1.1

1.1 Verify the dimensions, in both the *FLT* and *MLT* systems, of the following quantities which appear in Table 1.1: **(a)** volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

(a) volume $\doteq \underline{\underline{L^3}}$

(b) acceleration = time rate of change of velocity

$$\doteq \frac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$$

(c) mass $\doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

mass $\doteq \underline{\underline{FL^{-1}T^{2}}}$

(d) moment of inertia (area) = second moment of area

$$\doteq (L^2)(L^2) \doteq \underline{\underline{L^4}}$$

(e) work = force × distance

$$\doteq \underline{\underline{FL}}$$

or with $F \doteq MLT^{-2}$

work $\doteq \underline{\underline{ML^2T^{-2}}}$

1.2

1.2 Determine the dimensions, in both the *FLT* system and *MLT* system, for **(a)** the product of force times volume, **(b)** the product of pressure times mass divided by area, and **(c)** moment of a force divided by velocity.

(a) force × volume $\doteq (F)(L^3) \doteq \underline{\underline{FL^3}}$

Since $F \doteq MLT^{-2}$

force × volume $\doteq (MLT^{-2})(L^3) \doteq \underline{\underline{ML^4T^{-2}}}$

(b) $$\frac{\text{pressure} \times \text{mass}}{\text{area}} \doteq \frac{(FL^{-2})(M)}{L^2} \doteq \frac{(FL^{-2})(FL^{-1}T^2)}{L^2}$$

$$\doteq \underline{\underline{F^2L^{-5}T^2}}$$

$$\doteq \frac{(MLT^{-2})(L^{-2})(M)}{L^2}$$

$$\doteq \underline{\underline{M^2L^{-3}T^{-2}}}$$

(c) $$\frac{\text{moment of a force}}{\text{velocity}} \doteq \frac{FL}{LT^{-1}} \doteq \underline{\underline{FT}}$$

$$\doteq (MLT^{-2})(T) \doteq \underline{\underline{MLT^{-1}}}$$

1.3

1.3 Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table 1.1: **(a)** acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

(a) $\text{acceleration} = \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \underline{\underline{LT^{-2}}}$

(b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} = \underline{\underline{ML^{-1}T^{-2}}}$

(c) $\text{moment of a force} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

(d) $\text{volume} = (\text{length})^3 \doteq \underline{\underline{L^3}}$

(e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

1.4

1.4 If P is a force and x a length, what are the dimensions (in the FLT system) of **(a)** dP/dx, **(b)** d^3P/dx^3, and **(c)** $\int P\,dx$?

(a) $\frac{dP}{dx} \doteq \frac{F}{L} \doteq \underline{\underline{FL^{-2}}}$

(b) $\frac{d^3P}{dx^3} \doteq \frac{F}{L^3} \doteq \underline{\underline{FL^{-3}}}$

(c) $\int P\,dx \doteq \underline{\underline{FL}}$

1.5

1.5 If p is a pressure, V a velocity, and ρ a fluid density, what are the dimensions (in the MLT system) of (a) p/ρ, (b) $pV\rho$, and (c) $p/\rho V^2$?

(a) $\frac{p}{\rho} \doteq \frac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$

(b) $pV\rho \doteq (ML^{-1}T^{-2})(LT^{-1})(ML^{-3}) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$

(c) $\frac{p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0$ ($\underline{\underline{\text{dimensionless}}}$)

1.6

1.6 If V is a velocity, ℓ a length, and ν a fluid property having dimensions of L^2T^{-1}, which of the following combinations are dimensionless: **(a)** $V\ell\nu$, **(b)** $V\ell/\nu$, **(c)** $V^2\nu$, **(d)** $V/\ell\nu$?

(a) $V\ell\nu \doteq (LT^{-1})(L)(L^2T^{-1}) \doteq L^4T^{-2}$ (not dimensionless)

(b) $\dfrac{V\ell}{\nu} \doteq \dfrac{(LT^{-1})(L)}{(L^2T^{-1})} \doteq L^0T^0$ (dimensionless)

(c) $V^2\nu \doteq (LT^{-1})^2(L^2T^{-1}) \doteq L^4T^{-3}$ (not dimensionless)

(d) $\dfrac{V}{\ell\nu} \doteq \dfrac{(LT^{-1})}{(L)(L^2T^{-1})} \doteq L^{-2}$ (not dimensionless)

1.7

1.7 Determine the dimensions of the coefficients A and B which appear in the dimensionally homogeneous equation

$$\frac{d^2x}{dt^2} + A\frac{dx}{dt} + Bx = 0$$

where x is a length and t is time.

$$\frac{d^2x}{dt^2} + A\frac{dx}{dt} + Bx = 0$$

$$[LT^{-2}] + [A][LT^{-1}] + [B][L] \doteq 0$$

Since each term must have the same dimensions:

$$[A][LT^{-1}] \doteq [LT^{-2}]$$

so that

$$\underline{\underline{A \doteq T^{-1}}}$$

and

$$[B][L] \doteq [LT^{-2}]$$

or

$$\underline{\underline{B \doteq T^{-2}}}$$

1.8

1.8 The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8 \mu \ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $(FL^{-2}T)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] \frac{[L^4][FL^{-2}]}{[FL^{-2}T][L]}$$

$$[L^3 T^{-1}] \doteq \left[\frac{\pi}{8}\right] [L^3 T^{-1}]$$

The constant $\pi/8$ is dimensionless, and the equation is a general homogeneous equation that is valid in any consistent unit system. Yes.

1.9

1.9 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

$$h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$\left[\frac{FL}{F}\right] \doteq \left[0.04 \text{ to } 0.09\right]\left[\frac{L^4}{L^4}\right]\left[\frac{1}{2}\right]\left[\frac{L^2}{T^2}\right]\left[\frac{T^2}{L}\right]$$

$$[L] \doteq [0.04 \text{ to } 0.09][L]$$

Since each term in the equation must have the same dimensions, the constant term (0.04 to 0.09) must be dimensionless. Thus, the equation is a general homogeneous equation that is valid in any system of units. Yes.

1.10

1.10 The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity ($FL^{-2}T$), ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

$$\Delta p = K_v \frac{\mu V}{D} + K_u\left[\frac{A_0}{A_1} - 1\right]^2 \rho V^2$$

$$[FL^{-2}] \doteq [K_v]\left[\left(\frac{FT}{L^2}\right)\left(\frac{L}{T}\right)\left(\frac{1}{L}\right)\right] + [K_u]\left[\frac{(L^2)}{(L^2)} - 1\right]^2 \left[\frac{FT^2}{L^4}\right]\left[\frac{L}{T}\right]^2$$

$$[FL^{-2}] \doteq [K_v][FL^{-2}] + [K_u][FL^{-2}]$$

Since each term must have the same dimensions, K_v and K_u are dimensionless. Thus, the equation is a general homogeneous equation that would be valid in any consistent system of units. Yes.

1.11

1.11 Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_v, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_v)^a(\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See Eq. 1.19.)

$$c = (E_v)^a(\rho)^b$$

Since $c \doteq LT^{-1}$ $\quad E_v \doteq FL^{-2}$ $\quad \rho = FL^{-4}T^2$

$$\left[\frac{L}{T}\right] \doteq \left[\frac{F^a}{L^{-2a}}\right]\left[\frac{F^b T^{2b}}{L^{-4b}}\right] \qquad (1)$$

For a dimensionally homogeneous equation each term in the equation must have the same dimensions. Thus, the right hand side of Eq. (1) must have the dimensions of LT^{-1}. Therefore,

$a + b = 0$ (to eliminate F)

$2b = -1$ (to satisfy condition on T)

$2a + 4b = -1$ (to satisfy condition on L)

It follows that $a = \frac{1}{2}$ and $b = -\frac{1}{2}$

So that

$$c = \sqrt{\frac{E_v}{\rho}}$$

This result is consistent with the standard formula for the speed of sound. Yes.

1.14

1.14 Make use of Table 1.3 to express the following quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s², **(e)** 0.0234 lb·s/ft².

(a) $$10.2\ \frac{in.}{min} = \left(10.2\ \frac{in.}{min}\right)\left(2.540\times10^{-2}\ \frac{m}{in.}\right)\left(\frac{1\ min}{60\ s}\right)$$

$$= 4.32\times10^{-3}\ \frac{m}{s} = \underline{\underline{4.32\ \frac{mm}{s}}}$$

(b) $$4.81\ slugs = \left(4.81\ slugs\right)\left(1.459\times10\ \frac{kg}{slug}\right) = \underline{\underline{70.2\ kg}}$$

(c) $$3.02\ lb = \left(3.02\ lb\right)\left(4.448\ \frac{N}{lb}\right) = \underline{\underline{13.4\ N}}$$

(d) $$73.1\ \frac{ft}{s^2} = \left(73.1\ \frac{ft}{s^2}\right)\left(3.048\times10^{-1}\ \frac{\frac{m}{s^2}}{\frac{ft}{s^2}}\right) = \underline{\underline{22.3\ \frac{m}{s^2}}}$$

(e) $$0.0234\ \frac{lb\cdot s}{ft^2} = \left(0.0234\ \frac{lb\cdot s}{ft^2}\right)\left(4.788\times10\ \frac{\frac{N\cdot s}{m^2}}{\frac{lb\cdot s}{ft^2}}\right)$$

$$= \underline{\underline{1.12\ \frac{N\cdot s}{m^2}}}$$

1.15

1.15 Make use of Table 1.4 to express the following quantities in BG units: **(a)** 14.2 km, **(b)** 8.14 N/m³, **(c)** 1.61 kg/m³, **(d)** 0.0320 N·m/s, **(e)** 5.67 mm/hr.

(a) $14.2\ km = (14.2 \times 10^3\ m)\left(3.281\ \frac{ft}{m}\right) = \underline{\underline{4.66 \times 10^4\ ft}}$

(b) $8.14\ \frac{N}{m^3} = \left(8.14\ \frac{N}{m^3}\right)\left(6.366 \times 10^{-3}\ \frac{\frac{lb}{ft^3}}{\frac{N}{m^3}}\right) = \underline{\underline{5.18 \times 10^{-2}\ \frac{lb}{ft^3}}}$

(c) $1.61\ \frac{kg}{m^3} = \left(1.61\ \frac{kg}{m^3}\right)\left(1.940 \times 10^{-3}\ \frac{\frac{slugs}{ft^3}}{\frac{kg}{m^3}}\right) = \underline{\underline{3.12 \times 10^{-3}\ \frac{slugs}{ft^3}}}$

(d) $0.0320\ \frac{N \cdot m}{s} = \left(0.0320\ \frac{N \cdot m}{s}\right)\left(7.376 \times 10^{-1}\ \frac{\frac{ft \cdot lb}{s}}{\frac{N \cdot m}{s}}\right)$

$= \underline{\underline{2.36 \times 10^{-2}\ \frac{ft \cdot lb}{s}}}$

(e) $5.67\ \frac{mm}{hr} = \left(5.67 \times 10^{-3}\ \frac{m}{hr}\right)\left(3.281\ \frac{ft}{m}\right)\left(\frac{1\ hr}{3600\ s}\right)$

$= \underline{\underline{5.17 \times 10^{-6}\ \frac{ft}{s}}}$

1.16

1.16 Express the following quantities in SI units: (a) 160 acre, (b) 15 gallons (U.S.), (c) 240 miles, (d) 79.1 hp, (e) 60.3 °F.

(a) $160 \text{ acre} = (160 \text{ acre})\left(4.356 \times 10^{4} \frac{ft^2}{acre}\right)\left(9.290 \times 10^{-2} \frac{m^2}{ft^2}\right)$

$= \underline{\underline{6.47 \times 10^{5} m^{2}}}$

(b) $15 \text{ gallons} = (15 \text{ gallons})\left(3.785 \frac{liters}{gallon}\right)\left(10^{-3} \frac{m^3}{liter}\right) = \underline{\underline{56.8 \times 10^{-2} m^{3}}}$

(c) $240 \text{ mi} = (240 \text{ mi})\left(5280 \frac{ft}{mi}\right)\left(3.048 \times 10^{-1} \frac{m}{ft}\right) = \underline{\underline{3.86 \times 10^{5} m}}$

(d) $79.1 \text{ hp} = (79.1 \text{ hp})\left(550 \frac{ft \cdot lb/s}{hp}\right)\left(1.356 \frac{J}{ft \cdot lb}\right) = 5.90 \times 10^{4} \frac{J}{s}$

and $1 \frac{J}{s} = 1 W$ so that

$79.1 \text{ hp} = \underline{\underline{5.90 \times 10^{4} W}}$

(e) $T_c = \frac{5}{9}(60.3°F - 32) = 15.7°C$

$= 15.7°C + 273 = \underline{\underline{289 K}}$

1.17

1.17 Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter (g/m³). Assume that a cumulus cloud occupies a volume of one cubic kilometer, and its liquid water content is 0.2 g/m³. **(a)** What is the volume of this cloud in cubic miles? **(b)** How much does the water in the cloud weigh in pounds?

(a) $\forall olume = 1\,(km)^3 = 10^9\,m^3$

Since $1\,m = 3.281\,ft$

$$\forall olume = \frac{(10^9\,m^3)\left(3.281\,\frac{ft}{m}\right)^3}{\left(5.280\times10^3\,\frac{ft}{mi}\right)^3}$$

$$= \underline{\underline{0.240\,mi^3}}$$

(b) $\mathcal{W} = \gamma \times \forall olume$

$$\gamma = \rho g = \left(0.2\,\frac{g}{m^3}\right)\left(10^{-3}\,\frac{kg}{g}\right)\left(9.81\,\frac{m}{s^2}\right) = 1.962\times10^{-3}\,\frac{N}{m^3}$$

$$\mathcal{W} = \left(1.962\times10^{-3}\,\frac{N}{m^3}\right)\left(10^9\,m^3\right) = 1.962\times10^6\,N$$

$$= \left(1.962\times10^6\,N\right)\left(2.248\times10^{-1}\,\frac{lb}{N}\right) = \underline{\underline{4.41\times10^5\,lb}}$$

1.18

1.18 For Table 1.3 verify the conversion relationships for: **(a)** area, **(b)** density, **(c)** velocity, and **(d)** specific weight. Use the basic conversion relationships: 1 ft = 0.3048 m; 1 lb = 4.4482 N; and 1 slug = 14.594 kg.

(a) $$1\ ft^2 = (1\ ft^2)\left[(0.3048)^2 \frac{m^2}{ft^2}\right] = 0.09290\ m^2$$

Thus, multiply ft^2 by 9.290 E−2 to convert to m^2.

(b) $$1\ \frac{slug}{ft^3} = \left(1\ \frac{slug}{ft^3}\right)\left(14.594\ \frac{kg}{slug}\right)\left[\frac{1\ ft^3}{(0.3048)^3 m^3}\right]$$

$$= 515.4\ \frac{kg}{m^3}$$

Thus, multiply $slugs/ft^3$ by 5.154 E+2 to convert to kg/m^3.

(c) $$1\ \frac{ft}{s} = \left(1\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = 0.3048\ \frac{m}{s}$$

Thus, multiply ft/s by 3.048 E−1 to convert to m/s.

(d) $$1\ \frac{lb}{ft^3} = \left(1\ \frac{lb}{ft^3}\right)\left(4.4482\ \frac{N}{lb}\right)\left[\frac{1\ ft^3}{(0.3048)^3 m^3}\right]$$

$$= 157.1\ \frac{N}{m^3}$$

Thus, multiply lb/ft^3 by 1.571 E+2 to convert to N/m^3.

1.19

1.19 For Table 1.4 verify the conversion relationships for: **(a)** acceleration, **(b)** density, **(c)** pressure, and **(d)** volume flowrate. Use the basic conversion relationships: 1 m = 3.2808 ft; 1 N = 0.22481 lb; and 1 kg = 0.068521 slug.

(a) $$1\ \frac{m}{s^2} = \left(1\ \frac{m}{s^2}\right)\left(3.2808\ \frac{ft}{m}\right) = 3.281\ \frac{ft}{s^2}$$

Thus, multiply m/s^2 by $\underline{\underline{3.281}}$ to convert to ft/s^2.

(b) $$1\ \frac{kg}{m^3} = \left(1\ \frac{kg}{m^3}\right)\left(0.068521\ \frac{slugs}{kg}\right)\left[\frac{1\ m^3}{(3.2808)^3\ ft^3}\right]$$

$$= 1.940 \times 10^{-3}\ \frac{slugs}{ft^3}$$

Thus, multiply kg/m^3 by $\underline{\underline{1.940\text{ E-3}}}$ to convert to $slugs/ft^3$.

(c) $$1\ \frac{N}{m^2} = \left(1\ \frac{N}{m^2}\right)\left(0.22481\ \frac{lb}{N}\right)\left[\frac{1\ m^2}{(3.2808)^2\ ft^2}\right]$$

$$= 2.089 \times 10^{-2}\ \frac{lb}{ft^2}$$

Thus, multiply N/m^2 by $\underline{\underline{2.089\text{ E-2}}}$ to convert to lb/ft^2.

(d) $$1\ \frac{m^3}{s} = \left(1\ \frac{m^3}{s}\right)\left[(3.2808)^3\ \frac{ft^3}{m^3}\right] = 35.31\ \frac{ft^3}{s}$$

Thus, multiply m^3/s by $\underline{\underline{3.531\text{ E+1}}}$ to convert to ft^3/s.

1.20

1.20 Water flows from a large drainage pipe at a rate of 1200 gal/min. What is this volume rate of flow in (a) m^3/s, (b) liters/min, and (c) ft^3/s?

(a)

$$\text{flowrate} = \left(1200\ \frac{\text{gal}}{\text{min}}\right)\left(6.309\times10^{-5}\ \frac{\frac{m^3}{s}}{\frac{\text{gal}}{\text{min}}}\right)$$

$$= \underline{\underline{7.57\times10^{-2}\ \frac{m^3}{s}}}$$

(b) Since 1 liter $= 10^{-3}\ m^3$,

$$\text{flowrate} = \left(7.57\times10^{-2}\ \frac{m^3}{s}\right)\left(\frac{10^3\ \text{liters}}{m^3}\right)\left(\frac{60\,s}{\text{min}}\right)$$

$$= \underline{\underline{4540\ \frac{\text{liters}}{\text{min}}}}$$

(c) $$\text{flowrate} = \left(7.57\times10^{-2}\ \frac{m^3}{s}\right)\left(3.531\times10\ \frac{\frac{ft^3}{s}}{\frac{m^3}{s}}\right)$$

$$= \underline{\underline{2.67\ \frac{ft^3}{s}}}$$

1.21

1.21 A tank of oil has a mass of 25 slugs. **(a)** Determine its weight in pounds and in newtons at the earth's surface. **(b)** What would be its mass (in slugs) and its weight (in pounds) if located on the moon's surface where the gravitational attraction is approximately one-sixth that at the earth's surface?

(a) weight = mass × g

$$= (25\ \text{slugs})\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right) = \underline{\underline{805\ \text{lb}}}$$

$$= (25\ \text{slugs})\left(14.59\ \frac{\text{kg}}{\text{slug}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = \underline{\underline{3580\ \text{N}}}$$

(b) mass = $\underline{\underline{25\ \text{slugs}}}$ (mass does not depend on gravitational attraction)

$$\text{weight} = (25\ \text{slugs})\left(\frac{32.2\ \frac{\text{ft}}{\text{s}^2}}{6}\right) = \underline{\underline{134\ \text{lb}}}$$

1.22

1.22 A certain object weighs 300 N at the earth's surface. Determine the mass of the object (in kilograms) and its weight (in newtons) when located on a planet with an acceleration of gravity equal to 4.0 ft/s^2.

$$\text{mass} = \frac{\text{weight}}{g}$$

$$= \frac{300\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}} = \underline{\underline{30.6\ \text{kg}}}$$

For $g = 4.0\ \text{ft/s}^2$,

$$\text{weight} = (30.6\ \text{kg})\left(4.0\ \frac{\text{ft}}{\text{s}^2}\right)\left(0.3048\ \frac{\text{m}}{\text{ft}}\right)$$

$$= \underline{\underline{37.3\ \text{N}}}$$

1.23

1.23 An important dimensionless parameter in certain types of fluid flow problems is the *Froude number* defined as $V/\sqrt{g\ell}$, where V is a velocity, g the acceleration of gravity, and ℓ a length. Determine the value of the Froude number for $V = 10$ ft/s, $g = 32.2$ ft/s², and $\ell = 2$ ft. Recalculate the Froude number using SI units for V, g, and ℓ. Explain the significance of the results of these calculations.

In BG units,

$$\frac{V}{\sqrt{g\ell}} = \frac{10\ \frac{ft}{s}}{\sqrt{\left(32.2\ \frac{ft}{s^2}\right)(2\ ft)}} = \underline{\underline{1.25}}$$

In SI units:

$$V = \left(10\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = 3.05\ \frac{m}{s}$$

$$g = 9.81\ \frac{m}{s^2}$$

$$\ell = (2\ ft)\left(0.3048\ \frac{m}{ft}\right) = 0.610\ m$$

Thus,

$$\frac{V}{\sqrt{g\ell}} = \frac{3.05\ \frac{m}{s}}{\sqrt{\left(9.81\ \frac{m}{s^2}\right)(0.610\ m)}} = \underline{\underline{1.25}}$$

The value of a dimensionless parameter is independent of the unit system.

1.24

1.24 The specific gravity of mercury at 80 °C is 13.4. Determine its density and specific weight at this temperature. Express your answer in both BG and SI units.

$$\rho = SG \times \rho_{H_2O\,@\,4°C} \qquad \gamma = \rho g$$

In BG units

$$\rho = 13.4\left(1.94\ \frac{slugs}{ft^3}\right) = \underline{\underline{26.0\ \frac{slugs}{ft^3}}}$$

$$\gamma = \left(26.0\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right) = \underline{\underline{837\ \frac{lb}{ft^3}}}$$

In SI units:

$$\rho = 13.4\left(1000\ \frac{kg}{m^3}\right) = \underline{\underline{13.4 \times 10^3\ \frac{kg}{m^3}}}$$

$$\gamma = \left(13.4 \times 10^3\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right) = \underline{\underline{131\ \frac{kN}{m^3}}}$$

1.25

1.25 A *hydrometer* is used to measure the specific gravity of liquids. (See **Video V2.6.**) For a certain liquid a hydrometer reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight? Express your answer in SI units.

$$SG = \frac{\rho}{\rho_{H_2O\,@\,4°C}}$$

$$1.15 = \frac{\rho}{1000\ \frac{kg}{m^3}}$$

$$\rho = (1.15)\left(1000\ \frac{kg}{m^3}\right) = \underline{\underline{1150\ \frac{kg}{m^3}}}$$

$$\gamma = \rho g = \left(1150\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right) = \underline{\underline{11.3\ \frac{kN}{m^3}}}$$

1.26

1.26

1.26 An open, rigid-walled, cylindrical tank contains 4 ft^3 of water at 40 °F. Over a 24-hour period of time the water temperature varies from 40 °F to 90 °F. Make use of the data in Appendix B to determine how much the volume of water will change. For a tank diameter of 2 ft, would the corresponding change in water depth be very noticeable? Explain.

mass of water $= \forall \times \rho$

where $\forall$ is the volume and ρ the density. Since the mass must remain constant as the temperature changes

$$\forall_{40^\circ} \times \rho_{40^\circ} = \forall_{90^\circ} \times \rho_{90^\circ} \qquad (1)$$

From Table B.1 $\rho_{H_2O\,@\,40^\circ F} = 1.940 \dfrac{slugs}{ft^3}$

$$\rho_{H_2O\,@\,90^\circ F} = 1.931 \frac{slugs}{ft^3}$$

Therefore, from Eq. (1)

$$\forall_{90^\circ} = \frac{(4\,ft^3)\left(1.940 \frac{slugs}{ft^3}\right)}{1.931 \frac{slugs}{ft^3}} = 4.0186\ ft^3$$

Thus, the increase in volume is

$$4.0186 - 4.000 = \underline{\underline{0.0186\ ft^3}}$$

The change in water depth, $\Delta \ell$, is equal to

$$\Delta \ell = \frac{\Delta \forall}{area} = \frac{0.0186\ ft^3}{\frac{\pi}{4}(2ft)^2} = 5.92 \times 10^{-3} ft = 0.0710\ in.$$

This small change in depth would not be very noticeable. $\underline{\underline{No}}$.

Note: A slightly different value for $\Delta \ell$ will be obtained if specific weight of water is used rather than density. This is due to the fact that there is some uncertainty in the fourth significant figure of these two values, and the solution is sensitive to this uncertainty.

1.28

1.28 A beaker contains 10 in.^{3} of pure glycerin. If 2 in.^{3} of water is added to the glycerine, what is the specific gravity of the mixture?

$$\text{density of mixture} = \frac{\rho_{gly} \times (\text{volume})_{gly} + \rho_{H_2O} \times (\text{volume})_{H_2O}}{(\text{volume})_{gly} + (\text{volume})_{H_2O}}$$

$$= \frac{\left[\left(2.44\ \frac{slugs}{ft^3}\right)(10\ in.^3) + \left(1.94\ \frac{slugs}{ft^3}\right)(2\ in.^3)\right]\left(\frac{1\ ft^3}{1728\ in.^3}\right)}{(10\ in.^3 + 2\ in.^3)\left(\frac{1\ ft^3}{1728\ in.^3}\right)}$$

$$= 2.36\ \frac{slugs}{ft^3}$$

$$SG = \frac{\rho}{\rho_{H_2O @ 4°C}} = \frac{2.36\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}} = \underline{\underline{1.22}}$$

1.29

1.29 The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}} \qquad (1)$$

total weight = mass x g = $(0.369\,kg)(9.81\,\frac{m}{s^2}) = 3.62\,N$

weight of can = $0.153\,N$

Volume of fluid = $(355 \times 10^{-3} L)(10^{-3} \frac{m^3}{L}) = 355 \times 10^{-6}\,m^3$

Thus, from Eq. (1)

$$\gamma = \frac{3.62\,N - 0.153\,N}{355 \times 10^{-6}\,m^3} = \underline{\underline{9770\,\frac{N}{m^3}}}$$

$$\rho = \frac{\gamma}{g} = \frac{9770\,\frac{N}{m^3}}{9.81\,\frac{m}{s^2}} = 996\,\frac{N \cdot s^2}{m^4} = \underline{\underline{996\,\frac{kg}{m^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O@4°C}} = \frac{996\,\frac{kg}{m^3}}{1000\,\frac{kg}{m^3}} = \underline{\underline{0.996}}$$

For water at 20°C (see Table B.2 in Appendix B)

$$\gamma_{H_2O} = 9789\,\frac{N}{m^3}\,; \quad \rho_{H_2O} = 998.2\,\frac{kg}{m^3}\,; \quad SG = 0.9982$$

A comparison of these values for water with those for the pop shows that the specific weight, density, and specific gravity of the pop are all slightly lower than the corresponding values for water.

1.30 *

*1.30 The variation in the density of water, ρ, with temperature, T, in the range 20 °C ≤ T ≤ 50 °C, is given in the following table.

Density (kg/m^3)	998.2	997.1	995.7	994.1	992.2	990.2	988.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form $\rho = c_1 + c_2T + c_3T^2$ which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at 42.1 °C?

Fit the data to a second order polynomial using a standard curve-fitting program such as found in EXCEL. Thus,

$$\rho = 1001 - 0.0533T - 0.0041T^2 \qquad (1)$$

As shown in the table below, ρ (predicted) from Eq. (1) is in good agreement with ρ (given).

T, °C	ρ, kg/m^3	ρ, Predicted
20	998.2	998.3
25	997.1	997.1
30	995.7	995.7
35	994.1	994.1
40	992.2	992.3
45	990.2	990.3
50	988.1	988.1

At $T = 42.1\ °C$

$$\rho = 1001 - 0.0533\,(42.1\,°C) - 0.0041\,(42.1\,°C)^2 = \underline{991.5\ \frac{kg}{m^3}}$$

1.31

1.31 If 1 cup of cream having a density of 1005 kg/m^3 is turned into 3 cups of whipped cream, determine the specific gravity and specific weight of the whipped cream.

Mass of cream, $m = \left(1005 \frac{kg}{m^3}\right) \times \left(\forall_{cup}\right)$

where $\forall$ ~ volume.

Since $m_{cream} = m_{\substack{whipped \\ cream}}$

$$\rho_{\substack{whipped \\ cream}} = \frac{m_{\substack{whipped \\ cream}}}{\forall_{3\ cups}} = \frac{\left(1005 \frac{kg}{m^3}\right) \forall_{cup}}{\forall_{3\ cups}}$$

$$= \frac{1005 \frac{kg}{m^3}}{3} = 335 \frac{kg}{m^3}$$

$$SG = \frac{\rho_{\substack{whipped \\ cream}}}{\rho_{H_2O\ @\ 4^\circ C}} = \frac{335 \frac{kg}{m^3}}{1000 \frac{kg}{m^3}} = \underline{\underline{0.335}}$$

$$\gamma_{\substack{whipped \\ cream}} = \rho_{\substack{whipped \\ cream}} \times g = \left(335 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)$$

$$= \underline{\underline{3290 \frac{N}{m^3}}}$$

1.32

1.32 The density of nitrogen contained in a tank is 1.5 kg/m^3 when the temperature is 25° C. Determine the gage pressure of the gas if the atmospheric pressure is 97 kPa.

$$p = \rho R T = \left(1.5\,\frac{kg}{m^3}\right)\left(296.8\,\frac{J}{kg\cdot K}\right)\left[(25^\circ C + 273)K\right]$$

$$= 133\ kPa\ (abs)$$

$$p\,(gage) = p_{abs} - p_{atm} = 133\,kPa - 97\,kPa = \underline{\underline{36\ kPa}}$$

1.33

1.33 The temperature and pressure at the surface of Mars during a Martian spring day were determined to be −50 °C and 900 Pa, respectively. **(a)** Determine the density of the Martian atmosphere for these conditions if the gas constant for the Martian atmosphere is assumed to be equivalent to that of carbon dioxide. **(b)** Compare the answer from part (a) with the density of the earth's atmosphere during a spring day when the temperature is 18 °C and the pressure 101.6 kPa (abs).

(a) $$\rho_{Mars} = \frac{p}{RT} = \frac{900\,\frac{N}{m^2}}{\left(188.9\,\frac{J}{kg\cdot K}\right)\left[(-50^\circ C + 273)K\right]} = \underline{\underline{0.0214\,\frac{kg}{m^3}}}$$

(b) $$\rho_{earth} = \frac{p}{RT} = \frac{101.6\times10^3\,\frac{N}{m^2}}{\left(286.9\,\frac{J}{kg\cdot K}\right)\left[(18^\circ C + 273)K\right]} = 1.22\,\frac{kg}{m^3}$$

Thus,

$$\frac{\rho_{Mars}}{\rho_{earth}} = \frac{0.0214\,\frac{kg}{m^3}}{1.22\,\frac{kg}{m^3}} = 0.0175 = \underline{\underline{1.75\%}}$$

1.34

1.34

1.34 A closed tank having a volume of 2 ft^3 is filled with 0.30 lb of a gas. A pressure gage attached to the tank reads 12 psi when the gas temperature is 80 °F. There is some question as to whether the gas in the tank is oxygen or helium. Which do you think it is? Explain how you arrived at your answer.

Density of gas in tank $\rho = \dfrac{\text{weight}}{g \times \text{volume}} = \dfrac{0.30 \text{ lb}}{\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)(2 \text{ ft}^3)}$

$= 4.66 \times 10^{-3} \dfrac{\text{slugs}}{\text{ft}^3}$

Since $\rho = \dfrac{p}{RT}$ with $p = (12 + 14.7)$ psia

(atmospheric pressure assumed to be $\approx$ 14.7 psia)

and with $T = (80°\text{F} + 460)\,°\text{R}$ it follows that

$$\rho = \frac{\left(26.7 \frac{\text{lb}}{\text{in.}^2}\right)\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)}{R\,(540°\text{R})} = \frac{7.12}{R} \frac{\text{slugs}}{\text{ft}^3} \qquad (1)$$

From Table 1.7 $R = 1.554 \times 10^3$ for oxygen

and $R = 1.242 \times 10^4 \dfrac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot °\text{R}}$ for helium.

Thus, from Eq. (1) if the gas is oxygen

$$\rho = \frac{7.12}{1.554 \times 10^3} \frac{\text{slugs}}{\text{ft}^3} = 4.58 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}$$

and for helium

$$\rho = \frac{7.12}{1.242 \times 10^4} = 5.73 \times 10^{-4} \frac{\text{slugs}}{\text{ft}^3}$$

A comparison of these values with the actual density of the gas in the tank indicates that the gas must be oxygen.

1.36

1.36 A tire having a volume of 2.5 ft³ contains air at a gage pressure of 30 psi and a temperature of 70 °F. Determine the density of the air and the weight of the air contained in the tire.

$$\rho = \frac{p}{RT} = \frac{\left(30\,\frac{lb}{in.^2} + 14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1716\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)\left[\left(70^\circ F + 460\right)^\circ R\right]} = \underline{\underline{7.08 \times 10^{-3}\,\frac{slugs}{ft^3}}}$$

$$\text{weight} = \rho g \times \text{volume} = \left(7.08 \times 10^{-3}\,\frac{slugs}{ft^3}\right)\left(32.2\,\frac{ft}{s^2}\right)\left(2.5\,ft^3\right)$$

$$= \underline{\underline{0.570\ lb}}$$

1.37

1.37 A rigid tank contains air at a pressure of 90 psia and a temperature of 60 °F. By how much will the pressure increase as the temperature is increased to 110 °F?

$$p = \rho R T \qquad \text{(Eq. 1.8)}$$

For a rigid closed tank the air mass and volume are constant so ρ = constant. Thus, from Eq. 1.8 (with R constant)

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad (1)$$

where $p_1 = 90 \text{ psia}$, $T_1 = 60°F + 460 = 520°R$, and $T_2 = 110°F + 460 = 570°R$. From Eq. (1)

$$p_2 = \frac{T_2}{T_1} p_1 = \left(\frac{570°R}{520°R}\right)(90 \text{ psia}) = \underline{\underline{98.7 \text{ psia}}}$$

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1.38*

*1.38 Develop a computer program for calculating the density of an ideal gas when the gas pressure in pascals (abs), the temperature in degrees Celsius, and the gas constant in J/kg · K are specified.

For an ideal gas

$$p = \rho R T$$

so that

$$\rho = \frac{p}{RT}$$

where p is absolute pressure, R the gas constant, and T is absolute temperature. Thus, if the temperature is in °C then

$$T = {}^\circ C + 273.15$$

A spreadsheet (EXCEL) program for calculating ρ follows.

This program calculates the density of an ideal gas					
when the absolute pressure in Pascals, the temperature					
in degrees C, and the gas constant in J/kg-K are specified.					
To use, replace current values with desired values of					
temperature, pressure, and gas constant.					
A	B	C	D		
Pressure,	Temperature,	Gas constant,	Density,		
Pa	°C	J/kg·K	kg/m³		
1.01E+05	15	286.9	1.23	Row 10	
			Formula:		
			=A10/((B10+273.15)*C10)		

Example: Calculate ρ for $p = 200\ kPa$, temperature = $20°C$, and $R = 287\ J/kg \cdot K$.

A	B	C	D		
Pressure,	Temperature,	Gas constant,	Density,		
Pa	°C	J/kg·K	kg/m³		
2.00E+05	20	287	2.38	Row 10	

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1.39*

*1.39 Repeat Problem 1.38 for the case in which the pressure is given in psi (gage), the temperature in degrees Fahrenheit, and the gas constant in ft·lb/slug·°R.

For an ideal gas

$$p = \rho R T$$

so that

$$\rho = \frac{p}{RT}$$

where p is absolute pressure, and T is absolute temperature. Thus, if temperature in °F, and pressure in psi, then

$$T = {}^\circ F + 459.67 \quad \text{and} \quad p = \left[p(\text{psi}) + p_{atm}(\text{psia}) \right] \times 144 \frac{\text{in.}^2}{\text{ft}^2}$$

A spreadsheet (EXCEL) program for calculating ρ follows.

This program calculates the density of an ideal gas when the gage pressure in psi, the atmospheric pressure in psia, the temperature in degrees F, and the gas constant in ft·lb/slug·deg R are specified.
To use, replace current values with desired values of gage pressure, atmospheric pressure, temperature, and gas constant.

A	B	C	D	E	
Pressure,	Temperature,	Gas constant,	Atm. Pressure,	Density,	
psi	°F	ft lb/slug·°F	psia	slugs/ft³	
0	59	1716	14.7	0.00238	Row 12

Formula:
=((A12+D12)*144)/((C12)*(B12+459.67))

Example: Calculate ρ for $p = 40$ psi, temperature $= 100°F$, $p_{atm} = 14.7$ psia, and $R = 1716$ ft·lb/slug·°R.

A	B	C	D	E	
Pressure,	Temperature,	Gas constant,	Atm. Pressure,	Density,	
psi	°F	ft lb/slug °F	psia	slugs/ft³	
40	100	1716	14.7	0.00820	Row 12

1.40

1.40 Make use of the data in Appendix B to determine the dynamic viscosity of glycerin at 85 °F. Express your answer in both SI and BG units.

$$T_C = \frac{5}{9}\left(T_F - 32\right) = \frac{5}{9}\left(85°F - 32\right) = 29.4\,°C$$

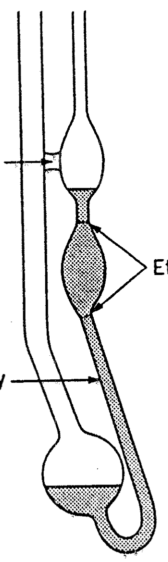
From Fig. B.1 in Appendix B:

$$\mu\,(\text{glycerin at } 85°F\,(29.4°C)) \approx \underline{\underline{0.6\ \frac{N \cdot s}{m^2}}} \quad (\text{SI units})$$

$$\mu \approx \left(0.6\ \frac{N \cdot s}{m^2}\right)\left(2.089 \times 10^{-2}\ \frac{\frac{lb \cdot s}{ft^2}}{\frac{N \cdot s}{m^2}}\right) \approx \underline{\underline{1.3 \times 10^{-2}\ \frac{lb \cdot s}{ft^2}}} \quad (\text{BG units})$$

1.41

1.41 One type of *capillary-tube viscometer* is shown in **Video V1.3** and in Fig. P1.41. For this device the liquid to be tested is drawn into the tube to a level above the top etched line. The time is then obtained for the liquid to drain to the bottom etched line. The kinematic viscosity, ν, in m^2/s is then obtained from the equation $\nu = KR^4t$ where K is a constant, R is the radius of the capillary tube in mm, and t is the drain time in seconds. When glycerin at 20°C is used as a calibration fluid in a particular viscometer the drain time is 1,430 s. When a liquid having a density of 970 kg/m^3 is tested in the same viscometer the drain time is 900 s. What is the dynamic viscosity of this liquid?



■ FIGURE P1.41

$$\nu = KR^4t$$

For glycerin @ 20°C $\nu = 1.19 \times 10^{-3} \, m^2/s$

$$\therefore \; 1.19 \times 10^{-3} \, m^2/s = (KR^4)(1{,}430 \, s)$$

$$KR^4 = 8.32 \times 10^{-7} \, m^2/s^2$$

For unknown liquid with $t = 900s$

$$\nu = (8.32 \times 10^{-7} \, m^2/s^2)(900 \, s)$$

$$= 7.49 \times 10^{-4} \, m^2/s$$

Since $\mu = \rho \nu$

$$= (970 \, kg/m^3)(7.49 \times 10^{-4} \, m^2/s)$$

$$= 0.727 \, \frac{kg}{m \cdot s} = \underline{\underline{0.727 \, \frac{N \cdot s}{m^2}}}$$

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1.42

1.42 The viscosity of a soft drink was determined by using a capillary tube viscometer similar to that shown in Fig. P1.41 and Video V1.3. For this device the kinematic viscosity, ν, is directly proportional to the time, t, that it takes for a given amount of liquid to flow through a small capillary tube. That is, $\nu = Kt$. The following data were obtained from regular pop and diet pop. The corresponding measured specific gravities are also given. Based on these data, by what percent is the absolute viscosity, μ, of regular pop greater than that of diet pop?

	Regular pop	Diet pop
t(s)	377.8	300.3
SG	1.044	1.003

$$\% \text{ greater} = \left[\frac{\mu_{reg} - \mu_{diet}}{\mu_{diet}}\right] \times 100 = \left[\frac{\mu_{reg}}{\mu_{diet}} - 1\right] \times 100$$

Since $\nu = \mu/\rho$, $\nu = kt$, and $\rho = (SG)\rho_{H_2O\,@\,4°C}$ it follows that

$$\% \text{ greater} = \left[\frac{(\nu\rho)_{reg}}{(\nu\rho)_{diet}} - 1\right] \times 100$$

$$= \left[\frac{(t \times SG)_{reg}}{(t \times SG)_{diet}} - 1\right] \times 100$$

$$= \left[\frac{(377.8\text{ s})(1.044)}{(300.3\text{ s})(1.003)} - 1\right] \times 100$$

$$= \underline{\underline{31.0\%}}$$

1.43

1.43 The time, t, it takes to pour a liquid from a container depends on several factors, including the kinematic viscosity, ν, of the liquid. (See Video V1.1.) In some laboratory tests various oils having the same density but different viscosities were poured at a fixed tipping rate from small 150 ml beakers. The time required to pour 100 ml of the oil was measured, and it was found that an approximate equation for the pouring time in seconds was $t = 1 + 9 \times 10^2\nu + 8 \times 10^3\nu^2$ with ν in m²/s. **(a)** Is this a general homogeneous equation? Explain. **(b)** Compare the time it would take to pour 100 ml of SAE 30 oil from a 150 ml beaker at 0°C to the corresponding time at a temperature of 60°C. Make use of Fig. B.2 in Appendix B for viscosity data.

(a) $$t = 1 + 9\times 10^2 \nu + 8\times 10^3 \nu^2 \qquad (1)$$

$$[T] \doteq [1] + [9\times 10^2]\left[\frac{L^2}{T}\right] + [8\times 10^3]\left[\frac{L^4}{T^2}\right]$$

Since each term in the equation must have the same dimensions the constants appearing in the equation must have dimensions, i.e.,

$$[1] \doteq [T] \qquad [9\times 10^2] \doteq \left[\frac{T^2}{L^2}\right] \qquad [8\times 10^3] \doteq \left[\frac{T^3}{L^4}\right]$$

Thus, with a change in units the value of the constants would change and this is not a general homogeneous equation. $\underline{\underline{No}}$.

(b) From Table B.2 in Appendix B:

(for SAE30 oil @ 0°C) $\nu = 2.3\times 10^{-3}\ m^2/s$

(for SAE30 oil @ 60°C) $\nu = 4.0\times 10^{-5}\ m^2/s$

Thus, from Eq. (1)

@ 0°C $\quad t = 1 + 9\times 10^2(2.3\times 10^{-3}) + 8\times 10^3(2.3\times 10^{-3})^2$

$= \underline{\underline{3.11\ s}}$

@ 60°C $\quad t = 1 + 9\times 10^2(4.0\times 10^{-5}) + 8\times 10^3(4.0\times 10^{-5})^2$

$= \underline{\underline{1.04\ s}}$

1.44

1.44 The viscosity of a certain fluid is 5×10^{-4} poise. Determine its viscosity in both SI and BG units.

From Appendix E $10^{-1}\ \frac{N \cdot s}{m^2} = 1$ poise. Thus,

$$\mu = \left(5 \times 10^{-4}\ \text{poise}\right)\left(\frac{10^{-1}\ \frac{N \cdot s}{m^2}}{\text{poise}}\right) = \underline{\underline{5 \times 10^{-5}\ \frac{N \cdot s}{m^2}}}$$

and From Table 1.4

$$\mu = \left(5 \times 10^{-5}\ \frac{N \cdot s}{m^2}\right)\left(\frac{2.089 \times 10^{-2}\ \frac{lb \cdot s}{ft^2}}{\frac{N \cdot s}{m^2}}\right) = \underline{\underline{10.4 \times 10^{-7}\ \frac{lb \cdot s}{ft^2}}}$$

1.45

1.45 The kinematic viscosity of oxygen at 20 °C and a pressure of 150 kPa (abs) is 0.104 stokes. Determine the dynamic viscosity of oxygen at this temperature and pressure.

$$\mu = \nu \rho$$

$$\rho = \frac{p}{RT} = \frac{150 \times 10^3\ \frac{N}{m^2}}{\left(259.8\ \frac{J}{kg \cdot K}\right)\left[\left(20^\circ C + 273\right)K\right]} = 1.97\ \frac{kg}{m^3}$$

$$\nu = 0.104\ \text{stokes} = 0.104\ \frac{cm^2}{s}$$

$$\mu = \left(0.104\ \frac{cm^2}{s}\right)\left(10^{-4}\ \frac{m^2}{cm^2}\right)\left(1.97\ \frac{kg}{m^3}\right)$$

$$= 2.05 \times 10^{-5}\ \frac{kg}{m \cdot s} = \underline{\underline{2.05 \times 10^{-5}\ \frac{N \cdot s}{m^2}}}$$

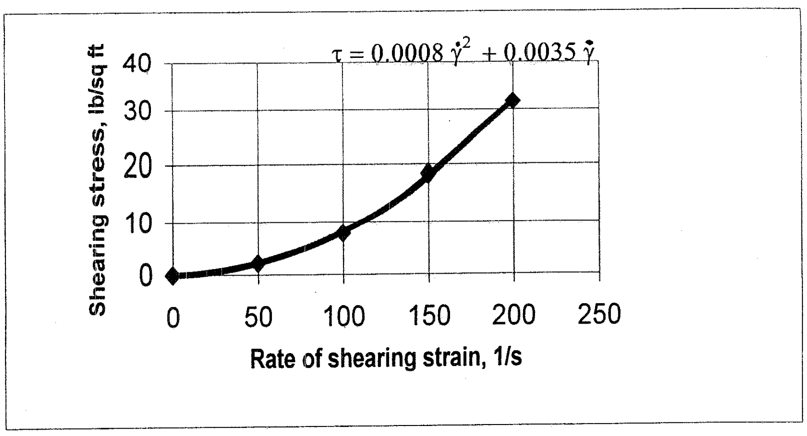
1.46*

*1.46 Fluids for which the shearing stress, τ, is not linearly related to the rate of shearing strain, $\dot{\gamma}$, are designated as non-Newtonian fluids. Such fluids are commonplace and can exhibit unusual behavior as shown in Video V1.4. Some experimental data obtained for a particular non-Newtonian fluid at 80 °F are shown below.

τ (lb/ft^2)	0	2.11	7.82	18.5	31.7
$\dot{\gamma}$ (s^{-1})	0	50	100	150	200

Plot these data and fit a second-order polynomial to the data using a suitable graphing program. What is the apparent viscosity of this fluid when the rate of shearing strain is 70 s^{-1}? Is this apparent viscosity larger or smaller than that for water at the same temperature?

Rate of shearing strain, 1/s	Shearing stress, lb/sq ft
0	0
50	2.11
100	7.82
150	18.5
200	31.7



From the graph $\tau = 0.0008\,\dot{\gamma}^2 + 0.0035\,\dot{\gamma}$ where τ is the shearing stress in lb/ft^2 and $\dot{\gamma}$ is the rate of shearing strain in s^{-1}.

$$\mu_{apparent} = \frac{d\tau}{d\dot{\gamma}} = (2)(0.0008)\dot{\gamma} + 0.0035$$

At $\dot{\gamma} = 70\ s^{-1}$

$$\mu_{apparent} = (2)\left(0.0008\ \frac{lb\cdot s^2}{ft^2}\right)\left(70\ s^{-1}\right) + 0.0035\ \frac{lb\cdot s}{ft^2}$$

$$= \underline{\underline{0.116\ \frac{lb\cdot s}{ft^2}}}$$

From Table B.1 in Appendix B, $\mu_{H_2O\,@\,80°F} = 1.791 \times 10^{-5}\ \frac{lb\cdot s}{ft^2}$, and since water is a Newtonian fluid this value is independent of $\dot{\gamma}$. Thus, the unknown non-Newtonian fluid has a much larger value.

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1.47

1.47 Water flows near a flat surface and some measurements of the water velocity, u, parallel to the surface, at different heights, y, above the surface are obtained. At the surface $y = 0$. After an analysis of the data, the lab technician reports that the velocity distribution in the range $0 < y < 0.1$ ft is given by the equation

$$u = 0.81 + 9.2y + 4.1 \times 10^3 y^3$$

with u in ft/s when y is in ft. **(a)** Do you think that this equation would be valid in any system of units? Explain. **(b)** Do you think this equation is correct? Explain. You may want to look at Video 1.2 to help you arrive at your answer.

(a)
$$u = 0.81 + 9.2\,y + 4.1 \times 10^3\, y^3$$

$$[LT^{-1}] \doteq [0.81] + [9.2][L] + [4.1 \times 10^3][L^3]$$

Each term in the equation must have the same dimensions. Thus, the constant 0.81 must have dimensions of LT^{-1}, 9.2 dimensions of T^{-1}, and 4.1×10^3 dimensions of $L^{-2}T^{-1}$. Since the constants in the equation have dimensions their values will change with a change in units. <u>No.</u>

(b) Equation cannot be correct since at $y=0$ $u = 0.81$ ft/s, a non-zero value which would violate the "no-slip" condition. <u>Not correct.</u>

1.48

1.48 Calculate the Reynolds numbers for the flow of water and for air through a 4-mm-diameter tube, if the mean velocity is 3 m/s and the temperature is 30 °C in both cases (see Example 1.4). Assume the air is at standard atmospheric pressure.

For water at 30°C (from Table B.2 in Appendix B):

$$\rho = 995.7 \ \frac{kg}{m^3} \qquad \mu = 7.975 \times 10^{-4} \ \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho V D}{\mu} = \frac{\left(995.7 \ \frac{kg}{m^3}\right)\left(3 \ \frac{m}{s}\right)\left(0.004 \ m\right)}{7.975 \times 10^{-4} \ \frac{N \cdot s}{m^2}} = \underline{\underline{15{,}000}}$$

For air at 30°C (from Table B.4 in Appendix B):

$$\rho = 1.165 \ \frac{kg}{m^3} \qquad \mu = 1.86 \times 10^{-5} \ \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho V D}{\mu} = \frac{\left(1.165 \ \frac{kg}{m^3}\right)\left(3 \ \frac{m}{s}\right)\left(0.004 \ m\right)}{1.86 \times 10^{-5} \ \frac{N \cdot s}{m^2}} = \underline{\underline{752}}$$

1.49

1.49 For air at standard atmospheric pressure the values of the constants that appear in the Sutherland equation (Eq. 1.10) are $C = 1.458 \times 10^{-6}\ \mathrm{kg/(m \cdot s \cdot K^{1/2})}$ and $S = 110.4$ K. Use these values to predict the viscosity of air at 10 °C and 90 °C and compare with values given in Table B.4 in Appendix B.

$$\mu = \frac{C T^{\frac{3}{2}}}{T+S} = \frac{\left(1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}}\right) T^{\frac{3}{2}}}{T + 110.4\ K}$$

For $T = 10°C = 10°C + 273.15 = 283.15\ K$,

$$\mu = \frac{(1.458 \times 10^{-6})(283.15\ K)^{3/2}}{283.15\ K + 110.4} = \underline{\underline{1.765 \times 10^{-5}\ \frac{N \cdot s}{m^2}}}$$

From Table B.4, $\mu = 1.76 \times 10^{-5}\ \frac{N \cdot s}{m^2}$

For $T = 90°C = 90°C + 273.15 = 363.15\ K$,

$$\mu = \frac{(1.458 \times 10^{-6})(363.15\ K)^{3/2}}{363.15\ K + 110.4} = \underline{\underline{2.13 \times 10^{-5}\ \frac{N \cdot s}{m^2}}}$$

From Table B.4, $\mu = 2.14 \times 10^{-5}\ \frac{N \cdot s}{m^2}$

1.50

1.50*

1.50* Use the values of viscosity of air given in Table B.4 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants C and S which appear in the Sutherland equation (Eq. 1.10). Compare your results with the values given in Problem 1.49. (*Hint*: Rewrite the equation in the form

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{C}\right)T + \frac{S}{C}$$

and plot $T^{3/2}/\mu$ versus T. From the slope and intercept of this curve C and S can be obtained.)

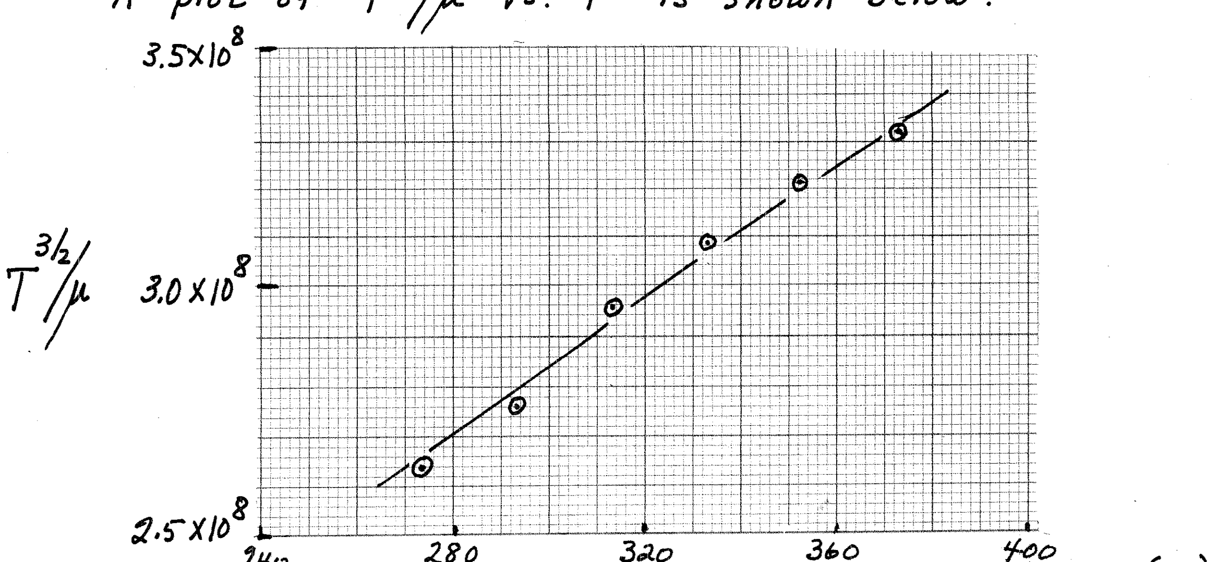
Equation 1.10 can be written in the form

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{C}\right)T + \frac{S}{C} \qquad (1)$$

and with the data from Table B.4:

$T(°C)$	$T(K)$	$\mu\ (N \cdot s/m^2)$	$T^{3/2}/\mu\ \left[K^{3/2}/(kg/m \cdot s)\right]$
0	273.15	1.71×10^{-5}	2.640×10^{8}
20	293.15	1.82×10^{-5}	2.758×10^{8}
40	313.15	1.87×10^{-5}	2.963×10^{8}
60	333.15	1.97×10^{-5}	3.087×10^{8}
80	353.15	2.07×10^{-5}	3.206×10^{8}
100	373.15	2.17×10^{-5}	3.322×10^{8}

A plot of $T^{3/2}/\mu$ vs. T is shown below:



(con't)

1.50* (Con't)

Since the data plot as an approximate straight line, Eq. (1) can be represented by an equation of the form

$$y = bx + a$$

where $y \sim T^{3/2}/\mu$, $x \sim T$, $b \sim 1/C$, and $a \sim S/C$.

Fit the data to a linear equation using a standard curve-fitting program such as found in EXCEL. Thus,

$$y = 6.969 \times 10^{5} x + 7.441 \times 10^{7}$$

and

$$\frac{1}{C} = b = 6.969 \times 10^{5}$$

so that
$$\underline{\underline{C = 1.43 \times 10^{-6} \ kg/(m \cdot s \cdot K^{1/2})}}$$

and
$$\frac{S}{C} = a = 7.441 \times 10^{7}$$

and therefore
$$\underline{\underline{S = 107 \ K}}$$

These values for C and S are in good agreement with values given in Problem 1.49.

1.51

1.51 The viscosity of a fluid plays a very important role in determining how a fluid flows. (See Video V1.1.) The value of the viscosity depends not only on the specific fluid but also on the fluid temperature. Some experiments show that when a liquid, under the action of a constant driving pressure, is forced with a low velocity, V, through a small horizontal tube, the velocity is given by the equation $V = K/\mu$. In this equation K is a constant for a given tube and pressure, and μ is the dynamic viscosity. For a particular liquid of interest, the viscosity is given by Andrade's equation (Eq. 1.11) with $D = 5 \times 10^{-7}\ \text{lb} \cdot \text{s/ft}^2$ and $B = 4000\ °R$. By what percentage will the velocity increase as the liquid temperature is increased from 40 °F to 100 °F? Assume all other factors remain constant.

$$V_{40°} = \frac{K}{\mu_{40°}} \qquad (1)$$

$$V_{100°} = \frac{K}{\mu_{100°}} \qquad (2)$$

$$\%\text{ increase in } V = \left[\frac{V_{100°} - V_{40°}}{V_{40°}}\right] \times 100 = \left[\frac{V_{100°}}{V_{40°}} - 1\right] \times 100$$

and from Eq. (1) & (2)

$$\%\text{ increase in } V = \left[\frac{K/\mu_{100°}}{K/\mu_{40°}} - 1\right] \times 100 = \left[\frac{\mu_{40°}}{\mu_{100°}} - 1\right] \times 100 \qquad (3)$$

From Andrade's equation

$$\mu_{40°} = 5 \times 10^{-7} e^{\frac{4000}{(40°F + 460)}}$$

and

$$\mu_{100°} = 5 \times 10^{-7} e^{\frac{4000}{(100°F + 460)}}$$

Thus, from Eq. (3)

$$\%\text{ increase in } V = \left[\frac{5 \times 10^{-7} e^{\frac{4000}{500}}}{5 \times 10^{-7} e^{\frac{4000}{560}}} - 1\right] \times 100$$

$$= \underline{\underline{136\%}}$$

1.52*

1.52* Use the value of the viscosity of water given in Table B.2 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants D and B which appear in Andrade's equation (Eq. 1.11). Calculate the value of the viscosity at 50 °C and compare with the value given in Table B.2. (*Hint*: Rewrite the equation in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D$$

and plot $\ln \mu$ versus $1/T$. From the slope and intercept of this curve B and D can be obtained. If a nonlinear curve fitting program is available the constants can be obtained directly from Eq. 1.11 without rewriting the equation.)

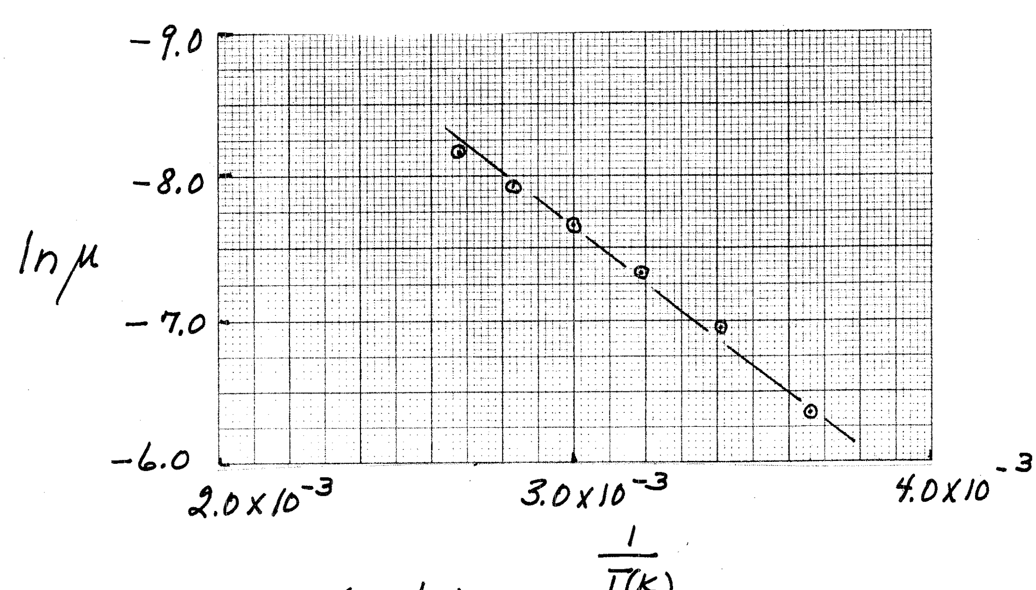
Equation 1.11 can be written in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D \qquad (1)$$

and with the data from Table B.2:

T (°C)	T (K)	$1/T$ (K)	μ ($N\cdot s/m^2$)	$\ln \mu$
0	273.15	3.661×10^{-3}	1.787×10^{-3}	−6.327
20	293.15	3.411×10^{-3}	1.002×10^{-3}	−6.906
40	313.15	3.193×10^{-3}	6.529×10^{-4}	−7.334
60	333.15	3.002×10^{-3}	4.665×10^{-4}	−7.670
80	353.15	2.832×10^{-3}	3.547×10^{-4}	−7.944
100	373.15	2.680×10^{-3}	2.818×10^{-4}	−8.174

A plot of $\ln \mu$ vs. $1/T$ is shown below:



(con't)

1.52*

(Con't)

Since the data plot as an approximate straight line, Eq. (1) can be used to represent these data. To obtain B and D, fit the data to an exponential equation of the form $y = ae^{bx}$ such as found in EXCEL.

Thus,

$$\underline{\underline{D = a = 1.767 \times 10^{-6} \ N \cdot s/m^2}}$$

and

$$\underline{\underline{B = b = 1.870 \times 10^{3} \ K}}$$

so that

$$\mu = 1.767 \times 10^{-6} \, e^{\frac{1870}{T}}$$

At 50°C (323.15K),

$$\mu = 1.767 \times 10^{-6} \, e^{\frac{1870}{323.15}} = \underline{\underline{5.76 \times 10^{-4} \ N \cdot s/m^2}}$$

From Table B.2, $\mu = 5.468 \times 10^{-4} \ N \cdot s/m^2$.

1.53

1.53 For a parallel plate arrangement of the type shown in Fig. 1.3 it is found that when the distance between plates is 2 mm, a shearing stress of 150 Pa develops at the upper plate when it is pulled at a velocity of 1 m/s. Determine the viscosity of the fluid between the plates. Express your answer in SI units.

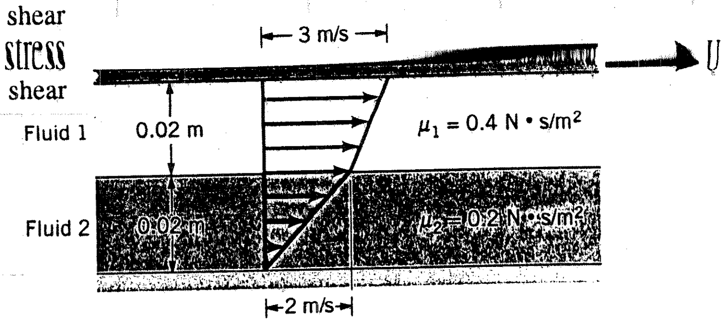
$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{U}{b}$$

$$\mu = \frac{\tau}{\left(\frac{U}{b}\right)} = \frac{150\ \frac{N}{m^2}}{\left(\frac{1\ \frac{m}{s}}{0.002\,m}\right)} = \underline{\underline{0.300\ \frac{N \cdot s}{m^2}}}$$

1.54

1.54 As shown in **Video V1.2**, the "no slip" condition means that a fluid "sticks" to a solid surface. This is true for both fixed and moving surfaces. Let two layers of fluid be dragged along by the motion of an upper plate as shown in Fig. P1.54. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the botton plate. Determine the ratio of these two shear stresses.



■ **FIGURE P1.54**

For fluid 1

$$\tau_1 = \mu_1 \left(\frac{du}{dy}\right)_{\text{top surface}} = \left(0.4 \frac{N \cdot s}{m^2}\right)\left(\frac{3\frac{m}{s} - 2\frac{m}{s}}{0.02\,m}\right) = 20 \frac{N}{m^2}$$

For fluid 2

$$\tau_2 = \mu_2 \left(\frac{du}{dy}\right)_{\text{bottom surface}} = \left(0.2 \frac{N \cdot s}{m^2}\right)\left(\frac{2\frac{m}{s}}{0.02\,m}\right) = 20 \frac{N}{m^2}$$

Thus,

$$\frac{\tau_{\text{top surface}}}{\tau_{\text{bottom surface}}} = \frac{20 \frac{N}{m^2}}{20 \frac{N}{m^2}} = \underline{\underline{1}}$$

1.55

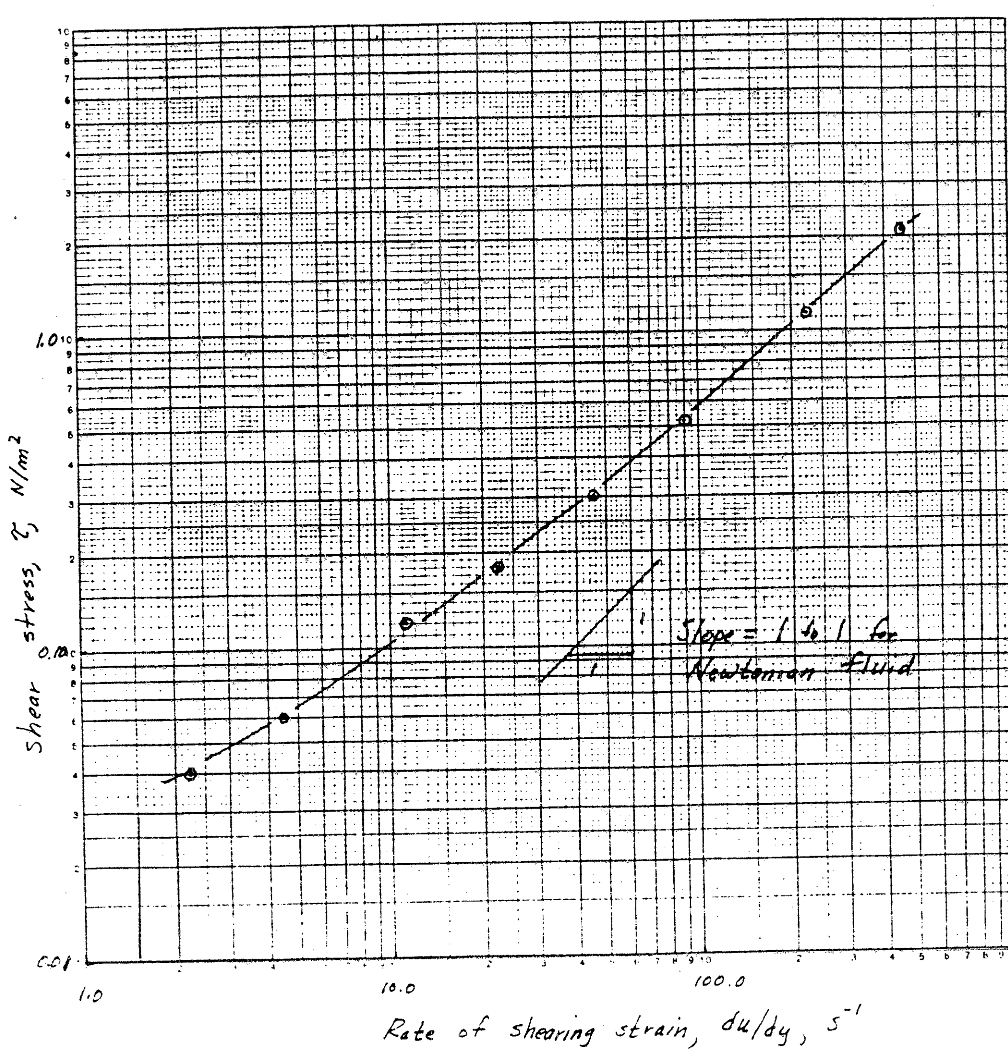
1.55 There are many fluids that exhibit non-Newtonian behavior (see for example **Video V1.4**). For a given fluid the distinction between Newtonian and non-Newtonian behavior is usually based on measurements of shear stress and rate of shearing strain. Assume that the viscosity of blood is to be determined by measurements of shear stress, τ, and rate of shearing strain, du/dy, obtained from a small blood sample tested in a suitable viscometer. Based on the data given below determine if the blood is a Newtonian or non-Newtonian fluid. Explain how you arrived at your answer.

τ (N/m^2)	0.04	0.06	0.12	0.18	0.30	0.52	1.12	2.10
du/dy (s^{-1})	2.25	4.50	11.25	22.5	45.0	90.0	225	450

For a Newtonian fluid the ratio of τ to du/dy is a constant. For the data given:

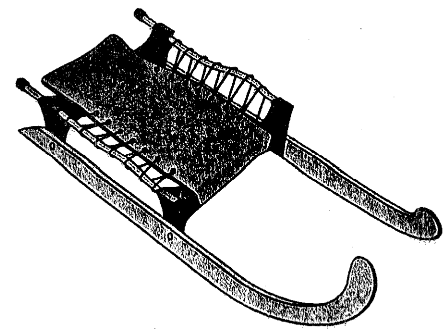
$\dfrac{\tau}{du/dy}$ (N·s/m^2)	0.0178	0.0133	0.0107	0.0080	0.0067	0.0058	0.0050	0.0047

The ratio is not a constant but decreases as the rate of shearing strain increases. Thus, this fluid (blood) is a non-Newtonian fluid. A plot of the data is shown below. For a Newtonian fluid the curve would be a straight line with a slope of 1 to 1.



1.56

1.56 The sled shown in Fig. P1.56 slides along on a thin horizontal layer of water between the ice and the runners. The horizontal force that the water puts on the runners is equal to 1.2 lb when the sled's speed is 50 ft/s. The total area of both runners in contact with the water is 0.08 ft^2, and the viscosity of the water is 3.5×10^{-5} lb s/ft^2. Determine the thickness of the water layer under the runners. Assume a linear velocity distribution in the water layer.



■ FIGURE P1.56

F (force) $= \tau A$

$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{d}$ where d = thickness of water layer

Thus,

$$F = \mu \frac{V}{d} A$$

and

$$d = \frac{\mu V A}{F} = \frac{\left(3.5\times10^{-5}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(50\ \frac{\text{ft}}{\text{s}}\right)\left(0.08\ \text{ft}^2\right)}{1.2\ \text{lb}}$$

$$= \underline{\underline{11.7\times10^{-4}\ \text{ft}}}$$

1.57

1.57 A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Fig. P1.57. The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of $8.0 \times 10^{-4}\ m^2/s$ and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.

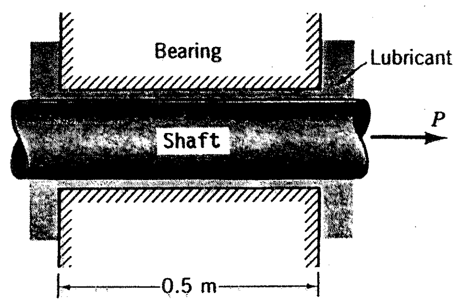
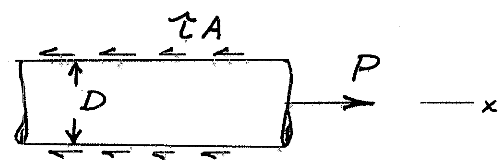


FIGURE P1.57



$\sum F_x = 0$

Thus,
$$P = \tau A$$

where $A = \pi D \times (\text{shaft length in bearing}) = \pi D \ell$

and
$$\tau = \mu \frac{(\text{velocity of shaft})}{(\text{gap width})} = \mu \frac{V}{b}$$

so that
$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell)$$

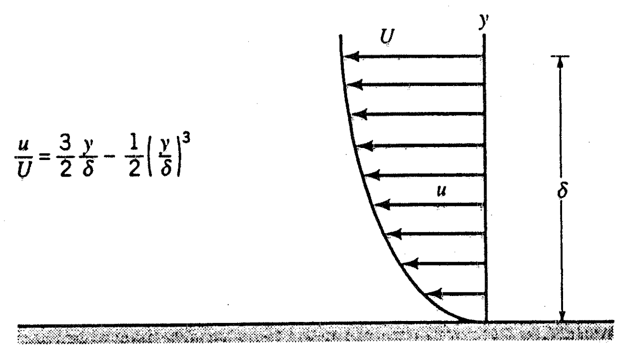
Since $\mu = \nu \rho = \nu (SG)(\rho_{H_2O @ 4°C})$,

$$P = \frac{(8.0 \times 10^{-4} \frac{m^2}{s})(0.91 \times 10^3 \frac{kg}{m^3})(3 \frac{m}{s})(\pi)(0.025 m)(0.5 m)}{(0.0003 m)}$$

$$= \underline{\underline{286\ N}}$$

1.58

1.58 A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4}\ m^2/s$ flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero (as shown in Video V1.2), and the velocity profile near the surface is shown in Fig. P1.58. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



■ FIGURE P1.58

$$\tau_{\substack{surface \\ (y=0)}} = \mu \left(\frac{du}{dy}\right)_{y=0}$$

$$\frac{du}{dy} = U\left(\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right)$$

$$@\ y=0, \quad \frac{du}{dy} = \frac{3}{2}\frac{U}{\delta}$$

Since, $\mu = \nu\rho$

$$\tau_{surface} = \nu\rho\left(\frac{3}{2}\frac{U}{\delta}\right)$$

$$= \left(4 \times 10^{-4}\frac{m^2}{s}\right)\left(0.92 \times 10^3 \frac{kg}{m^3}\right)\left(\frac{3}{2}\right)\frac{U}{\delta}$$

$$= \underline{\underline{0.552\frac{U}{\delta}\ N/m^2 \text{ acting to left on plate}}}$$

1.59

1.59 A layer of water flows down an inclined fixed surface with the velocity profile shown in Fig. P1.59. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for U = 2 m/s and h = 0.1 m.

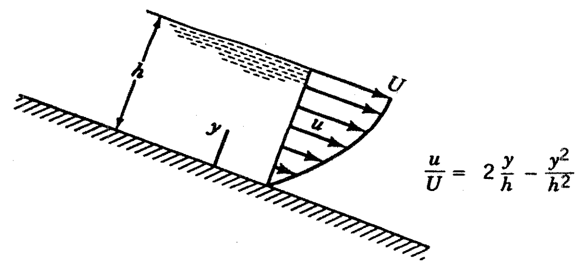


FIGURE P1.59

$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = U\left(\frac{2}{h} - \frac{2y}{h^2}\right)$$

Thus, at the fixed surface ($y=0$)

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{h}$$

so that

$$\tau = \mu\left(\frac{2U}{h}\right) = \left(1.12\times10^{-3}\ \frac{N\cdot s}{m^2}\right)(2)\ \frac{(2\ \frac{m}{s})}{(0.1m)}$$

$$= \underline{\underline{4.48\times10^{-2}\ \frac{N}{m^2}\ \text{acting in direction of flow}}}$$

1,60

1.60*

1.60* Standard air flows past a flat surface and velocity measurements near the surface indicate the following distribution:

y (ft)	0.005	0.01	0.02	0.04	0.06	0.08
u (ft/s)	0.74	1.51	3.03	6.37	10.21	14.43

The coordinate y is measured normal to the surface and u is the velocity parallel to the surface.

(a) Assume the velocity distribution is of the form

$$u = C_1 y + C_2 y^3$$

and use a standard curve-fitting technique to determine the constants C_1 and C_2. **(b)** Make use of the results of part (a) to determine the magnitude of the shearing stress at the wall ($y = 0$) and at $y = 0.05$ ft.

(a) Use nonlinear regression program, such as SAS-NLIN, to obtain coefficients C_1 and C_2. This program produces least squares estimates of the parameters of a nonlinear model. For the data given,

$$\underline{\underline{C_1 = 153\ s^{-1}}} \quad \text{and} \quad \underline{\underline{C_2 = 4350\ ft^{-2} s^{-1}}}$$

(b) Since,

$$\tau = \mu \frac{du}{dy}$$

it follows that

$$\tau = \mu \left(C_1 + 3 C_2 y^2 \right)$$

Thus, at the wall ($y=0$)

$$\tau = \mu C_1 = \left(3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)\left(153 \frac{1}{s}\right) = \underline{\underline{5.72 \times 10^{-5} \frac{lb}{ft^2}}}$$

At $y = 0.05$ ft

$$\tau = \left(3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)\left[153 \frac{1}{s} + 3\left(4350 \frac{1}{ft^2 s}\right)\left(0.05 ft\right)^2\right]$$

$$= \underline{\underline{6.94 \times 10^{-5} \frac{lb}{ft^2}}}$$

1.61

1.61 The viscosity of liquids can be measured through the use of a *rotating cylinder viscometer* of the type illustrated in Fig. P1.61. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω. The torque $\mathcal{T}$ required to develop ω is measured and the viscosity is calculated from these two measurements. Develop an equation relating μ, ω, $\mathcal{T}$, ℓ, R_o and R_i. Neglect end effects and assume the velocity distribution in the gap is linear.

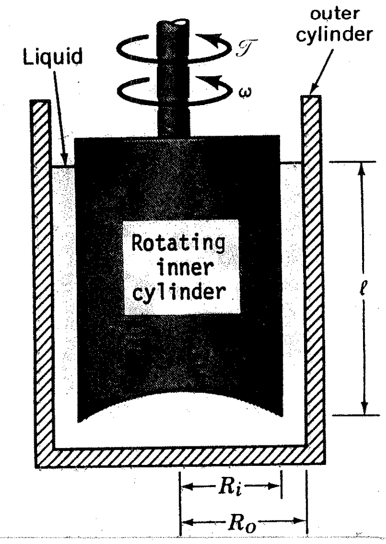


FIGURE P1.61

Torque, $d\mathcal{T}$, due to shearing stress on inner cylinder is equal to

$$d\mathcal{T} = R_i \, \tau \, dA$$

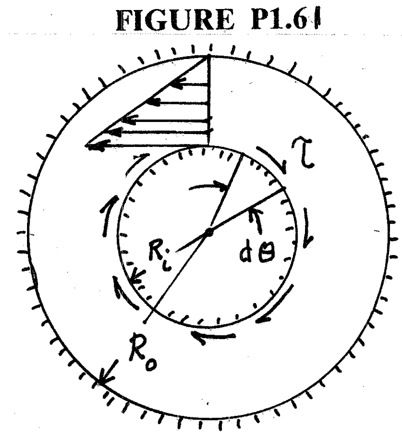
where $dA = (R_i \, d\theta)\ell$. Thus,

$$d\mathcal{T} = R_i^2 \, \ell \, \tau \, d\theta$$

and torque required to rotate inner cylinder is

$$\mathcal{T} = R_i^2 \, \ell \, \tau \int_0^{2\pi} d\theta$$

$$= 2\pi R_i^2 \, \ell \, \tau$$



top view

(ℓ ~ cylinder length)

For a linear velocity distribution in the gap

$$\tau = \mu \frac{R_i \omega}{R_o - R_i}$$

so that

$$\underline{\underline{\mathcal{T} = \frac{2\pi R_i^3 \, \ell \, \mu \, \omega}{R_o - R_i}}}$$

1.62

1.62

1.62 The space between two 6-in. long concentric cylinders is filled with glycerin (viscosity $= 8.5 \times 10^{-3}$ lb·s/ft²). The inner cylinder has a radius of 3 in. and the gap width between cylinders is 0.1 in. Determine the torque and the power required to rotate the inner cylinder at 180 rev/min. The outer cylinder is fixed. Assume the velocity distribution in the gap to be linear.

From Problem 1.61,

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu \omega}{R_o - R_i}$$

and with $\omega = \left(180 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)\left(\frac{1\ min}{60\ s}\right) = 6\pi \frac{rad}{s}$

then

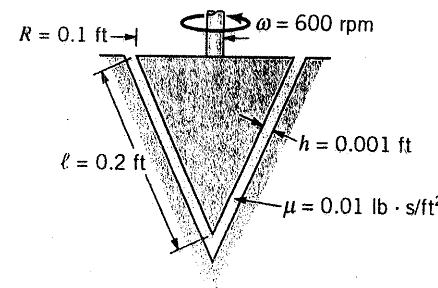
$$\mathcal{T} = \frac{2\pi \left(\frac{3}{12} ft\right)^3 \left(\frac{6}{12} ft\right)\left(8.5\times 10^{-3} \frac{lb\cdot s}{ft^2}\right)\left(6\pi \frac{rad}{s}\right)}{\left(\frac{0.1}{12} ft\right)} = \underline{\underline{0.944\ ft\cdot lb}}$$

Since $\quad power = \mathcal{T} \times \omega$ it follows that

$$power = (0.944\ ft\cdot lb)\left(6\pi \frac{rad}{s}\right) = \underline{\underline{17.8 \frac{ft\cdot lb}{s}}}$$

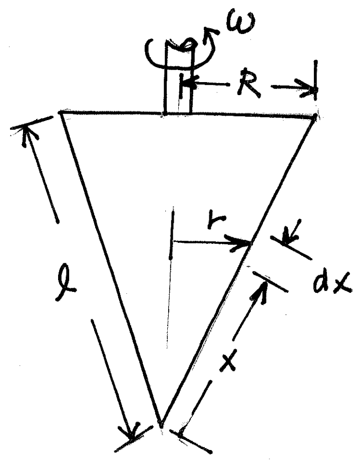
1.63

1.63 A conical body rotates at a constant angular velocity of 600 rpm in a container as shown in Fig. P1.63. A uniform 0.001-ft gap between the cone and the container is filled with oil that has a viscosity of 0.01 lb · s/ft². Determine the torque required to rotate the cone.



■ FIGURE P1.63

The force, dF, acting on the surface of the cone is equal to



$$dF = \tau\, dA = \tau\,(2\pi r\, dx)$$

and $\tau = \dfrac{\mu \omega r}{h}$

Thus, torque $\mathcal{T} = \int r\, dF = \int r \left(\dfrac{\mu \omega r}{h}\right)(2\pi r\, dx)$

$$= \frac{2\pi\omega\mu}{h}\int r^3 dx \qquad (1)$$

Also, $\dfrac{R}{\ell} = \dfrac{r}{x}$ so that $x = \dfrac{\ell}{R}\, r$

and $dx = \dfrac{\ell}{R}\, dr$

From Eq. (1)

$$\mathcal{T} = \frac{2\pi\omega\mu\ell}{R h}\int_0^R r^3 dr = \frac{\pi\omega\mu\ell R^3}{2h}$$

and with

$\ell = 0.2\text{ ft},\ R = 0.1\text{ ft},\ h = 0.001\text{ ft},\ \omega = 600\text{ rpm},\ \mu = 0.01\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}$

$$\mathcal{T} = \frac{\pi\left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right)\left(0.01\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)(0.2\text{ ft})(0.1\text{ ft})^3}{2\,(0.001\text{ ft})}$$

$$= \underline{\underline{0.197\text{ ft}\cdot\text{lb}}}$$

1.64*

1.64* The following torque–angular velocity data were obtained with a rotating cylinder viscometer of the type described in Problem 1.61.

Torque (ft·lb)	13.1	26.0	39.5	52.7	64.9	78.6
Angular velocity (rad/s)	1.0	2.0	3.0	4.0	5.0	6.0

For this viscometer $R_o = 2.50$ in., $R_i = 2.45$ in., and $\ell = 5.00$ in. Make use of these data and a standard curve-fitting program to determine the viscosity of the liquid contained in the viscometer.

1.61

The torque, $\mathcal{T}$, is related to the angular velocity, ω, through the equation,

$$\mathcal{T} = \frac{2\pi R_i^3 \ell \mu}{R_o - R_i}\,\omega \qquad (1)$$

(see solution to Problem 1.61). Thus, for a fixed geometry and a given viscosity, Eq. (1) is of the form

$$y = bx \qquad (y \sim \mathcal{T} \text{ and } x \sim \omega)$$

where b is a constant equal to

$$b = \frac{2\pi R_i^3 \ell \mu}{R_o - R_i} \qquad (2)$$

To obtain b fit the data to a linear equation of the form $y = bx$ using a standard curve-fitting program such as found in EXCEL.

Thus, from Eq. (2)

$$\mu = \frac{(b)(R_o - R_i)}{2\pi R_i^3 \ell}$$

and with the data given, $b = 13.08$ ft·lb·s, so that

$$\mu = \frac{(13.08 \text{ ft·lb·s})\left(\frac{2.50 - 2.45}{12} \text{ ft}\right)}{2\pi\left(\frac{2.45}{12} \text{ ft}\right)^3\left(\frac{5.00}{12} \text{ ft}\right)} = \underline{\underline{2.45 \ \frac{\text{lb·s}}{\text{ft}^2}}}$$

1.65

1.65 A 12-in.-diameter circular plate is placed over a fixed bottom plate with a 0.1-in. gap between the two plates filled with glycerin as shown in Fig. P1.65. Determine the torque required to rotate the circular plate slowly at 2 rpm. Assume that the velocity distribution in the gap is linear and that the shear stress on the edge of the rotating plate is negligible.

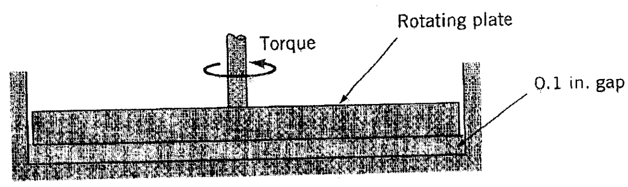


FIGURE P1.65

Torque, $d\mathcal{T}$, due to shearing stresses on plate is equal to

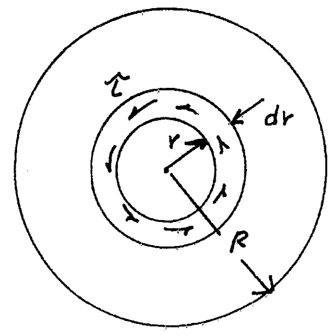
$$d\mathcal{T} = r\,\tau\,dA$$

where $dA = 2\pi r\,dr$. Thus,

$$d\mathcal{T} = r\,\tau\,2\pi r\,dr$$

and

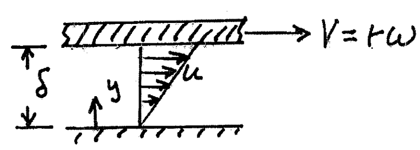
$$\mathcal{T} = 2\pi\int_0^R r^2\,\tau\,dr$$



stresses acting on bottom of plate

Since $\tau = \mu \frac{du}{dy}$, and for a linear velocity distribution (see figure)

$$\tau = \mu\frac{r\omega}{\delta}$$



$$\frac{du}{dy} = \frac{V}{\delta} = \frac{r\omega}{\delta}$$

velocity distribution

Thus,

$$\mathcal{T} = \frac{2\pi\mu\omega}{\delta}\int_0^R r^3\,dr = \frac{2\pi\mu\omega}{\delta}\left(\frac{R^4}{4}\right)$$

and with the data given

$$\mathcal{T} = \frac{2\pi\left(0.0313\,\frac{lb\cdot s}{ft^2}\right)\left(2\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{6}{12}\,ft\right)^4}{\left(\frac{0.1}{12}\,ft\right)(4)}$$

$$= \underline{\underline{0.0772\ ft\cdot lb}}$$

1.67

1.67 Estimate the increase in pressure (in psi) required to decrease a unit volume of mercury by 0.1%.

$$E_V = -\frac{dp}{d\forall/\forall} \qquad (Eq.\ 1.12)$$

Thus,

$$\Delta p \approx -\frac{E_V \Delta\forall}{\forall} = -\left(4.14 \times 10^6 \frac{lb}{in.^2}\right)(-0.001)$$

$$\Delta p \approx \underline{\underline{4.14 \times 10^3\ psi}}$$

1.68

1.68 A 1-m³ volume of water is contained in a rigid container. Estimate the change in the volume of the water when a piston applies a pressure of 35 MPa.

$$E_V = -\frac{dp}{d\forall/\forall} \qquad (Eq.\ 1.12)$$

Thus,

$$\Delta\forall \approx -\frac{\forall\,\Delta p}{E_V} = -\frac{(1\ m^3)(35 \times 10^6 \frac{N}{m^2})}{2.15 \times 10^9 \frac{N}{m^2}} = -0.0163\ m^3$$

or

$$\underline{\underline{\text{decrease}}} \text{ in volume} \approx \underline{\underline{0.0163\ m^3}}$$

1.69

1.69 Calculate the speed of sound in m/s for **(a)** gasoline, **(b)** mercury, and **(c)** seawater.

$$c = \sqrt{\frac{E_v}{\rho}} \qquad \text{(Eq. 1.19)}$$

(a) For gasoline: $c = \sqrt{\dfrac{1.3 \times 10^9 \frac{N}{m^2}}{680 \frac{kg}{m^3}}} = \underline{\underline{1.38 \frac{km}{s}}}$

(b) For mercury: $c = \sqrt{\dfrac{2.85 \times 10^{10} \frac{N}{m^2}}{1.36 \times 10^4 \frac{kg}{m^3}}} = \underline{\underline{1.45 \frac{km}{s}}}$

(c) For seawater: $c = \sqrt{\dfrac{2.34 \times 10^9 \frac{N}{m^2}}{1.03 \times 10^3 \frac{kg}{m^3}}} = \underline{\underline{1.51 \frac{km}{s}}}$

1.70

1.70

1.70 Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 25 psi. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process to be isothermal and the local atmospheric pressure to be 14.7 psi.

For isothermal compression, $\frac{p}{\rho} = \text{constant}$ so that

$$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus, $p_f = \frac{\rho_f}{\rho_i} p_i$

Since $\rho = \frac{\text{mass}}{\text{volume}}$, $\frac{\rho_f}{\rho_i} = \frac{\text{initial volume}}{\text{final volume}} = 3$ (for constant mass)

and therefore

$$p_f = (3)\left[(25 + 14.7)\text{ psi (abs)}\right] = 119 \text{ psi (abs)}$$

or

$$p_f\text{ (gage)} = (119 - 14.7)\text{psi} = \underline{\underline{104 \text{ psi (gage)}}}$$

1.71

1.71 Often the assumption is made that the flow of a certain fluid can be considered as incompressible flow if the density of the fluid changes by less than 2%. If air is flowing through a tube such that the air pressure at one section is 9.0 psi (gage) and at a downstream section it is 8.6 psi (gage) at the same temperature, do you think that this flow could be considered an incompressible flow? Support your answer with the necessary calculations. Assume standard atmospheric pressure.

For isothermal change in density

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}$$

so that

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}$$

The percent change in air densities between sections (1) & (2) is

$$\% \text{ change} = \frac{\rho_1 - \rho_2}{\rho_1} \times 100$$

$$= \left(1 - \frac{\rho_2}{\rho_1}\right) \times 100 = \left(1 - \frac{p_2}{p_1}\right) \times 100$$

Thus,

$$\% \text{ change} = \left[1 - \frac{(8.6 + 14.7)\,\text{psia}}{(9.0 + 14.7)\,\text{psia}}\right] \times 100$$

$$= 1.69\%$$

Since $1.69\% < 2\%$ the flow could be considered incompressible.
Yes.

1.72

1.72 Carbon dioxide at 30 °C and 300 kPa absolute pressure expands isothermally to an absolute pressure of 165 kPa. Determine the final density of the gas.

For isothermal expansion, $\frac{p}{\rho} = \text{constant}$ so that

$$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus,

$$\rho_f = \frac{p_f}{p_i}\rho_i$$

Also,

$$\rho_i = \frac{p_i}{RT_i} = \frac{300 \times 10^3 \frac{N}{m^2}}{\left(188.9 \frac{J}{kg \cdot K}\right)\left[(30°C + 273)K\right]} = 5.24 \frac{kg}{m^3}$$

so that

$$\rho_f = \left(\frac{165 \text{ kPa}}{300 \text{ kPa}}\right)\left(5.24 \frac{kg}{m^3}\right) = \underline{\underline{2.88 \frac{kg}{m^3}}}$$

1.73

1.73 Natural gas at 70 °F and standard atmospheric pressure of 14.7 psi (abs) is compressed isentropically to a new absolute pressure of 70 psi. Determine the final density and temperature of the gas.

For isentropic compression, $\frac{p}{\rho^k} = \text{constant}$ so that

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$ where $i \sim$ initial state and $f \sim$ final state.

Thus,

$$\rho_f^k = \frac{p_f}{p_i}\rho_i^k$$

or

$$\rho_f = \left(\frac{p_f}{p_i}\right)^{\frac{1}{k}} \rho_i$$

Also,

$$\rho_i = \frac{p_i}{R T_i} = \frac{\left(14.7 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(3.099 \times 10^3 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)\left[(70^\circ F + 460)^\circ R\right]} = 1.29 \times 10^{-3} \frac{slugs}{ft^3}$$

so that

$$\rho_f = \left[\frac{70 \text{ psi (abs)}}{14.7 \text{ psi (abs)}}\right]^{\frac{1}{1.31}} \left(1.29 \times 10^{-3} \frac{slugs}{ft^3}\right) = \underline{\underline{4.25 \times 10^{-3} \frac{slugs}{ft^3}}}$$

and

$$T_f = \frac{p_f}{\rho_f R} = \frac{\left(70 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(4.25 \times 10^{-3} \frac{slugs}{ft^3}\right)\left(3.099 \times 10^3 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)}$$

$$= 765 \ {}^\circ R$$

or

$$T_f = 765 \ {}^\circ R - 460 = \underline{\underline{305 \ {}^\circ F}}$$

1.74

1.74 Compare the isentropic bulk modulus of air at 101 kPa (abs) with that of water at the same pressure.

For air (Eq. 1.17),

$$E_v = kp = (1.40)(101 \times 10^3\,Pa) = 1.41 \times 10^5\,Pa$$

For water (Table 1.6)

$$E_v = 2.15 \times 10^9\,Pa$$

Thus,

$$\frac{E_v\,(water)}{E_v\,(air)} = \frac{2.15 \times 10^9\,Pa}{1.41 \times 10^5\,Pa} = \underline{\underline{1.52 \times 10^4}}$$

1.75

1.75 *

1.75* Develop a computer program for calculating the final gage pressure of gas when the initial gage pressure, initial and final volumes, atmospheric pressure, and the type of process (isothermal or isentropic) are specified. Use BG units. Check your program against the results obtained for Problem 1.70.

1.70

For compression or expansion,

$$\frac{p}{\rho^k} = \text{constant}$$

where $k=1$ for isothermal process, and k = specific heat ratio for isentropic process. Thus,

$$\frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$$

where $i \sim$ initial state, $f \sim$ final state, so that

$$p_f = \left(\frac{\rho_f}{\rho_i}\right)^k p_i \qquad (1)$$

Since

$$\rho = \frac{\text{mass}}{\text{volume}}$$

then

$$\frac{\rho_f}{\rho_i} = \frac{V_i}{V_f}$$

where V_i, V_f, are the initial and final volumes, respectively. Thus, from Eq. (1)

$$p_{fg} + p_{atm} = \left(\frac{V_i}{V_f}\right)^k \left(p_{ig} + p_{atm}\right) \qquad (2)$$

where the subscript g refers to gage pressure. Equation (2) can be written as

$$p_{fg} = \left(\frac{V_i}{V_f}\right)^k \left(p_{ig} + p_{atm}\right) - p_{atm} \qquad (3)$$

A spreadsheet (EXCEL) program for calculating the final gage pressure follows.

(con't)

1.75* (con't)

This program calculates the final gage pressure of an ideal gas when the initial gage pressure in psi, the initial volume, the final volume, the atmospheric pressure in psia, and the type of process (isothermal or isentropic) is specified. To use, replace current values and let k = 1 for isothermal process or k = specific heat for isentropic process.

A	B	C	D	E	F	
Initial gage pressure	Initial volume	Final volume	Atmospheric pressure		Final gage pressure	
p_{ig}(psi)	V_i	V_f	p_{atm}(psia)	k	p_{fg}(psi)	
25	1	0.3333	14.7	1	104.4	Row 10

Formula:
=((B10/C10)^E10)*(A10+D10)-D10

Data from Problem 1.70 are included in the above table, giving a final gage pressure of 104.4 psi.

1.76

1.76 An important dimensionless parameter concerned with very high speed flow is the *Mach number*, defined as V/c, where V is the speed of the object such as an airplane or projectile, and c is the speed of sound in the fluid surrounding the object. For a projectile traveling at 800 mph through air at 50 °F and standard atmospheric pressure, what is the value of the Mach number?

$$\text{Mach number} = \frac{V}{c}$$

From Table B.3 in Appendix B

$$c_{air\ @\ 50°F} = 1106\ \frac{ft}{s}$$

Thus

$$\text{Mach number} = \frac{(800\ mph)\left(5280\ \frac{ft}{mi}\right)\left(\frac{1\ hr}{3600\ s}\right)}{1106\ \frac{ft}{s}}$$

$$= \underline{\underline{1.06}}$$

1.77

1.77 Jet airliners typically fly at altitudes between approximately 0 to 40,000 ft. Make use of the data in Appendix C to show on a graph how the speed of sound varies over this range.

$$c = \sqrt{kRT} \qquad (\text{Eq. } 1.20)$$

For $k = 1.40$ and $R = 1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}$

$$c = 49.0 \sqrt{T({}^\circ R)}$$

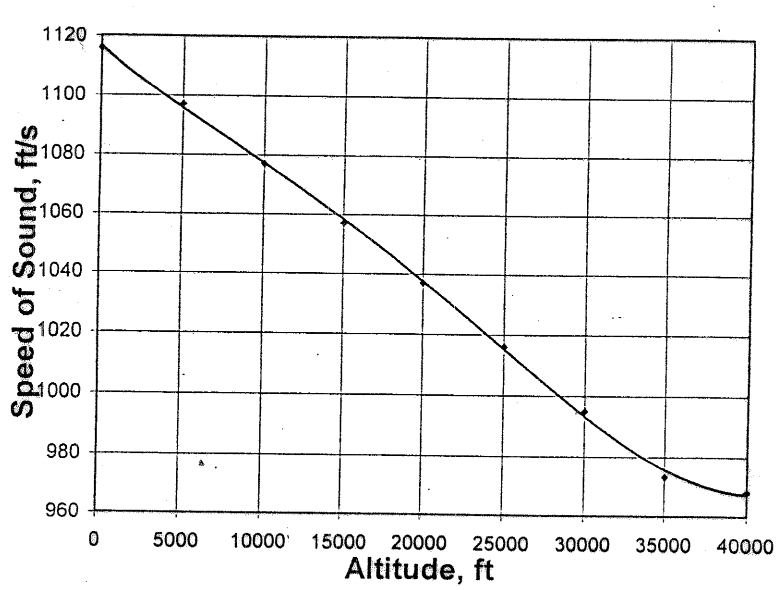
From Table C.1 in Appendix C at an altitude of 0 ft

$T = 59.00 + 460 = 519\,{}^\circ R$ so that

$$c = 49.0\sqrt{519\,{}^\circ R} = 1116 \frac{ft}{s}$$

Similar calculations can be made for other altitudes and the resulting graph is shown below.

Altitude, ft	Temp., °F	Temp., °R	c, ft/s
0	59	519	1116
5000	41.17	501.17	1097
10000	23.36	483.36	1077
15000	5.55	465.55	1057
20000	-12.26	447.74	1037
25000	-30.05	429.95	1016
30000	-47.83	412.17	995
35000	-65.61	394.39	973
40000	-69.7	390.3	968



1.78

1.78 When a fluid flows through a sharp bend, low pressures may develop in localized regions of the bend. Estimate the minimum absolute pressure (in psi) that can develop without causing cavitation if the fluid is water at 160 °F.

Cavitation may occur when the local pressure equals the vapor pressure. For water at 160 °F (from Table B.1 in Appendix B)

$$p_v = 4.74 \text{ psi (abs)}$$

Thus, minimum pressure $=$ $\underline{\underline{4.74 \text{ psi (abs)}}}$

1.79

1.79 Estimate the minimum absolute pressure (in pascals) that can be developed at the inlet of a pump to avoid cavitation if the fluid is carbon tetrachloride at 20 °C.

Cavitation may occur when the suction pressure at the pump inlet equals the vapor pressure.
For carbon tetrachloride at 20°C $p_v = 13 \text{ kPa (abs)}$.
Thus, minimum pressure $=$ $\underline{\underline{13 \text{ kPa (abs)}}}$

1.80

1.80 When water at 70 °C flows through a converging section of pipe, the pressure is reduced in the direction of flow. Estimate the minimum absolute pressure that can develop without causing cavitation. Express your answer in both BG and SI units.

Cavitation may occur in the converging section of pipe when the pressure equals the vapor pressure. From Table B.2 in Appendix B for water at 70°C, $p_v = 31.2 \text{ kPa (abs)}$. Thus,

$$\text{minimum pressure} = \underline{\underline{31.2 \text{ kPa (abs)}}} \text{ in SI units.}$$

In BG units

$$\text{minimum pressure} = \left(31.2 \times 10^3 \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{psi}{\frac{N}{m^2}}\right)$$

$$= \underline{\underline{4.52 \text{ psia}}}$$

1.81

1.81 At what atmospheric pressure will water boil at 35 °C? Express your answer in both SI and BG units.

The vapor pressure of water at 35°C is 5.81 kPa (abs) (from Table B.2 in Appendix B using linear interpolation). Thus, if water boils at this temperature the atmospheric pressure must be equal to $\underline{\underline{5.81 \text{ kPa (abs)}}}$ in SI units. In BG units,

$$\left(5.81 \times 10^3 \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{\frac{N}{m^2}}{\frac{lb}{in.^2}}\right) = \underline{\underline{0.842 \text{ psi (abs)}}}$$

1.82

1.82 Small droplets of carbon tetrachloride at 68 °F are formed with a spray nozzle. If the average diameter of the droplets is 200 μm what is the difference in pressure between the inside and outside of the droplets?

$$p = \frac{2\sigma}{R} \qquad \text{(Eq. 1.21)}$$

Since $\sigma = 2.69 \times 10^{-2} \frac{N}{m}$ at $68°F (=20°C)$,

$$p = \frac{2\left(2.69 \times 10^{-2} \frac{N}{m}\right)}{100 \times 10^{-6} \, m} = \underline{\underline{538 \text{ Pa}}}$$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

1.83

1.83 A 12-mm diameter jet of water discharges vertically into the atmosphere. Due to surface tension the pressure inside the jet will be slightly higher than the surrounding atmospheric pressure. Determine this difference in pressure.

For equilibrium (see figure),

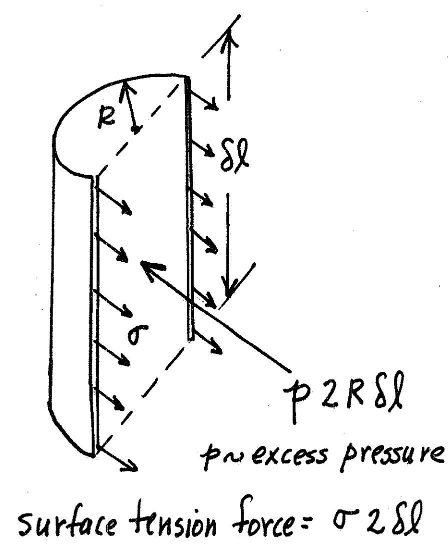
$$p(2R\delta\ell) = \sigma(2\delta\ell)$$

so that

$$p = \frac{\sigma}{R}$$

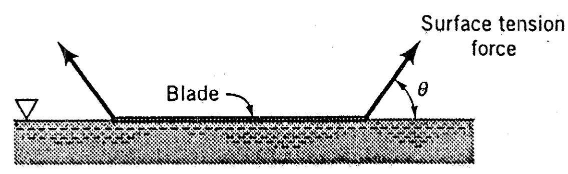
$$= \frac{7.34 \times 10^{-2} \frac{N}{m}}{\frac{12}{2} \times 10^{-3}\ m}$$

$$= \underline{\underline{12.2\ Pa}}$$

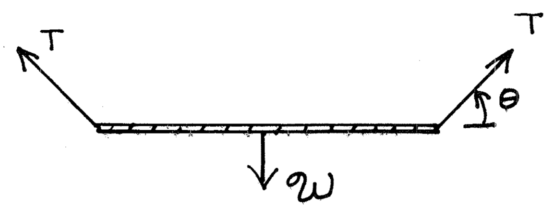


1.84

1.84 As shown in Video V1.5, surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water, but a single-edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in Fig. P1.84. (a) The mass of the double-edge blade is 0.64×10^{-3} kg, and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. (b) The mass of the single-edge blade is 2.61×10^{-3} kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.



■ FIGURE P1.84



(a) $\sum F_{vertical} = 0$

$$\mathcal{W} = T \sin\theta$$

where $\mathcal{W} = m_{blade} \times g$ and $T = \sigma \times$ length of sides.

$$\therefore \; (0.64 \times 10^{-3}\,kg)(9.81\,m/s^2) = (7.34 \times 10^{-2}\,\tfrac{N}{m})(0.206\,m)\sin\theta$$

$$\sin\theta = 0.415$$

$$\underline{\underline{\theta = 24.5^\circ}}$$

(b) For single-edge blade

$$\mathcal{W} = m_{blade} \times g = (2.61 \times 10^{-3}\,kg)(9.81\,m/s^2)$$
$$= 0.0256\,N$$

and

$$T\sin\theta = (\sigma \times \text{length of blade})\sin\theta$$
$$= (7.34 \times 10^{-2}\,N/m)(0.154\,m)\sin\theta$$
$$= 0.0113\sin\theta$$

In order for blade to "float" $\mathcal{W} < T\sin\theta$. Since maximum value for $\sin\theta$ is 1, it follows that $\mathcal{W} > T\sin\theta$ and single-edge blade will sink.

1.85

1.85 To measure the water depth in a large open tank with opaque walls, an open vertical glass tube is attached to the side of the tank. The height of the water column in the tube is then used as a measure of the depth of water in the tank. **(a)** For a true water depth in the tank of 3 ft, make use of Eq. 1.22 (with $\theta \simeq 0°$) to determine the percent error due to capillarity as the diameter of the glass tube is changed. Assume a water temperature of 80 °F. Show your results on a graph of percent error versus tube diameter, D, in the range 0.1 in. $< D <$ 1.0 in. **(b)** If you want the error to be less than 1%, what is the smallest tube diameter allowed?

(a) The excess height, h, caused be the surface tension is

$$h = \frac{2\sigma\cos\theta}{\gamma R} \qquad \text{(Eq. 1.22)}$$

For $\theta \cong 0°$ with $D = 2R$

$$h = \frac{4\sigma}{\gamma D} \qquad (1)$$

From Table B.1 in Appendix B for water at 80°F
$\sigma = 4.91 \times 10^{-3}$ lb/ft and $\gamma = 62.22$ lb/ft³.
Thus, from Eq. (1)

$$h\,(\text{ft}) = \frac{4\left(4.91 \times 10^{-3}\,\frac{\text{lb}}{\text{ft}}\right)}{\left(62.22\,\frac{\text{lb}}{\text{ft}^3}\right)\dfrac{D\,(\text{in.})}{12\text{ in./ft}}} = \frac{3.79 \times 10^{-3}}{D\,(\text{in.})} \qquad (2)$$

Since

$$\%\ \text{error} = \frac{h\,(\text{ft})}{3\ \text{ft}} \times 100 \quad (\text{with the true depth} = 3\ \text{ft})$$

it follows from Eq. (2) that

$$\%\ \text{error} = \frac{3.79 \times 10^{-3}}{3\,D\,(\text{in.})} \times 100$$

$$= \frac{0.126}{D\,(\text{in.})} \qquad (3)$$

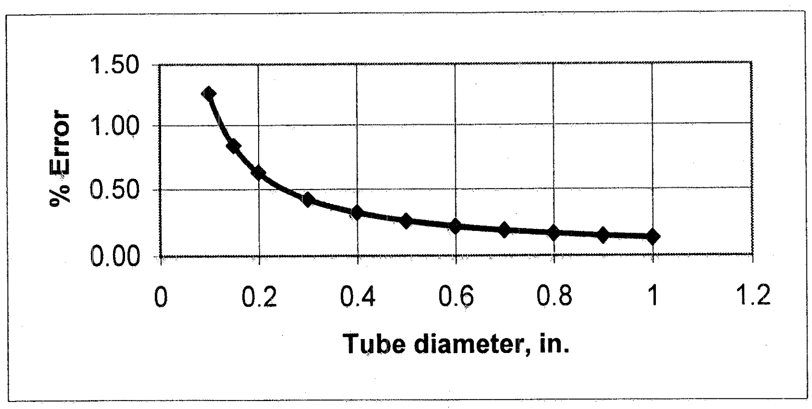
A plot of % error versus tube diameter is shown on the next page.

(con't)

1.85 (con't)

Diameter of tube, in.	% Error
0.1	1.26
0.15	0.84
0.2	0.63
0.3	0.42
0.4	0.32
0.5	0.25
0.6	0.21
0.7	0.18
0.8	0.16
0.9	0.14
1	0.13

Values obtained from Eq. (3)



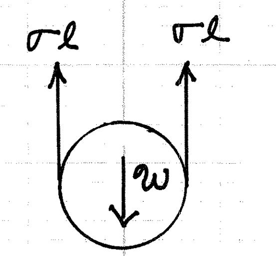
(b) For 1% error from Eq. (3)

$$1 = \frac{0.126}{D(\text{in.})}$$

$$D = \underline{\underline{0.126\ \text{in.}}}$$

1.86

1.86 Under the right conditions, it is possible, due to surface tension, to have metal objects float on water. (See Video V1.5.) Consider placing a short length of a small diameter steel (sp. wt. = 490 lb/ft^3) rod on a surface of water. What is the maximum diameter that the rod can have before it will sink? Assume that the surface tension forces act vertically upward. *Note:* A standard paper clip has a diameter of 0.036 in. Partially unfold a paper clip and see if you can get it to float on water. Do the results of this experiment support your analysis?



ℓ = rod length

In order for rod to float (see figure) it follows that

$$2\sigma\ell \geq w \geq \left(\frac{\pi}{4}\right)(D^2)\,\ell\,\gamma_{steel}$$

Thus, for the limiting case

$$D_{max}^2 = \frac{2\sigma\ell}{\left(\frac{\pi}{4}\right)\ell\,\gamma_{steel}} = \frac{8\sigma}{\pi\,\gamma_{steel}}$$

so that

$$D_{max} = \left[\frac{8\left(5.03\times10^{-3}\ \frac{lb}{ft}\right)}{\pi\left(490\ \frac{lb}{ft^3}\right)}\right]^{1/2} = 5.11\times10^{-3}\ ft$$

$$= \underline{\underline{0.0614\ in.}}$$

Since a standard steel paper clip has a diameter of 0.036 in., which is less than 0.0614 in., it should float. A simple experiment will verify this. <u>Yes</u>.

1.87

1.87 An open, clean glass tube, having a diameter of 3 mm, is inserted vertically into a dish of mercury at 20 °C. How far will the column of mercury in the tube be depressed?

$$h = \frac{2\sigma \cos\theta}{\gamma R} \qquad (\text{Eq. } 1.22)$$

For $\theta = 130°$,

$$h = \frac{2\left(4.66 \times 10^{-1} \frac{N}{m}\right) \cos 130°}{\left(133 \times 10^{3} \frac{N}{m^3}\right)(0.0015\ m)} = -3.00 \times 10^{-3}\ m$$

Thus, column will be depressed $\underline{\underline{3.00\ mm}}$

1.88

1.88 An open, clean glass tube ($\theta = 0°$) is inserted vertically into a pan of water. What tube diameter is needed if the water level in the tube is to rise one tube diameter (due to surface tension)?

$$h = \frac{2\sigma \cos\theta}{\gamma R} \qquad (\text{Eq. } 1.22)$$

For $h = 2R$ and $\theta = 0°$

$$2R = \frac{2\sigma(1)}{\gamma R}$$

and

$$R^2 = \frac{\sigma}{\gamma} = \frac{5.03 \times 10^{-3} \frac{lb}{ft}}{62.4 \frac{lb}{ft^3}}$$

$$R = 8.98 \times 10^{-3}\ ft$$

$$\text{diameter} = 2R = \underline{\underline{1.80 \times 10^{-2}\ ft}}$$

1.89

1.89 *

1.89* The capillary rise in a tube depends on the cleanliness of both the fluid and the tube. Typically, values of h are less than those predicted by Eq. 1.22 using values of σ and θ for clean fluids and tubes. Some measurements of the height, h, a water column rises in a vertical open tube of diameter, d, are given below. The water was tap water at a temperature of 60 °F and no particular effort was made to clean the glass tube. Fit a curve to these data and estimate the value of the product $\sigma \cos \theta$. If it is assumed that σ has the value given in Table 1.5 what is the value of θ? If it is assumed that θ is equal to 0° what is the value of σ?

d (in.)	0.3	0.25	0.20	0.15	0.10	0.05
h (in.)	0.133	0.165	0.198	0.273	0.421	0.796

From Eq. 1.22

$$h = \frac{2\sigma \cos\theta}{\gamma}\left(\frac{1}{R}\right) = \frac{4\sigma \cos\theta}{\gamma}\left(\frac{1}{d}\right) \qquad (1)$$

with $d = 2R$. Thus, Eq.(1) is of the form

$$h = b\, d' \qquad (2)$$

where:

$$b = \frac{4\sigma\cos\theta}{\gamma} \quad \text{and} \quad d' = \frac{1}{d}$$

The constant, b, can be obtained by a linear least squares fit of the given data (h and $1/d$).

$1/d$ (ft^{-1})	h (ft)
40	0.01108
48	0.01375
60	0.01650
80	0.02275
120	0.03508
240	0.06633

A plot of the data shows a good fit with a linear curve and a fit of the data using a standard curve-fitting program such as found in EXCEL gives

$$h = 2.799 \times 10^{-4}(1/d)$$

so that $b = 2.799 \times 10^{-4}\ ft^2$

(con't)

1.89* (con't)

Thus,

$$\sigma \cos\theta = \frac{b\,\gamma}{4}$$

$$= \frac{(2.799 \times 10^{-4}\ ft^2)(62.4\ \frac{lb}{ft^3})}{4} = \underline{\underline{4.37 \times 10^{-3}\ \frac{lb}{ft}}}$$

If $\sigma = 5.03 \times 10^{-3}$ lb/ft, then

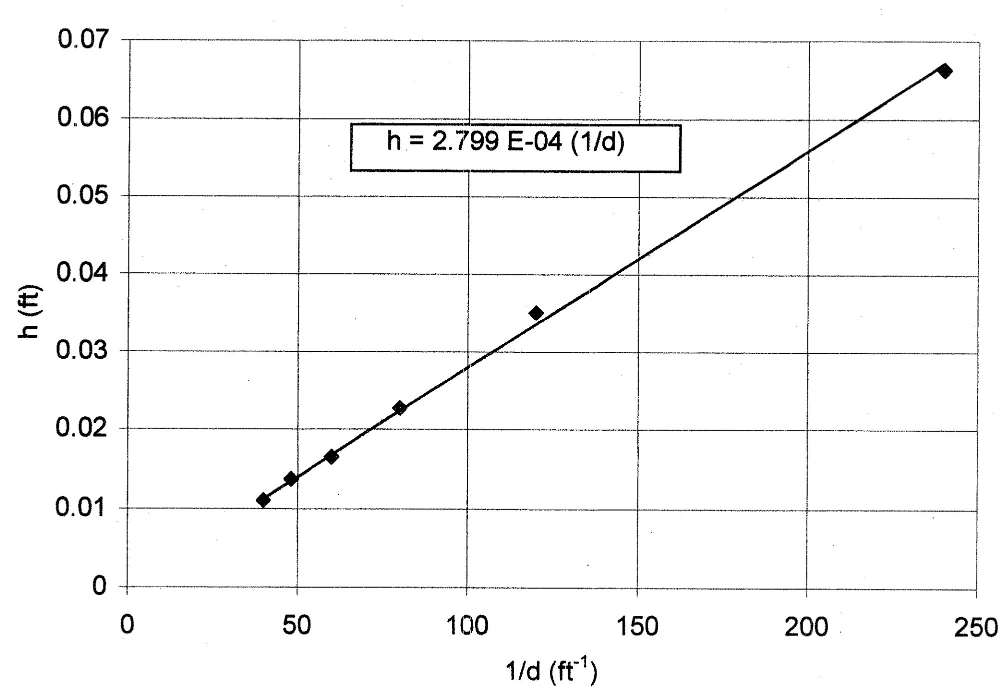
$$\cos\theta = \frac{4.37 \times 10^{-3}\ \frac{lb}{ft}}{5.03 \times 10^{-3}\ \frac{lb}{ft}} = 0.869$$

and

$$\underline{\underline{\theta = 29.7^\circ}}$$

If $\theta = 0^\circ$ Then $\cos\theta = 1.0$ and

$$\sigma = \frac{4.37 \times 10^{3}\ \frac{lb}{ft}}{1.0} = \underline{\underline{4.37 \times 10^{3}\ \frac{lb}{ft}}}$$



1.90

1.90 Fluid Characterization by Use of a Stormer Viscometer

Objective: As discussed in Section 1.6, some fluids can be classified as Newtonian fluids; others are non-Newtonian. The purpose of this experiment is to determine the shearing stress versus rate of strain characteristics of various liquids and, thus, to classify them as Newtonian or non-Newtonian fluids.

Equipment: Stormer viscometer containing a stationary outer cylinder and a rotating, concentric inner cylinder (see Fig. P1.90); stop watch; drive weights for the viscometer; three different liquids (silicone oil, Latex paint, and corn syrup).

Experimental Procedure: Fill the gap between the inner and outer cylinders with one of the three fluids to be tested. Select an appropriate drive weight (of mass m) and attach it to the end of the cord that wraps around the drum to which the inner cylinder is fastened. Release the brake mechanism to allow the inner cylinder to start to rotate. (The outer cylinder remains stationary.) After the cylinder has reached its steady-state angular velocity, measure the amount of time, t, that it takes the inner cylinder to rotate N revolutions. Repeat the measurements using various drive weights. Repeat the entire procedure for the other fluids to be tested.

Calculations: For each of the three fluids tested, convert the mass, m, of the drive weight to its weight, $W = mg$, where g is the acceleration of gravity. Also determine the angular velocity of the inner cylinder, $\omega = N/t$.

Graph: For each fluid tested, plot the drive weight, W, as ordinates and angular velocity, ω, as abscissas. Draw a best fit curve through the data.

Results: Note that for the flow geometry of this experiment, the weight, W, is proportional to the shearing stress, τ, on the inner cylinder. This is true because with constant angular velocity, the torque produced by the viscous shear stress on the cylinder is equal to the torque produced by the weight (weight times the appropriate moment arm). Also, the angular velocity, ω, is proportional to the rate of strain, du/dy. This is true because the velocity gradient in the fluid is proportional to the inner cylinder surface speed (which is proportional to its angular velocity) divided by the width of the gap between the cylinders. Based on your graphs, classify each of the three fluids as to whether they are Newtonian, shear thickening, or shear thinning (see Fig. 1.5).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

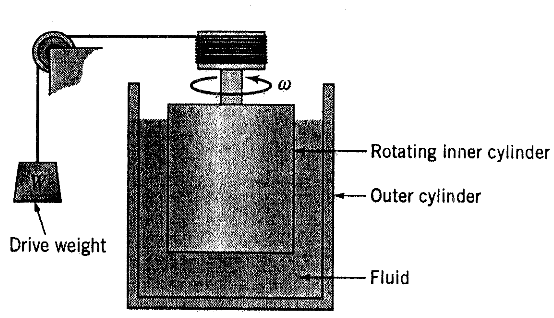


FIGURE P1.90

(con't)

1.90 (con't)

Solution for Problem 1.90: Fluid Characterization by Use of a Stormer Viscometer

m, kg	N, revs	t, s	ω, rev/s	W, N
Silicone Oil Data				
0.02	4	59.3	0.07	0.20
0.05	12	66.0	0.18	0.49
0.10	24	64.2	0.37	0.98
0.15	20	35.0	0.57	1.47
0.20	24	31.7	0.76	1.96
0.25	30	31.0	0.97	2.45
0.30	20	17.4	1.15	2.94
0.35	25	18.8	1.33	3.43
0.40	40	26.0	1.54	3.92
Corn Syrup Data				
0.05	1	28.2	0.04	0.49
0.10	2	27.5	0.07	0.98
0.20	4	27.2	0.15	1.96
0.40	8	25.7	0.31	3.92
Latex Paint Data				
0.02	2	32.7	0.06	0.20
0.03	2	20.2	0.10	0.29
0.04	5	32.2	0.16	0.39
0.05	10	47.3	0.21	0.49
0.06	10	37.2	0.27	0.59
0.07	10	29.8	0.34	0.69
0.08	10	24.6	0.41	0.78
0.09	10	20.1	0.50	0.88
0.10	20	34.0	0.59	0.98

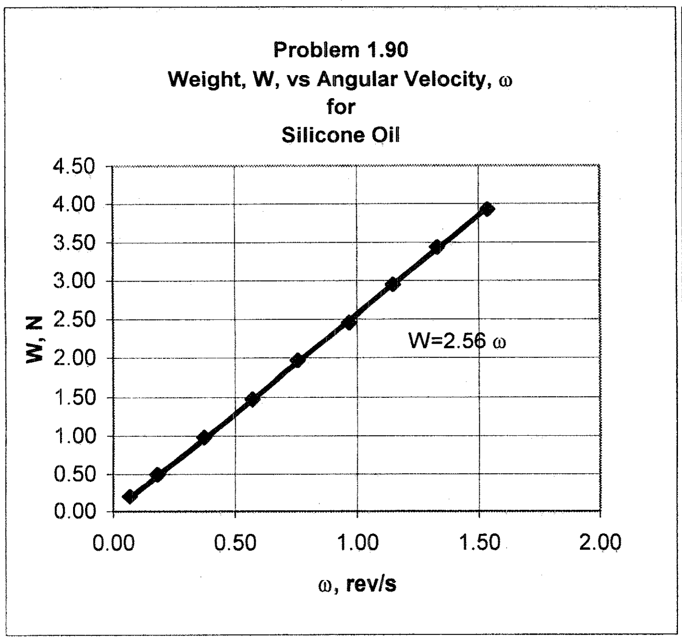
From the graphs:
Silicone oil is Newtonian
Corn Syrup is Newtonian
Latex paint is shear thinning

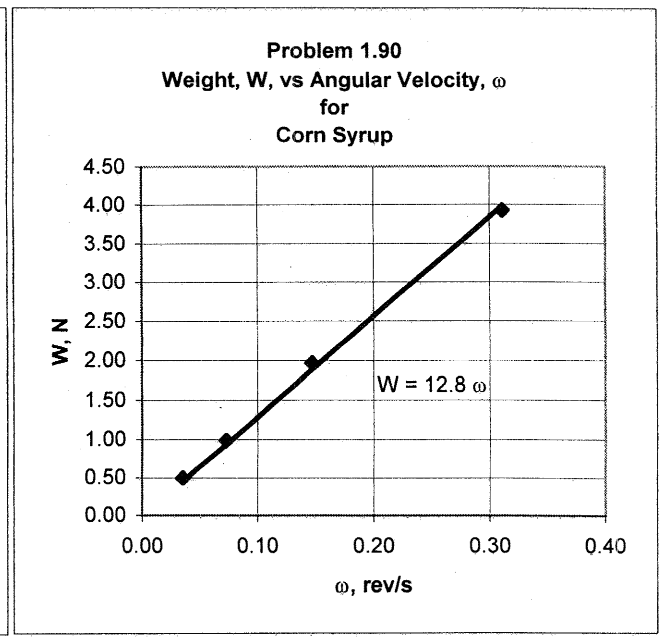
$\omega = N/t$

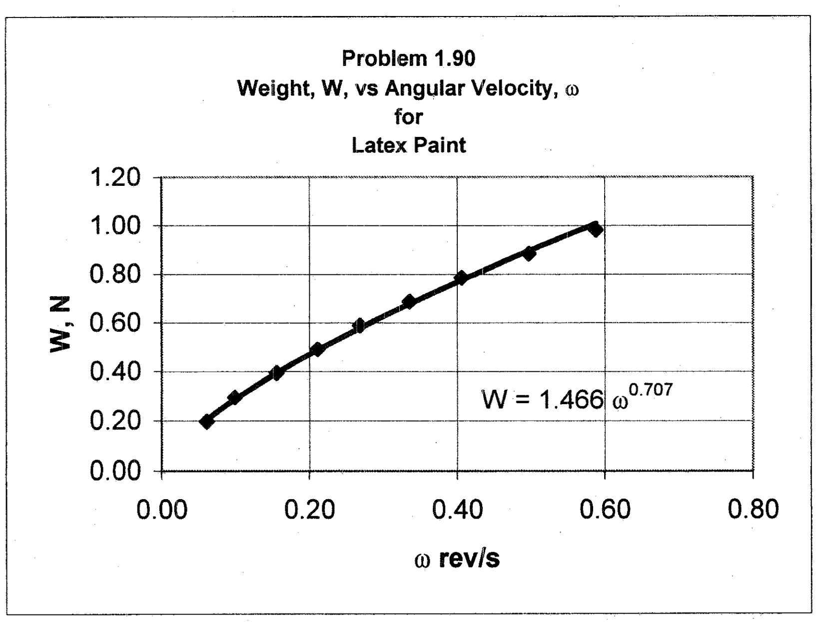
$W = mg$

(con't)

1.90 (con't)


Problem 1.90
Weight, W, vs Angular Velocity, ω
for
Silicone Oil
W=2.56 ω
W, N
ω, rev/s
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.00
0.50
1.00
1.50
2.00


Problem 1.90
Weight, W, vs Angular Velocity, ω
for
Corn Syrup
W = 12.8 ω
W, N
ω, rev/s
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.00
0.10
0.20
0.30
0.40


Problem 1.90
Weight, W, vs Angular Velocity, ω
for
Latex Paint
W = 1.466 ω^0.707
W, N
ω rev/s
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.00
0.20
0.40
0.60
0.80

1.91

1.91 Capillary Tube Viscometer

Objective: The flowrate of a viscous fluid through a small diameter (capillary) tube is a function of the viscosity of the fluid. For the flow geometry shown in Fig. P1.91, the kinematic viscosity, ν, is inversely proportional to the flowrate, Q. That is, $\nu = K/Q$, where K is the calibration constant for the particular device. The purpose of this experiment is to determine the value of K and to use it to determine the kinematic viscosity of water as a function of temperature.

Equipment: Constant temperature water tank, capillary tube, thermometer, stop watch, graduated cylinder.

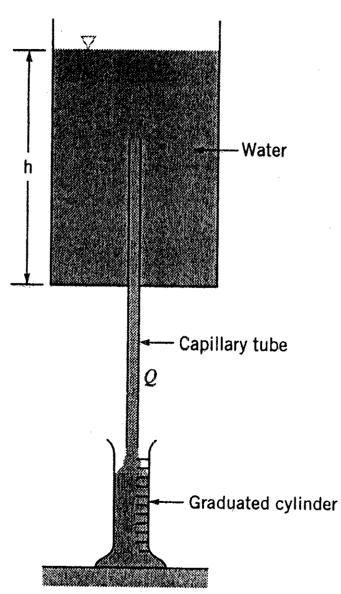
Experimental Procedure: Adjust the water temperature to 15.6°C and determine the flowrate through the capillary tube by measuring the time, t, it takes to collect a volume, V, of water in a small graduated cylinder. Repeat the measurements for various water temperatures, T. Be sure that the water depth, h, in the tank is the same for each trial. Since the flowrate is a function of the depth (as well as viscosity), the value of K obtained will be valid for only that value of h.

Calculations: For each temperature tested, determine the flowrate, $Q = V/t$. Use the data for the 15.6°C water to determine the calibration constant, K, for this device. That is, $K = \nu Q$, where the kinematic viscosity for 15.6°C water is given in Table 1.5 and Q is the measured flowrate at this temperature. Use this value of K and your other data to determine the viscosity of water as a function of temperature.

Graph: Plot the experimentally determined kinematic viscosity, ν, as ordinates and temperature, T, as abscissas.

Results: On the same graph, plot the standard viscosity-temperature data obtained from Table B.2.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P1.91

(con't)

1.91 (Con't)

Solution for Problem 1.91: Capillary Tube Viscometer

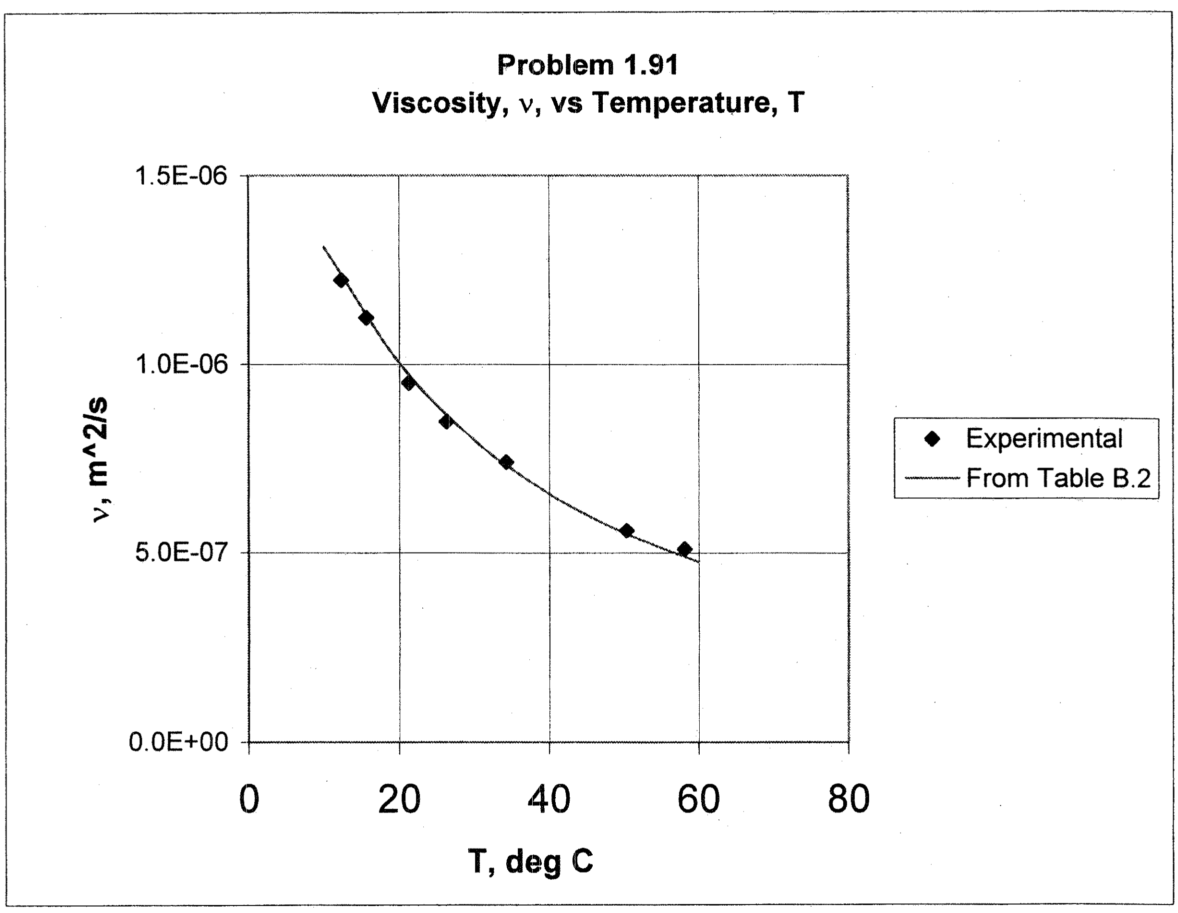
V, ml	t, s	T, deg C	Q, ml/s	ν, m^2/s
9.2	19.8	15.6	0.465	1.12E-06
9.7	15.8	26.3	0.614	8.49E-07
9.2	16.8	21.3	0.548	9.51E-07
9.1	21.3	12.3	0.427	1.22E-06
9.2	13.1	34.3	0.702	7.42E-07
9.4	10.1	50.4	0.931	5.60E-07
9.1	8.9	58.1	1.022	5.10E-07

From Table B.2

T, deg C	ν, m^2/s
10	1.31E-06
20	1.00E-06
30	8.01E-07
40	6.58E-07
50	5.53E-07
60	4.75E-07

ν = K/Q	K, m^2 ml/s^2	ν (at 15.6 deg C), m^2/s
	5.21E-07	1.12E-06

K = ν Q = 1.12E-6 m^2/s * 0.465 ml/s = 5.21E-7 m^2 ml/s^2



1.92

1.92 (See "A vital fluid," Section 1.6) Some measurements on a blood sample at 37 °C (98.6 °F) indicate a shearing stress of 0.52 N/m^2 for a corresponding rate of shearing strain of 200 s^{-1}. Determine the apparent viscosity of the blood and compare it with the viscosity of water at the same temperature.

$$\tau = \mu \frac{du}{dy} = \mu \dot{\gamma}$$

$$\mu_{blood} = \frac{\tau}{\dot{\gamma}} = \frac{0.52 \frac{N}{m^2}}{200 \frac{1}{s}} = \underline{\underline{26.0 \times 10^{-4} \frac{N \cdot s}{m^2}}}$$

From Table B.2 in Appendix B:

@ 30°C $\quad \mu_{H_2O} = 7.975 \times 10^{-4} \frac{N \cdot s}{m^2}$

@ 40°C $\quad \mu_{H_2O} = 6.529 \times 10^{-4} \frac{N \cdot s}{m^2}$

Thus, with linear interpolation, $\mu_{H_2O}(37°C) = 6.96 \times 10^{-4} \frac{N \cdot s}{m^2}$

and

$$\frac{\mu_{blood}}{\mu_{H_2O}} = \frac{26.0 \times 10^{-4} \frac{N \cdot s}{m^2}}{6.96 \times 10^{-4} \frac{N \cdot s}{m^2}} = \underline{\underline{3.74}}$$

1.93

1.93 (See "This water jet is a blast," Section 1.7.1) By what percent is the volume of water decreased if its pressure is increased to an equivalent to 3000 atmospheres (44,100 psi)?

$$E_v = -\frac{dp}{d\forall / \forall} \approx -\frac{\Delta p}{\Delta \forall / \forall} \qquad \text{(Eq. 1.12)}$$

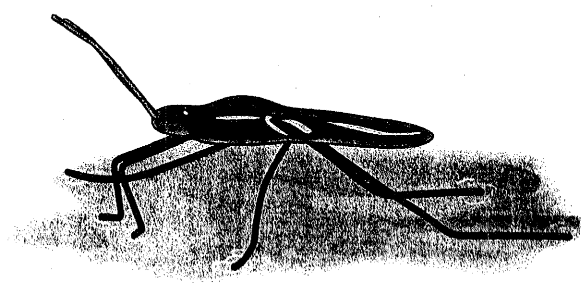
$$\frac{\Delta \forall}{\forall} = -\frac{\Delta p}{E_v} = -\frac{44{,}100 \text{ psia} - 14.7 \text{ psia}}{3.12 \times 10^5 \text{ psia}} = -0.141$$

Thus,

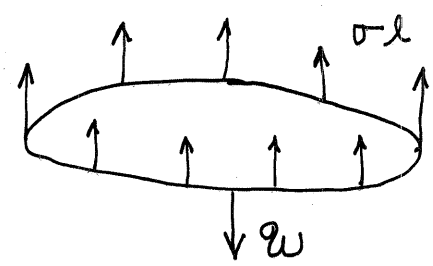
% decrease in volume = $\underline{\underline{14.1\%}}$

1.94

1.94 (See "Walking on water," Section 1.9.) **(a)** The water strider bug shown in Fig. P1.94 is supported on the surface of a pond by surface tension acting along the interface between the water and the bug's legs. Determine the minimum length of this interface needed to support the bug. Assume the bug weighs 10^{-4} N and the surface tension force acts vertically upwards. **(b)** Repeat part (a) if surface tension were to support a person weighing 750 N.



■ FIGURE P1.94



For equilibrium,

$$\mathcal{W} = \sigma \ell$$

$\mathcal{W} \sim$ weight
$\sigma \sim$ surface tension
$\ell \sim$ length of interface

(a) $$\ell = \frac{\mathcal{W}}{\sigma} = \frac{10^{-4}\ \text{N}}{7.34 \times 10^{-2}\ \frac{\text{N}}{\text{m}}}$$

$$= 1.36 \times 10^{-3}\ \text{m}$$

$$= (1.36 \times 10^{-3}\ \text{m})\left(10^{3}\ \frac{\text{mm}}{\text{m}}\right) = \underline{\underline{1.36\ \text{mm}}}$$

(b) $$\ell = \frac{750\ \text{N}}{7.34 \times 10^{-2}\ \frac{\text{N}}{\text{m}}} = \underline{\underline{1.02 \times 10^{4}\ \text{m}}} \quad (6.34\ \text{mi}\ !!)$$