TRIPLE BOND TEAM

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[المكتبة التخصصية]

مهندسون على الطريق

الاتجاه الإسلامي - البوليتكنك

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TRIPLE BOND TEAM

Fluid Mechanics for Chemical Engineers, Third Edition
Noel de Nevers
Solutions Manual

This manual contains solutions to all the problems in the text.

Many of those are discussion problems; I have tried to present enough guidance so that the instructor can lead a useful discussion of those problems.

In addition I have added discussion material to many of the computation problems. I regularly assign these as computation, and then after we have agreed that the computation is correct, asked the students what this computation tells them. That leads to discussion. Wherever I can, I begin a discussion of some topic with a computation problem which introduces the students to the magnitudes of various quantities, and thus requires them to read the part of the text covering that topic. Once we all know the magnitudes, and have all read that section of the text, we can have an interesting discussion of their meaning.

In this additional discussion I have presented reference when I could. Often I relied on industry "common knowledge", folklore and gossip. I hope I got it all right. If not, I apologize for leading you astray. Where I am not sure about the folklore, I have tried to make that clear in the discussion.

Those problems whose numbers are followed by an asterisk, * are ones whose answer is presented in the Answers to Selected Problems in Appendix D of the book.

Many of these problems have been class tested. Some, alas, have not. This manual, as well as the book, is certain to contain errors. I will be grateful to those who point these errors out to me, so that they can be corrected. I keep a running correction sheet, and send copies to anyone who asks for it. If you find such an error, please notify me at

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Many of the problems go beyond what is in the text, or show derivations which I have left out of the text to make it read easier. I suggest that the instructor tell the students to at least read all the problems, so that they will know what is contained there.

Some of the problems use spreadsheets. In the individual chapters I have copied the spreadsheet solutions into the text in table format. That is easy to see, but does not let the reader modify the spreadsheets. In the Folder labeled "Spreadsheets" I have included copies of all the spreadsheets shown in the individual chapters, and also the high velocity gas tables from the appendices. These are in Excel 4.0, which is compatible with all later versions.

Noel de Nevers
Salt Lake City, Utah, 2003

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Solutions Manual

Chapter 1 An * on a problem number means that the answer is given in Appendix D of the book.

1.1 Laws Used, Newton's laws of motion, conservation of mass, first and second laws of thermodynamics. Laws Not Used, third law of thermodynamics, all electrostatic and magnetic laws, all laws discussing the behavior of matter at the atomic or subatomic level, all relativistic laws.

1.2 By ideal gas law, for uranium hexafluoride

$$\rho = \frac{PM}{RT} = \frac{(1\ \text{atm})\cdot\left(352\dfrac{\text{g}}{\text{mol}}\right)}{\left(0.082\dfrac{\text{L atm}}{\text{mol K}}\right)\cdot(56.2+273.15\ \text{K})}\cdot\frac{\text{L}}{10^6\ \text{cm}^3} = 0.0130\frac{\text{g}}{\text{cm}^3} = 0.81\frac{\text{lbm}}{\text{ft}^3}$$

Here the high density results from the high molecular weight.

At its normal boiling point, 4 K, by ideal gas law helium has

$$\rho = \frac{PM}{RT} = \frac{1\cdot 4}{0.082\cdot 4} = 0.012\frac{\text{g}}{\text{cm}^3} = 0.76\frac{\text{lbm}}{\text{ft}^3}$$

Here the high density results from the very low absolute temperature. The densities of other liquids with low values are: liquid methane at its nbp, 0.42 gm/cm^3, acetylene at its nbp, 0.62, ethylene at its nbp, 0.57.

Discussion; the point of this problem is for the students to recognize that one of the principal differences between liquids and gases is the large difference in density. As a rule of thumb, the density of liquids is 1000 times that of gases.

1.3*

$$\rho = \sum \text{mass} \Big/ \sum \text{volume:} \quad \text{For 100 lbm}$$

$$\rho = 100\ \text{lbm} \Big/ \left(\frac{50\ \text{lbm}}{4.49\cdot 62.3\ \text{lbm/ft}^3} + \frac{50\ \text{lbm}}{62.3\ \text{lbm/ft}^3}\right) = 102\frac{\text{lbm}}{\text{ft}^3}$$

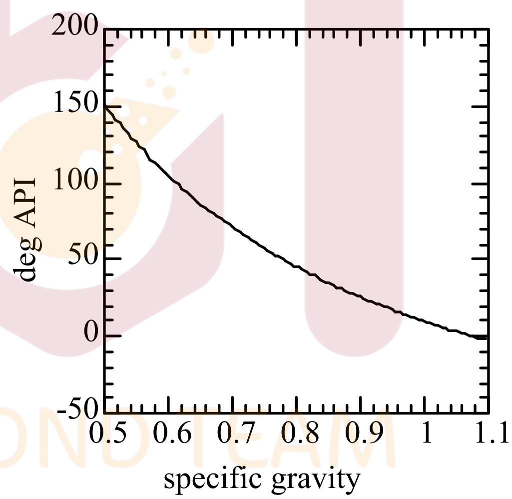
Discussion; this assumes no volume change on mixing. That is a good assumption here, and in many other cases. In a few, like ethanol and water have changes of up to a few %.

1.4 The maximum density of water occurs at 4°C, not at zero. The relation between the meter and the kg was defined to have the density of water at 4°C be 1.00 gm/cm^3. However for various historical reasons it has ended up that the density of water at 4°C is about 0.99995 gm/cm^3.

1.5* $$\rho = \frac{m_{\text{gross}} - m_{\text{tare}}}{V} = \frac{45\text{ g} - 17.24\text{ g}}{25\text{ cm}^3} = 1.110\frac{\text{g}}{\text{cm}^3}$$
$$\rho = \frac{m_{\text{gross}} - (m_{\text{tare}} + m_{\text{air}})}{V} = \frac{45\text{ g} - (17.24 + 0.03)\text{ g}}{25\text{ cm}^3} = 1.109\frac{\text{g}}{\text{cm}^3}$$

Omitting the weight of the air makes a difference of 0.001 = 0.1%. This is normally ignored, but in the most careful work it must be considered.

1.6 This scale has the advantage that it places a higher number on lower density oils. That matches the price structure for oil, where lower density crude oils have a higher selling price, because they are more easily converted to high-priced products (e.g. gasoline). Oil prices will often be quoted as (A + B·deg API) $/bbl. where B ≈ $0.01/deg API.



The plot covers the whole range of petroleum liquids, from propane (s.g. ≈ 0.5) to asphalts (s.g. ≈ 1.1). Water (s.g. = 1) has 10° API.

1.7 For ideal gases, $\begin{pmatrix}\text{specific}\\ \text{gravity}\end{pmatrix}_{\text{ideal gas}} = \frac{M_{\text{gas}}}{M_{\text{air}}}$. For methane and propane the values are $16/29 = 0.55$ and $44/29 = 1.51$. Propane is by far the most dangerous fuel in common use. If we have a methane leak, buoyancy will take it up and disperse it. If we have a leak of any liquid fuel, it will flow downhill on the ground, and be stopped by any ditch or other low spot. Propane, as a gas heavier than air, flows downhill, over small obstructions and depressions. It often finds an ignition source, which methane or gasoline would not find in the same situation.

1.8 $\frac{dV}{dy}\begin{bmatrix}\text{has dimension}\\ \text{of}\end{bmatrix}\frac{\text{ft/s}}{\text{ft}}=\frac{1}{\text{s}}$;

$$\tau\begin{bmatrix}\text{has dimension}\\ \text{of}\end{bmatrix}\frac{\text{lbf}}{\text{ft}^2}\cdot\frac{\text{lbf ft}}{\text{lbm sec}^2}[=]\frac{\text{lbm ft/sec}}{\text{ft}^2\text{ sec}}[=]\frac{\text{momentum}}{\text{area}\cdot\text{time}}=\text{momentum flux}$$

$$\mu\begin{bmatrix}\text{has dimension}\\ \text{of}\end{bmatrix}\frac{\tau}{dV/dy}[=]\frac{\text{lbf / ft}^2}{1/\text{s}}[=]\frac{\text{lbm}}{\text{ft}\cdot\text{s}}$$

1.9 Paints; not settle in the can. Inks; spread when you are writing, don't leak when you are not. Lipstick; spread when applied, then not move. Crayons; same as lipstick. Blood; corpuscles don't settle, viscous resistance is small in large arteries and veins. Chocolate; easy to spray on warm or apply by dipping, then does not run off until it cools. Oil well drilling fluids; low viscosity when pumped up and down the well, high viscosity when leaking out of the well into porous formations. Radiator sealants; low viscosity in pumped coolant, high viscosity in potential leaks. Aircraft de-icing fluids; stick to the plane at low speeds, flow off at high speeds. Plaster, spread easily, not run once applied while it is setting. Margarine, spread easily, but remain solid when not being spread.

1.10 (a) For $r = R$, $\quad V_\theta=\omega\left(\frac{k^2}{1-k^2}\right)\cdot\left(\frac{R^2}{R}-R\right)=0$

For $r = r_{\text{inner cylinder}} = kR$

$$V_\theta=\omega\left(\frac{k^2}{1-k^2}\right)\cdot\left(\frac{R^2}{kR}-kR\right)=\omega\left(\frac{k^2}{1-k^2}\right)\cdot R\frac{1-k^2}{k}=\omega kR$$

(b)

$$\frac{d}{dr}\left(\frac{V_\theta}{r}\right)=\omega\left(\frac{k^2}{1-k^2}\right)\cdot\frac{d}{dr}\left(\frac{R^2}{r^2}-1\right)=\omega\left(\frac{k^2}{1-k^2}\right)\cdot\left(\frac{-2R^2}{r^3}\right)\text{ and}$$

$$\sigma=r\frac{d}{dr}\left(\frac{V_\theta}{r}\right)=\omega\left(\frac{k^2}{1-k^2}\right)\cdot\left(\frac{-2R^2}{r^2}\right)$$

Here we can ignore the minus sign, because as discussed in the text its presence or absence is arbitrary. Then, at the surface of the inner cylinder $r = r_{\text{inner cylinder}} = kR$ and

$$\sigma_{\text{inner cylinder}}=\omega\left(\frac{k^2}{1-k^2}\right)\cdot\left(\frac{2R^2}{(kR)^2}\right)=\omega\left(\frac{2}{1-k^2}\right)$$

(c) Here $k=\frac{r_{\text{inner}}}{R}=\frac{D_{\text{inner}}}{D_{\text{outer}}}=\frac{25.15\text{ mm}}{27.62\text{ mm}}=0.9106$ so that

$$\sigma_{\text{inner cylinder}}=\frac{10\cdot2\pi}{\text{min}}\cdot\frac{\text{min}}{60\text{ s}}\left(\frac{2}{1-0.9106^2}\right)=\frac{12.26}{\text{s}}$$

1.11 Sphere, $\frac{S}{V} = \frac{\pi D^2}{\frac{\pi}{6} D^3} = \frac{6}{D}$**,** Cube with edge *E*, $\frac{S}{V} = \frac{6E^2}{E^3} = \frac{6}{E}$ Right cylinder

$$\frac{S}{V} = \frac{2\frac{\pi}{4} D^2 + \pi D^2}{\frac{\pi}{4} D^3} = \frac{6}{D}$$

This appears strange, but is correct. If we ask what is the surface area for each of these figures for a volume of 1 cm^3, we find that for the sphere $D = (6\ cm^3/\pi)^{1/3} = 1.24$ cm, for the cube $E = 1$ cm and for the right cylinder $D = (4\ cm^3/\pi)^{1/3} = 1.08$ cm. Thus for equal volumes the sphere has the least surface, the right cylinder the next least, and the cube the most.

1.12 Crystallization of supersaturated solutions, boiling in the absence of a boiling chip, a pencil balanced on its eraser, all explosives, a mixture of hydrogen and oxygen (in the absence of a spark), a balloon full of air (in the absence of a pin), a charged capacitor.

1.13

$$V = \frac{\pi}{6} D^3 = \frac{\pi}{6}\left(8000\,\text{mi}\cdot\frac{5280\,\text{ft}}{\text{mi}}\right)^3 = 3.95\cdot 10^{23}\ \text{ft}^3 = 1.12\cdot 10^{21}\text{m}^3$$

$$m = \rho V = \left(5.5\cdot 62.4\frac{\text{lbm}}{\text{ft}^3}\right)\cdot\left(3.95\cdot 10^{22}\ \text{ft}^3\right) = 1.35\cdot 10^{25}\text{lbm} = 6.14\cdot 10^{24}\text{kg}$$

There is no meaning to the term "the weight of the earth" because weight only has meaning in terms of a well-defined acceleration of gravity. For the earth there is no such well-defined acceleration of gravity.

1.14 (a) 62.3 lbm/ft^3 , **(b)** $\left(62.3\frac{\text{lbm}}{\text{ft}^3}\right)\left(\frac{6\ \text{ft}}{\text{s}^2}\right)\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}} = 11.6\,\text{lbf}$ **, (c)** same as (a).

1.15*

$$\left(\text{mile}\cdot\frac{5280\,\text{ft}}{\text{mi}}\right)^3\cdot\frac{1728}{231}\frac{\text{gal}}{\text{ft}^3} = 1.1\cdot 10^{12}\text{gal},\quad 30\cdot 10^9\,\text{gal}\cdot\frac{\text{mi}^3}{1.1\cdot 10^{12}\text{gal}} = 1.14\text{mi}^3$$

The value in cubic miles is surprisingly small. We use about 5 E 9 bbl/yr, of which we import about half. This suggests that at the current usage rate we have $(30/(0.5\cdot 5) \approx 12$ years of oil reserves. For the past half century this number has remained about constant, we have found it at about the rate we used it. In the past few years this has mostly been by improved recovery methods for existing fields, rather than finding new fields. In the 1950's the US could produce about twice as much oil as it consumed; now we can produce about half. This is mostly not due to a decline in production, but an increase in consumption.

1.16 $\text{coulomb} = \dfrac{\text{gm equivalent}}{96,500} \cdot \dfrac{6.02 \cdot 10^{23}\,\text{electrons}}{\text{gm equivalent}} = 6.2 \cdot 10^{8}\,\text{electrons}$

1.17 $J = 1 = \dfrac{778\ \text{ft} \cdot \text{lbf}}{\text{Btu}} = \dfrac{4.184\ \text{J}}{\text{cal}} = \dfrac{4.184\ \text{kJ}}{\text{kcal}}$ This J is easily confused with the J for a joule. See any thermodynamics text written before about 1960 for the use of this J to remind us to convert from ft·lbf to Btu.

1.18* $g_c = 1 = \dfrac{32.2\ \text{lbm ft}}{\text{lbf s}^2} = 1\dfrac{\text{lbm toof}}{\text{lbf s}^2};\text{toof} = 32.2\ \text{ft}$ or

$$g_c = 1 = \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} = 1\frac{\text{lbm ft}}{\text{lbf dnoces}^2}; \quad \text{dnoces} = \frac{\text{s}}{\sqrt{32.2}}$$

The toof and the dnoces are the foot and the second spelled backwards. No one has seriously proposed this, but it makes as much sense as the slug and the poundal.

1.19 $m_{\text{acre ft}} = \rho V = \left(62.3\dfrac{\text{lbm}}{\text{ft}^3}\right)(\text{ft})\left(\dfrac{5280^2\ \text{ft}^2}{\text{acre}}\right) = 2.71 \cdot 10^{6}\ \text{lbm}$

$$m_{\text{hectare meter}} = \rho V = \left(998.2\frac{\text{kg}}{\text{m}^3}\right)(\text{m})(100\text{m})^2 = 9.98 \cdot 10^{6}\,\text{kg}$$

The acre-ft is the preferred volume measure for irrigation because one knows, roughly, how many ft/yr of water one must put on a field, in a given climate to produce a given crop. Multiplying that by the acres to be irrigated gives the water demand. The Colorado River, over which arid Southwestern states have been fighting for a century flows about 20 E6 acre ft/yr. The Columbia flows about 200 E 6 acre-ft/yr. The south western states look to the Columbia with envy; the Northwestern states are in no hurry to give it up.

1.20 $c^2 = \left(186,000\dfrac{\text{mi}}{\text{s}}\right)^2\left(5280\dfrac{\text{ft}}{\text{mi}}\right)^2 \cdot \dfrac{\text{lbf s}^2}{32.2\ \text{lbm ft}} \cdot \dfrac{\text{BTU}}{778\ \text{ft lbf}} = 3.85 \cdot 10^{13}\ \dfrac{\text{BTU}}{\text{lbm}}$

$$c^2 = \left(2.998 \cdot 10^{8}\ \frac{\text{m}}{\text{s}}\right)^2 \cdot \frac{\text{N s}^2}{\text{kg} \cdot \text{m}} \cdot \frac{\text{J}}{\text{N} \cdot \text{m}} = 8.99 \cdot 10^{16}\ \frac{\text{J}}{\text{kg}}$$

1.21* 300 seconds is not the same as 300 lbf·s/lbm. So this terminology is wrong. But it is in very common usage.

$$I_{sp} = 300\frac{\text{lbf}\cdot\text{s}}{\text{lbm}}\cdot\frac{32.2\,\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 9660\frac{\text{ft}}{\text{s}}$$

As discussed in Ch. 7, this is the exhaust velocity if the exhaust pressure matches the atmospheric pressure. If they are different (the common case) then this value must be modified. However European rocket engineers use the "effective exhaust velocity" which takes the pressure into account the same way US engineers use the specific impulse.

1.22

$$\frac{q}{A} = \frac{1\text{cal}}{\text{cm}^2\text{s}}\cdot\frac{\text{Btu}}{252\,\text{cal}}\cdot\left(\frac{30.5\text{cm}}{\text{ft}}\right)^2\cdot\frac{3600\,\text{s}}{\text{hr}} = 1.33\cdot 10^4\frac{\text{Btu}}{\text{hr ft}^2}$$

$$\frac{q}{A} = \frac{1\text{J}}{\text{m}^2\text{s}}\cdot\frac{\text{Btu}}{1.055\,\text{J}}\cdot\left(\frac{\text{m}}{3.281\text{ft}}\right)^2\cdot\frac{3600\,\text{s}}{\text{hr}} = 0.317\frac{\text{Btu}}{\text{hr ft}^2}$$

1.23* $$R = \frac{DV\rho}{\mu} = \frac{(0.5\,\text{ft})\left(10\frac{\text{ft}}{\text{s}}\right)\left(62.3\frac{\text{lbm}}{\text{ft}^3}\right)}{1.002\,\text{cp}}\cdot\frac{\text{cp}\cdot\text{ft}^2}{2.09\cdot 10^{-5}\text{lbf}\cdot\text{s}}\cdot\frac{\text{lbf s}^2}{32.2\,\text{lbm ft}} = 4.6\cdot 10^5$$

or $$R = \frac{\left(\frac{0.5\text{ m}}{3.28}\right)\left(\frac{10}{3.28\frac{\text{m}}{\text{s}}}\right)\left(998.2\frac{\text{kg}}{\text{m}^3}\right)}{1.002\cdot 10^{-3}\,\text{Pa}\cdot\text{s}}\cdot\frac{\text{Pa}\cdot\text{m}^2}{\text{N}}\cdot\frac{\text{Nm}}{\text{kg}\cdot\text{s}^2} = 4.6\cdot 10^5$$

1.24 $$\text{Darcy} = \frac{\left(1\frac{\text{cm}}{\text{s}}\right)(1\text{ cp})}{\left(1\frac{\text{atm}}{\text{s}}\right)}\cdot\frac{0.01\,\text{gm}}{\text{cm}\cdot\text{s}\cdot\text{cp}}\cdot\frac{\text{atm}\cdot\text{cm}\cdot\text{s}^2}{1.01\cdot 10^6\,\text{gm}} = 0.99\cdot 10^{-8}\,\text{cm}^2 = 1.06\cdot 10^{-11}\,\text{ft}^2$$

1.25* $$F = 2\sigma l = 2\cdot 72.74\cdot 10^{-3}\frac{\text{N}}{\text{m}}\cdot 0.1\text{ m} = 0.0145\text{ N} = 0.0033\text{ lbf} = 1.48\text{ g(force)}$$

1.26 $$X = \frac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{F}}\cdot\frac{{}^\circ\text{C}}{\text{gm}\cdot\text{cal}}\cdot\frac{1.8\,{}^\circ\text{F}}{{}^\circ\text{C}}\cdot\frac{\text{lbm}}{454\,\text{g}}\cdot\frac{252\,\text{cal}}{\text{Btu}} = 1.0$$

The calorie and the Btu were defined in ways that make this 1.0. Some are confused because ${}^\circ\text{F} = 1.8\cdot{}^\circ\text{C} + 32$, but if we differentiate, $d{}^\circ\text{F} = 1.8\cdot d{}^\circ\text{C}$ which is the relation used above.

1.27 One standard atmosphere = 1.013 bar. Clearly the bar is a convenient approximate atmosphere. It is very commonly used in high pressure work, as an approximate atmosphere. Meteorologists express all pressures in millibar. Currently thermodynamic tables like steam tables and chemical thermodynamic tables show pressure in bars.

1.28 The US air pollution regulations are almost all written for concentrations in (g/m^3) and the models expect emission rates in (g/s). The available auto usage data is almost all in vehicle miles per hour (or day or second) so to get emission rates in (g/s) one multiplies the (vehicle miles/s) times the emission factor in (g/mi). The US-EPA would be happy to do it all in metric, but the mile doesn't seem to be going away very fast in the US.

1.29 $\frac{1\,\text{kg f}}{\text{cm}^2} \cdot \frac{9.81\,\text{N}}{\text{kg f}} \cdot \frac{10^4\,\text{cm}^2}{\text{m}^2} = 98.1\,\text{kPa} = 0.968\,\text{atm}$

This is very close to one atmosphere. The most common type of pressure gage testers, the dead-weight tester, balances a weight against the pressure in the gages. The direct observation is the values of the weights on the tester and the cross-sectional area of the piston on which they rest, which has the dimensions of kgf/cm^2. I think this is slowly losing out to the kPa, but it is not gone yet.

1.30 $a = \frac{1.59\,\text{kgf}}{4.54\,\text{kgm}} \cdot \frac{32.2\,\text{lbm ft}}{\text{lbf s}^2} \cdot \frac{\text{kg}}{2.2\,\text{lbm}} \cdot \frac{2.2\,\text{lbf}}{\text{kgf}} \cdot \left(60\frac{\text{s}}{\text{min}}\right)^2 = 40,597\frac{\text{ft}}{\text{min}^2}$

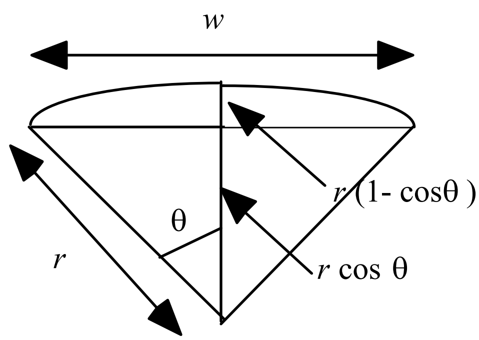
This leads to the conclusion that (kg/kgf) = (lbm/lbf).

Solutions, Chapter 2

2.1* As the sketch at the right shows, depth = r (1 - cos θ), and

$$\theta = \arcsin(0.5\ w / r) = \arcsin\frac{50\,\text{ft}}{4000\cdot 5280\,\text{ft}}$$
$$= \arcsin 2.3674\cdot 10^{-6} = 2.3674\cdot 10^{-6}$$

$$depth = 4000\cdot 5280\cdot(1-\cos\ 2.3674\cdot 10^{-6})$$
$$= 5.92\cdot 10^{-5}\ \text{ft} = 0.018\ \text{mm}$$



Discussion The point of this problem is that, for engineering purposes, for modest-sized equipment, the world is flat!

Older computers and some hand calculators do not carry enough digits to solve this problem as shown above. If we use the approximations

$$\theta \approx \sin\theta \approx \tan\theta = 0.5w / r \ \ \text{and}\ \ \cos\theta = 1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \frac{\theta^6}{6!}.....$$

we can drop the higher terms and write

$$depth \approx w^2 / 8r = \frac{(100\,\text{ft})^2}{8\cdot(4000\cdot 5280\,\text{ft})} = 5.92\cdot 10^{-5}\,\text{ft}$$

On my spreadsheet this approximation agrees with the above solution to 1 part in 50,000.

2.2 (a) $$\gamma = \rho g = 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\frac{\text{ft}}{\text{s}^2}\cdot\frac{\text{lbf s}^3}{32.2\ \text{lbm ft}} = 62.3\frac{\text{lbf}}{\text{ft}^3}$$

$$\gamma = \rho g = 998.2\frac{\text{kg}}{\text{m}^3}\cdot 9.81\frac{\text{m}}{\text{s}^2} = 9.792\frac{\text{kN}}{\text{m}^3}$$

(b) $$\gamma \approx 62.3\cdot 6\cdot\frac{1}{32.2} \approx 11.6\frac{\text{lbf}}{\text{ft}^3} = 998.2\cdot 2 = 1.996\frac{\text{kN}}{\text{m}^3}$$

2.3* $$\gamma = \rho g = 998.2\frac{\text{kg}}{\text{m}^3}\cdot 9.81\frac{\text{m}}{\text{s}^2} = 9.792\frac{\text{kN}}{\text{m}^3} = 998.2\frac{\text{kgf}}{\text{m}^3}$$

I don't know if civil engineers in metric countries use the specific weight at all, and if they do whether they use the proper SI value, 9,792 kN/m^3 or the intuitive 998.2 kgf/m^3 US civil engineering fluids textbooks show the SI value, 9,792 kN/m^3.

2.4

$$\frac{dP}{dz} = -\rho g = -62.3\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\frac{\text{ft}}{\text{s}^2}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\ \text{lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2} = -0.423\frac{\text{psi}}{\text{ft}}$$

This is close to -0.5 psi/ft, so most of us remember that *dP/dh* ≈ 0.5 psi/ft.

2.5

$$P = P_0 + \rho g h = 14.7\text{ psia} + \frac{62.3\dfrac{\text{lbm}}{\text{ft}^3}\cdot 32.17\dfrac{\text{ft}}{\text{s}^2}}{32.17\dfrac{\text{lbm ft}}{\text{lbf s}^2}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2} = 14.7 + 4.33 = 18.03\text{ psia} = 4.33\,\text{psig}$$

Discussion; 0.433 psi/ft ≈ 0.5 psi/ft in water is a useful number to remember.
One can also discuss why it hurts. The pressure inside our eardrums is ≈ the same as that in our lungs, which is practically atmospheric. So a pressure difference of about 5 psi across the eardrums is painful. Scuba divers have the same air pressure inside their lungs as that of the outside water. The air valve that does that automatically, invented by Jaques Costeau, made scuba diving possible.
People who dive deeply wearing a face mask must equalize the pressure in their lungs with that in the face mask, or the blood vessels in their eyes will burst. This is a problem for deep free divers.

2.6 $$h = \frac{\Delta P}{\rho g} = \frac{1000\ \dfrac{\text{lbf}}{\text{in}^2}}{(1.03)\left(62.3\dfrac{\text{lbm}}{\text{ft}^3}\right)\left(32.17\dfrac{\text{ft}}{\text{s}^2}\right)}\cdot 32.17\frac{\text{lbm ft}}{\text{lbf s}^2}\cdot\frac{144\ \text{in}^2}{\text{ft}^2} = 2244\text{ ft} = 684\text{ m}$$

2.7

$$P_2 = P_1 + \rho g h = 15\,\text{psig} + \frac{62.3\dfrac{\text{lbm}}{\text{ft}^3}\cdot 32.17\dfrac{\text{ft}}{\text{s}^2}\cdot 1450\,\text{ft}}{32.17\dfrac{\text{lbm ft}}{\text{lbt s}^2}\cdot\dfrac{144\,\text{in}^2}{\text{ft}^2}} = 15 + 627 = 642\text{ psig} = 4.43\text{ MPa}$$

This is a higher pressure than exists in any municipal water system, so in any such tall building the water, taken from the municipal water system must be pumped to the top. Furthermore, one cannot tolerate such a high pressure in the drinking fountains on the ground floor; they would put out people's eyes. Tall buildings have several zones in their internal water system, with suitable pressures in each, and storage tanks at the top of each zone.

2.8* $$P = P_{\text{atm}} + \rho g h = 101.3\,\text{kPa} + (1.03)\left(998.2\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(11{,}000\,\text{m})\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\cdot\frac{\text{Pa}\cdot\text{m}^2}{\text{N}}$$

$$P_{abs} = 1.013 \cdot 10^5 + 1.109 \cdot 10^8 \text{ Pa} = 1.1105 \cdot 10^5 \text{ kPa} = 1096 \text{ atm} = 16{,}115 \text{ psia}$$
$$P_{gauge} = 1.109 \cdot 10^5 \text{ kPa} = 1095 \text{ atm} = 16{,}100 \text{ psig}$$

2.9 $$\rho = \frac{\Delta P}{gh} = \frac{10{,}000\dfrac{\text{lbf}}{\text{in}^2}}{\left(32.17\dfrac{\text{ft}}{\text{s}^2}\right)(15000\text{ft})} \cdot \frac{144\text{ in}^2}{\text{ft}^2} \cdot \frac{32.17 \text{ lbm ft}}{\text{lbf s}^2} = 96.0\frac{\text{lbm}}{\text{ft}^3} = 1538\frac{\text{kg}}{\text{m}^3}$$

Discussion: When I assign this problem I sketch the flow in an oil-drilling rig for the students. High-pressure drilling fluid is pumped down the drill pipe, and flows back up the annulus between the drill pipe and the wellbore. This drilling fluid cools the drill bit and carries the rock chips up out of the well.

Ask the students how one would get such dense drilling fluids? The answer is to use slurries of barite, (barium sulfate) s.g. = 4.499. In extreme cases powdered lead, s.g. = 11.34 has been used.

The great hazard is that high pressure gas will enter the drilling fluid, expand, lower its density, and cause it all to be blown out. Most deep drilling rigs have mechanical "blowout presenters", which can clamp down on the drill stem and stop the flow in such an emergency. Even so one of the most common and deadly drilling accidents is the blowout, caused by drilling into an unexpected zone of high-pressure gas. If the gas is rich in H_2S, the rig workers are often unable to escape the toxic cloud.

2.10 The pressure at the gasoline-water interface is

$$P = \rho g h = 0.72 \cdot 62.3\frac{\text{lbm}}{\text{ft}^3} \cdot 32.17\frac{\text{ft}}{\text{s}^2} \cdot 20\text{ ft} \cdot \frac{\text{lbf s}^2}{32.17\text{ lbm ft}} \cdot \frac{\text{ft}^2}{144\text{ in}^2} = 6.23 \text{ psig}$$
$$= 20.93 \text{ psia} = 43.5 \text{ kPa gauge} = 144.9 \text{ kPa abs}$$

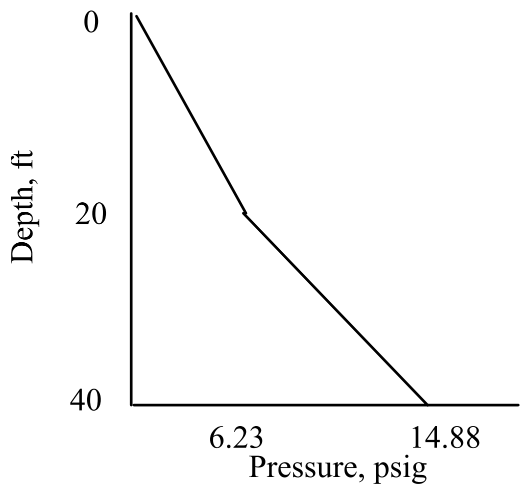
The increase in pressure from that interface to the bottom of the tank is

$$\Delta P = \rho g h = 62.3\frac{\text{lbm}}{\text{ft}^3} \cdot 32.17\frac{\text{ft}}{\text{s}^2} \cdot 20\text{ ft} \cdot \frac{\text{lbf s}^2}{32.17\text{ lbm ft}} \cdot \frac{\text{ft}^2}{144\text{ in}^2} = 8.65 \text{ psig}$$

So the pressure at the bottom is the sum,

$$P_{bottom} = 14.88 \text{ psig} = 29.58 \text{ psia} = 102.65 \text{ kPa gauge} = 203.95 \text{ kPa abs}$$

The figure is sketch, roughly to scale at the right.



2.11 (a) See example 2.7. The result is exactly one-half of that result, or $2.35 \cdot 10^5$ lbf. for internal pressure and exactly one-fourth of that result of $1.175 \cdot 10^5$ lbf for internal vacuum.

(b) Almost all pressure vessels can withstand much higher internal pressures than vacuums. An internal pressure blows the vessel up like a balloon, straightening out any non-uniformities. An internal vacuum collapses the tank, starting at a non-uniformity and magnifying it. Most students have see the demonstration in which a rectangular 5 gallon can is filled with steam and then collapsed by cooling to condense the steam. The maximum vacuum there is 14.7 psig. An internal pressure of the same amount will cause the sides and top of the can to bulge out somewhat, but the result is far less damaging than the vacuum collapse.

You might suggest the analogy of pulling and pushing a rope. If you pull a rope, it straightens out. If you push one the kinks are increased and it folds up.

2.12* Here let the depth to the bottom of the can be h_1, the height of the can be h_2, and the height to which water rises in the can be h_3. Then, after the water has risen, at the interface the pressure of the water and the pressure of the air are equal, $P_w = P_a$

$$P_w = P_{atm} + \rho g(h_1 - h_3), \qquad P_{air} = P_{atm} \cdot \left(\frac{h_2}{h_2 - h_3} \right),$$

The easiest way to solve the problem is to guess h_3, and solve for both pressures on a spreadsheet. Then one constructs the ratio of the two pressures, and uses the spreadsheet's root finding engine to find the value of h_3, which makes that ratio 1.00. The result of that procedure (using *Goal seek* on an excel spreadsheet) is $h_3 = 0.2236$ ft, and $P_{\text{interface}}$ = 18.93 psia. Before we had spreadsheets we could have done the same by manual trial and error, or done it analytically. If we equate the two pressures

$$P_{atm} \cdot \left(\frac{h_2}{h_2 - h_3} - 1 \right) = \rho g (h_1 - h_3); \qquad P_{atm} \cdot \left(\frac{h_3}{h_2 - h_3} \right) = \rho g (h_1 - h_3)$$

$\dfrac{P_{atm} h_3}{\rho g} = (h_1 - h_3) \cdot (h_2 - h_3) = h_2 h_1 - h_2 h_3 - h_1 h_3 + h_3^2$, which can be factored to

$h_3^2 + h_3 \left(-h_2 - h_1 - \dfrac{P_{atm}}{\rho g} \right) + h_2 h_1 = 0$; which is a simple quadratic equation. To simplify, let

$\left(-h_2 - h_1 - \dfrac{P_{atm}}{\rho g} \right) = A$, so that $h_3 = \dfrac{-A \pm \sqrt{A^2 - 4 h_1 h_2}}{2}$ Now we can insert values

$$\left(-h_2 - h_1 - \frac{P_{atm}}{\rho g} \right) = A = \left(-1\text{ft} - 10\text{ft} - \frac{14.7 \dfrac{\text{lbf}}{\text{in}^2}}{62.3 \dfrac{\text{lbm}}{\text{ft}^3} \cdot 32.2 \dfrac{\text{ft}}{\text{s}^2}} \cdot \frac{144\text{in}^2}{\text{ft}^2} \cdot \frac{32.2\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \right) = -33.98\text{ft}$$

and $h_3 = \dfrac{-(-33.98\text{ft}) \pm \sqrt{(-33.98\text{ft})^2 - 4 \cdot 10\text{ft} \cdot 1\text{ft}}}{2} = 0.223\text{ft} = 2.68\text{in}$

The liquid pressure is

$$P_{liquid} = 14.7 \text{ psia} + (10 - 0.223)\text{ft} \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot \frac{\text{ft}^2}{144\text{in}^2} \cdot \frac{\text{lbf} \cdot \text{s}^2}{32.2\text{lbm} \cdot \text{ft}} = 18.94\text{psia}$$

while the gas pressure is

$$P_{gas} = 14.7\text{psia} \cdot \left(\frac{1\text{ft}}{1\text{ft} - 0.223\text{ft}} \right) = 18.94\text{psia}$$

This has an elegance which the spreadsheet does not.

2.13 (a) See the solution to problem 2.8., P = 16,099 psig

(b) $$dP = \rho g dh = \rho_0 g \left[1 + \beta (P - P_0) \right] dh$$

$$\int_{P_0}^{P} \frac{dP}{1 + \beta P - \beta P_0} = \rho_0 g \int_0^h dh = \rho g h \Big|_0^h = \frac{1}{\beta} \ln (1 + \beta P - \beta P_0) \Big|_{P_0}^{P}$$

$$\ln\left(\beta\left(P-P_0\right)+1\right)=\beta\rho_0 gh; \qquad P-P_0=\frac{\left[\exp\left(\beta\rho_0 gh\right)\right]-1}{\beta}$$

From App. A.9, $\beta = 0.3\cdot 10^{-5}$ / psi, so that

$$\exp\left(\beta\rho_0 gh\right)=\left(\frac{0.3\cdot 10^{-5}}{\text{psi}}\right)\left(1.03\cdot 62.3\frac{\text{lbm}}{\text{ft}^3}\right)\left(\frac{32.17\text{ ft}}{\text{s}^2}\right)\left(11000\cdot 3.281\text{ft}\right)\cdot$$

$$\cdot\left(\frac{\text{ft}^2}{144\text{in}^2}\right)\left(\frac{\text{lbf s}^2}{32.17\text{ lbm ft}}\right)=1.0494$$

and

$$P-P_0=\frac{1.0494-1}{0.3\cdot 10^{-5}\text{ / psi}}=16477\text{psig}$$

The ratio of the answer taking compressibility into account to that which does not take it into account is $\text{Ratio}=\frac{16\,477}{16\,099}=1.023$.

Discussion: Here we see that even at the deepest point in the oceans, taking the compressibility of the water into account changes the pressure by only 2.3%.
One may do a simple plausibility check on this mathematics by computing the ratio of the density at half of the depth to the surface density. Using the data here, the density at a pressure of 8000 psi is about 1.024 times the surface density. For the depths of ordinary industrial equipment this ratio is almost exactly 1.00.

2.14 $P_2 = P_1\exp\left(\frac{-gMz}{RT}\right)$

$$=1\text{atm}\exp\left(\frac{-32.17\frac{\text{ft}}{\text{s}^2}\cdot 29\frac{\text{lbm}}{\text{lb mol}}\cdot 10000\text{ft}\cdot\frac{\text{lbf s}^2}{32.17\text{lbm ft}}\cdot\frac{\text{ft}^2}{144\text{in}^2}}{10.73\frac{\frac{\text{lbf}}{\text{in}^2\cdot\text{ft}^3}}{\text{lbmol}^\circ\text{R}}\cdot 400^\circ\text{R}}\right)$$

$$=1\text{atm}\exp\left(-0.4692\right)=0.625\text{ atm}$$

2.15 $P=P_0-\rho gz$; $\frac{dP}{dt}=-\rho g\frac{dz}{dt}$

$$\frac{dP}{dt}=0.075\frac{\text{lbm}}{\text{ft}^3}\cdot 32.17\frac{\text{ft}}{\text{s}^2}\cdot 2000\frac{\text{ft}}{\text{min}}\cdot\frac{\text{lbf s}^2}{32.17\text{lbm ft}}\cdot\frac{\text{ft}^2}{144\text{in}^2}=-1.04\frac{\text{psi}}{\text{min}}=-7.18\frac{\text{kPa}}{\text{min}}$$

2.16 $\dfrac{P}{\rho^k} = \text{const} = \dfrac{P}{\left(\dfrac{MP}{RT}\right)^k}$; $\left(\dfrac{RT}{M}\right)^k \cdot \text{const} = P^{(1-k)}$; $\dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$$dP = -\rho g dz = \frac{-PMg}{RT} dz\text{;} \quad \frac{dP}{P} = -\frac{gM}{RT} dz = \frac{-gM}{RT_1}\left(\frac{P_1}{P}\right)^{\left(\frac{k-1}{k}\right)} dz$$

$$P^{\left(\frac{k-1}{k}-1\right)} dP = -\frac{gM}{RT_1} P_1^{\frac{k-1}{k}} dz\text{;} \quad \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1 = -\left(\frac{k-1}{k}\right)\frac{gM}{RT_1}\Delta z$$

$$P_2 = P_1\left\{1 - \frac{k-1}{k}\frac{gM\Delta z}{RT_1}\right\}^{\frac{k}{k-1}} \qquad \text{(Eq. 2.17)}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)} = \left\{1 - \frac{k-1}{k}\frac{gM\Delta z}{RT_1}\right\} \qquad \text{(Eq. 2.18)}$$

2-17 (a) $T = 519^{\circ}\text{R} - \dfrac{59-(-69.7)^{\circ}\text{F}}{36\,150 \text{ ft}} z = 519^{\circ}\text{R} - \dfrac{0.00356^{\circ}\text{R}}{\text{ft}} \cdot z = a + bz$

$$\frac{dP}{P} = -\frac{gM}{RT} dz = -\frac{gM}{R}\frac{dz}{a+bz}; \quad \ln\frac{P}{P_0} = -\frac{gM}{bR}\ln(a+bz)\Big|_0^z = -\frac{gM}{bR}\ln\frac{T}{T_0};$$

$\dfrac{P}{P_0} = \left(\dfrac{T}{T_0}\right)^{-\frac{gM}{bR}}$ Here

$$\frac{gM}{bR} = \left(\frac{32.17\dfrac{\text{ft}}{\text{s}^2}\cdot\dfrac{29 \text{ lbm}}{\text{lb mole}}}{0.000356\dfrac{^{\circ}\text{R}}{\text{ft}}\cdot\dfrac{10.73\dfrac{\text{lbf}}{\text{in}^2}}{\text{lb mol}^{\circ}\text{R}}\cdot 32.17\dfrac{\text{lbm ft}}{\text{lbf s}^2}\cdot\dfrac{144\text{in}^2}{\text{ft}^2}}\right) = 5.272$$

(b) At the interface

$$P = P_0\left(\frac{390^{\circ}\text{R}}{518.7^{\circ}\text{R}}\right)^{0.572} = P_0(0.752)^{5.272} = 0.222 P_0$$

(c) In the stratosphere, T = constant, so we use the isothermal formula

$$P = P_{interface}\exp\left(\frac{-gM(z - z_{\text{interface}})}{R\,T_{\text{stratosphere}}}\right)$$

2.18* Using Eq. 2.18 we have $T_2 = 0$ when $\dfrac{k-1}{k}\dfrac{gM\Delta z}{RT_1} = 1$,

$$\Delta z = \frac{RT_1}{gM}\left(\frac{k}{k-1}\right) = \frac{\dfrac{10.73\dfrac{\text{lbf}}{\text{in}^2}\text{ft}^3}{\text{lbmol}^\circ\text{R}}\cdot 519^\circ\text{R}}{32.17\dfrac{\text{ft}}{\text{s}^2}\cdot 29\dfrac{\text{lbm}}{\text{lbmol}}\left(\dfrac{1.4}{0.4}\right)}\cdot\frac{144\text{in}^2}{\text{ft}^2}\cdot\frac{32.17\text{lbm ft}}{\text{lbf s}^2} = 96{,}783\text{ ft}$$

This is the height which corresponds to turning all internal energy and injection work (Pv) into gravitational potential energy, adiabatically. The fact that the atmosphere extends above this height shows that there is heat transfer in the atmosphere, which contradicts the adiabatic assumption.

At this elevation the predicted pressure from equation 2.17 is $P = 0$.

2.19 Taking values directly from the table in example 2.4,

(a) $P_{4300\text{ ft}} = P_1 \exp\left(-\dfrac{gM\Delta z}{RT_1}\right) = 1\text{atm}\left(-0.03616\cdot\dfrac{4300\text{ ft}}{1000\text{ ft}}\right) = 0.856\text{ atm}$

(b) is given in Table 2.1 = 0.697 atm

(c) $P_{29\,028\text{ft}} = P_1 \exp\left(-\dfrac{gM\Delta z}{RT_1}\right) = 1\text{atm}\left(-0.03616\cdot\dfrac{29\,028\text{ ft}}{1000\text{ ft}}\right) = 0.350\text{ atm}$

These are based on T_0 = 519·°R (the standard atmosphere). If one uses 528°R, the answers are 0.858, 0.701 and 0.356 atm.

Discussion: you might ask the students about the barometric pressures, which are regularly reported on the radio for places not at sea level. These are normally "corrected to sea level" so that, for example in Salt Lake City the reported barometric pressures are normally about 29.9 inches of mercury, which is about 5 inches higher than the actual barometric pressure ever gets. Those values are used to set altimeters in airplanes, which have a dial-in for the sea level barometric pressure, which they compare to the observed pressure to compute the altitude.

2.20* (a) $\dfrac{dT}{dz} = \dfrac{\Delta T}{\Delta z} = \dfrac{-69.7-(59)^\circ\text{F}}{36{,}150\text{ ft}} = -0.00356\dfrac{^\circ\text{F}}{\text{ft}}$

(b)

$$\frac{dT}{dz} = -\left(\frac{k-1}{k}\right)\frac{gM}{R} = -\left(\frac{0.4}{1.4}\right)\frac{32.17\frac{\text{ft}}{\text{s}^2}\cdot 29\frac{\text{lbm}}{\text{lbmol}}}{10.73\frac{\frac{\text{lbf}}{\text{in}^2}\cdot\text{ft}^3}{\text{lb mol}^\circ\text{R}}}\cdot\frac{\text{lbf s}^2}{32.17\text{ lbm ft}}\cdot\frac{\text{ft}^2}{144\text{in}^2} = -0.00536\frac{^\circ\text{R}}{\text{ft}}$$

Discussion: These are called *lapse rates* and the minus sign is normally dropped, so that the standard lapse rate is 3.56°F/ 1000 ft, and the adiabatic lapse rate is 5.36 °F/ 1000 ft, These terms are widely used in meteorology, and in air pollution modeling.

2.21 (a) From Table 2.1, 0.697 atm.

(b) $$T_2 = T_1\left[1-\frac{k-1}{k}\left(\frac{gM\Delta z}{RT}\right)\right] = T_1\left[1-\frac{0.4}{1.4}(0.3616)\right] = 519\cdot 0.8967$$

$$= 465.4^\circ\text{R} = 5.7^\circ\text{R}$$

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = 14.7\text{psia}\,(0.8964)^{\frac{1.4}{0.4}} = 10.04\text{ psia} = 0.683\,\text{atm}$$

(c) See the solution to Problem 2.17

$$T = 59 + \left(-0.00356\frac{^\circ\text{R}}{\text{ft}}\right)\left(10^4\text{ ft}\right) = 23.4^\circ\text{F} = 483.1^\circ\text{R}$$

$$P = P_0\left(\frac{T}{T_0}\right)^{\frac{-gM}{bR}} = 1\text{ atm}\left(\frac{483.1^\circ\text{R}}{519^\circ\text{R}}\right)^{5.272} = 1\text{ atm}\,(0.685) = 10.07\text{ psia}$$

2.22* Here the pressure at sea level is due to the weight of the atmosphere above it, from which we can easily conclude that there are 14.7 lbm of atmosphere above each square inch of the earth's surface. Then

$$m = \frac{m}{A}\cdot A = \frac{m}{A}\cdot 4\pi D^2 = 14.7\frac{\text{lbm}}{\text{in}^2}\cdot 4\cdot\pi\left[4000\text{mi}\cdot\frac{5280\text{ ft}}{\text{mi}}\cdot\frac{12\text{ in}}{\text{ft}}\right]^2 = 1.186\cdot 10^{19}\text{ lbm}$$

2-23 $$F = PA = \frac{\pi}{4}(120\text{ft})^2\cdot\frac{1\text{lbf}}{\text{in}^2}\cdot\frac{144\text{in}^2}{\text{ft}^2} = 1.63\cdot 10^6\text{lbf} = 7.24\text{MN}$$

This is normally enough to crush such a tank, which explains why such vents are of crucial importance, and why prudent oil companies pay high prices to get one that always works!

2.24* $P=\dfrac{F}{A}=\dfrac{1800\ \text{kg}\cdot 9.81\ \text{m/s}^2}{0.2\ \text{m}^2}\cdot\dfrac{\text{N s}^2}{\text{kg m}}\cdot\dfrac{\text{Pa m}^2}{\text{N}}=88.92\ \text{kPa}=12.81\ \text{psig}$

Discussion: This small value shows how all hydraulic systems work. A small pressure, acting over a modest area can produce an impressive force. The small air compressor in the service station easily lifts the heavy car, fairly rapidly.

You can ask the students whether this is psig or psia. You can also ask them about buoyancy. Is the piston a floating body? Answer; not in the common sense of that term. If you look into Archimedes principle for floating bodies, there is an unstated assumption that the top of the floating body is exposed to the same atmospheric pressure as the fluid is. Here for a confined fluid that is clearly not the case. If an immersed body is in a confined fluid that assumption plays no role, and the buoyant force, computed by Archimedes' principle is independent of the external pressure applied to the fluid.

2.25 (a) $P=\rho g h=998.2\dfrac{\text{kg}}{\text{m}^3}\cdot 9.81\dfrac{\text{m}}{\text{s}^2}\cdot 230\ \text{m}\dfrac{\text{N s}^2}{\text{kg m}}\dfrac{\text{Pa}}{\text{N/m}^2}=2.25\ \text{MPa}=326\ \text{psi}$

(b) See Ex. 2.8[

$$F=\frac{\rho g w h^2}{2}=\frac{998.2\dfrac{\text{kg}}{\text{m}^3}\cdot 9.81\dfrac{\text{m}}{\text{s}^2}\cdot 76\ m\cdot(230\ \text{m})^2}{2}\ \frac{\text{N s}^2}{\text{kg m}}=1.97\cdot 10^{10}\ \text{N}=4.42\cdot 10^9\ \text{lbf}$$

2-26 (a) The width, W, is linearly related to the depth, 20 m at the top, zero at h = 10 m,

so that $W=20\ \text{m}\left(1-\dfrac{h}{10\ \text{m}}\right)$

(b) Here, as in Ex. 2.8 the atmospheric-pressure terms cancel, so we may save effort by working the problem in gauge pressure, Direct substitution of this value for W leads to

$$F=\rho g\ 20\ \text{m}\int h\left(1-\frac{h}{10\ \text{m}}\right)dh=\rho g\ 20\ \text{m}\left[\frac{h^2}{2}-\frac{h^3}{3\cdot 10\ \text{m}}\right]_0^{10\ \text{m}}$$

(c) $F=998.2\dfrac{\text{kg}}{\text{m}^3}\cdot 9.81\dfrac{\text{m}}{\text{s}^2}\cdot 20\ \text{m}\left[\dfrac{(10\ \text{m})^2}{2}-\dfrac{(10\ \text{m})^3}{3\cdot 10\ \text{m}}\right]_0^{10\ \text{m}}\dfrac{\text{N s}^2}{\text{kg m}}=3.26\ \text{MN}=2.2\cdot 10^6\ \text{lbf}$

This is exactly 1/3 of the answer to Ex. 2.8.

(d) We begin by writing Eq. 2.M for the gauge pressure of a constant-density fluid:

$F=\rho g\int h dA$ Multiplying by A/A and rearranging, we find $F=\rho g A\dfrac{\int h dA}{A}$

But $\int h\, dA/A$ is the definition of h_c, the centroid of the depth measured from the free surface, so this equation may be simplified to $F = \rho g A h_c$

(e) $F = \rho g A h_c = \rho g \frac{Wh}{2}\frac{h}{3}$; A little algebra shows that this is identical to the formula in part (b).

2.27* (a) $F = 2\rho g \int depth\sqrt{r^2 - depth^2}\, d(depth) = 2\rho g \left[-\frac{1}{3}\sqrt{r^2 - depth^2} \right]_0^r = \frac{2\rho g r^3}{3}$

$$= \frac{(2)\left(998.2\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(50\text{m})^3}{3} \cdot \frac{\text{N s}^2}{\text{kg m}} \cdot \frac{\text{Pa m}^2}{\text{N}} = 8.16 \cdot 10^8 \text{N} = 1.83 \cdot 10^8 \text{lbf}$$

(b) $F = \rho g A h_c = \rho g \frac{1}{2}\frac{\pi}{4} D^2 \cdot \frac{2D}{3\pi} = \rho g \frac{D^3}{12} = \frac{2}{3}\rho g r^3$, the same as in part (a)

If this doesn't convince the students of the convenience of the centroid method, one wonders what would.

2.28 See Example 2.9. There, at 60 ft, the required thickness was 0.0825 in. The required thickness (for the assumptions in the problem) is proportional to the gauge pressure at the bottom of each band of plates, which is proportional to the depth,

(a) The values are

Depth, ft	**Pressure, psig**	**Design, *t*, in**
60	22.9	0.825
50	19.0833333	0.6875
40	15.2666667	0.55
30	11.45	0.4125
20	7.63333333	0.275
10	3.81666667	0.1375

(b) The calculated thickness for the top band is less than 0.25 inches, so a plate with 0.25 inches thickness would be used. Some designs call for 5/16 inch. It is not asked for in the problem, but the same industry source who gave me these values also told me that the maximum used is 1.75 inch, because if you make it thicker than that, you must heat-treat the welds, which is prohibitively expensive for a large tank.

(c) The first two columns of the following table repeat the first and third columns of the preceding table. However in the middle column the lowest value, 0.1375 has been replaced by 0.25, because the top plate has a minimum thickness of 0.25 inches. The rightmost column shows the average thickness of the tapered plate (except for the lowest entry, which is not tapered). So, for example, the lowest plate is 0.825 inches at the bottom and 0.6875 inches at the top, for an average thickness of 0.75625 inches.

Depth, ft	Wall thickness, inches, uniform thickness plates	Average thickness, inches, tapered plates.
60	0.825	0.75625
50	0.6875	0.61875
40	0.55	0.48125
30	0.4125	0.34375
20	0.275	0.2625
10	0.25	0.25

We can then compute the average thickness of the walls of the whole tank by summing the six values in the second and third columns, and dividing by 6, finding 0.500 inch and 0.452 inches. Thus the tapered plates reduce the weight of the plates, for equal safety factor, to 0.904 times the weight of the constant-thickness plates. The same industry source who gave me the other values in the problem also told me that the tapered plate solution is quite rare. It is apparently only done when there is an order for a substantial number of very large tanks all of one size.

2.29 As a general proposition, the larger the tank the smaller the cost per unit volume. Surprisingly the volume of metal in the wall of the tank is ≈ proportional to the cube of the tank diameter, for all three types of tank, because the surface area is proportional to the diameter squared and the required thickness ≈ proportional to the diameter. So it is not simply the cost of the metal, but the cost per unit stored of erection, foundations, piping, etc. that decrease as the size increases. I thank Mr. Terry Gallagher of CBI who spent a few minutes on the phone and shared his insights on this problem.

He said that the biggest flat bottomed tank he knew of was in the Persian Gulf, 412 ft (≈ 125 m) in diameter and 68 ft (≈ 20 m) high. Compare this to Example 2.9. Here the depth increases by (68/60) and the diameter by (412/120) so the calculated wall thickness would be

$$t = 0.825 \text{ in} \cdot \frac{68}{60} \cdot \frac{412}{120} = 3.21 \text{ in} = 8.15 \text{ cm}$$

which is thick enough to be a problem to handle, and would require heat treating of all the welds. He said that they stress-relieved this tank, in the field, after it was complete, by building an insulating tent around it, and heating to a stress relieving temperature. That costs, but the economics of having this big a tank apparently made it worthwhile.

For spherical tanks he and I think that the difficulties of erection become too much as the size goes over about 80 ft. I have seen an advertisement for Hyundai, showing one of their liquid natural gas tankers, which had some huge spherical tanks in it. LNG is transported at a low enough temperature that one must use expensive alloy steels, so the cost of those spherical tanks must have been justified.

For sausage-shaped tanks, the limits are presumably the difficulty of shipping, where roads and railroads have maximum height, width and length specifications. The diameter may be set by the largest diameter hemispherical heads which steel mills will regularly produce. The heads are one-piece, hot pressed, I think. Mr. Gallagher (who refers to this tank shape as "blimps") indicates that there are some field-erected tanks of this type, which are larger than the values I show. Most of the tanks of this type are factory assembled, with a significant cost saving over field erection, and then shipped to the user.

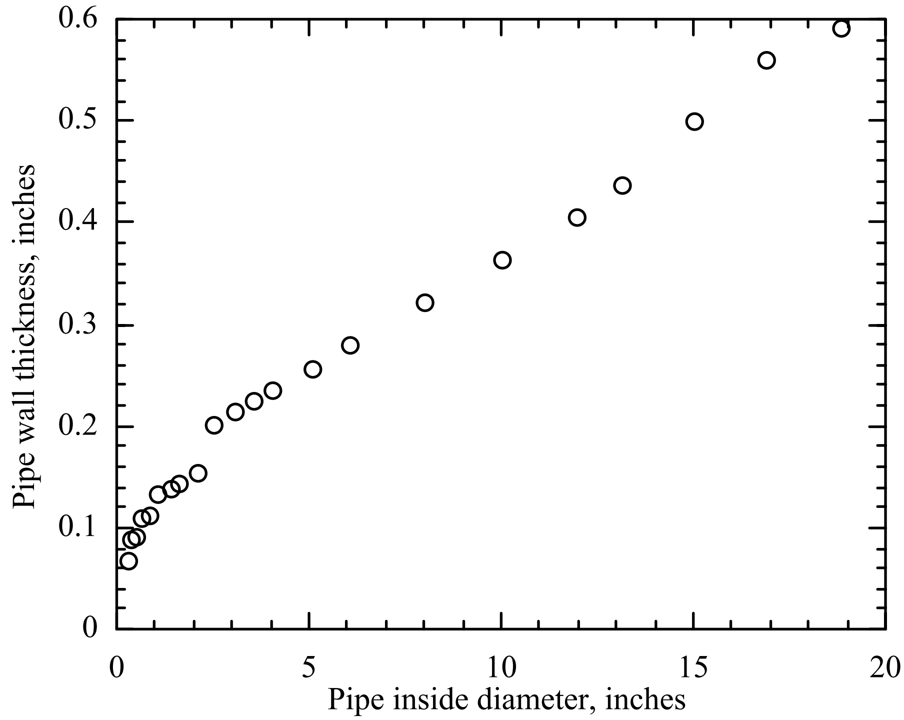
Two interesting web sites to visit on this topic are www.chicago-bridge.com and www.trinitylpg.com.

2.30* See Example 2.9.

$$t = \frac{1000 \frac{\text{lbf}}{\text{in}^2} \cdot 1 \text{ ft}}{2 \cdot 10{,}000 \frac{\text{lbf}}{\text{in}^2}} = 0.05 \text{ ft} = 0.60 \text{ in} = 1.52 \text{ cm}$$

From the larger table in *Perry's* equivalent to Appendix A.3 we see that a 12 inch diameter, schedule 80 pipe has a wall thickness of 0.687 inches. It would probably be selected.

2.31 The plot below shows all the values from App. A.2. Clearly one should not fit a single equation to that set of data. If one considers only the values for 2.5 inch nominal diameter and larger, then $t = 0.1366 \text{ in} + 0.0240\, D_{\text{inside}}$, $R^2 = 0.994$. Thus, for these pipes, $A \approx 0.1366$ inch and $B = 0.240$.



However if one fits the sizes from 1/8 to 2 inches nominal, one finds $t = 0.0719\text{ in} + 0.0454\, D_{\text{inside}}$, $R^2 = 0.897$. One can get $R^2 = 0.976$ by fitting a quadratic to this data set, showing that the values for the smaller pipes do not correspond very well to this equation.

If, as shown above, $A \approx 0.1366$ is the corrosion allowance, then the Eq. 2.25 can be rewritten $t - 0.1336\text{ in} = 0.024\,D = \dfrac{PD}{2\sigma}$, from which $\dfrac{P_{\text{allowable}}}{\sigma_{\text{allowable}}} = 0.048$. Comparing this will the definition of schedule number in App. A.2, we see that this corresponds to Sch 48, not Sch. 40. The difference is conservatism on the part of those defining the schedules.

2.32 Hooke's law says that the stress is proportional to the strain, which is the change in length divided by the original length. If as stated the increase in diameter is the same for all diameters, while the original diameter is greater, the further one moves from the center, then

$$\text{stress} = \begin{pmatrix}\text{Young's}\\ \text{modulus}\end{pmatrix} \cdot \frac{\begin{pmatrix}\text{increase in}\\ \text{diamter}\end{pmatrix}}{\begin{pmatrix}\text{original}\\ \text{diameter}\end{pmatrix}} = k\frac{\Delta D}{D}$$

and the stress decreases as (1/*D*) from the inner to the outer diameter. For $D_{\text{outer}} / D_{\text{inner}} = 1.5$ the stress at the outer wall would be (1/1.5 = 2/3) of the stress at the inner wall.

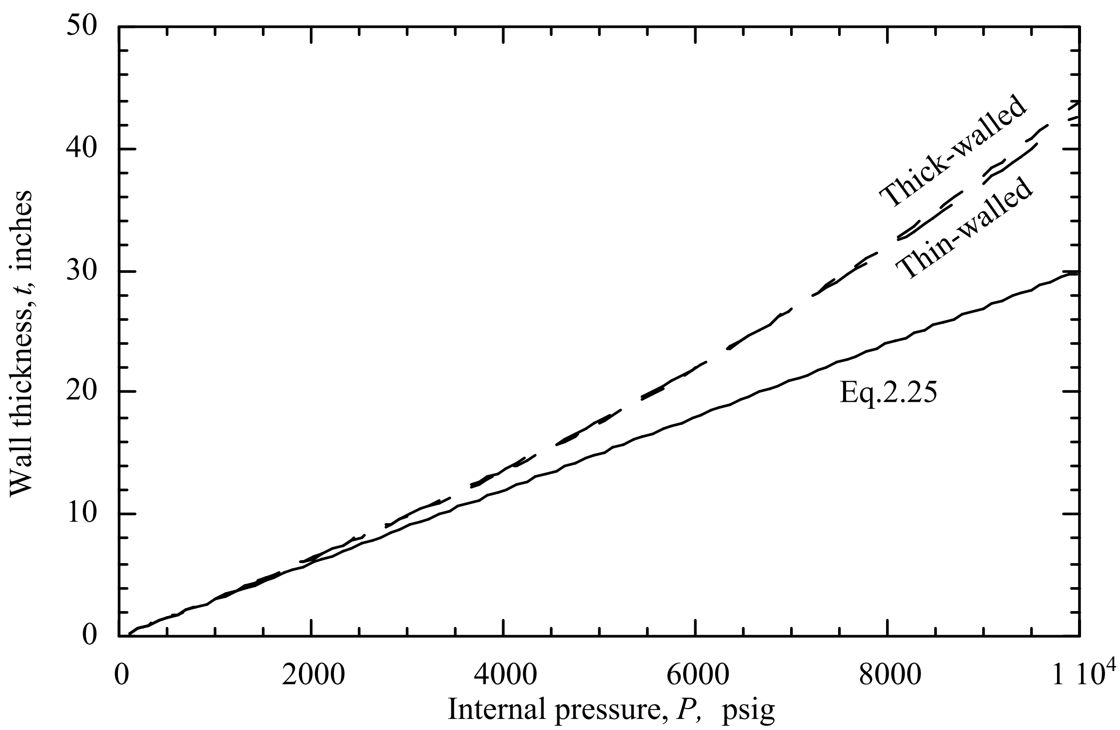
2.33 (a) Example 2.10 shows a wall thickness of 0.7500 inch. The thin-walled formula shows

$$t = \frac{P \cdot r_i}{SE_J - 0.6P} + C_c = \frac{250 \text{ psi} \cdot 5 \text{ ft}}{20{,}000 \text{ psi} \cdot 1 - 0.6 \cdot 250 \text{ psi}} + 0 = 0.6297 \text{ ft} = 0.7556 \text{ in}$$

(b) The thick-walled shows

$$t = r_i \left(\frac{SE_J + P}{SE_J - P} \right)^{1/2} - r_i + C_c = 5 \text{ ft} \left(\frac{20{,}000 \cdot 1 + 250}{20{,}000 \cdot 1 - 250} \right)^{1/2} - 5 \text{ ft} + 0 = 0.06290 \text{ ft} = 0.7547 \text{ in}$$

(c) The plot is shown below. We see that below about 2000 psi the three equations give practically the same result. Above that Eq. 2.25 gives values which increase linearly with pressure, while the thin-walled and thick-walled equations give practically the same value as each other, but larger values than Eq. 2.25.



2.34 (a) $t = \dfrac{P \cdot r_i}{SE_J - 0.6P} + C_c = \dfrac{50\,000 \text{ psi} \cdot 0.11 \text{ in}_{\text{i}}}{80\,000 \text{ psi} - 0.6 \cdot 50\,000 \text{ psi}} = 0.11 \text{ in}$

$$t = r_i\left(\frac{SE_J + P}{SE_J - P}\right)^{1/2} - r_i + C_c = 0.11\text{ in}\left(\frac{80\,000_J + 50\,000}{80\,000_J - 50\,000}\right)^{1/2} - 0.11\text{ in} = 0.1189\text{ in}$$

(b) $$t = 4\cdot\frac{50\,000\text{ psi}\cdot 0.11\text{ in}_i}{20\,000\text{ psi} - 0.6\cdot 50\,000\text{ psi}} = -2.20\text{ in ???}$$

$$t = 4\cdot 0.11\text{ in}\left(\frac{20\,000_J + 50\,000}{20\,000_J - 50\,000}\right)^{1/2} - 0.11\text{ in} = \left(\begin{matrix}\text{sq. rt of a}\\ \text{negative \#}\end{matrix}\right)\text{???}$$

Clearly this does not work. For Eq. 2.27

$$t = \left(\begin{matrix}\text{safety}\\ \text{factor}\end{matrix}\right)\cdot\frac{PD}{4\sigma_{\text{tensile}}} = 4\frac{80\,000\text{ psi}\cdot 0.22\text{ in}}{4\ 20\,000\text{ psi}} = 0.88\text{ in}$$

Which is much larger than the values shown in part (c)

(c) $$\left(\begin{matrix}\text{safety}\\ \text{factor}\end{matrix}\right) = \frac{t_{\text{thick end of barrel}}}{t_{\text{thin walled formula}}} = \frac{0.5\cdot(0.74 - 0.22)\text{ in}}{0.11\text{ in}} = 2.36\text{ and}$$

$$\left(\begin{matrix}\text{safety}\\ \text{factor}\end{matrix}\right) = \frac{t_{\text{thick end of barrel}}}{t_{\text{thick walled formula}}} = \frac{0.5\cdot(0.74 - 0.22)\text{ in}}{0.1189\text{ in}} = 2.19$$

The simple formulae used for pressure vessels give some indication about the design of guns, but this example shows it is only and indication.

2.35 The pressure at the bottom of the tank is 27.7 psig. From Eq. 2.25

$$t = \frac{27.7\text{ psi}\cdot 200\text{ ft}}{2\cdot 30\,000\text{ psi}} = 0.0922\text{ ft} = 1.1076\text{ in}$$

This differs by 1.4% from the value calculated by the API method.

2.36 (a) For the cylindrical container, remembering that the length of the cylindrical section is 6 times its diameter

$$V = \frac{\pi}{6}D^3 + \left(\frac{\pi}{4}\right)D^2\cdot 6D = \pi D^3\cdot\frac{10}{6}$$

$$D = \left(\frac{6}{10\pi}V\right)^{1/3} = \left(\frac{6}{10\pi}20{,}000\text{ gal}\cdot\frac{\text{ft}^3}{7.48\text{ gal}}\right)^{1/3} = 7.99\text{ ft}$$

$$t_{\text{cylindrical}} = \frac{PD}{2\sigma_{\text{tensile}}} = \frac{250\text{ psi}\cdot 7.99\text{ ft}}{2\cdot 20{,}000\text{ psi}} = 0.0499\text{ ft} = 0.599\text{ in}$$

The hemispherical ends have exactly one-half this thickness

$$\begin{pmatrix}\text{volume of}\\ \text{metal shell}\end{pmatrix} \approx \begin{pmatrix}\text{shell}\\ \text{thickness}\end{pmatrix} \cdot \begin{pmatrix}\text{shell}\\ \text{surface}\end{pmatrix} = t_{\text{spherical}} \pi D^2 + t_{\text{cylindrical}} \pi D \cdot 6D$$

$$= \pi D^2 \cdot \left(t_{\text{spherical}} + 6t_{\text{cylindrical}}\right) = 6.5\pi D^2 t_{\text{cylindrical}}$$

$$= 6.5\pi (7.99 \text{ ft})^2 \cdot 0.0499 \text{ ft} = 65.13 \text{ ft}^3$$

$$m_{\text{shell}} = \rho V = \left(7.9 \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3}\right) \cdot 65.13 \text{ ft}^3 = 32{,}050 \text{ lbm}$$

(b) For a spherical container,

$$V = \frac{\pi}{6} D^3; \quad D = \left(\frac{6}{\pi} V\right)^{1/3} = \left(\frac{6}{\pi} 20{,}000 \text{ gal} \cdot \frac{\text{ft}^3}{7.48 \text{ gal}}\right)^{1/3} = 17.22 \text{ ft}$$

$$t = \frac{PD}{4\sigma_{\text{tensile}}} = \frac{250 \text{ psi} \cdot 17.22 \text{ ft}}{4 \cdot 20{,}000 \text{ psi}} = 0.0538 \text{ ft} = 0.6456 \text{ in}$$

$$\begin{pmatrix}\text{volume of}\\ \text{metal shell}\end{pmatrix} \approx \begin{pmatrix}\text{shell}\\ \text{thickness}\end{pmatrix} \cdot \begin{pmatrix}\text{shell}\\ \text{surface}\end{pmatrix} = t\pi D^2$$

$$= 0.00538 \text{ ft} \cdot \pi \cdot (17.22 \text{ ft})^2 = 50.1 \text{ ft}^3$$

$$m_{\text{shell}} = \rho V = \left(7.9 \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3}\right) \cdot 50.1 \text{ ft}^3 = 24{,}650 \text{ lbm}$$

The spherical container requires only 77% as much metal (exclusive of corrosion allowances, foundations, etc.) However the sausage-shaped container is small enough to be shop assembled and shipped by rail or truck, while the spherical container could be shipped by barge, but at 17 ft diameter it could probably not be shipped by truck or rail.

2.37 For this size pipe the wall thickness is 0.258 in. From Eq. 2.25

$$P = \frac{2t\sigma}{D} = \frac{2 \cdot 0.258 \text{ in} \cdot 10\ 000 \text{ psi}}{5.047 \text{ in}} = 1022 \text{ psi}$$

But, as shown in Prob. 2.31, the sizes of Sch. 40 pipe seem to include a corrosion allowance of 01366 inches. Reworking the problem taking that into account, we find

$$P = \frac{2t\sigma}{D} = \frac{2 \cdot (0.258 - 0.1366) \text{ in} \cdot 10\ 000 \text{ psi}}{5.047 \text{ in}} = 481 \text{ psi}$$

This suggests that a new 5 inch Sch. 40 pipe would have a safe working pressure of 1022 psi, but that at the end of its corrosion life it would have one of 481 psi. The rule of thumb cited in App. A.2 suggests a safe working pressure of 400 psi.

2.38 Comparing Eq 2.33 and 2.35 we see that the axial stress in a cylindrical pressure vessel is exactly half the hoop stress (for the simple assumptions in those equations.). Grooved joint connectors (Perry's 7e Fig 10-139) use this fact. They cut a circumferential groove half-way through the pipe to be joined. The external clamp which fits into the groove supports the pipe against external expansion, so the doubling of the hoop stress is not a hazard. The clamp only operates on half the thickness of the wall, but that is enough to resist the axial stress, which is only half the hoop stress.

2.39* The buoyant force equals the weight of the water displaced

$$B.F = 5.0 - 4.725 = 0.275\text{N}$$

Thus the mass of water displaced was $m = \frac{F}{a} = \frac{0.275\text{ N}}{9.81\frac{\text{m}}{\text{s}^2}} \cdot \frac{\text{kg m}}{\text{N s}^2} = 2.8032 \cdot 10^{-2}\text{kg}$

and the volume of water displaced was $V = \frac{m}{\rho} = \frac{2.8032 \cdot 10^{-2}\text{ kg}}{998.2\frac{\text{kg}}{\text{m}^3}} = 2.80832 \cdot 10^{-5}\text{m}^3$

So the density of the crown was

$$\rho = \frac{m}{v} = \frac{\frac{F}{g}}{v} = \frac{\left(5\text{ N} / 9.81\frac{\text{m}}{\text{s}^2}\right)}{2.80832 \cdot 10^{-5}\text{m}^3} \cdot \frac{\text{kg m}}{\text{N s}^2} = 18149\frac{\text{kg}}{\text{m}^2} = 18.149\frac{\text{g}}{\text{cm}^3}$$

and its gold content was

$$\frac{\text{vol\% gold}}{100} = \frac{\rho_{\text{alloy}} - \rho_{\text{silver}}}{\rho_{\text{gold}} - \rho_{\text{silver}}} = \frac{18.149 - 10.5}{19.3 - 10.5} = 0.869 = 86.9\%$$

The corresponding wt % (mass %) gold is 92.4 %. If one computes the buoyant force of the air on the crown in the weighting in air, one finds 0.00033 N. If one then substitutes 5.00033 N for the 5 N in the problem one finds 86.7 vol % gold.

Discussion; This assumes no volume change on mixing of gold and silver. That is a very good, but not perfect assumption.

2.40* If you assign this problem make clear that one should not assume that air is a constant-density fluid, but must insert the perfect gas expression for its density. Then

$$Payload = BF - weight = Vg(\rho_{\text{air}} - \rho_{\text{gas}}) = Vg\left[\frac{P}{RT}(M_{\text{air}} - M_{\text{gas}})\right]$$

But $V = \frac{nRT}{P}$ so this becomes $Payload = ng(M_{\text{air}} - M_{\text{gas}})$

Thus the answers to parts (a,) (b) and (c) are the same, viz.

$$Payload = \frac{10\text{ lbm}}{4\frac{\text{lbm}}{\text{lbmole}}} \cdot \frac{32.2\frac{\text{ft}}{\text{s}^2}}{32.2\frac{\text{lbm ft}}{\text{lbf s}^2}}\left(\frac{29\text{ lbm}}{\text{lbmol}} - \frac{4\text{ lbm}}{\text{lbmol}}\right) = 62.5\text{ lbf} = 278\text{ N}$$

2.41 Starting with the solution to Ex. 2.11

$$\text{Payload} = 34\text{ lbf}\frac{\left(29\frac{\text{gm}}{\text{mole}} - 2\frac{\text{gm}}{\text{mole}}\right)}{\left(29\frac{\text{gm}}{\text{mole}} - 4\frac{\text{gm}}{\text{mole}}\right)} = 36.7\text{ lbf}$$

This is an increase of 7.6%, which is significant, but probably not enough to justify the increased hazard of hydrogen combustion in most cases. As far as I know no one uses hydrogen in balloons today.

2.42 $\text{Payload} = V_{\text{g}}(\rho_{\text{air}} - \rho_{\text{gas}});\qquad \rho_{\text{gas}} = \rho_{\text{air}} - \frac{\text{Payload}}{V_{\text{g}}}$

Here the students are confronted with the fact that the weight, in kg, is not an SI unit. One must make the distinction between kgf and kgm to solve here, finding

$$\rho_{\text{gas}} = 1.20\frac{\text{kgm}}{\text{m}^3} - \frac{200\text{kgf}}{\frac{\pi}{6}(20\text{m})^3 \cdot 9.81\frac{\text{m}}{\text{s}^2}} \cdot \frac{9.81\text{kgm m}}{\text{kgf s}^2} = 1.20 - 0.0477 = 1.152\frac{\text{kg}}{\text{m}^3}$$

$$T_{\text{gas}} = T_{\text{air}}\left(\frac{\rho_{\text{air}}}{\rho_{\text{gas}}}\right) = 293.15\text{K}\left(\frac{1.20}{1.152}\right) = 305.3\text{K} = 32.14^{\circ}\text{C} = 89.9^{\circ}\text{F}$$

2.43* The volumes of the lead and the brass weight are

$$V_{\text{lead}} = \frac{2.50\text{ lbm}}{62.3(11.3)\frac{\text{lbm}}{\text{ft}^3}} = 3.5512 \cdot 10^{-3}\text{ft}^3;\quad V_{\text{brass}} = \frac{2.5}{(62.3)(8.5)} = 4.721 \cdot 10^{-3}\text{ft}^3$$

and the difference in their volumes is $\Delta V = 4.721 \cdot 10^{-3} - 3.551 \cdot 10^{-3} = 1.170 \cdot 10^{-3}\text{ft}^3$

(a) The differential buoyant force is

$$\Delta BF = \rho\Delta V = 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{32.2\dfrac{\text{ft}}{\text{s}^2}}{32.2\dfrac{\text{lbm}}{\text{lbf}}\dfrac{\text{ft}}{\text{s}^2}}\cdot 1.170\cdot 10^3\,\text{ft}^3 = 7.29\cdot 10^{-2}\,\text{lbf}$$

so the indicated weight is $\textit{Indicated weight} = 2.500 + 7.29\cdot 10^{-2} = 2.573\,\text{lbf}$

(b) here the differential buoyant force is $\Delta BF = -0.075\cdot\frac{32}{32}\cdot 1.170\cdot 10^3 = -8.77\cdot 10^{-5}\,\text{lbf}$
so the indicated weight is

$$\textit{Indicated weight} = 2.500 + \left(-8.77\cdot 10^{-5}\right) = 2.49991\ \text{lbf}$$

2.44 $F_{\text{Buoyant}}\uparrow = F_{\text{Bottom}}\uparrow - F_{\text{Top}}\downarrow = A\left(P_B - P_T\right) = \rho_{\text{wood}}\, gA\, h_{\text{wood}}$

$$P_{\text{B}} = P_{\text{T}} + g\left[h_{\text{gasoline}}\,\rho_{\text{gas}} + h_{\text{water}}\,\rho_{\text{w}}\right]$$

$$\rho_{\text{wood}}\, gA\, h_{\text{wood}} = Ag\left[h_{\text{gas}}\,\rho_{\text{gas}} + h_{\text{water}}\,\rho_{\text{water}}\right]; \qquad h_{\text{gasoline}} = h_{\text{wood}} - h_{\text{water}}$$

$$\rho_{\text{wood}}\, h_{\text{wood}} = \left[h_{\text{wood}} - h_{\text{water}}\right]\rho_{\text{gasoline}} + h_{\text{water}}\,\rho_{\text{water}}$$

$$\frac{h_{\text{water}}}{h_{\text{wood}}} = \frac{\rho_{\text{wood}} - \rho_{\text{gasoline}}}{\rho_{\text{water}} - \rho_{\text{gasoline}}} = \frac{\text{SG}_{\text{wood}} - \text{SG}_{\text{gasoline}}}{1 - \text{SG}_{\text{gasoline}}}$$

Discussion; this shows that Archimedes principle does indeed give the right answer for two-fluid problems, and why the density of the gasoline does appear in the answer.

2.45 $\text{Gravity Force}\downarrow = \text{Buoyant Force}\uparrow + \text{Treading Force}$

$$\text{Gravity Force} = m_{\text{man}}\cdot g; \quad \text{Buoyant Force} = V_{\text{displaced}}\,\rho\text{g} = \frac{0.85\, m_{\text{man}}}{0.99\,\rho_{\text{water}}}\cdot\rho_{\text{whiskey}}$$

$$F_T = F_g - F_b = m_{\text{man}}\cdot\text{g} - m_{\text{man}}\cdot g\frac{(0.85)}{0.99}\frac{\rho_{\text{whiskey}}}{\rho_{\text{water}}}$$

$$= 150\ \text{lbm}\left(1 - 0.85\frac{0.92}{0.99}\right)\cdot\frac{32\dfrac{\text{ft}}{\text{s}^2}}{32\dfrac{\text{ft lbm}}{\text{s}^7\ \text{lbf}}} = 31.5\ \text{lbf} = 140.2\ \text{N}$$

2.46* $\textit{Payload} = V_{\text{g}}\left(\rho_{\text{w}} - \rho_{\text{logs}}\right)$

$$V = \frac{\textit{payload}}{g\left(\rho_{\text{w}} - \rho_{\text{logs}}\right)} = \frac{500\,\text{kgf}}{9.81\dfrac{\text{m}}{\text{s}^2}(1-0.8)998.2\dfrac{\text{kg}}{\text{m}^3}}\cdot\frac{9.81\,\text{kg m}}{\text{kgf}\,\text{s}^2} = 2.5045\,\text{m}^3$$

$$\textit{Mass of logs} = V\rho = 2.5045\ \text{m}^3\cdot 0.8\cdot 998.2\frac{\text{kg}}{\text{m}^3} = 2000\ \text{kg}$$

2.47 (a) Buoyant Force = weight, $g\left(V_{\text{tanks}} + V_{\text{ship}}\right) = m_{\text{ship}} \cdot g$

$$V_{\text{tanks}} = \frac{m_{\text{ship}}}{\rho_{\text{water}}} - V_{\text{ship}} = m_{\text{ship}}\left[\frac{1}{\rho_{\text{water}}} - \frac{1}{\rho_{\text{steel}}}\right]$$

$$= \frac{8 \cdot 10^7 \text{ lbm}}{62.3\dfrac{\text{lbm}}{\text{ft}^3}}\left[\frac{1}{1.03} - \frac{1}{0.79}\right] = 1.084 \cdot 10^6 \text{ ft}^3$$

(b) The cross-sectional area of the cable would be

$$A = \frac{\text{force}}{\text{stress}} = \frac{40{,}000 \cdot 2000 \text{ lbf}}{20{,}000 \text{ lbf / in}^2} = 4000 \text{ in}^2 \quad \text{and its diameter}$$

$$D = \sqrt{\frac{4}{\pi}A} = \sqrt{\frac{4}{\pi}4000\ in^2} = 71.4 \text{ in} \approx 6 \text{ ft}$$ which is not a "cable" in the ordinary sense. Whatever was pulling on it would have to have a buoyant force large enough to support its own weight, and also the weight of the battleship which it was raising.

(c) For a steel cable 1000 ft long, hung by its end in water, the stress at the top of the cable, due to the cable's weight is

$$\sigma = \frac{F}{A} \approx \frac{(\text{SG } -1)\rho_{\text{water}} LAg}{A}$$

$$= (7.9 - 1) \cdot 62.3\frac{\text{lbm}}{\text{ft}^3} \cdot 1000 \text{ ft} \cdot 32\frac{\text{ft}}{\text{s}^2} \cdot \frac{\text{lbf s}^2}{32 \text{ lbm ft}} = 4.3 \cdot 10^5 \frac{\text{lbf}}{\text{ft}^2} = 3000 \text{ psi}$$

This is 15% of the allowable stress. For deeper sunken vessels the fraction is higher. To the best of my knowledge the fist time this was ever tried was when the CIA attempted to raise a sunken Russian submarine for intelligence purposes during the cold war. They lowered a clamping device on multiple cables and tried to pull it up. It broke up while they were doing it, so they only got part of it, but that was said to have revealed lots of useful technical information. In 2001 the Russians successfully raised the submarine Kursk this way, with multiple cables.

2.48 $F_B = V\rho g = (5 \cdot 20 \cdot 30) \text{ ft}^3 \cdot 62.3\dfrac{\text{lbm}}{\text{ft}^3} \cdot \dfrac{32.2 \text{ ft / s}^2}{32.2 \text{ lbm ft / lbf s}^2}$

$$= 1.87 \cdot 10^5 \text{ lbf} = 8.31 \cdot 10^5 \text{ N}$$

This is a serious problem in areas with high water tables; empty swimming pools are regularly ruined by being popped out of the ground by buoyant forces.

2.49* $B.F. = weight = 200 \text{ lbf} = Vg(\rho_{\text{air}} - \rho_{\text{helium}})$

$$V = \frac{B.F.}{g(\rho_{\text{air}} - \rho_{\text{helium}})} = \frac{200 \text{ lbf}}{32\frac{\text{ft}}{\text{s}^2} \cdot 0.075\frac{\text{lbm}}{\text{ft}^3}\left(1 - \frac{4}{29}\right)} \cdot \frac{32 \text{ lbf ft}}{\text{lbf s}^2} = 3.093 \cdot 10^3 \text{ft}^3 = 87.6 \text{ m}^3$$

$$V_{\text{individual}} = \frac{V_{\text{total}}}{42} = \frac{3093 \text{ ft}^3}{42} = 73.65 \text{ ft}^3 = \frac{\pi}{6} D^3$$

$$D = \left(\frac{6}{\pi} \cdot 73.65 \text{ ft}^3\right)^{1/3} = 5.2 \text{ ft} = 1.58 \text{ m}$$

2.50 In the boat, the part of the boat's displacement due to the block is

$$V = \frac{m}{\rho_{\text{water}}} = \frac{100 \text{ lbm}}{62.3 \text{ lbm / ft}^3} = 1.605 \text{ ft}^3$$

In the water, the displacement due to the block is

$$V = \frac{m}{\rho_{\text{steel}}} = \frac{100 \text{ lbm}}{7.9 \cdot 62.3 \text{ lbm / ft}^3} = 0.203 \text{ ft}^3$$

So the volume of the pond decreases by $\Delta V_{\text{pond}} = 0.203 - 1.605 = -1.402 \text{ ft}^3$ and its elevation falls by

$$dz = \frac{\Delta V_{\text{pond}}}{A_{\text{pond}}} = \frac{-1.40 \text{ ft}^3}{(\text{p} / 4) \cdot \text{p}(10 \text{ ft})^2} = -0.0179 \text{ ft} = 0.214 \text{ in}$$

This is a counter-intuitive result, which has made this type of problem a favorite puzzle for many years.

2.51 (a) $P_2 - P_1 = \rho g(h_1 + h_2)$

$$= \left(1.93 \cdot 62.3\frac{\text{lbm}}{\text{ft}^3}\right) \cdot \frac{32\frac{\text{ft}}{\text{s}^2}}{32\frac{\text{lbm ft}}{\text{lbf } s^2}} \cdot (44 + 8) \text{ in} \cdot \frac{\text{ft}}{12 \text{ in}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2} = 3.62 \text{ psig} = 24.96 \text{ kPa gauge}$$

(b) $P_2 = 3.61 \text{ psig} + 14.7 \text{ psia} = 18.3 \text{ psia} = 126.3 \text{ kPa abs}$

2.52*

$$P_2 = P_1 + \rho_{\text{w}} g(z_1 - z_2)$$

$$P_3 = P_2 + \rho_{\text{Hg}} g(z_2 - z_3)$$

$$P_4 = P_3 + \rho_{\text{w}} g(z_3 - z_4)$$

$$P_2 = P_4$$

adding these equations, canceling and grouping produces

$$0 = \rho_{\text{w}} g(z_1 - z_2 + z_3 - z_4) + \rho_{\text{Hg}} g(z_2 - z_3)$$

but $(z_1 - z_4) = \Delta z$ and $(z_2 - z_3) = \Delta h$ so that $0 = \rho_{\text{w}} g\Delta z + (\rho_{\text{Hg}} - \rho_{\text{w}})g\Delta h$ and

$$\Delta h = \Delta z \frac{\rho_{\text{w}}}{\rho_{\text{Hg}} = \rho_{\text{w}}}$$

2.53 $P_1 = P_a + \rho_1 g h_1$

$P_1 = P_a - \rho_1 g h_1$

$P_2 = P_1 - \rho_2 g \Delta h$

Adding these and canceling like terms, $P_b = P_a - \rho_2 g \Delta h + \rho_1 g (h_1 - h_2)$

$$P_b - P_a = g \Delta h (\rho_1 - \rho_2)$$

For maximum sensitivity one chooses $(\rho_1 - \rho_2)$ as small as possible. However as this quantity becomes small, the two fluids tend to mix, and may emulsify. So there is a practical limit on how small it can be made.

2.54* Do part (b) first $\Delta P = \rho g \Delta z$

$$\Delta z = \frac{\Delta P}{\rho g} = \frac{0.1 \dfrac{\text{lbf}}{\text{in}^2} \, 32 \dfrac{\text{lbm ft}}{\text{lbf s}^2}}{62.3 \dfrac{\text{lbm}}{\text{ft}^3} \, 32 \dfrac{\text{ft}}{\text{s}^2}} \cdot \frac{144 \text{in}^2}{\text{ft}^2} = 0.231 \text{ ft} = 2.77 \text{ in} = 70 \text{ mm}$$

Then do part (a)

$$\frac{\Delta z}{\Delta L} = \sin\theta, \ \Delta L = \frac{\Delta z}{\sin\theta} = \frac{2.77 \text{ in}}{\sin 15^{\circ}} = 10.72 \text{ in} = 272 \text{ mm}$$

Discussion; this shows the magnification of the reading obtained with a "draft tube", and hence why these are widely used for low pressure differences.

2.55 $A_1 \Delta L_1 = A_2 \Delta L_2$

$$\Delta L_2 = \frac{A_1}{A_2} \Delta L_1 = \left(\frac{D_1}{D_2} \right)^2 L_1 = \left(\frac{\dfrac{1}{8} \text{in}}{2 \text{in}} \right)^2 \cdot 10 \text{in} = 0.039 \text{ in}$$

Which is small enough that it is regularly ignored.

2.56

$$\text{(a)} \, h = \frac{P}{\rho g} = \frac{14.699 \dfrac{\text{lbf}}{\text{in}^2}}{13.6 \cdot 62.4 \dfrac{\text{lbm}}{\text{ft}^3} \cdot 32 \dfrac{\text{ft}}{\text{s}^2}} \cdot 32 \frac{\text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} = 2.493 \text{ ft} = 29.29 \text{ in} = 760 \text{ mm}$$

This is the set of values for the standard atmosphere, which is defined exactly as 101.325 kPa. That corresponds to 14.69595 psia. To get values in terms of heights of mercury one must use the density of mercury at some temperature. The values above, which lead to the values shown inside the front cover of the book are for mercury at ≈ 0°C. At 20°C the density of mercury is 13.52 g/cm^3, compared to 13.6 at 0 °C. Laboratory barometers are supplied with conversion tables, allowing one to correct for the thermal expansion of the mercury.

(b) $$h=\frac{P}{\rho g}=\frac{14.696\,\dfrac{\text{lbf}}{\text{in}^2}}{62.4\,\dfrac{\text{lbm}}{\text{ft}^3}\cdot 32\,\dfrac{\text{ft}}{\text{s}^2}}\cdot 32\,\frac{\text{lbm ft}}{\text{lbf s}^2}\cdot\frac{144\ \text{in}^2}{\text{ft}^2}=33.91\ \text{ft}=407\ \text{in}=10.34\ \text{m}$$

which is impractically tall.

(c) The vapor pressure of water at 20°C = 68°F is 0.3391 psia. This is 0.023 atm = 2.3% of an atmosphere. Thus the above calculation would replace the atmospheric pressure with 14.696 - 0.3391, reducing all the computed values by 2.3%.

A water barometer would need a temperature correction not only for the thermal expansion of the liquid, but also for the vapor pressure of the liquid which is about 0.006 atm at 0°C.

2.57 The pressure at ground level for a non-moving atmosphere must be

$$P_{z=0}=\int_{z=0}^{z=\infty}\rho g dz=\int_{z=0}^{z=\infty}\frac{PM}{RT}g dz=\left(\frac{PM}{RT}\right)_{\text{avg}} g z_{\text{top}}$$

Here the average is over the elevation from the ground to z_{top} which is the elevation below which all the mass of the atmosphere is. We now differentiate

$$\frac{dP}{dT_{\text{avg}}}=\left(\frac{PM}{R}\right)_{\text{avg}} g z_{\text{top}}\left(\frac{-1}{T_{\text{avg}}^2}\right);\qquad \frac{\Delta P}{g z_{\text{top}}\left(\dfrac{PM}{RT}\right)_{\text{avg}}}=\frac{\Delta P}{P_{z=0}}=-\frac{\Delta T_{\text{avg}}}{T_{\text{avg}}}$$

$$\Delta T_{\text{avg}}=-T_{\text{avg}}\frac{\Delta P}{P_{z=0}}$$

The troposphere contains ≈ 77% of the mass of the whole atmosphere, and has an average temperature (See Fig 2.4) of -5.35 °F ≈ 455°R ≈ -20°C.

Here $$\frac{P_{\text{high}}-P_{\text{low}}}{P_{z=0}}=\frac{1.025-0.995\ \text{atm}}{1\ \text{atm}}=0.003$$ so that

$$\Delta T_{\text{avg}}=-T_{\text{avg}}\frac{\Delta P}{P_{z=0}}=-455°\text{R}\cdot 0.003=1.4°\text{R}$$

An average decrease in average temperature through the troposphere of 1.4°F would account for the difference in pressure between a high and a low.

Changing moisture content changes the average molecular weight. Following exactly the same logic, we could say that if the temperatures were the same between a high and a low, the difference in molecular weight would have to be

$$\Delta M_{\text{avg}} = M_{\text{avg}} \frac{\Delta P}{P_{z=0}} = 29 \frac{\text{g}}{\text{mol}} \cdot 0.003 = 0.087 \frac{\text{g}}{\text{mol}}$$

At the average temperature of 455° R the vapor pressure of water is slightly less than 1 torr ≈ 0.0013 atm. If the atmosphere were saturated with water at -20°C, its change in average molecular weight would be

$$\Delta M_{\text{avg}} = 0.0013 \cdot (29 - 18) \frac{\text{g}}{\text{mol}} = 0.02 \frac{\text{g}}{\text{mol}}$$

which is about 1/4 of the required amount. (If one runs this calculation at 20°C, where the vapor pressure of water is 0.023 atm, one finds a change in M_{avg} of 0.26, but most of the atmosphere is much colder than it is at the surface). Thus it appears that the principal cause of atmospheric highs and lows is the difference in average temperature between air masses.

2.58* The pressure at the bottom of the dip tube is

$$P_{\text{bottom}} = P_{\text{top}} + \rho g h = 2\,\text{psig} + \left(0.08 \frac{\text{lbm}}{\text{ft}^3}\right) \frac{32 \frac{\text{ft}}{\text{s}^2}}{32 \frac{\text{lbm ft}}{\text{lbf s}^2}} \cdot 6\,\text{ft} \cdot \frac{\text{ft}^2}{144\,\text{in}^2} = 2.0033\,\text{psig}$$

(a) $$h = \frac{P}{\rho g} = \frac{2.0033\,\text{psig}}{60 \frac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32 \frac{\text{lbm ft}}{\text{lbf s}^2}}{32 \frac{\text{ft}}{\text{s}^2}} \cdot \frac{144\,\text{in}^2}{\text{ft}^2} = 4.808\,\text{ft} = 1.465\text{ m}$$

That is the depth at the end of the dip tube. The total depth is 4.808 + 0.5 =5.308 ft.

(b) The difference is 0.008 ft or 0.15% of the total. If we use 2.000 psig, the calculated depth is 4.800 ft.

Discussion: One can discuss here the other ways to continually measure the level in a tank. One can use float gages, pressure difference gages, etc. If the gas flow can be tolerated, this is simpler.

2.59 If we ignore the gas density, then $\Delta P = (\rho g h)_{\text{fluid}} = (\rho g h)_{\text{manometer}}$

$$\rho_{\text{fluid}} = \frac{h_{\text{manometer}}}{h_{\text{fluid}}} \cdot \rho_{\text{manometer}} = \frac{1.5\,\text{m}}{1.0\,\text{m}} \cdot 998.2 \frac{\text{kg}}{\text{m}^3} = 1497.3 \frac{\text{kg}}{\text{m}^3} = 93.45 \frac{\text{lbm}}{\text{ft}^3}$$

If we account for the gas density, then

$$\Delta P = (gh)_{\text{fluid}}(\rho_{\text{fluid}} - \rho_{\text{gas}}) = (gh)_{\text{manometer}}(\rho_{\text{manometer}} - \rho_{\text{gas}})$$

$$\rho_{\text{fluid}} = \rho_m\left(\frac{h_m}{h_f}\right) - \left(\frac{h_m}{h_f} - 1\right)\rho_{\text{gas}} = 1497.3 - (1.5 - 1)\cdot 1.20 = 1496.7\frac{\text{kg}}{\text{m}^3}$$

which is a difference of 0.04%.

Discussion: one can determine densities with hydrometers, or with some sonic gages. This system is simple and continuous.

2.60* See Ex. 2.16 $\quad P_A - P_B = gz_1\frac{P}{R}\left[\left(\frac{M}{T}\right)_{\text{air}} - \left(\frac{M}{T}\right)_{\text{flue gas}}\right]$

$$= \frac{32\frac{\text{ft}}{\text{s}^2}}{32\frac{\text{lbm ft}}{\text{lbf s}^2}}\frac{(100\text{ ft})(14.7\text{ psia})}{10.73\frac{\text{psia ft}^3}{\text{lbmol}^\circ\text{R}}}\left[\frac{29\frac{\text{lb}}{\text{lbmol}}}{530^\circ\text{R}} - \frac{28\frac{\text{lb}}{\text{lbmol}}}{760^\circ\text{R}}\right]\cdot\frac{\text{ft}^2}{144\text{ in}^2}$$

$$= 0.0170\text{ psi} = 0.47\text{ in } H_2O = 0.117\text{ kPa}$$

Discussion; This introduces the whole idea of the use of stacks for draft. Currently we use stacks to disperse pollutants, more than we use them for draft, but historically the height of the stack was determined by the required draft to overcome the frictional resistance to gas flow in the furnace.

2.61 $\quad P_{\text{top}} = P_{\text{bot}} - \rho_{\text{oil}}gh; \qquad P_{\text{bot}} = \rho_{\text{water}}gh$

$$P_{\text{top}} = gh(\rho_{\text{water}} - \rho_{\text{oil}}) = \frac{32\frac{\text{ft}}{\text{s}^2}}{32\frac{\text{lbm ft}}{\text{lbf s}^2}}\cdot 10^4\text{ ft}(1.03\cdot 62.3 - 55)\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{\text{ft}^2}{144\text{ in}^2}$$

$$= 637\text{ psig} = 4.39\text{ MPa}$$

2.62* $\quad P_2 = P_1\exp\left(\frac{-gM\Delta z}{RT}\right)$

$$\left(\frac{-gM\Delta z}{RT}\right) = \left[\frac{-\left(32\frac{\text{ft}}{\text{s}^2}\right)\left(16\frac{\text{lb}}{\text{lb mol}^\circ\text{R}}\right)(-10^4\text{ ft})}{32\frac{\text{lbm}}{\text{lbf}}\frac{\text{ft}}{\text{s}^2}\left(10.73\frac{\frac{\text{lbf}}{\text{in}^2}\text{ft}^3}{\text{lbmol}^\circ\text{R}}\right)}\right] = 0.1954$$

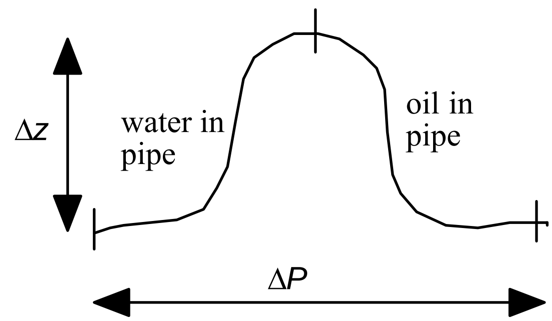
$$P_2 = 1014.7\text{ psia}\exp(+0.1954) = 1234\text{ psia} = 1219\text{ psig} = 8400\text{ kPa gauge}$$

(b) For constant density

$$P_2 = P_1\left(1 - \frac{g\,M\Delta z}{RT}\right) = 1014.7\,\text{psia}(1.1954) = 1213\,\text{psia} = 1198\,\text{psig} = \text{ 8257 kPa gauge}$$

$$\text{Error} = \frac{1234 - 1213}{1234} = 1.7\%$$

2.63 This is part of oil industry folklore; the pipeline was from the Central Valley of California to the Coast, and crossed the Coast Range of mountains. When the oil was slowly introduced, it flowed over the water, and bypassed water, leaving some in place. This led to situations like that sketched, in which the uphill leg of a rise was filled with water, while the downhill leg on the other side was filled with oil.



For any one such leg, the pressure difference is

$$\Delta P = gh\Delta\rho = \frac{32.2\text{ft / s}^2}{32.2\text{lbm ft / lbf s}^2}\cdot 200\text{ ft}\cdot(1-0.8)962.3\frac{\text{lbm}}{\text{ft}^3} = 17.3\text{ psig}$$

and for 10 such legs it is 173 psig.

2.64 $P_A = P_B - \rho g\Delta z$

$$= 14.7\,\text{psia} - 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{32\dfrac{\text{ft}}{\text{s}^2}}{32\dfrac{\text{lbm ft}}{\text{lbf s}^2}}\cdot 40\,\text{ft}\cdot\frac{\text{ft}^2}{144\text{ in}^2} = 14.7 - 17.3 = -2.6\,\text{psia ??}$$

This is an impossible pressure, the water would boil, and the pressure would be the vapor pressure of water, which at 70°F is 0.37 psia. This low a pressure will cause a normal pressure vessel to collapse. In oil industry folklore it is reported that this has happened several times. Normally a newly installed pressure vessel is hydrostatically tested, and then the workmen told to "drain the tank". They open valve B, and soon the tank crumples.

See: Noel de Nevers, "Vacuum Collapse of Vented Tanks", Process Safety Progress, 15, No 2, 74 - 79 (1996)

2.65* $P = \dfrac{F}{A} = \dfrac{25\text{ lbf}}{\dfrac{\pi}{4}(0.5\text{ in})^2} = 127.3\text{ psig}$

The point of this problem is to show the students how one calibrates spring-type pressure gauges. Common, cheap ones are not locally calibrated, but simply used with the factory design values. In the factory they are not individually calibrated because each one coming down the production line is practically the same at the next. For precise work, pressure gauges are calibrated this way.

2.66 No. This is a manometer, and if bubbles are present, they make the average density on one side of the manometer different from that on the other, leading to false readings. If all the bubbles are excluded, then this is an excellent way to get two elevations equal to each other.

2.67 $P_{\text{gage}} = \rho h\left(g + \dfrac{d^2z}{dz^2}\right) = \rho h\left(32.2\dfrac{\text{ft}}{\text{s}^2} - 32.2\dfrac{\text{ft}}{\text{s}}\right) = 0$

2.68* $P = \rho h\left(g + \dfrac{d^2z}{dz^2}\right)$

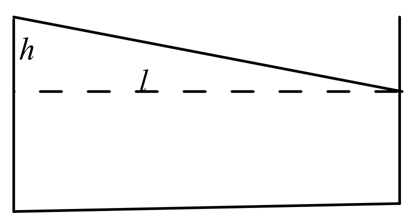
$$\frac{d^2z}{dz^2} = \frac{P}{\rho h} - g = \frac{5\dfrac{\text{lbf}}{\text{in}^2}}{62.3\dfrac{\text{lbm}}{\text{ft}^3}\cdot 8\text{ ft}} \cdot 32.2\frac{\text{lbm ft}}{\text{lbf s}^2}\cdot\frac{144\text{ in}^2}{\text{ft}^2} - 32.2\frac{\text{ft}}{\rho^2} = 46.5 - 32.2 = 14.3\frac{\text{ft}}{\text{s}^2} = 4.36\frac{\text{m}}{\text{s}^2}$$

The elevator is moving upwards. The horizontal part of the device plays no role.

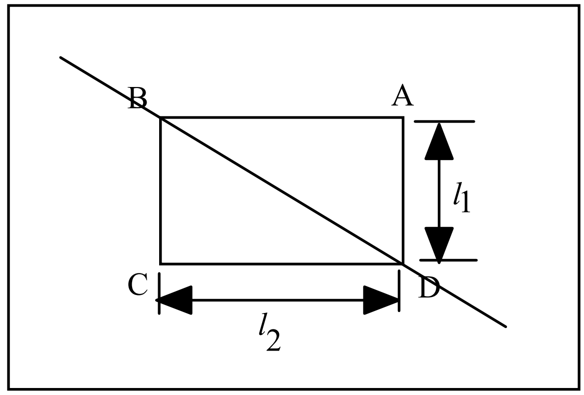
2.69 At spill, the elevation difference, h, from one end of the tank to the other is twice the original freeboard, or 2 ft.

$$\rho g h = \rho l \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{hg}{l} = \frac{2\text{ft}}{20\text{ ft}}\cdot 32.2\frac{\text{ft}}{\text{s}^2} = 3.32\frac{\text{ft}}{\text{s}^2}$$



2.70* As shown in the sketch at the right, the interface is a plane, passing through B and D. We compute the pressure at C two ways



$P_c = \rho_1 g l_1 + l_2 \rho_2 \dfrac{d^2x}{dt^2} + P_A = P_A + l_2 \rho_1 \dfrac{d^2x}{dt^2} + \rho_2 g l_1$ The pressure is the same either way we calculate it, so $\rho_1 g l_1 + l_2 \rho_2 \dfrac{d^2x}{dt^2} = +l_2 \rho_1 \dfrac{d^2x}{dt^2} + \rho_2 g l_1$ or

$g l_1 (\rho_1 - \rho_2) = l_2 \dfrac{d^2x}{dt^2} (\rho_1 - \rho_2)$ and $\dfrac{l_1}{l_2} = \dfrac{\frac{d^2x}{dt^2}}{g} = \tan\theta$ The division by $(\rho_1 - \rho_2)$ is only safe if this term is not zero. Physically, if the two fluids have exactly the same density, then they will not form a clean and simple interface. Most likely they will emulsify.

The numerical answer is

$$\theta = \arctan \frac{\frac{d^2x}{dt^2}}{g} = \arctan \frac{1\frac{\text{ft}}{\text{s}^2}}{32.2\frac{\text{ft}}{\text{s}^2}} = \arctan 0.03106\,\text{rad} = 0.03105\,\text{rad} = 1.779°$$

2.71 $P_{\text{gage}} = \rho\omega^2 \int_{14\,\text{m}}^{15\,\text{m}} r dr = \dfrac{\rho\omega^2}{2}\left[(15\,\text{in})^2 - (14\,\text{in})^2\right]$

$$= \frac{62.3\frac{\text{lbm}}{\text{ft}^3}}{2}\left[2\pi \frac{1000}{\text{min}} \cdot \frac{\text{min}}{60\,\text{s}}\right]^2 [225 - 196]\text{in}^2 \cdot \frac{\text{lbf s}^2}{32.17\,\text{lbm ft}} \cdot \left(\frac{\text{ft}^2}{144\,\text{in}^2}\right)^2$$

$= 14.85\,\text{psig} = 102.4\,\text{kPa gage}$

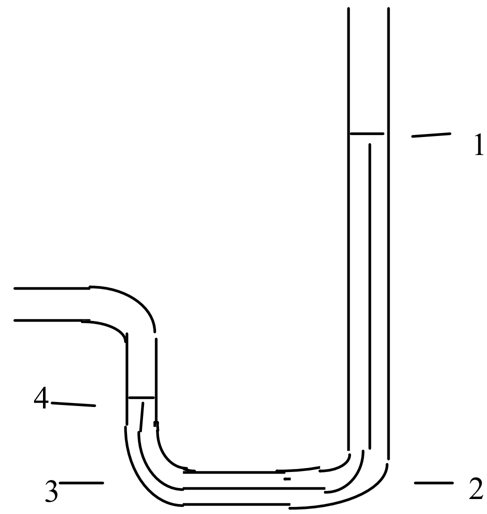
2.72* A rigorous solution would involve an integral of *P dA* over the whole surface. Here *P* is not linearly proportional to radius, so this is complicated. If we assume that over the size of this particle we can take an average radius, and calculate the "equivalent gravity" for that location, then we can use Archimedes' principle as follows.

(a) $BF=\rho V\omega^2 r=\left(62.3\dfrac{\text{lbm}}{\text{ft}^3}\right)\left(0.01\,\text{in}^3\right)\left(\dfrac{2\pi\cdot 1000}{\text{min}}\cdot\dfrac{\text{min}}{60\,\text{s}}\right)^2\left(15\,\text{in}\right)\dfrac{\text{lbf}\,\text{s}^2}{32.17\,\text{lbm}\,\text{ft}}\cdot\left(\dfrac{\text{ft}^2}{144\,\text{in}^2}\right)^2$

$=0.153\,\text{lbf}=0.683\ \text{N}$

(b) This points toward the axis of rotation.

2.73 See the sketch at the right. Work in gage pressure.



$P_1=0$
$P_2=P_1\rho g(h_2-h_1)$
$P_3=P_2+\dfrac{\rho\omega^2}{2}(r_3^2-r_2^2)$
$P_4=P_3+\rho g(h_4-h_3)$

Adding these and canceling like terms

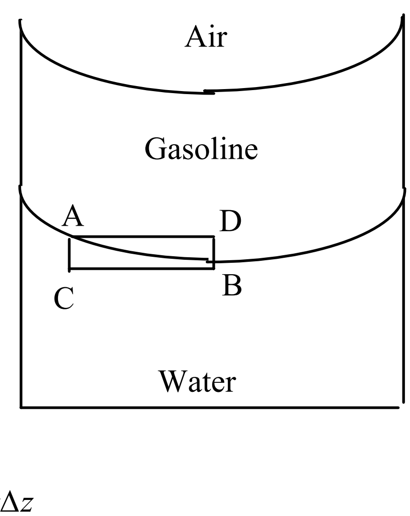
$$P_4=\rho\left[g(h_4-h_1)+\frac{\omega^2}{2}\left(r_3^2-r_2^2\right)\right]$$

$$\omega=\frac{2\pi\cdot 10}{60\ \text{s}}=1.0472\frac{1}{\text{s}}$$

$$P_4=62.3\frac{\text{lbm}}{\text{ft}^3}\left[32.2\frac{\text{ft}}{\text{s}^2}\cdot 4\ \text{ft}+\frac{1}{2}\cdot\left(\frac{1.0472}{\text{s}}\right)^2\cdot\left((22\ \text{ft})^2-(24\,\text{ft})^2\right)\right]\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2}$$

$$=62.3\cdot[128.8-50.44]\frac{}{32.2\cdot 144}=1.053\ \text{psig}=7.26\,\text{kPa gauge}$$

2.74 The gasoline-air interface is a parabola, as shown in example 2.19 Using the sketch at the right, we compute the pressure at C, two ways, finding



By path DBC

$$P_C=P_D+\rho_{\text{gasoline}}g\Delta z+\rho_{\text{water}}\frac{\omega^2}{2}(\Delta r)^2$$

By path DAC

$$P_C=P_D+\rho_{\text{gasoline}}\frac{\omega^2}{2}(\Delta r)^2+\rho_{\text{water}}g\Delta z$$

Subtracting the second from the first

$$0=\left(\rho_{\text{water}}-\rho_{\text{gasoline}}\right)\frac{\omega^2}{2}(\Delta r)^2-\left(\rho_{\text{water}}-\rho_{\text{gasoline}}\right)g\Delta z$$

or $$\Delta z = \frac{\omega^2 (\Delta r)^2}{2g}$$

Which is the same as Eq. 2.38, for air and water. Two parabolas are the same.

2-75 Yes, it will work! If one solves it as a manometer, from the top of the funnel to the nozzle of the jet, one will see that there is a pressure difference equal to the difference in height of the liquid in the two bottles times g times (density of water - density of air). This is clearly not a steady-state device; as it flows the levels become equal and then the flow stops.

Solutions, Chapter 3

3.1 Migrant agricultural workers, airline pilots who have homes or apartments at both ends of their regular route, military personnel on station, truck drivers passing through the state, rock artists in the state for a concert, traveling salespersons, construction workers, visiting drug dealers.

3.2 Boundaries: USA including money in bank vaults, in people's mattresses, etc.

Accumulation = + number printed - number destroyed (either intentionally by treasury department which destroys old bills, or unintentionally, for example in a fire in a supermarket) + flow in across borders (mostly with tourists, some by inter-bank shipments) - flow out across borders

Currently not too many $1 bills circulate outside the USA, but the US $100 bill is a common medium of exchange outside the USA, including both real and counterfeit bills.

3.3 Accumulation = amount refined (in sugar plants) - amount consumed (both by people who put it in the coffee or cereal and by food manufacturers who add it to processed foods) + flow in across state borders (normally in trucks or railroad cars) - flow out across state borders, either in bulk carriers or packaged for sale in stores.

3.4 Boundaries: the skin of the balloon, including the plane of the opening.

Accumulation = - flow out.

The boundaries are not fixed in space, nor are they fixed in size, but they are readily identifiable.

3.5 Boundaries: The auto including all parts, with boundaries crossing the openings in the grille and the exhaust pipe.

Accumulation = 0 - 0 + flow in (carbon dioxide and other carbon-bearing gases in inflowing air, carbon in bugs smashing into the windshield or in road oil thrown up on body, bird droppings falling on car) - flow out (carbon dioxide and carbon monoxide in exhaust gas, fuel leakage, oil leakage, anti-freeze leakage, tire wear, paint flaking off, carbon dioxide and other carbon compounds in driver and passenger's breaths, leaking out through windows, carbon in paper and trash thrown out the windows by litterers).

The point is that there is one main flow out, the carbon dioxide in the exhaust, which equals the negative accumulation of carbon in the fuel, and then a lot of other minor terms.

3.6 Here defining the boundaries is the big problem. If we choose as boundaries those atoms originally in the firecracker, then

accumulation = 0

If we choose the topologically smallest envelope that includes all the solid parts, then

accumulation = - flow out of gaseous products, which may move faster than solid products + flow in of air engulfed in the expanding envelope.

3-7* $$Q = VA = 1\frac{\text{ft}}{s}\cdot 10\,\text{ft}\cdot 50\,\text{ft}\cdot\frac{60\,\text{s}}{\text{min}}\cdot\frac{7.48\,\text{gal}}{\text{ft}^3} = 2.244\cdot 10^5\frac{\text{gal}}{\text{min}} = 500\frac{\text{ft}^3}{\text{s}} = 14.16\frac{\text{m}}{\text{s}}$$

$$V_2 = V_1\frac{A_1}{A_2} = 1\frac{\text{ft}}{\text{s}}\cdot\frac{500\,\text{ft}^2}{7\cdot 150\,\text{ft}^2} = 0.476\frac{\text{ft}}{\text{s}} = 0.145\frac{\text{m}}{\text{s}}$$

3.8 $$Q = 10^7\frac{\text{acre ft}}{\text{yr}}\cdot 4.35\cdot 10^4\frac{\text{ft}^3}{\text{acre ft}} = 4.35\cdot 10^{11}\frac{\text{ft}^3}{\text{yr}}$$

$$V = \frac{Q}{A} = \frac{4.35\cdot 10^{11}\frac{\text{ft}^3}{\text{yr}}}{2000\,\text{ft}^2} = 2.17\cdot 10^8\frac{\text{ft}}{\text{yr}} = 6.9\frac{\text{ft}}{\text{s}} = 4.7\frac{\text{mi}}{\text{hr}}$$

3.9 (a) $$V = \frac{Q}{A} = \frac{45\,000\frac{\text{ft}^3}{\text{s}}}{8\cdot(\pi/4)\cdot(8\,\text{ft})^2} = 111.9\frac{\text{ft}}{\text{s}}$$

(b) $$V = \frac{Q}{A} = \frac{45\,000\frac{\text{ft}^3}{\text{s}}}{200\,\text{ft}\cdot 8\,\text{ft}} = 22.5\frac{\text{ft}}{\text{s}} = 15.34\frac{\text{mi}}{\text{hr}} = 2.10\frac{\text{m}}{\text{s}}$$

The purpose of the artificial flood was to mimic the spring floods which regularly passed down the canyon before the construction of the Glen Canyon Dam (1963). That was supposed to reshape the beaches and in other ways make the canyon more like its pristine state. Those who organized the flood pronounced it a great success.

3.10 $$V_{\text{average}} = \frac{Q}{A} = \frac{\int_{r=0}^{r=r_{\text{wall}}} V\cdot 2\pi r\,dr}{\pi r_{\text{wall}}^2}$$

a) Substituting Eq. 3.21 and simplifying,

$$V_{\text{average}} = \frac{Q}{A} = \frac{\int_{r=0}^{r=r_{\text{wall}}} V_{\max}\left(\frac{r_w^2 - r^2}{r_w^2}\right) \cdot 2\pi r dr}{\pi r_{\text{wall}}^2} = \frac{2V_{\max}}{r_{\text{wall}}^4}\left[\frac{r_w^2 r^2}{2} - \frac{r^4}{4}\right]_0^{r_w} = \frac{V_{\max}}{2}$$

$$V_{\max} = 2V_{\text{average}}$$

(b) Substituting Eq. 3.22 and simplifying,

$$V_{\text{average}} = \frac{Q}{A} = \frac{\int_{r=0}^{r=r_{\text{wall}}} V_{\max}\left(\frac{r_w - r}{r_w}\right)^{1/7} \cdot 2\pi r dr}{\pi r_{\text{wall}}^2} = \frac{2V_{\max}}{r_{\text{wall}}^2}\int_{r=0}^{r=r_{\text{wall}}}\left(1 - \frac{r}{r_w}\right)^{1/7} \cdot r dr$$

$$= \frac{2V_{\max}}{r_{\text{wall}}^2} \cdot r_{\text{wall}}^2\left[\frac{\left(1-\frac{r}{r_w}\right)^{2\frac{1}{7}}}{2\frac{1}{7}} - \frac{\left(1-\frac{r}{r_w}\right)^{1\frac{1}{7}}}{1\frac{1}{7}}\right]_{r=0}^{r=r_w} = \frac{2}{2\frac{1}{7}\cdot 1\frac{1}{7}} V_{\max} = \frac{49}{60} V_{\max} = 0.8166 V_{\max}$$

$$V_{\max} = \frac{V_{\text{average}}}{0.8166} = 1.224 V_{\text{average}}$$

(c) $$V_{\text{average}} = \frac{Q}{A} = \frac{\int_{r=0}^{r=r_{\text{wall}}} V_{\max}\left(\frac{r_w - r}{r_w}\right)^{1/10} \cdot 2\pi r dr}{\pi r_{\text{wall}}^2} = \frac{2V_{\max}}{r_{\text{wall}}^2}\int_{r=0}^{r=r_{\text{wall}}}\left(1 - \frac{r}{r_w}\right)^{1/10} \cdot r dr$$

$$= \frac{2V_{\max}}{r_{\text{wall}}^2} \cdot r_{\text{wall}}^2\left[\frac{\left(1-\frac{r}{r_w}\right)^{2\frac{1}{10}}}{2\frac{1}{10}} - \frac{\left(1-\frac{r}{r_w}\right)^{1\frac{1}{10}}}{1\frac{1}{10}}\right]_{r=0}^{r=r_w} = \frac{2}{2\frac{1}{10}\cdot 1\frac{1}{10}} V_{\max} = \frac{200}{231} V_{\max} = 0.8656\ V_{\max}$$

$$V_{\max} = \frac{V_{\text{average}}}{0.8568} = 1.155\ V_{\text{average}}$$

3.11 (a) Substituting Eq. 3.21 in Eq. 3.23 and simplifying,

$$\begin{pmatrix}\text{average kinetic}\\ \text{energy, per}\\ \text{unit mass}\end{pmatrix} = \frac{\int_{r=0}^{r=r_{\text{wall}}}\left[V_{\max}\left(\frac{r_w^2 - r^2}{r_w^2}\right)\right]^3 \cdot r dr}{r_{\text{wall}}^2 V_{\text{average}}} = \frac{V_{\max}^3}{V_{\text{avg}} r_{\text{wall}}^8}\int_{r=0}^{r=r_{\text{wall}}}\left[\left(r_w^6 - 3r_w^4 r^2 + 3r_w^2 r^4 - r^6\right)\right] \cdot r dr$$

$$= \frac{V_{\max}^3}{V_{\text{avg}} r_{\text{wall}}^8} \cdot r_{\text{wall}}^8\left[\frac{1}{2} - \frac{3}{4} + \frac{3}{6} - \frac{1}{8}\right] = \frac{V_{\max}^3}{V_{\text{avg}}} \cdot \frac{1}{8}$$

But for laminar flow $V_{max} = 2V_{avg}$, so that

$$\begin{pmatrix}\text{average kinetic}\\ \text{energy, per}\\ \text{unit mass}\end{pmatrix}_{\text{laminar flow}} = \frac{(2V_{avg})^3}{V_{avg}} \cdot \frac{1}{8} = V_{avg}^2 = 2 \cdot \frac{V_{avg}^2}{2}$$

Substituting Eq. 3.22 and simplifying,

$$\begin{pmatrix}\text{average kinetic}\\ \text{energy, per}\\ \text{unit mass}\end{pmatrix} = \frac{\int_{r=0}^{r=r_{wall}} \left[V_{max}\left(1-\frac{r}{r_w}\right)^{1/7}\right]^3 \cdot rdr}{r_{wall}^2 V_{average}} = \frac{V_{max}^3}{V_{avg} r_{wall}^2} \int_{r=0}^{r=r_{wall}} \left(1-\frac{r}{r_w}\right)^{3/7} \cdot rdr$$

$$= \frac{V_{max}^3}{V_{avg} r_{wall}^2} \cdot r_{wall}^2 \left| \frac{\left(1-\frac{r}{r_w}\right)^{2\frac{3}{7}}}{2\frac{3}{7}} - \frac{\left(1-\frac{r}{r_w}\right)^{1\frac{3}{7}}}{1\frac{3}{7}} \right|_{r=0}^{r=r_w} = \frac{1}{2\frac{3}{7} \cdot 1\frac{3}{7}} \frac{V_{max}^3}{V_{avg}} = \frac{49}{170} \frac{V_{max}^3}{V_{avg}} = 0.2882 \frac{V_{max}^3}{V_{avg}}$$

But, from the preceding problem

$$V_{max} = 1.224 V_{average}$$

So that

$$\begin{pmatrix}\text{average kinetic}\\ \text{energy, per}\\ \text{unit mass}\end{pmatrix}_{\substack{\text{turbulent flow}\\ \text{1/7 th power}\\ \text{approximation}}} = 0.2882 \frac{V_{max}^3}{V_{avg}} = 0.2882 \frac{(1.224 V_{avg})^3}{V_{avg}} = 0.5285 V_{avg}^2 = 1.057 \frac{V_{avg}^2}{2}$$

(b) The total momentum flow in a pipe is given by

$$\begin{pmatrix}\text{total momentum}\\ \text{flow}\end{pmatrix} = \int_{\text{whole flow area}} Vdm = \int_{r=0}^{r=r_{wa;;}} V \cdot \rho V \cdot 2\pi rdr \qquad (3.??)$$

Substituting Eq 3.21 we find

$$\begin{pmatrix}\text{total momentum}\\ \text{flow}\end{pmatrix} = \int_{r=0}^{r=r_{wall}} \left[V_{max}\left(\frac{r_w^2 - r^2}{r_w^2}\right)\right]^2 \rho 2\pi rdr = \frac{V_{max}^2 2\pi\rho}{r_w^4} \int_{r=0}^{r=r_{wa;;}} \left[r_w^2 - r^2\right]^2 rdr$$

$$= \frac{V_{max}^2 2\pi\rho}{r_w^4} \left[\frac{r_w^4 r^2}{2} - 2\frac{r_w^2 r^4}{4} + \frac{r^6}{6}\right]_{r=0}^{r=r_w} = V_{max}^2 2\pi r_w^2 \rho \frac{1}{6}$$

But for laminar flow $V_{max} = 2V_{avg}$, so that

$$\begin{pmatrix}\text{total momentum}\\ \text{flow}\end{pmatrix}_{\text{laminar flow}} = V_{max}^2\, 2\pi r_w^2 \rho \frac{1}{9} = \left(2V_{avg}\right)^2 2\pi r_w^2 \rho \frac{1}{6} = 1.333\pi r_w^2 \rho V_{avg}^2$$

Substituting Eq 3.22, we find

$$\begin{pmatrix}\text{total momentum}\\ \text{flow}\end{pmatrix} = \int_{r=0}^{r=r_{wa;;}} \left[V_{max}\left(1-\frac{r}{r_0}\right)^{1/7}\right]^2 \rho 2\pi r dr = V_{max}^2 2\pi\rho r_{wall}^2 \left|\frac{\left(1-\frac{r}{r_w}\right)^{2\frac{21}{7}}}{2\frac{2}{7}} - \frac{\left(1-\frac{r}{r_w}\right)^{1\frac{21}{7}}}{1\frac{2}{7}}\right|_{r=0}^{r=r_w}$$

$$= V_{max}^2\, 2\pi\rho r_{wall}^2 \frac{1}{2\frac{2}{7}\cdot 1\frac{2}{7}} = V_{max}^2 2\pi\rho r_{wall}^2 \frac{49}{144} = 0.3403 V_{max}^2 2\pi\rho r_{wall}^2$$

But, from the preceding problem

$$V_{max} = 1.224 V_{average}$$

$$\begin{pmatrix}\text{total momentum}\\ \text{flow}\end{pmatrix}_{\substack{\text{turbulent flow}\\ 1/7\text{ power}\\ \text{approximatin}}} = 0.3403\left(1.224 V_{avg}\right)^2 2\pi\rho r_{wall}^2 = 1.0137\rho\pi r_{wall}^2 V_{average}^2$$

These are the values in Tab. 3.1

3.12 $\dot{m} = Q\rho = \frac{\pi}{4} D^2 V \cdot \frac{PM}{RT}$; $\quad V = \frac{\dot{m}RT}{(\pi/4)D^2}\frac{1}{PM}$

The velocity increases as the pressure falls. This has the paradoxical consequence that friction, which causes the pressure to fall, causes the velocity to increase. This type of flow is discussed in Ch. 8.

3.13* Assuming that the soldiers do not change their column spacing

$$Q = \rho_1 V_1 A_1 = \rho_2 V_2 A_2; \quad V_2 = V_1 \frac{A_1}{A_2} = \frac{4\text{ mi}}{\text{hr}} \cdot \frac{12}{10} = 4.8\frac{\text{mi}}{\text{hr}}$$

3.14* $\frac{dm}{dt} = 0 = \sum \dot{m} = \rho \sum Q; \quad Q_3 = Q_1 - Q_2$

$$Q_3 = \frac{\pi}{4}(1\text{ ft})^2 5\frac{\text{ft}}{\text{s}} - \frac{\pi}{4}\left(\frac{1}{2}\text{ft}\right)^2 7\frac{\text{ft}}{\text{s}} = \frac{\pi}{4}\left(5 - \frac{7}{4}\right) = 2.55\frac{\text{ft}^3}{\text{s}}$$

$$\dot{m}_3 = \rho Q_3 = 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot 2.55\frac{\text{ft}^3}{\text{s}} = 159\frac{\text{lbm}}{\text{s}}$$

$$V_3 = \frac{Q_3}{A_3} = \frac{2.55\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}\left(\frac{1}{2}\text{ ft}\right)^2} = 13\frac{\text{ft}}{\text{s}}$$

3.15 $P = \frac{\rho RT}{M} = \frac{m}{\upsilon}\frac{RT}{M};\qquad \frac{dP}{dt} = \frac{RT}{\upsilon m}\frac{dm}{dr} = \frac{RT}{MV}\dot{m}$

$$\frac{dP}{dt} = \frac{10.73\frac{\text{psi ft}^3}{\text{lb mol }^\circ\text{K}}\cdot 530^\circ\text{R}\cdot 10\frac{\text{lbm}}{\text{hr}}}{10\text{ ft}\cdot 29\frac{\text{lbm}}{\text{lb mole}}} = 196\frac{\text{psi}}{\text{hr}}$$

Students regularly try to find a way to include the current value of the pressure, 100 psig, in their answer. It plays no role. The answer would be the same if the initial pressure were any value, as long as the pressure was low enough for the ideal gas law to be appropriate.

3.16* From Eq. 3.AC, $P_{\text{steady state}} = 1\text{ atm}\cdot\frac{0.001\frac{\text{lbm}}{\text{ft}^3}}{0.075\frac{\text{lbm}}{\text{ft}^3}} = 0.0133\text{ atm}$

3.17 (a) Start with Eq 3.17, take the exponential of both sides

$$\frac{\rho_{\text{sys, final}} - \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}}}{\rho_{\text{sys, inital}} - \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}}} = \exp\left(-\frac{Q_{\text{out}}}{V_{\text{sys}}}\Delta t\right) = \alpha$$

where α is used to save writing.

Then $\rho_{\text{sys, final}} - \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}} = \alpha\left(\rho_{\text{sys, inital}} - \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}}\right)$: $\qquad \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}}(\alpha - 1) = \alpha\rho_{\text{sys, initiall}} - \rho_{\text{sys, final}}$

$\frac{\dot{m}_{\text{in}}}{Q_{\text{out}}} = \frac{\alpha\rho_{\text{sys, initial}} - \rho_{\text{sys, final}}}{\alpha - 1}$ Here $\rho_{\text{sys, inital}} = 0.075\text{ lbm / ft}^3$ and

$\rho_{\text{sys, final}} = 0.001\cdot 0.075\text{ lbm / ft}^3$

$$\alpha = \exp\left(-\frac{1\frac{\text{ft}^3}{\text{min}}}{10\text{ ft}^3}72\text{ min}\right) = 0.00074659$$

$$\frac{\dot{m}_{\text{in}}}{Q_{\text{out}}} = \frac{7.4659\cdot 10^{-5}\cdot 0.075\dfrac{\text{lbm}}{\text{ft}^3} - 0.001\cdot 0.075\dfrac{\text{lbm}}{\text{ft}^3}}{7.4659\cdot 10^{-5} - 1} = 1.902\cdot 10^{-5}\,\frac{\text{lbm}}{\text{ft}^3}$$

$$\dot{m}_{\text{in}} = 1.902\cdot 10^{-5}\,\frac{\text{lbm}}{\text{ft}^3}\cdot 10\frac{\text{ft}^3}{\text{min}} = 1.902\cdot 10^{-4}\,\frac{\text{lbm}}{\text{min}}$$

(b) To find the steady state pressure we set t = infinity, which leads to

$$\rho_{\text{steady state}} = \frac{\dot{m}_{\text{in}}}{Q_{\text{out}}} = 1.902\cdot 10^{-5}\,\frac{\text{lbm}}{\text{ft}^3}$$

$$\frac{P_{ss}}{P_0} = \frac{\rho_{ss}}{\rho_0} = \frac{1.902\cdot 10^{-5}\dfrac{\text{lbm}}{\text{ft}^3}}{0.075\dfrac{\text{lbm}}{\text{ft}^3}} = 2.536\cdot 10^{-4};\quad P_{ss} = 2.536\cdot 10^{-4}\ \text{atm}$$

3.18 $V\dfrac{d\rho}{dt} = -Q\rho + \dot{m}_{\text{in}} = -Q\rho + a(P_0 - P)$ where a is a place holder.

$$\frac{VM}{RT}\frac{dP}{dt} = -\frac{QM}{RT}P + a(P_0 - P)$$

$$\frac{dP}{dt} = -\frac{Q}{V}P + \frac{aRT}{VM}(P_0 - P) = P\left[-\frac{Q}{V} - \frac{aRT}{VM}\right] + \frac{P_0 aRT}{VM} = \alpha P + \beta$$

$$\int\frac{dP}{P + \dfrac{\beta}{\alpha}} = \alpha\int dt;\quad t - 0 = \frac{1}{\alpha}\ln\frac{P + \dfrac{\beta}{\alpha}}{P_0 + \dfrac{\beta}{\alpha}}$$

$$\alpha = -\frac{Q}{V} - \frac{aRT}{VM} = -\frac{\dfrac{1\,\text{ft}^3}{\text{min}}}{10\,\text{ft}^3} - \frac{5\cdot 10^{-4}\,\text{lbm}}{\text{min atm}}\cdot\frac{0.7302\,\text{atm ft}^3}{\text{lb mol}\,^\circ\text{R}}\,\frac{530^\circ\text{R}}{10\,\text{ft}^3\cdot 29\dfrac{\text{lbm}}{\text{lbmol}}}$$

$$\alpha = -0.10 - 6.672\cdot 10^{-4} = -\frac{0.100667}{\text{min}}\qquad \beta = 6.6672\cdot 10^{-4}\,\frac{\text{atm}}{\text{min}}$$

$$\frac{\beta}{\alpha} = -6.6628\cdot 10^{-3}\ \text{atm}$$

$$t = \frac{1}{-\dfrac{0.10067}{\text{min}}}\ln\frac{0.01\,\text{atm} - 0.006628\,\text{atm}}{1\,\text{atm} - 0.006628\,\text{atm}} = 56.5\,\text{min}$$

At steady state $\qquad P = -\beta/\alpha = -(-0.006628\,\text{atm}) = 0.006628\,\text{atm}$

If one assumes that the leak rate is 0.0005 lbm/min, independent of pressure, then repeating Ex. 3.8 we find a final pressure of 0.00666 atm instead of 0.00663 calculated here, and a time to 0.01 atm of 56.97 min, instead of the 56.50 min calculated here. All real vacuum systems have leaks whose flow rate is more or less proportional to the pressure difference from outside to in (assuming laminar flow in small flow passages) but this comparison shows that replacing Eq. 3.AN, which is a more reasonable representation of the leak rate with a constant leak rate makes very little difference in the computed final pressure and time to reach any given pressure. The reason is that the pressure in the system falls rapidly to a small fraction of an atmosphere, and thus the leakage rate quickly becomes ≈ independent of pressure.

3.19* $\frac{1}{\rho}\frac{dm}{dt} = \frac{dV}{dt} = \frac{1}{\rho}\left(\dot{m}_{\text{in}} - \dot{m}_{\text{out}}\right) = 10000 - 8000 = 2000\frac{\text{m}^3}{\text{s}}$

$$\frac{dh}{dt} = \frac{1}{A}\frac{dV}{dt} = \frac{2000\frac{\text{m}^3}{\text{s}}}{100 \cdot 10^6 \text{m}^2} = 2 \cdot 10^{-5}\frac{\text{m}}{\text{s}} = 72\frac{\text{mm}}{\text{hr}} = 2.8\frac{\text{in}}{\text{h}}\text{, rising}$$

3.20 All the flows are incompressible, so $\left(\frac{dV}{dt}\right)_{\text{sys}} = 0 = \sum Q_{\text{in and out}}$ and

$$Q_{\text{vent}} = -Q_{\text{in}} + Q_{\text{out}} = -0.5\,\text{ft}^2\left(12\frac{\text{ft}}{\text{s}}\right) + 0.3\,\text{ft}^2\left(16\frac{\text{ft}}{\text{s}}\right) = -1.2\frac{\text{ft}^3}{\text{s}}$$

The minus sign indicates a flow out from the tank through the vent, and

$$\dot{m}_{\text{vent}} = Q\rho = \left(-1.2\frac{\text{ft}^3}{\text{s}}\right)\left(0.075\frac{\text{lbm}}{\text{ft}^3}\right) = -0.090\frac{\text{lbm}}{\text{s}} = -0.0408\frac{\text{kg}}{\text{s}}$$

3.21* $\frac{dP}{dt} = \text{const} = 0.1\frac{\text{psi}}{\text{min}}$**:** $\qquad \frac{dP}{dt} = \frac{RT}{V}\frac{dn}{dt}$

$$\frac{dn}{dt} = \dot{n}_{\text{in}} = \frac{V}{RT}\frac{dP}{dt} = \frac{10\,\text{ft}^3 \cdot 0.1\frac{\text{psi}}{\text{min}}}{\frac{10.73\text{psi ft}^3}{\text{lb mol }^\circ\text{R}} \cdot 530^\circ\text{R}} = 1.758 \cdot 10^{-4}\frac{\text{lb mol}}{\text{min}}$$

$$\dot{m}_{\text{in}} = \dot{n}_{\text{in}} \cdot M = 1.758 \cdot 10^{-4} \cdot 29 = 0.0051\frac{\text{lbm}}{\text{min}} = 0.0023\frac{\text{kg}}{\text{min}}$$

3.22 $\frac{d(Vc)_{\text{system}}}{dt} = (cQ)_{\text{in}} - (cQ)_{\text{out}}$ where c is the concentration. Here

V_{sys} is constant and $c_{\text{out}} = c_{\text{sys}}$, and $c_{\text{in}} = 0$ so that $V\frac{dc}{dt} = -Qc$

$$\frac{dc}{c} = -\frac{Q}{V}dt; \quad \ln\frac{c}{c_0} = -\frac{Q}{V}(t-0) \text{ and}$$

$$c = c_0 \exp\left(-\frac{Q}{V}\Delta t\right) = 10\frac{\text{kg}}{\text{m}^3}\exp\left(-\frac{10\text{ m}^3/\text{min}}{1000\text{ m}}t\right)$$

which is the same as Eq 3.16 (after taking the exponential of both sides) with concentration replacing density, and the Q/V one tenth as large as in Ex. 3.7. One can look up the answers on Fig 3.5, if one takes the vertical scale as c/c_0 and multiplies the time values on the horizontal scale by 10.

3.23 $V\dfrac{dc}{dt} = -Qc + K; \quad K = \text{dissolution rate}$

$$\frac{dc}{c - K/Q} = -\frac{Q}{V}dt; \;\; \ln\left|\frac{c - K/Q}{c_0 - K/Q}\right| = -\frac{Q}{V}(t-0);$$

$$c = (c_0 - K/Q)\exp(-Qt/K) + K/Q$$

Comparing this solution line-by-line with Ex. 3.8, we see that it is the same, with the variables renamed. Its analogs also appear in heat and mass transfer.

3.24 $V_{\text{sys}} = a + bt; \quad \left(b = \dfrac{1\text{ m}^3}{\text{min}}\right)$

$$\frac{d(cV)_{\text{sys}}}{dt} = \frac{d(c(a+bt))}{dt} = (Qc)_{\text{in}} - (Qc)_{\text{out}} \;\; \text{but } (Qc)_{\text{in}} = 0 \text{ so that}$$

$$a\frac{dc}{dt} + bc + bt\frac{dc}{dt} = Q_{\text{our}}c; \quad (a+bt)\frac{dc}{dt} = -(Q_{\text{our}} + b)c \;;\; \frac{dc}{c} = -(Q+b)\frac{dt}{a+bt}$$

$$\ln\frac{c}{c_0} = -(Q_{\text{out}} + b)\frac{1}{b}\ln\frac{a+bt}{a}; \frac{c}{c_0} = \left(\frac{1}{1+(b/a)t}\right)^{\left(1+\frac{Q_{\text{our}}}{b}\right)}$$

3.25 $V_{\text{sys}} = a + bt; \; a = 10\text{ ft}^3, \; b = -0.1\dfrac{\text{ft}^3}{\text{min}};$

$$\frac{d(\rho V)_{sys}}{dt} = \frac{d(\rho(a+bt))_{sys}}{dt} = -m_{out} = -Q\rho$$

Comparing this to the solution to the preceding problem, we see that they are the same, with the variables renamed, so that we can simply copy,

$$\frac{\rho}{\rho_0} = \left(\frac{1}{1+(b/a)t}\right)^{\left(1+\frac{Q}{b}\right)}; \qquad 1 + (b/a)t = \frac{1}{(\rho/\rho_0)^{\frac{1}{1+Q/b}}}$$

$$t = \frac{a}{b}\left[\frac{1}{(\rho/\rho_0)^{\frac{1}{1+Q/b}}} - 1\right] = \frac{10\text{ ft}^3}{-0.1\dfrac{\text{ft}^3}{\text{min}}}\left[\frac{1}{(10^{-4})^{\left(\frac{1}{1+(1\text{ ft}^3/\text{min})/(-0.1\text{ ft}^3/\text{min})}\right)}} - 1\right]$$

$$= -100\,\text{min}\left\lfloor \frac{1}{2.78256} - 1 \right\rfloor = 64.06\,\text{min}$$

3.26* Assume that the earth is a sphere with radius 4000 miles, that it is 3/4 covered by oceans, and that the average depth is 1 mile. That makes the volume of the oceans be

$$V_{\text{ocean}} = \frac{3}{4} \cdot 4\pi \cdot (5280 \cdot 4000\ \text{ft})^2 \cdot (5280\,\text{ft}) \cdot \left(\frac{\text{m}}{3.281\,\text{ft}} \right)^3 \cdot \left(\frac{1000\,l}{\text{m}^3} \right) = 6.3 \cdot 10^{20}\ \text{liters}$$

$$\frac{\text{molecules}}{\text{liter}} \approx \frac{1000\,\text{gm}}{\dfrac{18\,\text{gm}}{\text{mol}}} \cdot 6.023 \cdot 10^{23}\,\frac{\text{molecules}}{\text{mole}} = 3.34 \cdot 10^{25}$$

This ignores the differences between pure water and ocean water, which is negligible compared to the other assumptions. Then, a randomly-selected liter contains

$\dfrac{3.34 \cdot 10^{25}}{6.3 \cdot 10^{20}} = 5.3 \cdot 10^4$ molecules which were in the liter that Moses examined.

The assumption of perfect mixing of the oceans since Moses' time is not very good. The deep oceans mix very slowly. The Red Sea does not mix much with the rest of the world ocean, except by evaporation and rainfall.

3.27 The mass of the atmosphere is

$$\text{mass} = 14.7\frac{\text{lbm}}{\text{in}^2} \cdot 4\pi(4000 \cdot 5280 \cdot 12\ \text{in})^2\, 1.186 \cdot 10^{19}\ \text{lbm}$$

$$\text{mols} = \text{mass} \cdot \frac{454\ \text{gm}}{\text{lbm}} \cdot \frac{\text{mol}}{29\ \text{gm}} = 1.857 \cdot 10^{20}\,\text{mols}$$

Caesar lived 56 years, so he breathed in his lifetime

$$56\ \text{years} \cdot \frac{365\ \text{day}}{\text{yr}} \cdot \frac{24\,\text{hr}}{\text{day}} \cdot \frac{60\,\text{min}}{\text{hr}} \cdot 10\frac{\text{breath}}{\text{min}} \cdot \frac{1\text{L}}{\text{breath}} \cdot \frac{\text{mol}}{22.4\ \text{L}} = 1.31 \cdot 10^7\ \text{mols}$$

Thus the fraction of the atmosphere that he breathed was

$$\text{Fraction he breathed} = \frac{1.31 \cdot 10^7}{1.85 \cdot 10^{20}} = 7.07 \cdot 10^{-14}$$

At 1 atm and 20°C one liter contains 2.50·E22 molecules so that the number of molecules in one liter of the atmosphere that he breathed must be

$$\frac{\text{molecules breathed by Caesar}}{\text{breath}} = 2.50 \cdot 10^{22}\,\frac{\text{molecules}}{\text{liter}} \cdot 7.07 \cdot 10^{-14} = 1.77 \cdot 10^9$$

I have carried more significant figures than the assumptions justify, because it is so easy to do on a spreadsheet. The logical answer is ≈ 2 billion. The perfect mixing assumption is much better for the atmosphere than for the ocean in the preceding problem..

Solutions, Chapter 4

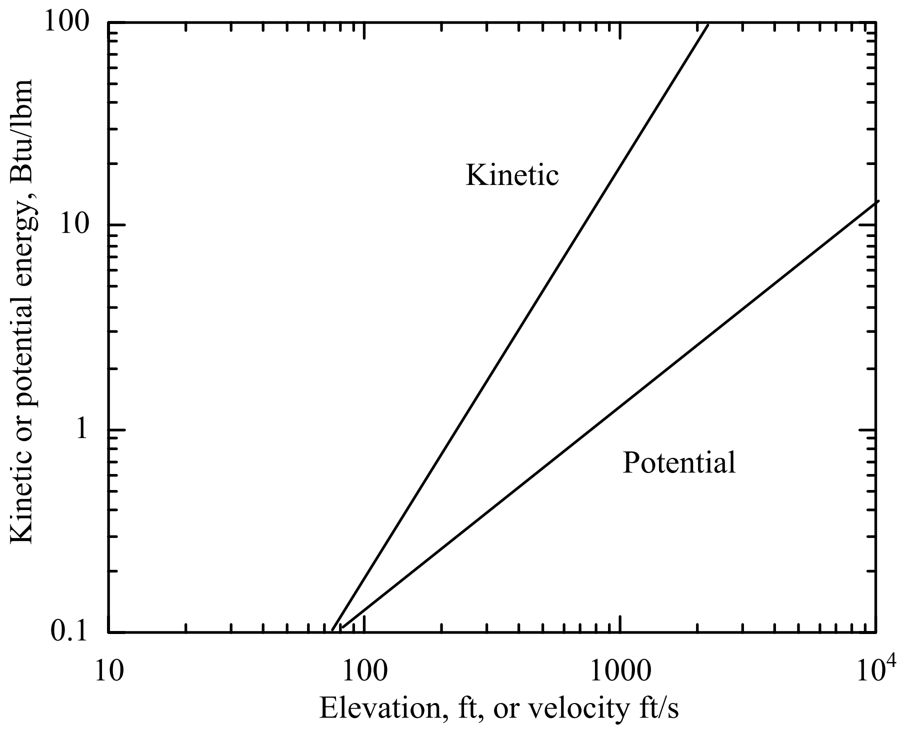
In working these problems I have used the and Keyes 1969 Steam Table for English Unit problems, and the NBS/NRC Steam Tables for SI problems. For Freon 12 I have used DuPont publication A-973.

4.1 The plots will be straight lines on log-log plots. The gz term will have slope 1, and the $V^2/2$ curve will have slope 2. For z = 100 ft, we have

$$PE = gz = 32.2\frac{\text{ft}}{\text{s}^2}\cdot 100\text{ ft}\cdot\frac{\text{Btu}}{778\text{ ft lbf}}\cdot\frac{\text{lbf s}^2}{32.2\text{ lbm ft}} = 0.129\frac{\text{Btu}}{\text{lbm}} = 0.299\frac{\text{kJ}}{\text{kg}}$$

and for V = 100 ft/s, we have

$$KE = \frac{V^2}{2} = \frac{(100\ ft/s)^2}{2}\cdot\frac{\text{Btu}}{778\text{ ft lbf}}\cdot\frac{\text{lbf s}^2}{32.2\text{ lbm ft}} = 0.200\frac{\text{Btu}}{\text{lbm}} = 0.464\frac{\text{kJ}}{\text{kg}}$$



4.2

(a) $$K.E. = \frac{mV^2}{2} = 0.02\text{ lbm}\cdot\frac{\left(2000\frac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\frac{\text{lbf s}^2}{32.2\text{ lbm ft}} = 1.242\cdot 10^3\text{ft lbf} = 1.684\text{ kJ}$$

(b) $$V = V_0 - gt; \quad V = 0 \text{ when } t = \frac{V_0}{g} = \frac{2000\frac{\text{ft}}{\text{s}}}{32.2\frac{\text{ft}}{\text{s}^2}} = 62.11\text{s}$$

$$z = \int V dt = V_0 t - \frac{1}{2} g t^2 = 2000 \frac{\text{ft}}{\text{s}} \cdot 62.1\,\text{s} - \frac{1}{2} 32.2 \frac{\text{ft}}{\text{s}^2} \cdot (62.1\,\text{s})^2 = 6.21 \cdot 10^4\ \text{ft}$$

(c) $$PE = mgz = 0.02\,\text{lbm} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 6.21 \cdot 10^4\ \text{ft} \cdot \frac{\text{lbf}\,\text{s}^2}{32.2\,\text{lbm}\,\text{ft}} = 6.21 \cdot 10^3\,\text{ft lbf} = 1.684\ \text{kJ}$$

The fact that the initial kinetic energy and the potential energy at the top of the trajectory are the same is not an accident. Air resistance complicates all of this, see Prob. 6.94.

4.3* $$W = F dx = mg\Delta z = 2.0\,\text{lbm} \cdot 6 \frac{\text{ft}}{\text{s}^2} \cdot 10\,\text{ft} \cdot \frac{\text{lbf}\,\text{s}^2}{32.2\,\text{lbm}\,\text{ft}} = 3.73\ \text{ft lbf} = 5.05\ \text{J}$$

4.4 Our system is the ball. $d\left[m\left(u + gz + \frac{V^2}{2}\right)\right]_{\text{sys}} = 0 - 0 + 0 - dW_{\text{n.f.}}$

Here the left hand term is zero, so there is no work done on the ball! This appears paradoxical, but the ball is simply converting one kind of energy into another. Work was done on the ball as the airplane lifted it from the ground. If one applied the above equation with the same system from takeoff until the ball hit the ground, then we would have

$$\Delta\left[m\left(u + gz + \frac{V^2}{2}\right)\right]_{\text{sys}} = 0 - 0 + \frac{mV_{\text{final}}^2}{2} - \Delta W_{\text{n.f.}}$$

with the final *KE* being equal to the work done by the airplane in lifting the ball.

4.5 (a) System, the water $d\left[m\left(u + gz + \frac{V^2}{2}\right)\right]_{\text{sys}} = 0 - 0 + 0 + dW_{\text{n.f.}}$

but for this system the changes in internal, potential and kinetic energies are all ≈ 0, so that $dW_{\text{n.f.}} \approx 0$. This seems odd, but the water is merely transferring work from the pump to the piston, rack and car. Here the $dW_{\text{n.f.}}$ is the algebraic sum of the work done on the water by the pump and the work done by the water on the piston, rack and car.

(b) System; water, piston, rack and car, $d\left[m\left(u + gz + \frac{V^2}{2}\right)\right]_{\text{sys}} = 0 - 0 + 0 + dW_{\text{n.f.}}$

with this choice of system the change in potential energy is significant,

$$dW_{\text{n.f}} = mg\Delta z = 4000\,\text{ft lbf} = 5423\ \text{N m} = 5.423\ \text{kJ}$$

(c) System; the volume of the hydraulic cylinder downstream of the pump

$$d\left[m\left(u + gz + \frac{V^2}{2}\right)\right]_{\text{sys}} = h_{\text{in}} dm_{\text{in}} - 0 + 0 + dW_{\text{n.f.}}$$

$dW_{\text{n.f.}} = d(mu)_{\text{sys}} - h_{\text{in}}\, dm_{\text{in}}$ if we assume no change in temperature (a good assumption) then $u_{\text{sys}} = u_{\text{in}} = \text{constant}$ and $dW_{\text{n.f.}} = -Pv_{\text{in}}\, dm_{\text{in}}$ here $dW_{\text{n.f.}}$ is the work done by this system on the car, rack and piston =- 4000 ft lbf. Solving for $v_{\text{in}}\, dm_{\text{in}}$

$$v_{\text{in}}\, dm_{\text{in}} = \frac{dW_{\text{n.f.}}}{-P} = \frac{-4000 \text{ft lbf}}{(1000 - 14.7) \text{lbf / in}^2} \cdot \frac{\text{ft}^2}{144 \text{ in}^2} = 0.0282 \text{ft}^3 = 7.93 \text{ E - 4 m}^3$$

Here we work the problem in gauge pressure. If we had worked it in absolute, then we would have needed to include a term for driving back the atmosphere as the car, rack and piston were driven up.

4.6* System; the water path through the dam, from inlet to outlet., steady flow

$$= -0.62 \frac{\text{Wh}}{\text{kg}} = -0.00062 \frac{\text{kWh}}{\text{kg}} = -2.8 \cdot 10^{-4} \frac{\text{kWh}}{\text{lbm}}$$

$$= -0.622 \frac{\text{Wh}}{\text{kg}} = -0.00062 \frac{\text{kWh}}{\text{kg}} = -2.8 \cdot 10^{-4} \frac{\text{kWh}}{\text{lbm}}$$

This is negative because it is work flowing **out** of the system.

4.7* System the pump, steady flow

$$\frac{dW_{\text{n.f.}}}{dm} = -\left(h_{\text{in}} - h_{\text{out}} + \frac{dQ}{dm} \right) = -(397.99 - 807.94 + (-2)) = 412 \frac{\text{kJ}}{\text{kg}}$$

The work is positive because it is work done on the system. The enthalpy values are from the NBS/NRC steam tables. If one does not have access to those tables, one can use

$$\Delta h \approx \frac{\Delta P}{\rho} + C_P \Delta T = \frac{(20 - 1)\text{bar}}{962 \dfrac{\text{kg}}{\text{m}^3}} \cdot \frac{100 \text{ kJ}}{\text{m}^3 \text{ bar}} + 4.184 \frac{\text{kJ}}{\text{kg °C}} \approx 2.0 + 401 \approx 403 \frac{\text{kJ}}{\text{kg}}$$

which is only approximate because the equation used is approximate and the values of the density and heat capacity are only approximate. But it shows how one would proceed if one did not have access to a suitable steam table.

4.8 System, the power plant, steady flow

$$\frac{dW_{a \cdot 0}}{dm} = g\Delta z + \Delta\left(\frac{V^2}{2}\right) = 32.2 \frac{\text{ft}}{\text{s}^2}(-80 \text{ ft}) + \left(\frac{\left(5 \dfrac{\text{ft}}{\text{s}}\right)^2}{2} - \frac{\left(100 \dfrac{\text{ft}}{\text{s}}\right)^2}{2} \right)$$

$$= -235\frac{\text{ft lbf}}{\text{lbm}} = -0.302\frac{\text{Btu}}{\text{lbm}} = -0.702\frac{\text{kJ}}{\text{kg}}$$

The minus sign goes with the new sign convention, work flows out of the power plant.

4.9* System; power plant, steady flow Po = $\dot{m}\dfrac{dW_{\text{n.f.}}}{dm}$

$$\frac{dW_{\text{n.f.}}}{dm} = \left(g\Delta z + \frac{\Delta V^2}{2}\right) = \left(9.81\frac{\text{m}}{\text{s}^2}\cdot(-40\text{ m}) + \frac{\left(15\frac{\text{m}}{\text{s}}\right)^2 - 9\left(\frac{\text{m}}{\text{s}}\right)^2}{2}\right) = (-392.4 + 72.0) = -320\frac{\text{m}^2}{\text{s}^2}$$

$$\text{Po} = \left(5000\frac{\text{kg}}{\text{s}}\right)\left(-320\frac{\text{m}^2}{\text{s}^2}\right)\cdot\frac{\text{N s}^2}{\text{kg m}}\cdot\frac{\text{J}}{\text{N m}}\cdot\frac{\text{W s}}{\text{J}} = -1.602\,\text{MW}$$

This is power leaving the system, hence negative according to the sign convention.

4.10 System; nozzle, steady flow, $0 = \left(h + \dfrac{V^2}{2}\right)_{\text{in}} - \left(h + \dfrac{V^2}{2}\right)_{\text{out}}$

$$\Delta h = C_P\Delta T = -\frac{\Delta V^2}{2} = \frac{\left(\left(2000\frac{\text{ft}}{\text{s}}\right)^2 - \left(300\frac{\text{ft}}{\text{s}}\right)^2\right)}{2}$$

$$= -1.955\cdot 10^6\frac{\text{ft}^2}{\text{s}^2} = -6.07\,\text{E4}\frac{\text{ft lbf}}{\text{lbm}} = -78.0\frac{\text{Btu}}{\text{lbm}}$$

$$\Delta T = \frac{-1.955\cdot 10^6\frac{\text{ft}^2}{\text{s}^2}}{0.3\frac{\text{Btu}}{\text{lbm}\,^\circ\text{F}}}\cdot\frac{\text{lbf s}^2}{32.2\,\text{lbm ft}}\cdot\frac{\text{Btu}}{778\,\text{ft lbf}} = -260\,^\circ\text{F}$$

$$T_{\text{out}} = 600 - 260 = 340^\circ\text{F} = 800^\circ\text{R} = 444\text{ K}$$

4.11* System; the tank up to the valve, unsteady state. Here the students must use the fact, stated at the beginning of this set of problems, that for a perfect gas u and h are functions of T alone, and not on the pressure.

$$\int d(mu)_{\text{sys}} = 0 - h_{\text{out}}\int dm_{\text{out}} + \int dQ - 0 \;:\quad \Delta Q = \Delta mu + h_{\text{out}}\Delta m_{\text{out}}$$

but $\Delta m_{\text{sys}} = -\Delta m_{\text{out}}$ so that

$$\Delta Q = \Delta m_{\text{out}}(h_{\text{out}} - u) = \Delta m_{\text{out}}(C_P - C_V)T = \Delta m_{\text{out}}RT$$

$$\Delta n_{\text{out}} = \frac{V}{RT}(P_1 - P_2)$$ here we switch from mass to moles, and use molar heat capacities

$$\Delta Q = \frac{V}{RT}(P_1 - P_2)RT = V(P_1 - P_2) = 1\,\text{ft}^3 \cdot 80\frac{\text{lbf}}{\text{in}^2}\cdot\frac{144}{778} = 14.8\ \text{Btu} = 15.62\ \text{kJ}$$

4.12 System, contents of the container, adiabatic. $d(mu)_{\text{sys}} = h_{\text{in}} dm_{\text{in}}$, and for ideal gases

$$T_{\text{final}} = T_{\text{in}}\frac{C_P}{C_V} = kT_{\text{in}}$$ This is the classic solution, which appears in almost all thermodynamics books, and in the first two editions of this book (see page 118 of the second edition). Unfortunately the adiabatic assumption cannot be realized, even approximately if one tries this in the laboratory. Even in the few seconds it takes to fill such a container the amount of heat transferred from the gas makes the observed temperature much less than one calculates this way. The amount of heat transferred is small, but the mass of gas from which it is transferred is also small so their ratio is substantial. This is explored in Noel de Nevers, "Non-adiabatic Container Filling and Emptying", *CEE 33(1)*, 26-31 (1999).

4.13 See the discussion in the solution to the preceding problem. In spite of that this is a classic textbook exercise, which is repeated here.

System, contents of the container, adiabatic

$$\int d(mu)_{\text{sys}} = h_{\text{in}}\int dm_{\text{in}}\ ;(mu)_f - (mu)_i = h_{\text{in}}(m_f - m_i)$$

$$C_V(m_f T_f - m_i T_i) = C_P T_{\text{in}}(m_f - m_i)$$

$$T_f = T_{\text{in}}\frac{C_P}{C_V}\left(\frac{m_f - m_i}{m_f}\right) = \frac{V\frac{P_i}{RT_i}}{V\frac{P_f}{RT_f}} = \frac{P_i T_f}{P_f T_i}$$; Substitute and rearrange to

$$T_f = \frac{T_{\text{in}}\left(\frac{C_P}{C_V}\right)}{1 - \frac{P_i}{P_f}\cdot\frac{1}{T_i}\left(T_i - \frac{C_P}{C_V}T_{\text{in}}\right)}$$ If $P_i = 0$, this simplifies to the solution to the preceding problem.

4.14 See the discussion with problem 4.12. Using the solution to problem 4.13 we have

$$T_f = \frac{\frac{C_P}{C_V}T}{1 - \frac{P_i}{P_f}\cdot\frac{1}{T_i}\left(T_i - \frac{C_P}{C_V}T_{\text{in}}\right)} = \frac{1.40\cdot 293.15K}{1 - \frac{0.5}{1.0}\cdot\frac{1}{293.15}(293.15 - 1.4\cdot 293.15)}$$

$$= 293.15\,\text{K}\,\frac{(1.4)}{1 - 0.5(-0.4)} = 342\,\text{K} = \ 68.9°\text{C} = 156°\text{F}$$

4.15* $c^2 dm = dQ$

$$dm = \frac{dQ}{c^2} = \frac{-14 \cdot 10^{12}\ \text{cal}}{3.85 \cdot 10^{13} \dfrac{\text{Btu}}{\text{lbm}}} \cdot \frac{\text{Btu}}{252\,\text{cal}} = -1.44 \cdot 10^{-3}\ \text{lbm} = \text{-}0.65\ \text{g}$$

If this won't convince the students that c^2 is a large number, I don't know what will!

4.16 $\Delta\left(\frac{V^2}{2}\right) = 1800\frac{\text{Btu}}{\text{lbm}}$

$$V = \sqrt{2\left(1800\frac{\text{Btu}}{\text{lbm}}\right) \cdot \frac{778\,\text{ft lbf}}{\text{Btu}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2}} = 9497\frac{\text{ft}}{\text{s}} = 1.8\frac{\text{mi}}{\text{s}} = 2.89\frac{\text{km}}{\text{s}}$$

Such velocities are observed in meteorites, earth satellites, ICBMs, and in the kinetic energy weapons proposed in the Star Wars defensive systems.

To stimulate class discussion you can tell the students that hydrocarbon fuels like gasoline have heating values of ≈ 18,000 Btu/lbm, and this problem shows that high explosives only release about one tenth of that. Ask them why. Some will figure out that the heating value is for the fuel plus the oxygen needed to burn it, taken from the air, and not included in the weight of the fuel. High explosives include their oxidizer in their weight. They do not release large amounts of energy per pound, compared to hydrocarbons. What they do is release it very quickly, much faster than ordinary combustion reactions. The propagation velocities of high explosives are of the order of 10,000 ft/s, compared to 1 to 10 ft/s for hydrocarbon flames. That is so fast that the reaction is complete before there is significant expansion, and the solid (or liquid) explosive turns into a high temperature gas, with ≈ the same density as the initial liquid or solid. You can estimate the pressure for this from the ideal gas law, finding amazing values.

4.17 This is a discussion problem. Fats are roughly $(CH_2)_n$. Carbohydrates are roughly $(CH_2O)_n$. For one C atom the ratio of weights is (14/30) = 0.47. So if fats have 9 kcal/gm, we would expect carbohydrates to have 9·0.47 = 4.2 ≈ 4 kcal/gm. This simplifies the chemistry a little, but not much.

You might ask your students what parts of plants have fats. Some will know that the seeds have fats, the leaves and stems practically zero. Then some will figure out why. The assignment of a seed is to find a suitable place, put down roots and put up leaves before it can begin to make its own food. It is easier to store the energy for that as a fat than as a carbohydrate. Our bodies make fats out of carbohydrates so that we can store them for future use (in case of famine). As fats an equal amount of food takes about 47%

as much space and weight as the same food as carbohydrates. Thermodynamics explains a lot of basic biology!

4.18 System; contents of calorimeter, unsteady state. Note that the metal walls of the calorimeter are outside the system.

$$d(mu) = 0 - 0 + dQ + 0$$

$$= -500\,\text{g} \cdot 5^{\circ}\text{C} \cdot \frac{0.12\,\text{cal}}{\text{g}\,^{\circ}\text{C}} - 5000\,\text{g} \cdot 5^{\circ}\text{C} \cdot \frac{1.0\,\text{cal}}{\text{g}\,^{\circ}\text{C}}$$

$$= -300 - 25\,000 = -25\,300\ \text{cal}$$

$$\Delta u = \frac{\Delta Q}{m} = \frac{-25\,300\,\text{cal}}{4\,\text{g}} = -6325\frac{\text{cal}}{\text{g}} = -11\,395\frac{\text{Btu}}{\text{lbm}}$$

Sources of error: heating of gases in calorimeter, condensation of water vapor produced on combustion, accurate measurement of small temperature difference.

The value here is between that for fats and for carbohydrates. It corresponds to a good grade of coal,

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4.19 System: earth, unsteady state. The heat flow is

$$\frac{dQ}{dt} = kA\left(\frac{dT}{dx}\right) = \frac{1\ \text{Btu}}{\text{hr}\ {}^{\circ}\text{F}\ \text{ft}} \cdot \pi\left(8000\,\text{mi} \cdot \frac{5280\,\text{ft}}{\text{mi}}\right)^2 \cdot \frac{-0.02\,^{\circ}\text{F}}{\text{ft}} = -1.12 \cdot 10^{14}\frac{\text{Btu}}{\text{hr}}$$

The amount of mass converted to energy is

$$\frac{dm}{dt} = \frac{\frac{dQ}{dt}}{c^2} = \frac{-1.12 \cdot 10^{14}\frac{\text{Btu}}{\text{hr}}}{3.85 \cdot 10^{13}\frac{\text{Btu}}{\text{lb}}} = -2.9\frac{\text{lb}}{\text{hr}}$$

4.20 System; the boundaries of the sun, assuming negligible outflow of mass. This latter is not quite correct, because particles are blown off the sun by solar storms, but their contribution to the energy balance of the sun is small compared to the outward energy flux due to radiation.

$$c^2 dm_{\text{sys}} = dQ$$

4.21* System; the power plant from inlet to outlet, steady flow

$$\frac{dW_{\text{n.f.}}}{dm} = \left(\Delta h + g\Delta z + \Delta\frac{V^2}{2}\right)$$

$$= \left[1\frac{\text{Btu}}{\text{lbm}} + \left(32\frac{\text{ft}}{\text{s}^2}(-75\,\text{ft}) \cdot \frac{\text{Btu}}{778\ \text{ft lbf}} \cdot \frac{\text{lbf s}^2}{32.2\ \text{lbm ft}} \right) + \left(\frac{\left(50\frac{\text{ft}}{\text{s}}\right)^2 - \left(400\frac{\text{ft}}{\text{s}}\right)^2}{2} \right) \cdot \frac{1}{778} \cdot \frac{1}{32.2} \right]$$

$$= (1 - 0.096 - 3.144) = -2.24\frac{\text{Btu}}{\text{lbm}} = -1.07\frac{\text{kJ}}{\text{kg}}$$

4.22 (a) System, valve and a small section of adjacent piping, steady flow. $h_f = h_i$

(b) System; one kg of material flowing down the line.
$\Delta u = 0 + \Delta W$; $u_f - u_c = (P_i v_i - P_f v_f)$; $h_f = h_i$

The results are the same, as they must be. What nature does is independent of how we think about it.

4.23 None of these violate the first law, all violate the second law. We may show that they do not violate the first law by making an energy balance for each.

(a) $$\Delta(u + g\Delta z) = 0 = \left(-0.01284\frac{\text{Btu}}{\text{lb}} + 32.2\frac{\text{ft}}{\text{s}^2} \cdot 10\ \text{ft} \cdot \frac{\text{Btu}}{778\ \text{ft lbf}} \cdot \frac{\text{lbf s}^2}{32.2\ \text{ft lbm}} \right) \backslash$$

$$= -0.01284 + 0.01284 = 0$$

(b) $u_{\text{final}} = 0.105\,\text{lbm} \cdot \left(-143.34\frac{\text{Btu}}{\text{lbm}} \right) + 0.895\ \text{lbm} \cdot \left(1232.2\frac{\text{Btu}}{\text{lbm}} \right) = 1088\frac{\text{Btu}}{\text{lbm}}$ which is equal to the initial internal energy.

(c) The upstream and downstream enthalpies are the same, as they must be for an adiabatic throttle.

These all violate the second law. They all violate common sense. If you doubt that, put a baseball on a table and watch it, waiting to see it spontaneously jump to a higher elevation and cool. Be patient!

Solutions, Chapter 5

5.1* $d(u+gz)=0$

$$\Delta u = -g(\Delta z) = -32.2\frac{\text{ft}}{\text{s}^2} - (1000\text{ ft})\frac{\text{lbf s}^2}{32.2\text{ lbm ft}}\cdot\frac{\text{Btu}}{778\text{ ft lbf}} = 1.29\frac{\text{Btu}}{\text{lbm}}$$

(a) $$\Delta T = \frac{\Delta u}{C_{V,\text{steel}}} = \frac{\left(1.29\frac{\text{Btu}}{\text{lbm}}\right)}{\left(0.12\frac{\text{Btu}}{\text{lbm }^\circ\text{F}}\right)} = 11^\circ\text{F}$$

(b) $$\Delta T = \frac{\Delta u}{C_{V,\text{water}}} = \frac{\left(1.29\frac{\text{Btu}}{\text{lbm}}\right)}{\left(1.0\frac{\text{Btu}}{\text{lbm }^\circ\text{F}}\right)} = 1.3^\circ\text{F}$$

Discussion; we do not observe friction heating unless it is concentrated, e.g. smoking brakes, spinning tires on drag racers, hot drill bits or saw blades, boy scouts making fire by friction.

5.2 The head form is $\frac{\Delta P}{\rho g} + \Delta z + \frac{\Delta V^2}{2g} = \frac{dW_{\text{n.f.}}}{g\,dm} - \frac{\mathcal{F}}{g}$; $z\begin{pmatrix}\text{has dimension}\\ \text{of}\end{pmatrix}$ ft

$$\frac{\Delta P}{\rho g}\begin{pmatrix}\text{has dimension}\\ \text{of}\end{pmatrix}\frac{\frac{\text{lbf}}{\text{ft}^2}}{\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{\text{ft}}{\text{s}^2}}\frac{32.2\text{ lbm ft}}{\text{lbf s}^2}[=]\frac{\text{ft}^3}{\text{ft}^2} = \text{ft}$$

$$\frac{V^2}{2g}\begin{pmatrix}\text{has dimension}\\ \text{of}\end{pmatrix}\frac{\left(\frac{\text{ft}}{\text{s}}\right)^2}{\frac{\text{ft}}{\text{s}^2}}[=]\text{ ft}$$

$$\frac{dW_{\text{n.f.}}}{g\,dm}\begin{pmatrix}\text{has dimension}\\ \text{of}\end{pmatrix}\frac{\text{ft}\cdot\text{lbf}}{\frac{\text{ft}}{\text{s}^2}\cdot\text{lbm}}\cdot\frac{32.2\text{ lbm ft}}{\text{lbf s}^2}[=]\text{ft}$$

$$\frac{\mathcal{F}}{g}\begin{pmatrix}\text{has dimension}\\ \text{of}\end{pmatrix}\frac{\frac{\text{ft}^2}{\text{s}^2}}{\frac{\text{ft}}{\text{s}^2}}[=]\text{ft}$$

5.3* (a) $$\Delta P = \frac{\rho V^2}{2} = \frac{\left(998.2\frac{\text{kg}}{\text{m}^3}\right)\left(8\frac{\text{m}}{\text{s}}\right)^2}{2}\cdot\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\cdot\frac{\text{Pa}\cdot\text{m}}{\text{N}} = 31.94\text{ kPa} = 4.64\text{ psi}$$

$\Delta u = 0$

(b) $\Delta P = 0.9 \cdot (\text{answer from part a}) = 28.7 \text{ kPa} = 4.28 \text{ psi}$

$$\Delta u = \frac{0.1\Delta P_{\text{revrsible}}}{\rho} = \frac{(0.1)(31.94 \text{ kPa})}{998.2\frac{\text{kg}}{\text{m}^3}} \cdot \frac{\text{N}}{\text{Pa} \cdot \text{m}^2} \cdot \frac{\text{J}}{\text{N} \cdot \text{m}} = 3.20\frac{\text{J}}{\text{kg}}$$

(c) $\Delta P = 0; \qquad \Delta u = 32.0\frac{\text{J}}{\text{kg}}$

5.4 $$\Delta P = \frac{\rho(V_1^2 - V_2^2)}{2} = \frac{\rho}{2}\left[\left(10\frac{\text{ft}}{\text{s}}\right)^2 - \left(\frac{10}{3}\frac{\text{ft}}{\text{s}}\right)^2\right] = \rho\frac{\left(100\frac{\text{ft}^2}{\text{s}^2}\right)}{9/4}$$

(a) For water

$$\Delta P = 62.3\frac{\text{lbm}}{\text{ft}^3}\left(\frac{4}{9}\right)\left(100\frac{\text{ft}^2}{\text{s}^2}\right) \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2} = 0.60 \text{ psi} = 4.1 \text{ kPa}$$

(b) For air

$$\Delta P = \text{above answer} \cdot \frac{0.075}{62.3} = 7.2 \cdot 10^{-4} \text{psi} = 0.0050 \text{ kPa}$$

5.5 The answers are the answers in Example 5.2 multiplied by $\sqrt{\frac{P_1}{\left(\frac{(P_1 + P_{atm})}{2}\right)}}$ with all pressures absolute. For 1 psig, this factor is

$$\sqrt{\frac{P_1}{\left(\frac{(P_1 + P_{atm})}{2}\right)}} = \sqrt{\frac{15.7 \text{ psia}}{\left(\frac{(15.7 \text{ psia} + 14.7 \text{ psia})}{2}\right)}} = 1.016$$ and the calculated velocity is

$$V = 340\frac{\text{ft}}{\text{s}} \cdot 1.016 = 345.5\frac{\text{ft}}{\text{s}}$$

Below is a table comparing the three solutions, carrying more significant figures than are shown in Table 5.1.

delta P, psi	V, simple BE ft/s	V, Ch. 8, ft/s	V, this Problem ft/s
0.01	35.094	35.090	35.100
0.1	110.640	110.747	110.827
0.3	190.352	190.992	191.311
0.6	266.546	268.384	269.198
1	339.697	343.602	345.239
2	465.799	476.269	480.405

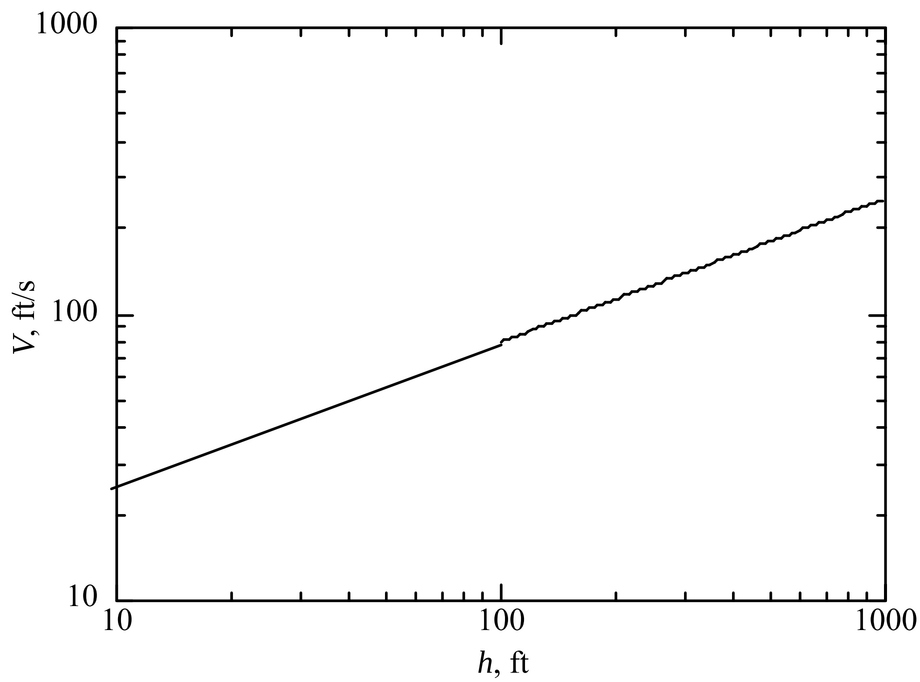
3	554.136	572.306	579.222
5	678.100	713.154	725.709

From the table we see that while the simple approach in Example 5.2 underestimates the velocity, the approach in this problem overestimates it, but by a smaller percentage than the simple approach in Example 5.2. If one is not going to use the approach in Ch. 8, then this approach is more accurate than the simpler one in Example 5.2.

5.6 The plot will be a straight line with slope 1/2 on log-log paper. For a height of 100 ft

$$V = \sqrt{2gh} = \sqrt{2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 100 \text{ ft}} = 80.2 \frac{\text{ft}}{\text{s}}$$

The figure is shown below



5.7* $V_2 = \sqrt{2 \cdot 32 \cdot 2 \frac{\text{ft}}{\text{s}} \cdot 12 \text{ ft}} = 27.8 \frac{\text{ft}}{\text{s}} = 8.47 \frac{\text{m}}{\text{s}}$

$$Q = VA = 27.8 \frac{\text{ft}}{\text{s}} \cdot 2 \text{ ft}^2 = 55.6 \frac{\text{ft}^3}{\text{s}} = 1.57 \frac{\text{m}^3}{\text{s}}$$

5.8 The density of the fluid does not enter Torricelli's equation, so

$$V = \sqrt{2gh} = \sqrt{2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 30 \text{ ft}} = 44.0 \frac{\text{ft}}{\text{s}} = 13.4 \frac{\text{m}}{\text{s}}$$

Discussion; ask the students if you dropped a can of water from 30 ft, with zero air resistance, what the velocity at $h = 0$ would be. The ask about a can of gasoline.

5.9* $$V = \left(2gh \Big/ \left(1 - \left(\frac{A_2}{A_1}\right)^2\right)\right)^{\frac{1}{2}} = \frac{\text{answer to Prob. 5.7}}{\left[1 - \left(\frac{2\,\text{ft}^2}{5\,\text{ft}^2}\right)^2\right]^{\frac{1}{2}}} = \frac{27.8\,\frac{\text{ft}}{\text{s}}}{[1 - 0.16]^{\frac{1}{2}}} = 30.3\,\frac{\text{ft}}{\text{s}} = 9.24\,\frac{\text{m}}{\text{s}}$$

$$Q = 30.3\,\frac{\text{ft}}{\text{s}} \cdot 2\,\text{ft}^2 = 60.6\,\frac{\text{ft}^3}{\text{s}} = 1.71\,\frac{\text{m}^3}{\text{s}}$$

5.10 $$V = \sqrt{2gh} = \sqrt{2 \cdot 32.2\,\frac{\text{ft}}{\text{s}^2} \cdot 726\,\text{ft}} = 216\,\frac{\text{ft}}{\text{s}} = 65.9\,\frac{\text{m}}{\text{s}} = 147\,\frac{\text{mi}}{\text{hr}}$$

Discussion; the dam is fairly thick at its base, so the pipe would be long, and friction would be significant. Torricelli's Eq. is not reliable here. See Chapter 6.

5.11 $$V = \sqrt{2 \cdot 9.81\,\frac{\text{m}}{\text{s}^2} \cdot 10\,\text{m}} = 14.01\,\frac{\text{m}}{\text{s}} = 46\,\frac{\text{ft}}{\text{s}}$$

$$Q = VA = \left(14.01\,\frac{\text{m}}{\text{s}}\right)\left(5\,\text{m}^2\right) = 70.0\,\frac{\text{m}^3}{\text{s}} = 2474\,\frac{\text{ft}^3}{\text{s}}$$

I'm sure all of you liked the movie "Titanic", particularly the scene in which the boat's designer, resigning himself to death. says that the design allowed for the boat to survive flooding of two compartments, but not four or five. Apparently the safety doors divide the boat's hull into some number (7 on the Titanic?) of compartments, all of which are open at the top. So if the hull is punctured, as in this problem, and the doors close properly, one compartment will fill with water up to the level of the water outside, but the other compartments will remain dry. Apparently the impact with the iceberg opened the hull to more compartments than the ship could survive, even with the safety doors closed. Presumably if the compartments were sealed at the top, then even that accident would have been survivable, but it is much easier to provide closed doors on passageways parallel to the axis of the boat, in which there is little traffic, than on vertical passageways (stairs, elevators, etc.) on which there is considerable traffic.

5.12 From Eq. 5.12 $\quad V_2 = \sqrt{2gh} = 43.9\,\frac{\text{ft}}{\text{s}}$ which is absurd.

From Eq 5.22 $\quad V_2 = \sqrt{2gh\left(1 - \frac{\rho_{\text{air}}}{\rho_{\text{air}}}\right)} = 0$ which is quite plausible.

5.13* $\dfrac{\Delta P}{\rho} + g\Delta z + \dfrac{\Delta V^2}{2} = 0; \quad V_2 = \sqrt{2\left(-\dfrac{\Delta P}{\rho} - g\Delta z\right)}$

$$\Delta P = P_2 - P_1 = -\rho_{\text{air}} g\Delta z; \quad V_2 = \sqrt{2g(-\Delta z)\left(\frac{\rho_{\text{air}}}{\rho_{\text{He}}} - 1\right)}$$

The densities are proportional to molecular weights, so

$$V_2 = \sqrt{(2)\cdot\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\cdot(40\,\text{ft})\cdot\left(\frac{29}{4} - 1\right)} = 127\frac{\text{ft}}{\text{s}} = 39\frac{\text{m}}{\text{s}}$$

This looks strange, because the velocity is higher than the velocity from Torricelli's equation. However it is correct. The reason is the very low density of the helium, which makes the difference in atmospheric pressure seem like a large driving force.

5.14 $V = \sqrt{2gh}; \quad Q = VA_{\text{outlet}} = V_{\substack{\text{tank}\\\text{surface}}} A_{\text{tank}} = A_{\text{tank}}\left(-\dfrac{dh}{dt}\right)_{\text{tank}}$

$$\left(-\frac{dh}{dt}\right)_{\text{tank}} = \frac{A_{\text{outlet}}}{A_{\text{tank}}}\sqrt{2gh} = 0.01\sqrt{2\cdot 9.81\frac{\text{m}}{\text{s}}\cdot 10\ \text{m}} = 0.14\frac{\text{m}}{\text{s}} = 0.46\frac{\text{ft}}{\text{s}}$$

Some students will say you should use $V = \sqrt{\dfrac{2gh}{1-(A_2/A_1)^2}}$. That multiplies the above answers by 1.00005, which is clearly negligible compared to the uncertainties introduced by the standard Torricelli's assumptions.

5.15 $V_2 = \sqrt{2\left(-\dfrac{\Delta P}{\rho} - g\Delta z\right)}$, but $\Delta P = \rho_{\text{gasoline}} g\Delta z$ so that

$$V_2 = \sqrt{2g(-\Delta z)\left(1 - \frac{\rho_{\text{gas}}}{\rho_{\text{w}}}\right)} = \sqrt{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(30\,\text{ft})(1-0.72)} = 23.3\frac{\text{ft}}{\text{s}} = 7.09\frac{\text{m}}{\text{s}}$$

This is the same as Ex. 5.5, with different fluids.

5.16* $V_2 = \sqrt{-\dfrac{\Delta P}{\rho_w}} = \sqrt{\dfrac{P_1 - P_2}{\rho_w}}$

$$P_1 = 30\,\text{ft}\cdot g\rho_{\text{water}}; \quad P_2 = 10\,\text{ft}\cdot g\rho_{\text{water}} + 20\,\text{ft}\cdot g\rho_{\text{oil}}$$

$$P_1 - P_2 = 20\,\text{ft}\cdot g(\rho_{\text{water}} - \rho_{\text{oil}})$$

$$V = \sqrt{2(20\,\text{ft})\cdot\left(32.2\frac{\text{ft}}{s^2}\right)\cdot(1-0.9)} = 11.3\frac{\text{ft}}{\text{s}} = 3.46\frac{\text{m}}{\text{s}}$$

5.17 $\mathcal{F} = -\left(\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta V^2}{2}\right)$ Assuming the flow is from left to right

$$\mathcal{F} = \left(-\frac{8\frac{\text{lbf}}{\text{in}^2}}{62.3\frac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} + 32.2\frac{\text{ft}}{\text{s}^2} \cdot 20\ \text{ft} + 0\right)$$

$$= \left(-595\frac{\text{ft}}{\text{s}^2} + 644\frac{\text{ft}^2}{\text{s}^2}\right) = -48.6\frac{\text{ft}^2}{\text{s}^2} = -4.51\frac{\text{m}^2}{\text{s}^2}$$

The negative value of the friction heating makes clear that the assumed direction of the flow is incorrect, and that the flow is from right to left.

5.18* $V_2 = \sqrt{2\left(-\frac{\Delta P}{\rho} - g\Delta z\right)}$

$$V_2 = \sqrt{2\left(\frac{20\frac{\text{lbf}}{\text{in}^2}}{62.3\frac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\,\text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\,\text{in}^2}{\text{ft}^2} + 5\,\text{ft} \cdot 32.2\frac{\text{ft}}{\text{s}^2}\right)} = 57.4\frac{\text{ft}}{\text{s}} = 17.5\frac{\text{m}}{\text{s}}$$

5.19 $\left(\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta V_2}{2}\right) = 0$; $\left(\frac{0 - P_1}{\rho} + g(-h) + \frac{V_2^2 - 0}{2}\right) = 0$

$$P_1 = \rho\left(g(-h) + \frac{V_2^2}{2}\right) = 62.3\frac{\text{lbm}}{\text{ft}^3}\left(32.2\frac{\text{ft}}{\text{s}^2} \cdot (-5\,\text{ft}) + \frac{100\frac{\text{ft}^2}{\text{s}^2}}{2}\right) \cdot \frac{\text{lbf s}^2}{32.2\ \text{lbm ft}} \cdot \frac{\text{ft}^2}{144\ \text{in}^2}$$

$= (-2.16 + 0.67) = -1.49$ psig = -10.3 kPa gage

This indicates that at that velocity, there is a vacuum in the top of the vessel.

5.20 $V_2 = \sqrt{2\left(-\frac{\Delta P}{\rho} - g\Delta z\right)} = \sqrt{2\left(-\frac{-50\frac{\text{lbf}}{\text{in}^2}}{62.3\frac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\,\text{in}^2}{\text{ft}^2} - 32.2\frac{\text{ft}}{\text{s}^2} \cdot 30\ \text{ft}\right)}$

$= 74.2\frac{\text{ft}}{\text{s}} = 22.6\frac{\text{m}}{\text{s}}$

5.21* Let the water-air interface be (1), the mercury-water interface be (2) and the outlet be (3). Then $V_3 = \sqrt{2\left(\left(\dfrac{P_2 - P_3}{\rho_{Hg}}\right) + g(z_2 - z_3)\right)}$

but $P_3 = P_1$ and $P_2 = h_2 \rho_w g$, so that

$$V_3 = \sqrt{2\left[\left(h_2 g \frac{\rho_w}{\rho_{Hg}}\right) + g h_1\right]}$$

$$= \sqrt{2\left[(8 \text{ m}) \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot \frac{1}{13.6} + 9.81 \frac{\text{m}}{\text{s}^2} \cdot 1 \text{ m}\right]} = 5.58 \frac{\text{m}}{\text{s}} = 18.3 \frac{\text{ft}}{\text{s}}$$

5.22 Let the water-air interface be (1) and the outlet be (2). Then

$$V_2 = \sqrt{2\left(\frac{P_1 - P_2}{\rho} + g(z_1 - z)\right)}$$

$$= \sqrt{2\left(\frac{10 \dfrac{\text{lbf}}{\text{in}^2}}{62.3 \dfrac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2 \text{ lbm ft}}{\text{lbf s}^2} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} + 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 10 \text{ ft}\right)} = 46.2 \frac{\text{ft}}{\text{s}} = 14.1 \frac{\text{m}}{\text{s}}$$

5.23* $V_2 = \sqrt{-\dfrac{2\Delta P}{\rho}}$; $\Delta P = \int \rho(r\omega^2)dr = \dfrac{\rho\omega^2}{2}(r_2^2 - r_1^2)$

$$V_2 = \sqrt{\frac{2}{\rho}\left[\frac{\rho\omega^2}{2}(r_2^2 - r_1^2)\right]} = \sqrt{\omega^2(r_2^2 - r_1^2)}$$

$$= \sqrt{\left(\frac{2\pi\, 2000}{\text{min}} \cdot \frac{\text{min}}{60 \text{ s}}\right)^2 \left((21 \text{ in})^2 - (20 \text{ in})^2\right)} = 1341 \frac{\text{in}}{\text{s}} = 112 \frac{\text{ft}}{\text{s}} = 34 \frac{\text{m}}{\text{s}}$$

5.24 In Bernoulli's equation devices (orifice meters, venturi meters, pitot tubes) the flow rate is proportional to the square of the pressure difference. These devices most often send a signal which is proportional to the pressure difference. If this is shown directly on an indicator or on a chart, the desired information, the flow rate, is proportional to the square of the signal. One solution to this problem is to have chart paper with the markings corresponding to the square of the signal. For example if one inch of chart covers the range from zero to 20% of full scale, then four inches of chart will correspond to the range from zero to 40% of full scale. The chart manufacturers refer to these as "square root" charts, meaning that in reading them one is automatically extracting the square root of the signal.

With recent advances in microelectronics, many new flow recorders extract the square root of the signal electronically, and use a chart with a linear scale.

5.25* $$V_{max} = \sqrt{2gh} = \sqrt{2 \cdot 32.2\frac{\text{ft}}{\text{s}} \cdot 10\,\text{ft}} = 25.4\frac{\text{ft}}{\text{s}} = 17.3\frac{\text{mi}}{\text{hr}} = 7.7\frac{\text{m}}{\text{s}}$$

These have actually been sold; boat fans will buy anything. For speedboats the pitot tube is normally connected directly to a bourdon-tube pressure gage, which is marked to read the speed directly in mi/hr or equivalent.

5.26 $$V_2 = \sqrt{\frac{2gh}{\rho_{air}}(\rho_{fluid} - \rho_{air})}$$

$$V_{min} = \sqrt{(2)\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{0.5\,\text{ft}}{12}\right)\left(\frac{62.3 - 0.075}{0.075}\right)} = 47.2\frac{\text{ft}}{\text{s}} = 32.2\frac{\text{mi}}{\text{hr}} = 14.4\frac{\text{m}}{\text{s}}$$

Low-speed flows of gases are hard to measure with Bernoulli's equation devices, because the pressure differences are so small. Moving blade or moving cup anemometers are most often used for low-speed gas flows, e.g. meteorological measurements. The students have certainly seen these in weather stations.

5.27* see preceding problem

$$V_2 = \sqrt{2gh\left(\frac{\rho_{\text{manometer fluid}}}{\rho_{\text{gasoline}}} - 1\right)} = \sqrt{(2)\cdot\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\cdot\left(\frac{0.1}{12}\,\text{ft}\right)\cdot\left(\frac{1}{0.72} - 1\right)} = 0.46\frac{\text{ft}}{\text{s}} = 0.14\frac{\text{m}}{\text{s}}$$

5.28 $$V = \sqrt{2\frac{\Delta P}{\rho}}$$

(a) $$V = \sqrt{\frac{2\left(0.3\frac{\text{lbf}}{\text{in}^2}\right)}{0.075\frac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\ \text{in}^2}{\text{ft}^2}} = 192.6\frac{\text{ft}}{\text{s}} = 131\frac{\text{mi}}{\text{hr}} = 58.7\frac{\text{m}}{\text{s}}$$

(b) $$V = \text{Above answer} \cdot \sqrt{\frac{0.075}{0.057}} = 220.9\frac{\text{ft}}{\text{s}} = 151\frac{\text{mi}}{\text{hr}} = 67.3\frac{\text{m}}{\text{s}}$$

This is one of my favorite discussion problems. On the dashboard of an airplane (private or commercial) is an "indicated air speed" dial. The indicated airspeed is = $V(\rho/\rho_{\text{sea level}})^{1/2}$ which is found by putting this pressure difference across a diaphragm, and using a linkage to drive the pointer around a dial. This indicated air speed is the velocity of interest for pilots. The lift is directly proportional to the indicated air speed, so this is also effectively a lift indicator, see Secs. 6.14 and 7.6.

You can ask the students why commercial airliners fly as high as they can. For a given weight, there is only one indicated air speed at which they can fly steadily in level flight. As the air density goes down, the corresponding absolute velocity goes up. So by going high, they go faster. This means fewer hours between takeoff and landing. That means less fuel used, fewer hours to pay the pilots and crews for, and happier customers who do not like to sit too long in an aircraft.

5.29

$$\Delta P = 1002\frac{\text{kg}}{\text{m}^3}\frac{\left(60\ \frac{\text{km}}{\text{hr}}\cdot\frac{\text{hr}}{3600\ \text{s}}\cdot\frac{1000\ \text{m}}{\text{km}}\right)^2}{2}\cdot\frac{\text{N s}^2}{\text{kg m}}\cdot\frac{\text{Pa m}^2}{\text{N}} = 139.2\ \text{kPa} = 20.2\ \text{psig}$$

5.30 Equation 5.BN predicts that for a *VP* (which we would call ΔP) of 1 inch of water = 0.03615 psig the velocity is 4005 ft/min. Using the methods in this book, we find for that pressure difference

$$V = \sqrt{2\frac{0.03615\frac{\text{lbf}}{\text{in}^2}}{0.075\frac{\text{lbm}}{\text{ft}^2}}\cdot\frac{144\ \text{in}^2}{\text{ft}^2}\cdot\frac{32.2\ \text{lbm ft}}{\text{lbf s}^2}} = 66.85\frac{\text{ft}}{\text{s}} = 4011\frac{\text{ft}}{\text{min}}$$

This is 1.0016 times the 4005 in Eq. 5.BN. The form of the equations is the same, V proportional to the square root of ΔP, so for any value of ΔP the prediction of this "practical" equation is within 0.16% of the value from Eq. 5.16.

5.31 Example 5.8 shows that for a pressure difference of 1 psig the volumetric flow rate is 2.49 ft^3/s. We can get other values by ratio, e.g. for 1 ft^3/s

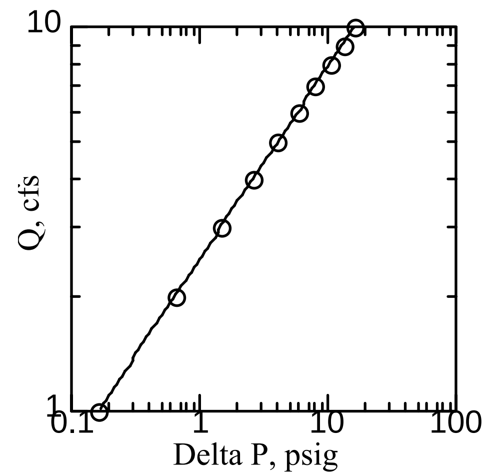
$$\Delta P = \Delta P_0\left(\frac{Q}{Q_0}\right)^2 = 1\ \text{psig}\left(\frac{1\ \text{ft}^3/\text{s}}{2.49\ \text{ft}^3/\text{s}}\right)^2 = 0.161\ \text{psig}$$

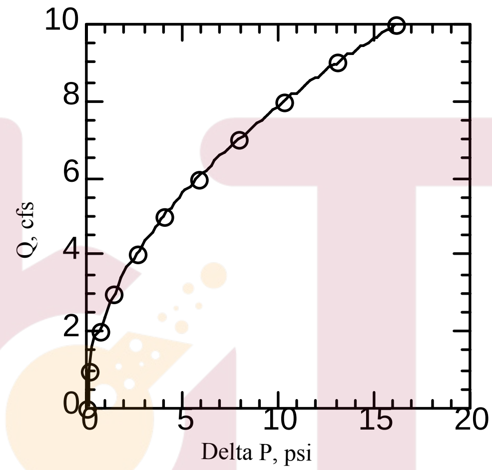
and similarly for other values.

The plot is shown at the right in two forms. The upper one, on log-log coordinates is probably the more useful, because the fractional uncertainty in the reading is practically the same over the whole range.

The lower one, on arithmetic coordinates is probably easier for non-technical people to use, and would probably be selected if technicians were to use it.

Here I have chosen the pressure drop as the independent variable, because that is the observational instrument reading.





5.32 $V_2 = \frac{Q}{A} = \sqrt{\frac{2gh(\rho_{\text{manometer}} - \rho)/\rho}{1-(D_2/D_1)^4}} \approx \sqrt{\frac{2 \cdot 32.2\frac{\text{ft}}{\text{s}^2} \cdot 1\ \text{ft} \cdot 62.3\frac{\text{lbm}}{\text{ft}^3} / 0.075\frac{\text{lbm}}{\text{ft}^3}}{\left(1-(0.5)^4\right)}}$

$= 239\frac{\text{ft}}{\text{s}} = 72.8\frac{\text{m}}{\text{s}}$

One may compute that the Reynolds number at 1 is ≈ 3.7 E 5. From Fig 5.11 one would estimate $C_V \approx 0.984$ so that if we take this correction into account we would report $V_2 = 235\frac{\text{ft}}{\text{s}} = 71.6\frac{\text{m}}{\text{s}}$ In most common work we would probably ignore this difference.

5.33 We use Eq. 5.19

$$V_2 = \sqrt{\frac{2 \cdot 32.2\frac{\text{ft}}{\text{s}^2} \cdot 1\,\text{ft} \cdot 62.3\frac{\text{lbm}}{\text{ft}^3}(1-0.72)}{62.3\frac{\text{lbm}}{\text{ft}^3}(0.72)\left(1-(0.5)^4\right)}} = 5.17\frac{\text{ft}}{\text{s}} = 1.58\frac{\text{m}}{\text{s}}$$

$$Q = V_2 A_2 = 5.17\frac{\text{ft}}{\text{s}}\left(\frac{\pi}{4}(0.5\,\text{ft})^2\right) = 1.01\frac{\text{ft}^3}{\text{s}} = 0.029\frac{\text{m}^3}{\text{s}}$$

5.34* $$V_2 = \sqrt{\frac{\frac{2(P_1 - P_2)}{\rho} + 2g(z_1 - z_2)}{1 - (A_1 / A_2)^2}}$$

$$= \sqrt{\frac{\frac{2(7-5)\frac{\text{lbf}}{\text{in}^2}}{0.72(62.3)\frac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} + 2 \cdot 32.2\frac{\text{ft}}{\text{s}^2} \cdot 2\ \text{ft}}{1 - (0.5)^4}} = 24.0\frac{\text{ft}}{\text{s}} = 7.33\frac{\text{m}}{\text{s}}$$

$$Q = VA = 24.0\frac{\text{ft}}{s} \cdot \frac{\pi}{4}(0.5\,\text{ft})^2 = 4.72\frac{\text{ft}^3}{\text{s}} = 0.13\frac{\text{m}}{\text{s}}$$

5.35 Let (1) be in the pipe opposite the top leg of the manometer, (2) be in the vertical section opposite the lower leg of the manometer. See Eq 5.19. Here $D_2 / D_1 = \sqrt{0.5 / 2} = 0.5$

$$V_2 = \sqrt{\frac{2 \cdot 32.2\frac{\text{ft}}{\text{sec}} \cdot \left(\frac{2}{12}\,\text{ft}\right) \cdot (13.6 - 1) \cdot 62.3\frac{\text{lbm}}{\text{ft}^3}}{62.3\frac{\text{lbm}}{\text{ft}^3}\left(1 - (0.5)^4\right)}} = 12.0\frac{\text{ft}}{\text{s}} = 3.66\frac{\text{m}}{\text{s}}$$

$$Q = V_2 A_2 = 12.0\frac{\text{ft}}{\text{s}} \cdot 0.5\,\text{ft}^2 = 6.0\frac{\text{ft}^3}{\text{s}} = 0.17\frac{\text{m}^3}{\text{s}}$$

From the figure there is no way to tell which way the water if flowing.

5.36* $$V = C_v\sqrt{\frac{2(P_1 - P_2)}{\rho\left(1 - (A_2 / A_1)^2\right)}}\ ; \quad P_{\substack{\text{water-mercury}\\ \text{interface}}} = P_2 + \rho_{\text{waater}} g \cdot 2\,\text{in}$$

$$P_{\substack{\text{mercury-air}\\ \text{interface}}} = P_1 = P_{\substack{\text{mercury-water}\\ \text{interface}}} + \rho_{\text{Hg}} g \cdot 0.1\ \text{in}$$

$$P_1 - P_2 = 2\ \text{in}\ \rho_{\text{w}} g + 0.1\,\text{in}\,\rho_{\text{Hg}} g = \rho_{\text{w}} g \cdot (2 + 1.36)\,\text{in}$$

$$V = 1.0\sqrt{\frac{2 \cdot 32.2\frac{\text{ft}}{\text{s}^2} \cdot 62.3\frac{\text{lbm}}{\text{ft}^3} \cdot \left(\frac{3.36}{12}\,\text{ft}\right)}{\left(0.075\frac{\text{lbm}}{\text{ft}^3}\right)\left(1 - 0.1^2\right)}} = 123\frac{\text{ft}}{\text{s}} = 37.5\frac{\text{m}}{\text{s}}$$

$$Q = VA = 123\frac{\text{ft}}{\text{s}} \cdot 1\,\text{ft}^2 = 123\frac{\text{ft}^3}{\text{s}} = 3.48\frac{\text{m}^3}{\text{s}}$$

5.37 (a) $AF = \frac{\dot{m}_A}{\dot{m}_F} = \frac{A_A V_A \rho_A}{A_F V_F \rho_F}$; $\frac{V_A}{V_F} = \frac{\sqrt{2\frac{\Delta P}{\rho_A}}}{\sqrt{2\frac{\Delta P}{\rho_F}}} = \sqrt{\frac{\rho_F}{\rho_F}}$

We cancel the pressure difference because both the incoming air and the liquid gasoline in the reservoir are at the same pressure. Then $\frac{A_A}{A_F} = \left(\frac{D_A}{D_F}\right)^2$ and

$$AF = \left(\frac{D_A}{D_F}\right)^2 \left(\frac{\rho_F}{\rho_A}\right)^{\frac{1}{2}} \left(\frac{\rho_A}{\rho_F}\right) = \left(\frac{D_A}{D_F}\right)^2 \left(\frac{\rho_A}{\rho_F}\right)^{\frac{1}{2}}$$

(b) As the above equation shows, the air-fuel ratio should be constant, independent of the air flow rate. This explains why this type of carburetor was the practically-exclusive choice of automobile engine designers for about 80 years. With a very simple device, one gets a practically constant air-fuel ratio, independent of the throttle setting. The rest of the carburetor was devoted to those situations in which one wanted some other air-fuel ratio, mostly cold starting and acceleration, for which one wants a lower air fuel ratio (" rich ratio").

(c) $$15 = \left(\frac{D_A}{D_F}\right)^2 \left(\frac{0.075\frac{\text{lbm}}{\text{ft}^3}}{0.72 \cdot 62.3\frac{\text{lbm}}{\text{ft}^3}}\right)^{\frac{1}{2}}; \qquad \frac{D_A}{D_F} = 19.15; \qquad \frac{D_J}{D_2} = \frac{1}{19.15} = 0.052$$

(d) At 5280 ft the atmospheric pressure is about 0.83 atm. The air density falls while the fuel density does not, so the air fuel ratio would become

$$AF = 15\sqrt{\frac{0.83\text{ atm}}{1\text{ atm}}} = 13.7\left(\frac{\text{lbm}_{\text{air}}}{\text{lbm}_{\text{fuel}}}\right)$$

and the engine would run "rich". High altitude conversion kits (smaller diameter jets) are available to deal with this problem. Rich combustion leads to increased emissions of CO and hydrocarbons; there are special air pollution rules for autos at high elevations.

Demands for higher fuel economy and lower emissions are causing the carburetor to be replaced by the fuel injector. In a way it is sad; the basic carburetor is a really clever, simple, self-regulating device.

5.38 By Bernoulli's equation the velocity of a jet is proportional to the square root of the pressure drop/density. The density is proportional to the molecular weight. For the values shown, the predicted jet velocity in the burners will be the same for propane and natural gas. Because of the higher density of propane, the diameter of the jets will normally be reduced, to maintain a constant heat input. But the velocities of the individual gas jets are held the same, to get comparable burner aerodynamics.

5.39 $V_2 = C_v \sqrt{\dfrac{2(-\Delta P)}{\rho\left(1-\left(\dfrac{D}{D_0}\right)^4\right)}}$; $\quad -\Delta P = \dfrac{\rho V_2^2}{2C_v^2}\left(1-\left(\dfrac{D}{D_0}\right)^4\right)$

$$V_2 = \frac{1\ \text{ft}}{\text{s}}\left(\frac{\frac{\pi}{4}(1\ \text{in})^2}{\frac{\pi}{4}(0.2\,\text{in})^2}\right) = 25\frac{\text{ft}}{\text{s}}$$

$$-\Delta P = \frac{\left(55\frac{\text{lbm}}{\text{ft}^3}\right)\left(25\frac{\text{ft}}{\text{s}}\right)^2\left(1-(0.2)^4\right)}{(2)(0.6)^2}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2} = 10.3\ \frac{\text{lbf}}{\text{in}^2} = 70.9\ \text{kPa}$$

5.40 Here for $D_2 / D_1 = 0.8$ $\quad \left(1-\dfrac{A_2^2}{A_1^2}\right) = \left(1-0.8^4\right) = 0.768$

and for $D_2 / D_1 = 0.2$, $\quad \left(1-\dfrac{A_2^2}{A_1^2}\right) = \left(1-0.2^4\right) = 0.9992$

So the curves for C would simply be the corresponding curves for C_V divided by these values (or multiplied by their reciprocals). For 0.2 the multiplier is 1.0008, or practically 1, while for 0.8 it is 1.30. This is a simple scale change for each curve on Fig. 5.14.

5.41 This is simplest by trial and error. First we guess that $D_2 / D_1 = 0.5$. Then

$$V_2 = V_1\cdot\left(\frac{D_1}{D_2}\right)^2 = 1\frac{\text{ft}}{\text{s}}\cdot\left(\frac{1}{0.5}\right)^2 = 4\frac{\text{ft}}{\text{s}} \text{ and}$$

$$-\Delta P = \left(1-\left(\frac{D_2}{D_1}\right)^4\right)\frac{\rho}{2}\cdot\left(\frac{V}{C_v}\right)^2$$

$$= \left(1-0.5^4\right)\frac{13.6\cdot 62.3\frac{\text{lbm}}{\text{ft}^3}}{2}\cdot\left(\frac{4\frac{\text{ft}}{\text{s}}}{0.62}\right)^2\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2} = 3.57\frac{\text{lbf}}{\text{in}^2}$$

This is done on a spreadsheet. We then ask the spreadsheet's search engine to find the value of D_2/D_1 which makes the press drop = 3 psi. Doing this by "goal seek" on excel one finds $D_2/D_1 = 0.5205$.

5.42* (a)

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left(1-(A_2/A_1)^2\right)}} = \sqrt{\frac{2\cdot(20-0)\frac{\text{lbf}}{\text{in}^2}}{62.3\frac{\text{lbm}}{\text{ft}^3}}\cdot\frac{32.2\ \text{lbm ft}}{\text{lbf s}^2}\cdot\frac{144\,\text{in}^2}{\text{ft}^2}} = 63.0\frac{\text{ft}}{\text{s}} = 19.2\frac{\text{m}}{\text{s}}$$

(b) $$V_2 = \begin{pmatrix} V_2 \text{ from} \\ \text{part (a)} \end{pmatrix}\cdot\sqrt{\frac{(20-11.5)}{20}} = 63.0\sqrt{\frac{8.5}{20}} = 41.1\frac{\text{ft}}{\text{s}} = 12.5\frac{\text{m}}{\text{s}}$$

5.43 $$\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta V^2}{2} = 0;\ \Delta z = -\frac{\Delta P}{\rho g} - \frac{\Delta V^2}{2g}$$

$$\Delta z = \frac{-(1-14.7)\frac{\text{lbf}}{\text{in}^2}}{62.3\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\frac{\text{ft}}{\text{s}^2}}\cdot\frac{32.2\ \text{lbm ft}}{\text{lbf s}^2}\cdot\frac{144\,\text{in}^2}{\text{ft}^2} - \frac{\left(10\frac{\text{ft}}{\text{s}}\right)^2}{2\cdot 32.2\frac{\text{ft}}{\text{s}^2}}$$

$$= +31.67 - 1.55 = 30.11\ \text{ft} = 9.18\ \text{m}$$

5.44 The velocities will be the same as in Ex. 5.12. Solving Eq. 5.AX for $(z_2 - z_1)$, we find

$$(z_2 - z_1) = \frac{P_1 - P_2}{\rho g} - \frac{V_2^2}{2g}$$

$$= \frac{(14.7-0.34)\frac{\text{lbf}}{\text{in}^2}}{62.3\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\frac{\text{ft}}{\text{s}^2}}\cdot 32.2\frac{\text{lbm ft}}{\text{lbf s}^2}\cdot\frac{144\ \text{in}^2}{\text{ft}^2} - \frac{\left(25.3\frac{\text{ft}}{\text{s}}\right)^2}{2\cdot 32.2\frac{\text{ft}}{\text{s}^2}}$$

$$= 33.19 - 9.94 = 23.25\,\text{ft} = 7.09\ \text{m}$$

5.45 This follows Ex. 5.12

$$V_3 = \left[2g(h_1 - h_3)\right]^{1/2} = \left(2\cdot 32.2\frac{\text{ft}}{\text{s}}\cdot 10\ \text{ft}\right)^{1/2} = 25.3\frac{\text{ft}}{\text{s}} = 7.71\frac{\text{m}}{\text{s}}$$

One may make the next step applying BE from 1 to 2 or from 2 to 3. Either gives the same result. Working from 2 to 3, we have

$$\frac{P_3 - P_2}{\rho} + (z_3 - z_2)g + \frac{V_3^2}{2}\cdot\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = 0$$

$$P_2 - P_3 = \rho(z_3 - z_2)g + \rho\frac{V_3^2}{2}\cdot\left(1 - \left(\frac{A_3}{A_2}\right)^2\right)$$

$$= 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot(-5\ \text{ft})\cdot 32.2\frac{\text{ft}}{\text{s}^2}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2}$$

$$+62.3\frac{\text{lbm}}{\text{ft}^3}\frac{\left(25.3\frac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\left(1-\left(\frac{1.5\,\text{ft}^2}{1.0\,\text{ft}^2}\right)^2\right)\cdot\frac{\text{lbf s}^2}{32.2\text{ lbm ft}}\cdot\frac{\text{ft}^2}{144\text{ in}^2}$$

$$=-2.16-5.38=-7.54\frac{\text{lbf}}{\text{in}^2}=-7.54\text{ psig}=+7.16\text{ psia}=49.4\text{ kPa abs}$$

5.46* Take the ocean surface as (1) the tip of the propeller at its highest point as (2)

$$V_{2_{\max}}=\sqrt{\frac{2(P_1-P_2)}{\rho}+2g(z_1-z_2)}$$

$$=\sqrt{\frac{2\left(14.7-0.26\frac{\text{lbf}}{\text{in}^2}\right)}{62.3\frac{\text{lbm}}{\text{ft}^3}}\cdot\frac{32.2\text{ lbm ft}}{\text{lbf }s^2}\cdot\frac{144\text{ in}^2}{\text{ft}^2}+2\cdot 32.2\frac{\text{ft}}{\rho}\cdot 4\text{ ft}}=49.1\frac{\text{ft}}{\text{s}}=15.0\frac{\text{m}}{\text{s}}$$

$$\omega_{\max}=\frac{V_{\max}}{\pi D}=\frac{49.1\frac{\text{ft}}{\text{s}}}{\pi\cdot 15\,\text{ft}}=\frac{1.04}{\text{s}}=62\,\text{RPM}$$

5.47 See solution to Prob. 5.46*. Clearly, the greater the depth, the higher the speed at which the propeller can turn without cavitation, so this is not as severe a problem for submarines (submerged) as it is for surface ships. However the noise from propeller cavitation is a serious problem for submarines, because it reveals the position of the submarine to acoustic detectors. Submarines which do not wish to be detected operate with their propellers turning slower than their minimum cavitation speed.

5.48 See Ex. 5.14. As in that example take (1) at the upper fluid (gasoline) surface and (2) at the outlet jet. Take (2a) at the interface between the gasoline-water interface. Then, applying BE from 1 to 2a, we have $\frac{\Delta P}{\rho_{\text{gasoline}}}+g\Delta z+\frac{\Delta V^2}{2}=0$, Here the velocities are both negligible, so that $P_{2a}=\rho_{\text{gasoline}}g(z_1-z_{2a})$ Then applying BE from 2a to 3 we find

$$\frac{V_3^2}{2}=\frac{\rho_{\text{gasoline}}g(z_1-z_{2a})}{\rho_{\text{water}}}+g(z_{2a}-z_3)$$

At this point we can observe that as long as the gasoline-water interface is above the nozzle, the pressure at 2a will always be that due to 20 m of gasoline, which is the same as that of 14.4 m of water. If we replaced the 20 m of gasoline with 14.4 m of water, then the flow at any instant would be unchanged. So this is really the same as Ex. 5.14 with the initial height being 24.4 m and the final height being (14.4 +1) 15.4 m. Thus this is a plug-in,

$$\Delta t = \frac{-2\left[(15.4\,\text{m})^{1/2} - (24.4\,\text{m})^{1/2}\right]}{\frac{(\pi/4)\cdot(1\,\text{m})^2}{(\pi/4)\cdot(10\,\text{m})^2}\cdot\left(2\cdot 9.81\frac{\text{m}}{\text{s}^2}\right)^{1/2}} = 45.8\,\text{s} = 0.76\ \text{min}$$

An alternative, which may please some students better, is to write $\frac{V_3^2}{2} = \frac{\rho_{\text{gasoline}} g(z_1 - z_{2a})}{\rho_{\text{water}}} + g(z_{2a} - z_3) = \alpha + g(z_{2a} - z_3)$ where α is a constant (as long as the interface is above the outlet). Then one repeats the whole derivation in Ex 5.14, finding the same result.

One can also compute the time from when the interface has passed the exit to when the surface is 1 m above the exit, finding the same time as for water, because in this period the density of the flowing fluid is the same as that of gasoline.

The period when the interface (either gasoline-water or gasoline-air) is close to the nozzle is not easy to predict by simple BE because the Δz becomes comparable to the diameter of the opening, and the non-uniform flow phenomena discussed in Sec. 5.11 come into play.

See Prob. 5.53. I have the device described in that problem. I regularly assign the problem, then run the demonstration. One can estimate well, down to an interface one or two diameters above the nozzle, but not lower.

5.49* See the solution to the preceding problem. The easiest way to work the problem is to conceptually convert the 10 ft of gasoline to 7.2 ft of water. Then this is the same as Ex. 5.13 , asking the time for the level to fall from 17.2 ft to 7.2 ft.

$$\Delta t = \frac{-2}{\frac{A_2}{A_1}\sqrt{2g}}\left(\sqrt{h_2} - \sqrt{h_1}\right) = \frac{2}{\frac{1}{100}\sqrt{2\cdot\frac{32.2\,\text{ft}}{\text{s}^2}}}\left(\sqrt{17.2\,\text{ft}} - \sqrt{7.2\,\text{ft}}\right) = 36.5\ \text{s}$$

5.50 (a) See the preceding two problems. 20 psig is the equivalent of 46.19 ft of water, so that

$$\Delta t = \frac{-2\left[(1 + 46.19\,\text{ft})^{1/2} - (5 + 46.19\ \text{ft})^{1/2}\right]}{\frac{(\pi/4)\cdot(1\ \text{ft})^2}{(\pi/4)\cdot(10\ \text{ft})^2}\cdot\left(2\cdot 32.2\frac{\text{ft}}{\text{s}^2}\right)^{1/2}} = 7.11\ \text{s} = 0.12\ \text{min}$$

(b) In this case we have $P = P_0\frac{V_0}{V} = (100 + 14.7)\text{psia}\frac{A\cdot 1\ \text{ft}}{A\cdot(6-h)\text{ft}} = 114.7\ \text{psia}\frac{1}{(6 - h/\ \text{ft})}$ where h is the height of the interface above the nozzle, initially 5 ft, finally 1 ft. The final pressure is 22.94 psia = 8.24 psig. This gives the answer to part (c); the 5-fold expansion of the gas lowers its absolute pressure by a factor of 5, which would produce a vacuum and stop the flow if the initial pressure were 20 psig.

At any instant, the velocity is given by $V_{\text{outlet}} = \sqrt{2 \cdot \left(\frac{-\Delta P}{\rho} + gh \right)}$ where $-\Delta P$ = the above absolute pressure minus 14.7 psia. If we substitute that equation in the velocity expression we get an equation of the form $V_{\text{outlet}} = \sqrt{\frac{a}{b-h} + ch}$ where *a, b,* and *c* are constants, and an integral of the form $\frac{dh}{\sqrt{\frac{a}{b-h} + ch}}$. This might be reducible to one of the forms in my integral table, but not easily. Instead we proceed by a spreadsheet numerical integration. For time zero we have

$$V_{\text{outlet}} = \sqrt{2 \cdot \left(\frac{100 \frac{\text{lbf}}{\text{in}^2}}{62.3 \frac{\text{lbm}}{\text{ft}^3}} \cdot 32.2 \frac{\text{lbm ft}}{\text{lbf s}^2} \cdot 144 \frac{\text{in}^2}{\text{ft}^2} + 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 5 \text{ ft} \right)} = 122.66 \frac{\text{ft}}{\text{s}}$$

and $\frac{dh}{dt} = -\frac{A_{\text{out}}}{A_{\text{tank}}} V_{\text{outlet}} = -0.01 \cdot 122.66 \frac{\text{ft}}{\text{s}} = -1.23 \frac{\text{ft}}{\text{s}}$. The time for the surface to fall from 5 to 4.75 ft above the outlet would be $\Delta t = \frac{\Delta h}{\frac{dh}{dt}} = \frac{-0.25 \text{ ft}}{1.23 \frac{\text{ft}}{\text{s}}} = 0.203 \text{ s}$ if the velocity remained constant over this height. But as the following table shows, the velocity declines, so we cover this height step using the average of the velocity above, and the velocity at *h* = 4.75 ft, which is -1.07 ft/s. Then we make up the following spreadsheet;

h, ft	*P*, psia	*P* psig	*V* inst, ft/s	*dh/dt*, ft/s	delta *t*	Cumulative *t*
5	114.700	100.000	122.664	-1.227		0.000
4.75	91.760	77.060	107.813	-1.078	0.217	0.217
4.5	76.467	61.767	96.639	-0.966	0.245	0.461
4.25	65.543	50.843	87.778	-0.878	0.271	0.733
4	57.350	42.650	80.482	-0.805	0.297	1.030
3.75	50.978	36.278	74.302	-0.743	0.323	1.353
3.5	45.880	31.180	68.949	-0.689	0.349	1.702
3.25	41.709	27.009	64.227	-0.642	0.375	2.077
3	38.233	23.533	59.997	-0.600	0.403	2.480
2.75	35.292	20.592	56.159	-0.562	0.430	2.910
2.5	32.771	18.071	52.636	-0.526	0.460	3.370
2.25	30.587	15.887	49.368	-0.494	0.490	3.860
2	28.675	13.975	46.310	-0.463	0.523	4.383
1.75	26.988	12.288	43.422	-0.434	0.557	4.940
1.5	25.489	10.789	40.673	-0.407	0.595	5.534
1.25	24.147	9.447	38.033	-0.380	0.635	6.170
1	22.940	8.240	35.479	-0.355	0.680	6.850

Finding a time of 6.85 s. The original spreadsheet carries more digits than will fit on this table. One may test the stability of this solution, by rerunning it with smaller height increments. For increments of 0.25 ft, shown in the table, the time is 6.8498 s. For increments of 0.1 ft it is 6.8563 s. Both round to 6.85 s.

(c) See the discussion at the top of part (b).

5.51 $V_{out} = \sqrt{2gh}; \quad Q_{out} = A_{exit} V_{out} = -A_{tank} \dfrac{dh}{dt}$

$$A_{tank} = 50 \text{ ft}^2 + 50 \text{ ft}^2 \left(\frac{h}{20 \text{ ft}} \right) = 50 \text{ ft}^2 + 2.5 \text{ ft} \cdot h = a + bh$$

where a and b are constants.

$$\frac{dh}{dt} = \frac{-A_{out}\sqrt{2gh}}{a+bh}; \qquad -A_{out}\sqrt{2g}\int_0^{\Delta t} dt = \int_{h_0}^{h} \frac{a\,dh}{\sqrt{h}} + b\int_{h_0}^{h} \sqrt{h}\,dh$$

$$\Delta t = \frac{1}{A_{out}\sqrt{2g}} \left[\frac{a}{\frac{1}{2}}\sqrt{h} + \frac{b}{\frac{3}{2}} h^{\frac{3}{2}} \right]_{h=0}^{h=h_{top}}$$

$$= \frac{1}{1 \text{ ft}^2 \sqrt{2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2}}} \left[\frac{50 \text{ ft}^2}{\frac{1}{2}} \sqrt{20 \text{ ft}} + \frac{2.5 \text{ ft}}{\frac{3}{2}} (20 \text{ ft})^{\frac{3}{2}} \right] = 74.3 \text{ s}$$

5.52* The instantaneous depth in the tank is h. Let $h' = h_1 - h$. Then this becomes the same as Ex. 5.13

$$\Delta t = \frac{-2\left(\sqrt{h_2} - \sqrt{h_1}\right)}{\frac{A_2}{A_1}\sqrt{2g}}; \qquad h_2 = 0, \; h_1 = h_1$$

$$\Delta t = \frac{2\sqrt{h_1}}{\frac{A_2}{A_1}\sqrt{2g}} = \frac{2\sqrt{h_1}}{\frac{0.5}{20} \cdot \sqrt{2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2}}} = 9.97 \frac{\text{s}}{\sqrt{\text{ft}}} \cdot \sqrt{h_1}$$

5.53 This is the same as Ex. 5.14, with much smaller dimensions, which match a Plexiglas demonstrator which I have used regularly in class. Following Ex. 5.14

$$\Delta t = \frac{-2\left[(1 \text{ in})^{1/2} - (11 \text{ in})^{1/2}\right]}{\frac{(\pi/4)\cdot(0.3 \text{ in})^2}{6 \text{ in} \cdot 5.5 \text{ in}} \cdot \left(2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2}\right)^{1/2}} \cdot \sqrt{\frac{\text{ft}}{12 \text{ in}}} = 78 \text{ s} = 1.30 \text{ min}$$

When I have run this in a classroom (with a sink) the times are typically 84 to 88 s. Much of the time is spent in the last inch, where the assumption that the diameter of the nozzle is negligible compared to the height of the fluid above it becomes poor (see Sec 5.11). When I do this I assign the problem and ask each student to write her/his name and predicted time on the blackboard before I run it. Some students like that, others don't.

5.54 (a) See Ex. 5.5

$$V_2 = C\sqrt{2gh\left(\frac{\rho_{\text{air}}}{\rho_{\text{gas}}} - 1\right)} = 0.6\sqrt{2 \cdot 32.2\frac{\text{ft}}{\text{s}^2} \cdot \frac{7.5\text{ ft}}{12} \cdot \left(\frac{29}{16} - 1\right)} = 3.43\frac{\text{ft}}{\text{s}}$$

Here we introduce the orifice coefficient because the hole drilled in the lid of the can is much more like a flat-plate orifice than like the rounded nozzles in the Torricelli examples.

(b) For totally unmixed flow the densities above and below the interface between gas and the air which has flowed in from below are constant, so that

$\dfrac{V_2}{V_{2,\text{ initial}}} = \sqrt{\dfrac{h}{h_0}}$. We then square both sides and differentiate w.r.t., finding

$\dfrac{2V_2}{(V_{2,\text{ initial}})^2}\dfrac{dV_2}{dt} = \dfrac{1}{h_0}\dfrac{dh}{dt}$. Then using the same ideas as in Ex 5.14 we write that

$\dfrac{dh}{dt} = -V_2\dfrac{A_{\text{outlet}}}{A_{\text{tank cross section}}}$. We substitute this in the preceding equation and simplify

to $$\frac{dV_2}{dt} = \frac{-(V_{2,\text{ initial}})^2}{2h_0}\frac{A_{\text{outlet}}}{A_{\text{tank cross section}}}$$

which we can then separate and integrate from start to any time, finding

$$\frac{V_2}{V_{2,\text{original}}} = 1 - \frac{V_{2,\text{original}}A_{\text{outlet}}t}{2h_0 A_{\text{tank cross section}}}$$

This is a straight line on a plot of $\dfrac{V_2}{V_{2,\text{original}}}$ vs. t. Its value is zero when

$$t = \frac{2h_0 A_{\text{tank cross section}}}{V_{2,\text{ original}}A_{\text{outlet}}} = \frac{2 \cdot \left(\frac{7.5\text{ ft}}{12}\right) \cdot \left(\frac{\pi}{4} \cdot \left(\frac{6.5\text{ ft}}{12}\right)^2\right)}{3.43\frac{\text{ft}}{\text{s}} \cdot \left(\frac{\pi}{4} \cdot \left(\frac{0.25\text{ ft}}{12}\right)^2\right)} = 246\ /\ \text{s}$$

For the totally mixed model we have

$$\frac{V_2}{V_{2,\text{ initial}}} = \sqrt{\frac{\left(\dfrac{\rho_{\text{air}}}{\rho_{\text{gas mixture in can}}}\right)_{\text{instantaneous}} - 1}{\left(\dfrac{\rho_{\text{air}}}{\rho_{\text{gas mixture in can}}}\right)_{\text{initial}} - 1}}$$

By material balance, taking the can as our system. In this problem two Vs appear, the volume of the can and the instantaneous velocity. Following the nomenclature, both are italic. When a *V* is the volume of the can, it has a "can" subscript.

$$V_{\text{can}} \frac{d\rho_{\text{mixture}}}{dt} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = V_2 A_{\text{outlet}} \left(\rho_{\text{air}} - \rho_{\text{mixture}}\right)$$

Which we rearrange to

$$\frac{d\rho_{\text{mixture}}}{\left(\rho_{\text{air}} - \rho_{\text{mixture}}\right)} = \frac{V_2 A_{\text{outlet}}}{V_{\text{can}}} dt$$

and integrate to

$$\ln\left(\frac{\left(\rho_{\text{air}} - \rho_{\text{mixture}}\right)}{\left(\rho_{\text{air}} - \rho_{\text{mixture}}\right)_{\text{initial}}}\right) = \frac{-A_{\text{outlet}}}{V_{\text{can}}} \int_0^t V_2 dt = -\frac{Q}{V_{\text{can}}}$$

where $\frac{Q}{V_{\text{can}}}$ is the number of can volumes which have flowed into and out of the tank since time zero. We take the exp of both sides and rearrange, finding

$$\rho_{\text{mixture}} = \rho_{\text{air}} - \left(\rho_{\text{air}} - \rho_{\text{mixture}}\right)_{\text{initial}} \exp\left(-\frac{Q}{V_{\text{can}}}\right)$$

We then substitute this into the velocity ratio equation and simplify to

$$\frac{V_2}{V_{2,\text{ initial}}} = \sqrt{\frac{\left(\dfrac{\rho_{\text{gas initial}}}{\rho_{\text{air}}}\right) \exp\left(-\dfrac{Q}{V_{\text{can}}}\right)}{1 - \left(1 - \dfrac{\rho_{\text{gas initial}}}{\rho_{\text{air}}}\right) \exp\left(-\dfrac{Q}{V_{\text{can}}}\right)}}$$

In principle, one should be able to eliminate *Q* from this relationship by $\frac{-A_{\text{outlet}}}{\mathrm{V}_{\text{can}}} \int_0^t V_2 dt = -\frac{Q}{V_{\text{can}}}$ to get $\frac{V_2}{V_{2,\text{ initial}}}$ as an explicit function of *t*, but I haven't been able to do so, nor has anyone shown me how. Try it, you'll be impressed! However it is easy to solve this on a spreadsheet. The solution is shown below. The first line is obvious, at time zero there has been no flow in and $V = V_0 = 3.431$ s. Most of the spreadsheet carries more significant figures than are shown here.

For $Q/V = 0.1$, we can easily compute that $\exp(-Q/V) = 0.905$. Then

$$\frac{V_2}{V_{2,\text{ initial}}} = \sqrt{\frac{(16/29)\cdot 0.905}{1 - (1 - 16/29)\cdot 0.905}} = 0.916 \text{ and}$$

$V_2 = 0.916 \cdot 3.341 \text{ ft/s} = 3.145 \text{ ft/s}$ We get delta *t* by differentiating the material balance, above, finding

$$\Delta t = \frac{V_{\text{can}}}{A_{\text{outlet}}}\frac{\Delta(Q/V_{\text{can}})}{V_{2,\text{ average}}} = \frac{\frac{\pi}{4}\cdot\frac{7.5\text{ ft}}{12}\cdot\left(\frac{6.5\text{ ft}}{12}\right)^2}{\frac{\pi}{4}\cdot\left(\frac{0.25\text{ ft}}{12}\right)^2}\frac{(0.1-0)}{0.5\cdot(3.341+3.145)\frac{\text{ft}}{\text{s}}} = 12.85\text{ s}$$

Each number in the rightmost column is the sum of the one above it and the one to its immediate left.

Q/*V*	exp(-*Q*/*V*)	V/V_0	*V*, ft/s	delta *t*, s	*t* cum.
0	1	1	3.431		0
0.1	0.905	0.916	3.145	12.850	12.850
0.2	0.819	0.845	2.899	13.983	26.833
0.3	0.741	0.782	2.684	15.136	41.969
0.4	0.670	0.727	2.495	16.316	58.285
0.5	0.607	0.678	2.326	17.527	75.812
0.6	0.549	0.634	2.174	18.776	94.588
0.7	0.497	0.594	2.037	20.065	114.653
0.8	0.449	0.557	1.912	21.399	136.052
0.9	0.407	0.524	1.797	22.784	158.836
1	0.368	0.493	1.692	24.221	183.057
1.1	0.333	0.465	1.594	25.717	208.774
1.2	0.301	0.438	1.504	27.275	236.049
1.3	0.273	0.414	1.420	28.899	264.948
1.4	0.247	0.391	1.342	30.593	295.541
1.5	0.223	0.370	1.269	32.363	327.904
1.6	0.202	0.350	1.201	34.213	362.117
1.7	0.183	0.331	1.137	36.147	398.264
1.8	0.165	0.314	1.077	38.171	436.435
1.9	0.150	0.297	1.020	40.289	476.725

I tested the stability of this solution by reducing the increment size from 0.1 to 0.05 and found that the time to $Q/V = 1.9$ changed from 476.725 s to 476.994 s which is certainly a minimal change.

(c) From the above table we may interpolate that 1.1 ft/s corresponds to ≈ 420 s, which is a tolerable match with the observed ≈ 325 s. For the plug flow model the agreement is not good as good. We can see that the required value of $\frac{V}{V_0} = \frac{1.1}{3.43} = 0.321$. We solve for it by

$$\frac{V_2}{V_{2\text{ ,original}}} = 0.321 = 1 - \frac{V_{2\text{ ,original}}A_{\text{outlet}}t}{2h_0A_{\text{tank cross section}}}; \qquad t = \frac{(1-0.321)\cdot 2h_0A_{\text{tank cross section}}}{V_{2\text{ ,original}}A_{\text{outlet}}}$$

$$t = \frac{(1-0.321)\cdot 2\frac{7.5\text{ ft}}{12}}{3.43\frac{\text{ft}}{\text{s}}}\left(\frac{6.5\text{ in}}{0.25\text{ in}}\right)^2 = 167\text{ s}$$

We always tell the students that the totally mixed and plug flow models will normally bracket the observed behavior of nature. That happens here.

In making up this solution I found two errors in [7]. I hope the version here is error free. As of spring 2003 you could watch a film clip of a version of this demonstration at

http://chemmovies.unl.edu/Chemistry/DoChem/DoChem049.html

The combustion specialists in Chemical Engineering at the U of Utah like this demonstration very much. For me it is a fine demonstration of unsteady-state flow. They see all sorts of interesting combustion-related issues in it.

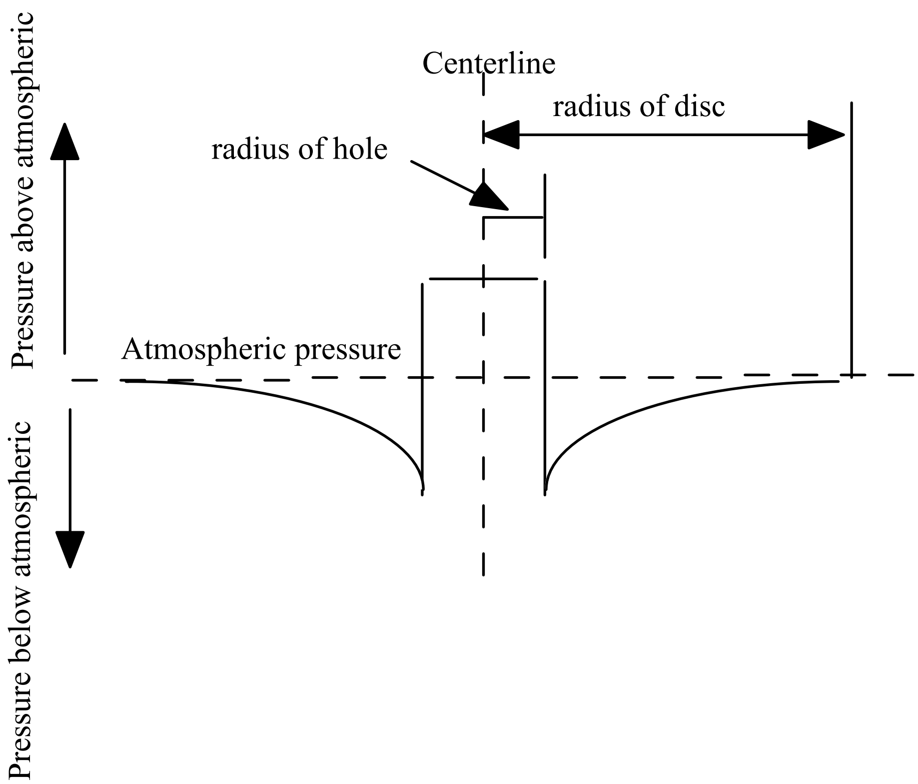
5.55 Replacing the flowing liquid with a gas heavier than air requires us to combine Ex. 5.14 with Ex. 5.5. We see that the instantaneous velocity is given by

$$V_2 = \left[2gh\left(1 - \frac{\rho_{\text{air}}}{\rho_{\text{propane}}}\right)\right]^{1/2} = \sqrt{2gh}\cdot\sqrt{\left(1-\frac{29}{44}\right)} = 0.584\cdot\sqrt{2gh}$$

This is true at all values of h so we can simply substitute this value and find that the required time is the answer to that example divided by 0.584 = 5.77 minutes.

I give this problem as an introduction to a safety lecture. Propane is by far the most dangerous of the commonly-used fuels. If there is a large leak of natural gas, buoyancy will take it up away from people and away from ignition sources. If there is leak of any liquid fuel (gasoline, diesel, heating oil) it will fall on the ground and flow downhill. But the ground will absorb some, and ditches, dikes or low spots will trap some or all of it. But released propane forms a gas heavier than air, which flows downhill, and flows over ditches, dikes and low spots, looking for an ignition source, and then burning at the elevation where people are. See "Chemical Engineers as Expert Witnesses in Accident Cases" Chem. Eng. Prog. 84(6), 22-27 (1988). and "Propane Overfilling Fires," Fire Journal 81, No 5, 80-82 and 124-126 (1987).

5.56 At the periphery, where the air flows out, its pressure must be the same as that of the atmosphere (subsonic jet!). By material balance (assuming that the width of the flow channel between the cardboard and the spool is constant) the velocity is proportional to (1/radius) so the velocity decreases with radial distance. Thus by BE, the pressure must be rising steadily in the radial direction. In the center hole the pressure must be higher than atmospheric, in order to give the gas its initial velocity and to overcome the frictional effect of the entrance into the channel between the cardboard and the spool. Thus the figure is as sketched below. This is a very simple, portable, cheap, dramatic demonstration to use in class.



5.57 $Q = \frac{1\text{ L}}{2\text{ s}} = 0.5\frac{\text{L}}{\text{s}}$ The area of the periphery is

$A = \pi D h = \pi \cdot 35\text{ mm} \cdot 0.2\text{ mm} = 22.0\text{ mm}^2$, so the velocity at the perimeter is

$$V = \frac{Q}{A} = \frac{0.5\frac{\text{L}}{\text{s}}}{22\text{ mm}^2} \cdot \frac{10^6\text{ mm}^3}{\text{L}} = 22\ 736\frac{\text{mm}}{\text{s}} = 22.7\frac{\text{m}}{\text{s}}$$

The velocity at the edge of the center hole is

$$V_{\text{hole}} = V_{\text{perimeter}}\frac{D_{\text{perimeter}}}{D_{\text{hole}}} = 22.7\frac{\text{m}}{\text{s}}\frac{35\text{ mm}}{7.1\text{ mm}} = 112\frac{\text{m}}{\text{s}}$$

Then we apply BE from the edge of the hole to the perimeter, finding

$$\Delta P = \frac{\rho}{2}\cdot\left(V_{\text{periphery}}^2 - V_{\text{hole}}^2\right) = \frac{1.20\frac{\text{kg}}{\text{m}^3}}{2}\cdot\left(\left(22.7\frac{\text{m}}{\text{s}}\right)^2 - \left(112\frac{\text{m}}{\text{s}}\right)^2\right)\cdot\frac{\text{N s}^2}{\text{kg m}}\cdot\frac{\text{Pa m}^2}{\text{N}}$$

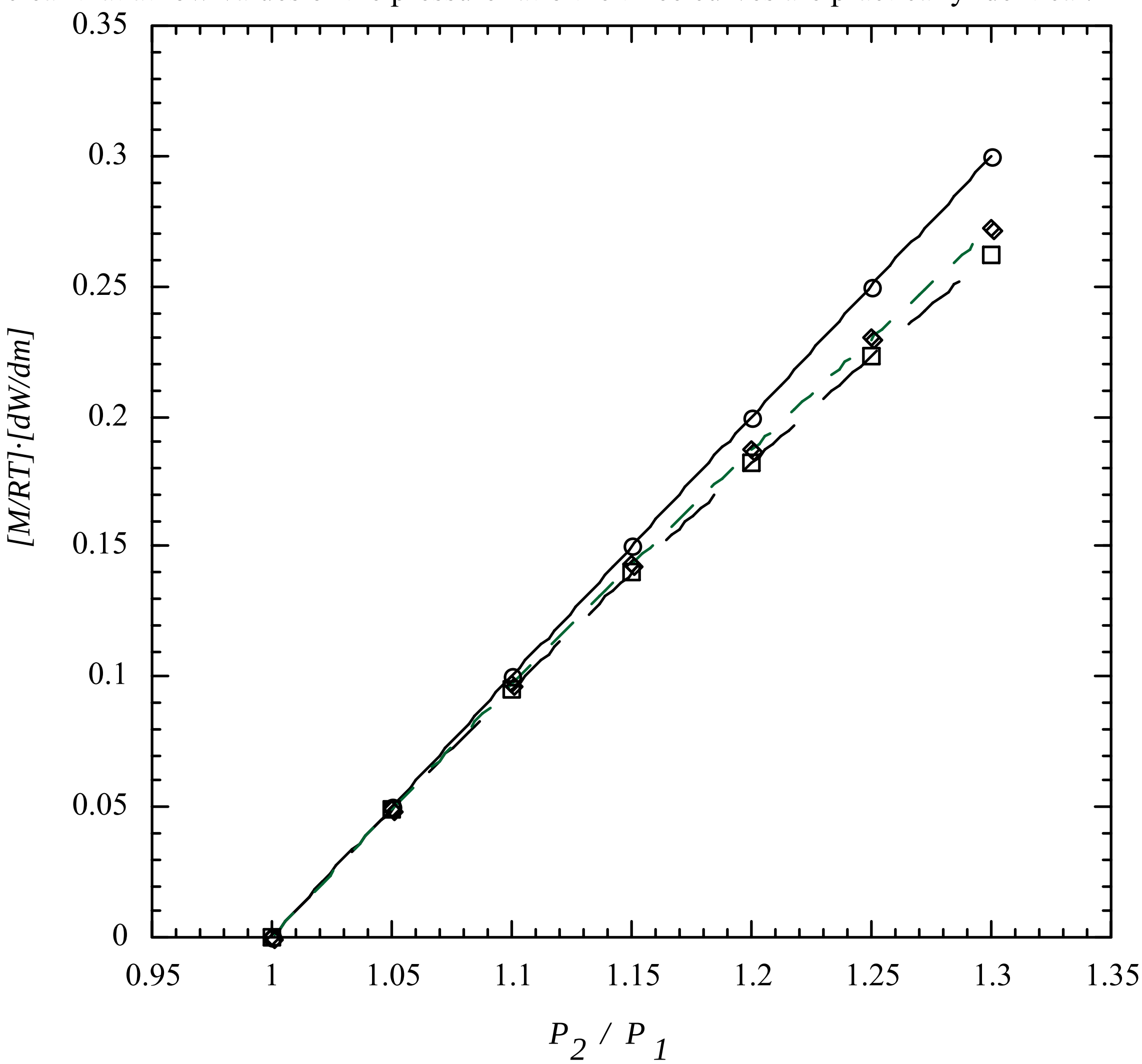
$$= 7.2\text{ kPa} = 1.05\text{ psi}$$

At the periphery the pressure is 0.00 psig. Since the pressure **increases** by 1.05 psi from center to periphery, the pressure at the edge of the center hole must be minus 1.05 psig, i.e. a vacuum of 1.05 psi.

5.58 The numerical values are shown below:

P_2/P_1	incompressible	isothermal	adiabatic
1	0	0.000	0.000
1.05	0.05	0.049	0.049
1.1	0.1	0.095	0.097
1.15	0.15	0.140	0.143
1.2	0.2	0.182	0.187
1.25	0.25	0.223	0.230
1.3	0.3	0.262	0.272

These values are plotted below, with smooth curves drawn through them. It must be clear that at low values of the pressure ratio the three curves are practically identical.



5.59 The pressure difference needed to support the load is $-\Delta P = \dfrac{W}{A} = \dfrac{W}{\dfrac{\pi}{4}D^2}$

The velocity under the skirt is $V = \sqrt{\dfrac{2(-\Delta P)}{\rho}}$; thus

$$Q = VA = V\pi Dh = \pi h\sqrt{\frac{2W}{\frac{\pi}{4}\rho}} = \frac{0.01}{12}\,\text{ft}\,\pi\sqrt{\frac{8\cdot 5000\ \text{lbf}}{\pi\cdot 0.075\dfrac{\text{lbm}}{\text{ft}^3}}\cdot\frac{32.2\,\text{lbm ft}}{\text{lbf s}^2}} = 6.1\frac{\text{ft}^3}{\text{s}}$$

$$\text{Po} = \dot{m}\frac{dW_{\text{n.f.}}}{dm} = \rho Q\frac{RT_1}{M}\ln\frac{P_2}{P_1} = P_1 Q\ln\frac{P_2}{P_1}$$

$$P_2 = \frac{W}{A} = \frac{5000\ \text{lbf}}{\left(\frac{\pi}{4}\right)(10\,\text{ft})^2}\cdot\frac{\text{ft}^2}{144\ \text{in}^2} = 0.44\,\text{psi} = 3.05\ \text{kN}$$

$$\text{Po} = \left(14.7\frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{6.1\,\text{ft}^3}{\text{s}}\right)\ln\left(\frac{14.7+0.44}{14.7}\right)\cdot\frac{144\ \text{in}^2}{\text{ft}^2}\cdot\frac{\text{hp s}}{550\ \text{ft lbf}} = 0.69\ \text{hp} = 0.52\ \text{kW}$$

As far as I know these devices were quite popular for a while, but have recently declined in popularity. As far as I know there are still ferry boats that use this system to stay above the water, regularly crossing English Channel.

5.60* (a) The gauge pressure inside the structure must equal the weight of the roof per unit area, P = 1.0 lbf/in^2 = 0.007 psig = 0.036 in H_2O = 0.0479 kPa

(b) $$V = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2\cdot 1.0\dfrac{\text{lbf}}{\text{ft}^2}}{0.075\dfrac{\text{lbm}}{\text{ft}^3}}\,\frac{32.2\ \text{lbm ft}}{\text{lbf s}^2}} = 29.3\frac{\text{ft}}{\text{s}} = 8.93\frac{\text{m}}{\text{s}}$$

$$Q = VA = 29.3\frac{\text{ft}}{\text{s}}\cdot 5\,\text{ft}^2 = 146.5\frac{\text{ft}^3}{\text{s}} = 8760\ \text{cfm} = 4.15\frac{\text{m}^3}{\text{s}}$$

(c) $$\text{Po} = \frac{Q\Delta P}{\eta} = \frac{146.2\dfrac{\text{ft}^3}{\text{s}}\cdot 1.0\dfrac{\text{lbf}}{\text{ft}^2}}{0.75}\cdot\frac{\text{HP s}}{550\ \text{ft lbf}} = 0.355\ \text{HP} = 0.264\ \text{kW}$$

These structures are widely used as winter covers for outdoor tennis courts, and some other applications. There are two put up each winter within five miles of my house.

5.61 (a), using Eq. 5.22 with the coefficient described below it;

$$Q = \frac{0.67W\sqrt{2g}}{3/2}h^{\frac{3}{2}}$$

$$h = \left[\frac{\frac{3}{2} \cdot Q}{0.67 W \sqrt{2g}} \right]^{\frac{2}{3}} = \left[\frac{\left(\frac{3}{2}\right)\left(\frac{100\,\mathrm{m}^3}{\mathrm{s}}\right)}{0.67 \cdot 50\,\mathrm{m} \sqrt{2 \cdot 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}}} \right]^{\frac{2}{3}} = 1.007\,\mathrm{m} = 3.3\ \mathrm{ft}$$

(b) $$V_1 = \frac{Q}{A} = \frac{100 \frac{\mathrm{m}^3}{\mathrm{s}}}{(5 \cdot 50)\,\mathrm{m}^2} = 0.4 \frac{\mathrm{m}}{\mathrm{s}}$$

(c) Here, looking at the equation 5.BI, we see that the average value of $2gh$ is

$$(2gh)_{\mathrm{avg}} = 2 \cdot 9.81 \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \left(\frac{0 + 1.007}{2} \right) \mathrm{m} = 9.88 \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

while the value of $V_1{}^2/2$ is constant at 0.08 $\mathrm{m}^2/\mathrm{s}^2$, so the ratio of the neglected term to the one retained is

$$\text{Ratio} = \frac{0.08 \frac{\mathrm{m}^2}{\mathrm{s}^2}}{9.88 \frac{\mathrm{m}^2}{\mathrm{s}^2}} = 0.008 = 0.8\%$$

5.62 $$V_{\mathrm{top}} = \sqrt{2 \cdot 32.2 \frac{\mathrm{ft}}{\mathrm{s}^2} (30 - 0.25)\,\mathrm{ft}} = 43.77 \frac{\mathrm{ft}}{\mathrm{s}}$$

$$V_{\mathrm{bottom}} = \sqrt{2 \cdot 32.2 (30 + 0.25)} = 44.14 \frac{\mathrm{ft}}{\mathrm{s}}$$

$$\frac{V_{\mathrm{top}}}{V_{\mathrm{bottom}}} = 0.9917$$

Clearly we make a negligible error by ignoring this in the standard Torricelli's problems.

5.63 $$\dot{m}_1 = \dot{m}_2 = \rho A V \ ; \qquad \frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{D_2}{D_1} \right)^2 ; \qquad \frac{V_2^2}{2} = \frac{V_1^2}{2} - g \Delta z$$

$$\frac{V_2}{V_1} = \sqrt{1 - \frac{2 g \Delta z}{V_1^2}} = \sqrt{1 - \frac{2\left(32.2 \frac{\mathrm{ft}}{\mathrm{s}^2}\right)(-1\,\mathrm{ft})}{\left(1 \frac{\mathrm{ft}}{\mathrm{s}}\right)^2}} = 8.087$$

$$\frac{D_2}{D_1} = \sqrt{\frac{1}{8.087}} = 0.3516 \, ; \qquad D_2 = 0.25\ \mathrm{in}\ (0.3516) = 0.088\ \mathrm{in} = 2.23\ \mathrm{mm}$$

5.64 See the preceding problem. I have the solution on a spreadsheet, so I can simply duplicate that, and ask the spreadsheet's numerical engine to find the value of Δz for which

$D_2 / D_1 = 0.1 / 0.25 = 0.4$, finding $-\Delta z = 0.594\text{ ft}$. By hand, we rearrange the above equations to

$$-\Delta z = \frac{V_0^2}{2g} \cdot \left(\left(\frac{D_1}{D_2} \right)^4 - 1 \right) = \frac{\left(1\frac{\text{ft}}{\text{s}} \right)^2}{2 \cdot 32.2\frac{\text{ft}}{\text{s}^2}} \cdot \left(\left(\frac{0.25\text{ in}}{0.1\text{ in}} \right)^4 - 1 \right) = 0.594\text{ ft}$$

At first I was puzzled that ≈ 0.6 ft produces $D_2 / D_1 = 0.4$ while 1 ft produces $D_2 / D_1 = 0.35$. However we see that $D_2 / D_1 \propto (-\Delta z)^{1/4}$ so that the diameter decrease is rapid at the nozzle, and becomes much slower as the stream accelerates due to gravity.

5.65 I got this quote out of a book somewhere. I wish I remembered where. As the following calculation shows, by simple B.E. one would estimate a much higher velocity. However choosing 1.013 bar as atmospheric pressure may be too high for a major hurricane; the pressure at which the velocity is negligible may be substantially below that. In any event,

$$V = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2 \cdot (1.013 - 0.850) \cdot 10^5\text{Pa}}{1.20\frac{\text{kg}}{\text{m}^3}} \cdot \frac{\text{N m}^2}{\text{Pa}} \cdot \frac{\text{kg m}}{\text{N s}^2}}$$

$$= 164\frac{\text{m}}{\text{s}} = 538\frac{\text{ft}}{\text{s}} = 367\frac{\text{mi}}{\text{hr}}$$

Solutions Chapter 6

For all problems in this chapter, unless it is stated to the contrary, friction factors are computed from Eq. 6.21.

6.1 For non-horizontal flow one simply replaces the (P_1 - P_2) term with

$$(P_1 - P_2) + g\rho(z_1 - z_2)$$

which makes the equivalent of Eq. 6.4 become.

$$\tau = \frac{-r\left[(P_1 - P_2) + g\rho(z_1 - z_2)\right]}{2\Delta x}$$

and the equivalent of Eq. 6.9 becomes

$$Q = \frac{\pi}{\mu}\frac{D_0^4}{128}\left(\frac{(P_1 - P_2) + \rho g \Delta z}{\Delta x}\right)$$

For vertical flow $\Delta z/\Delta x$ is plus or minus 1; for some other angle it is cos θ so that Eq. 6.9 becomes

$$Q = \frac{\pi}{\mu}\frac{D_0^4}{128}\left(-\frac{dP}{dx} + \rho g\cos\theta\right)$$

6-2 Assuming a transition Reynolds number of 2000,

$$V = \frac{\mathsf{R}\cdot\mu}{D\cdot\rho} = \frac{2000\cdot 0.018\ \text{cP}}{\left(\frac{1}{12}\right)\text{ft}\cdot 0.075\frac{\text{lbm}}{\text{ft}^3}}\cdot\frac{6.72\cdot 10^{-4}\ \text{lbm}}{\text{ft s cP}} = 3.87\frac{\text{ft}}{\text{s}} = 1.18\frac{\text{m}}{\text{s}}$$

$$\frac{\Delta P}{\Delta x} = \frac{4f}{D}\rho\frac{V^2}{2}; \qquad f = \frac{16}{\mathsf{R}} = 0.008$$

$$\frac{\Delta P}{\Delta x} = \frac{(4)(0.008)\left(0.075\frac{\text{lbm}}{\text{ft}^3}\right)\left(3.87\frac{\text{ft}}{\text{s}}\right)^2}{\left(\frac{1}{12}\right)\text{ft}\cdot 2}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2}$$

$$= 4.7\cdot 10^{-5}\frac{\text{psi}}{\text{ft}} = 0.0011\frac{\text{kPa}}{\text{m}}$$

These values are low enough that air is very rarely in laminar flow in industrial pipes.

6.3* Again assuming a transition Reynolds number of 2000,

$$V = \frac{2000\cdot 1.002\ \text{cP}}{\left(\frac{1}{12}\right)\text{ft}\cdot 62.3\frac{\text{lbm}}{\text{ft}^3}}\cdot\frac{6.72\cdot 10^{-4}\,\text{lbm}}{\text{ft s cP}} = 0.259\frac{\text{ft}}{\text{s}} = 0.079\frac{\text{m}}{\text{s}}$$

$$\frac{\Delta P}{\Delta x} = \frac{128}{\pi}\frac{\mu}{D^4} = \frac{128}{\pi}\cdot\frac{2.09\cdot 1\text{e}-5\ \text{lbf s / ft}^2}{(\text{ft}/12)^4}$$

$$= 0.0250\frac{\text{lbf}}{\text{ft}^3} = 0.0001735\frac{\text{psi}}{\text{ft}} = 0.0039\frac{\text{kPa}}{\text{m}}$$

or, taking $f = 16/\mathcal{R} = 0.008$, and using the friction factor formulation

$$\frac{\Delta P}{\Delta x}=\frac{4\cdot 0.008\cdot 62.3\cdot(0.259)^2}{2\cdot(1/12)}\cdot\frac{1}{32.2}\cdot\frac{1}{144}=1.735\cdot 10^{-4}\frac{\text{psi}}{\text{ft}}=0.92\frac{\text{psi}}{\text{mile}}$$

These values show that water is occasionally, but not often, in laminar flow in industrial-sized equipment. It is often in laminar flow in laboratory or analytical sized equipment.

6.4 $$V_{\text{avg}}=\frac{Q}{A}=\frac{-\frac{\Delta P}{\Delta x}\cdot\frac{\pi}{4}\cdot\frac{r_0^4}{8}}{\pi r_0^2}=-\frac{\Delta P}{\Delta x}\frac{1}{\mu}\frac{r_0^2}{8}\qquad \text{From Eq. 6.8}$$

$$V_{\max}=-\frac{\Delta P}{\Delta x}\frac{1}{\mu}\frac{r_0^2}{4}\qquad \text{so that}\qquad \frac{V_{\text{avg}}}{V_{\max}}=2\quad \text{which is Eq. 6.10.}$$

$$ke_{\text{avg}}=\frac{\int V^3 r dr}{2\int V r dr}=\left[\frac{\left(-\frac{\Delta P}{\Delta x}\cdot\frac{1}{4\mu}\right)^2}{2}\right]\frac{\int_0^{r_0}\left(r_0^2-r^2\right)^3 r dr}{\int_0^{r_0}\left(r_0^2-r^2\right) r dr}$$

$$=[\text{etc}]\frac{\int_0^{r_0}\left(r_0^6-3r_0^4r^2+3r_0^2r^4-r^6\right) r dr}{\int_0^{r_0}\left(r_0^2-r^2\right) r dr}=[\text{etc}]\frac{r_0^8\left[\frac{1}{2}-\frac{3}{4}+\frac{3}{6}-\frac{1}{8}\right]}{r_0^4\left(\frac{1}{2}-\frac{1}{4}\right)}=[\text{etc}]\frac{r_0^4}{2}$$

$$=\frac{1}{4}\left(V_{\max}\right)^2=\left(V_{\text{avg}}\right)^2\qquad \text{which is Eq. 6.11.}$$

6.5 $$\mu=\frac{\rho g(-\Delta z)\pi D_0^4}{Q\,\Delta x\,128};\qquad \text{Take ln of both sides and differentiate, finding}$$

$$d\ln\mu=\frac{d\mu}{\mu}=\frac{d\rho}{\rho}+\frac{d(-\Delta z)}{-\Delta z}+\frac{4dD_0}{D_0}-\frac{dQ}{Q}-\frac{d(\Delta x)}{\Delta x}$$

(a) and (b) $\frac{d\mu}{\mu}\alpha\frac{dQ}{Q}$ or $\frac{d\rho}{\rho}$ so that a 10% error in Q or ρ causes a 10% error in μ

(c) $\frac{d\mu}{\mu}\alpha 4\frac{dD_0}{D_0}$ so that a 10% error in D_0 causes a 40% error in μ.

This marked sensitivity to errors in diameter shows why this type of viscometer is almost always treated as a calibrated device.

6.6 $$\mathsf{F}=-g\Delta z=\Delta u=-\frac{9.81\,\text{m}}{\text{s}^2}\cdot(-0.12\text{ m})\cdot\frac{\text{J}}{\text{N m}}\cdot\frac{\text{N s}^2}{\text{kg m}}=1.77\frac{\text{J}}{\text{kg}}$$

$$\Delta T = \frac{\Delta u}{C_V} = \frac{1.77\frac{\text{J}}{\text{kg}}}{\frac{2.14\,\text{kJ}}{\text{kg}\,^{\text{o}}\text{C}}} = 0.55 \cdot 10^{-3}\ ^{\text{o}}\text{C} = 0.99 \cdot 10^{-3}\ ^{\text{o}}\text{F}$$

For much higher viscosities this temperature rise can be significant. The corresponding viscometer for very high viscosity fluids replaces gravity with a pump, and attempts with cooling to hold the whole apparatus and fluid isothermal. All serious viscometry is done inside constant temperature baths, see Fig. 1.5.

6.7* $$Q = \frac{-\Delta P}{\Delta x}\frac{\pi}{128}\frac{D^4}{\mu} = 1\frac{\text{lbf / in}^2}{\text{ft}}\frac{\pi}{128}\frac{(2\text{ ft / }12)^4}{1\text{e}5 \cdot 6.72\text{e}-4\frac{\text{lbm}}{\text{ft s}}} \cdot 32.2\frac{\text{lbm ft}}{\text{lbf s}^2} \cdot \frac{\text{ft}}{144\text{ in}}$$

$$= 0.00133\frac{\text{ft}^3}{\text{s}} = 3.75 \cdot 10^{-5}\frac{\text{m}^3}{\text{s}}$$

$$V = \frac{Q}{A} = \frac{0.00133\,\text{ft}^3\text{ / s}}{(\pi / 4) \cdot (2\,\text{ft / }12)^2} = 0.0608\frac{\text{ft}}{\text{s}} = 0.0185\frac{\text{m}}{\text{s}}$$

One may check finding $\mathcal{R} \approx 0.01$, so this is clearly a laminar flow.

6.8 $$;V = \frac{10^{-8}\frac{\text{m}^3}{\text{s}}}{\frac{\pi}{4}\left(10^{-3}\text{ m}\right)^2} = 0.0127\frac{\text{m}}{\text{s}}$$

$$\text{R} = \frac{DV\rho}{\mu} = \frac{\left(10^{-3}\text{m}\right)\left(0.0127\frac{\text{m}}{\text{s}}\right)\left(1050\frac{\text{kg}}{\text{m}^3}\right)}{0.0303\text{ Pa s}} \cdot \frac{\text{Pa m}^2}{\text{N}} \cdot \frac{\text{N s}^2}{\text{kg} \cdot \text{m}} = 0.44$$

If the fluid density does not change then the lowest viscosity is one would correspond to $\mathcal{R}$= 2000, and

$$\mu_{\min} = \mu \cdot \frac{\text{R}}{2000} = 3.03\text{ cp} \cdot \frac{0.44}{2000} = 0.0067\,\text{cp}$$

App. A1 shows that this low a viscosity is rarely encountered in liquids, so that this kind of viscometer can be applied to most liquids. Cryogenic liquids, however, have very low viscosities, so there might be a laminar-turbulent transition problem using this particular viscometer on them.

6.9 The volume of fluid between the two marks is

$$V = \Delta z \cdot \frac{\pi}{4}D^2 = 1\text{ cm} \cdot \frac{\pi}{4}\left(1\text{ cm}\right)^2 = \frac{\pi}{4}\text{cm}^3$$

The volumetric flow rate in the example is for the level at the top. The volumetric flow rate is proportional to the elevation, so at mid elevation of the test

$$Q = Q_{\text{Ex.6.2}} \frac{11.5\,\text{cm}}{12\ \text{cm}} = 10^{-8}\frac{\text{m}^3}{\text{s}}\frac{11.5}{12} = 0.958\cdot 10^{-8}\frac{\text{m}^3}{\text{s}}$$

and

$$\Delta t = \frac{V}{Q} = \frac{\frac{\pi}{4}\ \text{cm}^3}{0.958\cdot 10^{-8}\frac{\text{m}^3}{\text{s}}}\cdot\frac{\text{m}^3}{10^6\ \text{cm}^3} = 82.0\ \text{s}$$

Many common industrial viscometers are of this "read the time between the marks" type, and the viscosities are reported in "seconds", e.g. Saybolt Seconds Universal, SSU which is the standard unit of viscosity for high boiling petroleum fractions.

6.10* $F = ma = m\dfrac{dV}{dt} = m\dfrac{\Delta V}{\Delta t}$ here $\Delta v\ =\ 40\text{ - }(\text{-}40) = 80\dfrac{\text{mi}}{\text{hr}}$ so that

$$F = \frac{\left(\frac{5}{16}\text{lbm}\right)\left(80\frac{\text{mi}}{\text{hr}}\right)}{10\ \text{s}}\cdot\frac{5280\frac{\text{ft}}{\text{mi}}}{3600\frac{\text{s}}{\text{hr}}}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}} = 0.114\,\text{lbf}$$

6.11 If the balls are thrown in the I direction, then the I component is independent of the *y* component. The *x* component force depends on the speed of the train, and how often the balls are thrown back and forth, but not on their velocity.

6.12 Start with Eq. 6.4, which is a general force balance, applicable to any kind of flow. and equate the shear stress to the value shown in the problem

$$\tau = \frac{\text{-}r(P_1 - P_2)}{2\Delta x} = f\rho\frac{V^2}{2} = \frac{D}{4}\left(-\frac{\Delta P}{\Delta x}\right)$$

Then, for horizontal flow use Eq. 6.B with both sides divided by Δx

$\dfrac{\text{-F}}{\Delta x} = \dfrac{1}{\rho}\cdot\dfrac{\Delta P}{\Delta x}$. Eliminate $\dfrac{\Delta P}{\Delta x}$ between these two equations finding

$f\rho\dfrac{V^2}{2} = \dfrac{D}{4}\left(\rho\dfrac{\text{F}}{\Delta x}\right)$ which is rearranged to $f = \dfrac{\text{F}}{4(\Delta x\ /\ D)\cdot(V^2\ /\ 2)}$

Which is Eq. 6.18. This shows how the 4*f* appears naturally in the Fanning friction factor.

6.13 $f = \left(-\dfrac{\Delta P}{\Delta x}\right)\dfrac{D}{2\rho V^2}$; In Poisueille's equation $V_{\text{avg}} = \dfrac{D_0^2}{32\mu}\left(-\dfrac{\Delta P}{\Delta x}\right)$

Eliminating $\dfrac{\Delta P}{\Delta x}$ between these two we find

$$f = \left(\frac{32\mu V_{avg}}{D_0^2} \right) \frac{D_0}{2\rho V_{avg}^2} = \frac{16\mu}{D\rho V_{avg}} = \frac{16}{\mathsf{R}}$$

6.14* (a) The flow is laminar, so doubling the flow rate doubles the pressure drop to 20 psi/ 1000 ft.
(b) At this high a Reynolds number we are on the flat part of the lines of constant relative roughness for all but the largest pipe sizes, so that the pressure drop is proportional to the velocity squared. Doubling the velocity quadruples the pressure drop to 40 psi/1000 ft

One would think students would all see this. They don't. Assign this problem, after you have discussed laminar and turbulent flow, and you will be appalled at how few of the students can solve it. After they have struggled with it, they will be embarrassed when you show them how trivial it is. Maybe that way they will learn about the different relation between pressure drop and velocity in laminar and turbulent flow!

6.15* $f_{\text{Darcy-Weisbach}} = 4 f_{\text{Fanning}}$, See Eq. 6.19. Poisueille's equation can be written as

$$f_{\text{Darcy-Weisbach}} = 4 f_{\text{Fanning}} = 4 \cdot \frac{16}{\mathsf{R}} = \frac{64}{\mathsf{R}}$$

If you look at the equivalent of Fig. 6.10 in any civil or mechanical engineering fluids book, you will see that the laminar flow line is labeled $f = 64 / \mathsf{R}$.

6.16 $$\mathsf{R} = \frac{\left(\frac{6.065}{12} \text{ ft} \right) \left(7 \frac{\text{ft}}{\text{s}} \right) \left(62.3 \frac{\text{lbm}}{\text{ft}^3} \right)}{1.002 \text{ cp} \cdot 6.72 \cdot 10^{-4} \frac{\text{lbm}}{\text{ft s cp}}} = 3.27 \cdot 10^5$$

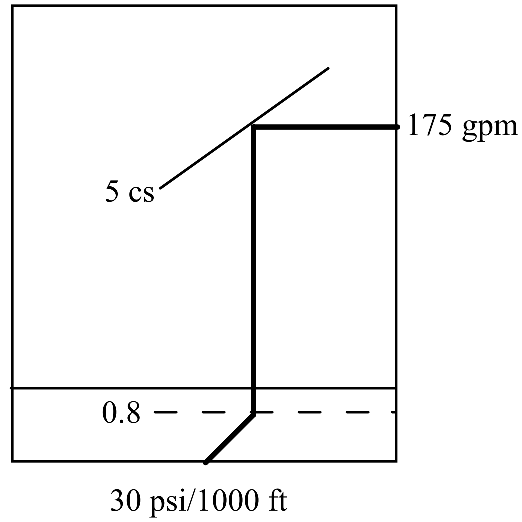
$$\frac{\varepsilon}{D} = \frac{0.0018 \text{ in}}{6.065 \text{ in}} = 0.0003; \quad f = 0.0042$$

$$\frac{\Delta P}{\Delta x} = \frac{(4)(0.0042) \left(62.3 \frac{\text{lbm}}{\text{ft}^3} \right) \left(7 \frac{\text{ft}}{\text{s}} \right)^2}{\left(\frac{6.065}{12} \text{ ft} \right)(2)} \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 1.12 \cdot 10^{-2} \frac{\text{psi}}{\text{ft}} = 11.2 \frac{\text{psi}}{1000 \text{ ft}} = 0.253 \frac{\text{Pa}}{\text{m}}$$

by linear interpolation in Table A.3 $\quad \frac{-\Delta P}{\Delta x} \approx 11.1 \frac{\text{psi}}{1000 \text{ ft}}$

6.17 (a) The easiest way to work this problem is to use Fig. 6.12. As sketched at the right one enters at the bottom at 30 psi/1000 ft, reads diagonally up to the 0.8 s.g. line, then vertically to the 5 cs line, and then reads horizontally, finding about 175 gpm.



(b) To do it using Fig 6.10 one sees that it is the same type of problem as Ex. 6.5, the Type 2 problem in Table 6.3, which leads to a trial-and error solution. This is very similar to that shown in Table 6.5, and is shown below

Variable	**First guess**	**Solution**
D, ft	0.256	0.256
L, ft	1000.000	1000.000
delta *P*, psig	-30.000	-30.000
e, inches	0.002	0.002
r, lbm/ft3	49.840	49.840
n, cs	5.000	5.000
m cp	4.000	4.000
f, guessed	0.005	0.006
V, ft/s	8.447	7.685
R	40044.302	36428.733
e/D	0.001	0.001
f, computed	0.006	0.006
f, computed/*f* guessed	1.189	1.000
Q, gal/min		176.746

The answer, 176.7 gpm is slightly more than the 175 we got in part (a), but no one should believe any calculation this type to better than ± 10%, as discussed in the text.

6.18 This is easily solved on Fig. 6.12. One enters at 200 gpm, reads horizontally to the zero viscosity boundary (which corresponds to the flat part of the curves on Fig. 6.10), then down to s.g .= 0.75, and diagonally to ≈ 23 psi/1000 ft. This is less than the available 28, so a 3 inch pipe would be satisfactory.

To solve using Fig. 6.10 (or Eq. 6.21) we compute

$$V = \frac{200 \text{ gpm}}{23.0 \dfrac{\text{gpm}}{\text{ft/s}}} = 8.70 \frac{\text{ft}}{\text{s}}$$; we calculate that $\mathcal{R}$= 1.5106 and *e/D* ≈0.006, so that

f=0.00456, and

$$\frac{-\Delta P}{\Delta x} = \frac{(4)(0.00456)\left(0.75 \cdot 62.3 \dfrac{\text{lbm}}{\text{ft}^3}\right)\left(8.70 \dfrac{\text{ft}}{\text{s}}\right)^2}{\left(\dfrac{3.068}{12}\right)\text{ft} \cdot 2} \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 0.0271 \frac{\text{psi}}{\text{ft}} = 27.1 \frac{\text{psi}}{1000 \text{ ft}}$$

This is adequate agreement with Fig. 6.13 and indicates that a 3 inch pipe is big enough.

6.19* (a) $V = \dfrac{150 \text{ gpm}}{23 \dfrac{\text{gpm}}{(\text{ft/s})}} = 6.52 \dfrac{\text{ft}}{\text{s}}$ **;** $\mathcal{R} = \dfrac{0.256 \text{ ft} \cdot 54.2 \dfrac{\text{lbm}}{\text{ft}^3} \cdot \left(6.52 \dfrac{\text{ft}}{\text{s}}\right)^2}{1.5 \cdot 6.72\text{e}-4 \dfrac{\text{lbm}}{\text{ft s}}} = 89{,}650$

f ≈ 0.53, so

$$\Delta P = 4f \frac{L}{D} \frac{\rho V^2}{2} = \frac{(4)(0.0053)(1000 \text{ ft})\left(0.87 \cdot 62.3 \dfrac{\text{lbm}}{\text{ft}^3}\right)\left(6.52 \dfrac{\text{ft}}{\text{s}}\right)^2}{\left(\dfrac{3.068}{12} \text{ft}\right) \cdot 2} \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 20.5 \text{ psi}$$

(b) by chart look up on Fig. 6.13 the pressure drop is 19 psi. Fig. 6.13 generally predicts pressure drop values for turbulent flow somewhat less than does Fig 6.10, most likely because it uses a lower value for the absolute roughness. The most likely reason for the lower roughness in Fig 6.13 is that most petroleum products are slightly acid, and will smooth out a steel pipe in use, while most waters will rust or leave deposits on steel pipes, so that they become more rough over time. The value for the roughness of steel pipe Table 6.2 is presumably more in accord with the experience with water in steel pipes than the experience with various petroleum products in steel pipes.

6.20 $\dot{\mathcal{F}} = \Delta u = -g\Delta z = -\left(32.2 \dfrac{\text{ft}}{\text{s}^2}\right)(-20 \text{ ft}) \cdot \dfrac{\text{Btu}}{778 \text{ ft lbf}} \cdot \dfrac{\text{lbf s}^2}{32.2 \text{ lbm ft}}$

$$= 0.026 \frac{\text{Btu}}{\text{lbm}} = 0.060 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta T = \frac{\Delta u}{C_V} = \frac{0.026 \dfrac{\text{Btu}}{\text{lbm}}}{0.6 \dfrac{\text{Btu}}{\text{lbm }^\circ\text{F}}} = 0.043\ ^\circ\text{F} = 0.024^\circ\text{C}$$

Here again we see that the temperature rise is negligible.

6.21 The table is shown below. The first three columns are the same as those in Table 6.5, except for the number of digits shown. The bottom 4 rows are in addition to what is shown in Table 6.5. The right column is made by copying the column to its left, inserting -30 in place of - 10 for Δz, and then using the spreadsheet's numerical solution engine ("Goal Seek" on Excel spreadsheets) to make the *f* computed/guessed =1.00 by manipulating the value of *f* guessed.

Variable	First guess	Solution	Prob. 6.21
D, m	0.1	0.1	0.1
L, m	100	100	100
Δz, m	-10	-10	-30
ε, inches	0.0018	0.0018	0.0018
ρ, kg/m3	720	720	720
μ, cp	0.6	0.6	0.6
f, guessed	0.0050	0.0044	0.0044
V, m/s	4.4294	4.7102	8.2232
$\mathcal{R}$	531533.63	565222.127	986784.836
e/D	0.0005	0.0005	0.0005
f, computed	0.0044	0.0044	0.0044
f, computed/*f* guessed	0.8870	1.0007	1.0001
Q, m/s	0.0348	0.0370	0.0646
ft3/s	1.2277	1.3055	2.2792
gal/s	9.1838	9.7659	17.0497
gal/min	551.0298	585.9539	1022.9792

With this number of significant figures (chosen to fit the page) it would appear that the value of *f* did not change from Ex. 6.5 to Prob. 6.21. Looking at the original spreadsheet we see it went from 0.004425 to 0.004352, which is practically a negligible change, indicating that we are on an almost-flat part of Fig. 6.10. If there had been no change in *f* then the volumetric flow rate in this example would be √3 times the value in Ex. 6.5, or 1015 gpm, vs the 1025 gpm computed here. Again, remember the ± 10% uncertainty in all such calculations.

6.22 The table is shown below. The first three columns are the same as those in Table 6.6, except for the number of digits shown. The bottom 2 rows are in addition to what is shown in Table 6.6. The right column is made by copying the column to its left, inserting 2000 in place of 500 for *Q* and then using the spreadsheet's numerical solution engine ("Goal Seek" on Excel spreadsheets) to make the ΔP, computed/guessed =1.00 by manipulating the value of *D* guessed.

Variable	First guess	Solution	Prob. 6.22
Q, cfm	500	500	2000
D, ft	1	0.66700631	1.11911808
L, ft	800	800	800
ε, inches	0.00006	0.00006	0.00006
ρ, lbm/ft3	0.08	0.08	0.08
μ, cp	0.017	0.017	0.017
V, ft/s	10.6103	23.8489	33.8872
$\mathcal{R}$	74301.8538	111396.028	265572.884
e/D	0.000005	7.4962E-06	4.4678E-06
f, computed	0.00465346	0.0042	0.0035
ΔP, calculated	0.01446184	0.1000	0.1000
Allowed ΔP,	0.1	0.1000	0.1000
ΔP, calc/ΔP, allow	0.14461836	1.0000	1.0002
D calc, m		0.2033	0.3411
inches		8.0041	13.4294

Here f declines substantially, mostly because e/D declines due to the larger diameter. If there were no change in f then quadrupling the volumetric flow rate would cause the required diameter to double. As shown here it increases by a factor of 1.68.

6.23* This is the same as Ex. 6.4, with different numerical values. By trial and error one finds V= 10.5 ft/s, $\mathcal{R}$= 6.46 E5, f= 0.0039, Q = 1639 gpm.

To use Table A.3. we compute $-\mathcal{F} = -g\Delta z = -\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\cdot(-200\text{ ft}) = 6440\frac{\text{ft}}{\text{s}^2}$

Then, for horizontal flow

$$-\frac{\Delta P}{\Delta x} = \frac{\rho}{\Delta x}(-F) = \frac{62.3\frac{\text{lbm}}{\text{ft}^3}}{5000\,\text{ft}}\left(6440\frac{\text{ft}^2}{\text{s}^2}\right)\cdot\frac{\text{lbf s}^2}{32.2\text{ lbm ft}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2} = 0.0173\frac{\text{psi}}{\text{ft}} = 1.73\frac{\text{psi}}{100\,\text{ft}}$$

Then entering the table for 8 inch pipe we find for Q = 1600 gpm, $-dP/dx$ = 1.65 psi/100 ft, and for Q = 1800 gpm, $-dP/dx$ = 2.08 psi/100 ft, By linear interpolation, 1.73 psi/100 ft corresponds to $Q \approx$ 1630 gpm.

6.24 See the solution to Prob. 6.23. With the same pressure drop (1.73 psi/1000 ft) we enter Table A.4, and find that to get 10,000 gpm, we need an 18 inch pipe.

6.25* Assuming the flow is from the first to the second tanks,

$$\frac{\Delta P}{\rho} + g\Delta z = -\mathcal{F}$$

$$\mathsf{F} = -\left(\frac{10\dfrac{\text{lbf}}{\text{in}^2}}{0.85 \cdot 62.3\dfrac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} + 32.2\frac{\text{ft}}{\text{s}^2}(-20\,\text{ft}) \right) = -(876 - 644) = -232\frac{\text{ft}^2}{\text{s}^2}$$

This negative value of the friction heating indicates the above assumption is incorrect, the flow must be from the second tank to the first. Then we compute the equivalent pressure gradient for a horizontal pipe, taking the flow from right to left

$$-\frac{\Delta P}{\Delta x} = \frac{\rho}{\Delta x}\mathsf{F} = \frac{(0.85)\left(62.3\dfrac{\text{lbm}}{\text{ft}^3}\right)}{500\ \text{ft}}\left(232\frac{\text{ft}^2}{\text{s}^2}\right) \cdot \frac{\text{lbf s}^2}{32.2\ \text{lbm ft}} \cdot \frac{\text{ft}^2}{144\,\text{in}^2} = 0.00529\frac{\text{psi}}{\text{ft}} = 5.29\frac{\text{psi}}{1000\ \text{ft}}$$

and read from Fig 6.13, at $\nu = \dfrac{100}{0.85} = 117.6\text{cs}$ $Q \approx 18$ gal, min. This is only an approximate read, because to the need to interpolate the 117.6 cs line. The result is in the laminar flow region, so we could solve directly from Pouisueille's equation,

$$Q = \frac{5.29\dfrac{\text{lbf}}{\text{in}^2}}{1000\,\text{ft}} \cdot \frac{\pi}{128} \cdot \frac{(3.068\,\text{ft}/12)^4}{100\,\text{cp}} \cdot \frac{\text{ft}^2\ \text{cP}}{2.09\text{e}-5\,\text{lbf s}} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} = 0.038\frac{\text{ft}^3}{\text{s}} = 17.2\frac{\text{gal}}{\text{min}}$$

6.26 $\mathsf{R} = \dfrac{DV\rho}{\mu} = \dfrac{\left(\dfrac{3.068\ \text{ft}}{12}\right)\left(40\dfrac{\text{ft}}{\text{s}}\right)\left(62.3\dfrac{\text{lbm}}{\text{ft}^3}\right)}{1.002\ \text{cP} \cdot 6.72 \cdot 10^{-4}\dfrac{\text{lbm}}{\text{ft s cp}}} = 9.5 \cdot 10^5$

From Fig 6.10, or by trial and error in Eq. 6.21, for this Reynolds number and friction factor

$$\frac{\varepsilon}{D} \cong 0.000065; \qquad \varepsilon = (0.000065)(3.068\,\text{in}) = 0.00020\ \text{in}$$

See Table 6.2. This new pipe is not as smooth as drawn tubing, ($\varepsilon = 0.00006$ in**)** but is smoother than any of the other types of pipe shown in that table.

6.27 There are easier and most satisfactory ways for finding them. The density is easily measured in simple pyncnometers, to much greater accuracy than it could possibly be measured by any kind of flow experiment. The viscosity is measured in laminar flow experiments, as shown in Ex. 6.2.

Furthermore, in a flow experiment we would presumably choose as independent variables the choice of fluid, the pipe diameter and roughness and the velocity. We would measure the pressure drop, and compute the friction factor. If we were at a high Reynolds number, i.e. at the right side of Fig. 6.10, we would see that the friction factor was independent of the Reynolds number, so the measurements would be independent of the viscosity. The friction factor would also be independent of the density (which is part of the Reynolds number), although the calculation of the friction factor from the observed pressure drop would require us to know the density in advance.

We may restate this by saying that for really high Reynolds numbers, Figure 6-10 really only relates three variables, *f, D* and *ε*.

6.28 For Ex 6.5, the equations to be combined are Eq. 6.Q and 6.21. Eliminating *f* between these, we have

$$V = \left(\frac{2g(-\Delta z)}{4 \cdot 0.001375 \cdot \left[1 + \left(20{,}000 \frac{\varepsilon}{D} + \frac{10^6 \mu}{DV\rho} \right)^{1/3} \right]} \cdot \frac{D}{\Delta x} \right)^{1/2}$$

The only unknown in this equation is V which appears on both sides, (to the 1/6 power on the right). I doubt that this has an analytical solution; it would be tolerably easy numerically, but the procedure shown in Ex. 6.5 is certainly easier.

For Ex. 6.6 we combine Eq. 6.X and 6.21,

$$\Delta P = \rho_{\text{air}} \left(-4 \cdot 0.001375 \cdot \left[1 + \left(20{,}000 \frac{\varepsilon}{D} + \frac{10^6 \mu}{DV\rho} \right)^{1/3} \right] \frac{\Delta x}{D} \cdot \frac{V^2}{2} \right)$$

and then replace both V's with $Q / \left((\pi / 4) \cdot (D^2) \right)$. The resulting equation gives ΔP as a function of D and variables specified in the problem. Again, I doubt that this has an analytical solution; it would be tolerably easy numerically, but the procedure shown in Ex. 6.6 is certainly easier.

6.29 If you assign this problem, you may want to give some hints, otherwise the students flounder with it. First you should point out that Fig. 6.12 is a log-log plot, but that it is not plotted on normal log-log paper. It has a scale ratio of about 1.5 vertically to 1 horizontally. If one re-plotted it on normal log-log paper, one would see that it has three families of parallel straight lines, the laminar flow lines with slope 1, the turbulent flow lines with slope about 0.55 and the transition and zero viscosity lines with slope 0.5.

(a) for the zero viscosity boundary we estimate $\varepsilon/D = 0.0018/2 = 0.0009$. The from Fig. 6.10 we read $f = 0.0048$. Then for a velocity of 1 ft/s and s.g. = 1.00 we compute

$$-\frac{\Delta P}{\Delta x} = \frac{4f}{D} \rho \frac{V^2}{2} = \frac{4 \cdot 0.0048 \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3} \cdot \left(1 \frac{\text{ft}}{\text{s}} \right)^2}{2 \cdot \left(\frac{2.067 \text{ ft}}{12} \right)} \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 0.00077 \frac{\text{psi}}{\text{ft}} = \frac{0.77 \text{ psi}}{1000 \text{ ft}}$$

From Table A.2 we see that 1 ft/s in a 2 in pipe corresponds to 10.45 gpm, so we plot this pressure gradient at that volumetric flow rate. We draw a line of proper slope through it completes the zero viscosity boundary.

(b) We can get both the laminar flow region and the transition region from one calculation, by choosing our point as a Reynolds number of 2000 and a viscosity of 40 cs. We compute

$$V = \frac{2000\nu}{D} = \frac{(2000)(40 \text{ cS})}{\left(\frac{2.067 \text{ ft}}{12}\right)} \cdot 1.08 \cdot 10^{-5} \frac{\text{ft}^2}{\text{s cS}} = 5.02 \frac{\text{ft}}{\text{s}} \quad \text{Then from Table A.2}$$

$Q = 10.45 \cdot 5.02 = 52.4$ gpm; Then, for s.g. = 1, μ = 40 cP, and

$$-\frac{\Delta P}{\Delta x} = \frac{8 \cdot V_{\text{avg}} \cdot \mu}{r_0^2} = \frac{8 \cdot 5.02 \frac{\text{ft}}{\text{s}} \cdot 40 \text{ cP}}{\left(\frac{2.067}{24} \text{ft}\right)^2} \cdot 6.72 \cdot 10^{-4} \frac{\text{lbm}}{\text{ft s cP}} \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 0.0313 \frac{\text{psi}}{\text{ft}} = 31.3 \frac{\text{psi}}{1000 \text{ft}} \quad \text{for s.g.} = 1.00$$

This point lies on both the 40 cS laminar curve, and on the transition curve. We draw a line with slope 1 through it, for the laminar region, and a line with slope 0.5 through it for the laminar-turbulent transition. Then we complete the laminar region, by drawing lines parallel to the first line, for various viscosities. In the laminar region, at constant flow rate the pressure drop is proportional to the viscosity, so, for example, the 20 cs line is parallel to the 40 cs line, but shifted to the left by a factor of two, and the 80 cs line is parallel to the 40 cs line, but shifted to the right by a factor of two.

(c) For the turbulent region we again need to calculate one point. We must guess a value of the kinematic viscosity for which the line will fall between the transition and zero viscosity lines. From Fig. 6.12 it seems clear that the 1 cS line is likely to meet that requirement. For 1 ft/s, Q = 10.45 gpm. Then for s.g. = 1.00

$$\mathsf{R} = \frac{\frac{2.067 \text{ ft}}{12} \cdot 1 \frac{\text{ft}}{\text{s}}}{1 \text{cS} \cdot 1.08 \cdot 10^{-5} \frac{\text{ft}^2}{\text{s cS}}} = 1.59 \cdot 10^4; \; f = 0.0073$$

$$-\frac{\Delta P}{\Delta x} = \frac{4 \cdot 0.0073 \cdot \left(1 \frac{\text{ft}}{\text{s}}\right)^2 \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3}}{2 \cdot \frac{2.067 \text{ ft}}{12}} \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{in}^2}$$

$$= 0.00114 \frac{\text{psi}}{\text{ft}} = \frac{1.14 \text{ psi}}{1000 \text{ ft}}$$

We then draw a line with slope 0.55 through that point. We could compute similar lines for other kinematic viscosities if we wished.

(d) is taken care of in (b) above.

The problem does not ask the students to compute the lower section which corrects for differences in density. You might bring that up in class discussion. The lines there would have slope 1 on an ordinary piece of log-log paper. The values calculated above are all for s.g. = 1.00, which corresponds to the bottom of the figure, s.g. = 1.00. If, for example, the true s.g. is 0.5, then that bottom section shows that the pressure gradient will be 0.5 times the value for s.g. = 1.00. This is obvious for turbulent flow, where we set the density to 62.3 lbm/ft^3. For laminar flow it is a bit more subtle. If we rewrote Poisueille's equation replacing μ with $\rho\nu$ we would see that the pressure gradient is proportional to the kinematic viscosity times the density. In making up the laminar part of the plot we used the kinematic viscosity, with an assumed s.g. = 1.00. Thus, in this formulation the laminar pressure gradient is also proportional to the specific gravity.

Here I have used 2000 as the transition Reynolds number. The authors of Fig. 16.2 (and the others in that series) used 1600, see Prob. 6.31.

6.30 Here

$$\nu = \frac{\mu}{\rho} = \frac{0.018 \text{ cP}}{\left(0.075 \frac{\text{lbm}}{\text{ft}^3}\right)} \cdot \frac{\text{c St} \cdot 62.3 \frac{\text{lbm}}{\text{ft}^3}}{\text{cP}} = 15.0 \, \text{cSt}$$

Entering the chart at 100 gpm, and reading left to this kinematic viscosity (interpolating between 10 and 20 in the turbulent region) , and then reading upward to the head loss scale on the top of the chart, we find

$$\left(\begin{matrix}\text{head loss,} \\ \text{ft / 1000 ft}\end{matrix}\right) = \frac{-\frac{\Delta P}{\Delta x}}{\rho g} \approx 40 \frac{\text{ft}}{1000 \, \text{ft}} = 0.040$$

Then the pressure drop is

$$-\frac{\Delta P}{\Delta x} = 0.040 \cdot 0.075 \frac{\text{lbm}}{\text{ft}^3} 32.2 \frac{\text{ft}}{\text{s}^2} \cdot \frac{\text{lbf s}^2}{32.2 \, \text{lbm ft}} \cdot \frac{\text{ft}^2}{144 \, \text{in}^2}$$

$$= 2.08 \cdot 10^{-5} \frac{\text{psi}}{\text{ft}} = \frac{0.0208 \text{ psi}}{1000 \, \text{ft}}$$

It is most unlikely that anyone would use Fig. 6.12 to solve this type of problem. But this problem shows that one could. If one solves this by standard friction factor methods, one finds $\mathcal{R}$ = 6872, f = 0.0088, and $-dP/dx$ = 0.021 psi/1000 ft.

6-31* The 20 cSt line enters the transition region at 31 gpm and exits at 87 gpm. The corresponding Reynolds numbers are

$$\mathsf{R}_{\text{transition}} = \frac{\left(\frac{3.068\text{ ft}}{12}\right)\cdot\left(\frac{31\,\text{gpm}}{23.0\frac{\text{gpm}}{(\text{ft / s})}}\right)}{(20\text{ cSt})\cdot\left(1.08\cdot10^{-5}\frac{\text{ft}^2}{\text{s cSt}}\right)} = 1595 \approx 1600$$

and

$$\mathsf{R}_{\text{turb}} = \text{above}\cdot\frac{87\,\text{gpm}}{31\,\text{gpm}} = 4477 \approx 4500$$

6.32 The lowest velocity value for 1/2 inch pipe, (0.3 gpm) has a calculated Reynolds number of 1346 (taking into account that the viscosity at 60°F is 113% of that at 68°F). For this value we can compute the pressure gradients for laminar and turbulent equations, finding 0.062 and 0.073 psi/100 ft. The value in the table is 0.061, showing that this is a laminar value. One can also see from a plot of $\Delta P/\Delta x$ vs Q that there is break in the curve for 1/2 inch pipe between this value and all the others, indicating that it is laminar. A few of the other smallest velocity values are also laminar. All the other values are for turbulent flow.

6.33 One may compute that the friction factor falls from 0.0036 to 0.0031 over the range of values, which corresponds to Reynolds numbers from 0.56 million to 2.78 million. These match Fig. 6.10 reasonably well for an ε/D of 0.00007. Comparing them to the vales from Eq. 6.21, we see that for the lowest flow the ratio of the value n Tab. A.3 is 104% of that from Eq. 6.21. For increasing flow rates this ratio diminished. For velocities greater than 9.58 ft/s it is 1.00 ± 1%.

As a further comparison I ran the friction factors from Eq. 6.21, using $\varepsilon/D = 0$, finding that the values in table A.3 are 117% to 131% of the smooth tube values. Clearly even at this size pipe, we do not have smooth tube behavior.

6.34 Solving the problem either by Fig. 6.10 or by App. A.3, we have

$$\mathsf{F} = 4f\frac{\Delta x}{D}\frac{V^2}{2} = g(-\Delta z) = 9.81\frac{\text{m}}{\text{s}}(10\,\text{m}) = 98.1\frac{\text{m}^2}{\text{s}^2}$$

Either way the value of the friction heating per pound and of $\Delta x/D$ are the same, so if we assume an equal friction factor we would conclude that the velocities were equal. Then, for water

$$-\frac{\Delta P}{\Delta x} = \frac{\mathsf{F}\rho}{\Delta x} = \frac{98.1\frac{\text{m}^2}{\text{s}^2}\cdot 998.2\frac{\text{kg}}{\text{m}^3}}{100\text{ m}}\cdot\frac{\text{N s}^2}{\text{kg m}}\cdot\frac{\text{Pa}}{\text{N m}^2} = 0.979\frac{kPa}{m} = 4.32\frac{\text{psi}}{100\text{ ft}}$$

In App. A.3 we must interpolate between 425 and 450 gpm, finding about 436 gpm, compared to the 386 gpm in Ex. 6.5. Here the ratio of the velocities is proportional to the square root of the ratio of the friction factors. This is tolerable agreement. The main reason for this is that in turbulent flow, the volumetric flow rate is proportional to D^5. Thus for the two different size pipes $\frac{D_{\text{4 in Sch. 40}}}{D_{\text{Ex. 6.5}}} = \frac{4.03\text{ in}}{3.94\text{ in}} = 1.0228$ and

$$\left(\frac{D_{\text{4 in Sch. 40}}}{D_{\text{Ex. 6.5}}}\right)^5 = 1.1196$$ Thus our best estimate of Q from Table A.3 is

$$Q = \frac{436\text{ gpm}}{1.1196} = 389\text{ gpm}$$ which agrees well with the 386 gpm in Ex. 6.5.

This problem shows that the friction factor is practically the same for gasoline and water in this geometry, so that we can solve the problem using data for water, if we take the difference in diameters into account.

6.35 Entering Fig. 6.14 at the left at 1000 cfm, reading horizontally to the 12 inch line, and then vertically to the pressure drop scale we find 0.2 inches of water per 100 ft, so for 1000 ft of pipe the pressure drop is 2 inches of water.

6.36 Drawing a horizontal line on Fig. 6.14 at 100 cubic meters per hour, and a vertical line at 1 Pa per meter, we see they intersect between the 0.1 and the 0.125 meter diameter lines. By visual interpolation the required pipe diameter is about 0.115 m.

6-37 Drawing a vertical line on Fig. 6.14 from 5 Pa/m to the 0.125 m diameter line, and then a horizontal line from there to the right hand of the figure we find a flow rate of 300 cubic meters per hour.

6.38(a) $$V = \frac{1000\frac{\text{ft}^3}{\text{min}}}{\frac{\pi}{4}\left[\frac{6.065}{12}\text{ft}\right]^2} \cdot \frac{\text{min}}{60\text{ s}} = 83.1\frac{\text{ft}}{\text{s}}$$ From App. A.1 $\mu = 0.009\text{cP}$

A.1 is hard to read, but my HPCP, 71e shows a value of 0.009 at 300 K, so this reading is close to right.

$$\rho = 0.075\frac{\text{lbm}}{\text{ft}^3}\left(\frac{2}{29}\right) = 0.0052\frac{\text{lbm}}{\text{ft}^3}$$

$$\mathsf{R} = \frac{\frac{6.065\text{ ft}}{12} \cdot 83.1\frac{\text{ft}}{\text{s}} \cdot 0.0052\frac{\text{lbm}}{\text{ft}^3}}{0.009\text{ cP} \cdot 6.72 \cdot 10^{-4}\frac{\text{lbm}}{\text{ft s cP}}} = 3.6 \cdot 10^4; \quad \varepsilon = 0.003; \qquad f = 0.0058$$

$$-\frac{\Delta P}{\Delta x} = \frac{4 \cdot 0.0065 \cdot 0.0052\frac{\text{lbm}}{\text{ft}^3} \cdot \left(83.1\frac{\text{ft}}{\text{s}}\right)^2}{\frac{6.065\text{ ft}}{12} \cdot 2} \cdot \frac{\text{lbf s}^2}{32.2\text{ lbm ft}} \cdot \frac{\text{ft}^2}{144\text{ in}^2}$$

$$= 0.000177\frac{\text{psi}}{\text{ft}} = 0.0049\frac{\text{in H}_2\text{O}}{\text{ft}} = 0.49\frac{\text{in H}_2\text{O}}{100\text{ ft}} = 4.01\frac{\text{Pa}}{\text{m}}$$

(b) For air at the same pressure gradient and pipe size we draw a horizontal line from 1000 cfm to the 6 inch line, and then down vertically to read about 6.5 inches of water per 100 ft. If the friction factors are the same, we would expect the pressure gradients to be proportional to the densities. Taking the ratio of the densities as the same as the ratio of the molecular weights, we estimate

$$-\frac{\Delta P}{\Delta x} = \frac{6.5\text{ in H}_2\text{O}}{100\text{ ft}} \cdot \frac{2}{29} = \frac{0.45\text{ in H}_2\text{O}}{100\text{ ft}}$$

The answers to parts (a) and (b) differ by ≈10%, which is within the range of uncertainty of any friction factor calculation. The principal reason for the difference is that the kinematic viscosity of hydrogen is ≈ 7 times that of air, so the Reynolds number is 1/7 that of air. If we repeat part (a) for air, we find $\mathcal{R} = 2.6 \cdot 10^5$ and $f = 0.0043$.

Thus, this is only an approximate way of estimating the behavior of hydrogen. For gases with properties more like those of air it works better.

6.39 (a) From Fig 6.1 (before I tried Eq. 6.21), $f \approx 0.0048$
(b) By Colebrook, the solution is a trial and error, shown in the following spreadsheet;

	First Guess	Solved values
f guessed	0.5	0.00482514
left side	1.41421356	14.3961041
right side	15.0676463	14.3909702
ratio	0.09385763	1.00035674

I intentionally made a bad first guess, but the "Goal Seek" routine on excel, when asked to make the ratio of the left to right sides = 1.00 had no trouble finding $f \approx 0.0048$

(c) By straightforward plug into Eq. 6.60 we find $f \approx 0.0049$

(d) By straightforward plug into Eq. 6.21, we find $f \approx 0.0048$

(e) From Eq. 6.61, which I found in the source listed, and for which that book gives no reference, by straightforward plug in one finds $f \approx 0.0045$.

All of these give practically the same answer. Observe that in the original publications give the equations for the darcy-weisbach friction factor = 4 · fanning friction factor.

6.40 (a) $-\dfrac{\Delta P}{\Delta x} = 4\dfrac{f}{D}\rho\dfrac{V^2}{2}$ If everything on the right except V is constant, then this should plot as a straight line with slope 2 on that plot.

(b) For 5 inches and less, the curves are parallel straight lines (as best the eye can see). To find the slopes I read the 4 inch line as beginning at 15 and ending at 430. Then

$$\log\left(\frac{\Delta P_2}{\Delta P_1}\right) = n\log\left(\frac{Q_2}{Q_1}\right); \qquad n = \log(1000)/\log(430/15) \approx 2.05.$$

This result is sensitive to chart reading, but the exponent is very close to 2.00.

(c) For all the pipe sizes greater than 5 inches the curves are concave downward. The curvature seems to increase with increasing pipe size.

(d) Those making up the charts concluded that for small diameter pipes the normal flows were on the flat part of the constant *e/D* curves on Figure 6.10. However as D increases, with constant e one goes to lower and lower values of *e/D* and hence lower curves on Fig. 6.10. These do not flatten out until higher values of the velocity, so the curves correspond to a value of f which decreases slowly with increasing Q, giving the curves shown.

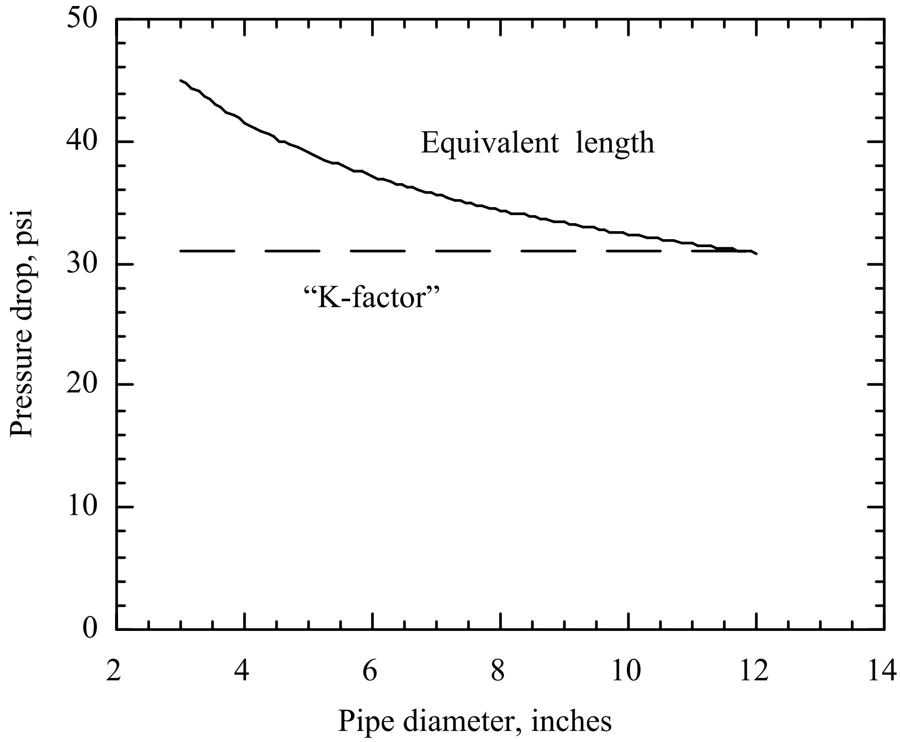
(e) $$V = \frac{1000\dfrac{\text{ft}^3}{\text{min}}}{\dfrac{\pi}{4}[1.0\text{ ft}]^2}\cdot\frac{\text{min}}{60\text{ s}} = 21.22\frac{\text{ft}}{\text{s}}; \qquad \mathsf{R} = \frac{1\text{ ft}\cdot 21.22\dfrac{\text{ft}}{\text{s}}\cdot 0.075\dfrac{\text{lbm}}{\text{ft}^3}}{0.018\text{ cP}\cdot 6.72\cdot 10^{-4}\dfrac{\text{lbm}}{\text{ft s cP}}} = 1.32\cdot 10^5$$

$$f = \frac{-\dfrac{\Delta P}{\Delta x}}{\dfrac{4}{D}\rho\dfrac{V^2}{2}} = \frac{\left(\dfrac{0.2\text{ in H}_2\text{O}}{100\text{ ft}}\right)\cdot 1\text{ ft}\cdot 2\cdot 32.2\dfrac{\text{lbm ft}}{\text{lbf s}^2}\cdot 144\dfrac{\text{in}^2}{\text{ft}^2}\cdot 0.03615\dfrac{\text{psi}}{\text{in H}_2\text{O}}}{4\cdot 0.075\dfrac{\text{lbm}}{\text{ft}^2}\cdot\left(21.22\dfrac{\text{ft}}{\text{s}}\right)^2} = 0.00496$$

Substituting this value in Eq. 6.21 and solving we find $e/D \approx 0.00050$, $e = 0.0005$ ft.

(f) From Table 6.2 we see that this corresponds to the value for "galvanized iron" which is ≈ 3 times the value for commercial steel. From the caption for Fig. 6.14 we see that it is for "galvanized metal ducts" etc. So this matches Table 6.2 fairly well.

6.41 From Ex. 6.12 we know that for the "K-factor" method the pressure drop due to the fittings is 31 psi, independent of pipe diameter. For the equivalent length method we must calculate the Reynolds number , *e/D*, f and the equivalent length for each pipe size. For all sizes *L/D* is constant at 1089. The computations are easy on a spreadsheet. Considering the 12 inch pipe we find that $\mathcal{R} \approx 24{,}100$, $e/D = 0.00015$, $f = 0.0062$, L_{equiv} = 1089 ft, and $dP \approx 31$ psi. The plot, covering the range from 3 to 12 ft is shown below.



We see that the equivalent length estimate is higher than the "K-factor" estimate for small pipes, but that the two values cross at about 1 ft. I ran the value for 24 inches on the same spreadsheet program, finding $dP \approx 6.5$ psi. I don't know which is best, other than to follow Lapple's suggestion in the text.

Another peculiarity of this comparison is that the estimate by the "K-factor" method is uninfluenced by a change in fluid viscosity, as long as the flow is turbulent. But the equivalent length method is sensitive to changes in viscosity. I reran the spreadsheet that generated the values for the above plot, taking the fluid viscosity as 1 cP instead of the 50 cP in those examples. That raised the Reynolds numbers by a factor of 50, reducing the computed f's by almost a factor of 2. That reduced the estimated delta P by roughly a factor of 2, going from 23.7 psi for 3 inch pipe to 17.4 psi for 12 inch pipe. In this case the curve on the above plot ends up nearly horizontal; the computed value for 24 inches is 15.2 psi.

6.42* From Table A.4 we read that $V = 6.10$ ft/s and $-\dfrac{\Delta P}{\Delta x} = \dfrac{0.466\,\text{psi}}{100\,\text{ft}}$

For the equivalent length method

$$\Delta x_{\text{equivalent}} = 50\,\text{ft} + \left(\frac{10.02}{12}\text{ft}\right)\cdot(2\cdot 30 + 135) = 212\,\text{ft}$$

$$-\Delta P = \left(-\frac{\Delta P}{\Delta x}\right)\cdot \Delta x_{\text{eq}} = \frac{0.466\ \text{psi}}{100\ \text{ft}}\cdot 212\ \text{ft} = 0.992\ \text{psi}$$

For the K method, using the values from Table 6.7 $\sum K = 2\cdot 0.74 + 2 = 3.48$

$$-\Delta P_{\text{pipe only}} = \left(-\frac{\Delta P}{\Delta x}\right)\cdot \Delta x_{\text{pipe}} = \frac{0.466\ \text{psi}}{100\ \text{ft}}\cdot 50\ \text{ft} = 0.233\ \text{psi}$$

$$-\Delta P_{\text{fittings}} = \sum K\frac{\rho V^2}{2} = 3.48\cdot\frac{62.3\dfrac{\text{lbm}}{\text{ft}^2}\cdot\left(6.1\dfrac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\frac{1}{32.2\dfrac{\text{lbm ft}}{\text{lbf s}^2}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2} = 0.870\frac{\text{lbf}}{\text{in}^2}$$

and $-\Delta P_{\text{total}} = 0.233 + 0.870 = 1.103\ \text{psi} = 7.61\ \text{kPa}$ We find very good, but not exact agreement between the predictions of the two methods.

You might ask your students what a check valve is. Normally there will be one visible in the vicinity of your building, as part of the lawn sprinklers, which you can point out.

6.43 Let (1) be the upstream end, (2) be the 2 inch pipe as it enters the sudden expansion, (3) be the 3 inch pipe as it leaves that expansion and (4) be the far end of the pipe. Writing BE from (1) to (4) and multiplying through by ρ we have

$$P_4 - P_1 = \rho\left(-\mathcal{F} - \frac{V_4^2 - V_1^2}{2}\right)$$

The $\rho\,\mathcal{F}$ term has three parts, the first pipe, the second pipe and the expansion. We can simply look up the two pipe terms, in Table A.3 for 100 gpm in each of the two pipe sizes, multiplied by the lengths, finding 7.59 and 0.525 psi. For the expansion term we first observe that $\dfrac{D_3}{D_2} = \dfrac{2.067\ \text{in}}{3.068\ \text{in}} = 0.674$. Then we read Fig. 6.16 (or use the equation printed on it) to find that $K = 0.298 \approx 0.3$. From Table A.3 we find that the velocities before and after the expansion are 9.56 and 4.34 ft/s. Then for the expansion

$$\rho\mathcal{F} = -\Delta P = 0.3\cdot 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{\left(9.56\dfrac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2} = 0.18\,\text{psi}$$

Finally

$$\rho\frac{V_4^2 - V_1^2}{2} = 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{\left(4.34\dfrac{\text{ft}}{\text{s}}\right)^2 - \left(9.56\dfrac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2} = -0.487\,\text{psi}$$

So that

$$P_4 - P_1 = -7.56 - 0.525 - 0.18 + 0.487 = -7.81\,\text{psi}$$

6.44* $-\dfrac{\Delta P}{\rho} = \mathcal{F} = \dfrac{V^2}{2}\left(4f\dfrac{L}{D} + K_{\text{exp}} + K_{\text{cont}}\right) \quad V = \sqrt{\dfrac{2\Delta P}{\rho\left(4f\dfrac{L}{D} + K_{\text{e}} + K_{\text{c}}\right)}}$

The expansion and contraction coefficients are 1.0 and 0.5. The solution is a trial and error, in which one guesses f and then does a suitable trial and error. This goes well on a spreadsheet. Here $\frac{\varepsilon}{D} = 0.006$ and our first guess of $f \approx 0.004$. Then

$$V_{\text{first guess}} = \sqrt{\frac{2\left(30\frac{\text{lbf}}{\text{in}^2}\right)\cdot\frac{144\ \text{in}^2}{\text{ft}^2}\cdot 32.2\frac{\text{lbm ft}}{\text{lbf s}^2}}{\left(62.3\frac{\text{lbm}}{\text{ft}^3}\right)\left(4(0.004)\left(\frac{10}{3.068/12}\right)+1+0.5\right)}} = 45.8\frac{\text{ft}}{\text{s}}$$

This corresponds to $\mathcal{R} \approx 1.09\text{e}6$ and $f = 0.00458$. Subsequent trial and error, (easy on a spreadsheet) leads to $V = 44.9$ ft/s, $\mathcal{R} \approx 1.07\text{e}6$ and $f = 0.00458$, and $Q = 23.0 \cdot 44.9 = 1032$ gpm.

6.45 Writing BE from surface to outlet, we have

$$\frac{\Delta P}{\rho} + g\Delta z + \frac{V^2}{2} = -\mathcal{F} = -4f\frac{\Delta x}{D}\frac{V^2}{2} - \frac{K V_2^2}{2}\ ; \ V = \sqrt{\frac{2g(-\Delta z)}{1 + 4f\frac{\Delta x}{D} + K}}$$

Here $\frac{\varepsilon}{D} = 0.006$ and $\mathcal{R}$ is large, so $f \approx 0.0045$, and for a large diameter ratio $K_{\text{contraction}} \approx 0.5$, and

$$V = \sqrt{\frac{2\cdot 32.3\frac{\text{ft}}{\text{s}}\cdot 20\ \text{ft}}{1 + 4\cdot 0.0045\left(\frac{10\,\text{ft}}{3.086\,\text{ft}/12}\right) + 0.5}} = 24.2\frac{\text{ft}}{\text{s}} = 7.36\frac{\text{m}}{\text{s}}$$

$$Q = \left(23.0\frac{\text{gpm}}{\text{ft/s}}\right)\cdot 24.2\frac{\text{ft}}{\text{s}} = 556\ \text{gpm} = 0.262\frac{\text{m}^3}{s}$$

To be safe, we check the Reynolds number, finding $\mathcal{R} = 5.74\cdot 10^5$ and $f = 0.00467$ which is close enough to the assumed 0.0045 to make another iteration unnecessary.

The three terms in the denominator under the radical have values 1, 0.70 and 0.5. Each contributes to the answer.

6.46* From table A.4 for 500 gpm in a 6 in pipe, $-\frac{\Delta P}{\Delta x} = \frac{0.720\,\text{psi}}{100\,\text{ft}}$

$$\mathcal{F} = -\frac{\Delta P}{\rho} = -\frac{\Delta P}{\Delta x}\frac{\Delta x}{\rho} = g\Delta z$$

$$\frac{\Delta z}{\Delta x} = -\frac{\Delta P}{\Delta x}\frac{1}{\rho g} = \frac{0.720\,\text{psi}}{100\,\text{ft}}\cdot\frac{\text{ft}^3}{62.3\,\text{lbm}}\cdot\frac{\text{s}^2}{32\ \text{ft}}\cdot\frac{32\ \text{lbm ft}}{\text{lbf s}^2}\cdot\frac{144\ \text{in}^2}{\text{ft}^2}$$

$$= 0.0166 = 0.0166\frac{\text{ft}}{\text{ft}} = \frac{1.66\ \text{ft}}{100\ \text{ft}}$$

6.47 This is a demonstration, which our shop made for me, with a clear plastic window to let the students watch the falling level. In a lecture room with a sink one can fill the tank with a piece of tygon tubing, and let the students watch the results. I assign this as a homework problem, and ask each student to write her/his answer to part (a) on the board with her/his name. Then we do the calculations, then run the test.

(a) See the solution to Prob. 6.43. $V_{\text{exit}} = \sqrt{\dfrac{2g(z_1 - z_2)}{\left(1 + K_e + 4f\dfrac{\Delta x}{D}\right)}}$

let $z_1 - z_2 = h$

Then see Ex. 5.14, from which we have directly $\Delta t = \dfrac{2\left(\sqrt{h_1} - \sqrt{h_2}\right)\left(\dfrac{A_1}{A_2}\right)}{\sqrt{\dfrac{2g}{\left(1 + K_e + 4f\dfrac{\Delta x}{D}\right)}}}$

$$\frac{A_1}{A_2} = \frac{(6.5 \cdot 4)\ \text{in}^2}{0.104\,\text{in}^2} = 250.8; \qquad K_e = 0.5; \qquad \frac{\varepsilon}{D} = 0.016; \qquad f \approx 0.013$$

Students argue with me about relative roughness. The above value is for galvanized pipe, as shown in the problem. They use the value for steel pipe, getting a much lower value.

$$\Delta t = \frac{2\left(\sqrt{\frac{7}{12}\text{ft}} - \sqrt{\frac{1}{12}\text{ft}}\right) \cdot 250}{\sqrt{\dfrac{2 \cdot 32.2\dfrac{\text{ft}}{\text{s}^2}}{1 + 0.5 + 4 \cdot 0.013 \cdot \dfrac{24\ \text{in}}{0.364\ \text{in}}}}} = 66\ \text{s}$$

When I first did this in class it took about 75 sec for that change in elevation. As it was used more and more (and the pipe rusted) the time climbed slowly to 85 sec. Cleaning it with a wire brush got it back to its original value.

(b) For a Reynolds number of 2000 we find

$$V = \frac{2000 \cdot 1.077 \cdot 10^{-5}\dfrac{\text{ft}^2}{\text{s}}}{\dfrac{0.364}{12}\text{ft}} = 0.71\frac{\text{ft}}{\text{s}} = 0.22\frac{\text{m}}{\text{s}}$$

$$\Delta z = \frac{V^2\left(1 + K_e + 4f\left(\dfrac{\Delta x}{D}\right)\right)}{2g} = \frac{\left(0.71\dfrac{\text{ft}}{\text{s}}\right)^2}{2 \cdot 32.2\dfrac{\text{ft}}{\text{s}^2}}[4.91] = 0.039\ \text{ft} = 0.46\ \text{in} = 0.012\ \text{m}$$

This would suggest that the transition would not be seen. But the transition region is entered at a Reynolds number of about 4000, for which one would calculate

$$\Delta z = \text{above} \cdot \left(\frac{4000}{2000}\right)^2 = 1.85\,\text{in}$$

Sometimes when I have used the demonstration, this has worked well, and been easily observed, others not. The fundamental cussedness of inanimate objects.... To show this clearly I replaced the pipe in the problem with a three-foot length of 1/8 inch sch 40 steel pipe, for which the calculated elevation for $\mathcal{R}$= 2000 is 1.2 inches. This works well, one can see several oscillations in the flow rate. One can sketch Fig. 6.2 on the board, and show that this oscillation represents a horizontal oscillation between the two curves in the transition region.

6.48 By steady-state force balance

$$(P_1 - P_2)\cdot 2\cdot l\cdot y = 2\cdot l\cdot \Delta x\cdot \tau_y;\ \tau_y = \left(-\frac{dP}{dx}\right)\cdot y = -\mu\frac{dV}{dy}$$

$$\int dV = -\left(-\frac{dP}{dx}\right)\frac{1}{\mu}\int y dy; \qquad V = \frac{dP}{dx}\frac{1}{\mu}\frac{y^2}{2} + C$$

$$@\, y = \frac{h}{2},\ V = 0; \qquad C = -\frac{dP}{dx}\frac{1}{2\mu}\left(\frac{h}{2}\right)^2; \quad V = -\frac{dP}{dx}\frac{1}{2\mu}\left[\left(\frac{h}{2}\right)^2 - y^2\right]$$

$$Q = \int V dA = \left(-\frac{dP}{dx}\right)\frac{1}{2\mu}\cdot 2\cdot l\cdot\int_0^{\frac{h}{2}}\left[\left(\frac{h}{2}\right)^2 - y^2\right]dy = \left(-\frac{dP}{dx}\right)\frac{l}{\mu}\left[\left(\frac{h}{2}\right)^2 y - \frac{y^3}{3}\right]_0^{\frac{h}{2}}$$

$$= \left(-\frac{dP}{dy}\right)\frac{l}{\mu}\frac{h^3}{12} = \text{Eq. 6.28.}$$

6.49 This is derived in detail on pages 51-54 of Bird et. al. first edition, and page 53-56 of the second edition.

6.50 The flow rate is proportional to the third power of the thickness of the leakage path, so that $\frac{h_2}{h_1} = \left(\frac{Q_2}{Q_1}\right)^{\frac{1}{3}} = (3.5)^{\frac{1}{3}} = 1.508;$

$$h_2 = 0.0001\ \text{in.}\cdot 1.508 = 0.00015\ \text{in.}$$

True leakage paths are certainly more complex than the uniform annulus assumed here, but this description of the leakage path is generally correct.

In that example I treated the annular space between the stem and the packing as if it were a rectangular slit. You might ask bright students how much error is introduced that way? I checked by comparing the slit solution to that for an annulus with inside diameter 0.25 inches and outside diameter 0.2502 inches, using Eq. 6.29. The computed flow for the slit is 1.0004 that for the annulus.

6.51 Here D_o = 0.2502 in. The other values are taken from Ex. 6.13

$$Q = \left(\frac{P_1 - P_2}{\Delta x} \cdot \frac{1}{\mu}\right) \cdot \frac{\pi}{128}\left(D_o^2 - D_i^2\right)\left(D_o^2 + D_i^2 - \frac{D_o^2 - D_i^2}{\ln(D_o / D_i)}\right)$$

$$= \frac{100\dfrac{\text{lbf}}{\text{in}^2}}{1\text{ in} \cdot 0.6\text{ cP}} \cdot \frac{\pi}{128} \cdot \left(0.2502^2 - 0.2500^2\right)\text{in}^2 \left(0.2502^2 + 0.2500^2 - \frac{0.2502^2 - 0.2500^2}{\ln\dfrac{0.2502}{0.2500}}\right)\text{in}$$

$$\cdot \frac{\text{cP} \cdot \text{ft}^2}{2.09 \cdot 10^{-5}\text{ lbf} \cdot \text{s}} \cdot \frac{144\text{ in}^2}{\text{ft}^2} = 7.52 \cdot 10^{-5}\,\frac{\text{in}^3}{\text{s}}$$

To see the difference between the two solutions, one must copy more significant figures off the spreadsheets, finding $\dfrac{Q_{\text{Ex.6.13}}}{Q_{\text{this problem}}} = \dfrac{7.5158}{7.5188} = 1.0004$

This problem shows that this is one of a large class of problems which can be greatly simplified (and thus made much safer from error) by replacing some other geometry with a planar geometry.

6.52 (a) Here the $l = \pi D$ and $h = (D_o - D_i)/2$. Making those substitutions in Eq. 6.28 produces Eq. 6.30.

(b) Dividing and canceling like terms produces

$$\frac{Q_{6.30}}{Q_{6.28}} = \frac{128}{12} \cdot \frac{D \cdot \left(\dfrac{D_o - D_i}{2}\right)^3}{\left(D_o^2 - D_i^2\right)\left(D_o^2 + D_i^2 - \dfrac{D_o^2 - D_i^2}{\ln(D_o / D_i)}\right)}$$

The D in the numerator can be taken as D_i without much problem.

(c) The values of this ratio for $D_o / D_i = 1.1, 1.01, 1.001, 1.0001$ are 0.954, 0.9951, 0.9995, 1.00004. For smaller values of D_o/D_i the spreadsheet produces values with oscillate between -132 and $1.59 \cdot 10^9$, almost certainly because it does not carry enough significant digits to handle the 0/0 limit of the last term in the denominator.

I tried to reduce this ratio algebraically, with no success, and then tried by introducing $D_o = D_i + \Delta$ and simplifying with and without dropping higher-order terms in Δ, again without much success. If one of you will work that out satisfactorily and send me a copy, I will be grateful.

6.53* This is flow in a slit, for which

$$Q = \left(-\frac{dP}{dx}\right)\frac{1}{12}\frac{1}{\mu}lh^3$$

$$= \frac{0.01\frac{\text{lbf}}{\text{in}^2}}{2\text{m}}\cdot\frac{1}{12}\cdot\frac{1}{0.018\text{ cP}}\cdot 24\text{ in}\cdot\left(10^{-3}\text{ in}\right)^3\cdot\frac{\text{cP ft s}}{6.72\cdot 10^{-4}\text{lbm}}\cdot\frac{32.2\text{ lbm ft}}{\text{lbf s}^2}\cdot\frac{\text{ft}}{12\text{ in}}$$

$$= 2.22\cdot 10^{-6}\frac{\text{ft}}{\text{s}} = 6.28\cdot 10^{-8}\frac{\text{m}}{\text{s}}; \qquad V_{\text{avg}} = \frac{Q}{A} = 0.0133\frac{\text{ft}}{\text{s}} = 0.0040\frac{\text{m}}{\text{s}}$$

this is a very low-leakage window. A gap of 0.00 inches makes a very good seal. The flow is laminar.

6.54 This is really the same as the last problem, after one sees that the ratio of the vessel radius to the length of the flow path is large enough that one may treat it as a linear problem rather than a cylindrical one.

$$Q = \left(-\frac{\Delta P}{\Delta x}\right)\frac{1}{\mu}\frac{1}{12}lh^3 = \left(\frac{1000\frac{\text{lbf}}{\text{in}^2}}{1\text{ in}}\right)\cdot\frac{1}{1.002\text{cP}}\cdot\frac{1}{12}\cdot\frac{10\pi\text{ ft}\cdot\left(10^5\text{in}\right)^3}{2.09\cdot 10^{-5}\frac{\text{lbf}}{\text{ft s cP}}}$$

$$= 1.25\cdot 10^{-7}\frac{\text{ft}^3}{\text{s}} = 4.5\cdot 10^{-4}\frac{\text{ft}^3}{\text{hr}} = 0.93\cdot 10^{-6}\frac{\text{gal}}{\text{min}} = 3.5\cdot 10^{-9}\frac{\text{m}^3}{\text{s}}$$

This low flow rate shows why we can use mating surfaces which are practically smooth as seals. All real gaskets are the equivalent of this; the flow rate through them is not zero, it is simply too small to detect.

6.55 $\text{HR} = \dfrac{\text{Area}}{\text{Wetted perimeter}}$

(a) $\text{HR} = \dfrac{\frac{1}{2}\frac{\pi}{4}D^2}{\frac{1}{2}\pi D + D} = \dfrac{D}{4}\dfrac{1}{2+\pi}$

(b) $\text{HR} = \dfrac{\frac{1}{2}\frac{\pi}{4}D^2}{\frac{1}{2}\pi D} = \dfrac{D}{4}$ which is the same as for a circle.

(c) $\text{HR} = \dfrac{D^2}{4D} = \dfrac{D}{4}$ which is also the same as for a circle

(d) $$\text{HR} = \frac{\frac{\pi}{4}\left(D_2^2 - D_1^2\right)}{\pi\left(D_2 + D_1\right)} = \frac{D_2^2 - D_1^2}{4\left(D_2 + D_1\right)} = \frac{D_2 - D_1}{4}$$

6-56* As in Ex. 6.5, this is a trial and error. We first assume that a square duct with 8 inch sides will be used.

$$\text{HR} = \frac{(8\,\text{in})^2}{4(8\,\text{in})} = 2\,\text{in}; \qquad 4\,\text{HR} = 8\text{ in}$$

$$V = \frac{Q}{A} = \frac{500\frac{\text{ft}^3}{\text{min}}}{(8\text{ ft}/12)^2} = 1125\frac{\text{ft}}{\text{min}} = 18.75\frac{\text{ft}}{\text{s}}; \quad \frac{\varepsilon}{D} = \frac{0.00006\,\text{in}}{8\,\text{in}} = 0.000075 \approx 0$$

$$\mathcal{R} = \frac{\left(\frac{8}{12}\text{ft}\right)\left(18.75\frac{\text{ft}}{\text{s}}\right)\left(0.080\frac{\text{lbm}}{\text{ft}^3}\right)}{0.017\text{cP}\cdot 6.72\cdot 10^{-4}\frac{\text{lbm}}{\text{ft s cP}}} = 8.75\cdot 10^4; \qquad f \approx 0.00447$$

$$-\Delta P = \frac{(4)(0.00447)\left(18.75\frac{\text{ft}}{\text{s}}\right)^2\left(0.08\frac{\text{lbm}}{\text{ft}^2}\right)(800\,\text{ft})}{2\left(\frac{8}{12}\text{ft}\right)} \cdot \frac{\text{lbf s}^2}{32.2\,\text{lbm ft}} \cdot \frac{\text{ft}^2}{144\,\text{in}^2} = 0.065\text{ psi}$$

Then we trial-and-error n D to find the value which makes $-\Delta P = 0.1$ psi**,** finding $D =$ 7.31 in, $V =$ 22.4 ft/s, $\mathcal{R} = 9.57\cdot 10^4$ and $f = 0.00438$.

For equal cross-sectional area $A = D_{\text{square}}^2 = \frac{3.1416}{4} D_{\text{circle}}^2$**,** $\frac{D_{\text{square}}}{D_{\text{circle}}} = \sqrt{\frac{3.1416}{4}} = 0.886$

$$\frac{Perimeter_{\text{square}}}{Perimeter_{\text{circle}}} = \frac{4D_{\text{square}}}{\pi D_{\text{circle}}} = \frac{4}{\pi}\sqrt{\frac{\pi}{4}} = \sqrt{\frac{4}{\pi}} = 1.128$$

This is also the ratio of weights for equal wall thickness. In this problem the computed square diameter is $\frac{7.31}{8} = 0.914$ of the corresponding circular duct, somewhat more than the equal area value of 0.886. This shows that the friction effect of a square duct is more than that of a circular one, and we need a little more than equal area.

6.57* The velocity is proportional to 1/(square root of the friction factor, so

$$\frac{f_{\text{used in actual design}}}{f_{\text{estimated in Ex. 6.15}}} = \sqrt{\frac{V_{\text{Ex. 6.15}}}{V_{\text{actual design}}}} = \sqrt{\frac{4.28\text{ ft/s}}{3.89\,\text{ft/s}}} = 1.049$$

and

$$f_{\substack{\text{used in actual}\\ \text{design}}} = 0.0024 \cdot 1.049 = 0.00252$$

From Eq 6.21 we find that this corresponds to a relative roughness of 2.07e-5, which corresponds to an absolute roughness of 0.0014 ft (compared to the 0.001 estimated in the example).

6.58 (a) Combining the equations, we have $V = C\sqrt{\text{HR}\cdot\frac{-\Delta z}{\Delta x}} = \sqrt{\frac{2\cdot\text{HR}\cdot g}{f}\cdot\frac{-\Delta z}{\Delta x}}$

These are the same if $C = \sqrt{\frac{2g}{f}}$

(b) $$C = \sqrt{\frac{2g}{f}} = \alpha\frac{\text{HR}^{1/6}}{n}; \quad \alpha = \frac{n}{\text{HR}^{1/6}}\sqrt{\frac{2g}{f}} = \frac{0.012}{(17.09\text{ ft})^{1/6}}\sqrt{\frac{2\cdot 32.2\text{ ft/s}^2}{0.0024}} = 1.22\frac{\text{ft}^{1/3}}{\text{s}}$$

6.59* Here the velocity will be much too high for us to use any of the convenient methods, so

$$V = \sqrt{\frac{2\left(-\frac{\Delta P}{\rho} - g\Delta z\right)}{1 + 4f\frac{\Delta x}{D}}}$$

As a first trial, for $\varepsilon / D = 0.0018\text{ in} / 3.068\text{ in} = 0.0006$ and a large Reynolds number, try $f = 0.0040$. Then

$$V_{\text{first guess}} = \sqrt{\frac{2\left(1000\frac{\text{lbf}}{\text{in}^2}\cdot\frac{144\text{ in}^2}{\text{ft}^2}\cdot\frac{32.2\text{ lbm ft}}{\text{lbf s}^2} + 32.2\frac{\text{ft}}{\text{s}^2}\cdot 10\text{ ft}\right)}{1 + 4(0.004)\left(\frac{10}{\left(\frac{3.068}{12}\right)}\right)}} = 303\frac{\text{ft}}{\text{s}}$$

The corresponding $\mathcal{R}$ = 7.198e6 and from Eq. 6.21 f = 0.00453. Then, using the search engine on my spreadsheet, we find the velocity at which the guessed and calculated values of f are equal, finding V = 295.7 ft/s, $\mathcal{R}$ = 7.102e6 and from Eq. 6.21 f = 0.00453.

6.60 (a) $\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta V^2}{2} = -\mathcal{F} + \frac{dW_{\text{n.f.}}}{dm}$ Here $\frac{\Delta V^2}{2}$ is zero, so

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{\Delta P_{\text{pump}}}{\rho} = \mathcal{F} + \frac{\Delta P}{\rho} + g\Delta z; \qquad \Delta P_{\text{pump}} = \rho\mathcal{F} + \Delta P_{12} + \rho g\Delta z_{1-2}$$

From Appendix A.4, for 150 gpm in a 3 inch pipe

$$\rho \mathcal{F} = \frac{2.24 \text{ psi}}{100 \text{ ft}} \cdot 2300 \text{ ft} = 51.5 \text{ psi}$$

$$\Delta P_{\text{pump}} = 51.5 \text{psi} + 20 \text{psi} + 62.3 \frac{\text{lbm}}{\text{ft}^3} \cdot 32.2 \frac{\text{ft}}{2^2} \cdot 230 \text{ ft} \cdot \frac{\text{lbf s}^2}{32 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{in}^2}$$

$$= 51.5 + 20 + 99.5 = 171 \text{ psi} = 1.18 \text{MPa}$$

(b) $$\text{Po} = \frac{dW_{\text{n.f.}}}{dm} \cdot \dot{m} = \frac{\Delta P_{\text{pump}}}{\rho} \cdot Q\rho = \Delta P_{\text{pump}} \cdot Q$$

$$= 171 \frac{\text{lbf}}{\text{in}^2} \cdot 150 \frac{\text{gal}}{\text{min}} \cdot \frac{\text{ft}^3}{7.48 \text{ gal}} \cdot \frac{144 \text{ in}^2}{\text{ft}^2} \cdot \frac{\text{hp min}}{33\,000 \text{ ft lbf}} = 15.0 \text{ hp} = 11.1 \text{ kW}$$

According to the sign convention in this book, this is power entering the system, in this case by driving the pump.

6.61* $V = \sqrt{\dfrac{2(-g\Delta z)}{1 + 4f\dfrac{\Delta x}{D}}}$ As a first trial, for $\varepsilon / D = 0.0018 \text{ in} / 5.047 \text{ in} = 0.00036$ and a large Reynolds number, try $f = 0.0040$. Then

$$V_{\text{first guess}} = \sqrt{\frac{2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 10 \text{ ft}}{1 + 4(0.004)\left(\frac{10}{(5.047/12)}\right)}} = 48.3 \frac{\text{ft}}{\text{s}}$$

The corresponding $\mathcal{R}$ = 1.89e6 and from Eq. 6.21 f = 0.004094. Then, using the search engine on my spreadsheet, we find the velocity at which the guessed and calculated values of f are equal, finding V = 48.14 ft/s, $\mathcal{R}$ = 1.88e6 and from Eq. 6.21 f = 0.004094.

6.62 Let (1) be the air inlet, and (2) the exit from the stack. Then applying BE from (1) to (2), assuming that (1) is far enough from the furnace that the velocity is negligible, we find

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^2}{2} = -\mathcal{F} = -\frac{V_2^2}{2} \cdot \left(K_{\text{furnace}} + 4f\left(\frac{L}{D}\right)_{\text{stack}} \right)$$

The first term is $\dfrac{P_2 - P_1}{\rho} = -\dfrac{\rho_{\text{air}}}{\rho_{\text{stack}}} g(z_2 - z_1)$ so that

$$g(z_2 - z_1) \cdot \left(1 - \frac{\rho_{\text{air}}}{\rho_{\text{stack}}} \right) = -\frac{V_2^2}{2} \cdot \left(1 + K_{\text{furnace}} + 4f\left(\frac{L}{D}\right)_{\text{stack}} \right)$$

$$V_2 = \left[\frac{2g(z_2 - z_1)\left(\frac{\rho_{\text{air}}}{\rho_{\text{stack}}} - 1\right)}{1 + K_{\text{furnace}} + 4f\left(\frac{L}{D}\right)_{\text{stack}}} \right]^{1/2} \quad \text{Here } \left(\frac{\rho_{\text{air}}}{\rho_{\text{stack}}} - 1\right) = \left(\frac{29}{28} \cdot \frac{810°\text{R}}{528°\text{R}} - 1\right) = 0.589$$

$$K_{\text{furnace}} = 3.0 \text{ and } \left(\frac{L}{D}\right)_{\text{stack}} = \frac{100 \text{ ft}}{5 \text{ ft}} = 20$$

To solve the problem we need an estimate of f. We assume that the stack is made of concrete (a common choice), so from table 6.2 we estimate that $\varepsilon \approx 0.02$ in, and $\varepsilon / D \approx 0.00033$ This leads to a first guess of $f_{\text{first guess}} = 0.005$. Then

$$V_{2,\text{ first guess}} = \left[\frac{2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 100 \text{ ft} \cdot 0.589}{1 + 3 + 4 \cdot 0.005\left(\frac{100 \text{ ft}}{5 \text{ ft}}\right)_{\text{stack}}} \right]^{1/2} = 29.4 \frac{\text{ft}}{\text{s}}$$

This corresponds to $\mathcal{R} = 1.3 \cdot 10^7$ and $f = 0.0040$. (We estimated the viscosity of the stack gas as 0.025 cP ≈ the value for air at 350°F from Fig. A.1, and the gas density from the ideal gas law). Then, with our spreadsheet's numerical engine we seek the guess of f which makes the guessed and calculated values equal, finding $V = 29.7$ ft/s, and $\mathcal{R} = 1.3 \cdot 10^7$ and $f = 0.0040$.

Students are generally uncomfortable with the idea that the pressure drop though the furnace is estimated by a constant times the kinetic energy based on the flue gas temperature. For the highest-quality estimate we would follow the flow through the furnace, taking into account the changes in density, viscosity and velocity from point to point in the furnace. That is now possible with CFD programs. But the simple estimating method here is widely used, and reasonably reliable.

6.63* Assume the flow is from left to right, $\frac{\Delta P}{\rho} + g\Delta z = -\mathcal{F}$

$$\mathcal{F} = -\left(\frac{30 \frac{\text{lbf}}{\text{in}^2}}{60 \frac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2 \text{ lbm ft}}{\text{lbf s}^2} \cdot \frac{144 \text{in}^2}{\text{ft}^2} + 32.2 \frac{\text{ft}}{\text{s}^2}(-30 \text{ ft}) \right) = -(2318 - 966) = -1352 \frac{\text{ft}^2}{\text{s}^2}$$

The minus sign indicates that the assumed flow direction is wrong, flow is from right to left! The equivalent dP/dx is

$$\left(\frac{dP}{dx}\right)_{\text{equivalent}} = \left(\frac{dP}{\rho}\right)_{\text{equivalent}} \frac{\rho}{\Delta x} = \frac{1352 \frac{\text{ft}^2}{\text{s}^2} \cdot 60 \frac{\text{lbm}}{\text{ft}^3}}{1000 \text{ft}} \cdot \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}} \cdot \frac{\text{ft}^2}{144 \text{in}^2} = 0.0175 \frac{\text{psi}}{\text{ft}}$$

If we enter Figure 6.13 at 17.5 psi/1000 ft, read diagonally up to SG =0.96, then vertically to 100 cSt and horizontally we find $Q \approx 57$ gpm. We also see that this is in the laminar flow region, which we could solve by Poisueille's equation, (first computing a viscosity of 96.3 cP) finding

$$Q = \frac{17.5\dfrac{\text{lbf}}{\text{in}^2}}{1000\,\text{ft}} \cdot \frac{\pi}{128} \cdot \frac{(3.068\,\text{ft}/12)^4}{96.3\,\text{cp}} \cdot \frac{\text{ft}^2\ \text{cp}}{2.09\text{e}-5\,\text{lbf s}} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} = 0.126\frac{\text{ft}^3}{\text{s}} = 56.7\frac{\text{gal}}{\text{min}}$$

6.64 (a) We must specify the maximum flow rate, in this case 200 gpm.

(b) For all conditions

$$\left(\frac{\Delta P}{\rho g}\right)_{\text{pump}} = \frac{dW_{\text{n.f.}}}{g\,dm} = \frac{\Delta P}{\rho g} + \Delta z + \frac{\mathcal{F}}{g}$$

The highest required head will correspond to the highest elevation and highest pressure in vessel 2, and the lowest elevation and pressure in vessel 1. The required head will be highest for the lowest specific gravity (0.80) and for the highest kinematic viscosity (5 cSt). The "equivalent lengths" are $(6 \cdot 30) + (4 \cdot 13) + (1 \cdot 340) = 572$ so that

$$L_{\text{effective}} = 672 + 572\left(\frac{3.086}{12}\right) = 774\ \text{ft}$$

On Fig. 6.12, we enter at the right at 200 gpm, read horizontally to the 5 cSt line and then vertically to the top finding $\left(\begin{array}{c}\text{loss in head}\\ \text{ft per 1000 ft}\end{array}\right) \approx 105 = \frac{1}{L} \cdot \frac{\mathcal{F}}{g}$ from which

$$\left(\frac{\mathcal{F}}{g}\right)_{\max} = 773\ \text{ft} \cdot \left(\frac{105\ \text{ft}}{1000\ \text{ft}}\right) = 81\ \text{ft}$$

$$\left(\frac{\Delta P}{\rho g}\right)_{\text{pump}} = \frac{(81-8)\dfrac{\text{lbf}}{\text{in}^2}}{\left(0.8 \cdot 62.3\dfrac{\text{lbm}}{\text{ft}^3}\right)} \cdot \frac{\text{s}^2}{32.2\ \text{ft}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\,\text{in}^2}{\text{ft}^2} + (127-21)\ \text{ft} + 81\ \text{ft}$$

$$= 211 + 106 + 81 = 398\ \text{ft} = 121\ \text{m}$$

The entrance and exit losses, which we ignored are ≈ 0.65 psi ≈ 0. So we would specify a pump with a design point of 200 gpm, and 398 ft of head. (This is a high head for a centrifugal pump at this volumetric flow rate; we would probably have to specify some other type of pump).

6.65* The maximum corresponds to the highest pressure and elevation in vessel 2, and the lowest in vessel 1, and to the lowest values of the specific gravity and viscosity. The minimum corresponds to the opposite of those conditions. Here the open globe valve adds 340 x (3.068 / 12) = 87 ft, so the effective length of the pipe is 860 ft. In both parts of the problem we have a trial-and-error for the velocity. The values shown are those at the end of that trial and error. For both parts

$$\frac{\Delta P}{\rho} + g\Delta z = \frac{V_2^2}{2} = -\mathcal{F} = -4f\frac{L}{D}\frac{V^2}{2}; \qquad V = \sqrt{\frac{-2\left(\frac{\Delta P}{\rho} + g\Delta z\right)}{1 + 4f\frac{L}{D}}}$$

For the maximum,

$$V_{max} = \sqrt{\frac{2\left[\frac{(81-8)\text{psi}}{0.8\cdot 62.3\frac{\text{lbm}}{\text{ft}^2}}\cdot\frac{32.2\text{ lbm ft}}{\text{lbf s}^2}\cdot\frac{144\text{in}^2}{\text{ft}^2} + 32.2\frac{\text{ft}}{\text{s}^2}(106\text{ ft})\right]}{1 + 4(0.00488)\frac{860\text{ ft}}{3.068\text{ ft}/12}}} = 17.5\frac{\text{ft}}{\text{s}}$$

$$Q_{max} = \frac{23.0\text{gpm}}{\text{ft/s}}\cdot 17.5\frac{\text{ft}}{\text{s}} = 402\text{gpm}$$

For the minimum

$$V_{min} = \sqrt{\frac{2\left[\frac{(47-20)}{0.85\cdot 62.3}\cdot 32.2\cdot 144 + 32.2(100-43)\right]}{1 + 4(0.0057)\frac{860\text{ ft}}{3.068\text{ ft}/12}}} = 10.35\frac{\text{ft}}{\text{s}}$$

$$Q_{max} = \frac{23.0\text{gpm}}{\text{ft/s}}\cdot 10.3\frac{\text{ft}}{\text{s}} = 238\text{gpm}$$

6.66 We begin by assuming that the pressure at *A* will not be low enough to cause boiling. Then we solve for the velocity applying BE. with friction from water level to outlet. Here the adjusted length of the pipe is

$L_{adjusted} = 60\text{ ft} + 2\cdot 20\cdot\frac{10.020\text{ ft}}{12} = 93.3\text{ ft}$ By BE, with no pressure difference,

we have $g\Delta z + \frac{V_2^2}{2} = -\mathcal{F} = -\left(K_e + 4f\frac{L}{D}\right)\cdot\frac{V_2^2}{2}$ or $V_2 = \sqrt{\frac{2g(-\Delta z)}{1 + K_e + 4f\frac{L}{D}}}$

$\frac{\varepsilon}{D} = 0.00018$ and $\mathcal{R}$ is large so $f \approx 0.0034$ and $K_e = 1/2$, so

$$V_2 = \sqrt{\frac{2\cdot 32\frac{\text{ft}}{\text{s}^2}\cdot 10\text{ ft}}{1 + 0.5 + 4(0.0034)\frac{93.3}{10/12}}} = 14.6\frac{\text{ft}}{\text{s}}; \quad Q = \frac{246\text{ gpm}}{(\text{ft/s})}\cdot 14.6\frac{\text{ft}}{\text{s}} = 3592\text{ gpm}$$

At this point we check finding $\mathcal{R} = 1.1\cdot 10^6$ and by numerical solution find that f =0.0036, and V = 14.3 ft/s = 4.44 m/s, and Q = 3526 gpm. Then, using those values

$$\frac{P_a - P_1}{\rho} + g(z_a - z_1) + \frac{V_a^2}{2} = -\mathsf{F} = -\frac{V_a^2}{2}\left(K_e + 4f\frac{L_a}{D}\right)$$

$$P_a = P_1 - \rho\left[g(z_a - z_1) + \frac{V_a^2}{2}\left(1 + K_e + 4f\frac{L_a}{D}\right)\right]$$

$$= 14.7 - \frac{62.3\,\text{lbm}}{\text{ft}^3}\left[32.2\frac{\text{ft}}{\text{s}}\cdot 20\,\text{ft} + \frac{\left(14.6\frac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\left(1 + 0.5 + 4(0.00364)\frac{41.66}{10/12}\right)\right]$$

$$\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2} = 14.7 - 11.7 = 3.0\ \text{psia}$$

This siphon would probably work satisfactorily, although the absolute pressure is low enough that dissolved air might come out of solution and cause problems.

6.67 This was actually done in the 1950s. At that time high quality plastic pipe had not developed to the extent it has now, so the pipe was made of aluminum. One can show that in terms of (strength/weight) high quality aluminum alloys are still the best (which is why we make airplanes of them). If you walk down the trail from the North Rim to the river, you can see the occasional valve boxes, which allow the pipe, which is buried by the trail, to be shut down and drained for maintenance and repair when needed.

$$g\Delta z = -\mathsf{F} = -\left(\frac{\Delta P}{\rho}\right)_{\text{friction}}\cdot\frac{\Delta x}{\Delta x} = -\left(\frac{\Delta P}{\Delta x}\right)_{\text{friction}}\cdot\frac{\Delta x}{\rho}$$

$$\left(-\frac{\Delta P}{\Delta x}\right)_{\text{friction}} = \rho g\frac{\Delta z}{\Delta x} = 62.3\frac{\text{lbm}}{\text{ft}^2}\cdot 32.2\frac{\text{ft}}{\text{s}^2}\cdot\frac{3000\ \text{ft}}{(14\cdot 5280\ \text{ft})}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2}$$

$$= 0.0176\frac{\text{psi}}{\text{ft}} = \frac{1.76\ \text{psi}}{100\ \text{ft}}$$ From Table A.3 we see that this requires pipe larger than 6 inches and smaller than 8 inches. I don't know if they had a special pipe made, or used an 8 inch pipe.

$$P_{\text{max}} = \rho g\Delta z = 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot 32.2\frac{\text{ft}}{\text{s}^2}(7000 - 2500)\,\text{ft}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2}$$

$$= 1948\ \text{psi} = 13.4\ \text{MPa}$$

thickness $= \frac{PD}{2\sigma}$, for an 8 inch diameter pipe and a fairly conservative value of $\sigma = 10\,000$ psi, we compute thickness $= \frac{1947\ \text{psi}\cdot 8\ \text{in}}{2\cdot 10\,000\,\text{psi}} = 0.78\ \text{in} = 20\ \text{cm}$

6.68 This is Ex. 6.15 of the first edition (pages 193-195), where a trial and error solution based on Table A.3 is show through four trials. The final flow rates are A 285 gpm, B 60 gpm, C 225 gpm.

The following spreadsheet solution is longer, because it requires nested numerical solutions, but is probably more illustrative. The actual spreadsheet carries and displays more significant figures than are shown in this abbreviated table.

We begin by defining $\alpha = \left(\dfrac{P_1 - P_2}{\rho g} + (z_1 - z_2)\right)$

Then we write BE for each of the three pipe segments, and solve for V

$$V_A = \left[\frac{2\alpha g}{1 + 0.5 + 4f(L/D)}\right]^{0.5} \; ; \; V_B = \left[\frac{2(z_3 - z_1 - \alpha)g}{1 + 0.5 + 4f(L/D)}\right]^{0.5}$$

and

$$V_C = \left[\frac{2(z_4 - z_1 - \alpha)g}{1 + 0.5 + 4f(L/D)}\right]^{0.5}$$

where the values of f, L and D are those for the individual pipe sections. Then we guess a value of α, solve for the three flows, and compute the algebraic sum of the flows into point 2. The correct choice of α makes that algebraic sum zero. One need not know the individual values of the two components of α. If the pipes were flexible and we raised or lowered point 2, the flows would be unchanged because the two parts of α would change, but their sum would not.

The second column of the following table is the solution for α = 10 ft. For each of the three pipe sections we have a trial and error for the velocity, done using the spreadsheet's numerical engine. In this case the guessed value is Q, and the check value is the ratio of the V based on that guess, to the one computed from BE, as shown above We see that for α = 11.5 ft the algebraic sum of the flows into point 2 = -1.16 gpm ≈ 0, and the three flows are 304 gpm, 64 gpm and 241 gpm. These are somewhat higher than those shown in the example in the first edition, but within the accuracy of interpolations in Table A.3.

This is a somewhat clumsy way to solve this class of problems. Real solutions would use nested DO loops in FORTRAN or the equivalent.

(P2/rho g+z2) , guessed ft	10	20	12	11.5
Section A				
L, ft	2000	2000	2000	2000
D, ft	0.5054	0.5054	0.5054	0.5054
e/D	0.0003	0.0003	0.0003	0.0003
Q, guessed, gal/min	281.1204	413.4696	311.2119	303.9138
V guessed, ft/s	3.1236	4.5941	3.4579	3.3768
Re	146583.	215593.	162273.	158468.
f, equation 6.21	0.0040	0.0037	0.0039	0.0039
V, BE, ft/s	3.1235	4.5941	3.4579	3.3768
V BE/Vguessed	1.0000	1.0000	1.0000	1.0000
Section B				
L, ft	2000	2000	2000	2000

D, ft	0.2557	0.2557	0.2557	0.2557
e/D	0.0006	0.0006	0.0006	0.0006
Q, guessed, gal/min	66.5264	44.9489	62.6858	63.6623
V guessed, ft/s	2.8925	1.9543	2.7255	2.7679
Re	68663.	46392.	64699.	65707.
f, equation 6.21	0.0047	0.0052	0.0048	0.0048
V, BE, ft/s	2.8924	1.9543	2.7255	2.7679
V BE/Vguessed	1.0000	1.0000	1.0000	1.0000
Section C				
L, ft	1000	1000	1000	1000
D, ft	0.3355	0.3355	0.3355	0.3355
e/D	0.0004	0.0004	0.0004	0.0004
Q, guessed, gal/min	248.3668	198.4037	239.0666	241.4192
V guessed, ft/s	6.2719	5.0102	6.0370	6.0964
Re	195377.	156074.	188061	189912.
f, equation 6.21	0.0037	0.0039	0.0038	0.0038
V, BE, ft/s	6.2719	5.0102	6.0370	6.0964
V BE/Vguessed	1.0000	1.0000	1.0000	1.0000
Sum of flows to point 2, gpm	-33.7729	170.1170	9.4594	-1.1677

6.69 See the solution to Prob. 6.68. Here the flow in section B is from the reservoir toward point 2. In the above spreadsheet, we must write $V_B = \left[\dfrac{-2(z_3 - z_1 - \alpha)g}{1 + 0.5 + 4f(L/D)}\right]^{0.5}$ so that the term in brackets will be positive. Then in the sum of flows into point 2, we must take flow B as positive rather than negative. With these changes the trial-and-error on α is the same type as in Prob. 6.68, with solution $\alpha = 7.5$ ft. The three Q's are 239.4, 20.4 and 259.6 gpm. The algebraic sum of the flows into point 2 = 0.17 gpm ≈ zero.

6-70* Writing BE. from 1 to 2, we see that there is no change in elevation, so

$$\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} = -\mathcal{F} \;; \qquad \frac{\Delta P}{\rho} = -\frac{\Delta V^2}{2} - \mathcal{F}$$

The velocity out the individual tubes is proportional to the square root of the pressure in the manifold below them. Thus, if ΔP is positive, the far tube with squirt the highest, while if it is negative the near tube will squirt higher. For multiple tubes one rewrites these equations for successive pairs of tubes. In all cases the velocity is declining in the flow direction, because each tube bleeds off some of the flow. So for all cases ΔV^2 is negative.

(a) If $\mathcal{F} = 0$ then ΔP is positive, and the water will squirt highest out the farthest tube.

(b) If the absolute value of $\mathcal{F}$ is greater than that of $\Delta V^2 / 2$, then ΔP is negative and the water will squirt highest out the nearest (upstream) tube.

One can build the device to the dimensions show in the article, and see that these predictions are observed.

(c) First, maintain your humility. Second, if the experienced workers in some facility you are new at are goading you into betting on something, don't bet very much.

6.71 We guessed $f = 0.0042$. For 200 gpm in a 3.24 in diameter pipe, $\mathcal{R} = 96,500$, $e/D = 0.0005$, and from Eq. 6.21, $f = 0.0051$. If we take the ratio of this value to 00042 to the 1/6 power we find that the economic diameter would be 1.035 times the value in that example. For an assumed $f = 0.01$ it would be 1.155 times the value in that example. The point of this problem is that because of the 1/6 power, the computed diameter is quite insensitive to modest changes in f

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6.72
$$\begin{pmatrix}\text{Annual}\\ \text{cost}\end{pmatrix} = A \cdot m \cdot B + \frac{CI^2 r \Delta x}{\frac{\pi}{4} D^2} = AB\Delta x \frac{\pi}{4} D^2 \rho + \frac{CI^2 r \Delta x}{\frac{\pi}{4} D^2}$$

$$\frac{d(\text{annual cost})}{dD} = 2AB\Delta x \frac{\pi}{4} \rho D - \frac{2CI^2 r \Delta x}{\frac{\pi}{4} D^3} = 0$$

$$D^4_{\text{econ}} = \frac{CI^2 r}{\left(\frac{\pi}{4}\right)^2 AB\rho}$$

This is for low voltage transmission, where the resistive losses are the only serious factor. In modern, high voltage, transmission lines corona losses are apparently more significant, so this formula no longer applies. It is, however an interesting historical example of the apparently first use of simple economics to find an optimum.

Inside our houses and other buildings we set the wire diameter for safety reasons. We want the maximum ΔT due to resistive heating to be low enough to pose no fire hazard.

6.73 (a)
$$\begin{pmatrix}\text{total annual}\\ \text{cost}\end{pmatrix} = \text{PC} \cdot \frac{\dot{m}^3\, 2 f \Delta x (4/\pi)^2}{\rho^2 D^5} + \text{CC} \cdot \text{PP} \cdot D \cdot \Delta x\ ;\ f = \frac{16}{\mathcal{R}} = \frac{4\pi D\mu}{\dot{m}}$$

Substituting this value for f in the first term, and simplifying we have

$$\begin{pmatrix}\text{total annual}\\ \text{cost}\end{pmatrix} = \text{PC} \cdot \frac{\dot{m}^2\, 8\pi\mu\Delta x (4/\pi)^2}{\rho^2 D^4} + \text{CC} \cdot \text{PP} \cdot D \cdot \Delta x$$

Taking the derivative with respect to D, and setting it equal to zero we find

$$\mathrm{PC}\cdot\frac{\dot{m}^2 8\pi\mu\Delta x(4/\pi)^2}{\rho^2}\cdot\frac{4}{D^5}=\mathrm{CC}\cdot\mathrm{PP}\cdot\Delta x$$

$D_{\text{econ}}^5=\dfrac{\mathrm{PC}\cdot\dot{m}^2 32\pi\mu(4/\pi)^2}{\rho^2\mathrm{CC}\cdot\mathrm{PP}}$ but $\dfrac{\dot{m}^2}{\rho^2}=Q^2$ Substituting this and taking the fifth root produces Eq. 6.63

(b) For 200 gpm, 2000 cP, s.g = 1 and the economic values on Figure 6.23

$$D_{\text{econ}}=\left[\frac{\dfrac{\$0.04}{\text{kWh}}\cdot 32\pi\cdot 2000\text{ cP}\cdot\left(\dfrac{200\text{ gal}}{\text{min}}\right)^2}{\dfrac{0.04}{\text{yr}}\cdot\dfrac{\$2}{\text{in ft}}}\right]^{1/5}\cdot$$

$$\cdot\left[\frac{8760\text{ hr}}{\text{yr}}\cdot\left(\frac{\text{ft}^3}{7.48\text{ gal}}\right)^2\cdot\frac{\text{kW s}}{737.6\text{ ft lbf}}\cdot\frac{2.09\cdot 10^{-5}\text{lbf s}}{\text{cP ft}^2}\cdot\frac{\text{ft}}{12\text{ in}}\right]^{1/5}$$

$$=0.582\text{ ft}=6.98\text{ in}$$

From Fig. 6.23 for 200 gpm and 2000 cS we interpolate between the 6 in and 8 in lines, finding ≈ 6.5 inches, which is ≈ the same as calculated here.

(c) For constant viscosity, Eq. 6.63 becomes $D_{\text{econ}}\propto\left(Q^2\right)^{1/5}=Q^{0.4}$ and

$V_{\text{econ}}\propto\dfrac{Q_{\text{econ}}}{\left(D_{\text{econ}}\right)^2}=\dfrac{Q_{\text{econ}}}{\left(Q^{0.4}\right)^2}=Q_{\text{econ}}^{0.2}$ which is the slope on Fig 6.23

(d) The density does not appear in Eq. 6.63.

6.74 (a) Based on Table 6.8 we would conclude that the velocity should be about 40 ft/s. This is not quite right, because it is for pipes with *L/D* much larger than 100. Here we will see that *L/D* is of the order of 25, so that this is a short pipe, for which the entrance loss is significantly greater than the *4fL/D* term. However 40 ft/s is still a plausible estimate.

(b) $$Q=\frac{4083\text{ mi}^2\cdot 2000\text{ ft}}{24\text{ hr}}\cdot\left(\frac{5280\text{ ft}}{\text{mi}}\right)^2\cdot\frac{\text{hr}}{60\text{ min}}=1.58\cdot 10^{11}\frac{\text{ft}^3}{\text{min}}$$

$$D=\sqrt{\frac{4}{\pi}\frac{Q}{V}}=\sqrt{\frac{4}{\pi}\frac{1.58\cdot 10^{11}\dfrac{\text{ft}^3}{\text{min}}}{\left(40\cdot 60\dfrac{\text{ft}}{\text{min}}\right)}}=9\,158\text{ ft}$$

(c) $$\frac{\Delta P}{\rho}+\frac{V_2^2}{2}=-\mathcal{F}=-\left(4f\frac{L}{D}+K_e\right)V_2^2;\quad -\Delta P=\frac{\rho V^2}{2}\left(1+K_e+4f\frac{L}{D}\right)$$

Here $\mathcal{R}\approx 2\cdot 10^9$ and $\varepsilon/D\approx 1.6\cdot 10^{-8}$ (for steel pipe) so that $f\approx 0.0015$ and

$$4f\frac{L}{D}=4\cdot 0.0015\cdot\left(\frac{50\cdot 5280\text{ ft}}{9158\text{ ft}}\right)=0.17$$

$$\Delta P = 0.075\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{\left(40\frac{\text{ft}}{\text{s}}\right)^2}{2}(1+0.5+0.17)\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{\text{ft}^2}{144\ \text{in}^2} = 0.0022\ \text{psi}$$

(d) $\dot{m} = 1.58\cdot 10^{11}\frac{\text{ft}^3}{\text{min}}\cdot 0.075\frac{\text{lbm}}{\text{ft}^3} = 1.185\cdot 10^{10}\frac{\text{lbm}}{\text{min}}$

For an isothermal compressor, (See Prob. 5.58)

$$\frac{dW_{\text{n.f.}}}{dm} = \frac{RT}{M}\ln\frac{P_2}{P_1} = \frac{1.987\frac{\text{Btu}}{\text{lb mol }^{\circ}\text{R}}\cdot 528^{\circ}\text{R}}{29\frac{\text{lbm}}{\text{lb mole}}}\ln\left(\frac{14.7+0.022}{14.7}\right) = 0.053\frac{\text{Btu}}{\text{lbm}}$$

$$\text{Po} = \dot{m}\left(\frac{dW_{\text{n.f.}}}{dm}\right) = \left(1.185\cdot 10^{10}\frac{\text{lbm}}{\text{min}}\right)\left(0.053\frac{\text{Btu}}{\text{lbm}}\right)\left(\frac{\text{hp hr}}{2545\ \text{Btu}}\right)\left(\frac{60\ \text{min}}{\text{hr}}\right)$$

$$= 1.49\cdot 10^7\ \text{hp} = 11\ 100\ \text{MW}$$

(e) These values are all wildly impractical. How would you build a pipe with that diameter? The power requirement is roughly that needed for a population of 5 million people, including residential, commercial and industrial uses. The only practical solution is to prevent emissions at the source.

6.75 Based on economic velocity calculations (See Fig. 6.23 or Table 6.8) a velocity of about 7 ft/s would be used. This may be a bit low for this large a pipe, because the friction factor will be lower than that in typical industrial pipes, but as shown by the solution to Prob. 6.71, this makes little difference. Then the pipe diameter would be

$$D = \sqrt{\frac{4Q}{\pi V}} = \sqrt{\frac{4\left(10^7\cdot 4.356\cdot 10^4\right)\frac{\text{ft}^3}{\text{yr}}}{\pi\left(7\frac{\text{ft}}{\text{s}}\right)}\cdot\frac{\text{yr}}{365\frac{1}{4}\cdot 24\cdot 3600\ \text{s}}} = 50.1\ \text{ft}$$

For this diameter and velocity $\mathcal{R} \approx$ 3.2910^8 and $e/D \approx 3\cdot 10^{-6}$. From Eq. 6.21 we compute that $f = 0.002$, although for this large a pipe that equation may not be very accurate. The pressure drop would be

$$-\Delta P = 4f\rho\frac{L}{D}\frac{V^2}{2}$$

$$= 4(0.0020)\left(62.3\frac{\text{lbm}}{\text{ft}^3}\right)\left(\frac{1000\cdot 5280}{50}\right)\frac{\left(7\frac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}}\cdot\frac{144\ \text{in}^2}{\text{ft}^2} = 278\ \text{psi}$$

and the pumping power

$$\text{Po} = Q\Delta P = \left(10^7\cdot 4.356\cdot 10^4\right)\frac{\text{ft}^3}{\text{yr}}\cdot 278\frac{\text{lbf}}{\text{in}^2}\cdot\frac{\text{hp min}}{33\,000\ \text{ft lbf}}\cdot\frac{144\ \text{in}^2}{\text{ft}^2}\cdot\frac{\text{yr}}{365\frac{1}{4}\cdot 24\cdot 60\ \text{min}}$$

$$= 1.00\cdot 10^6\ \text{hp} \approx 746\ \text{MW}$$

As an engineering problem this is fairly easy. Los Angeles draws about half this amount from the Colorado River and pumps it about a tenth as far, in several parallel pipes, none this large in diameter. If we did this project it would probably involve several parallel pipes, and several pumping stations.

The politics of such large water movements is much more difficult (and much more interesting)! At one point Senator Warren Magnuson of Washington managed to pass a law forbidding the federal government from even *studying* such a transfer.

6.76 Here we can use the result from Ex. 6.18, and the ratio of PP values

$$D_{\text{econ}} = 3.24\,\text{in}\left(\frac{1}{10}\right)^{\frac{1}{6}} = 2.21\ \text{in}$$

Probably we would choose at 2.5 inch pipe. From App. A.3 we can compute

$$\Delta P = \frac{11.68\ \text{psi}}{100\ \text{ft}} \cdot 5000\ \text{ft} = 584\ \text{psi}$$

And the pump horsepower is

$$\text{Po} = Q\Delta P = 200\frac{\text{gal}}{\text{min}} \cdot 584\frac{\text{lbf}}{\text{in}^2} \cdot \frac{\text{ft}^3}{7.48\ \text{gal}} \cdot \frac{\text{hp min}}{33\,000\ \text{ft lbf}} \cdot \frac{144\ \text{in}^2}{\text{ft}^2} = 68.1\ \text{hp}$$

These latter two values are not asked for in the problem statement, but are included here to show the full example.

6.77 The product of the velocity and the cube root of the density (ft/s)(lbm/ft^3)$^{1/3}$ goes from 23.4 to 16.9 as we go from top to bottom of the table. The largest value is 1.40 times the smallest. This is not quite constant, but certainly close.

The reason for the decline seems to be that as the density falls, the product of the density and the economic velocity falls faster than the viscosity falls, so the economic Reynolds number falls, which causes the corresponding friction factor to increase, which raises the economic diameter and lowers the economic velocity. The effect is small, but real.

6.78 The 3.24 inches calculated in Ex. 6.18 corresponds to a velocity of 7.78 ft/s for 200 gpm. From Table 6.8 one would interpolate a velocity of perhaps 6.5 ft/s. From Fig. 6.23 one reads a value of $\approx$ 7 ft/s. These are close to the same.

The disagreement between Fig 6.213 and Table 6.8 results from two factors; Fig 6.21 is for a fluid of specific gravity 0.8, (almost exactly 50 lbm/ft^3) and Fig. 6.28 uses a lower value of the pipe roughness than does Table 6.8.

6.79 We wish to minimize the sum of the mass of the fuel line, the mass of the pump and the mass of the fuel which must be expended to pump that fuel through the line. The mass of the pump is probably practically independent of the pipe diameter, although its mass depends weakly on the pressure it must develop, which is a function of the pipe diameter. Leaving it out of consideration, we can say that

$$m_{\text{pipe}} = \rho_{\text{pipe}} DL \cdot (\text{pipe wall thickness})$$

$$m_{\text{fuel to pump}} = \text{Volume of fuel to pump} \cdot \Delta P_{\text{pipe}} \cdot \text{Heating value of pump fuel} \cdot \eta_{\text{pump}}$$

For a given allowable pressure, the pipe wall thickness is proportional to D and $\Delta P \alpha \dfrac{1}{D^5}$

so that $m_{\text{pipe}} = C_1 D^2$; and $m_{\text{pump fuel}} = \dfrac{C_2}{D^5}$; $m_{\text{total}} = C_1 D^2 + \dfrac{C_2}{D^5}$

$$\frac{dm_{\text{total}}}{dD} = 2C_1 D - \frac{5C_2}{D^6} = 0\,; \qquad D_{\text{minimum combined mass}} = \left[\frac{5C_2}{2C_1}\right]^{\frac{1}{7}}$$

6.80 (a) Following the instructions in the problem $\dfrac{dV}{dt} = -\dfrac{18\mu V}{D^2\rho} = \dfrac{VdV}{dx}$

$\displaystyle\int_{V_0}^{0} dV = -\frac{18\mu}{D^2\rho}\int_0^{x_{\text{Stokes stopping}}} dx$ carrying out the integration and inserting the limits leads to Eq. 6.66. The stokes stopping distance appears often in the fine particle literature.

(b) $$x_{\text{Stokes stopping}} = \frac{V_0 D^2 \rho}{18\,\mu} = \frac{10\dfrac{\text{m}}{\text{s}} \cdot \left(10^{-6}\,\text{m}\right)^2 \cdot 2000\dfrac{\text{kg}}{\text{m}^3}}{18 \cdot 0.018\,\text{cp} \cdot \dfrac{0.001\text{ kg}}{\text{m s cp}}} = 0.061\text{ mm} = 0.002\text{ in}$$

This startlingly small value shows that for particles this small (about the size of air pollution interest) the air is very stiff, and particles come to rest quickly. This is example 8.5 page 225 of Noel de Nevers, "Air Pollution Control Engineering, 2e", McGraw-Hill 2000. There it is shown that for a particle of this size $C \approx 1.12$, so taking the Cunningham correction factor into account raises the computed value by 12%. The behavior of particles small enough to be of air pollution interest is discussed in more detail in that book.

(c) If we return to Eq 6.63, and separate variables and integrate, we find

$$\int_{V_0}^{V} \frac{dV}{V} = -\frac{18\mu}{D^2\rho}\int_0^t dt\,; \qquad \ln\frac{V_0}{V} = \frac{18\mu}{D^2\rho}\cdot t$$

from this we see that $V = 0$ corresponds to infinite time. For 1% of V_0

$$t = \frac{D^2\rho}{18\mu}\cdot\ln\frac{V_0}{V} = \frac{\left(10^{-6}\text{m}\right)^2 \cdot 2000\dfrac{\text{kg}}{\text{m}^3}}{18 \cdot 0.018\,\text{cp} \cdot \dfrac{0.001\text{ kg}}{\text{m s cp}}} \cdot \ln 100 = \frac{x_{\text{Stokes stopping}}}{V_0} \cdot \ln 100 = 2.8 \cdot 10^{-5}\text{ s}$$

While the particle theoretically moves forever, it loses 99% of its initial velocity in 28 microseconds.

(d) From example 6.9 we know that the gravitational settling velocity is 6.05·10^-5 m/s, so

$$\Delta z = V_{\text{terminal, gravity}}\Delta t = 6.05 \cdot 10^{-5}\frac{\text{m}}{\text{s}} \cdot 2.8 \cdot 10^{-5}\text{ s} = 1.7 \cdot 10^{-9}\text{m}$$

This is 3·10^-5 times the stokes stopping distance (for this example) and clearly negligible.

6.81* $$V = \frac{(0.0001\text{ in})^2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(100-62.3)\frac{\text{lbm}}{\text{ft}^3}}{18 \cdot 1.002\,\text{cP} \cdot 6.72 \cdot 10^{-4}\frac{\text{lbm}}{\text{ft s cP}}} \cdot \frac{\text{ft}^2}{144\text{ in}^2} = 6.96 \cdot 10^{-6}\frac{\text{ft}}{\text{s}} = 0.6\frac{\text{ft}}{\text{day}}$$

One may check that $\mathcal{R}_p \approx 5$ e-6, so the stokes law assumption is safe. On the time scale of geology this particles settles rapidly, but on a human time scale it practically doesn't settle.

6.82 As a first trial we assume $C_d = 0.4$. Then, copying from that example;

$$V = \sqrt{\frac{4(0.02\text{ m})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left(7.85-\frac{78.5}{62.3}\right)\left(998.2\frac{\text{kg}}{\text{m}^3}\right)}{3\left(998.2\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{78.5}{62.3}\right)(0.4)}} = 1.84\frac{\text{m}}{\text{s}}$$

$$\mathsf{R}_p = \frac{(0.02\text{m})\left(998.2 \cdot \frac{78.5}{62.3}\right)\left(1.84\frac{\text{m}}{\text{s}}\right)}{800 \cdot 10^{-3}\text{ PaS}} = 58.1$$

From Fig. 6.24 se see that for this particle Reynolds number, $C_d \approx 1.5$. Using the approximate equation in the caption of that figure we find

$$C_d = (24/58.1) \cdot (1 + 0.14 \cdot 58.1^{0.7}) = 1.406$$

which is within our ability to read Fig. 6.24 of the 1.5 above. Then we use our spreadsheet's numerical engine to find the value of C_d in the above equation which makes the assumed and calculated values equal. We find $V = 0.776$ m/s = 2.55 ft/s, $\mathcal{R}_p$ = 24.4 and C_d = 2.27.

Be cautioned that the approximation formula is only applicable to spheres (although the discs curve is very close to the spheres curve) and only in the $\mathcal{R}_p$ range shown.

6.83* $$F = \frac{\pi}{4}D^2\rho C_d\frac{V^2}{2}; \qquad V = \sqrt{\frac{2F}{\frac{\pi}{4}D^2\rho C_d}}$$

As a first trial we assume $C_d = 0.4$

$$V_{\substack{\text{first}\\\text{trial}}} = \sqrt{\frac{2(0.1\text{ lbf})}{\frac{\pi}{4}(10\text{ ft})^2\left(0.075\frac{\text{lbm}}{\text{ft}^3}\right)(0.4)} \cdot \frac{32.2\text{ lbm ft}}{\text{lbf s}^2}} = 1.65\frac{\text{ft}}{\text{s}}$$

$$\mathsf{R}_P = \frac{(10\text{ ft})\left(1.65\frac{\text{ft}}{\text{s}}\right)}{1.613\cdot 10^{-4}\frac{\text{ft}^2}{\text{s}}} = 1.02\cdot 10^5$$ From Fig 6.24 we see that this corresponds to

$C_d \approx 0.5$ so that $V_{\substack{\text{second}\\\text{guess}}} = 1.65\frac{\text{ft}}{\text{s}}\left[\frac{0.4}{0.5}\right]^{1/2} = 1.48\frac{\text{ft}}{\text{s}}$ and $\mathsf{R} = 1.02\cdot 10^5\left(\frac{1.48}{1.65}\right) = 0.92\cdot 10^5$

For this particle Reynolds number, $C_d \approx 0.5$ so we accept the second guess.

6.84 (a) $\mathsf{R}_p = \dfrac{\left(\frac{2.9}{12}\text{ft}\right)\left(\frac{100\cdot 5280}{3600}\frac{\text{ft}}{\text{s}}\right)}{\left(1.613\cdot 10^{-4}\frac{\text{ft}^2}{\text{s}}\right)} = 2.2\cdot 10^5$; From Fig. 6.24 we read

that $C_d \approx 0.5$

$$F = \frac{\pi}{4}D^2\rho C_d\frac{V^2}{2}$$

$$= \frac{\left(\frac{\pi}{4}\right)\left(\frac{2.9}{12}\text{ft}\right)^2\left(0.075\frac{\text{lbm}}{\text{ft}^3}\right)(0.5)\left(\frac{100\cdot 5280}{3600}\frac{\text{ft}}{\text{s}}\right)^2}{2}\cdot\frac{\text{lbf s}^2}{32.2\text{ lbm ft}} = 0.57\text{ lbf} = 2.55\text{ N}$$

(b) $$m\frac{dV}{dt} = mV\frac{dV}{dx} = -F = -\frac{\frac{\pi}{4}D^2\rho C_d}{2}V^2$$

$$\frac{dV}{V} = -\left(\frac{\frac{\pi}{4}D^2\rho C_d}{2\,m}\right)dx;$$

$$\ln\frac{V}{V_0} = -\left(\frac{\frac{\pi}{4}D^2\rho C_D\Delta x}{2\ m}\right) = -\frac{\frac{\pi}{4}\left(\frac{2.9}{12}\text{ft}\right)^2\left(0.075\frac{\text{lbm}}{\text{ft}^3}\right)(0.5)(60\text{ ft})}{(2)(0.32\text{ lbm})} = -0.161$$

$$V = V_0\exp(-0.161) = 85\frac{\text{mi}}{\text{hr}} = 125\frac{\text{ft}}{\text{s}} = 38\frac{\text{m}}{\text{s}}$$

Stitching on the ball, spin, gravity and wind all influence the speed and curvature of the ball's flight path.

6.85 As the ball began its flight $\mathsf{R}_p = \dfrac{36\dfrac{\text{m}}{\text{s}}\cdot 0.223\text{ m}}{1.488\cdot 10^{-5}\dfrac{\text{m}^2}{\text{s}}} = 5.4\cdot 10^5$ From Fig. 6.24 we see that this is very close to the particle Reynolds number at which the C_d curve has a sudden change. At higher particle Reynolds numbers, $C_d \approx 0.1$, at lower particle Reynolds numbers, $C_d \approx 0.5$. If we assume that this transition occurred half-way from the kicker to the goal, we can then compute that at that half-way point (see preceding problem solution) that

$$\ln\frac{V}{V_0} = -\left(\frac{\dfrac{\pi}{4}D^2\rho C_D \Delta x}{2\ m}\right) = -\frac{\dfrac{\pi}{4}(0.223\text{ m})^2\left(1.20\dfrac{\text{kg}}{\text{m}^3}\right)(0.1)(13.5\text{ m})}{(2)(0.425\text{ kh})} = -0.074$$

and $V = 36\dfrac{\text{m}}{\text{s}}\cdot\exp(-0.074) = 33.4\dfrac{\text{m}}{\text{s}}$

For the second half of the ball's flight we substitute $C_d \approx 0.5$, finding

$$\ln\frac{V}{V_0} = -0.372 \text{ and } V = 33.4\frac{\text{m}}{\text{s}}\cdot\exp(-0.372) = 23.0\frac{\text{m}}{\text{s}}$$

In the first half of its flight, the ball lost about 7% of its initial velocity. In the second half it lost about 31% of its remaining velocity. The result was apparently very dramatic.

6.86 The world record speed for the 100 m dash is ≈ 10 m/s, while that for the 100 m freestyle swim is ≈ 2 m/s, so the ratio is ≈ 5. If we assume that the characteristic dimension for both running and swimming is ≈ 1 m, then we can compute the particle Reynolds numbers for both, finding $0.6\cdot 10^6$ and $2\cdot 10^6$. This suggests that for both running and swimming we are to the right of the sudden transition on Fig. 6.24 (but see the preceding problem) so that $C_d \approx 0.1$, for both. This says that we are in the flow regime in which the viscosity plays no role, but the drag force is ≈ proportional to the fluid density. The density of water is ≈ 833 times that of air, so the difference is mostly due to the difference in densities.

The projected areas are quite different, because the swimmer lies flat in the water, while the runner stands mostly upright. If we assume the sprinter has four times the projected area of the swimmer, and assume that the muscular power output is the same for the sprinter and the swimmer and all goes to overcome fluid resistance, then we would expect $\text{Po} = A\Delta PV \propto A\rho V^3$ to be the same for both. Surprisingly the values are quite similar

$$\frac{\text{Po}_{\text{sprinter}}}{\text{Po}_{\text{swimmer}}} = 4\cdot\frac{1.2\dfrac{\text{kg}}{\text{m}^3}}{1000\dfrac{\text{kg}}{\text{m}^3}}\cdot\frac{\left(10\dfrac{\text{m}}{\text{s}}\right)^3}{\left(2\dfrac{\text{m}}{\text{s}}\right)^3} = 0.60$$

This may be a coincidence.. I think that the competitive swimmer is mostly expending muscular energy to overcome water's fluid mechanic resistance, while the sprinter, while sensitive to aerodynamic resistance, is mostly expending energy to overcome the internal resistance of her/his own muscles and joints. If readers have comments on this I would be interested to hear them.

This was actually tested in Aug 2003 ("Swimmers take slimy dive for an experiment at U", Minneapolis Star Tribune, Aug 19, 2003). Professor Ed. Cussler of the U of M had swimmers timed for 25 yards in water and in a swimming pools whose viscosity had been doubled by dissolving guar gum in it. The swimmers had to modify their position, holding their heads out of the water because of the gum. The swimming speeds were the same, within experimental error.

6.87* Here we cannot use Eq. 6.58 because it is specific for a sphere. We derive its equivalent for a general falling body, $m\dfrac{dV}{dt} = 0 = mg - C_d A\rho_{air}\dfrac{V_t^2}{2}$; $\quad V_t = \sqrt{\dfrac{2\text{mg}}{C_d A\rho_{\text{air}}}}$

$$\frac{dV}{dt} = g - \frac{C_d A\rho}{2\,\text{m}}V^2; \quad \frac{dV}{V_t^2 - V^2} = \frac{g}{V_t^2}dt; \quad \frac{1}{2V_t}\ln\frac{(V_t + V)}{(V_t - V)} = \frac{gt}{V_t^2}; \; t = \frac{V_t}{2\text{g}}\ln\left(\frac{V_t + V}{V_t - V}\right)$$

$$\frac{dV}{dt} = V\frac{dV}{dx}; \quad dx = \frac{VdV}{g - \dfrac{C_d A\rho V^2}{2\,\text{m}}}; \quad \frac{g}{V_t^2}dx = \frac{VdV}{V_t^2 - V^2}; \quad \frac{gx}{V_t^2} = -\frac{1}{2}\ln\left[\frac{V^2 - V_t^2}{0 - V_t^2}\right]$$

$$x = \frac{V_t^2}{2g}\ln\frac{V_t^2}{V_t^2 - V^2}$$

(a)
$$V_t = \sqrt{\frac{(2)(120\text{ lbf})}{(0.7)(1\text{ ft}^2)\left(0.075\dfrac{\text{lbm}}{\text{ft}^3}\right)}\cdot\frac{32.2\text{ lbm ft}}{\text{lbf s}^2}} = 384\frac{\text{ft}}{\text{s}}$$

$$t = \frac{V_t}{2\text{g}}\ln\left(\frac{V_t + V}{V_t - V}\right) = \frac{384\dfrac{\text{ft}}{\text{s}}}{2\cdot 32.2\dfrac{\text{ft}}{\text{s}^2}}\ln\frac{1.99\,V_t}{0.01\,V_t} = 31.6\text{ s}$$

$$x = \frac{V_t^2}{2g}\ln\frac{V_t^2}{V_t^2 - V^2} = \frac{\left(384\dfrac{\text{ft}}{\text{s}}\right)^2}{2\cdot 32.2\dfrac{\text{ft}}{\text{s}^2}}\ln\frac{1^2}{1^2 - 0.01^2} = 8954\text{ ft}$$

(b) Repeating the calculation with $A = 6\text{ ft}^2$ and $C_d = 1.5$ leads to 107 ft/s, 8.8 s and 696 ft.

6.88 $\quad V_{\text{allowable}} = \sqrt{2gx} = \sqrt{2\cdot 32.2\dfrac{\text{ft}}{\text{s}^2}\cdot 10\,\text{ft}} = 25.4\dfrac{\text{ft}}{\text{s}} = 7.73\dfrac{\text{m}}{\text{s}}$

See the preceding problem. $V_t = \sqrt{\dfrac{2\,mg}{C_d A \rho_{\text{air}}}}$;

$$A = \frac{2mg}{C_d V_t^2 \rho_{\text{air}}} = \frac{(2)(150\,\text{lbm})\left(32.2\dfrac{\text{ft}}{\text{s}^2}\right)}{(1.5)\left(25.4\dfrac{\text{ft}}{\text{s}}\right)^2\left(0.075\dfrac{\text{lbm}}{\text{ft}^3}\right)} = 133\ \text{ft}^2 = 12.4\ \text{m}^2$$

$$D = \sqrt{\frac{4}{\pi}A} = \sqrt{\frac{4}{\pi}133\ \text{ft}^2} = 13\ \text{ft} = 4.0\ \text{m}$$

6.89 (a)

$$F = A\rho_{\text{air}} C_d \frac{V^2}{2} = \frac{(6\cdot 5)\text{ft}^2 \cdot 0.075\dfrac{lbm}{ft^3}\cdot 0.3\left(\dfrac{70\cdot 5280}{3600}\dfrac{\text{ft}}{s}\right)^2}{2}\cdot\frac{\text{lbf s}^2}{32.2\ \text{lbm ft}} = 110\ \text{lbf}$$

(b) $$\text{Po} = FV = (110\ \text{lbf})\left(\frac{70\cdot 5280}{3600}\frac{\text{ft}}{\text{s}}\right)\cdot\frac{\text{hp s}}{550\ \text{ft lbf}} = 20.6\ \text{hp} = 15.4\ \text{kW}$$

This is not asked for in the problem but can be used for discussion. If this auto gets 30 mi/gal at 70 mph, then the fuel consumption is

$$\begin{pmatrix}\text{fuel}\\ \text{consumption}\end{pmatrix} = \frac{\text{gal}}{30\ \text{mi}}\cdot 70\frac{\text{mi}}{\text{hr}}\cdot 6.0\frac{\text{lb}}{\text{gal}} = 14\frac{\text{lbm}}{\text{hr}}$$

The energy flow in the fuel is $\begin{pmatrix}\text{fuel energy}\\ \text{flow}\end{pmatrix} \approx 14\dfrac{\text{lbm}}{\text{hr}}\cdot 19\ 000\dfrac{\text{Btu}}{\text{lbm}} = 266\ 000\dfrac{\text{Btu}}{\text{hr}}$

If we assume a 30% thermal efficiency for the engine, then

$$\text{Po}_{\text{engine}} \approx 0.3 * 266\ 000\frac{\text{Btu}}{\text{hr}}\cdot\frac{\text{hp hr}}{2545\ \text{Btu}} = 31.4\ \text{hp} = 23.4\ \text{kW}$$

and roughly 2/3 of the power output of the engine is going to overcome air resistance.

6.90 If we guessed Stokes law, and did the calculation, we would find a velocity much higher than the correct value. Then, on evaluating the Reynolds number we would have a higher value than the real value, so we would be certain to conclude we were outside the Stokes law range. Thus the procedure shown in those examples is conservative; it can never lead to an incorrect answer if used as shown.

6.91 In those examples the calculated terminal velocities were $2\cdot 10^{-4}$ and 6.21 ft/s, For Ex. 6.19 for a 1 micron particle with s.g. = 2 setting in air, we read $V_t \approx 2.4\cdot 10^{-4}\ \text{ft/s}$. The difference between the two is partly due to the Cunningham correction factor, see p222 of Noel de Nevers, "Air Pollution Control Engineering, 2e", McGraw-Hill 2000. If we applied it to the result of Example 619 we would have found $V_t \approx 2.2\cdot 10^{-4}\ \text{ft/s}$. The rest of the difference is inaccuracy in making up the chart. For Ex. 6.20 we must interpolate between the s.g. = 5 and =10 lines, and must extrapolate off the right side of the figure to 20,000 microns. Doing so, we find $V_t \approx 6 \text{ ft/s}$ which matches the result in Ex. 6.20.

Fig. 6.26 is really useful; students should become familiar with it.

6.92 (0.001 in = 25.4 microns) From Fig. 6.26 we read directly a velocity of 0.06 ft/s. By direct substitution in Stokes law one finds a velocity of 0.064 ft/s, and a Reynolds number of 0.03. Droplets this small occur in fog and clouds, which settle very slowly. Most drops which we call rain have diameters at least ten times as large. Droplets this size are spherical, while larger ones are deformed from the spherical shape.

6.93 $\frac{dV}{dt} = V\frac{dx}{dt} = -\frac{C_d A \rho V^2}{2m}$; $dx = \frac{-2m}{C_D A\rho}\frac{dV}{V}$

$$\Delta x = \frac{-2m}{C_d A\rho}\ln\frac{V_2}{V_1} = -\frac{2\cdot 0.027 \text{ lbm}}{0.1\cdot\left(\frac{\pi}{4}\right)(0.5 \text{ in})^2\left(62.3\frac{\text{lbm}}{\text{ft}^3}\right)}\cdot\frac{144 \text{ in}^2}{\text{ft}^2}\ln\frac{100}{1000} = 14.5 \text{ ft} = 4.42 \text{ m}$$

My firearms consultant (Ray Cayias) says that pistols and submachine gun bullets are 800 to 1000 ft/s, so this velocity is plausible for the submachine guns normally involved in James Bond movies. Rifles and hunting guns fire 2500-3000 ft/s. He also tells me that if you can see a fish in the water, and you have a rifle, you can kill it with a bullet. You must aim below where you see the fish to correct for the refraction of the image at the air-water interface.

6.94 (a) For zero air resistance $V = V_0 - gt$; $t = \frac{V_0 - V}{a} = \frac{2700\frac{\text{ft}}{\text{s}} - 0}{32.2\frac{\text{ft}}{\text{s}}} = 84 \text{ s}$

$$z = z_0 + \int_0^t V dt = z_0 + \int_0^t (2700 - 32.2t)dt = 0 + 2700\frac{\text{ft}}{\text{s}}t - \frac{32.2\frac{\text{ft}}{\text{s}^2}}{2}t^2$$

$$z = 2700\frac{\text{ft}}{\text{s}}\cdot 84\,s - \frac{32.2\frac{\text{ft}}{\text{s}^2}}{2}(84s)^2 = 113\,000 \text{ ft}$$

With no air resistance, it would take just as long to return to earth, and it would return with its initial velocity.

(b) With air resistance (and an assumed constant drag coefficient)

$$dV = -\left(g + \frac{\pi}{4}D^2\frac{C_d}{m}\rho\frac{V^2}{2}\right)dt = -(a + bV^2)dt$$

where a and b are constants introduced to reduce our bookkeeping. Then

$$\frac{dV}{a+bV^2} = -dt; \qquad \frac{dV}{\frac{a}{b}+V^2} = -bdt$$ this is a standard form I can look up in my

integral tables, finding $$\int_{V_0}^{V} \frac{dV}{\frac{a}{b}+V^2} = \left| \sqrt{\frac{b}{a}} \tan^{-1} V \sqrt{\frac{b}{a}} \right|_{V_0}^{V} = -b(t-t_0)$$ or

$$t = \sqrt{\frac{1}{ab}} \cdot \left(\tan^{-1} V_0 \sqrt{\frac{b}{a}} - \tan^{-1} V \sqrt{\frac{b}{a}} \right)$$ for $V = 0$ this becomes

$$t_{\substack{\text{for top of}\\ \text{trajectory}}} = \sqrt{\frac{1}{ab}} \cdot \tan^{-1}\left(V_0 \sqrt{\frac{b}{a}} \right)$$

The unknown, C_d is buried in b. I doubt that there is an analytical solution of the above equation for a known t. But numerically it is quite easy. First we guess that $C_d = 0.6$. Then, $a = 32.2$ ft/s^2, and

$$b = \frac{\pi}{4} D^2 \frac{C_d}{m} \rho = \frac{\pi}{4} \cdot \left(\frac{0.3\text{ ft}}{12} \right)^2 \frac{0.6}{0.0214\text{ lbm}} \cdot 0.075 \frac{\text{lbm}}{\text{ft}^3} = \frac{0.00051}{\text{ft}}$$

and $$t_{\substack{\text{for top of}\\ \text{trajectory}}} = \sqrt{\frac{1}{ab}} \cdot \tan^{-1}\left(V_0 \sqrt{\frac{b}{a}} \right) = 7.76\text{ s} \cdot \tan^{-1}(10.80) = 11.5\text{ s}$$

We then use the spreadsheet's numerical engine to find the value of C_d which makes this value = 18 s, finding $C_d = 0.225$. We may check this by calculating the height, using this constant value of C_d. We must use

$$z = z_0 + \int_0^t V dt \approx 0 + \sum V_{avg} \Delta t$$

and Euler's first method on a spreadsheet. Thus for the first 0.1 second we compute that

$$V_{0.1\text{ s}} = V_0 - (a + bV^2)\Delta t = 2700 - 1442 \cdot 0.1 = 2555.7 \frac{\text{ft}}{\text{s}}$$

and $$\Delta z = 0.5 \cdot \left(V_0 + V_{0.1\text{ s}} \right) \cdot \Delta t = 262\text{ ft}$$

Continuing this on a spreadsheet (switching to 1 s intervals after 2 s, we find that V becomes zero between 16 and 17 s, at an elevation of 8880 ft, which is an excellent check on the reported values. The initial particle Reynolds number is $4.2 \cdot 10^5$ which is close to the transition on Fig 6.24, so without any data we would assume the drag coefficient would start at ≈ 0.1 and change to ≈ 0.5 during the flight. Thus the 0.225 value is in reasonable accord with what we would estimate from Fig. 6.24.

For going down we cannot use this high a value of the drag coefficient, because there is no supersonic flow. As a first approximation, we assume that the velocity at ground level represents a terminal velocity, and solve for

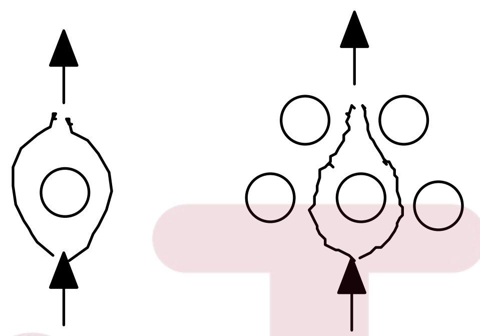
$$mg = \frac{\pi}{4} D^2 C_d \rho \frac{V_{\text{terminal}}^2}{2} ,;$$

$$C_d = \frac{mg \cdot 8}{\pi D^2 \rho V_{\text{terminal}}^2} = \frac{0.021\text{ lbm} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \cdot 8}{\pi \left(\frac{0.3\text{ ft}}{12} \right)^2 \cdot 0.075 \frac{\text{lbm}}{\text{ft}^3} \left(300 \frac{\text{ft}}{\text{s}} \right)^2} = 0.408$$

We can test this value by repeating the above numerical integration (changing the sign on the drag term!). Using 1 s intervals, we find that after 31 s the velocity is 302 ft/s, and the bullet has fallen 7500 ft. This is a reasonable, but not perfect check on the observed 300 ft/s, and 9000 ft in 31 s.

For the downward trip the particle Reynolds number begins at zero, and ends at 46,500, for which we would read a drag coefficient of ≈ 0.45 from Fig 6.24. Thus we could have calculated this part fairly well from Fig. 6.24 alone.

6.95 (a) The sketch is shown at the right. In the left figure the flow is around a single particle, and extends to infinity in all directions. In the right figure there is a regular array of particles, so that the local flow between them must go faster than that around a single particle, and the flow leaving one particle must turn to miss the particle behind it. These two effects cause the net velocity of fluid relative to particles to be smaller.



(b) For the particle by itself, we look up the result on Fig. 6.25 finding 0.0015 ft/s. The term on the right in Eq. 6.65 is $(1-c)^n = (1-0.4)^{4.65} = 0.093$, so that the expected setting velocity is $V_{\text{hindered settling}} = 0.0015\frac{\text{ft}}{\text{s}} \cdot 0.093 = 0.00014\frac{\text{ft}}{\text{s}}$.
