are zero, and V_x is a negative number. Therefore, in our scalar equations we will have to be more careful of algebraic signs than we were with the mass and energy balances.

7.2 THE MOMENTUM BALANCE

In Chap. 3 we saw that the general balance equation (Eq. 3.2) can be applied to any extensive property—any property that is proportional to the amount of matter present. Since momentum is proportional to the amount of matter present, it is an extensive property and must obey a balance equation. Here, as in all other balance equations, we must be careful to choose and define our system.

Figure 7.1 shows the system used to state the momentum balance; it consists of some tank or vessel with flow of matter in or out and system boundaries as shown. The momentum contained within the system boundaries is

$$\begin{pmatrix}\textbf{Momentum inside}\\ \textbf{system boundaries}\end{pmatrix} = \int_{\substack{\text{all mass}\\ \text{in system}}} \mathbf{V}\,dm \tag{7.8}$$

We simplify this by assuming that all the mass inside the system has the same velocity, so that this integral simplifies to $(m\mathbf{V})_{\text{sys}}$. The momentum-accumulation term becomes

$$\textbf{Momentum accumulation} = d(m\mathbf{V})_{\text{system}} \tag{7.9}$$

†Although in an absolute sense energies can never be negative, energies, relative to an arbitrary datum can be negative. In one-component systems, such as steam power plants or refrigeration systems, the datum usually is so chosen that none of the energy terms is negative. However, the common datum for combustion work and chemical-reaction problems results in negative energy terms.

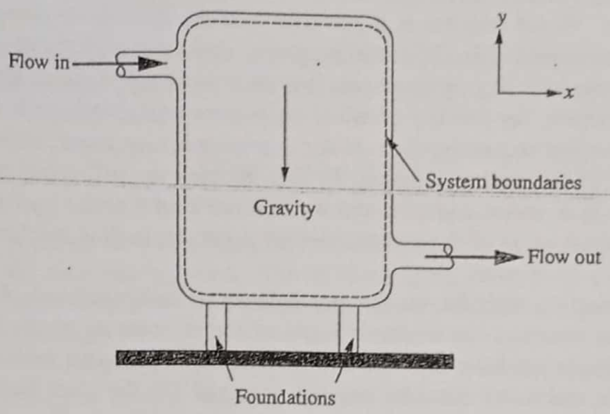


FIGURE 7.1
System used for stating the momentum balance.

For one flow in and one flow out, as in the figure, the momentum flow in minus momentum flow out is

$$\textbf{Momentum flow in} - \textbf{momentum flow out} = \mathbf{V}_{\text{in}}\, dm_{\text{in}} - \mathbf{V}_{\text{out}}\, dm_{\text{out}} \tag{7.10}$$

If there is more than one flow in or out, there will be summation terms for momentum flows in and out, just as there are summation terms for mass and energy flows into and out of a system in the mass and energy balances.

In Sec. 3.4 we discussed velocity distributions in flows in pipes. There we showed that simplifying the problems by replacing actual flows, which have a velocity gradient from centerline to edge, with flows all at one velocity, $V = V_{\text{avg}}$, changed the calculated kinetic energy in the flow by about 6 percent for most turbulent flows, which we considered negligible. Table 3.1 shows that a similar simplification changes the calculated momentum in such a flow by about 1 percent, which we will also consider negligible. Thus, for the rest of this chapter we will replace the real flow in pipes, channels, and jets, which has some nonuniform velocity distribution, with a flow with a uniform velocity distribution, $V = V_{\text{avg}}$. If there is any question whether this is permissible, please refer to Table 3.1 and the problems associated with it.

Now, to account for the creation or destruction of momentum, we invoke Eq. 7.1, which can be rewritten

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{V}}{dt} \tag{7.11}$$

The possible changes of mass of the system are accounted for by the flow-in or flow-out terms, and the creation or destruction terms must apply equally well to constant-mass and variable-mass systems. For a constant-mass system, we can take the m inside the differential sign in the last equation and rearrange, to show that

$$d(m\mathbf{V})_{\text{sys}} = \mathbf{F}\, dt \tag{7.12}$$

so that the momentum creation or destruction term is $\mathbf{F}\, dt$.*

If more than one force is acting, the $\mathbf{F}$ in Eq. 7.12 must be replaced with a sum of forces. Usually there are several forces acting in fluid-flow problems, so we write the momentum balance with a $\Sigma\,\mathbf{F}$. The forces acting on the system shown in Fig. 7.1 are the external pressure on all parts of its exterior and the force of gravity. Other forces that we might consider are electrostatic or magnetic forces. If we had chosen our system such that the boundary passed through the foundations in Fig. 7.1, then there would be a compressive force in the structural members of the foundation, which would have to be taken into account.

Writing all the terms together, we find the vector form of the momentum balance:

$$d(m\mathbf{V})_{\text{sys}} = \mathbf{V}_{\text{in}}\, dm_{\text{in}} - \mathbf{V}_{\text{out}}\, dm_{\text{out}} + \sum \mathbf{F}\, dt \tag{7.13}$$

Here we have not included a destruction term, because the $\Sigma\,\mathbf{F}$ in the equation is the vector sum of all the forces acting on the system. If this sum is in the opposite direction of the velocities, then the $\Sigma\,\mathbf{F}\, dt$ term is a momentum destruction term; it will enter with a minus sign. Most often we divide Eq. 7.13 by dt to find the *rate form* of the momentum balance:

$$\frac{d(m\mathbf{V})_{\text{sys}}}{dt} = \mathbf{V}_{\text{in}}\, \dot{m}_{\text{in}} - \mathbf{V}_{\text{out}}\, \dot{m}_{\text{out}} + \sum \mathbf{F} \tag{7.14}$$

This is not a *derivation* of the momentum balance, but simply a restatement of Newton's second law in a convenient form. Furthermore, Newton's laws, like the laws of thermodynamics and the law of conservation of mass, are underivable; they cannot be demonstrated from any prior principle but rest solely on their ability to predict correctly the outcome of any experiment ever run to test them.

Equations 7.13 and 7.14 are balance equations entirely analogous to the mass and energy balances we discussed in Chaps. 3 and 4. They have the same basic restriction of those balances, namely, that they may be applied only to a carefully defined system.

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*Equation 7.12 implies that momentum is creatable, which can be misleading. If a person standing on the earth throws a ball, the momentum of the ball is increased in one direction and the momentum of the earth in the opposite direction by an equal amount. Thus, the momentum of the earth-ball system is unchanged. Because the mass of the earth is so much larger than that of the ball, we do not perceive this change in the earth's velocity (Prob. 7.1). However, Eq. 7.12 is correct, because creating momentum in one system results in creating equal and opposite momentum in some other system (usually the earth); so the net change of momentum of the universe is zero.

Modern physicists prefer Eq. 7.12 to Eq. 7.1 as the basic statement of Newton's law of motion. The reason is that, as the velocity of a body approaches the speed of light, the force exerted on it results mostly in an increase in mass rather than an increase in velocity. Thus, Eq. 7.1 is limited to constant-mass systems and excludes any system that is being accelerated to a speed near that of light, whereas Eq. 7.12 applies not only to constant-mass systems but also to bodies being accelerated to speeds near that of light, such as the particles in linear accelerators.

With the mass balance we can choose any system in which we can account for all flows of matter across the boundaries. With the energy balance we must choose a system in which we can account not only for any flows of matter across the boundaries but also for heat flows across the boundaries and work due to electric current, changing magnetic fields, moving boundaries, and rotating and reciprocating shafts. In applying the momentum balance we must choose a system in which it is possible to account for all flows of matter across the boundaries and also for all external forces acting on the system. In most fluid-flow problems this means that it must be possible to calculate the pressure on every part of the system boundary. Notice, however, that there is no term in Eq. 7.13 or 7.14 for heat flows, rotating shafts, or electric current flows, so we may choose systems for the momentum balance without necessarily being able to calculate those quantities over the boundaries of the system (we must account for electrostatic or magnetic fields, if they are significant). As we do with mass and energy balances, we may consider closed systems, in which the $\dot{m}$ terms are zero, or steady-flow systems, in which the accumulation is zero. Skill in applying the momentum balance is largely a matter of skill in choosing a system in which one can conveniently calculate all the terms in the balance.

Equations 7.13 and 7.14 are vector equations; each of them can be represented by three scalar equations, showing the components of the vectors in the x, y, and z directions or the r, θ, and z directions or in spherical coordinates. The x-component scalar equations equivalent to them are

$$d(mV_x)_{\text{sys}} = V_{x_{\text{in}}}\, dm_{\text{in}} - V_{x_{\text{out}}}\, dm_{\text{out}} + \sum F_x\, dt \tag{7.15}$$

$$\frac{d(mV_x)_{\text{sys}}}{dt} = V_{x_{\text{in}}}\, \dot{m}_{\text{in}} - V_{x_{\text{out}}}\, \dot{m}_{\text{out}} + \sum F_x \tag{7.16}$$

The corresponding y and z equations can be found from these simply by replacing all the x subscripts with y or z subscripts. The r, θ, and z component equations for cylindrical coordinates are shown in App. C.

To illustrate the application of the momentum balance, we consider first two very simple examples not involving fluids.

Example 7.1. A baseball is thrown in a horizontal direction. What terms of the momentum balance apply?

Taking the ball as our system and using the x component of the momentum balance, we see that there is no flow of matter in or out; therefore,

$$d(mV_x)_{\text{sys}} = (m\, dV_x)_{\text{sys}} = F_x\, dt; \qquad F_x = m\frac{dV_x}{dt} = ma_x \tag{7.B}$$

This is a simple restatement of $F = ma$ for a constant-mass system. ■

Example 7.2. A duck has a mass of 3 lbm and is flying due west at 15 ft / s. The duck is struck by a bullet with a mass of 0.05 lbm, which is moving due east at 1000 ft / s. The bullet comes to rest in the duck's gizzard. What is the final velocity of the duck-bullet system?

Here the problem is one-dimensional, so we work with the x-directed scalar-component equation, Eq. 7.15, and choose east as the positive x direction. First we work the problem by taking as our system the combined bullet and duck. No matter is flowing in or out of this system, nor does any external force act on it (we ignore the wind force, if a wind is blowing); so Eq. 7.15 becomes

$$d(mV_x)_{\text{sys}} = 0 \tag{7.C}$$

$$(mV_x)_{\text{sys, fin}} = (mV_x)_{\text{duck, init}} + (mV_x)_{\text{bullet, init}} \tag{7.D}$$

When we solve for $V_{x_{\text{sys, fin}}}$, we find

$$V_{x_{\text{sys, fin}}} = \frac{(mV_x)_{\text{duck, init}} + (mV_x)_{\text{bullet, init}}}{m_{\text{duck-bullet, fin}}}$$

$$= \frac{[3 \text{ lbm} \cdot (-15 \text{ ft / s})] + (0.05 \text{ lbm} \cdot 1000 \text{ ft / s})}{3.05 \text{ lbm}}$$

$$= +1.6 \frac{\text{ft}}{\text{s}} = +0.73 \frac{\text{m}}{\text{s}} \tag{7.E}$$

Now we do the problem over, taking the duck as our system. In this case there is mass flow into the system, so we have

$$d(mV_x)_{\text{sys}} = V_{x_{\text{in}}}\, dm_{\text{in}} \tag{7.F}$$

Integrating, we find

$$(mV_x)_{\text{sys, fin}} - (mV_x)_{\text{sys, init}} = V_{x_{\text{in}}}\, m_{\text{in}} \tag{7.G}$$

When we solve for $V_{x_{\text{sys, fin}}}$, we find

$$V_{x_{\text{sys, final}}} = \frac{V_{x_{\text{in}}}\, m_{\text{in}} + (mV_x)_{\text{sys, init}}}{m_{\text{sys, fin}}} \tag{7.H}$$

This is exactly the same as the result we found by taking the combined system. ■

This example shows the great advantage of the momentum balance; the details of the collision are very complicated when we wish to know the exact distance-time-shape history of the bullet in traversing the various feathers, bones, muscles, and internal organs of the duck, but from the momentum balance alone we can write down the final velocity of the bullet-duck system without knowing those details. It also shows that signs are important in the momentum balance. For the system chosen, the duck's initial velocity was −15 ft / s. If we had omitted that minus sign we would have calculated a final velocity of 31 ft / s, which would have been the final velocity if the bullet had been moving in the same direction as the duck (and overtaken it). That is the correct answer to a different problem. The signs in the momentum balance seem to be a permanent problem for students; pay attention to them!

This example also illustrates that some problems can be solved by the momentum balance but not by the energy balance. If we write the energy balance for Example 7.2, taking the combined duck-bullet as our system and neglecting the small change in volume of the system as the bullet enters the duck, we find it reduces to

$$\left[m\left(u+\frac{V^2}{2}\right)\right]_{\text{sys, fin}} = \left[m\left(u+\frac{V^2}{2}\right)\right]_{\text{duck, init}} + \left[m\left(u+\frac{V^2}{2}\right)\right]_{\text{bullet, init}} \tag{7.I}$$

Here we know the mass of the system and the initial kinetic and internal energies of its two parts, but these alone do not allow us to solve Eq. 7.I for either the final internal energy or the final kinetic energy. However, from the momentum balance we were able to find the final velocity, and we can then use it in Eq. 7.I to find the final internal energy. In this chapter we will see several other examples in which the momentum balance must be applied before the energy balance can be used (see Prob. 7.3).

7.3 SOME STEADY-FLOW APPLICATIONS OF THE MOMENTUM BALANCE

If we choose as our system some pipe, duct, channel, or jet with steady flow through it in one direction, e.g., the x direction, then Eq. 7.16 becomes

$$0 = \dot{m}(V_{x_{\text{in}}} - V_{x_{\text{out}}}) + \sum F_x \qquad \text{[steady flow]} \tag{7.17}$$

The application of this steady-flow one-dimensional momentum balance will be illustrated by several examples.

7.3.1 Jet-Surface Interactions

Many applications of Eq. 7.17 involve jets. A jet is a stream of fluid that is not confined within a pipe, duct, or channel; examples are the stream of water issuing from a garden hose and the exhaust gas stream from a jet engine. If any jet is flowing at a subsonic velocity, its pressure will be the same as the pressure of the surrounding fluid. If a jet enters or leaves a system or device at subsonic speed, it will enter and leave at the pressure of the surrounding fluid, although its pressure may be different inside the device. Sonic and supersonic jets are discussed in Chap. 8.

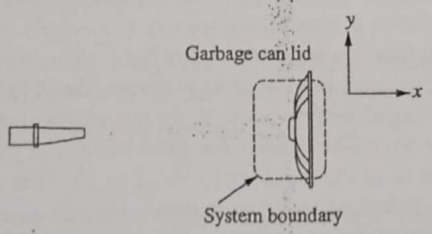


FIGURE 7.2
Interaction of a jet with a surface perpendicular to it.

Example 7.3. The police are using fire hoses to disperse an unruly crowd. The fire hoses deliver 0.01 m^3/s of water at a velocity of 30 m/s. A member of the crowd has picked up a garbage can lid and is using it as a shield to deflect the flow. She is holding it vertically, so the jet splits into a series of jets going off in the y and z directions, with no x component of the velocity; see Fig. 7.2. What force must she exert to hold the garbage can lid?

By applying Eq. 7.17 in the x direction and taking the lid and the adjacent fluid as our system, as sketched in Fig. 7.2, we find

$$F_x = -0.01\,\frac{\text{m}^3}{\text{s}} \cdot 998.2\,\frac{\text{kg}}{\text{m}^3} \cdot \left[30\,\frac{\text{m}}{\text{s}} - 0\right] \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = -299.5\ \text{N}$$
$$= -67.3\ \text{lbf} \qquad (7.J)$$

■

Here the pressure around the external boundary of the system is all atmospheric, so this force is simply the force exerted by the arms of the woman holding the lid. It is negative, because she is exerting this force in a direction opposite to the x axis. Here we could also have chosen as our system the fluid alone. Then to solve for the force we would have had to calculate the pressure exerted on it by the garbage can lid at every point of the system boundary. To do this we would have needed a detailed description of the flow. From such a detailed description, if it were available, we could calculate

$$F_x = \int_{\text{all area}} P\,dA \qquad (7.K)$$

for the lid, finding the same answer. Thus, by a proper choice of system we can find the desired force without a detailed description of the flow; we can apply the momentum balance "from the outside."

Example 7.4. The member of the crowd in Ex. 7.3 now turns the lid around so that she can hold it by the handle. However, because of the shape of the lid the flow goes off as shown in Fig. 7.3, with an average x component of the velocity of -15 m/s. What force must she exert?

Applying Eq. 7.17 exactly as before, we find

$$F_x = -0.01\,\frac{\text{m}^3}{\text{s}} \cdot 998.2\,\frac{\text{kg}}{\text{m}^3} \cdot \left[30\,\frac{\text{m}}{\text{s}} - \left(-15\,\frac{\text{m}}{\text{s}}\right)\right] \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$
$$= -449.3\ \text{N} = -101\ \text{lbf} \qquad (7.L)$$

■

See Prob. 7.7!

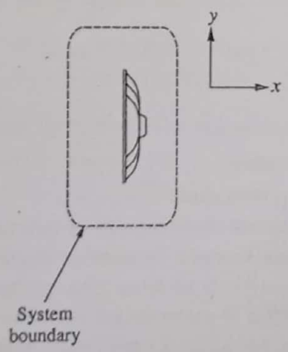


FIGURE 7.3
A curved surface turns the jet back toward its source.

7.3.2 Forces in Pipes

In the previous examples, the jets were open to the atmosphere, so their gauge pressure was always zero. Thus, we had no difficulty with deciding on the sign of pressure forces. The next two examples involves pressure forces inside pipes; they require us to consider the sign of the pressure forces. The easiest way to decide on the proper sign of a pressure force is to take the system boundary perpendicular to the axis in which we are

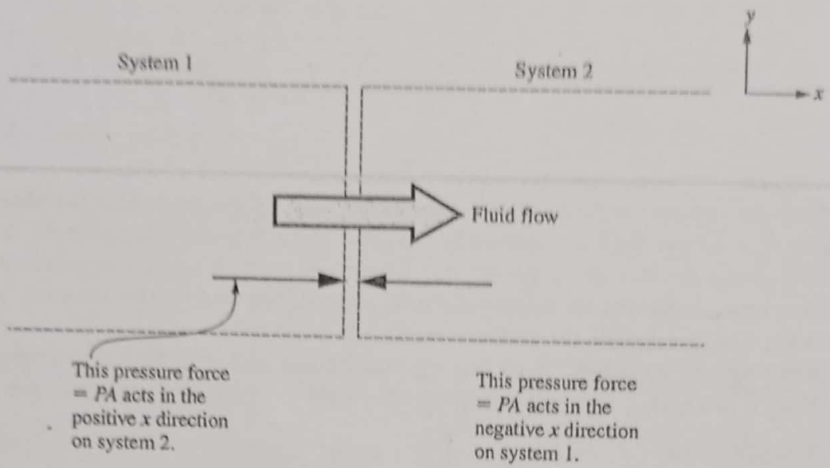


FIGURE 7.4
At the boundary between two systems, the pressure force acts inward on both systems.

applying the momentum balance, e.g., perpendicular to the x-direction for an x-directed momentum balance. If we do that, we will see that the pressure force acts *inward* on our system and simultaneously *outward* on the surroundings. The direction of the pressure force in a flowing fluid is independent of the direction of the flow. This is illustrated in Fig. 7.4.

If two systems adjoin each other, then an equal and opposite pressure force acts inward on each of them, as shown in Fig. 7.4. If you squeeze a coil spring between your thumb and forefinger, it exerts equal forces on thumb and forefinger, acting in opposite directions. It is the same with pressure forces on adjoining systems (or two systems we create by drawing a system boundary across a flow).

Example 7.5. A nozzle is attached to a fire hose by a bolted flange; see Fig. 7.5. What is the force tending to tear apart that flange when the valve in the nozzle is closed?

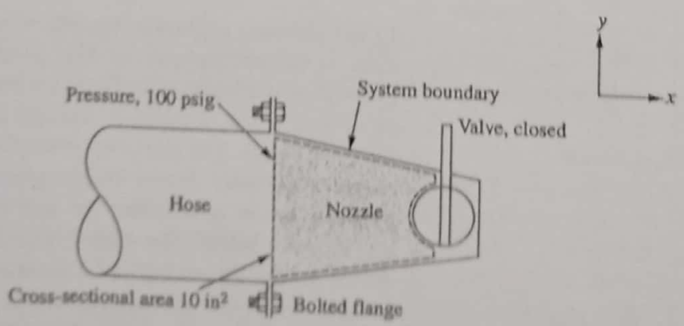


FIGURE 7.5
A hose nozzle, with the valve closed.

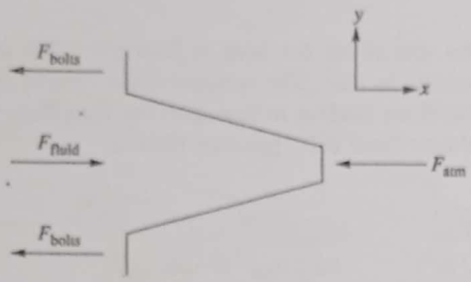


FIGURE 7.6
Forces acting on the nozzle in Fig. 7.5.

We take as our system the mass of fluid that is enclosed in the nozzle from the plane of the flange to the valve. Applying Eq. 7.17, we see that $\dot{m}$ is zero, so the summation of the x components of the forces on this body of fluid must be zero. In this case the summation of forces is the summation of the pressure forces in the x direction. In the plane of the flange the fluid outside the system exerts a pressure force on the system equal to PA. This is all in the x direction, because this surface is normal to the x axis. The x component of the pressure force exerted by the nozzle must be equal and opposite to this pressure force. The magnitude of these forces is

$$F_x = PA = (P_g + P_{atm})A = (100\ \text{lbf}/\text{in}^2 \cdot 10\ \text{in}^2) + P_{atm}A$$
$$= 1000\ \text{lbf} + P_{atm}A \qquad (7.M)$$

where P_g is the gauge pressure and P_{atm} is the atmospheric pressure.

Now we choose as a second system the nozzle itself. From Eq. 7.17 we see that for it, too, the sum of the x components of the forces must be zero. The forces acting on it are sketched in Fig. 7.6. We have previously calculated the force that it exerts on the fluid; by Newton's third law we know that the fluid exerts an equal and opposite force on it, which is given as F_{fluid} in the figure. The bolts also exert a force, as shown, and the atmosphere exerts a pressure on all those parts not exposed to the fluid. The atmospheric pressure force is not all in the x direction but, as we showed in Chap. 2, we could compute the x component of the atmospheric pressure force, which would be $-PA_x$, where A_x is the x projection of the net area exposed to the atmosphere. (The force is negative, because it acts opposite to the x direction.) Therefore, summing the forces shown in Fig. 7.6, setting the sum equal to zero, and solving for F_{bolts}, we find

$$F_{bolts} = -F_{liq} - F_{atm} = -(1000\ \text{lbf} + P_{atm}A) - (-P_{atm}A)$$
$$= -1000\ \text{lbf} = -4.448\ \text{kN} \qquad (7.N)$$

The bolt force shown is negative because it acts on the nozzle in the negative x direction. ■

We see that in this problem the atmospheric pressure terms canceled. Because this is a common occurrence, engineers ordinarily work such problems in gauge pressures and thereby only need to show pressure forces on those parts of the boundary of the system where the pressure is different from atmospheric. If we had done that here, we would have found exactly the same answer.

We could have solved this problem more easily by not using the momentum balance, but it illustrates the method, which will be useful in the next example.

Example 7.6. The valve on the end of the fire hose in Example 7.5 is now opened. The area of the outlet nozzle is 1 in^2. The pressure at the flanged joint is still 100 psig. Now what is the force tending to tear apart the flange?

We estimate the outlet velocity from B.E., ignoring friction:

$$V = \left[\frac{2(-\Delta P)}{\rho(1-(A_2/A_1)^2)}\right]^{1/2}$$

$$= \left[\frac{2(100\ \text{lbf}/\text{in}^2)}{(62.3\ \text{lbm}/\text{ft}^3)(1-0.1^2)} \cdot \frac{32.2\ \text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} \cdot \frac{144\ \text{in}^2}{\text{ft}^2}\right]^{1/2}$$

$$= 122.6\frac{\text{ft}}{\text{s}} = 37.4\frac{\text{m}}{\text{s}} \tag{7.O}$$

And by material balance we know that the inlet velocity is $\frac{1}{10}$ of this value.

Again, we choose as our system the fluid enclosed in the nozzle. From Eq. 7.17 we have

$$0 = \dot{m}(V_{x_{\text{in}}} - V_{x_{\text{out}}}) + \sum F_x \tag{7.P}$$

From the mass balance for steady flow we have

$$\dot{m} = \rho A_{\text{out}} V_{\text{out}} = 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot 1\ \text{in}^2 \cdot 122.6\frac{\text{ft}}{\text{s}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2}$$

$$= 53.1\frac{\text{lbm}}{\text{s}} = 24.1\frac{\text{kg}}{\text{s}} \tag{7.Q}$$

So the net force on the fluid in the x direction is

$$\sum F_x = -\dot{m}(V_{x_{\text{in}}} - V_{x_{\text{out}}}) = -53.1\frac{\text{lbm}}{\text{s}}\cdot\left(\frac{12.3\ \text{ft}}{\text{s}} - \frac{122.6\ \text{ft}}{\text{s}}\right)\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}$$

$$= 182\ \text{lbf} = 815\ \text{N} \tag{7.R}$$

This is the sum of the forces on the fluid, as sketched in Fig. 7.7.

As discussed in Chap. 5, the fluid leaving such a nozzle will be at the same pressure as the surrounding atmosphere if the flow is subsonic, which it is here. Therefore, if we use gauge pressure, then the pressure force on the system as the stream leaves the nozzle is zero, and the 182 lbf given is the algebraic sum of the pressure force exerted on the system at its left boundary and the x component of the force exerted by the nozzle. Thus,

$$182\ \text{lbf} = PA - F_{x_{\text{nozzle}}} \tag{7.S}$$

and the force exerted by the nozzle on the fluid is −818 lbf. By comparing

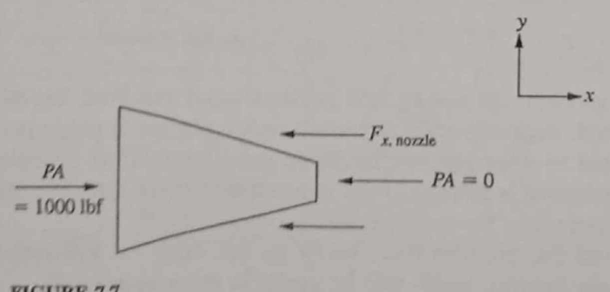


FIGURE 7.7
Forces acting on the fluid in the nozzle.

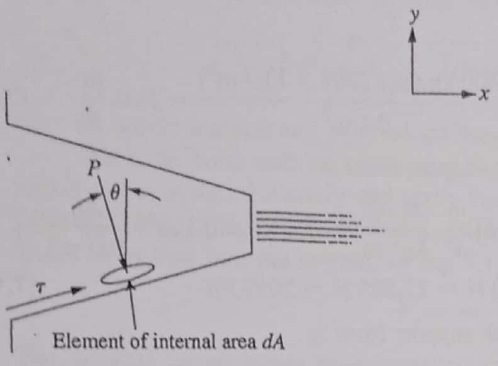


FIGURE 7.8
An alternative way of computing force on nozzle with fluid flowing.

this result with the one in Example 7.6, we can readily deduce that the force on the nozzle bolts in this case is −818 lbf. ■

Why is the force acting on the bolts less in this case? The pressure force acting on the system at its left boundary is the same as in the no-flow case (Example 7.5). However, when there is this flow, some of that force is being used to accelerate the fluid and hence is not being resisted by the nozzle bolts.

This example also illustrates how the momentum balance helps us solve problems from the outside without looking inside. We could also have found the force on the nozzle by determining the pressure and shear stress at every point on the internal surface of the nozzle; see Fig. 7.8. The x components of these pressure forces and shear forces are equal to the x component of the force on the nozzle:

$$-F_{x_{\text{nozzle}}} = \int (P \sin\theta + \tau \cos\theta)\, dA \tag{7.18}$$

Determining the local values of P and τ for all the internal surface of even a relatively simple device like this nozzle is a formidable task. For a complicated shape, it is beyond our current ability. Nevertheless, we computed the overall force (which we were seeking) by the momentum balance fairly easily.

Example 7.7. The pipe bend in Fig. 7.9 is attached to the rest of the piping system by two flexible hoses, which transmit no forces. Water enters in the $+x$ direction and leaves in the $-y$ direction. The flow rate is 500 kg / s, and the cross-sectional area of the pipe is constant = 0.1 m^2. The pressure throughout the pipe is 200 kPa gauge. Calculate F_x and F_y, the x and y components of the force in the pipe support.

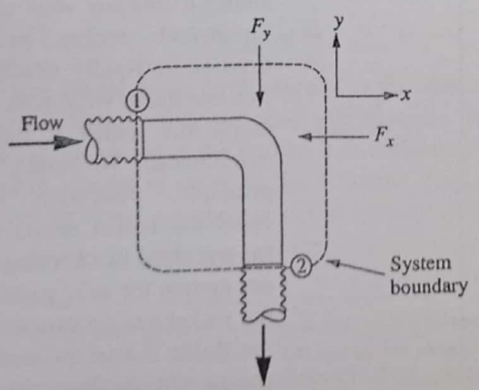


FIGURE 7.9
Forces on a pipe bend.

Applying Eq. 7.17 in the x direction to the system shown in Fig 7.9 and using gauge pressure, we find

$$-F_x = \dot{m}(V_{x_{\text{in}}} - V_{x_{\text{out}}}) + P_1A_1 \tag{7.T}$$

and for the y direction

$$-F_y = \dot{m}(V_{y_{\text{in}}} - V_{y_{\text{out}}}) + P_2A_2 \tag{7.U}$$

The velocity is constant,

$$V = \frac{Q}{A} = \frac{\dot{m}/\rho}{A} = \frac{(500\ \text{kg/s})/(998.2\ \text{kg/m}^3)}{0.1\ \text{m}^2} = 5.01\ \frac{\text{m}}{\text{s}} \tag{7.V}$$

so that the x component of the support force is

$$-F_x = 500\frac{\text{kg}}{\text{s}}(5.01 - 0)\frac{\text{m}}{\text{s}} \cdot \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} + 200\ \text{kPa} \cdot 0.1\ \text{m}^2 \cdot \frac{\text{N}}{\text{Pa} \cdot \text{m}^2}$$
$$= (2505 + 20{,}000)\ \text{N} = 22{,}505\ \text{N} = 5059\ \text{lbf} \tag{7.W}$$

whereas the y component of the support force is

$$-F_y = 500\frac{\text{kg}}{\text{s}}[0 - (-5.01)]\frac{\text{m}}{\text{s}} \cdot \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} + 200\ \text{kPa} \cdot 0.1\ \text{m}^2 \cdot \frac{\text{N}}{\text{Pa} \cdot \text{m}^2}$$
$$= (2505 + 20{,}000)\ \text{N} = 22{,}505\ \text{N} = 5059\ \text{lbf} \tag{7.X}$$

■

We see that the support force components are equal and point in the $-x$ and $-y$ directions, as sketched on the figure. In this example, and most piping-force problems, the pressure terms are larger than the fluid-acceleration terms. A few moments spent examining the signs of all the terms in this example is time well spent. Piping designers must support piping properly to deal with these forces, as well as those of thermal expansion. Failure to do so has led to serious process plant accidents, including the 1974 Flixboro disaster [1].

7.3.3 Rockets and Jets

Rockets are easy to analyze by means of the steady-flow one-dimensional momentum balance. Figure 7.10 shows a cutaway view of a liquid-fuel rocket being fired while rigidly attached to a test stand. What goes on inside the rocket is fairly complicated. We could, in principle, determine the force the rocket exerts on the test stand by choosing as our system the solid parts of the rocket and by excluding the fluids inside the tanks, pumps, combustion chamber, and nozzle. If we could determine the pressure and shear stress at every point in

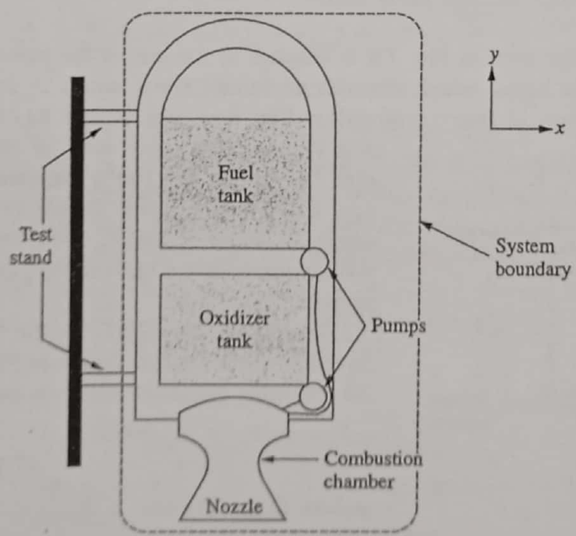


FIGURE 7.10
Simplified cross section of a liquid-fuel rocket.

the system, we could determine the total force by taking the integral

$$F = \int_{\text{int.ext.surf}} (P \sin\theta + \tau \cos\theta)\, dA \tag{7.19}$$

over the whole internal and external surfaces; that would be a giant task.

But if we want only to know what this force is, we can take the outside of the rocket as our system boundary and apply the momentum balance. With this boundary $[d(mV_y)/dt]_{\text{system}}$ is zero, because the system momentum is not changing with time.* There is no flow into the system, so Eq. 7.17 reduces to

$$F_y = V_{y_{\text{out}}}\, \dot{m}_{\text{out}} \tag{7.20}$$

The external forces acting on the system are the pressure forces around the entire boundary of the system shown and the force exerted by the test-stand support structure. The latter force is the one we are seeking, so we can split the F_y in Eq. 7.20 into two parts and rearrange to

$$F_{y_{\text{t.stand}}} = V_{y_{\text{out}}}\, \dot{m}_{\text{out}} - \begin{pmatrix} y \text{ component of pressure} \\ \text{force on system} \end{pmatrix} \tag{7.21}$$

In Sec. 5.5 we noted that for flows moving slower than the velocity of sound we can safely assume that a flow leaving an enclosed system and flowing into the atmosphere is at the same pressure as the atmosphere. We have made this assumption in the foregoing examples of this chapter. However, in the case of the rocket the flow leaving the system is generally supersonic, so we can no longer make this assumption. From Fig. 7.10 it is obvious that the pressure on the outside of the system is atmospheric everywhere except across the exit of the nozzle. Thus, the net y component of the pressure force on the system is

$$\begin{pmatrix} y \text{ component of pressure} \\ \text{force on system} \end{pmatrix} = A_{\text{exit}} \cdot (P_{\text{exit}} - P_{\text{atm}}) = A_{\text{exit}} \cdot P_{\text{exit, gauge}} \tag{7.22}$$

Since we have chosen the y direction as positive upward, both $F_{y_{\text{t.stand}}}$ and $V_{y_{\text{out}}}$ are negative. Multiplying Eq. 7.21 by -1, we find

$$-F_{y_{\text{t.stand}}} = +F_{y_{\text{rocket}}} = -V_{y_{\text{out}}}\, \dot{m}_{\text{out}} + A_{\text{exit}} P_{\text{exit, gauge}} \tag{7.23}$$

Here we show the force exerted by the rocket as the negative force exerted by the test stand, because they are equal and opposite. The force exerted by the rocket is referred to as the *thrust* of the rocket. It is commonly believed that rockets need something to push against in order to fly. Equation 7.23 shows that this is false. If it were true, rockets could not operate in the vacuum of outer space.

---

*Although the system as a whole is not moving in the y direction, some parts of it are, because of the internal fuel flows. Thus, the overall system has some y-directed momentum, but since this presumably is not changing with time, we have $[d(mV_y)/dt]_{\text{system}} = 0$. During the motor-starting period this simplification is not correct, but for a rocket standing still this term is always small compared with the other terms in the momentum balance.

Example 7.8. A rocket on a test stand is sending out 1000 kg / s of exhaust gas at a velocity of −3000 m / s (negative, because it is in the −y direction). The exit area of the nozzle is 7 m^2, and the pressure at the exhaust-nozzle exit is 35 kPa gauge. What is the thrust of the rocket?

From Eq. 7.23 we find

$$F_{\text{rocket}} = \text{thrust} = -\left(-3000 \frac{\text{m}}{\text{s}} \cdot 1000 \frac{\text{kg}}{\text{s}}\right) + (7 \text{ m}^2 \cdot 35 \text{ kPa})$$

$$= 3 \text{ MN} + 0.245 \text{ MN} = 3.25 \text{ MN} = 0.73 \cdot 10^6 \text{ lbf} \tag{7.Y}$$

■

From this it is clear that, the higher the exhaust velocity, the greater the thrust per unit mass of fuel consumed. If the exit pressure were exactly atmospheric pressure, $P_{\text{exit, gauge}} = 0$, then the thrust would be directly proportional to the exhaust velocity. For horizontally firing rockets, such as artillery rockets and airplane-assist rockets, the atmospheric pressure remains constant during burning, and one could, in principle, design the rocket nozzle for $P_{\text{exh}} = P_{\text{atm}}$. However, this generally results in an impractically large nozzle (too much air resistance or too difficult to fabricate or launch), so the nozzle is usually designed for a P_{exh} significantly greater than the P_{atm} at the exit of the nozzle.

Vertically firing rockets, such as ballistic missiles and satellite launchers, must operate over a wide range of atmospheric pressures, from those at sea level to those in outer space. The nozzle is designed for some average pressure; therefore, P_{exh} could equal P_{atm} only at one particular altitude in the flight. Normally $P_{\text{exh}} > P_{\text{atm}}$ for the whole duration of the rocket flight.

Ignoring this complication for the moment, we can say that for $P_{\text{exh}} = P_{\text{atm}}$ the exhaust velocity is a simple, reliable, direct method of comparing the efficiency of various rocket engines. The early German workers in rocketry used it as a comparison basis. U.S. workers, on the other hand, have preferred to use the *specific impulse*, I_{sp}, as a comparison basis:

$$\begin{pmatrix}\text{Specific}\\ \text{impulse}\end{pmatrix} = I_{\text{sp}} = \frac{\text{lbf of thrust produced}}{(\text{fuel} + \text{oxidizer}) \text{ flow in lbm / s}} \tag{7.24}$$

If $P_{\text{exh}} = P_{\text{atm}}$, then

$$\text{Thrust} = I_{\text{sp}}\, \dot{m} = -V_{y_{\text{out}}}\, \dot{m} \tag{7.25}$$

indicating that I_{sp} must be exactly the same as $-V_{y_{\text{out}}}$ except for a conversion of units. By inspection this must be the conversion involving force, mass, length and time.

Example 7.9. For a rocket with an exhaust velocity of −3000 m / s and with $P_{\text{exh}} = P_{\text{atm}}$, what is the value of I_{sp}?

$$I_{\text{sp}} = -(V_{y_{\text{exh}}}) = -\left(-3000 \frac{\text{m}}{\text{s}}\right) \cdot \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} = 3 \frac{\text{kN} \cdot \text{s}}{\text{kg}} = 305.9 \frac{\text{lbf} \cdot \text{s}}{\text{lbm}} \tag{7.Z}$$

■

It is common practice in the U.S. rocket industry simply to write this as 305.9 s. This is incorrect, because 1 lbf does not equal 1 lbm, and they should not be canceled. Nonetheless, the cancellation is common in rocket publications.

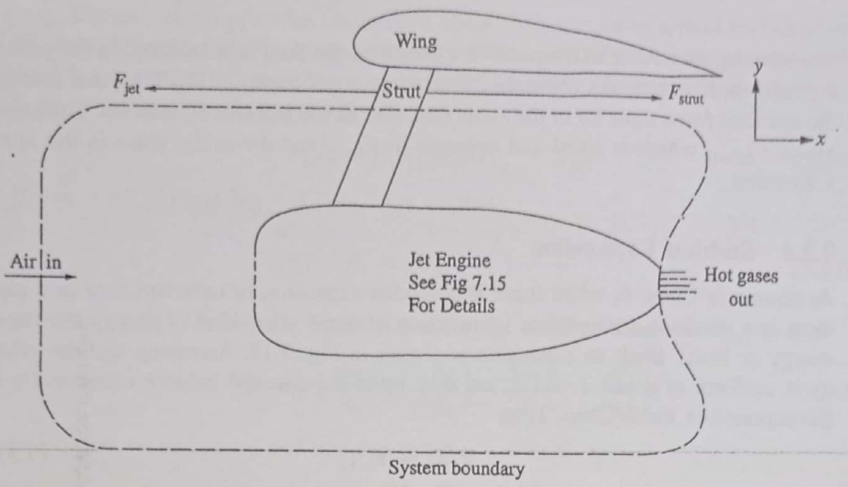


FIGURE 7.11
Simplified view of a jet engine. The fuel flows down the strut from wing tanks to engine.

Our discussion has concerned only rockets fixed to a test stand; we will consider moving rockets in Sec. 7.5.

Figure 7.11 shows a schematic of a jet engine, attached by a strut to the wing of a modern commercial airliner. Fuel flows in from tanks in the wing. Air flows in the front, is compressed, mixed with fuel and burned, then expanded to a high velocity and exhausted to the rear. The engine exerts a force, F_{jet}, called the *thrust* on the airplane, through the strut; the airplane exerts an equal and opposite force on the engine, F_{strut}, through the strut. Applying Eq. 7.17 to the system shown, for steady flow, we find

$$0 = \dot{m}_{air}(V_{x_{in}} - V_{x_{out}}) + \dot{m}_{fuel}(V_{x_{in}} - V_{x_{out}}) + F \tag{7.AA}$$

The fuel flow rate is much less than the air flow rate, so for simple analyses it is normally set equal to zero. The F in Eq. 7.AA is the force exerted by the strut on the engine, which is equal and opposite to the thrust of the engine, so

$$-F_{strut} = F_{thrust} \approx \dot{m}_{air}(V_{x_{in}} - V_{x_{out}}) \tag{7.26}$$

Example 7.10. A modern jet engine has an inlet velocity of almost zero and an exhaust velocity of about 1350 ft / s. Medium-sized ones produce a thrust of 20,000 lbf. What is the air flow rate required by such an engine?

Solving Eq. 7.26 for the mass flow rate, we find

$$\dot{m}_{air} \approx \frac{-F_{strut}}{(V_{x_{in}} - V_{x_{out}})} = \frac{-20{,}000 \text{ lbf}}{(0 - 1350) \text{ ft/s}} \cdot \frac{32.2 \text{ lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}$$

$$= 477 \frac{\text{lbm}}{\text{s}} = 216 \frac{\text{kg}}{\text{s}} \tag{7.AB}$$

■

The strut exerts a force in the positive x direction, the fluid is accelerated in the positive x direction. If one thinks about this from the system shown in Fig 7.11, one sees that the external force must be in the same direction as the increase in velocity. The engine exerts F_{thrust}, which is equal and opposite to F_{strut}, and drives the plane in the minus x direction.

7.3.4 Sudden Expansion

As shown in Chap. 6, when there is a sudden expansion in turbulent flow in a pipe, there is a resulting friction loss (conversion of some other kind of energy to internal energy or heat). Such an expansion is shown in Fig. 7.12. Assuming that the velocity is uniform at points 1 and 2, we may write the material balance equation for an incompressible fluid (Chap. 3) as

$$V_1A_1 = V_2A_2 \tag{7.AC}$$

and B.E. for horizontal flow (Chap. 5) as

$$\frac{P_2 - P_1}{\rho} + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) = -\mathcal{F} \tag{7.AD}$$

Given V_1, A_1, and A_2, we can calculate V_2, but we cannot calculate $(P_2 - P_1)$ unless we know $-\mathcal{F}$. However, we can apply the steady-flow momentum balance, Eq. 7.17, to this flow. When we take the system as the fluid from point 1 to point 2, we find

$$\sum F_x = -\dot{m}(V_{x_1} - V_{x_2}) \tag{7.AE}$$

Here

$$\sum F_x = P_1A_1 + P_{1a}A_{1a} - P_2A_2 - \int \tau \, dA_w \tag{7.AF}$$

where P_{1a} is the average pressure over the annular area, and the integral $\int \tau \, dA_w$ is the total shear force at the walls of the pipe, due to viscous friction. For a sudden expansion the other terms in the momentum balance are large compared with $\int \tau \, dA_w$, so we will drop it.

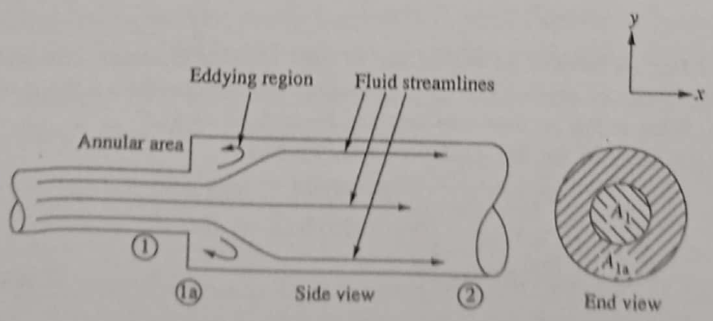


FIGURE 7.12
A sudden expansion in pipe flow.

Because of our previous discussions about the pressure of a fluid leaving a vessel, it is plausible that P_{1a} is approximately the same as P_1: i.e., the pressure is the same for the entire cross section at point 1. Making this assumption and substituting in Eq. 7.AF and then in Eq. 7.AE, we find

$$P_2A_2 - P_1A_2 = \dot{m}(V_{x_1} - V_{x_2}) \tag{7.AG}$$

But $\dot{m} = \rho V_{x_2} A_2$ and $V_{x_2} = V_{x_1}(A_1 / A_2)$, so that

$$P_2 - P_1 = \rho V_{x_1}^2 \frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right) \tag{7.AH}$$

Substituting Eq. 7.AH in Eq. 7.AD and using Eq. 7.AC to eliminate V_{x_2}, we find

$$-\mathcal{F} = V_{x_1}^2 \frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right) - \frac{V_{x_1}^2}{2}\left(1 - \frac{A_1^2}{A_2^2}\right) \tag{7.AI}$$

which may be regrouped and factored to give

$$\mathcal{F} = \frac{V_{x_1}^2}{2} \cdot \left(1 - \frac{A_1}{A_2}\right)^2 \tag{7.27}$$

Comparing this equation with Eq. 6.25, which describes the same situation, we see that the two equations are the same if

$$K = \left(1 - \frac{A_1}{A_2}\right)^2 \tag{7.28}$$

This is the function plotted in Fig. 6.16. Experimental tests indicate that Eq. 7.28 is indeed a good predictor of experimental results, so the assumption of uniform pressure across the cross section at point 1 seems a good one.

It is interesting to compare what we did here with what we did in Sec. 6.3, where we applied a force balance to find Poiseuille's equation. That kind of force balance was usable in that case because there was no acceleration of any part of the fluid, so that the sum of the forces acting on any part the fluid was zero. Here the fluid is decelerated, so the sum of the forces acting on some part of it is not zero. The simple force balance used in Sec. 6.3 is a strongly restricted form of the momentum balance; with the complete momentum balance we can deal with much more complex flows, such as the one examined here, and in the next section.

7.3.5 Eductors, Ejectors, Aspirating Burners, Jet Mixers, and Jet Pumps

Figure 7.13 shows a cross section of a typical laboratory Bunsen burner. In it a jet of fuel gas flows upward. By momentum exchange with the air in the tube, it creates a slight vacuum at the level of the jet (labeled "2" on the figure). This sucks air in through the air inlet. The gas and air mix in the mixing tube, and flow out together into the flame. This is the most common type of gas burner, called an *aspirating burner*. The burners of all household gas furnaces, water heaters, and stoves are of this type, as are hand-held propane torches and small- to medium-sized industrial burners. (The largest industrial burners have combustion air driven by fans and thus do not "aspirate" it as this burner does.) This burner produces only enough vacuum to suck in the combustion air. The same basic device using high-pressure steam as motive fluid can produce industrially useful vacuums. Such devices, called *eductors* or *ejectors*, are widely used industrially as vacuum pumps. Modified forms are also used as *jet mixers* and *jet pumps*. All devices of this type work by exchanging momentum between a centrally located, high-velocity jet and a circumferential, slower-moving flow. The Bunsen burner leads to a simple analysis, because its body is a simple, cylindrical tube. The eductors, ejectors, and jet pumps and most simple burners are shaped like the venturis in Chapter 5. They are more efficient than the straight tube in a Bunsen burner but lead to a much more complex analysis. The simple analysis in this section is correct for any straight-tube version of this type of device and shows intuitively what goes on in the more complex geometries but is not directly applicable to them.

To analyze such a device we begin by making a momentum balance, choosing as our system the section of the mixing tube between 2 and 3 in Fig. 7.13. We assume

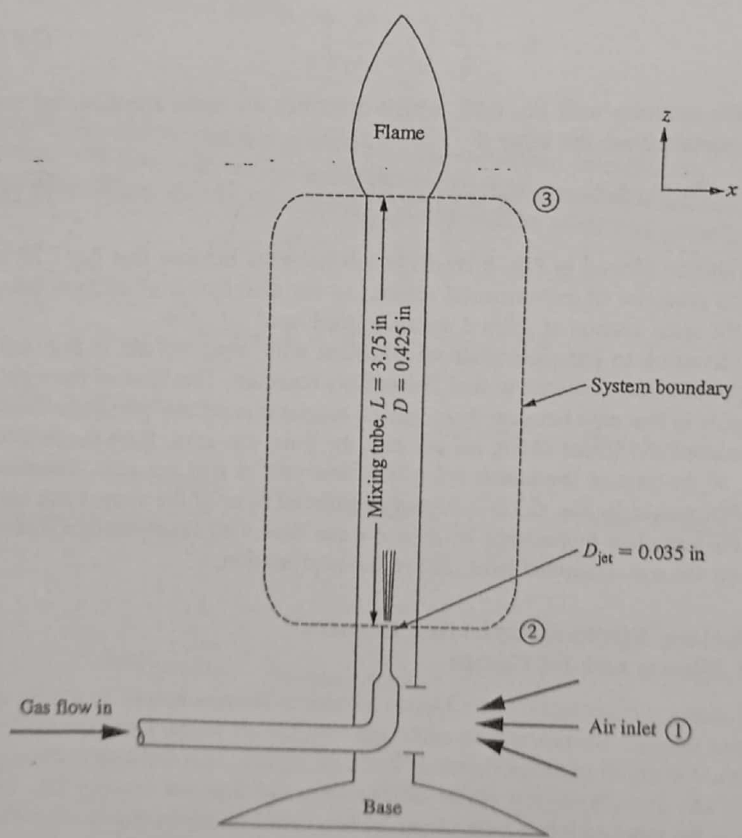


FIGURE 7.13
A simple Bunsen burner. Here the gas jet is shown above the air inlet, which makes the analysis simple. Often the gas jet is at the level of the air inlet, which works well but makes the analysis more complex.

steady flow in the positive z direction, dropping the direction subscripts, so that Eq. 7.17. becomes

$$0 = \dot{m}_{air}(V_3 - V_{air,2}) + \dot{m}_{gas}(V_3 - V_{gas,2}) + P_2A_2 - P_3A_3 - \tau_{wall}\pi D_3\,\Delta x \quad (7.29)$$

At point 3 the pressure must be atmospheric, whereas at point 2 it is not. We work the problem in gauge pressure, which makes the P_3A_3 term = zero. The term for the shear stress at the wall is negligible for low-velocity devices like the Bunsen burner in Fig. 7.13 (however, for high-velocity applications it is important). For the rest of this section we will assume $\tau \approx 0$ and drop the rightmost term in Eq. 7.29. Then

$$P_2 = -\frac{\dot{m}_{air}(V_3 - V_{air,2}) + \dot{m}_{gas}(V_3 - V_{gas,2})}{A_2}$$
$$= -\frac{\rho_{air}V_{air,2}A_{air,2}(V_3 - V_{air,2}) + \rho_{gas}V_{gas,2}A_{gas,2}(V_3 - V_{gas,2})}{(A_{air,2} + A_{gas,2})} \quad (7.30)$$

We also make a steady-flow material balance

$$\dot{m}_{air\ in} + \dot{m}_{gas\ in} = \dot{m}_{air\text{–}gas\ mixture\ out} \quad (7.31)$$

We then substitute ρAV for each of the $\dot{m}$ terms and rearrange to

$$V_3 = \frac{\dfrac{\rho_{gas}}{\rho_{mixture\ out}}V_{gas}A_{gas} + \dfrac{\rho_{air}}{\rho_{mixture\ out}}V_{air}A_{air}}{A_3} \quad (7.32)$$

If we know P_2, we can calculate the velocity of the fuel gas from B.E., and the known pressure in the gas supply line, and can also calculate the velocity of the air at point 2 from B.E. The equations to be solved simultaneously are one momentum balance, one mass balance and two B.E., as shown in Table 7.1. This set of

TABLE 7.1

Equations to be solved in a simple, cylindrical aspirating burner, eductor, jet mixer, or jet pump, for velocities well below sonic*

Equation type	Region	
Momentum balance	Point 2 to point 3	$P_2 = -\dfrac{\rho_{air}V_{air,2}A_{air,2}(V_3 - V_{air,2}) + \rho_{gas}V_{gas,2}A_{gas,2}(V_3 - V_{gas,2})}{(A_{air,2} + A_{gas,2})}$
Mass balance	Point 2 to point 3	$V_3 = \dfrac{\frac{\rho_{gas}}{\rho_{mixture\ out}}V_{gas}A_{gas} + \frac{\rho_{air}}{\rho_{mixture\ out}}V_{air}A_{air}}{A_3}$
B.E.	High pressure gas to point 2	$V_{gas,2} = \left(2\dfrac{P_{gas} - P_2}{\rho_{gas}}\right)^{1/2}$
B.E.	Outside air to point 2	$V_{air,2} = \left(2\dfrac{P_{atm} - P_2}{\rho_{air}}\right)^{1/2}$

*Pressures are gauge, not absolute.

equations is applicable to any cylindrical device of this type, whether the fluids are gases or liquids or slurries. If they are not gases, simply replace all the quantities with "gas" subscript with the quantity for the high-velocity (driver) stream and all those quantities with "air" subscript with the quantity for the low-velocity (driven) stream. If the gas velocities are close to the speed of sound, then the simple B.E. used here must be replaced by the high-velocity gas flow relations from Chap. 8. In the following example we assume that all the flowing fluids are ideal gases at low velocities.

Example 7.11. For the Bunsen burner shown in Fig. 7.13, with the dimensions shown, estimate P_2, V_3, and $\dot{m}_{air} / \dot{m}_{gas}$. The gas is assumed to be natural gas, which in the United States is distributed inside buildings at $P = 4$ in $H_2O = 0.145$ psig $= 1.00$ Pa gauge, and which has a density $\approx (16/29)$ times that of air. We also know that $A_{tube} = 0.142\ \text{in}^2$ and $A_{jet} = 0.00096\ \text{in}^2$.

We begin by guessing $P_{2,\ \text{first guess}} = -0.01$ psig. Then by simple B.E. we compute that $V_{\text{air},\ 2,\ \text{first guess}} = 35.2$ ft / s and by simple B.E. we know that the gas jet velocity ≈ 186 ft / s. The densities are computed from the ideal gas law by

$$\rho_{20°\text{C}} = 0.075 \frac{\text{lbm}}{\text{ft}^3} \cdot \frac{P(\text{abs.})}{14.7\ \text{psia}} \cdot \frac{29(\text{g/mol})}{M} \tag{7.AJ}$$

The mass flow rates of the two inlet streams, calculated from $\dot{m} = \rho AV$, are $2.6 \cdot 10^{-3}$ and $5.15 \cdot 10^{-5}$ lbm / s for air and gas, respectively. From these we can compute that

$$M_{\text{air-gas mixture at outlet}} = 28.5 \frac{\text{lbm}}{\text{lbmol}} \tag{7.AK}$$

Then we substitute values in Eq. 7.32 (dropping the dimensions on the As, which are all in in^2, and on the velocities, which are all in ft / s), so that

$$V_{3,\ \text{first guess}} = \frac{(16/28.5)\cdot(14.69/14.70)\cdot 186 \cdot 0.142 + 35.2\cdot(29/28.5)\cdot(14.69/14.70)\cdot 0.00096}{(0.142 + 0.00096)}$$
$$= 36.0 \frac{\text{ft}}{\text{s}} \tag{7.AL}$$

Then from Eq. 7.29,

$$P_{2,\ \text{calculated}} = \frac{\begin{bmatrix}(0.075\ \text{lbm/ft}^3)\cdot(35.2\ \text{ft/s})\cdot(0.142\ \text{in}^2)\cdot(36.0 - 35.2)\ \text{ft/s} \\ +\ (0.041\ \text{lbm/ft}^3)\cdot(186\ \text{ft/s})\cdot(0.00096\ \text{in}^2)\cdot(36.0 - 186)\ \text{ft/s}\end{bmatrix}}{0.142\ \text{in}^2}$$
$$\cdot \frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}} \cdot \frac{\text{ft}^2}{144\ \text{in}^2} = -0.00044 \frac{\text{lbf}}{\text{in}^2} \tag{7.AM}$$

This is much less than the assumed -0.01 psig, so we use the numerical solution routine on the spreadsheet that we used to generate these values and find the solution, as shown in Table 7.2. We see that all the equations are

TABLE 7.2
Numerical solution to Example 7.11

Variable	Type	First guess	Solution
D_{tube}, in	Given	0.425	0.425
$D_{gas\ jet}$, in	Given	0.035	0.035
A_{tube}, in^2	Calculated	0.142	0.142
A_{jet}, in^2	Calculated	0.00096	0.00096
P_{gas}, in H_2O: gauge	Given	4	4
: psig	Given	0.14461	0.14461
$P_{2,\ guessed}$, psig	**Guessed**	**−0.01**	**−4.43E-05**
ρ_{air} at 2, lbm / ft^3	Calculated	0.0749	0.0750
ρ_{gas} at 2, lbm / ft^3	Calculated	0.04135	0.0414
V_{gas}, ft / s	Calculated	186.21	180.05
V_{air}, ft / s	Calculated	35.18	2.341
$\dot{m}_{air}$, lbm / s	Calculated	0.0026	0.000173
$\dot{m}_{gas}$, lbm / s	Calculated	5.14E-05	4.98E-05
M, air-gas mix at 3, lbm / lbmol	Calculated	28.5	24.5
$\rho_{gas\text{-}air\ mix}$ at 3, lbm / ft^3	Calculated	0.0738	0.0635
V_3, ft / s	Calculated	35.95	3.52
$P_{2,\ calculated}$, psig	Calculated	−0.00044	−4.43E-05
$P_{2,\ calc} / P_{2,\ guessed}$	**Check value**	0.044	**1.0000**
$\dot{m}_{air} / \dot{m}_{gas} = A / F$ ratio	Calculated	50.48	3.48

solved if

$$P_{2,\ \text{solution}} = -4.43 \cdot 10^{-5}\ \text{psig} = -0.0003\ \text{Pa} = -\ 0.0012\ \text{in}\ H_2O \quad (7.AN)$$

$$V_3 = 3.52\,\frac{\text{ft}}{\text{s}} = 1.07\,\frac{\text{m}}{\text{s}} \quad (7.AO)$$

The mass flow rates in Table 7.2 are computed from $\dot{m} = \rho A V$; we see that

$$\dot{m}_{air} / \dot{m}_{gas} = A / F\ \text{ratio} = 3.48 \quad (7.AP)$$

From this example we see the following.

1. One could, in principle, use algebra to solve the four simultaneous equations explicitly, but the spreadsheet solution is quick, simple, and best of all, shows intermediate values that can be checked for plausibility.
2. The calculated air and air-gas mixture velocities are low. Most such burners have velocities of the magnitude shown here.
3. The A / F ratio is 3.48 (lbm / lbm). This is about 25 percent of the stoichiometric air-fuel ratio, which is a typical value for such burners. They all have adjustable shutters on the air inlet; one sets the shutters for the lowest air flow rate which gives a blue (non-smoky, non CO-producing) flame. Normally this requires about 25 percent of the air to be premixed with the natural gas.
4. The vacuum produced is minuscule. For vacuum pumps operating on the same principle, one substitutes high-pressure steam for the low-pressure natural gas,

finding velocities of several thousand ft / s for the central jet, and uses much smaller ratios of (driven fluid / driving fluid).

5. This is a very simple device; most of you have several of them in heaters in your house. To compute its behavior, we needed the momentum balance, a material balance, and two B.E.
6. See Probs. 7.31 to 7.34.

7.4 RELATIVE VELOCITIES

All the examples in the previous section concerned systems fixed in space. When a system is moving, the momentum balance still applies, but it is often convenient to introduce the idea of a relative velocity. Figure 7.14 shows a student on the ground throwing a ball to a student on a moving cart. The velocity of the ball, V, is 10 m / s. The cart is moving with a velocity V_{sys} of 5 m / s. As seen by the student who threw it, the ball is moving 10 m / s. As seen by the student who catches it, the ball is moving 5 m / s, because that is the velocity with which it is overtaking the cart. In general,

$$\mathbf{V} = \mathbf{V}_{sys} + \mathbf{V}_{rel} \tag{7.33}$$

where **V** is the velocity of a body or a stream of fluid relative to some set of fixed coordinates, $\mathbf{V}_{sys}$ is the velocity of the system (the cart in this case) relative to the same set of fixed coordinates, and $\mathbf{V}_{rel}$ is the velocity of the body or stream of fluid as seen by an observer riding on the moving system. When velocities are near the speed of light, this becomes more complicated, but such velocities seldom occur in fluid mechanics. We will see in this section that it is often practical to switch back and forth from the viewpoint of the fixed observer to that of the observer riding on some device or with some part of the flow, most often on a wave of some kind moving through a system.

Equation 7.33 is a vector equation; like all other vector equations it is simply a shorthand way of writing three scalar equations. In this text we will use only its scalar equivalents, such as

$$V_x = V_{x_{sys}} + V_{x_{rel}} \tag{7.34}$$

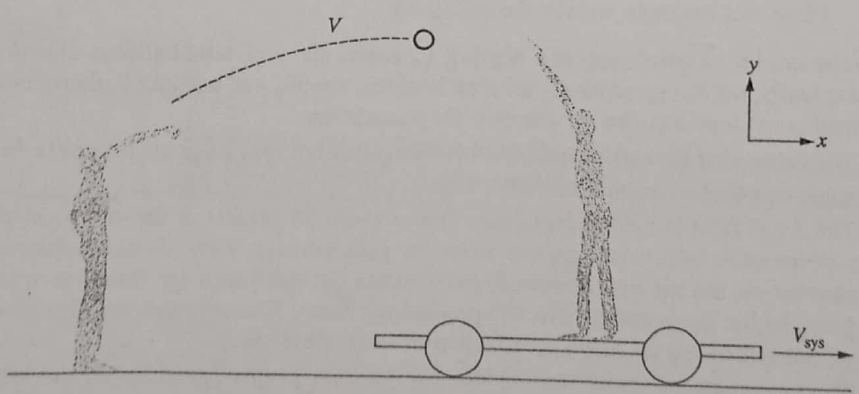


FIGURE 7.14
Relative velocities, illustrated by throwing a ball from a stationary pitcher to a catcher on a moving cart.

To illustrate the utility of this equation, let us consider a rocket in horizontal flight with no air resistance. We choose the rocket as our system and simplify by letting $P_{exh} = P_{atm}$. Then, since there is no flow into the system or any external force acting in the x direction, Eq. 7.13 becomes

$$d(mV_x)_{sys} = -V_{x_{out}}\, dm_{out} \tag{7.35}$$

Expanding the left side and substituting for $V_{x_{out}}$ from Eq. 7.34, we have

$$m_{sys}\, dV_{x_{sys}} + V_{x_{sys}}\, dm_{sys} = -(V_{x_{sys}} + V_{x_{rel,\,out}})\, dm_{out} \tag{7.36}$$

Because all the velocities are in the plus or minus x direction, we can drop the x subscripts. Now we note that $dm_{out} = -dm_{sys}$. Making this substitution and canceling like terms, we can divide by m_{sys} to find

$$dV_{sys} = V_{rel,\,out}\,(dm_{sys} / m_{sys}) \tag{7.37}$$

If the exhaust velocity relative to the rocket is constant (which is practically true of most rockets), then we can readily integrate this to

$$(V_{fin} - V_{init})_{sys} = V_{rel,\,out} \ln \frac{m_{fin}}{m_{init}} \tag{7.38}$$

This equation, often referred to as the *rocket equation* or the *burnout velocity equation*, indicates the limitation on possible speeds of various kinds of rockets.

Example 7.12. A single-stage rocket is to start from rest; $V_{init} = 0$. The mass of fuel is 0.9 of the total mass of the loaded rocket; $m_{fin} / m_{init} = 0.1$. The specific impulse of the fuel is 430 lbf · s / lbm, and the pressures are $P_{exh} = P_{atm}$. What speed will this rocket attain in horizontal flight if there is no air resistance?

From Eq. 7.25 we have

$$V_{rel,\,out} = -I_{sp} = -430 \frac{\text{lbf} \cdot \text{s}}{\text{lbm}} \cdot 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}$$

$$= -13{,}850 \frac{\text{ft}}{\text{s}} = -4220 \frac{\text{m}}{\text{s}} \tag{7.AQ}$$

Therefore,

$$V_{fin} = -13{,}850 \frac{\text{ft}}{\text{s}} \cdot \ln 0.1 = 31{,}900 \frac{\text{ft}}{\text{s}} = 9730 \frac{\text{m}}{\text{s}} \tag{7.AR}$$

■

This example shows the probable maximum speed attainable with single-stage rockets using chemical fuels. It appears that the maximum I_{sp} for chemical fuels is about 430 lbf · s / lbm. Better structural design may reduce the value of m_{fin} / m_{init}, but it is unlikely to go much under 0.1 if there is any significant payload involved. For higher velocities, staged rockets are needed. Equation 7.38 does not include the effects of gravity or air resistance. These can be included, making the equation somewhat more complicated (Prob. 7.36). For more on rockets see Sutton [2] and Ley [3].

Another example of the utility of the relative-velocity concept concerns the interaction of a jet of fluid and a moving blade. Such interactions are the basis of

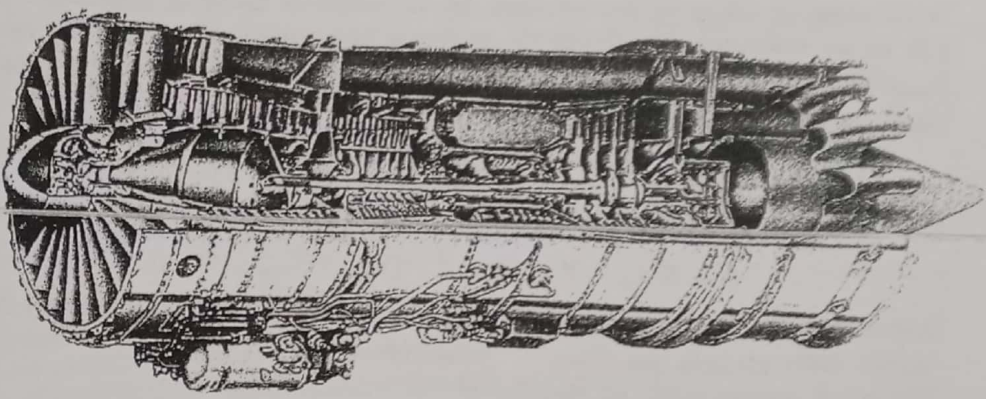


FIGURE 7.15
Cutaway of a modern jet engine. This is a Pratt and Whitney JT8D-219, whose basic parameters are shown in Example 7.10. Observe the large number of rotating and fixed blades that interact with the moving fluids. (Courtesy of Pratt and Whitney, a United Technologies Company.)

turbines and rotating compressors as used in turbojet and gas turbine engines and in the steam and water turbines that produce almost all of the world's electricity. Figure 7.15 shows a cutaway of a modern aircraft jet engine. In it the fluid interacts with multiple sets of blades, some of which do work on the fluid, increasing its pressure, and some of which extract work from the fluid, to be used in other parts of the engine. The rest of this section considers the interaction of a fluid with a single moving blade, one of the many blades in Figure 7.15.

In Examples 7.3 and 7.4 we saw how the interaction of a jet and solid surface produces a force on the surface. For this force to do work it must move through a distance. The work will be given by $dW = F\,dx$, and the power (or rate of doing work) is given by $\text{Po} = dW/dt = F\,dx/dt$. The latter is equal to the force times the velocity of the system, $\text{Po} = FV_{\text{sys}}$.

A curved blade is moving in the x direction and deflecting a stream of fluid; see Fig. 7.16. Consider this first from the viewpoint of an observer riding with the blade. As far as the observer can tell, the blade is standing still; no work is being done. Therefore, with no change in pressure and elevation, B.E. tells the observer that

$$\frac{V_{\text{out}}^2}{2} - \frac{V_{\text{in}}^2}{2} = -\mathscr{F} \qquad \text{(7.AS)}$$

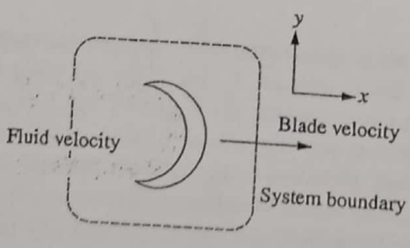


FIGURE 7.16
Simplest possible jet-blade interaction.

and that, if there is no friction, the outlet velocity is equal in magnitude to the inlet velocity but in a different direction. If there is friction, the outlet velocity will be less, but it cannot possibly be more, from the viewpoint of an observer riding on the blade. Here in the energy balance V^2 is a scalar, so we have no concern about the signs of V_{in} and V_{out}.

Now assume frictionless flow and that the blade in the figure is so shaped that the inlet and outlet streams have only x velocity and no y velocity. Then, by applying Eq. 7.17 and dropping the x subscript, because all the velocities are in the plus or minus x direction, we write

$$F = \dot{m}(V_{\text{out}} - V_{\text{in}}) \tag{7.AT}$$

This is the F exerted by the blade. The fluid exerts an equal and opposite force on the blade. Substituting Eq. 7.34 twice, we find

$$F = \dot{m}(V_{\text{rel, out}} + V_{\text{blade}} - V_{\text{rel, in}} - V_{\text{blade}}) = \dot{m}(V_{\text{rel, out}} - V_{\text{rel, in}}) \tag{7.39}$$

We see that the velocity of the blade cancels out of the force equation, so the force is the same whether viewed by an observer riding on the blade or by an observer standing still.

The work* done by the fluid per unit time (the power) is

$$\text{Po} = \frac{dW}{dt} = -F\frac{dx}{dt} = -FV_{\text{blade}} = -\dot{m}(V_{\text{rel, out}} - V_{\text{rel, in}})V_{\text{blade}} \tag{7.40}$$

and therefore the work done per unit mass of fluid is

$$\frac{dW}{dm} = (V_{\text{rel, in}} - V_{\text{rel, out}})V_{\text{blade}} \tag{7.41}$$

As shown above, from B.E. for frictionless flow we know that $V_{\text{rel, out}} = -V_{\text{rel, in}}$; therefore,

$$\frac{dW}{dm} = (2V_{\text{rel, in}})V_{\text{blade}} \tag{7.42}$$

Now suppose that the velocity of the jet is fixed. This would occur if it were a jet of water entering the power plant at the base of a dam with constant upstream water level (in which case we could calculate the jet velocity by B.E.) or if it were a jet of steam from a boiler with constant steam temperature and pressure (in which case we could calculate the jet velocity by the methods to be developed in Chap. 8). In Eq. 7.42 we replace $V_{\text{rel, in}}$ by $V_{\text{jet}} - V_{\text{blade}}$ and divide both sides by $V_{\text{jet}}^2/2$ to find

$$\frac{dW/dm}{V_{\text{jet}}^2/2} = 4\left(1 - \frac{V_{\text{blade}}}{V_{\text{jet}}}\right)\cdot\frac{V_{\text{blade}}}{V_{\text{jet}}} \tag{7.43}$$

The left side of Eq. 7.43 is the ratio of the work extracted from the fluid per pound of fluid to the kinetic energy per pound of the fluid in the jet. We may think of it as the fractional efficiency of the blade in converting jet kinetic energy into useful work (of the rotating turbine shaft). The right-hand side of Eq. 7.43 is plotted versus $V_{\text{blade}}/V_{\text{jet}}$ in Fig. 7.17.

*The work terms in this chapter are all exclusive of injection work and would have the symbol $W_{\text{n.f.}}$ in Chaps. 4, 5, and 6. Here we drop the subscript because it causes no confusion to do so.

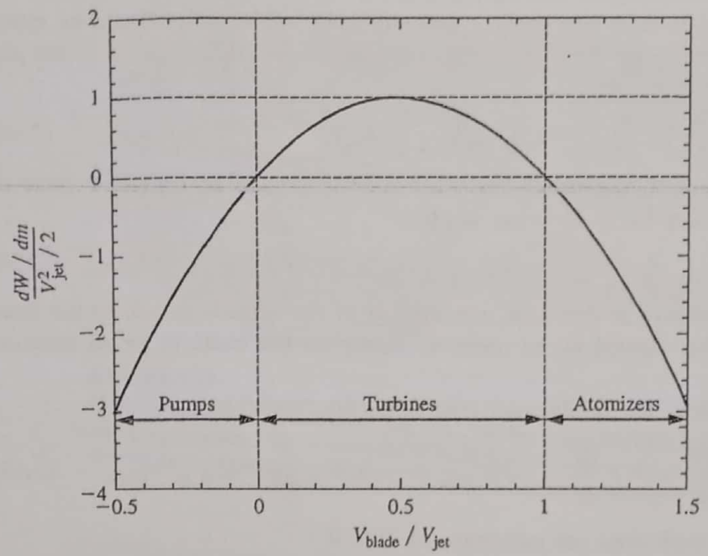


FIGURE 7.17
The ratio of work produced per pound to inlet kinetic energy per pound for a range of values of blade speed/jet speed.

From this figure we see:

1. For $V_{blade}/V_{jet} = 0$ the work extracted per pound = 0. The blade is standing still and resisting the flow but not extracting any work from it.
2. For $V_{blade}/V_{jet} = 1.00$, the work extracted per pound = 0. The blade is moving at the same speed as the jet (like a person walking through a revolving door that is turning at exactly the person's walking speed) and has no force interaction with the jet.
3. For V_{blade}/V_{jet} between 0 and 1.00 the work extracted is positive. For $V_{blade}/V_{jet} = 0.5$ it is a maximum, and $(dW/dm)/(V_{jet}^2/2) = 1.00$. At this condition all of the kinetic energy in the jet is being extracted by the blade and converted to work. If we consider Fig. 7.16 from the viewpoint of the person riding on the blade, then the fluid is overtaking us at 0.5 times the jet speed and leaving at that speed, in the opposite direction. From the viewpoint of a fixed observer watching us ride by, the jet leaves the blade at zero velocity; all of its kinetic energy has been extracted. (In a practical turbine of this kind the jet leaves with a little y velocity to get out of the way of the next batch of fluid that follows it; if it left with only x velocity, it would run into the part of the jet behind it.)
4. At the left of the figure for $V_{blade}/V_{jet} < 0$, the blade is moving in the opposite direction from that shown in Fig. 7.16. In this case it is doing work on the jet. From the viewpoint of someone riding the jet, the exit velocity is still the same as the inlet velocity, but from the viewpoint of a stationary observer the exit velocity is greater than the initial jet velocity. The sign of the work has changed because, instead of the jet doing work on the blade, the blade is doing work on the jet. This

is the description of a pump or compressor; the fluid leaving the blade at a high velocity is passed through some kind of diffuser (Sec. 5.5) and slowed, thus increasing in pressure.

5. At the right of the figure for $V_{\text{blade}} / V_{\text{jet}} > 1.00$ the blade is going faster than the jet and picks up the fluid and expels it at a higher velocity. This occurs in rotating disk atomizers, widely used to produce small drops for spray dryers. The sign of the work is the same as for pumps, and the opposite of that for turbines, because the rotating blade does work on the fluid. For this application, the orientation of the blade in Fig. 7.16 is rotated 180°.

This is a simple discussion of the interactions of jets of fluid and moving blades. The interactions in real turbines, compressors, and atomizers are more complex, but almost all of these devices are based on the transfer of momentum between a moving jet of fluid and a moving blade, as sketched here.

7.5 STARTING AND STOPPING FLOWS

The previous examples have been for steady flows. The momentum balance is powerful enough to deal with unsteady flows as well. Several very simple examples will illustrate this power.

7.5.1 Starting Flow in a Pipe

Example 7.13. Figure 7.18 shows a large water reservoir that discharges through a long, horizontal pipe, at the end of which is a valve. What is the velocity-time behavior of this system when the valve is suddenly opened?

Here the steady-state velocity, V_∞, can be found by B.E. from point 1 to point 3, finding

$$V_\infty = \left[\frac{(P_2 - P_3)}{\rho} \cdot \frac{D}{2fL} \right]^{1/2} \tag{7.AU}$$

Using the values in Table A.3, we find that steady-state velocity in the pipe is 2.45 m / s (8.03 ft / s) and the steady-state friction factor is 0.0042.

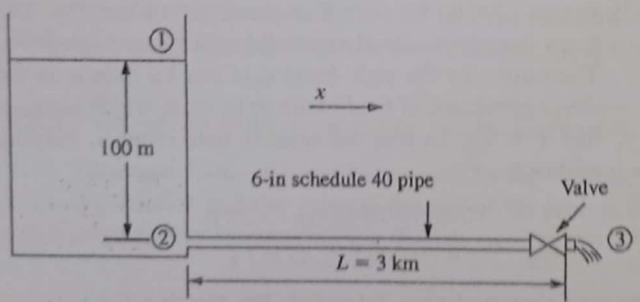


FIGURE 7.18
Long pipe with quick-opening and -closing valve.

To estimate the starting behavior, we take as our system the pipe from its entrance, point 2, to its exit, point 3. Here we can assume that the pressure at point 2 does not change during the starting of the flow and is given by $P_2 = \rho g(z_1 - z_2)$. Applying the x-directed momentum balance (Eq. 7.15), we assume that the density of the fluid does not change, so the mass of fluid in the system is constant and the mass flow rates and velocities in and out at any instant are equal. Then

$$m_{sys}\, dV_{sys} = \sum F\, dt = \left[(P_2 - P_3)\frac{\pi}{4}D^2 - \tau\pi DL\right] dt \tag{7.44}$$

Here the shear force acts in the direction opposite to the pressure force; at steady state they will be equal. Replacing τ by its expression in terms of the friction factor, and expressing the mass of the system in terms of its volume and density, we find

$$\rho\frac{\pi}{4}D^2L\, dV = \sum F\, dt = \left[(P_2 - P_3)\frac{\pi}{4}D^2 - f\rho\frac{V^2}{2}\pi DL\right] dt \tag{7.45}$$

$$dV = \left[\frac{(P_2 - P_3)}{\rho L} - \frac{4f}{D}\cdot\frac{V^2}{2}\right] dt = \frac{D}{2f}(V_\infty^2 - V^2)\, dt \tag{7.46}$$

and

$$\frac{dV}{(V_\infty^2 - V^2)} = \frac{D}{2f}dt \tag{7.47}$$

Here f is not constant because the flow starts in the laminar region, so that f is initially large, then declines, then increases sharply during the transition, and then declines slowly in the turbulent region. But the term involving f is only significant near the end of the starting transient, so we can treat f as a constant and perform the indicated integration, finding

$$t = \frac{D}{4fV_\infty}\ln\frac{V_\infty + V}{V_\infty - V} + C \tag{7.48}$$

Here at $t = 0$, $V = 0$, so the ln term on the right is ln 1 = 0, from which it follows that the constant of integration $C = 0$. We may also check to see that Eq. 7.48 gives the correct steady-state solution by setting $t = \infty$. The only way the right-hand side can be infinite is for the denominator of the ln term to be zero, which requires that $V = V_\infty$. To find the velocity-time relation, first we evaluate

$$\frac{D}{4fV_\infty} = \frac{[6.065 / 12\text{ ft}]\cdot \text{m} / 3.28\text{ ft}}{4\cdot 0.0042\cdot 2.45\text{ m/s}} = 3.74\text{ s} \tag{7.AV}$$

and then make up Table 7.3. We see that the velocity increases quickly at first and then asymptotically increases to the steady-state value. ■

TABLE 7.3
Flow starting behavior in Example 7.13

Velocity, m / s	Time, s
0.1	0.31
1	3.24
2	8.57
2.4	17.11
2.44	23.1
2.449	31.8
2.45	Infinite

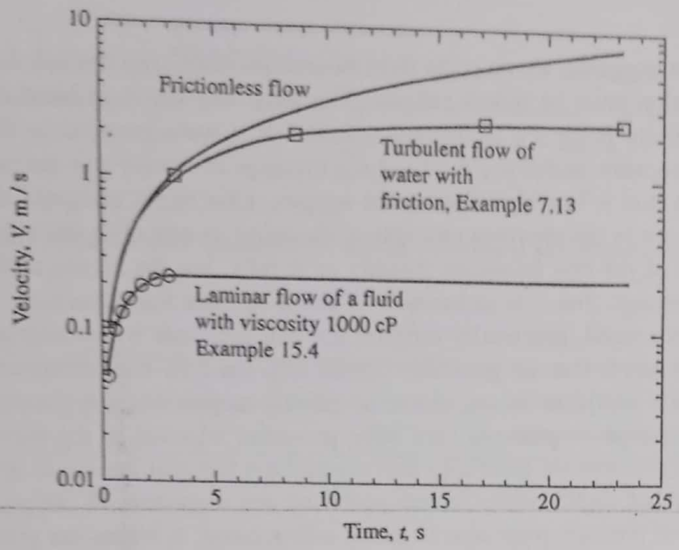


FIGURE 7.19
Flow-starting behavior for three situations. These all correspond to Fig. 7.18. The top curve corresponds to zero friction. The middle curve is the solution to Example 7.13; the square points shown are those from Table 7.3. The bottom curve is from Example 15.4, which is the same as Example 7.13 except that the water has been replaced by a fluid 1000 times as viscous as water.

Figure 7.19 shows the result of this example and compares it to two other results. The values from Table 7.3 are shown as squares, with a smooth curve through them. Above that curve is a frictionless curve, which results from setting $f = 0$ in Eq. 7.45. For the first few seconds it is identical to the curve from this example, indicating that the effects of friction on the starting behavior are negligible until the velocity begins to approach its final value. The lower curve shows the results of Example 15.4, in which the water in Example 7.13 has been replaced with a fluid 1000 times as viscous as water, which makes the flow laminar. That problem cannot be solved by the one-dimensional approach used here; it requires the two- and three-dimensional approach shown in Part IV of this book. We see that for this viscous a fluid the velocity is less than for Example 7.13 at all times, and that the final value is about a tenth of that for water. The starting transient is shorter for that case than for Example 7.13.

7.5.2 Stopping Flow in a Pipe; Water Hammer

Example 7.14. Repeat Example 7.13 for the case in which the fluid is flowing steadily and the valve at the end of the pipe is instantaneously closed. Here we begin by rearranging Eq. 7.44 to

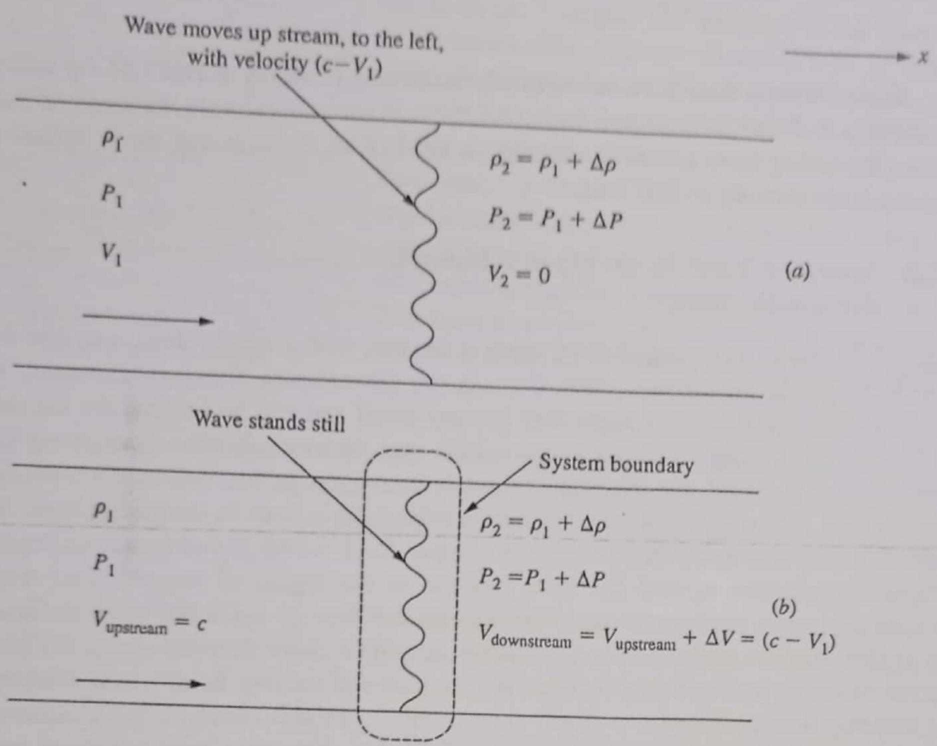
$$\frac{dV_{sys}}{dt} = \frac{\left[(P_2 - P_3)\frac{\pi}{4}D^2 - \tau\pi DL\right]}{m_{sys}} \tag{7.49}$$

If, as the problem suggests, we stop the fluid instantaneously, then the left-hand side of this equation must be minus infinity! The only way the right-hand side can be minus infinity is for P_3 to become infinite! If it were possible to stop the fluid instantaneously, and if the fluid did not increase in density nor the pipe wall stretch, then that is exactly what would happen. One might compare this situation and the one in the previous example to dropping an egg off a tall building. The velocity of the egg increases steadily as it falls, but the forces acting on it are gentle enough that it is unharmed. When it reaches the pavement, its deceleration is very rapid, practically infinite; the egg responds by splattering. The observational fact is that we generally cannot stop the flow instantaneously but that with readily available valves, closed as quickly as possible, we can stop the fluid quickly enough to generate very large pressures adjacent to the valve.

To solve the problem we must take into account the fact that the liquid will compress, slightly but significantly. In real problems the expansion of the pipe due to the increased pressure must also be taken into account; it makes the pressure less than the value we will compute here. If we are able to stop the flow by shutting the valve at point 3 instantaneously, then the layer of fluid adjacent to the valve will be stopped. It will stop the next layer, and so the region of stopped fluid will propagate backward up the pipe to the reservoir. (This is analogous to the big freeway pileups that occur during heavy fogs. Someone slows down and is hit by a faster-moving car coming from behind. The first crash produces a pile of stopped, wrecked cars. This pile then enlarges in the upstream direction as more and more cars pile into the stopped wreckage.) The rate of propagation of the boundary between stopped and moving fluid (assuming rigid pipe walls) will be the local speed of sound. That is not proven here, but will seem clearer after we have discussed the speed of sound in Chap. 8. From Chap. 8 we can borrow the fact that for water the speed of sound is about $c = 5000 \text{ ft/s}$ (1520 m/s) so that the stopped layer of water will reach the reservoir in ($t = L/c = 3000 \text{ m} / 1520 \text{ m/s}$) or about 2 s after the valve is closed.

To compute the pressure in the stopped fluid we take the viewpoint of the person riding on the interface between the moving fluid and the stopped fluid. Figure 7.20 shows this change of viewpoint and its consequences. We will apply this same logic several times again in this book. The upper part of the figure, from the viewpoint of a stationary observer, shows the wave passing from right to left, against the flow, with velocity $(c - V_1)$. The speed of sound, shown above, describes how fast a sound wave moves into a stationary fluid; here the fluid is moving, with the result shown. The lower part of the figure shows the viewpoint of someone riding the wave. From that viewpoint we are standing still, and the fluid is moving toward us with $V_{\text{upstream}} = c$. Changing the viewpoint does not change P or ρ at any point. Changing the viewpoint does change both the perceived upstream and downstream velocities. But it does not alter the velocity change, ΔV, across the wave. The merit of this change of viewpoint is that it changes an unsteady-state problem to a steady-state one. To see the advantage, try working out the rest of this problem and the problem in the next section from the viewpoint of the stationary observer.

Taking the viewpoint of the observer riding the wave, and using the system shown in the lower part of Fig. 7.20, we find that the x-directed, steady-state



In both coordinate systems, $(V_{upstream} - V_{downstream}) = V_1 - 0 = V_1$

FIGURE 7.20
A wave as seen (*a*) passing by a stationary observer and (*b*) by an observer moving at the same speed as the wave.

momentum balance, Eq. 7.15, becomes

$$0 = \dot{m}_{in}(V_{in} - V_{out}) + \sum F = c\rho A\,\Delta V - A\,\Delta P \tag{7.50}$$

and

$$\Delta P = c\rho\,\Delta V \tag{7.51}$$

where ΔP is the pressure change across the moving boundary and ΔV is the velocity change across it, which in this case is the velocity of the fluid that has not been stopped yet, minus zero, or 2.45 m / s, from Example 7.13. Inserting numerical values, we find

$$\Delta P = c\rho\,\Delta V = 1520\,\frac{\text{m}}{\text{s}} \cdot 998.2\,\frac{\text{kg}}{\text{m}^3} \cdot 2.45\,\frac{\text{m}}{\text{s}} \cdot \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \cdot \frac{\text{Pa}}{\text{N}/\text{m}^2}$$
$$= 3.72\ \text{kPa} = 539\ \text{psi} \tag{7.AW}$$

■

This is a very large pressure, and it explains why this phenomenon, called *water hammer*, can be a serious problem, particularly in large hydroelectric structures. One often produces the same result at home by closing a faucet quickly; a pounding sound in the plumbing indicates that a high pressure has been generated. The treatment here is the simplest possible; many more interesting details and complications are shown in the books on this subject [4].

Some students find it more intuitively satisfying to arrive at Eq. 7.51 by asking how long ($t = L/c$) the pressure force ($A\Delta P$) must act to decelerate the mass of fluid in the pipe ($AL\rho$) from its initial velocity to zero (ΔV). Substituting those values in $F = ma$ leads directly to Eq. 7.51.

7.5.3 Stopping Flow in an Open Channel: Hydraulic Jump

Figure 7.21 shows a sloping channel open at the top, with a steady flow in it. For the flow to be steady the channel must slope in the downstream direction. However, as shown in Example 6.15, that slope may be very small and will be ignored for the rest of this section. The figure shows a gate, which may be suddenly closed, stopping the flow. When it is closed, the situation is directly analogous to that in Sec. 7.5.2 except that the fluid pressure cannot increase because the fluid is open to the atmosphere. So instead, it increases in depth. The layer of stopped fluid, which has an increased depth, propagates upstream against the flow, just the as the region of stopped fluid with increased pressure propagated upstream against the flow in the water hammer case. As in that case, the mathematics are greatly simplified if we take the viewpoint of a person riding on the boundary between the stopped and moving fluid, which converts an unsteady-flow problem to a steady-flow problem. To that observer, the phenomenon is as sketched in cross section in Figure 7.22. This transition from a shallow, fast flow to a deeper, slower flow is called *hydraulic jump*. It is easily observed in gutters during heavy rainstorms and at the bottom of chutes and spillways. Before we begin

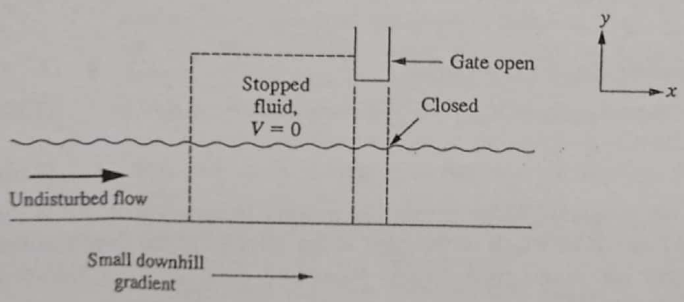


FIGURE 7.21
An open-channel flow stopped by closing a gate.

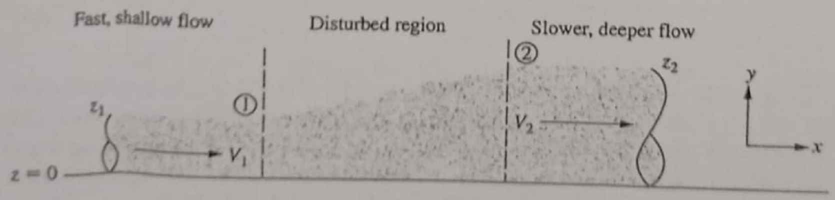


FIGURE 7.22
Hydraulic jump in a linear flow.

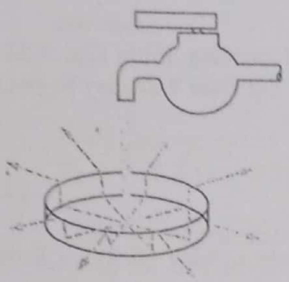


FIGURE 7.23
Hydraulic jump in a radial outward flow, as seen in any sink.

the analysis, I suggest that you look at this phenomenon in a kitchen sink. Figure 7.23 shows a jet of water from a faucet striking the bottom of a sink. It flows outward in a shallow, fast flow until it enters a hydraulic jump, which changes the flow to a deeper, slower flow than the flow near the faucet. This is easy see in any sink. The steady, one-dimensional version sketched in Fig. 7.22 is easiest to treat mathematically, so we return to it. The mathematics treat the wave as a step function, but as Fig. 7.22 (and observation of the jump in a sink) shows, the observed behavior is that the change in depth and velocity occurs over a short distance, not as a sharp step.

In Fig. 7.22 the cross section through the jump extends into and out of the paper. Consider a section l ft thick into the paper, and assume that the velocity across any section perpendicular to the flow is uniform. Then for steady flow of an incompressible fluid such as water, the mass balance gives

$$V_1 z_1 = V_2 z_2 \tag{7.AX}$$

In a typical problem we might know two of the four quantities here. This equation provides a relation for finding a third; one more is needed. B.E. written between states 1 and 2 shows

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^2 - V_1^2}{2} = -\mathscr{F} \tag{7.AY}$$

Here, as in Sec. 5.11, the fluid does not all enter or leave at the same z or the same P, so we must use appropriate average values for the zs and Ps. If the friction term were negligible, this equation would supply the needed extra relation, but mathematical analysis and experimental tests indicate that friction is quite large; so, although B.E. supplies an additional relation between the unknowns in Eq. 7.AY, it does us no good because it introduces another unknown, $\mathscr{F}$.

Equation 7.17, however, can supply the needed relationship. Taking as our system the section of the fluid between points 1 and 2, we see that the only forces acting in the x direction are the shear force on the bottom, which is negligibly small and will be ignored, and the pressure forces on each side of the liquid in the system, which are each of the form

$$F = \int P\,dA = l\int_{z=0}^{z_{surf}} g\rho(z_{surf} - z)\,dz = lg\rho\,\frac{(z_{surf})^2}{2} \tag{7.AZ}$$

Since the flow at points 1 and 2 is all in the x direction, we may write Eq. 7.17 and drop the x subscripts to find

$$0 = l\rho z_1 V_1(V_1 - V_2) + \frac{l\rho g}{2}(z_1^2 - z_2^2) \tag{7.BA}$$

Equations 7.AX and 7.BA can be solved for z (see App. B.2), which gives us Eq. 7.52,

$$z_2 = \frac{-z_1}{2} \pm \sqrt{\left(\frac{z_1}{2}\right)^2 + \frac{V_1^2 z_1}{g}} \tag{7.52}$$

Here the minus sign before the radical has no physical meaning. From Eqs. 7.52 and 7.AY we can calculate the value of $\mathcal{F}$ (see Prob. 7.52). Equation 7.52 may be put into an interesting form by dividing through by z_1:

$$\frac{z_2}{z_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2\frac{V_1^2}{gz_1}} \tag{7.53}$$

The dimensionless group V_1^2/gz_1 is called the Froude number (William Froude, 1810–1879); its significance is discussed in Chap. 9.

From the foregoing it is clear that z_2/z_1 must always be greater than or equal to 1 (a value of 1 would correspond to a jump of negligible height, i.e., one that was vanishingly small). One may verify from Eq. 7.53 that $z_2/z_1 = 1$ for a Froude number of 1, and that z_2/z_1 is greater than 1 for any Froude number greater than 1. In studying normal shock waves in gases, we see that another dimensionless group, the Mach number, plays a similar role.

This topic is traditionally included in fluid mechanics books for the following reasons:

1. Hydraulic jump is readily observed in nature.
2. Hydraulic jump is an interesting example of a problem that cannot be solved without using the momentum balance.
3. Shock waves and hydraulic jumps are very similar, as we will see when we study shock waves in high-velocity gas flow. Hydraulic jumps are easily demonstrated in any kitchen sink and easily studied in any well-equipped hydraulics laboratory. Shock waves are much harder to demonstrate and study. Therefore, from visual observation and mathematical analysis of hydraulic jumps we can gain an intuitive understanding of shock waves. We will return to their similarity in Chap. 8.

Equations 7.52 and 7.AY are equally well satisfied whether a flow is from left to right or from right to left in Fig. 7.22. However, if we calculate $\mathcal{F}$ for both we see that right-to-left flow in Fig. 7.22 (deep, slow flow to shallow, fast flow) results in a negative value of $\mathcal{F}$. This is forbidden by the second law of thermodynamics, so the flow can be only in the sense indicated in the figure. We see here a strong parallel with what we will see concerning shock waves, in which the continuity, energy, and momentum equations are also satisfied by flow in either direction; but the second law of thermodynamics shows that only one direction is possible. We also see that a hydraulic jump is only possible if the upstream value of the Froude number is greater than 1; when we study normal shock waves we will see that a normal shock wave is only possible if the upstream Mach number is greater than 1.

Example 7.15. A steady water flow as shown in Fig. 7.22 has $V_1 = 4$ ft / s, and $z_1 = 0.0005$ ft (= 0.006 inches). What are the values of V_2 and z_2?

First we calculate

$$\text{Froude number} = \mathcal{F}r = \frac{V_1^2}{gz_1} = \frac{(4\text{ ft / s})^2}{(32.2\text{ ft / s}^2)\cdot 0.0005\text{ ft}} = 993.8 \tag{7.BB}$$

and then

$$\frac{z_2}{z_1} = -0.5 + \sqrt{0.25 + 2 \cdot 993.8} = 45.08 \qquad (7.BC)$$

From this we easily compute that $z_2 = 0.0225$ ft $= 0.27$ in $= 6.86$ mm, and $V_2 = 0.09$ ft / s $= 0.017$ m / s. ■

These are representative values for the bathroom sink flow shown in Fig. 7.23. If you plug the sink so that the water cannot escape, you will see that as the water depth downstream of the jump increases, the radius at which the jump occurs will move inward. The shallow, fast flow decreases in velocity as it moves outward (which the material balance for steady flow says it must do if its depth remains constant, which it practically does), so the fluid is solving Eq. 7.53 for the place where V_1 corresponds to the value of (z_2 / z_1) set by the rising water level in the sink. Eventually the radius of the jump becomes small enough that the low, swift flow is "drowned," and the hydraulic jump disappears.

Returning to the straight, rectangular channel in Fig. 7.21 we can consider a more typical example, a fluid flow with $V_1 = 10$ ft / s as seen by an observer moving along with the jump, and $z_1 = 1$ ft. Straightforward calculations show that for this case the Froude number $= 3.1$, $V_2 = 3.24$ ft / s, and $z_2 = 3.25$ ft. Now if we switch back to the viewpoint of a stationary observer, we see that the value of z_2 is the same for a stationary or a moving observer, so our change in viewpoint did not affect that value. But the value of V_1, which we guessed, is the *sum* of the velocity of the upstream flow, as seen by a fixed observer, and the velocity at which the jump moves upstream. From the viewpoint of a stationary observer, the downstream flow is standing still, so we know that the jump must be moving upstream at $V_{\text{jump}} = 3.24$ ft / s and that, from the viewpoint of a stationary observer, the upstream velocity is $V_1 = 10 - 3.24 = 6.76$ ft / s. The corresponding problem in which we know the upstream velocity in Fig. 7.22 and want to know how fast the jump moves upstream requires a trial-and-error solution; see Prob. 7.53.

7.6 A VERY BRIEF INTRODUCTION TO AERONAUTICAL ENGINEERING

Chapters 5 and 6 were devoted to problems that could be most easily understood by applying the energy balance, and this chapter has been devoted to other problems, which could be most easily understood by applying the momentum balance. Some problems are most easily understood by applying *both* the energy and the momentum balances. A very interesting example is the elementary analysis of flight, which helps explain the behavior of airplanes and helicopters, and also birds and insects.

An airplane (or a bird or a flying insect) is a fluid-mechanical device; it flies by making a fluid, the air, move. Consider an airplane in constant-velocity, level flight; see Fig. 7.24. The airplane has no acceleration in either the x or the y directions; therefore, the sum of the forces acting on the airplane in each of these directions is zero. These forces are shown in the figure and given their common aeronautical-engineering names.

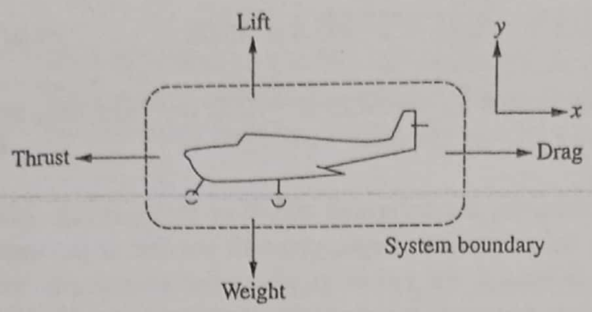


FIGURE 7.24
Airplane in constant-velocity, level flight.

The force that gravity exerts, the weight, acts downward. To counteract this, the air must exert an equal and opposite upward force, called the lift. It is the function of the airplane's wings to make the air exert this force.

To see how the wing does this, we make a momentum balance around the airplane and take as our system the airplane plus an envelope of air around it, large enough for the pressure on the outside of the envelope to be constant. This system boundary is also shown in Fig. 7.24. We base our coordinate system on the airplane, so the airplane appears to stand still and the air to flow toward it. Applying the y component of the momentum balance in constant-velocity flight, we see that there is no accumulation: we have $d(mV)_{\text{sys}} = 0$.* We have assumed that the pressure around the outside of the system is uniform; then the only external y-direction forces acting on the plane are the force of gravity and the force exerted by the air. Thus,

$$F = \text{weight of the plane} = \dot{m}(V_{y_{\text{out}}} - V_{y_{\text{in}}}) \tag{7.54}$$

Since not all the air comes in or goes out at the same velocity, the two V_y terms in this equation must be some appropriate average velocities, obtained by an integral of the flow per unit surface area over the entire surface of the system. However, we need not worry about this integration, if we merely think of these velocities as some appropriate average.

In the direction of the $+y$ axis, F_y is negative. The flow through $\dot{m}$ is positive, so $(V_{y_{\text{out}}} - V_{y_{\text{in}}})$ must be negative; the air must be accelerated in the $-y$ direction, downward. Thus, we see that, to stay in level flight, the airplane must accelerate the surrounding air downward. This is precisely what a swimmer does in treading water—by accelerating the water downward, the swimmer stays up.

Example 7.16. An airplane with a loaded mass of 1000 kg (and thus a weight of 9810 N) is flying in constant-velocity, horizontal flight at 50 m / s. Its wingspread is 15 m, and we assume that it influences a stream of air as wide as its wingspread and 3 m thick. How much average vertical downward velocity must it give this air? Assume that the air comes in at zero vertical velocity.

$$\dot{m} = \rho A V_{x_{\text{in}}} = 1.21\,\frac{\text{kg}}{\text{m}^3}\cdot(15\text{ m}\cdot 3\text{ m})\cdot 50\,\frac{\text{m}}{\text{s}} = 2723\,\frac{\text{kg}}{\text{s}} = 6000\,\frac{\text{lbm}}{\text{s}} \tag{7.BD}$$

$$V_{y_{\text{out,avg}}} = \frac{F_y}{\dot{m}} = \frac{9810\text{ N}}{(2723\text{ kg / s})}\cdot\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} = 3.60\,\frac{\text{m}}{\text{s}} = 11.8\,\frac{\text{ft}}{\text{s}} \tag{7.BE}$$

■

*In the most exact work, we would have to consider the decrease in mass due to the burning of fuel, but that is small enough to neglect here.

This is the approach "from the outside." Without knowing any details of the flow around the various parts of the airplane, we can find the average downward velocity it must give the air that it influences to stay in level flight. If we chose as our system the airplane itself, we would see that it has no flow in or out, excluding the negligible engine intake and exhaust, and therefore the sum of the forces on it must be zero. For the airplane as the system, these forces are the gravity force and the pressure force integrated over its entire surface. To find the latter it is necessary to analyze the flow in detail around the entire airplane. It can be done in the case of some simple structures, such as certain types of wings, by means of B.E.; this type of analysis is introduced in Chaps. 16 and 17. Briefly, the result of the detailed calculation is that the wing is shaped so that the pressure over its top surface is less than that over its bottom surface. From the balance of forces in the x direction we see that, in order to fly, the plane must overcome the air resistance, which is called drag. In constant-velocity, level flight the drag is equal and opposite to the forward force, or thrust, developed by the power plant.

In elementary and high-school science classes students are taught that the wing is curved so that the air flow over the top is faster than that over the bottom, and by B.E. the pressure is lower over the top of the wing, producing lift. It is true that the pressure is lower over the top of the wing, but if this were the correct explanation of how lift occurs, then flat-winged aircraft could not fly. We all have seen that flat-winged paper gliders fly very well. One could build full-sized airplanes with flat wings, and they would fly. However, a flat wing has much more drag than a properly curved one producing an equivalent lift. This was discovered by birds in their evolution and later by the pioneers in aviation. By careful analysis and much experimentation, wings have been built that have ratios of lift to drag as high as 20. The turbine and compressor blades shown in Fig. 7.15 are shaped like the wings of aircraft, because their function is the same, to turn an air flow and in so doing extract or impart work to it, with the lowest possible friction (drag). The design of airplane wings and of turbine or compressor blades uses the same mathematics as also does the design of sailboat sails.

Example 7.17. A light plane is being designed with an overall aircraft lift / drag ratio of 10. The available power plants have thrust / weight ratios of 2. What percentage of the total loaded weight of the aircraft will be power plant? Assume that the plane will be used only in constant-velocity, level flight.

Under these circumstances, lift equals gross weight and drag equals thrust; therefore, lift / drag is weight / thrust; then

$$\frac{\text{Weight}}{\text{Engine weight}} = \frac{\text{weight}}{\text{thrust}} \cdot \frac{\text{thrust}}{\text{engine weight}} = 10 \cdot 2 = 20 \qquad \text{(7.BF)}$$

■

Suppose that we wish to design a helicopter using a power plant with the same thrust / weight ratio. For a helicopter in hovering flight, thrust is vertically upward and equals the weight. Thus, with this engine 50 percent of the gross weight of the helicopter must be engine. This illustrates the fact that horizontal flight, with a wing, is much more efficient than hovering flight. But why is this so? From

Eq. 7.54, we see that the upward force is equal to the velocity change of the air times the mass flow rate of the air. Thus, we may lift a given load by making a small velocity change in a large flow rate of air or a large velocity change in a small flow rate of air.

Now consider the work that must be done to accelerate this quantity of air. We will apply B.E. to the system shown in Fig. 7.24. Here again there is negligible change in pressure in the air passing through the system and negligible change in elevation. Solving for the external work gives

$$\frac{dW}{dm} = -\mathscr{F} - \frac{\Delta V^2}{2} \tag{7.55}$$

This is the negative work which must be done on the air by the airplane's power plant. Here the ΔV^2 is the change in the square of the average value of the velocity, which is given by $V = (V_x^2 + V_y^2)^{1/2}$. The power is

$$\text{Po} = \frac{dW}{dm}\dot{m} = \dot{m}\left(-\mathscr{F} - \frac{\Delta V^2}{2}\right) \tag{7.56}$$

If we neglect the friction term, we see that the power required is proportional to the change in the square of the velocity.

Thus, from the momentum balance, Eq. 7.54, we see that an airplane with a given weight can be lifted by any flow that has the proper combination of $\dot{m}\,\Delta V_y$ but that the power to be supplied by the engine is proportional to $\dot{m}\,\Delta V^2$. So, to lift the maximum weight with the minimum power, one should make $\dot{m}$ as large as possible and thereby make ΔV_y and ΔV^2 as small as possible. This is the same as saying that the wings should be as long and thin as possible. However, long, thin wings are difficult to build and are not very satisfactory for high-speed flight. The fliers most interested in efficiency are soaring birds and human glider fliers; both have settled on the longest, thinnest wings that seem structurally feasible. Commercial aircraft designers have sacrificed some of this efficiency for better high-speed performance and sturdier wings. The first airplane to fly around the world without refueling had even longer and narrower wings than any glider or any bird; this feat became possible only when the development of fiber-reinforced plastics made it possible to build extremely long, thin, lightweight wings.

This same consideration of the energy and momentum effects of an aircraft shows why the helicopter is an inefficient weight-lifting device. In hovering flight a helicopter can move only as much air as it can suck into its blades, so it must give that air a very large velocity change. This leads to a high power requirement. It also explains why helicopters hover as little as possible; as soon as possible they move forward so that the amount of air influenced by their rotors is increased by their forward motion. The same arguments explain why bees and very small birds like hummingbirds can hover by wing beating, but larger birds cannot. Insects and hummingbirds have small values of mass / unit wing area, so they can stay up in inefficient hovering flight. Big birds have higher values of mass / unit wing area, and cannot hover by wing beating [5]. (Soaring birds like eagles seek out rising air currents: They do not hover by beating their wings.)

Finally this same consideration explains why the commercial aircraft industry is replacing simple jet engines with fan-jet or high-bypass engines, which move more air than a simple jet engine, making a smaller change in its velocity, and hence getting better fuel efficiency. Figure 7.15 shows this. About 40 percent of the air passing the first compressors passes through the combustion chambers; the remaining 60 percent passes through the channel around the outside of the engine. These flows mix at the tail, so that the overall air flow is greater and the change in velocity smaller than was seen in the earliest engines of this type, which did not have this "bypass".

7.7 THE ANGULAR-MOMENTUM BALANCE; ROTATING SYSTEMS

In the study of rotating systems it is convenient to define a quantity called the angular momentum of a body:

$$\begin{pmatrix}\text{Angular momentum}\\ \text{of a body, } L\end{pmatrix} = \begin{pmatrix}\text{mass of}\\ \text{a body, } m\end{pmatrix} \cdot \begin{pmatrix}\text{tangential}\\ \text{velocity, } \omega\end{pmatrix}; \quad L = m\omega \tag{7.57}$$

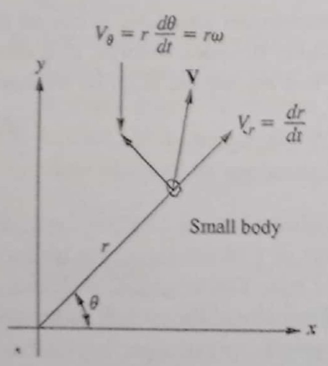


FIGURE 7.25
Velocity components in polar coordinates. Here **V** is the velocity vector, V_θ is the tangential component of the velocity, or simply the tangential velocity, and V_r is the radial component of the velocity, or simply the radial velocity.

The geometric significance of these terms is most easily seen by examining a body in motion and using polar coordinates; see Fig. 7.25.* We see from Eq. 7.57 that the angular momentum of a body depends not only on the body's velocity and mass but also on the point chosen for the origin of the coordinate system. This causes no confusion if we always make clear what choice of origin we make. Since the idea of angular momentum is used most often in rotating systems, generally it is easiest to choose the origin so that it coincides with the axis of rotation.

The r of a large body is not constant over the entire mass, so we must find the angular momentum by integrating over the entire mass:

$$L = \int_{\text{entire mass}} rV_\theta \, dm \tag{7.58}$$

As shown in Fig. 7.25, the tangential velocity V_θ is equal to $r\omega$; therefore, for constant angular velocity over an entire body (i.e., a rotating rigid body), this

*This equation is really one component of a vector equation, which is written $\mathbf{L} = m\mathbf{r} \times \mathbf{V}$. The other components of this vector equation refer to motions of the axis of rotation, which are significant in systems such as gyroscopes but seldom important in fluid mechanics. Therefore, we refer the reader to texts on mechanics for the three-dimensional vector form of this equation.

simplifies to

$$L = \omega \int_{\text{entire mass}} r^2 dm = \omega I \tag{7.59}$$

where I is the angular moment of inertia, $\int_{\text{entire mass}} r^2\, dm$.

It can be readily shown that angular momentum, like linear momentum, obeys the balance equation, but with the difference that in place of force acting we have torque acting, where

$$\text{Torque} = \text{tangential force} \cdot \text{radius}; \qquad \Gamma = F_\theta r \tag{7.60}$$

So the angular momentum balance (for a fixed axis of rotation) becomes

$$dL = (rV_\theta)_{\text{in}}\, dm_{\text{in}} - (rV_\theta)_{\text{out}}\, dm_{\text{out}} + \Gamma\, dt \tag{7.61}$$

Again we can divide by dt to find the rate form:

$$\left(\frac{dL}{dt}\right)_{\text{sys}} = (rV_\theta)_{\text{in}}\, \dot{m}_{\text{in}} - (rV_\theta)_{\text{out}}\, \dot{m}_{\text{out}} + \Gamma \tag{7.62}$$

This equation, often referred to as the *moment-of-momentum equation,* is one of the basic tools in the analysis of rotating fluid machines, turbines, pumps, and other devices [6]. In steady-state flow $(dL/dt)_{\text{sys}}$ is zero and $\dot{m}_{\text{in}}$ equals $\dot{m}_{\text{out}}$; so we have

$$\Gamma = \dot{m}[(rV_\theta)_{\text{out}} - (rV_\theta)_{\text{in}}]_{\text{sys}} \tag{7.63}$$

which is *Euler's turbine equation.*

Example 7.18. A centrifugal water-pump impeller rotates at 1800 rev / min; see Fig. 7.26. The water enters the blades at a radius of 1 in and leaves the blades at a radius of 6 in. The total flow rate is 100 gal / min. The tangential velocities in and out may be assumed equal to the tangential velocity of the rotor at those radii. What is the steady-state torque exerted on the rotor?

$$\dot{m} = 100\,\frac{\text{gal}}{\text{min}} \cdot 8.33\,\frac{\text{lbm}}{\text{gal}} = 833\,\frac{\text{lbm}}{\text{min}} = 378\,\frac{\text{kg}}{\text{min}} \tag{7.BG}$$

$$(V_\theta)_{\text{in}} = r_{\text{in}}\omega, \qquad (V_\theta)_{\text{out}} = r_{\text{out}}\omega \tag{7.BH}$$

From Eq. 7.63, we write

$$\Gamma = \dot{m}\omega(r_{\text{out}}^2 - r_{\text{in}}^2) \tag{7.BI}$$

$$\Gamma = 833\,\frac{\text{lbm}}{\text{min}} \cdot \frac{2\pi \cdot 1800}{\text{min}} \cdot \left[\left(\frac{6}{12}\,\text{ft}\right)^2 - \left(\frac{1}{12}\,\text{ft}\right)^2\right] \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2\ \text{lbm} \cdot \text{ft}} \cdot \frac{\text{min}^2}{3600\ \text{s}^2}$$

$$= 19.8\ \text{ft} \cdot \text{lbf}$$

$$= 26.8\ \text{N} \cdot \text{m} \tag{7.BJ}$$

■

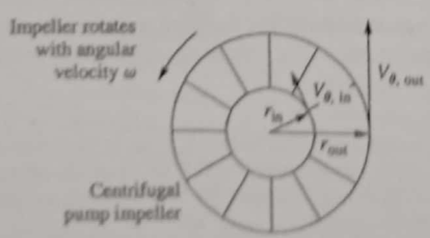


FIGURE 7.26
Centrifugal pump impeller.

This is the net torque acting on the rotor; the algebraic sum of the positive torque exerted by the shaft driving the rotor and the negative torque exerted by friction between the rotor and the surrounding fluid. If we wished to know the total torque applied to the shaft, we would need to know the frictional resistance; that is a much harder problem than this one.

7.8 SUMMARY

1. Momentum is the product of mass and velocity.
2. The momentum balance is simply the restatement of Newton's second law, $F = ma$, in a form that is convenient for fluid-flow problems.
3. The momentum balance is useful in allowing us to solve some fluid-flow problems from the outside, without having to know in detail what goes on inside.
4. The momentum balance, as we show, is applicable to unsteady flows like starting and stopping flows. B.E. is of no use for such flows!
5. The momentum balance is useful for flows in which two streams at different velocities mix and exchange momentum, e.g., the Bunsen burner. B.E. is of no use for such flows!
6. For rotating systems it is convenient to introduce an additional defined quantity called the angular momentum, which obeys a simple balance equation. It is used to analyze rotating systems like pumps and compressors.
7. In Part IV we will apply the momentum balance to three-dimensional flows and show some of its applications.

PROBLEMS

See the Common Units and Values for Problems and Examples, inside the back cover! An asterisk (*) on a problem number indicates that its answer is shown in App. D.

7.1.*The earth has a mass of roughly 10^{25} lbm. A person standing on the earth throws a 1 lbm rock vertically upward, in a direction perpendicular to the earth's motion about the sun, at a velocity of 20 ft / s.

(*a*) How much does the velocity of the earth increase in the direction opposite the throw?

(*b*) When the rock has fallen back to earth, what is the velocity of the earth compared with its velocity before the rock was thrown?

(*c*) When the rock has fallen back to earth, is the earth back on the same orbital path it had before the rock was thrown, or has its orbital path been shifted?

7.2. A 5 lbm gun fires a 0.05 lbm bullet. The bullet leaves the gun at a velocity of 1500 ft / s in the x direction. If the gun is not restrained, what is its velocity just after the bullet leaves? Work this problem two ways:

(*a*) Taking the gun as the system.

(*b*) Taking the combined gun and bullet as the system.

7.3. In Example 7.2, what fractions of the initial kinetic energy of the duck and of the bullet are converted to internal energy?

7.4. In movies and TV thrillers the hero shoots the villain, and the force of the bullet throws the villain into the air, causing his corpse to land several feet from his original position. If we assume that the bullet remains in the villain, what is the relation of the momentum transferred by the gun to the hand of the hero, and the momentum transferred by the bullet to the body of the villain? Are movies and TV a good place to learn one's physics?

7.5.*A fire hose directs a stream of water against a vertical wall. The flow rate of the water is 50 kg / s, and its incoming flow velocity is 80 m / s. The flow away from the impact point has zero velocity in the x direction. What is the force exerted by this stream on the wall?

7.6. Repeat Prob. 7.5, but instead of a fire hose the exhaust from a jet engine flows against the wall. Its velocity is 400 m / s, and its mass flow rate 200 kg / s.

7.7. In Examples 7.3 and 7.4 the analysis was simple because the jet was at right angles to the solid surface. You can observe with a garden hose that such jets, perpendicular to walls or sidewalks, go off radially in all directions, with more or less circular symmetry. You will also observe that there is a region near the jet that is much shallower than the rest, as shown in Fig. 7.23 and described in Sec. 7.5.3. A more interesting and complex problem is the flow of a jet against a flat surface that is not perpendicular to it; you can also observe this flow with a garden hose. The flow is more or less circularly symmetrical, but much more goes away in the direction away from the hose than in the direction toward the hose. But why does any of it flow back in the direction toward the hose?

We can understand this if we replace the three-dimensional problem (circular jet, moving in x, y, and z) directions with a two-dimensional jet (as might issue from a rectangular slot) that is constrained to move only in the x and y directions (by directing it into an open rectangular channel, which prevents flow in the z direction). This flow is sketched in Fig. 7.27. Friction is assumed to be negligible, so from B.E. (ignoring gravity) we see that both streams flowing along the wall must have the same velocity as that in the jet, V_1. In the figure the stream going off to the upper right, (2), is larger than that going to the lower left, (3), in accord with the observation described above. If the flow is frictionless, then there can be no shear stress on the wall, so the resisting force must act normal to the surface as shown. We could attempt to solve for these flows by writing the x and y components of the steady flow momentum balance, but that adds more terms and only makes the analysis harder (try it!).

Instead, we choose a new set of axes for our momentum balance, with one axis, the s direction, parallel to the plate and the other, the r direction, perpendicular to the plate, as sketched on Fig. 7.27. We now apply Eq. 7.17 in the s direction, finding

$$0 = \dot{m}_1 V_1 \cos\theta - \dot{m}_2 V_2 - \dot{m}_3 V_3 + F_s \tag{7.64}$$

From the assumption of frictionless flow, we can see that $F_s = 0$, that the absolute magnitudes of V_1, V_2, and V_3 are the same, but that V_3 is in the minus s direction, so that it is equal to $-V_1$. Making these substitutions, and dividing by V_1, we find

$$0 = \dot{m}_1 \cos\theta - \dot{m}_2 + \dot{m}_3 \tag{7.BK}$$

(*a*) Using the material balance to eliminate $\dot{m}_3$, show the equation for $\dot{m}_2 / \dot{m}_1$.
(*b*) Show the equation for the force exerted on the wall in the r direction.

7.8. In a steady-state methane-air flame at approximately atmospheric pressure the temperature is raised from 68°F to 3200°F. The incoming air-gas mixture and the products of

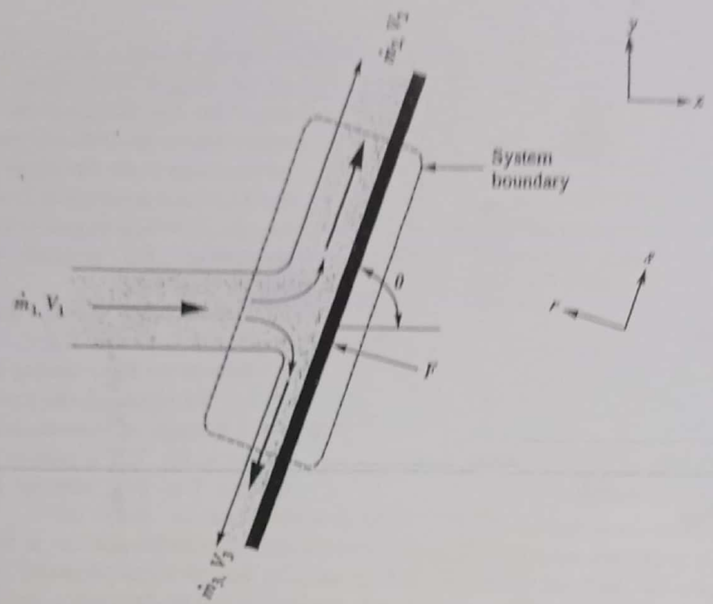


FIGURE 7.27
A jet impinging on a surface not perpendicular to it. We solve in the r-s coordinate system, instead of the x-y system.

combustion may both be considered ideal gases with a molecular weight of ≈28 g / mol. The flame is a thin, flat region perpendicular to the gas flow. If the flow comes into the flame at a velocity of 2 ft / s, what is the pressure difference from one side of the flame to the other? This problem and its consequences are discussed in Lewis and Von Elbe [7].

7.9. A new type of elevator is sketched in Fig. 7.28. The stream of water from a geyser will be regulated to hold the elevator at whatever height is required. If we assume that the maximum flow of the jet is 500 lbm / s at a velocity of 200 ft / s, what is the relation between the weight of the elevator and the maximum height to which the jet can lift it?

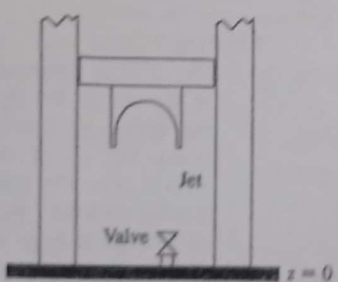


FIGURE 7.28
Hydraulic jet elevator.

7.10.* A sailboat is moving in the y direction. The wind approaches the boat at an angle of 45° to the y direction and is turned by the sails such that it leaves in exactly the minus y direction.

(*a*) If we assume that the average velocity of the incoming and outgoing wind is 10 m / s and that the mass flow rate of air being turned by the boat's sails is 200 kg / s, what are the x and y components of the force exerted by the boat's sails on the air?

(*b*) These are the opposite of the forces exerted by the air on the boat. The y component of the wind force drives the boat in its direction of travel. What does the x component do?

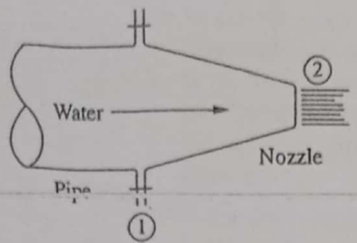
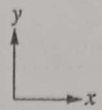


FIGURE 7.29
Nozzle, bolted to pipe.



7.11.*A nozzle is bolted onto a pipe by the flanged joint shown in Fig. 7.29. The flowing fluid is water. The cross-sectional area perpendicular to the flow at point 1 is 12 in^2 and at point 2 is 3 in^2. At point 2 the flow is open to the atmosphere. The pressure at point 1 $\approx$ 40 psig.

(*a*) Estimate the velocity and mass flow rate by B.E.

(*b*) What is the force tending to tear the nozzle off the pipe?

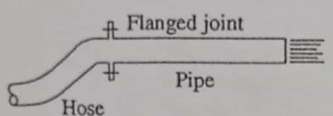
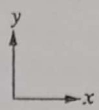


FIGURE 7.30
Pipe used as a sprayer.



7.12. The 5 ft length of 1-in schedule 40 pipe in Fig. 7.30 is used on a sprayer. The flow velocity is 100 ft / s.

(*a*) What is the pressure at the flanged joint, calculated by B.E. and the friction methods in Chap. 6?

(*b*) What is the force tending to tear the flange apart?

(*c*) How is this force transmitted by the fluid to the pipe?

7.13. Repeat Example 7.7 for 3-in schedule 40 pipe with water flowing at 8 ft / s and pressure 30 psig throughout.

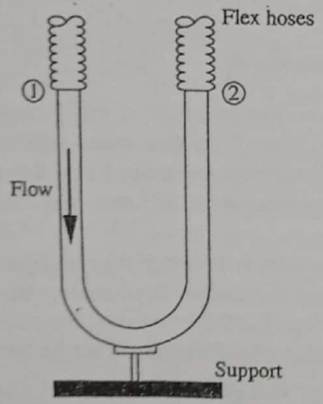
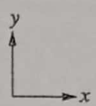


FIGURE 7.31
Vertical pipe U-bend.



7.14. The pipe U-bend in Fig. 7.31 is connected to a flow system by flexible hoses that transmit no force. The pipe has an ID of 3 in. Water is flowing through the pipe at a rate of 600 gal / min. The pressure at point 1 is 5 psig and at point 2 is 3 psig. What is the vertical component of the force in the support? Neglect the weight of the pipe and fluid.

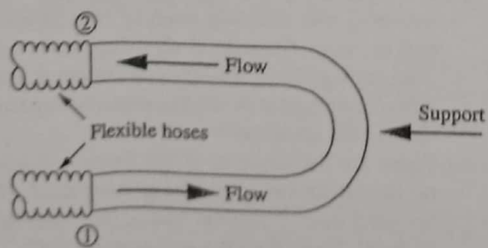


FIGURE 7.32
Horizontal pipe U-bend.

7.15.*The U-bend shown in Fig. 7.32 is connected to the rest of the piping system by flexible hoses. The ID of the pipe is 3 in. The fluid flowing is water, with an average velocity of 50 ft / s. The gauge pressure at point 1 is 30 psig and at point 2 is 20 psig. What is the horizontal component of the force in the support?

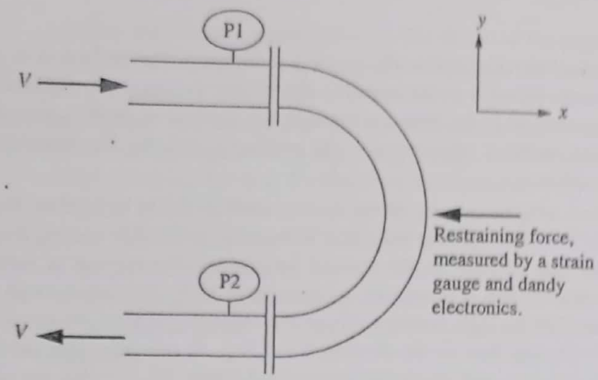


FIGURE 7.33
U-bend flow meter.

7.16. A new type of flow meter is sketched in Fig. 7.33. In it we read the two pressure gauges and the force on the pipe bend (using strain gauges and dandy electronics). From those three readings we compute the fluid velocity in the pipe. The cross-sectional area of the pipe and the couplings and of the bend is 1.000 in^2. The flowing fluid is water. P1 reads 20 psig, P2 reads 18 psig. The restraining force, measured by the strain gauge and the dandy electronics, is 45 lbf, acting in the minus x direction. The couplings between the straight sections of pipe and the bend are of a magical variety that transmit no forces and allow no leakage. For this problem, the acceleration of gravity is zero (we are in a space capsule). What is the fluid velocity in the pipe?

7.17.*A pump, together with the electric motor that drives it, is mounted on a wheeled cart that can be rolled about to various places in our plant for various pumping tasks. It is connected to the vessels to be pumped by flexible hoses that transmit no forces and is connected electrically by a flexible cord that transmits no forces. The pump inlet and outlet are parallel both to each other and to the x axis. The inlet pipe diameter is 4.00 in and the outlet pipe diameter is 3.00 in. The flow rate through the pump is 230 gal / min of water. The pressures at the inlet and outlet are 10 psig and 50 psig, respectively.

(*a*) How much force must we exert on the pump-motor-cart assembly to keep it from moving?

(*b*) In which direction must we exert this force?

7.18. A large rocket engine ejects 200 kg / s of exhaust gases at a velocity of 4000 m / s. The pressure of the exhaust gas is equal to the atmospheric pressure. What thrust does the engine produce?

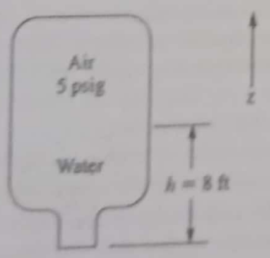


FIGURE 7.34
Compressed-air rocket.

7.19.*Calculate I_{sp} for the rocket in Example 7.8. Why is this different from the result in Example 7.9?

7.20. The compressed-air-driven water rocket shown in Fig. 7.34 is ejecting water vertically downward through a frictionless nozzle. The exit area of the nozzle is 1 in^2. When the pressure and elevation are as shown, how much thrust does the rocket produce?

7.21. A spherical toy balloon has diameter (when inflated) of 6 in and an internal pressure of 1 psig. The neck of the balloon has a diameter of 0.5 in. The skin of the balloon has a mass of 0.0075 lbm. At time zero we release the neck of the balloon, allowing the air to escape in the positive x direction. The balloon is at rest at time zero and then moves away in the negative x direction. Estimate the acceleration of the balloon at time zero, when the flow out the neck has reached its steady-state value but the balloon has not started to move yet. For this problem ignore any air resistance to the movement of the balloon.

7.22.* A typical high-pressure oxygen cylinder of the type commonly found in welding shops and laboratories falls over. The valve at the top is broken off in the fall, making a hole with a cross-sectional area of 1 in^2. At the time of the accident the cylinder is full, so the internal pressure is 2000 psia. The internal temperature is 70°F. The flow through the nozzle cannot be described by B.E., because it is a high-velocity gas flow. By using the methods of high-velocity gas flow (to be developed in Chap. 8) one may estimate that the outlet velocity is 975 ft / s, that the outlet density is 7.0 lbm / ft^3, and that the pressure in the plane of the outlet is 1060 psia.

(*a*) How much thrust does the oxygen cylinder exert?

(*b*) Is it a worthwhile safety practice to fasten these cylinders so they cannot fall over?

(*c*) The tops of high-pressure gas cylinders are protected either by collars (propane cylinders) or screwed-on thick-walled caps (gases like oxygen or hydrogen). Explain this practice.

7.23. The following is an incorrect solution to the preceding problem. Where is its error?

Incorrect solution. The pressure everywhere in the container is 2000 psig, except over the area of the outlet. So the pressure forces cancel, except for a section of 1 in^2, which has 2000 psig pointing away from the nozzle and 1060 psig pointing the other way at the nozzle. Thus, the net force is (2000 − 1060) psig · 1 in^2 = 940 lbf.

7.24. A typical garden hose has an inside diameter of $\frac{3}{4}$ in. The water stream flowing from it has a velocity of 10 ft / s.

(*a*) If such a hose is left loose, will the end move about?

(*b*) Would it move about if it were perfectly straight, or must it be curved?

(*c*) What is the maximum plausible value of the force involved in any such motion?

7.25. The 3-ft-diameter, horizontal main water cooling line from a nuclear reactor breaks. The pressure inside the reactor is 1000 psia, and the water surface inside the reactor is 20 ft above the broken line. The exiting fluid (a steam-water mixture) has a density of 50 lbm / ft^3. Estimate the horizontal force on the pipe-reactor system due to the flow through the broken pipe. Assume frictionless flow and B.E.

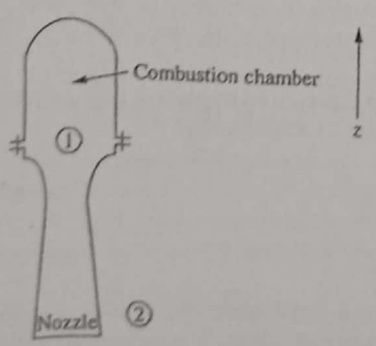


FIGURE 7.35
Solid-fueled rocket.

7.26.* The rocket motor sketched in Fig. 7.35 has the nozzle bolted to the combustion chamber, which contains the fuel. The flow rate is 300 lbm / s. At section 1 the cross-sectional area = 5 ft^2, the pressure is 300 psia, and the velocity is 300 ft / s. At section 2 the corresponding values are 1.5 ft^2, 40 psia, and 4600 ft / s. Estimate the following:

(*a*) The thrust of the rocket motor.

(*b*) The force (compressive or tensile) at the joint between the nozzle and combustion chamber.

7.27. In Example 7.10 we took the system boundary far enough away from the engine that the velocity there was negligible. If we take our system boundaries right at the inlet and outlet of the engine, then the outlet velocity will be unchanged, but the inlet velocity will become about 500 ft / s. The thrust of the engine is the same, independent of what system we choose (the engine does not care what we have in mind). Repeat Example 7.10, making this change in system. *Hint:* Consider the inlet pressure.

7.28. In Example 7.10 we ignored the mass flow of the fuel, which we asserted was negligible. Actual jet engines have $\dot{m}_{fuel} / \dot{m}_{air} \approx 0.02$. For the same air flow rate and inlet and outlet velocities, by what fraction does the computed thrust of the engine increase if we take this additional flow into account? Assume that the fuel crosses our system boundary in the y direction, so that its inlet x velocity $= 0$.

7.29. If the sudden expansion in Fig. 7.12 were replaced with a gradually outward-tapering transition, then the friction losses would be very small, often practically zero. Show how the momentum balance for that flow differs from the momentum balance for the sudden expansion.

7.30. Check several values of Fig. 6.16 to see if the sudden expansion curve was actually made up from Eq. 7.28.

7.31. (*a*) Set up the spreadsheet solution to Example 7.11, and show that the numerical solution does converge to the values shown in Table 7.2.

(*b*) Using the spreadsheet program, rerun the solution to Example 7.11 on the assumption that we should have used an orifice coefficient of 0.6 (see Sec. 5.8) in the B.E. for the gas flow. How does this change the calculated values of P_2, V_3, and $\dot{m}_{air} / \dot{m}_{gas}$?

7.32. (*a*) Using the spreadsheet prepared in the previous problem, repeat Example 7.11 for propane as a fuel. In the United States propane is distributed inside houses at 11 in of water, compared to the 4 in of water for natural gas. What are the calculated values of P_2, V_3, and $\dot{m}_{air} / \dot{m}_{gas}$?

(*b*) The values found in part (*a*) show that simple connection of a natural gas appliance to a normal propane supply line will produce a flame much larger and smokier than the appliance was designed to produce with natural gas. To solve the fuel-conversion problem, appliance manufacturers supply conversion kits. To convert a natural gas appliance to propane, the conversion kit normally replaces the fuel jets with ones with ≈45 percent as large a cross-sectional area as the jets for natural gas. Using the spreadsheet program used in the previous problem, rerun part (*a*) of this problem with a gas orifice area 45 percent as large as that in Example 7.12. What are the calculated values of P_2, V_3, and $\dot{m}_{air} / \dot{m}_{gas}$?

7.33. In our treatment of the Bunsen burner in Example 7.11,

(*a*) Calculate the Reynolds number of the gas jet, the incoming airflow, and the mixed flow (simplify by using the kinematic viscosity of air for all three flows).

(*b*) Which of these flows are laminar? Which are turbulent?

(*c*) Sketch the velocity distribution in such a flow. Here an intuitive sketch will do.

(*d*) In applying Eq. 7.29 we asserted that the rightmost term, which involves the shear stress at the wall was negligible. Check that assumption as follows; First, show that if we assume laminar flow we can substitute

$$\tau = f\rho\frac{V^2}{2} = \frac{16}{\mathcal{R}}\rho\frac{V^2}{2} = \frac{16\mu}{DV\rho}\rho\frac{V^2}{2} = \frac{8\mu V}{D} \qquad (7.BL)$$

which makes the rightmost term $8\pi\mu V_3 \Delta x$. Second, evaluate the magnitude of this term for the values in Example 7.11. Your answer should have the dimension of lbf. Third, evaluate the magnitude of the $\dot{m}_{gas}(V_3 - V_{gas,2})$ term for the same example, also in lbf. Show the ratio of this "momentum of the gas jet" term to the wall shear term.

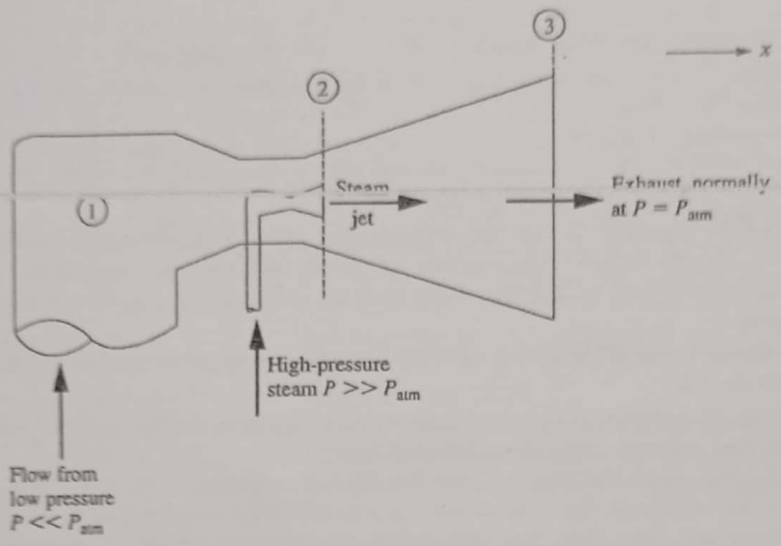


FIGURE 7.36
Steam-jet ejector or vacuum pump.

7.34. Figure 7.36 is a sketch of a *steam-jet ejector* of a type widely used to produce vacuums in process equipment. Conceptually, it is the same as the Bunsen burner in Fig. 7.14. A faster-moving central flow, produced in this case by high-pressure steam, exchanges momentum with a slower-moving surrounding flow, in this case the gas being removed at a vacuum; and the mixed stream discharges at a pressure intermediate between that of the incoming driver fluid and the lower-pressure driven fluid, in this case at atmospheric pressure. If we write the momentum balance (analogous to Eq. 7.29) for the system consisting of the inside of the device from 2 to 3, what term will appear in the momentum balance that does not appear in Eq. 7.29?

7.35. When a rocket is moving in the positive x direction with velocity V_1 and this velocity is equal and opposite to the velocity of the exhaust gas relative to the rocket, then the exhaust velocity relative to fixed surroundings is zero. Thus, according to Eq. 7.37, $d(mV)/dt = 0$. Does this mean that the rocket is not accelerating? Explain.

7.36. Show the equivalent of Eq. 7.38 for vertical flight with a constant value of the acceleration of gravity and zero air resistance.

7.37. The rocket in Example 7.12 is now fired vertically. The specific impulse and mass ratio are the same as in that example. The rocket consumes all its fuel in 1 min. Calculate its velocity at burnout, taking gravity into account.

7.38.*A rocket starts from rest on the ground and fires vertically upward. During the entire upward firing the velocity of the exhaust gas, measured relative to the rocket, is 4000 m/s. The pressure in the exit plane of the rocket nozzle is always exactly equal to the surrounding atmospheric pressure. The mass of the rocket and fuel before launching is 100,000 kg. The mass of the burned-out rocket is 20,000 kg. The entire burning process takes 50 s. What is the velocity of the rocket at burnout? Ignore air resistance.

7.39. A cylindrical tank, shown in Fig. 7.37, is sitting on a platform that in turn rests on absolutely frictionless wheels on a horizontal plane. There is no air resistance. At time zero the level in the tank is 10 ft above the outlet, and the whole system is not moving.

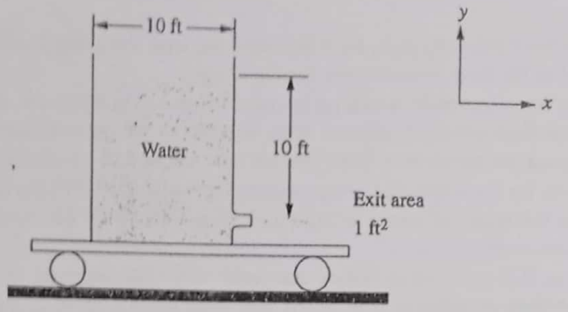


FIGURE 7.37
Draining tank on frictionless cart.

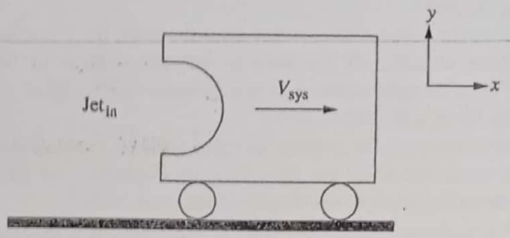


FIGURE 7.38
Starting a cart with a fire hose jet.

Then the outlet is opened and the system allowed to accelerate to the left. The flow through the outlet nozzle is frictionless. What is the final velocity, assuming that

(*a*) The mass of the tank and cart is zero?

(*b*) The mass of the tank and cart is 3000 lbm?

7.40. The cart in Fig. 7.38 has a mass of 2000 kg. It is resting on frictionless wheels on a solid, level surface and encounters no air resistance. At time zero it is standing still, and a jet from a fire hose is used to start it moving. The mass flow rate of the fluid from the fire hose is 100 kg / s, and its velocity relative to fixed coordinates is 50 m / s. The cup on the rear of the cart turns the jet around so that it leaves in the minus x direction with the same velocity relative to the cart with which it entered. Calculate the velocity-time behavior of the cart; assume the jet is unaffected by gravity. (This is not a very practical problem, but it is analogous to the more complex and interesting problem of starting a large turbine from rest. All such turbines must be occasionally shut down for maintenance; their starting and stopping behavior is more complex than their behavior running at a steady speed.)

7.41. Repeat Prob. 7.40 with the following change. Instead of the cart turning the jet around by 180° so that it flows out in the minus x direction, the cart only turns the jet by 90°, so that it flows out to the side at a right angle to the x axis. How long does it take the cart to reach a velocity of 40 m / s?

7.42. In Fig. 7.17 we show that the maximum efficiency for the simple blade-jet interaction in Fig. 7.16 occurs when the blade speed is exactly one-half of the jet speed. One can also show this by rewriting Eq. 7.42 as

$$\frac{dW}{dm} = 2(V_{\text{jet}} - V_{\text{blade}})V_{\text{blade}} \tag{7.BM}$$

Differentiate both sides of this equation with respect to V_{blade} and set the derivative equal to zero. Show that doing so leads to the same conclusion.

7.43. In Example 7.13 the pipe was long enough that we ignored the kinetic energy in the fluid leaving the downstream end of the pipe and the entrance loss.

(*a*) Rework the example, taking that kinetic energy and entrance loss into account, and show that the change is negligible.

(*b*) Rework the example for a 1-ft-long pipe, for which case the term involving f will be negligible compared to the term involving the kinetic energy.

7.44. If a fluid were absolutely incompressible, which no materials known to humans are, then the speed of sound in that fluid would be infinite. What happens to the pressure rise in Example 7.14 as we replace the water with fluids that are less and less compressible?

7.45.*Repeat Example 7.14, with the flowing liquid being propane, for which (at 70°F) the density is 31.1 lbm/ft^3 and the speed of sound is 2150 ft/s. The velocity of the flowing fluid is the same as in that example.

7.46. Example 7.14 makes clear that rapid valve closure can cause very high pressures at the valve. That raises the obvious question of how slowly one must close a valve to avoid water hammer. The approximate answer is that if the time to close is longer than the time for a sound wave to make a round trip from the valve to the reservoir, then no significant water hammer will occur. Again, see Parmakian [4] for more details.

(*a*) In Example 7.14, how long is this?

(*b*) Why is the time required that for a round trip, rather than the time for a one-way trip? *Hint:* For instantaneous closure, all the fluid is brought to rest in time ($t = L/c$). When it has all been brought to rest it is at a pressure much higher than the pressure in the reservoir. What will happen then?

7.47. Problem 7.46 shows that the time required for a valve to close without causing water hammer is linearly proportional to the length of the pipe, whereas Example 7.14 shows that the expected pressure rise is independent of the length of the pipe. Why?

7.48.*Water is flowing at a depth of 2 ft and a velocity of 50 ft/s. It undergoes a hydraulic jump. What are the depth and velocity after the jump?

7.49. Water is flowing at a depth of z_1. What is the minimum velocity at which this water could undergo a hydraulic jump? Why?

7.50.*Water is flowing steadily in a river 20 ft deep. If an obstruction is placed in the river, increasing the depth at the obstruction, what is the lowest river velocity at which the obstruction will cause a hydraulic jump to occur? If the velocity is less than this, what will happen?

7.51. In Eq. 7.AZ, we use gauge pressure, leaving out the atmospheric pressure force terms. Is this permissible? The areas on which the fluids exert pressure are not the same. Explain why that all works out satisfactorily.

7.52. Show that the friction heating in a hydraulic jump is given by

$$\mathcal{F} = \frac{g(z_2 - z_1)^3}{4z_1z_2} \tag{7.65}$$

7.53. Water is flowing in a horizontal gutter with velocity 10 ft/s and depth 0.1 ft. We now place a large brick in the gutter, which stops the flow. The flow is stopped by a hydraulic jump, which then moves upstream from the brick. The brick is large enough that there is no flow over it. The fluid between the brick and the jump has zero velocity. The jump moves upstream, with a velocity V_j. What is the numerical value of V_j?

This is a messy problem analytically. It is fairly easy on a spreadsheet if one takes the viewpoint of someone riding on the jump (the lagrangian viewpoint) and solves by trial and error for the jump velocity that satisfies the hydraulic jump equation in the moving frame of reference.

7.54.*Water flows through a hydraulic jump, entering at a velocity of 50 ft/s and a depth of 10 ft. How much does the temperature of the water increase in this jump? For water, $C_V = du/dt = 1.0$ Btu/lbm · °F.

7.55. Figure 7.23 shows a hydraulic jump, in radial geometry, which is easily demonstrated in a kitchen sink. If the sink drain is open, then the flow is steady and the depths and velocities