3.1

3.1 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.1. The velocity is given by $\mathbf{V} = 10(1 + x)\hat{\mathbf{i}}$ ft/s, where x is in feet. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 50 psi, determine the pressure at (2) by: (i) integration of the pressure gradient obtained in **(a)**; (ii) application of the Bernoulli equation.

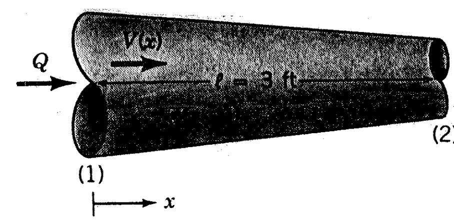


FIGURE P3.1

(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $V = 10(1+x)$ ft/s

$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s}$ or $\frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} = -\rho\,(10(1+x))(10)$

Thus, $\frac{\partial p}{\partial x} = -1.94 \frac{slugs}{ft^3}\left(10 \frac{ft}{s}\right)^2 (1+x)$, with x in feet

$= \underline{\underline{-194(1+x) \frac{lb}{ft^3}}}$

(b)(i) $\frac{dp}{dx} = -194(1+x)$ so that $\int_{p_1 = 50psi}^{p_2} dp = -194 \int_{x_1=0}^{x_2=3} (1+x)\,dx$

or $p_2 = 50\,psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\,ft^2}{144\,in.^2}\right) = 50 - 10.1 = \underline{\underline{39.9\,psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = z_2$

$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$ where $V_1 = 10(1+0) = 10 \frac{ft}{s}$

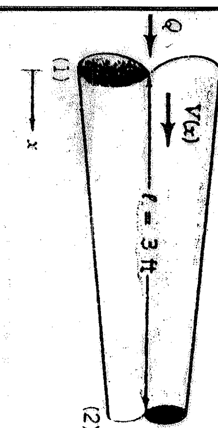
$V_2 = 10(1+3) = 40 \frac{ft}{s}$

Thus,

$p_2 = 50\,psi + \frac{1}{2}\left(1.94 \frac{slugs}{ft^3}\right)(10^2 - 40^2)\frac{ft^2}{s^2}\left(\frac{1\,ft^2}{144\,in^2}\right) = \underline{\underline{39.9\,psi}}$

3.2

3.2 Repeat Problem 3.1 if the pipe is vertical with the flow *down.*



(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ with $\theta = -90^\circ$ and $V = 10(1+x)\ \frac{ft}{s}$

$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} + \gamma$ or $\frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} + \gamma = -\rho(10(1+x))(10) + \gamma$

Thus, $\frac{\partial p}{\partial x} = -1.94\ \frac{slugs}{ft^3}\left(10\ \frac{ft}{s}\right)^2(1+x) + 62.4\ \frac{lb}{ft^3}$, with x in feet

$= \underline{\underline{-194(1+x) + 62.4\ \frac{lb}{ft^3}}}$

(b)(i) $\frac{dp}{dx} = -194(1+x) + 62.4$ so that $\int_{p_1=50psi}^{p_2} dp = \int_{x_1=0}^{x_2=3} [-194(1+x) + 62.4]\,dx$

or $p_2 = 50psi - 194\left(3 + \frac{3^2}{2}\right)\frac{lb}{ft^2}\left(\frac{1\ ft^2}{144\ in.^2}\right) + 62.4(3)\frac{lb}{ft^2}\left(\frac{1 ft^2}{144\ in.^2}\right)$

$= 50 - 10.1 + 1.3 = \underline{\underline{41.2 psi}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = 0$, $z_2 = -3\ ft$

and $V_1 = 10(1+0) = 10\ \frac{ft}{s}$, $V_2 = 10(1+3) = 40\ \frac{ft}{s}$

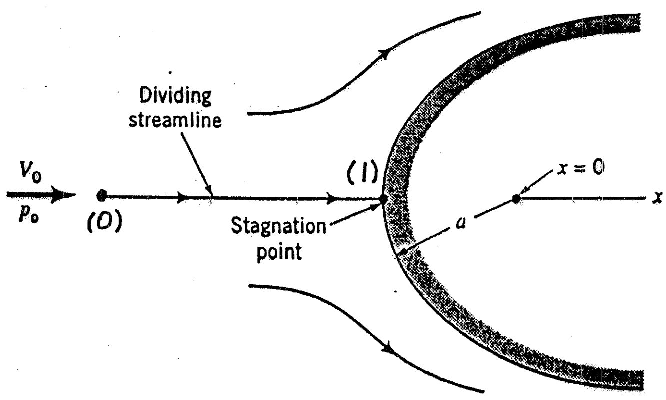
$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) - \gamma z_2$

$= 50psi + \frac{1}{2}\left(1.94\ \frac{slug}{ft^3}\right)(10^2 - 40^2) - 62.4\ \frac{lb}{ft^3}(-3\ ft)$

$= \underline{\underline{41.2 psi}}$

3.3

3.3 An incompressible fluid with density ρ flows steadily past the object shown in Video V3.3 and Fig. P3.3. The fluid velocity along the horizontal dividing streamline $(-\infty \leq x \leq -a)$ is found to be $V = V_0(1 + a/x)$, where a is the radius of curvature of the front of the object and V_0 is the upstream velocity. (a) Determine the pressure gradient along this streamline. (b) If the upstream pressure is p_0, integrate the pressure gradient to obtain the pressure $p(x)$ for $-\infty \leq x \leq -a$. (c) Show from the result of part (b) that the pressure at the stagnation point $(x = -a)$ is $p_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.



■ FIGURE P3.3

(a) $\frac{dp}{ds} = -\rho V \frac{dV}{ds}$ where $V = V_0\left(1 + \frac{a}{x}\right)$

Thus, $\frac{dV}{ds} = \frac{dV}{dx} = -\frac{V_0 a}{x^2}$

or

$$\frac{dp}{ds} = \frac{dp}{dx} = -\rho V_0\left(1 + \frac{a}{x}\right)\left(-\frac{V_0 a}{x^2}\right) = \underline{\underline{\rho a V_0^2\left(\frac{1}{x^2} + \frac{a}{x^3}\right)}}$$

(b) $\int_{p_0}^{p} dp = \int_{x=-\infty}^{x} \frac{dp}{dx}\,dx = \rho a V_0^2 \int_{-\infty}^{x}\left(\frac{1}{x^2} + \frac{a}{x^3}\right)dx$ Note: $p = p_0$ at $x = -\infty$

or

$$p - p_0 = \rho a V_0^2\left[-\frac{1}{x} - \frac{1}{2}\frac{a}{x^2}\right]_{-\infty}^{x}$$

Thus,

$$\underline{\underline{p = p_0 - \rho a V_0^2\left[\frac{1}{x} + \frac{a}{2x^2}\right]}}$$

(c) From part (b), when $x = -a$

$$p\Big|_{x=-a} = p_0 - \rho a V_0^2\left[-\frac{1}{a} + \frac{a}{2a^2}\right] = \underline{\underline{p_0 + \frac{1}{2}\rho V_0^2}}$$

From the Bernoulli equation $p_0 + \frac{1}{2}\rho V_0^2 = p_1 + \frac{1}{2}\rho V_1^2$

where

$$V_1 = V\Big|_{x=-a} = V_0\left(1 + \frac{a}{(-a)}\right) = 0$$

Thus, $p_1 = p_0 + \frac{1}{2}\rho V_0^2$ as expected.

3.4

3.4 What pressure gradient along the streamline, dp/ds, is required to accelerate water in a horizontal pipe at a rate of 30 m/s^2?

$$\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s} \quad \text{where} \quad \theta = 0 \quad \text{and} \quad V\frac{\partial V}{\partial s} = a_s = 0 \frac{m}{s^2}$$

Thus,

$$\frac{\partial p}{\partial s} = -\rho a_s = -999 \frac{kg}{m^3}\left(30 \frac{m}{s^2}\right) = -30{,}000\left(\frac{N}{m^2}\right)/m$$

or

$$\frac{\partial p}{\partial s} = \underline{\underline{-30.0 \ kPa/m}}$$

3.5

3.5 At a given location the air speed is 20 m/s and the pressure gradient along the streamline is 100 N/m^3. Estimate the air speed at a point 0.5 m further along the streamline.

If neglect gravity, $\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s}$ or $\frac{\partial V}{\partial s} = -\frac{\partial p}{\partial s}/\rho V$

or $\frac{\partial V}{\partial s} = -100 \frac{N}{m^3} / \left(1.23 \frac{kg}{m^3}\right)\left(20 \frac{m}{s}\right) = -4.07 \frac{1}{s}$

Thus,

$\delta V \approx \frac{\partial V}{\partial s}\delta s = \left(-4.07 \frac{1}{s}\right)(0.5 m) = -2.03 \frac{m}{s}$, so that $V + \delta V = 20 \frac{m}{s} - 2.03 \frac{m}{s}$

or $V \approx \underline{\underline{18.0 \frac{m}{s}}}$

3.6

3.6 What pressure gradient along the streamline, dp/ds, is required to accelerate water upward in a vertical pipe at a rate of 30 ft/s^2? What is the answer if the flow is downward?

$$\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s}$$

where $\theta = 90°$ for up flow, $\theta = -90°$ for down flow, and $V\frac{\partial V}{\partial s} = a_s = 30 \frac{ft}{s^2}$

Thus, for upflow

$$\frac{\partial p}{\partial s} = -62.4(1)\frac{lb}{ft^3} - 1.94 \frac{slugs}{ft^3}\left(30 \frac{ft}{s^2}\right) = -120.6\left(\frac{lb}{ft^2}\right)/ft = \underline{\underline{-0.838 \frac{psi}{ft}}}$$

and for downflow

$$\frac{\partial p}{\partial s} = -62.4(-1)\frac{lb}{ft^3} - 1.94 \frac{slugs}{ft^3}\left(30 \frac{ft}{s}\right) = 4.20 \left(\frac{lb}{ft^3}\right)/ft = \underline{\underline{0.0292 \frac{psi}{ft}}}$$

3.7

3.7 Consider a compressible fluid for which the pressure and density are related by $p/\rho^n = C_0$, where n and C_0 are constants. Integrate the equation of motion along the streamline, Eq. 3.6, to obtain the "Bernoulli equation" for this compressible flow as $[n/(n-1)]p/\rho + V^2/2 + gz = \text{constant}$.

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant along a streamline}$$

and

$$\rho^n = \frac{p}{C_0} \quad \text{or} \quad \rho = \frac{p^{1/n}}{C_0^{1/n}} \quad \text{so that}$$

$$\int \frac{dp}{\rho} = C_0^{\frac{1}{n}} \int \frac{dp}{p^{\frac{1}{n}}} = C_0^{\frac{1}{n}} \int p^{-\frac{1}{n}} dp = C_0^{\frac{1}{n}} \frac{1}{(1-\frac{1}{n})} p^{1-\frac{1}{n}} + \text{const.}$$

Thus,

$$\int \frac{dp}{\rho} = \frac{n}{n-1} p \left(\frac{C_0}{p}\right)^{\frac{1}{n}} = \frac{n}{n-1} \frac{p}{\rho}$$

Hence: $\underline{\underline{\frac{n}{n-1} \frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{constant along a streamline}}}$

***3.8**

*3.8 A wind of velocity V_0 blows past a smokestack of radius $a = 2.5$ ft as shown in Fig. P3.8. The fluid velocity along the dividing streamline $(-\infty \leq x \leq -a)$ is found to be $V = V_0(1 - a^2/x^2)$. Plot the pressure distribution from a distance 30 ft ahead of the smokestack to the stagnation point on the smokestack for wind speeds of $V_0 = 0, 10, 20, 30, 40,$ and 50 mph.

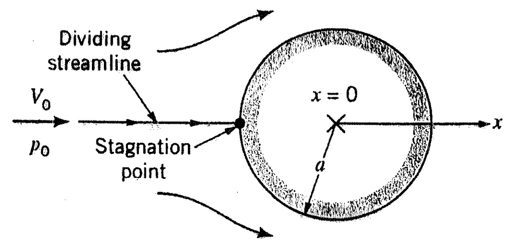


FIGURE P3.8

From the Bernulli eqn. with z=constant:

$$p_0 + \frac{1}{2}\rho V_0^2 = p + \frac{1}{2}\rho V^2, \text{ or with } p_0 = 0:$$

$$p = \frac{1}{2}\rho[V_0^2 - V^2] = \frac{1}{2}\rho[V_0^2 - V_0^2(1 - a^2/x^2)^2]$$

$$= \frac{1}{2}\rho V_0^2[1 - 1 + 2(a/x)^2 - (a/x)^4]$$

or

$$p = \frac{1}{2}\rho V_0^2[2(a/x)^2 - (a/x)^4]$$

Hence, with the given data:

$$p = \frac{1}{2}(0.00238\ slugs/ft^3)[2(2.5\ ft/x)^2 - (2.5\ ft/x)^4]V_0^2(mph)^2(88\ ft/s/60mph)^2$$

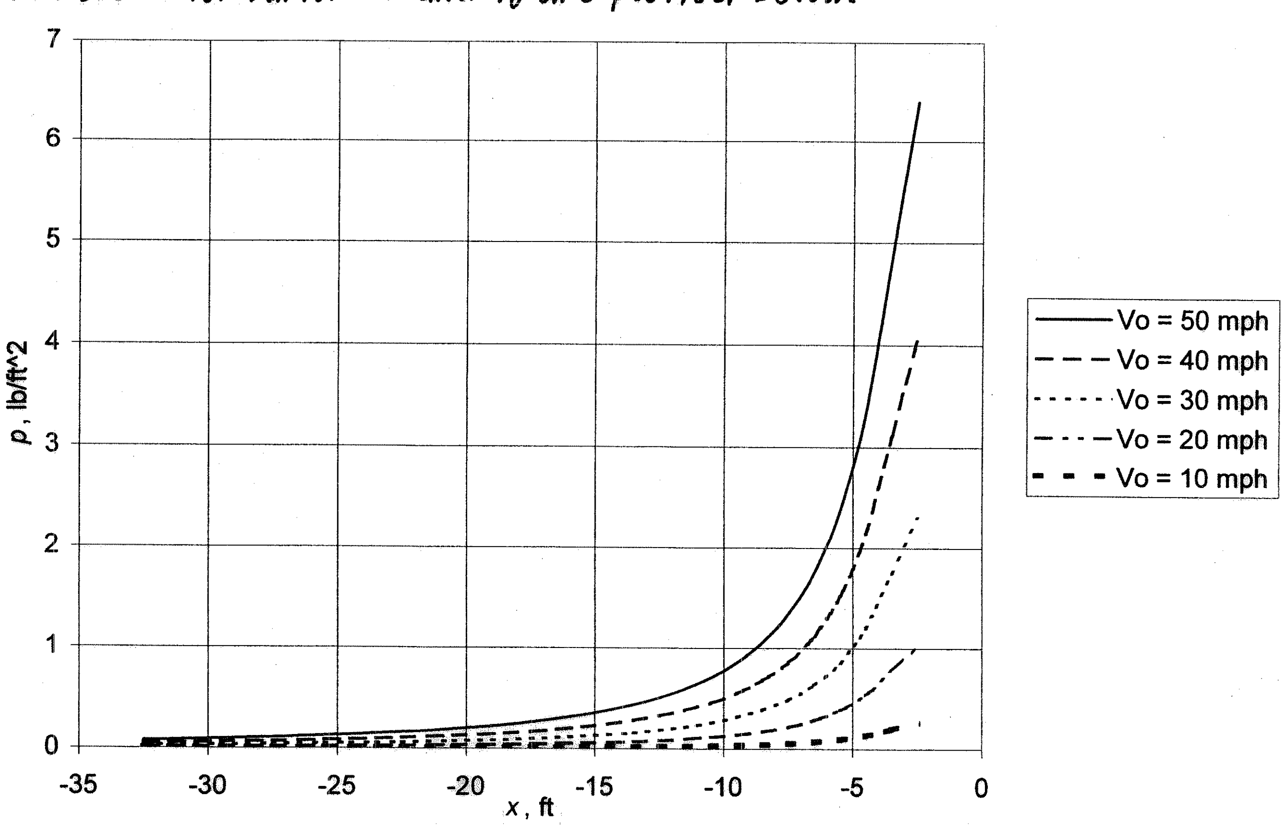
or

$$p = 0.00256[2(2.5/x)^2 - (2.5/x)^4]V_0^2\ lb/ft^2, \text{ where } V_0 \sim mph \text{ and } x \sim ft$$

For example, with $x = -3.5$ ft and $V_0 = 50$ mph,

$$p = 0.00256[2(2.5/3.5)^2 - (2.5/3.5)^4](50)^2 = 4.86\ lb/ft^2$$

The results for various x and V_0 are plotted below.



Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

3.9

3.9 Consider a compressible liquid that has a constant bulk modulus. Integrate "$\mathbf{F} = m\mathbf{a}$" along a streamline to obtain the equivalent of the Bernoulli equation for this flow. Assume steady, inviscid flow.

From Eq. 3.6

$dp + \frac{1}{2}\rho\, d(V^2) + \gamma\, dz = 0$ where $\gamma = \rho g$

and $dp = E_v \frac{d\rho}{\rho}$ where

E_v = bulk modulus = constant

(see Eq. 1.13)

Thus, along a streamline:

$$E_v \frac{d\rho}{\rho} + \frac{1}{2}\rho\, d(V^2) + \rho g\, dz = 0 \quad \text{or}$$

$$E_v \frac{d\rho}{\rho^2} + d\left(\frac{1}{2}V^2\right) + g\, dz = 0$$

which can be integrated between between points (1) and (2) to give

$$E_v \int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho^2} + \int_{V_1}^{V_2} d\left(\frac{1}{2}V^2\right) + \int_{z_1}^{z_2} g\, dz = 0$$

or

$$-E_v\left[\frac{1}{\rho_2} - \frac{1}{\rho_1}\right] + \frac{1}{2}\left[V_2^2 - V_1^2\right] + g\left[z_2 - z_1\right] = 0$$

Hence:

$$gz - \frac{E_v}{\rho} + \frac{V^2}{2} = \text{constant along a streamline}$$

3.10

3.10 Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in Fig. P3.10. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

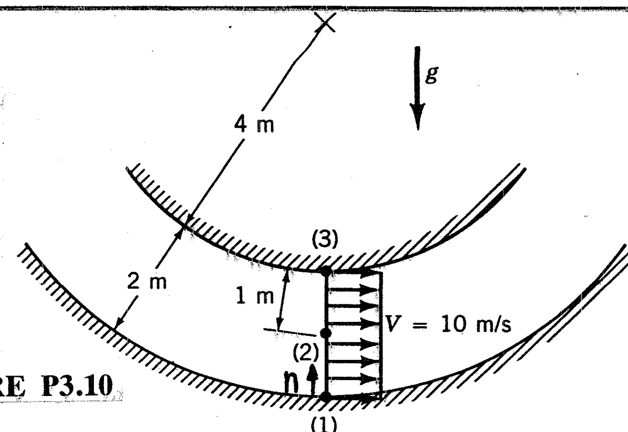


FIGURE P3.10

$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}} \quad \text{with } \frac{dz}{dn} = 1 \text{ and } V = 10\,m/s$$

Thus, with $\mathcal{R} = 6-n$

$$\frac{dp}{dn} = -\gamma - \frac{\rho V^2}{6-n} \quad \text{or}$$

$$\int_{n=0}^{n} \frac{dp}{dn}\,dn = -\int_{n=0}^{n} \gamma\,dn - \int_{n=0}^{n} \frac{\rho V^2\,dn}{6-n}$$

so that since γ and V are constants

$$p - p_1 = -\gamma n - \rho V^2 \int_{n=0}^{n} \frac{dn}{6-n}$$

Thus,

$$p = p_1 - \gamma n - \rho V^2 \ln\left(\frac{6}{6-n}\right)$$

With $p_1 = 40\,kPa$ and $n_2 = 1\,m$: $p_2 = 40\,kPa - 9.8\times10^3\,\frac{N}{m^3}(1\,m) - 999\,\frac{kg}{m^3}\left(10\,\frac{m}{s}\right)^2 \ln\left(\frac{6}{5}\right)$

or

$$p_2 = \underline{\underline{12.0\,kPa}}$$

and

with $p_1 = 40\,kPa$ and $n_3 = 2\,m$: $p_3 = 40\,kPa - 9.80\times10^3\,\frac{N}{m^3}(2\,m) - 999\,\frac{kg}{m^3}\left(10\,\frac{m}{s}\right)^2 \ln\left(\frac{6}{4}\right)$

or

$$p_3 = \underline{\underline{-20.1\,kPa}}$$

3.11

3.11 It can be shown that if viscous and gravitational effects are neglected, the fluid velocity along the surface of a circular cylinder of radius a is $V = 2V_0 \sin\theta$, where V_0 is the upstream velocity and $s = a\theta$ is the distance measured along the streamline that coincides with the cylinder (see Fig. P3.11). For a fluid of density ρ, determine the pressure gradient in the radial direction, $\partial p/\partial r$, on the surface of the cylinder. Assume the axis of the cylinder is vertical. Is $\partial p/\partial r$ positive or negative? Explain physically. For what θ is $\partial p/\partial r$ the maximum? Explain why.

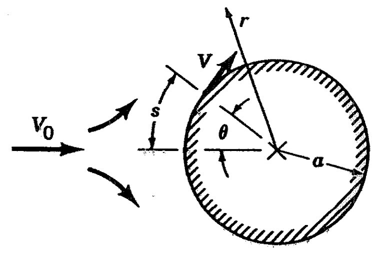


FIGURE P3.11

$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}} \quad \text{with } \frac{\partial}{\partial n} = \frac{\partial}{\partial r}\frac{\partial r}{\partial n} = -\frac{\partial}{\partial r} \text{ and } \frac{dz}{dn} = 0$$

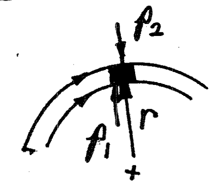
so that

$$\frac{\partial p}{\partial r} = \frac{\rho V^2}{\mathcal{R}} = \frac{\rho V^2}{a} \quad \text{where } V = 2V_0 \sin\theta \text{ and } \mathcal{R} = a$$

Thus,

$$\underline{\underline{\frac{\partial p}{\partial r} = 4\rho V_0^2 \sin^2\theta / a}}$$

Note that for any location (i.e. θ) it follows that $\underline{\underline{\frac{\partial p}{\partial r} > 0}}$, except at $\theta = 0$ or $\theta = 180$ deg where $\frac{\partial p}{\partial r} = 0$

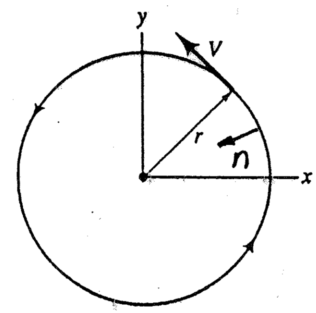


Must have $p_2 > p_1$ if fluid is to follow a curved path, except where $V = 0$ (as at $\theta = 0$ or $\theta = 180$ deg.)

Maximum $\frac{\partial p}{\partial r}$ occurs at $\theta = 90$ deg (i.e. maximum of $\sin^2\theta$) since that is the location of maximum normal acceleration.

3.12

3.12 Water in a container and air in a tornado flow in horizontal circular streamlines of radius r and speed V as shown in Video V3.2 and Fig. P3.12. Determine the radial pressure gradient, $\partial p/\partial r$, needed for the following situations: (a) The fluid is water with $r = 3$ in. and $V = 0.8$ ft/s. (b) The fluid is air with $r = 300$ ft and $V = 200$ mph.



■ FIGURE P3.12

For curved streamlines,

$$-\frac{dp}{dn} = \frac{\rho V^2}{\mathcal{R}} + \gamma \frac{dz}{dn}, \text{ or with } \frac{dz}{dn} = 0 \text{ (horizontal streamlines), } \mathcal{R} = r,$$

and $\frac{d}{dn} = -\frac{d}{dr}$ this becomes

$$\frac{dp}{dr} = \frac{\rho V^2}{r}$$

a) With $r = \frac{3}{12}$ ft and $V = 0.8\,\frac{ft}{s}$ and water ($\rho = 1.94\,\frac{slugs}{ft^3}$),

$$\frac{dp}{dr} = \frac{1.94\,\frac{slugs}{ft^3}\left(0.8\,\frac{ft}{s}\right)^2}{\left(\frac{3}{12}\,ft\right)} = 4.97\,\frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{4.97\,\frac{lb}{ft^3}}}$$

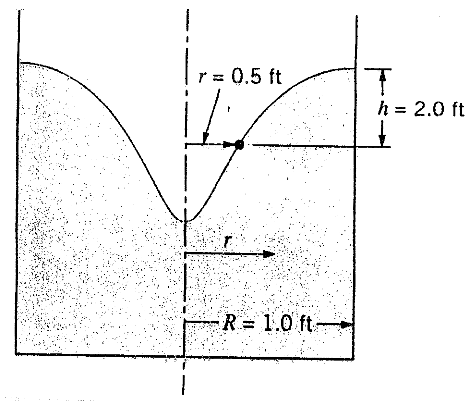
(b) With $r = 300$ ft and $V = 200\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 293\,\frac{ft}{s}$

and air ($\rho = 0.00238\,\frac{slugs}{ft^3}$),

$$\frac{dp}{dr} = \frac{0.00238\,\frac{slugs}{ft^3}\left(293\,\frac{ft}{s}\right)^2}{300\,ft} = 0.681\,\frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{0.681\,\frac{lb}{ft^3}}}$$

3.13

3.13 As shown in Fig. P3.13 and Video V3.2, the swirling motion of a liquid can cause a depression in the free surface. Assume that an inviscid liquid in a tank with an $R = 1.0$ ft radius is rotated sufficiently to produce a free surface that is $h = 2.0$ ft below the liquid at the edge of the tank at a position $r = 0.5$ ft from the center of the tank. Also assume that the liquid velocity is given by $V = K/r$, where K is a constant. **(a)** Show that $h = K^2[(1/r^2) - (1/R^2)]/(2g)$. **(b)** Determine the value of K for this problem.



■ **FIGURE P3.13**

(a) $-\dfrac{dp}{dn} = \dfrac{\rho V^2}{R}$ or $\dfrac{dp}{dr} = \dfrac{\rho V^2}{r} = \dfrac{\rho K^2}{r^3}$

Thus, $\displaystyle\int_p^{p_o} dp = \rho K^2 \int_r^R \frac{dr}{r^3}$ or $p_o - p = -\dfrac{\rho K^2}{2}\left[\dfrac{1}{R^2} - \dfrac{1}{r^2}\right]$

But $p_o = \gamma h$ and $p = 0$ at r on the free surface.

Thus,

$$\gamma h = -\frac{\rho K^2}{2}\left[\frac{1}{R^2} - \frac{1}{r^2}\right] \quad \text{or since } \gamma = \rho g,$$

$$h = \frac{K^2}{2g}\left[\frac{1}{r^2} - \frac{1}{R^2}\right] \qquad (1)$$

(b) With $h = 2$ ft, $R = 1$ ft, and $r = 0.5$ ft Eqn. (1) gives

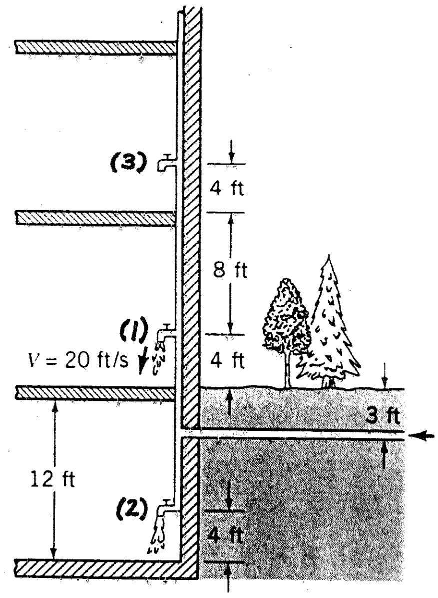
$$2\text{ ft} = \frac{K^2}{2(32.2\text{ ft/s}^2)}\left[\frac{1}{(0.5\text{ ft})^2} - \frac{1}{(1\text{ ft})^2}\right]$$

or

$$K = 6.55\ \frac{\text{ft}^2}{\text{s}}$$

3.14

3.14 Water flows from the faucet on the first floor of the building shown in Fig. P3.14 with a maximum velocity of 20 ft/s. For steady inviscid flow, determine the maximum water velocity from the basement faucet and from the faucet on the second floor (assume each floor is 12 ft tall).



■ FIGURE P3.14

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$

Thus, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $p_2 = p_1 = 0$ (free jet) and $V_1 = 20\,ft/s$, $z_1 = 4\,ft$, $z_2 = -8\,ft$

or

$$\frac{(20\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 4\,ft = \frac{V_2^2}{2(32.2\frac{ft}{s^2})} + (-8\,ft)$$

or $V_2 = \underline{\underline{34.2\,\frac{ft}{s}}}$

and $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ with $p_3 = p_1 = 0$ (free jet) and $V_1 = 20\,\frac{ft}{s}$, $z_1 = 4\,ft$, $z_3 = 16\,ft$

or

$$\frac{(20\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} + 4\,ft = \frac{V_3^2}{2(32.2\frac{ft}{s^2})} + 16\,ft$$

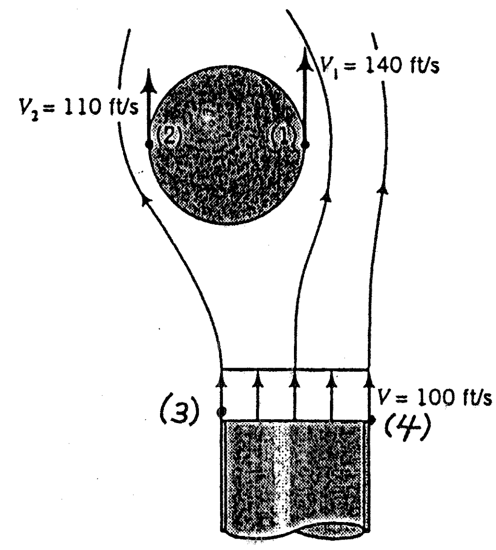
or

$V_3 = \sqrt{20^2 - 2(32.2)(12)} = \sqrt{-373}$ Impossible! No flow from second floor faucet.

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

3.16

3.16 A 100 ft/s jet of air flows past a ball as shown in Video V3.1 and Fig. P3.16. When the ball is not centered in the jet, the air velocity is greater on the side of the ball near the jet center [point (1)] than it is on the other side of the ball [point (2)]. Determine the pressure difference, $p_2 - p_1$, across the ball if $V_1 = 140$ ft/s and $V_2 = 110$ ft/s. Neglect gravity and viscous effects.



■ FIGURE P3.16

The Bernoulli equation from point (3) to (2) and (4) to (1) with gravity neglected gives

$$p_3 + \frac{1}{2}\rho V_3^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \text{and} \quad p_4 + \frac{1}{2}\rho V_4^2 = p_1 + \frac{1}{2}\rho V_1^2$$

But $p_3 = p_4 = 0$ and $V_3 = V_4$

Thus, even though points (1) and (2) are not on the same streamline,

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

or

$$p_1 - p_2 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \frac{1}{2}\left(0.00238\,\frac{\text{slugs}}{\text{ft}^3}\right)\left[\left(140\,\frac{\text{ft}}{\text{s}}\right)^2 - \left(110\,\frac{\text{ft}}{\text{s}}\right)^2\right]$$

$$= 8.93\,\frac{\text{slugs}}{\text{ft}\cdot\text{s}^2} = \underline{\underline{8.93\,\frac{\text{lb}}{\text{ft}^2}}}$$

3.17

3.17 Several holes are punched into a tin can as shown in Fig. P3.17. Which of the figures represents the variation of the water velocity as it leaves the holes? Justify your choice.

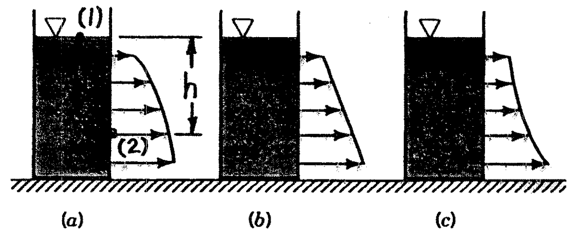


FIGURE P3.17

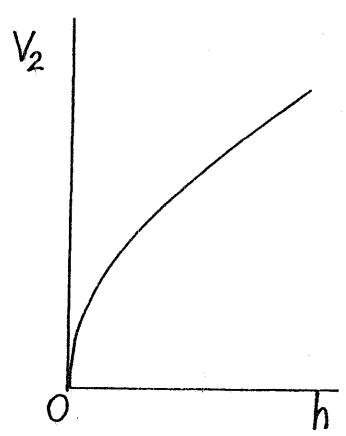
$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$ so that with $V_1 = 0$, $p_1 = 0$ and $z_1 = h_1$ at the free surface, then

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ or with $p_2 = 0$ (free jet) and $z_2 = h_2$

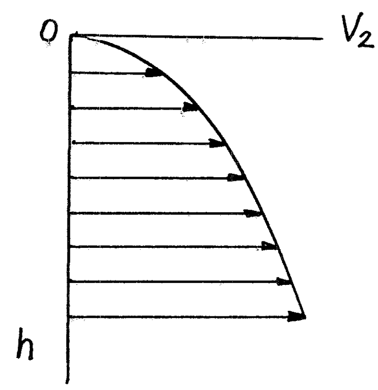
or

$h_1 = \frac{V_2^2}{2g} + h_2$ so that $V_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2gh}$

Thus,



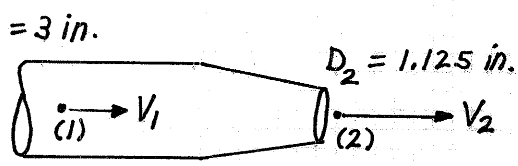
or



<u>Fig. (a) is correct distribution</u>

3.18

3.18 A fire hose nozzle has a diameter of $1\frac{1}{8}$ in. According to some fire codes, the nozzle must be capable of delivering at least 250 gal/min. If the nozzle is attached to a 3-in.-diameter hose, what pressure must be maintained just upstream of the nozzle to deliver this flowrate?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with $z_1 = z_2$, $p_2 = 0$

and $Q = \left(250 \frac{gal}{min}\right)\left(231 \frac{in^3}{gal}\right)\left(\frac{1\ ft^3}{1728\ in^3}\right)\left(\frac{1\ min}{60\ s}\right) = 0.557 \frac{ft^3}{s}$

Thus,

$$p_1 = \frac{\gamma}{2g}\left[V_2^2 - V_1^2\right] \quad \text{where} \quad V_2 = \frac{Q}{A_2} = \frac{0.557 \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{1.125}{12}\right)^2 ft^2} = 80.7 \frac{ft}{s}$$

and

$$V_1 = \frac{Q}{A_1} = \frac{0.557 \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2 ft^2} = 11.34 \frac{ft}{s}$$

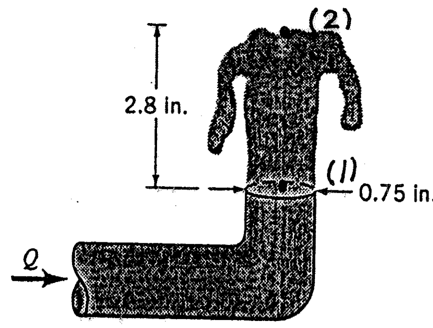
so that with $\frac{\gamma}{g} = \rho$

$$p_1 = \frac{1}{2}\left(1.94 \frac{slugs}{ft^3}\right)\left[80.7^2 - 11.34^2\right] \frac{ft^2}{s^2}$$

$$= 6190 \frac{lb}{ft^2} = \underline{\underline{43.0\ psi}}$$

3.19

3.19 Water flowing from the 0.75-in.-diameter outlet shown in Video V8.6 and Fig. P3.19 rises 2.8 inches above the outlet. Determine the flowrate.



■ FIGURE P3.19

The flowrate is $Q = A_1 V_1$, where from the Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Thus, with $p_1 = p_2 = z_1 = V_2 = 0$ we obtain

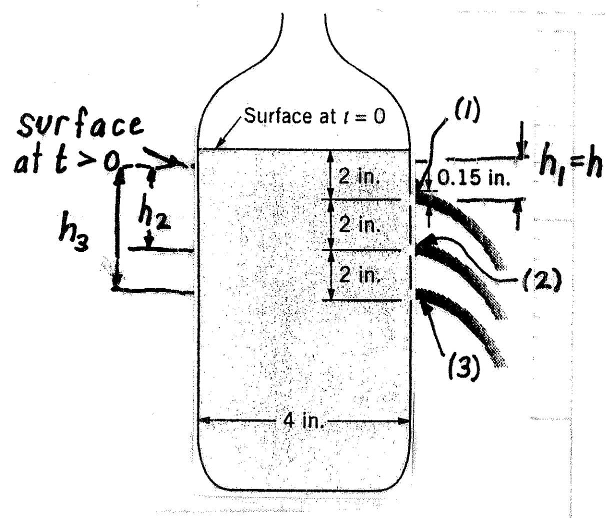
$$V_1 = \sqrt{2 g z_2} = \sqrt{2\,(32.2\ \text{ft/s}^2)\,(2.8/12)\,\text{ft}} = 3.88\ \text{ft/s}$$

so that

$$Q = A_1 V_1 = \frac{\pi}{4}\left(\frac{0.75}{12}\ \text{ft}\right)^2 \left(3.88\ \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.0119\ \frac{\text{ft}^3}{\text{s}}}}$$

3.20

3.20 Pop (with the same properties as water) flows from a 4-in. diameter pop container that contains three holes as shown in Fig. P3.20 (see Video 3.5). The diameter of each fluid stream is 0.15 in., and the distance between holes is 2 in. If viscous effects are negligible and quasi-steady conditions are assumed, determine the time at which the pop stops draining from the top hole. Assume the pop surface is 2 in. above the top hole when $t = 0$. Compare your results with the time you measure from the video.



■ FIGURE P3.20

$$Q = Q_1 + Q_2 + Q_3 = -A_T \frac{dh}{dt}$$

where $Q_i = V_i A_i = \sqrt{2gh_i}\, A_i \quad (i=1,2,3)$ and $A_1 = A_2 = A_3 = \frac{\pi}{4}\left(\frac{0.15}{12}\text{ft}\right)^2 = 1.227\times10^{-4}\text{ft}^2$

$$A_T = \frac{\pi}{4}\left(\frac{4}{12}\text{ft}\right)^2 = 0.0873\text{ ft}^2$$

Thus,

$\sqrt{2g}\, A_1\left[\sqrt{h_1} + \sqrt{h_2} + \sqrt{h_3}\right] = -A_T \frac{dh}{dt}$, where $h_1 = h$, $h_2 = h+L$, $h_3 = h+2L$ and $L = 2\text{ in.}$

Hence,

$$-\left(\sqrt{2g}\,A_1/A_T\right)\int_0^t dt = \int_L^0 \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

where t is the time it take for the free surface to reach the upper hole $(h=0)$.

or

$$t = \frac{A_T}{A_1\sqrt{2g}} \int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

$$= \frac{0.0873\text{ ft}^2}{(1.227\times10^{-4}\text{ft}^2)\left[(2)(32.2\text{ ft/s}^2)\right]^{\frac{1}{2}}} \int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)}$$

Thus,

$$t = 88.7 \int_0^L \frac{dh}{\left(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L}\right)} \quad \text{where } L = \frac{2}{12}\text{ ft} = 0.1667\text{ ft}$$

Note: With L in feet, this equation gives t in seconds.

(con't)

3.20	(con't)

The numerical value of the integral is obtained by using the trapezoidal rule since the closed form analytical solution is not given in integral tables. The EXCEL spread sheet used for this is given below.

$$t = 88.7\int_0^L f(h)\,dh \quad \text{where} \quad f(h) = \frac{1}{(\sqrt{h} + \sqrt{h+L} + \sqrt{h+2L})}$$

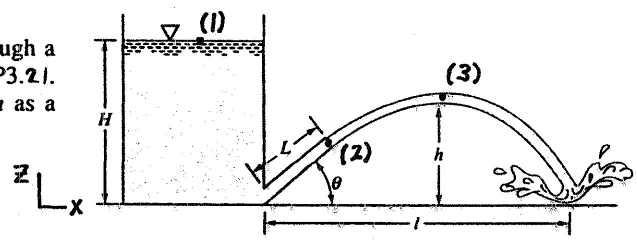
$$\approx 88.7\left[\frac{1}{2}\sum_{i=1}^{20}(f_i + f_{i+1})(h_{i+1} - h_i)\right] = \left(88.7\ \frac{s}{\sqrt{ft}}\right)\left[0.120\sqrt{ft}\right] = \underline{\underline{10.7 s}}$$

h, in.	h, ft	f(h), 1/ft$^{1/2}$	(1/2)*(f_i + f_{i+1})*(h_{i+1} - h_i), ft$^{1/2}$	*i*
0.0	0.0000	1.015	0.00804	1
0.1	0.0083	0.914	0.00743	2
0.2	0.0167	0.870	0.00711	3
0.3	0.0250	0.837	0.00686	4
0.4	0.0333	0.810	0.00665	5
0.5	0.0417	0.786	0.00646	6
0.6	0.0500	0.764	0.00629	7
0.7	0.0583	0.745	0.00614	8
0.8	0.0667	0.728	0.00600	9
0.9	0.0750	0.712	0.00587	10
1.0	0.0833	0.697	0.00575	11
1.1	0.0917	0.684	0.00564	12
1.2	0.1000	0.671	0.00554	13
1.3	0.1083	0.659	0.00544	14
1.4	0.1167	0.647	0.00535	15
1.5	0.1250	0.637	0.00526	16
1.6	0.1333	0.627	0.00518	17
1.7	0.1417	0.617	0.00510	18
1.8	0.1500	0.608	0.00503	19
1.9	0.1583	0.599	0.00496	20
2.0	0.1667	0.591		21
	Sum of column = integral =		0.12011	

Thus, t = 88.7*0.12011 = 10.7 s

3.21

3.21 Water flows from a large tank of depth H, through a pipe of length L, and strikes the ground as shown in Fig. P3.21. Viscous effects are negligible. Determine the distance h as a function of θ.



■ FIGURE P3.21

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\,,\quad \text{where } p_1=p_2=0\,,\ z_1=H,\ z_2=L\sin\theta,\ \text{and } V_1=0$$

Hence, $H=\frac{V_2^2}{2g}+L\sin\theta$

or $\quad V_2=\sqrt{2g(H-L\sin\theta)} \qquad (1)$

Also since from (2) to (3) the only acceleration the particle feels is that of gravity, it follows that $a_x=0$. Thus, $V_3=V_{2x}=V_2\cos\theta \qquad (2)$

From the Bernoulli equation between (1) and (3),

$$\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3=\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1\,,\quad \text{where } p_1=p_3=0\,,\ V_1=0\,,\ z_1=H,\ \text{and } z_3=h$$

or

$$H=\frac{V_3^2}{2g}+h$$

By using Eqs. (1) and (2) this gives

$$H=\frac{V_2^2\cos^2\theta}{2g}+h=\frac{2g(H-L\sin\theta)\cos^2\theta}{2g}+h$$

Thus,

$$h=H(1-\cos^2\theta)+L\sin\theta\cos^2\theta$$

or since $1-\cos^2\theta=\sin^2\theta$, $\quad \underline{\underline{h=H\sin^2\theta+L\sin\theta\cos^2\theta}}$

Note: 1) If $\theta=0$, then $h=0$

2) If $\theta=90^\circ$, then $h=H$

3) If $L\sin\theta>H$, then the above is not valid since $V_2=\sqrt{\text{negative number}}$ (see Eq. 1), which is not possible. Why is this so?

3.22

3.22 A person thrusts his hand into the water while traveling 3 m/s in a motor boat. What is the maximum pressure on his hand?

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with } z_1 = z_2$$

$V_1 = 3 \frac{m}{s}$

$p_1 = 0 \,,\ V_2 = 0$

Thus,

$$p_2 = \frac{\gamma}{2g} V_1^2 = \frac{1}{2}\rho V_1^2 \quad \text{or} \quad p_2 = \frac{1}{2}\left(999 \frac{kg}{m^3}\right)\left(3\frac{m}{s}\right)^2 = 4500 \frac{N}{m^2} = \underline{\underline{4.50\ kPa}}$$

3.23

3.23 A differential pressure gage attached to a Pitot-static tube (see Video V3.4) is calibrated to give speed rather than the difference between the stagnation and static pressures. The calibration is done so that the speed indicated on the gage is the actual fluid speed if the fluid flowing past the Pitot-static tube is air at standard sea level conditions. Assume the same device is used in water and the gage indicates a speed of 200 knots. Determine the water speed.

$$\Delta p = \frac{1}{2}\rho V^2$$

In air, $\Delta p_{air} = \frac{1}{2}\left(0.00238 \frac{slugs}{ft^3}\right)(200\ knots)^2$

In water, $\Delta p_{water} = \frac{1}{2}\left(1.94 \frac{slugs}{ft^3}\right)(V)^2$

so that with $\Delta p_{air} = \Delta p_{water}$,

$$\frac{1}{2}(0.00238)(200)^2 = \frac{1}{2}(1.94)V^2$$

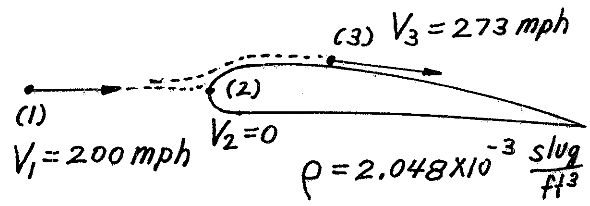
or

$V = \underline{\underline{7.01\ knots}}$

3.24

3.24 When an airplane is flying 200 mph at 5000-ft altitude in a standard atmosphere, the air velocity at a certain point on the wing is 273 mph relative to the airplane. What suction pressure is developed on the wing at that point? What is the pressure at the leading edge (a stagnation point) of the wing?

(a) $p + \frac{1}{2}\rho V^2 + z = \text{constant}$



Thus, with $z_1 \approx z_2 \approx z_3$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^2, \text{ but } p_1 = 0 \text{ so that}$$

$$p_3 = \frac{1}{2}\rho\left[V_1^2 - V_3^2\right] \quad \text{where } V_1 = 200\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 293\,\frac{ft}{s}$$

and

$$V_3 = 273\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 400\,\frac{ft}{s}$$

or

$$p_3 = \frac{1}{2}\left(2.05 \times 10^{-3}\,\frac{slugs}{ft^3}\right)\left[293^2 - 400^2\right]\frac{ft^2}{s^2}$$

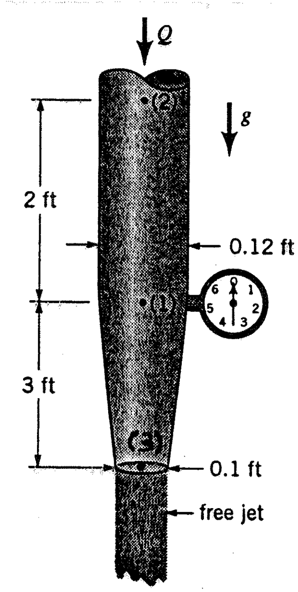
$$= \underline{\underline{-76.0\,\frac{lb}{ft^2}}} \quad (gage)$$

(b) Also,

$$p_2 = \frac{1}{2}\rho V_1^2 = \frac{1}{2}\left(2.05 \times 10^{-3}\,\frac{slugs}{ft^3}\right)\left(293\,\frac{ft}{s}\right)^2 = \underline{\underline{88.0\,\frac{lb}{ft^2}}} \ (gage)$$

3.25

3.25 Water flows steadily downward through the pipe shown in Fig. P3.25. Viscous effects are negligible, and the pressure gage indicates the pressure is zero at point (1). Determine the flowrate and the pressure at point (2).



■ FIGURE P3.25

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$$

where $z_1 = 3\text{ ft},\ z_3 = 0,\ p_1 = p_3 = 0$

and

$$V_1 = \frac{A_3}{A_1} V_3 = \left(\frac{\frac{\pi}{4}(0.1\text{ ft})^2}{\frac{\pi}{4}(0.12\text{ ft})^2}\right) V_3 = 0.694\, V_3$$

Thus,

$$\frac{(0.694)^2 V_3^2}{2(32.2\text{ ft/s}^2)} + 3\text{ ft} = \frac{V_3^2}{2(32.2\text{ ft/s}^2)} \quad \text{or} \quad V_3 = 19.3\,\frac{\text{ft}}{\text{s}}$$

so that

$$Q_3 = A_3 V_3 = \frac{\pi}{4}(0.1\text{ ft})^2 \left(19.3\,\frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.152\,\frac{\text{ft}^3}{\text{s}}}}$$

Also,

$$\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} = \frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}$$

where $p_1 = 0$ and since $A_1 = A_2$ it follows that $V_2 = V_1$

Thus,

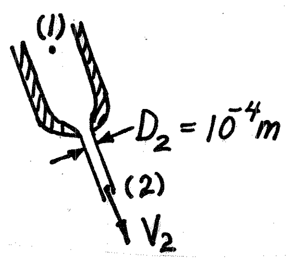
$$z_2 - z_1 = -\frac{p_2}{\gamma} \quad \text{or} \quad \frac{p_2}{\gamma} = -2\text{ ft}$$

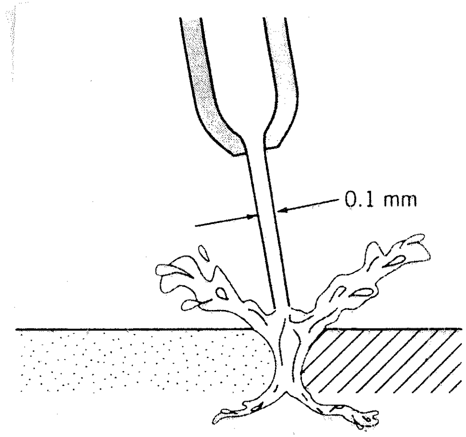
or

$$p_2 = -2\text{ ft}\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right) = \underline{\underline{-125\,\frac{\text{lb}}{\text{ft}^2}}}$$

3.26

3.26 Small-diameter, high-pressure liquid jets can be used to cut various materials as shown in Fig. P3.26. If viscous effects are negligible, estimate the pressure needed to produce a 0.10-mm-diameter water jet with a speed of 700 m/s. Determine the flowrate.





■ FIGURE P3.26

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } V_1 \approx 0,\ z_1 \approx z_2, \text{ and } p_2 = 0$$

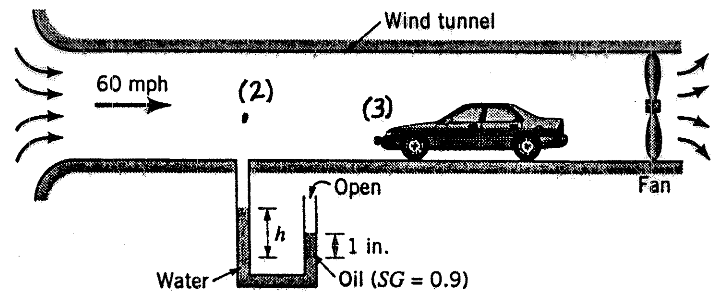
Thus $p_1 = \frac{1}{2}\frac{\gamma}{g}V_2^2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}(999\,\frac{kg}{m^3})(700\,\frac{m}{s})^2 = \underline{\underline{2.45 \times 10^5\,\frac{kN}{m^2}}}$

Also,

$$Q = V_2 A_2 = 700\,\frac{m}{s}\left[\frac{\pi}{4}(10^{-4} m)^2\right] = \underline{\underline{5.50 \times 10^{-6}\,\frac{m^3}{s}}}$$

3.27

3.27 Air is drawn into a wind tunnel used for testing automobiles as shown in Fig. P3.27. **(a)** Determine the manometer reading, h, when the velocity in the test section is 60 mph. Note that there is a 1-in. column of oil on the water in the manometer. **(b)** Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section.



■ FIGURE P3.27

(a) $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

where

$z_1 = z_2$, $p_1 = 0$, and $V_1 \approx 0$

Thus, with $V_2 = 60\text{ mph} = 88\frac{ft}{s}$,

$\frac{p_2}{\gamma} = -\frac{V_2^2}{2g}$ or

$p_2 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}(0.00238\frac{slugs}{ft^3})(88\frac{ft}{s})^2 = -9.22\frac{lb}{ft^2}$

But $p_2 + \gamma_{H_2O}h - \gamma_{oil}(\frac{1}{12}ft) = 0$ where $\gamma_{oil} = 0.9\,\gamma_{H_2O} = 0.9(62.4\frac{lb}{ft^3})$
$= 56.2\frac{lb}{ft^3}$

Thus,

$-9.22\frac{lb}{ft^2} + 62.4\frac{lb}{ft^3}(h\text{ ft}) - 56.2\frac{lb}{ft^3}(\frac{1}{12}ft) = 0$, or $h = \underline{\underline{0.223\text{ ft}}}$

(b) $\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$

where

$z_2 = z_3$ and $V_3 = 0$

Thus,

$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_3}{\gamma}$ or

$p_3 - p_2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}(0.00238\frac{slugs}{ft^3})(88\frac{ft}{s})^2 = \underline{\underline{9.22\frac{lb}{ft^2}}}$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

3.28

3.28 A loon is a diving bird equally at home "flying" in the air or water. What swimming velocity under water will produce a dynamic pressure equal to that when it flies in the air at 40 mph?

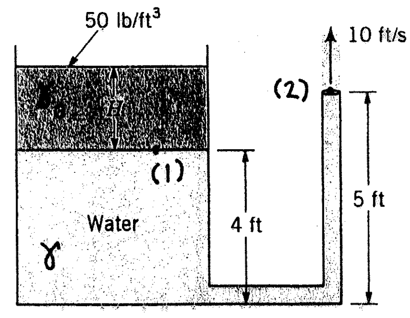
$$\frac{1}{2}\rho_{air}V_{air}^2 = \frac{1}{2}\rho_{H_2O}V_{H_2O}^2 \quad \text{or} \quad V_{H_2O} = \left[\frac{\rho_{air}}{\rho_{H_2O}}\right]^{\frac{1}{2}} V_{air}$$

Thus,

$$V_{H_2O} = \left[\frac{2.38\times10^{-3}\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}}\right](40\ mph) = \underline{\underline{1.40\ mph}}$$

3.29

3.29 Water (assumed inviscid and incompressible) flows steadily with a speed of 10 ft/s from the large tank shown in Fig. P3.29. Determine the depth, H, of the layer of light liquid (specific weight = 50 lb/ft^3) that covers the water in the tank.



■ FIGURE P3.29

From the Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = \gamma_0 H$, $V_1 = 0$, $p_2 = 0$, $z_1 = 4\text{ ft}$, and $z_2 = 5\text{ ft}$

Thus,

$$\frac{\gamma_0}{\gamma} H + z_1 = \frac{V_2^2}{2g} + z_2 \quad \text{so that with } V_2 = 10\text{ ft/s},$$

$$\left(\frac{50\text{ lb/ft}^3}{62.4\text{ lb/ft}^3}\right) H + 4\text{ ft} = \frac{(10\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 5\text{ ft}$$

Therefore,

$$H = \underline{\underline{3.19\text{ ft}}}$$

3.30

3.30 Water flows through the pipe contraction shown in Fig. P3.30. For the given 0.2-m difference in manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

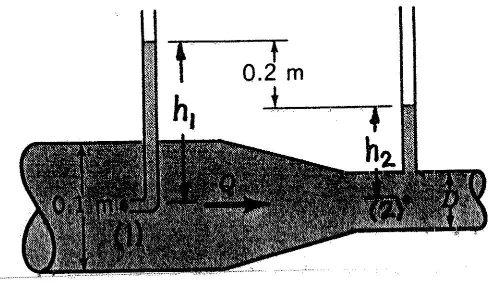


FIGURE P3.30

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ or with $z_1=z_2$ and $V_1=0$

$$V_2=\sqrt{2g\frac{(p_1-p_2)}{\gamma}}$$

but $p_1=\gamma h_1$ and $p_2=\gamma h_2$ so that $p_1-p_2=\gamma(h_1-h_2)=0.2\gamma$

Thus,

$$V_2=\sqrt{2g\frac{0.2\gamma}{\gamma}}=\sqrt{2g(0.2)}$$

or

$$Q=A_2V_2=\frac{\pi}{4}D^2V_2=\frac{\pi}{4}D^2\sqrt{2(9.81)(0.2)}=1.56\,D^2\ \frac{m^3}{s}\ \text{when } D\sim m$$

3.31

3.31 Water flows through the pipe contraction shown in Fig. P3.31. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

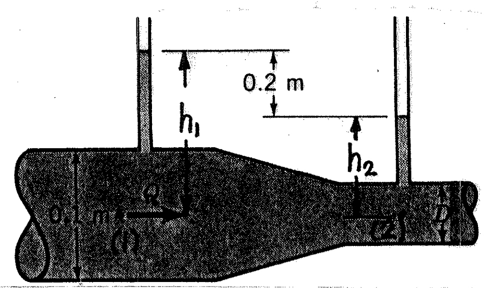


FIGURE P3.31

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad \text{with } A_1V_1 = A_2V_2$$

$$\text{or} \quad V_2 = \frac{\left(\frac{\pi}{4}D_1^2\right)}{\left(\frac{\pi}{4}D_2^2\right)}V_1 = \left(\frac{0.1}{D}\right)^2 V_1$$

Thus, with $z_1 = z_2$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g}$$

but

$p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

$$\frac{0.2\gamma}{\gamma} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{\frac{0.2\,(2g)}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

and

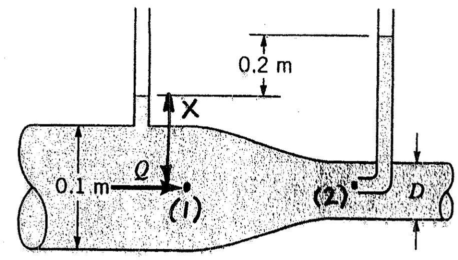
$$Q = A_1V_1 = \frac{\pi}{4}(0.1)^2\sqrt{\frac{0.2\,(2\,(9.81))}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

or

$$\underline{\underline{Q = \frac{0.0156\,D^2}{\sqrt{(0.1)^4 - D^4}}}} \ \frac{m^3}{s} \quad \text{when } D \sim m$$

3.32

3.32 Water flows through the pipe contraction shown in Fig. P3.32. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.



■ FIGURE P3.32

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$ and $V_2 = 0$.

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma}$$

But

$\frac{p_1}{\gamma} = X$ and $\frac{p_2}{\gamma} = 0.2\,m + X$ so that

$X + \frac{V_1^2}{2g} = 0.2\,m + X$ or

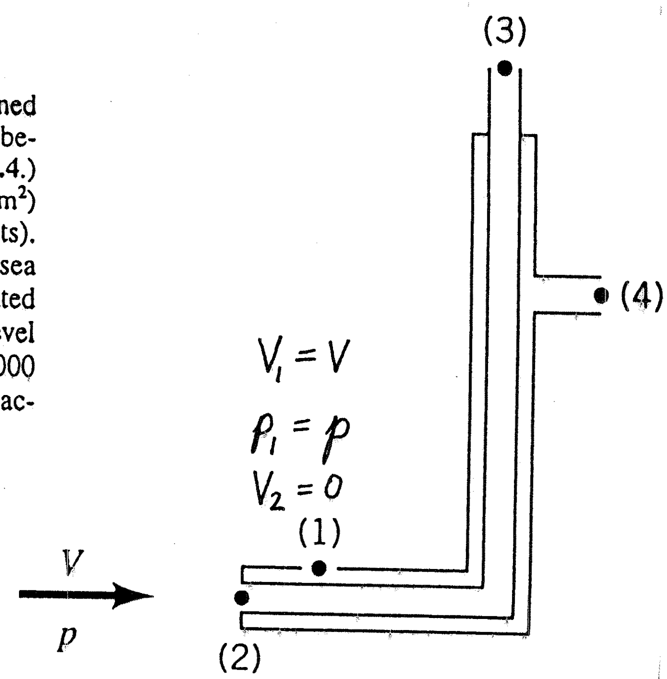
$$V_1 = \sqrt{2g\,(0.2\,m)} = \left(2\left(9.81\,\frac{m}{s^2}\right)(0.2\,m)\right)^{1/2} = 1.98\,\frac{m}{s}$$

Thus,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.1\,m)^2\left(1.98\,\frac{m}{s}\right) = \underline{\underline{0.0156\,\frac{m^3}{s} \text{ for any } D}}$$

3.33

3.33 The speed of an airplane through the air is obtained by use of a Pitot-static tube that measures the difference between the stagnation and static pressures. (See Video V3.4.) Rather than indicating this pressure difference (psi or N/m^2) directly, the indicator is calibrated in speed (mph or knots). This calibration is done using the density of standard sea level air. Thus, the air speed displayed (termed the indicated air speed) is the actual air speed only at standard sea level conditions. If the aircraft is flying at an altitude of 20,000 ft and the indicated air speed is 220 knots, what is the actual air speed?



For the Pitot-static tube shown

$V = \sqrt{2(p_3 - p_4)/\rho}$. Thus, $p_3 - p_4 = \frac{1}{2}\rho V^2$ so that with the same indicated airspeed $(p_3 - p_4)_{standard} = (p_3 - p_4)_{20,000}$ or

$\frac{1}{2}\rho_{standard} V_{standard}^2 = \frac{1}{2}\rho_{20,000} V_{20,000}^2$. Hence,

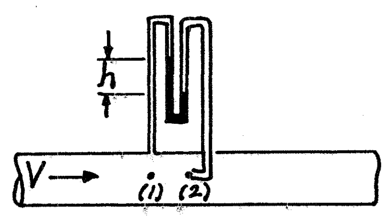
$$V_{20,000} = V_{standard}\left[\frac{\rho_{standard}}{\rho_{20,000}}\right]^{\frac{1}{2}} = 220 \text{ knots}\left[\frac{0.00238 \frac{slugs}{ft^3}}{0.001267 \frac{slugs}{ft^3}}\right]^{\frac{1}{2}}$$

or

$$V_{20,000} = \underline{\underline{302 \text{ knots}}}$$

3.34

3.34 A Pitot-static tube is used to measure the velocity of helium in a pipe. The temperature and pressure are 40 °F and 25 psia. A water manometer connected to the Pitot-static tube indicates a reading of 2.3 in. Determine the helium velocity. Is it reasonable to consider the flow as incompressible? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with $z_1 = z_2$, $V_1 = V$, and $V_2 = 0$

Thus,

$$V_1 = \sqrt{2g\frac{(p_2 - p_1)}{\gamma}} = \sqrt{\frac{2(p_2-p_1)}{\rho}}$$

where

$$\rho = \frac{p}{RT} = \frac{25\,\frac{lb}{in.^2}\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.242\times10^4\,\frac{ft\cdot lb}{slugs\cdot{}^\circ R}\right)(460+40)\,{}^\circ R} = 5.80\times10^{-4}\,\frac{slugs}{ft^3}$$

and since $\gamma_{H_2O} \gg \gamma_{He}$

$$p_2 - p_1 = \gamma_{H_2O}\,h = 62.4\,\frac{lb}{ft^3}\left(\frac{2.3}{12}\,ft\right) = 11.96\,\frac{lb}{ft^2}$$

Thus,

$$V_1 = \sqrt{\frac{2\left(11.96\,\frac{lb}{ft^2}\right)}{5.80\times10^{-4}\,\frac{slugs}{ft^3}}} = \underline{\underline{203\,\frac{ft}{s}}}$$

Note: $M = \frac{V}{c}$ where $c = \sqrt{kRT}$

Thus,

$$c = \left[1.66\,(1.242\times10^4)\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\,(460+40)\,{}^\circ R\right]^{1/2} = 3\,210\,\frac{ft}{s}$$

or

$$M = \frac{203\,\frac{ft}{s}}{3\,210\,\frac{ft}{s}} = 0.063 \ll 0.3$$

Thus, the flow can be considered incompressible.

3.35

3.35 A 0.15-m-diameter pipe discharges into a 0.10-m-diameter pipe. Determine the velocity head in each pipe if they are carrying 0.12 m^3/s of kerosene.

$$V_1 = \frac{Q}{A_1} = \frac{0.12\,\frac{m^3}{s}}{\frac{\pi}{4}(0.15m)^2} = 6.79\,\frac{m}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{0.12\,\frac{m^3}{s}}{\frac{\pi}{4}(0.10m)^2} = 15.27\,\frac{m}{s}$$

Thus,

$$\frac{V_1^2}{2g} = \frac{(6.79\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})} = \underline{\underline{2.35\,m}}$$

and

$$\frac{V_2^2}{2g} = \frac{(15.27\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})} = \underline{\underline{11.9\,m}}$$

3.36

3.36 Carbon tetrachloride flows in a pipe of variable diameter with negligible viscous effects. At point A in the pipe the pressure and velocity are 20 psi and 30 ft/s, respectively. At location B the pressure and velocity are 23 psi and 14 ft/s. Which point is at the higher elevation and by how much?

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \qquad \text{with } \gamma = 99.5\ \frac{lb}{ft^3}$$

or

$$z_B - z_A = \frac{p_A - p_B}{\gamma} + \frac{V_A^2 - V_B^2}{2g} = \frac{(20-23)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)}{99.5\ \frac{lb}{ft^3}} + \frac{(30^2 - 14^2)\frac{ft^2}{s^2}}{2\left(32.2\ \frac{ft}{s^2}\right)}$$

or

$$z_B - z_A = \underline{\underline{6.59\ ft,\quad B \text{ is above } A}}$$

3.37

3.37 Water flows from the large open tank shown in Fig. P3.37. If viscous effects are neglected, determine the heights, h_1, h_2, and h_3, to which the three streams rise.

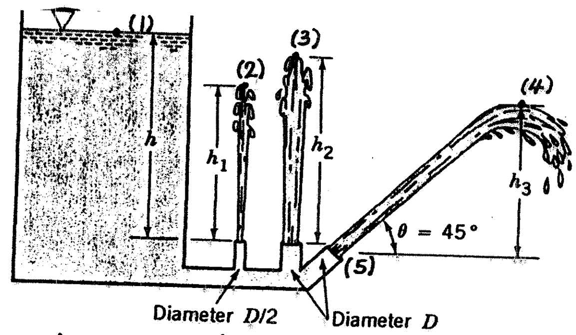


FIGURE P3.37

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = h,\ z_2 = h_1,\ p_1 = p_2 = 0 \text{ and } V_1 = V_2 = 0$$

Thus,

$$\underline{\underline{h_1 = h}}$$

Similarly, since

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad \text{with } z_3 = h_2 \text{ and } V_3 = 0$$

$$\underline{\underline{h_2 = h}}$$

Also,

$$\frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4 = \frac{p_5}{\gamma} + \frac{V_5^2}{2g} + z_5 \quad \text{with } p_4 = p_5 = 0,\ z_5 = 0 \text{ and } z_4 = h_3$$

or

$$h_3 = \frac{V_5^2}{2g} - \frac{V_4^2}{2g} \qquad (1)$$

but $V_5 = \sqrt{2gh}$ and $V_4^2 = V_{4x}^2 + V_{4y}^2 = V_{4x}^2$ since $V_{4y} = 0$

also

$V_{4x} = V_{5x}$ since the acceleration in the horizontal direction is zero.

Thus,

$$V_{4x} = V_5 \cos\theta = \sqrt{2gh}\cos\theta, \quad V_{5y} = \sqrt{2gh}\sin\theta$$

so that Eq. (1) becomes

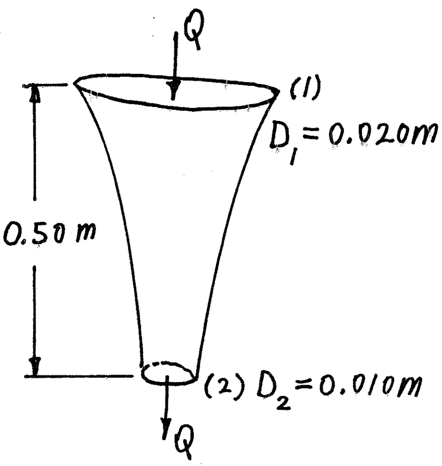
$$h_3 = \frac{V_{5x}^2 + V_{5y}^2}{2g} - \frac{V_{4x}^2 + V_{4y}^2}{2g} = \frac{1}{2g}V_{5y}^2 = \frac{1}{2g}(2gh)\sin^2\theta$$

or

$$h_3 = h\sin^2\theta = h\sin^2 45° = \underline{\underline{0.5\,h}}$$

3.38

3.38 The circular stream of water from a faucet is observed to taper from a diameter of 20 mm to 10 mm in a distance of 50 cm. Determine the flowrate.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = p_2 = 0$, $z_2 = 0$, $z_1 = 0.50m$

and

$$V_1 = \frac{Q}{A_1}, \quad V_2 = \frac{Q}{A_2}$$

Thus,

$$\left(\frac{Q}{A_1}\right)^2 + 2gz_1 = \left(\frac{Q}{A_2}\right)^2 \text{ or } Q = \left[\frac{2gz_1}{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)}\right]^{\frac{1}{2}} = \frac{A_2\sqrt{2gz_1}}{\sqrt{1 - (A_2/A_1)^2}}$$

or since

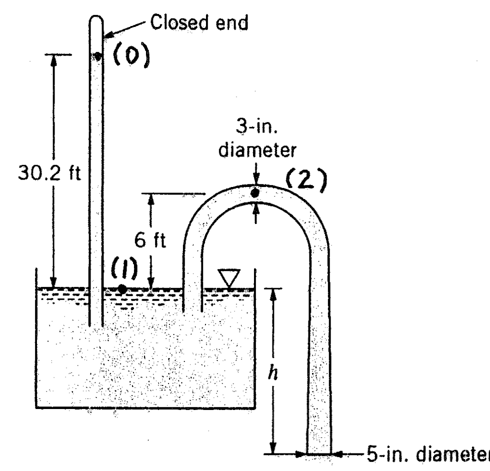
$$\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 \text{ we obtain}$$

$$Q = A_2 \frac{\sqrt{2gz_1}}{\sqrt{1 - (D_2/D_1)^4}} = \frac{\pi}{4}(0.010\, m)^2 \left[\frac{2(9.81\,\frac{m}{s^2})(0.50m)}{1 - \left(\frac{0.010}{0.020}\right)^4}\right]^{\frac{1}{2}}$$

$$= \underline{\underline{2.54 \times 10^{-4}\, \frac{m^3}{s}}}$$

3.39

3.39 Water is siphoned from the tank shown in Fig. P3.39. The water barometer indicates a reading of 30.2 ft. Determine the maximum value of h allowed without cavitation occurring. Note that the pressure of the vapor in the closed end of the barometer equals the vapor pressure.



■ FIGURE P3.39

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } p_1=0,\ V_1=0,\ p_2=p_{vapor}$$

Thus, $\quad z_1=0,\ z_2=6\ ft$

$$0=\frac{p_{vapor}}{\gamma}+\frac{V_2^2}{2g}+6\ ft$$

but $p_0+30.2\ ft\ \gamma=p_1$ or since $p_0=p_{vapor}$, $\frac{p_{vapor}}{\gamma}=-30.2\ ft$

Hence,

$$0=-30.2\ ft+\frac{V_2^2}{2g}+6\ ft \quad \text{or} \quad \frac{V_2^2}{2g}=24.2\ ft \quad \text{or} \quad V_2^2=\left[2\left(32.2\frac{ft}{s^2}\right)(24.2\ ft)\right]$$

Thus,

$$V_2=39.5\frac{ft}{s}$$

Since $V_3A_3=V_2A_2$, $\quad V_3=\frac{A_2}{A_3}V_2=\frac{D_2^2}{D_3^2}V_2=\left(\frac{3\ in.}{5\ in.}\right)^2\left(39.5\frac{ft}{s}\right)$

or

$$V_3=14.2\frac{ft}{s}$$

However,

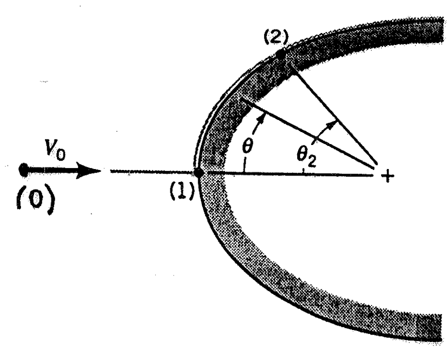
$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3 \quad \text{or} \quad V_3=\sqrt{2gh}$$

Thus,

$$14.2\frac{ft}{s}=\sqrt{2\left(32.2\frac{ft}{s^2}\right)h\ ft} \quad \text{or} \quad \underline{\underline{h=3.13\ ft}}$$

3.40

3.40 An inviscid fluid flows steadily along the stagnation streamline shown in Fig. P3.40 and Video V3.3, starting with speed V_0 far upstream of the object. Upon leaving the stagnation point, point (1), the fluid speed along the surface of the object is assumed to be given by $V = 2\,V_0 \sin\theta$, where θ is the angle indicated. At what angular position, θ_2, should a hole be drilled to give a pressure difference of $p_1 - p_2 = \rho V_0^2/2$? Gravity is negligible.



■ FIGURE P3.40

$$p_0 + \tfrac{1}{2}\rho V_0^2 = p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

where $V_1 = 0$

Thus,

$$p_1 - p_2 = \tfrac{1}{2}\rho\,(V_2^2 - V_1^2) = \tfrac{1}{2}\rho V_2^2$$

so that if

$$p_1 - p_2 = \tfrac{1}{2}\rho V_0^2 \quad \text{then } V_2 = V_0$$

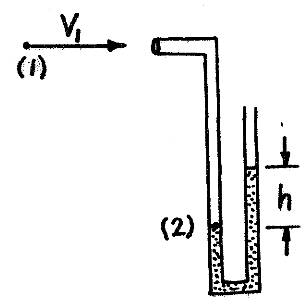
That is:

$$V_2 = 2\,V_0 \sin\theta_2 = V_0 \quad \text{or} \quad \sin\theta_2 = \tfrac{1}{2}$$

Hence, $\theta_2 = \underline{\underline{30^\circ}}$

3.41

3.41 A water-filled manometer is connected to a Pitot-static tube to measure a nominal airspeed of 50 ft/s. It is assumed that a change in the manometer reading of 0.002 in. can be detected. What is the minimum deviation from the 50 ft/s airspeed that can be detected by this system? Repeat the problem if the nominal airspeed is 5 ft/s.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = 0$, $V_2 = 0$

$z_1 \approx z_2$, and $p_2 = \gamma_{H_2O} h$

Thus,

$$\frac{V_1^2}{2g} = \frac{\gamma_{H_2O} h}{\gamma} \quad \text{or} \quad h = \frac{\rho V_1^2}{2\,\gamma_{H_2O}} = \frac{(0.00238 \frac{slugs}{ft^3})(V_1^2 \frac{ft^2}{s^2})(12 \frac{in.}{ft})}{2\,(62.4 \frac{lb}{ft^3})}$$

Hence, $h = 2.29 \times 10^{-4} V_1^2$, where $V_1 \sim ft/s$ and $h \sim in.$

For $V_1 = 50 \frac{ft}{s}$ this gives

$h = 2.29 \times 10^{-4} (50)^2 = 0.573$ in.

while for $V_1 = 5$ ft/s it gives

$h = 2.29 \times 10^{-4} (5)^2 = 0.00573$ in.

With $h \pm 0.002$ in. from these nominal values we obtain

h, in.	V_1, ft/s
0.571	49.9
0.573	50.0
0.575	50.1
0.00373	4.04
0.00573	5.00
0.00773	5.81

Thus, with $V_1 = 50$ ft/s the minimum air speed deviation that can be detected is $\underline{\underline{\pm 0.1 \text{ ft/s}}}$; for $V_1 = 5$ ft/s it is $\underline{\underline{\pm 0.81 \text{ ft/s}}}$.

3.42

3.42 An inviscid fluid flows steadily through the contraction shown in Fig. P3.42. Derive an expression for the fluid velocity at (2) in terms of D_1, D_2, ρ, ρ_m, and h if the flow is assumed incompressible.

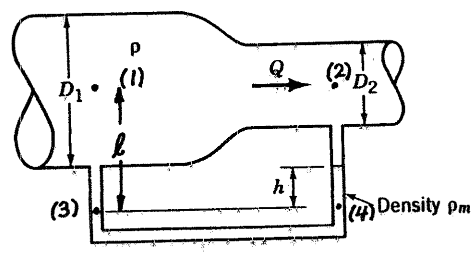


FIGURE P3.42

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2$$

and $V_1 A_1 = V_2 A_2$ or $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$

Thus,

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2}{2g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \quad (1)$$

but

$$p_3 = p_1 + \gamma \ell = p_4 = p_2 + \gamma(\ell - h) + \gamma_m h$$

or

$$p_1 - p_2 = \gamma\ell - \gamma h + \gamma_m h - \gamma\ell = (\gamma_m - \gamma)h = g(\rho_m - \rho)h$$

or

$$\frac{p_1 - p_2}{\gamma} = \left(\frac{\rho_m}{\rho} - 1\right)h \quad (2)$$

Combine Eqs. (1) and (2) to obtain

$$V_2 = \sqrt{\frac{2g\,(p_1 - p_2)/\gamma}{1 - \left(\frac{D_2}{D_1}\right)^4}} = \underline{\underline{\sqrt{\frac{2g\left(\frac{\rho_m}{\rho} - 1\right)h}{1 - \left(\frac{D_2}{D_1}\right)^4}}}}$$

3.43

3.43 A smooth plastic, 10-m-long garden hose with an inside diameter of 20 mm is used to drain a wading pool as is shown in Fig. P3.43. If viscous effects are neglected, what is the flowrate from the pool?

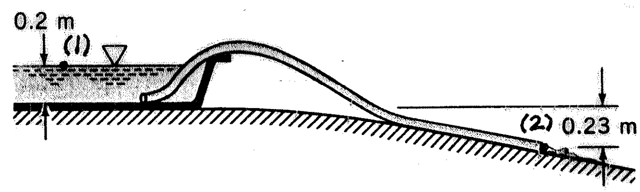


FIGURE P3.43

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = p_2 = 0$, $z_1 = 0.2\,m$, $z_2 = -0.23\,m$, and $V_1 = 0$

Thus,

$$V_2 = \sqrt{2g(z_1 - z_2)} = \left(2\left(9.81\,\frac{m}{s^2}\right)(0.2\,m - (-0.23\,m))\right)^{\frac{1}{2}}$$

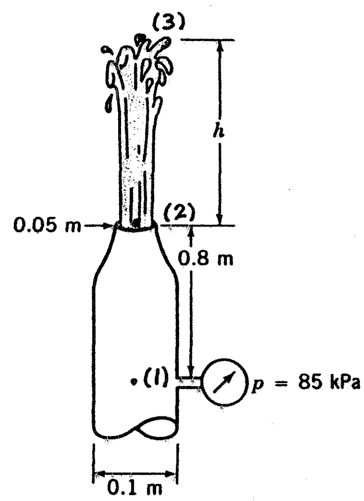
$$= 2.90\,\frac{m}{s}$$

or

$$Q = A_2 V_2 = \frac{\pi}{4}(0.020\,m)^2\left(2.90\,\frac{m}{s}\right) = \underline{\underline{9.11 \times 10^{-4}\,\frac{m^3}{s}}}$$

3.44

3.44 Water flows without viscous effects from the nozzle shown in Fig. P3.44. Determine the flowrate and the height, h, to which the water can flow.



■ FIGURE P3.44

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $z_1 = 0$, $p_2 = 0$

and $A_1 V_1 = A_2 V_2$ or $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{0.05\,m}{0.10\,m}\right)^2 V_2 = 0.25\,V_2$

Thus,

$$V_2 = \sqrt{2g\left(\frac{p_1}{\gamma} - z_2\right) \Big/ \left(1-(0.25)^2\right)}$$

or

$$V_2 = \left[2\left(9.81\frac{m}{s^2}\right)\left(\frac{85\times10^3\frac{N}{m^2}}{9.80\times10^3\frac{N}{m^3}} - 0.8\,m\right) \Big/ \left(1-(0.25)^2\right)\right]^{\frac{1}{2}} = 12.84\,\frac{m}{s}$$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4}(0.05\,m)^2\left(12.84\,\frac{m}{s}\right) = \underline{\underline{0.0252\,\frac{m^3}{s}}}$$

Also,

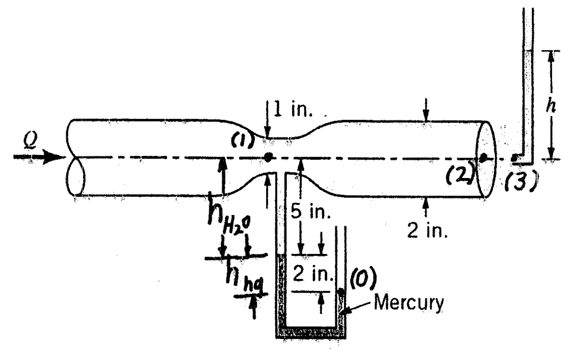
$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$ where $p_3 = 0$, $V_3 = 0$, and $z_3 = z_2 + h$

Thus,

$$h = \frac{V_2^2}{2g} = \frac{\left(12.84\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} = \underline{\underline{8.40\,m}}$$

3.45

3.45 Water flows through a converging–diverging nozzle as shown in Fig. P3.45. Determine (**a**) the volumetric flowrate, Q, through the nozzle and (**b**) the height, h, of the water in the Pitot tube inserted into the free jet. Viscous effects are negligible.



■ **FIGURE P3.45**

(a) From the Bernoulli equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2, \text{ where } z_1 = z_2 \text{ and } p_2 = 0 \qquad (1)$$

Also, $p_1 = p_0 - \gamma_{Hg} h_{Hg} - \gamma_{H_2O} h_{H_2O}$, where $p_0 = 0$ so that

$$p_1 = -13.6(62.4\,\text{lb/ft}^3)(\tfrac{2}{12}\,\text{ft}) - 62.4\,\text{lb/ft}^3(\tfrac{5}{12}\,\text{ft}) = -167\,\text{lb/ft}^2$$

In addition, $A_1 V_1 = A_2 V_2$ or $V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{2\,\text{in.}}{1\,\text{in.}}\right)^2 V_2 = 4V_2$

Hence, Eq. (1) becomes

$$-167\,\text{lb/ft}^2 + \tfrac{1}{2}(1.94\,\text{slugs/ft}^3)(4V_2)^2 = \tfrac{1}{2}(1.94\,\text{slugs/ft}^3)V_2^2$$

or

$$V_2 = 3.39\,\text{ft/s}$$

Thus, $Q = A_2 V_2 = \frac{\pi}{4}\left(\frac{2}{12}\,\text{ft}\right)^2(3.39\,\text{ft/s}) = \underline{\underline{0.0739\,\text{ft}^3/\text{s}}}$

(b) From the Bernoulli equation,

$$p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2 = p_3 + \tfrac{1}{2}\rho V_3^2 + \gamma z_3, \text{ where } p_2 = 0,\ z_2 = 0,\ V_3 = 0, \text{ and } z_3 = 0$$

Thus,

$$\tfrac{1}{2}\rho V_2^2 = p_3, \text{ where } p_3 = \gamma h \text{ so that}$$

$$\tfrac{1}{2}(1.94\,\text{slugs/ft}^3)(3.39\,\text{ft/s})^2 = (62.4\,\text{lb/ft}^3)h$$

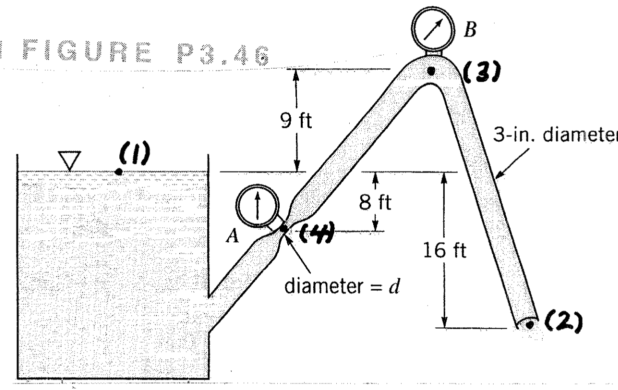
or

$$h = \underline{\underline{0.179\,\text{ft} = 2.14\,\text{in.}}}$$

3.46

3.46 Water flows steadily from a large open tank and discharges into the atmosphere through a 3-in.-diameter pipe as shown in Fig. P3.46. Determine the diameter, d, in the narrowed section of the pipe at A if the pressure gages at A and B indicate the same pressure.

FIGURE P3.46



$p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$, where $z_2 = 0$ and $p_2 = 0$

Thus, since $p_3 = p_4$

$p_3 + \frac{1}{2}\rho V_4^2 + \gamma z_4 = \frac{1}{2}\rho V_2^2$ (1)

However, $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$, where $p_1 = p_2 = V_1 = z_2 = 0$

so that

$\frac{1}{2}\rho V_2^2 = \gamma z_1$ or $V_2 = \sqrt{2\frac{\gamma}{\rho}z_1} = \sqrt{2gz_1} = \left[2(32.2\frac{ft}{s^2})(16\,ft)\right]^{\frac{1}{2}} = 32.1\,ft/s$

But

$p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ where $V_2 = V_3$ since $A_2 = A_3$

Thus,

$p_3 = -\gamma z_3 = -(16+9)ft(62.4\,lb/ft^3) = -1560\,lb/ft^2$ (2)

From Eqs. (1) and (2):

$-1560\frac{lb}{ft^2} + \frac{1}{2}(1.94\frac{slugs}{ft^3})V_4^2 = \frac{1}{2}(1.94\frac{slugs}{ft^3})(32.1\frac{ft}{s})^2$

or

$V_4 = 46.1\,ft/s$

Since $A_4 V_4 = A_2 V_2$ it follows that

$\frac{\pi}{4}d^2 V_4 = \frac{\pi}{4}D_2^2 V_2$

or

$d = D_2\sqrt{\frac{V_2}{V_4}} = (3\,in.)\sqrt{\frac{32.1\,ft/s}{46.1\,ft/s}} = \underline{\underline{2.50\,in.}}$

3.47

3.47 Determine the flowrate through the pipe in Fig. P3.47.

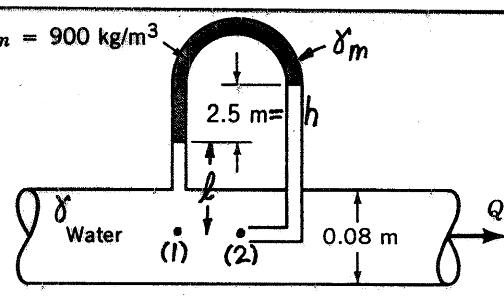


FIGURE P3.47

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2 \text{ and } V_2 = 0$$

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} \quad \text{or} \quad V_1 = \sqrt{2g\frac{(p_2 - p_1)}{\gamma}}$$

but,

$$p_1 - \gamma\ell - \gamma_m h + \gamma(\ell + h) = p_2 \quad \text{or} \quad p_2 - p_1 = (\gamma - \gamma_m)h$$

so that

$$V_1 = \sqrt{2g\left(1 - \frac{\gamma_m}{\gamma}\right)h} = \left[2\left(9.81\frac{m}{s^2}\right)\left(1 - \frac{900\ \frac{kg}{m^3}}{999\ \frac{kg}{m^3}}\right)(2.5\ m)\right]^{1/2}$$

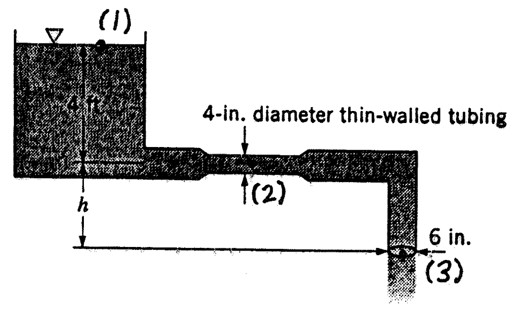
$$= 2.20\ \frac{m}{s}$$

Thus,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.08\ m)^2\left(2.20\ \frac{m}{s}\right) = \underline{\underline{0.0111\ \frac{m^3}{s}}}$$

3.48

3.48 Water flows steadily with negligible viscous effects through the pipe shown in Fig. P3.48. It is known that the 4-in. diameter section of thin-walled tubing will collapse if the pressure within it becomes less than 10 psi below atmospheric pressure. Determine the maximum value that h can have without causing collapse of the tubing.



■ FIGURE P3.48

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = 0,\ V_1 = 0,\ z_2 = 0,$ and $p_2 = -10\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right) = -1440\frac{lb}{ft^2}$

Thus, with $z_1 = 4\,ft$,

$$4\,ft = \frac{-1440\,lb/ft^2}{62.4\,lb/ft^3} + \frac{V_2^2}{2(32.2\,ft/s^2)}$$

or $V_2 = 41.7\,\frac{ft}{s}$

Also,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g}$$

where

$p_3 = 0,\ z_3 = -h,$ and $V_3 = \frac{A_2 V_2}{A_3} = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{4\,in.}{6\,in.}\right)^2 \left(41.7\,\frac{ft}{s}\right)$

$= 18.5\,\frac{ft}{s}$

Thus,

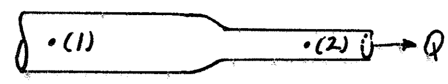
$$4\,ft = -h + \frac{(18.5\,ft/s)^2}{2(32.2\,ft/s^2)}$$

or

$h = \underline{\underline{1.31\,ft}}$

3.49

3.49 Helium flows through a 0.30-m-diameter horizontal pipe with a temperature of 20 °C and a pressure of 200 kPa (abs) at a rate of 0.30 kg/s. If the pipe reduces to 0.25-m-diameter determine the pressure difference between these two sections. Assume incompressible, inviscid flow.



$D_1 = 0.3\,m$ $\quad D_2 = 0.25\,m$

$p_1 = 200\,kPa\ abs$

$T_1 = 20\,°C$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$

Thus,

$$(1) \quad p_1 - p_2 = \tfrac{1}{2}\rho\left(V_2^2 - V_1^2\right) \qquad \text{where } \rho = \frac{p_1}{RT_1} = \frac{200 \times 10^3 \frac{N}{m^2}}{\left(2077 \frac{N\cdot m}{kg\cdot K}\right)(273+20)\,K}$$

or

$$\rho = 0.329 \frac{kg}{m^3}$$

Also,

$$\dot{m} = \rho A_1 V_1 = 0.30 \frac{kg}{s}$$

so that

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{0.30 \frac{kg}{s}}{\left(0.329 \frac{kg}{m^3}\right)\frac{\pi}{4}(0.3\,m)^2} = 12.9 \frac{m}{s}$$

and

$A_1 V_1 = A_2 V_2$ or

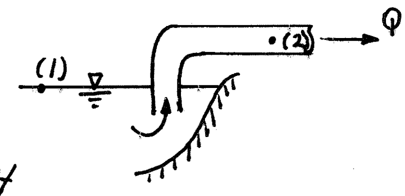
$$V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{0.3\,m}{0.25\,m}\right)^2 \left(12.9 \frac{m}{s}\right) = 18.6 \frac{m}{s}$$

Thus, from Eq. (1):

$$p_1 - p_2 = \frac{1}{2}\left(0.329 \frac{kg}{m^3}\right)\left(18.6^2 - 12.9^2\right)\frac{m^2}{s^2} = \underline{\underline{29.5\,Pa}}$$

3.50

3.50 Water is pumped from a lake through an 8-in. pipe at a rate of 10 ft³/s. If viscous effects are negligible, what is the pressure in the suction pipe (the pipe between the lake and the pump) at an elevation 6 ft above the lake?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = 0$, $V_1 = 0$, $z_1 = 0$, $z_2 = 6.0\ ft$
and

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D_2^2} = \frac{4\left(10\ \frac{ft^3}{s}\right)}{\pi\left(\frac{8}{12}\ ft\right)^2} = 28.6\ \frac{ft}{s}$$

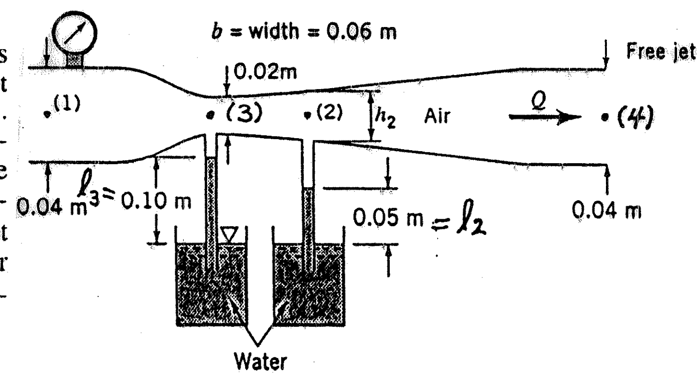
Thus,

$$p_2 = -\gamma z_2 - \frac{1}{2}\rho V_2^2 = -62.4\ \frac{lb}{ft^3}(6.0\ ft) - \frac{1}{2}\left(1.94\ \frac{slugs}{ft^3}\right)\left(28.6\ \frac{ft}{s}\right)^2$$

$$= -1168\ \frac{lb}{ft^2} = \underline{\underline{-8.11\ psi}}$$

3.51

3.51 Air flows through a Venturi channel of rectangular cross section as shown in **Video V3.6** and Fig. P3.51. The constant width of the channel is 0.06 m and the height at the exit is 0.04 m. Compressibility and viscous effects are negligible. **(a)** Determine the flowrate when water is drawn up 0.10 m in a small tube attached to the static pressure tap at the throat where the channel height is 0.02 m. **(b)** Determine the channel height, h_2, at section (2) where, for the same flowrate as in part **(a)**, the water is drawn up 0.05 m. **(c)** Determine the pressure needed at section (1) to produce this flow.



■ FIGURE P3.51

(a) For steady, inviscid, incompressible flow: ($\gamma = 12.0 \frac{N}{m^3}$)

(1) $\frac{p_3}{\gamma} + \frac{V_3^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_3 = -\gamma_{H_2O}\ell_3 = 9.80\times10^3 \frac{N}{m^3}(0.1m)$
$= -980 \frac{N}{m^2}$

Also,

$A_3 V_3 = A_4 V_4$ so that $V_3 = \frac{(0.04m \times 0.06m)}{(0.02m \times 0.06m)} V_4 = 2V_4$

Thus, Eqn. (1) becomes

$$\frac{-980 \frac{N}{m^2}}{12.0 \frac{N}{m^3}} + \frac{4V_4^2}{2(9.81\frac{m}{s^2})} = \frac{V_4^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_4 = 23.1 \frac{m}{s}$$

Hence,

$$Q = A_4 V_4 = (0.04m \times 0.06m)(23.1\tfrac{m}{s}) = \underline{\underline{0.0554 \tfrac{m^3}{s}}}$$

(2) (b) $\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_2 = -\gamma_{H_2O}\ell_2 = 9.80\times10^3 \frac{N}{m^3}(0.05m)$
$= -490 \frac{N}{m^2}$

From part (a), $V_4 = 23.1 \frac{m}{s}$

Thus, Eqn. (2) becomes

$$\frac{-490 \frac{N}{m^2}}{12.0 \frac{N}{m^3}} + \frac{V_2^2}{2(9.81\frac{m}{s^2})} = \frac{(23.1\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_2 = 36.5 \frac{m}{s}$$

But $V_2 A_2 = V_4 A_4$ so that

$$(36.5\tfrac{m}{s})(0.06m)h_2 = (23.1\tfrac{m}{s})(0.06m)(0.04m) \quad \text{or} \quad h_2 = \underline{\underline{0.0253 m}}$$

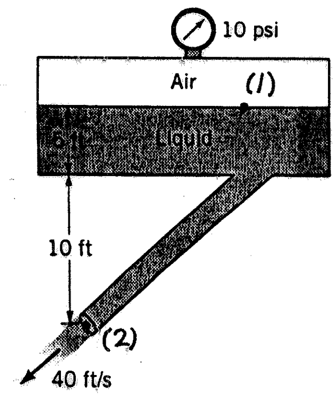
(3) (c) Also, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$ and $A_1 V_1 = A_4 V_4$

But since $A_1 = (0.04m \times 0.06m) = A_4$ then $V_1 = V_4$ and Eqn. (3) gives

$p_1 = p_4 = \underline{\underline{0}}$

3.52

3.52 An inviscid, incompressible liquid flows steadily from the large pressurized tank shown in Fig. P.3.52. The velocity at the exit is 40 ft/s. Determine the specific gravity of the liquid in the tank.



■ FIGURE P3.52

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = 10 \frac{lb}{in.^2}\left(144 \frac{in.^2}{ft^2}\right) = 1440 \frac{lb}{ft^2}$, $p_2 = 0$,

$z_1 = 15\,ft$, $z_2 = 0$, $V_1 = 0$, and $V_2 = 40 \frac{ft}{s}$

Thus,

$$\frac{1440\ lb/ft^2}{\gamma} + 15\,ft = \frac{(40\,ft/s)^2}{2(32.2\,ft/s^2)}$$

or

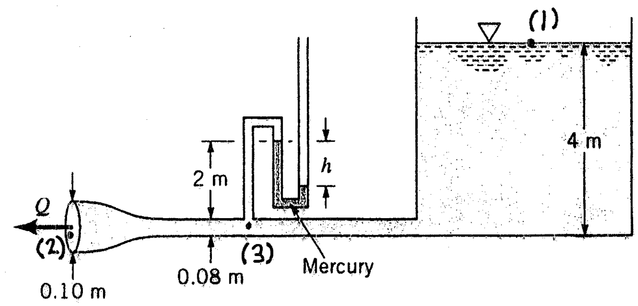
$$\gamma = 146.3 \frac{lb}{ft^3}$$

Hence,

$$SG = \frac{\gamma}{\gamma_{H_2O}} = \frac{146\ lb/ft^3}{62.4\ lb/ft^3} = \underline{\underline{2.34}}$$

3.53

3.53 Water flows steadily from the large open tank shown in Fig. P3.53. If viscous effects are negligible, determine (**a**) the flowrate, Q, and (**b**) the manometer reading, h.



■ FIGURE P3.53

(a) From the Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \text{, where } p_1 = p_2 = 0,\ V_1 = 0,\ z_1 = 4m, \text{ and } z_2 = 0.$$

Thus,

$$\gamma z_1 = \frac{1}{2}\rho V_2^2, \text{ or } \rho g z_1 = \frac{1}{2}\rho V_2^2 \text{ so that } V_2 = \sqrt{2 g z_1}$$

or

$$V_2 = \sqrt{2(9.81 m/s^2)(4m)} = 8.86\ m/s$$

Hence,

$$Q = A_2 V_2 = \frac{\pi}{4}(0.10m)^2(8.86m/s) = \underline{\underline{0.0696\ m^3/s}}$$

(b) From the Bernoulli equation,

$$p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_3 \text{, where } z_2 = z_3 \text{ and } p_2 = 0$$

so that

$$p_3 = \frac{1}{2}\rho (V_2^2 - V_3^2)$$

Also, $A_2 V_2 = A_3 V_3$ so that $V_3 = \frac{A_2}{A_3} V_2 = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{0.1m}{0.08m}\right)^2 8.86 m/s = 13.84 m/s$

Hence,

$$p_3 = \frac{1}{2}(999\ kg/m^3)\left[(8.86m/s)^2 - (13.84m/s)^2\right] = -56{,}500\ N/m^2 \qquad (1)$$

Also, from the manometer,

$$p_3 = -\gamma_{Hg} h + \gamma_{H_2O}(2m + (0.08/2)m)$$

$$= -(133 \times 10^3 N/m^3)h + (9.80 \times 10^3 N/m^3)(2.04m)$$

$$= -133 \times 10^3 h + 1.99 \times 10^4\ N/m^2, \text{ where } h \sim m \qquad (2)$$

Thus, from Eqs. (1) and (2):

$$-5.65 \times 10^4 N/m^2 = -133 \times 10^3 h + 1.99 \times 10^4 N/m^2$$

or

$$h = \underline{\underline{0.574\ m}}$$

***3.54**

*3.54 Water flows from a pressurized tank through six equally spaced outlets on the vertical spray tower shown in Fig. P3.54. The diameter of the lowest outlet is 13 mm. Determine the diameters of the other five outlets if the flowrate is to be the same from each of the outlets when the pressure in the tank is $p_1 = 200$ kPa. With outlets of these diameters, the flowrates from each outlet will not be equal to each other if the pressure in the tank is not equal to 200 kPa. On one graph plot each of the six flowrates as a function of p_1 for 100 kPa $< p_1 <$ 300 kPa. What range in p_1 is acceptable if it is required that the flowrate from each outlet must be within 10% of the flowrate from any of the other outlets?

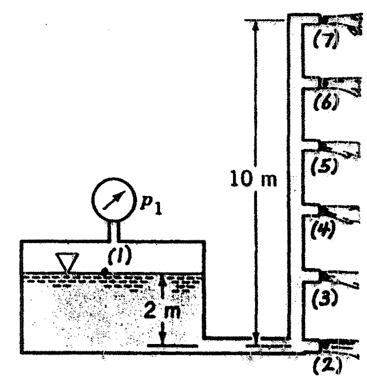


FIGURE P3.54

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_i}{\gamma} + \frac{V_i^2}{2g} + z_i, \text{ where } V_1 = 0,\ p_1 = 200\,kPa,\ p_i = 0,\ z_1 = 2m \quad (0)$$

or

$$V_i = \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - z_i\right)} = \left\{2\left(9.81\frac{m}{s^2}\right)\left[\left(\frac{200\frac{kN}{m^2}}{9.80\frac{kN}{m^3}}\right) + 2m - z_i\right]\right\}^{1/2}$$

Thus,

$$V_i = 4.43\sqrt{22.4 - z_i}\ \frac{m}{s}, \text{ where } z_i \sim m \quad (1)$$

Hence, $Q_2 = A_2 V_2 = \frac{\pi}{4}(0.013\,m)^2(4.43)\sqrt{22.4+0}\ \frac{m}{s} = 0.00278\frac{m^3}{s}$

In general, $Q_i = Q_2$ or

$\frac{\pi}{4}D_i^2 V_i = Q_2$ Thus, $D_i = \left[\frac{4Q_2}{\pi V_i}\right]^{1/2}$ with V_i from Eq. (1)

That is,

$$D_i = \left[\frac{4(0.00278)\frac{m^3}{s}}{\pi(4.43)\sqrt{22.4 - z_i}\ \frac{m}{s}}\right]^{1/2} \text{ or } D_i = \frac{0.0283}{(22.4 - z_i)^{1/4}}\,m, \text{ where } z_i \sim m \quad (2)$$

The following values are obtained:

i	z_i, m	D_i, mm
2	0	13.0
3	2	13.3
4	4	13.7
5	6	14.1
6	8	14.5
7	10	15.1

Consider the various Q_i for $100\,kPa \le p_1 \le 300\,kPa$

From Eq. (0) we obtain

$$V_i = \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - z_i\right)} = \left[2\left(9.81\frac{m}{s^2}\right)\left(\frac{p_1}{9.80\frac{kN}{m^3}} + 2m - z_i\right)\right]^{1/2}$$

or

$$V_i = 4.43\sqrt{\frac{p_1}{9.80} + 2 - z_i}\ \frac{ft}{s}, \text{ where } p_1 \sim kPa \text{ and } z_i \sim m \quad (3)$$

(con't)

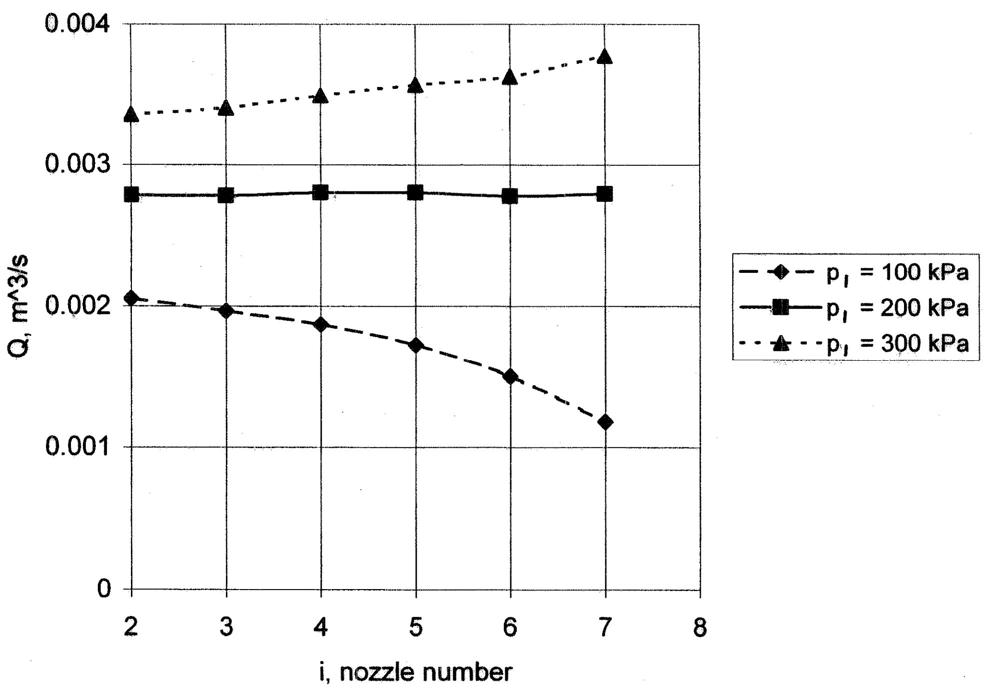
*3.54 (con't)

Hence, $Q_i = A_i V_i = \frac{\pi}{4} D_i^2 V_i$

or

$Q_i = 3.48\, D_i^2 \left(\frac{p_1}{9.80} + 2 - z_i\right)^{\frac{1}{2}}\ \frac{m^3}{s}$ for $i = 2,3,\ldots,7$ with $p_1 \sim kPa$, $z_i \sim m$ and $D_i \sim m$ (4)

The calculated results are plotted below.



Determine p_1 range so that all Q_i values are within 10% of each other for a given value of p_1. For given p_1, $Q_{max} = Q_2$ (i.e, the bottom nozzle) and $Q_{min} = Q_7$ (i.e, the top nozzle), provided $p_1 < 200$ kPa. For $p_1 = 200$ kPa all the Q_i are equal. For $p_1 > 200$ kPa, $Q_{max} = Q_7$ and $Q_{min} = Q_2$. (see below)

Thus, the minimum p_1 for which all flowrates are with 10% of each other is that for which $Q_2/Q_7 = 1.10$, and the maximum p_1 is $Q_7/Q_2 = 1.10$. From Eq. (4)

$$Q_2 = 3.48\, D_2^2 \left(\frac{p_1}{9.81} + 2\right)^{\frac{1}{2}} \text{ and } Q_7 = 3.48\, D_7^2 \left(\frac{p_1}{9.80} - 8\right)^{\frac{1}{2}}$$

or

$$\frac{Q_2}{Q_7} = \left(\frac{D_2}{D_7}\right)^2 \left[\frac{\frac{p_1}{9.80} + 2}{\frac{p_1}{9.80} - 8}\right]^{\frac{1}{2}} = \left(\frac{13.0}{15.1}\right)^2 \left[\frac{p_1 + 19.6}{p_1 - 78.4}\right]^{\frac{1}{2}}$$

(con't)

*3.54 (con't)

or

$$\frac{Q_2}{Q_7} = 0.741\left(\frac{p_1+19.6}{p_1-78.4}\right)^{1/2} \qquad (5)$$

For $\frac{Q_2}{Q_7} = 1.10$ Eq. (5) gives $p_1 = 160\ kN$

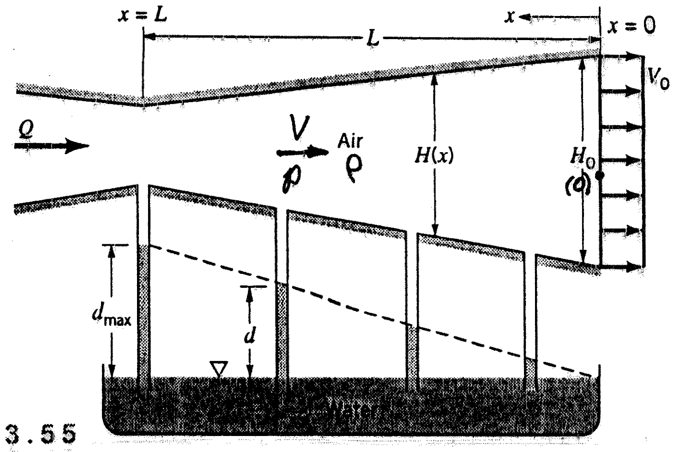
while for

$\frac{Q_7}{Q_2} = 1.10$ Eq. (5) gives $p_1 = 272\ kN$

Thus, with $\underline{\underline{160\ kN \le p_1 \le 272\ kN}}$ the flowrate from any nozzle (for a given value of p_1) is within 10% of that from any other nozzle. Note: This is consistent with the calculated results shown above.

3.55

3.55 Air flows steadily through a converging–diverging rectangular channel of constant width as shown in Fig. P3.55 and Video V3.6. The height of the channel at the exit and the exit velocity are H_0 and V_0, respectively. The channel is to be shaped so that the distance, d, that water is drawn up into tubes attached to static pressure taps along the channel wall is linear with distance along the channel. That is, $d = (d_{max}/L)\,x$, where L is the channel length and d_{max} is the maximum water depth (at the minimum channel height; $x = L$). Determine the height, $H(x)$, as a function of x and the other important parameters.



■ FIGURE P3.55

$$p + z\gamma + \tfrac{1}{2}\rho V^2 = p_0 + z_0\gamma + \tfrac{1}{2}\rho V_0^2 \quad \text{where } \rho = \text{air density}$$

where

$$z = z_0,\ p_0 = 0,\ p = -\gamma_{H_2O}\, d = -\gamma_{H_2O}\frac{d_{max}}{L}x$$

Thus,

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho V_0^2$$

But

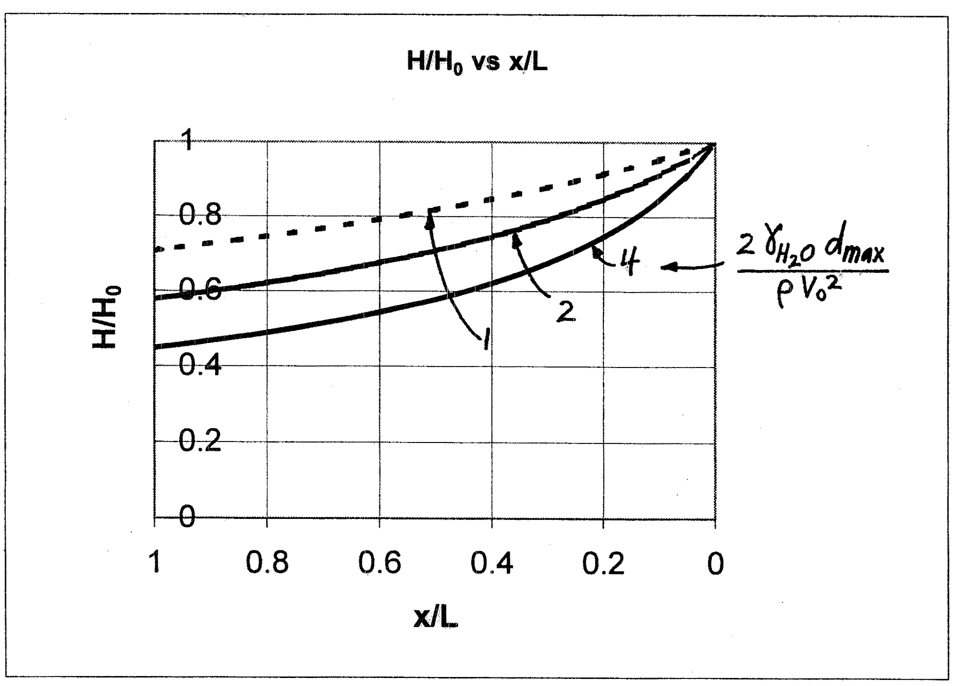
$$AV = A_0V_0, \text{ or } V = \frac{A_0}{A}V_0 = \frac{H_0}{H}V_0 \text{ so that}$$

$$-\gamma_{H_2O}\frac{d_{max}}{L}x + \tfrac{1}{2}\rho\left(\frac{H_0}{H}V_0\right)^2 = \tfrac{1}{2}\rho V_0^2$$

or

$$\frac{H}{H_0} = \frac{1}{\sqrt{1 + \left(\dfrac{2\gamma_{H_2O}\, d_{max}}{\rho V_0^2}\right)\dfrac{x}{L}}}$$

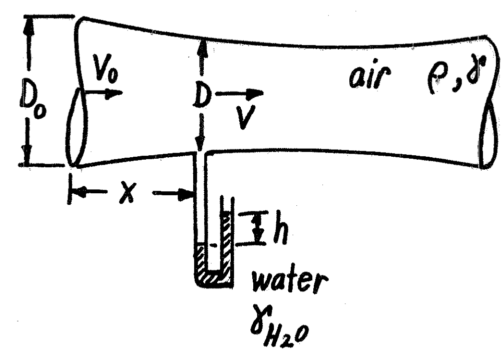
Typical shapes are shown below.



***3.56**

***3.56** Air flows through a horizontal pipe of variable diameter, $D = D(x)$, at a rate of 1.5 ft^3/s. The static pressure distribution obtained from a set of 12 static pressure taps along the pipe wall is as shown below. Plot the pipe shape, $D(x)$, if the diameter at $x = 0$ is 1, 2, or 3 in. Neglect viscous and compressibility effects.

x (in.)	p (in. H_2O)	x (in.)	p (in. H_2O)
0	1.00		
1	0.72	7	0.44
2	0.16	8	0.51
3	−0.96	9	0.65
4	−0.31	10	0.78
5	0.27	11	0.90
6	0.39	12	1.00



$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p}{\gamma} + \frac{V^2}{2g} + z \text{, where } z_0 = z$$

Thus,

$$V = \sqrt{V_0^2 + \frac{2(p_0 - p)}{\rho}} \quad \text{with } V_0 = \frac{Q}{A_0} = \frac{1.5 \frac{ft^3}{s}}{\frac{\pi}{4} D_0^2} = \frac{1.91}{D_0^2} \frac{ft}{s} \text{, where } D_0 \sim ft$$

and

$$p_0 - p = \gamma_{H_2O}(h_0 - h) = \frac{62.4 \frac{lb}{ft^3}}{12 \frac{in.}{ft}} (1 \text{ in.} - h) = 5.20(1-h) \frac{lb}{ft^2}\text{, with } h \sim in.$$

Hence, with $\rho = 2.38 \times 10^{-3} \frac{slug}{ft^3}$ we obtain

$$V = \left[\left(\frac{1.91}{D_0^2}\right)^2 + \frac{10.4(1-h)}{2.38 \times 10^{-3}}\right]^{\frac{1}{2}} = \left[\frac{3.65}{D_0^4} + 4370(1-h)\right]^{\frac{1}{2}} \qquad (1)$$

Also, $AV = Q$ or $\frac{\pi}{4} D^2 V = Q$ so that

$$D = \left[\frac{4Q}{\pi V}\right]^{\frac{1}{2}} = \left[\frac{4(1.5 \frac{ft^3}{s})}{\pi V \frac{ft}{s}}\right]^{\frac{1}{2}} = \frac{1.382}{\sqrt{V}}\text{, or when combined with Eq.(1)}$$

$$D = \frac{1.382}{\left[\frac{3.65}{D_0^4} + 4370(1-h)\right]^{\frac{1}{4}}} \text{ ft, where } D_0 \sim ft,\ h \sim in. \qquad (2)$$

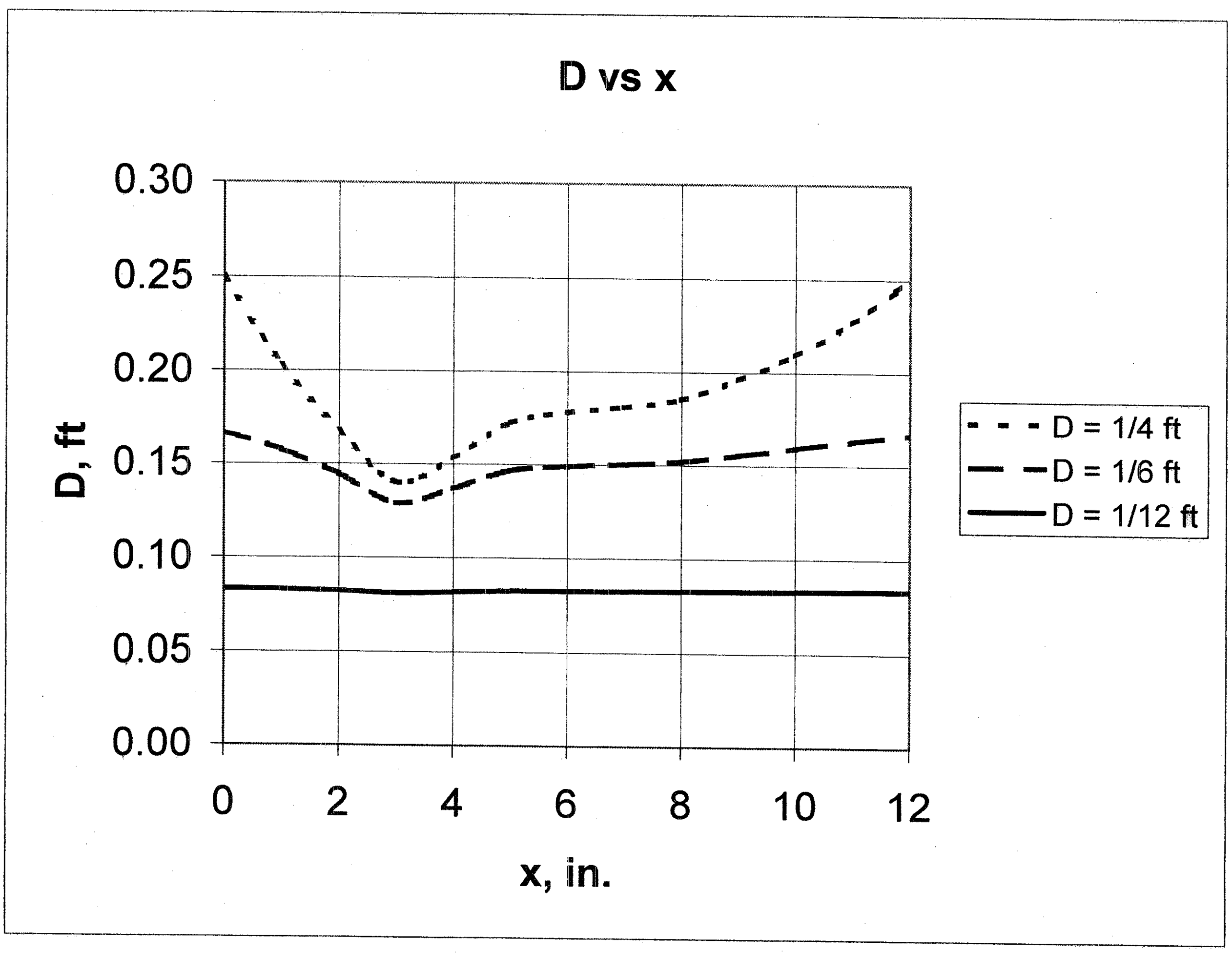
Plot $D = D(x)$ with $D_0 = \frac{1}{12}, \frac{1}{6}$, and $\frac{1}{4}$ ft, using the values of $h = h(x)$ from the table. Note: h is the same a "p (in. H_2O)" in the table.
An EXCEL program was used to obtain the following results.

(con't)

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

*3.56 (con't)

x, in.	D, ft (D_0 = 1/4 ft)	D, ft (D_0 = 1/6 ft)	D, ft (D_0 = 1/12 ft)	p, in. H_2O
0	0.24996	0.16664	0.08332	1.00
1	0.20277	0.15733	0.08299	0.72
2	0.16776	0.14435	0.08234	0.16
3	0.13999	0.12870	0.08112	-0.96
4	0.15299	0.13667	0.08182	-0.31
5	0.17245	0.14649	0.08247	0.27
6	0.17841	0.14902	0.08260	0.39
7	0.18123	0.15015	0.08266	0.44
8	0.18558	0.15179	0.08274	0.51
9	0.19616	0.15537	0.08291	0.65
10	0.20944	0.15911	0.08306	0.78
11	0.22710	0.16300	0.08320	0.90
12	0.24996	0.16664	0.08332	1.00



3.57

3.57 If viscous effects are neglected and the tank is large, determine the flowrate from the tank shown in Fig. P3.57.

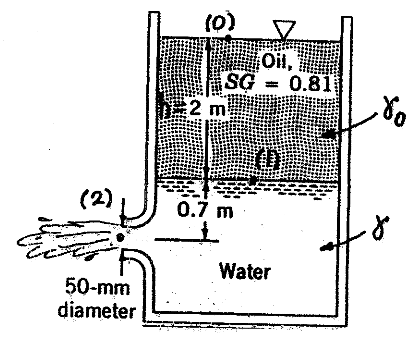


FIGURE P3.57

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = p_0 + \gamma_0 h = \gamma_0 h$

$z_1 = 0.7\,m$, $z_2 = 0$, and $V_1 = 0$

Thus,

$$\frac{\gamma_0 h}{\gamma} + z_1 = \frac{V_2^2}{2g} \quad \text{or} \quad V_2 = \sqrt{2g\left(\frac{\gamma_0 h}{\gamma} + z_1\right)}$$ where $\frac{\gamma_0}{\gamma} = 0.81$

and

$$Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2$$

Thus,

$$Q = \frac{\pi}{4}(0.050\,m)^2 \sqrt{2\left(9.81\,\frac{m}{s}\right)\left(0.81(2\,m) + 0.7\,m\right)} = \underline{\underline{0.0132\,\frac{m^3}{s}}}$$

3.58

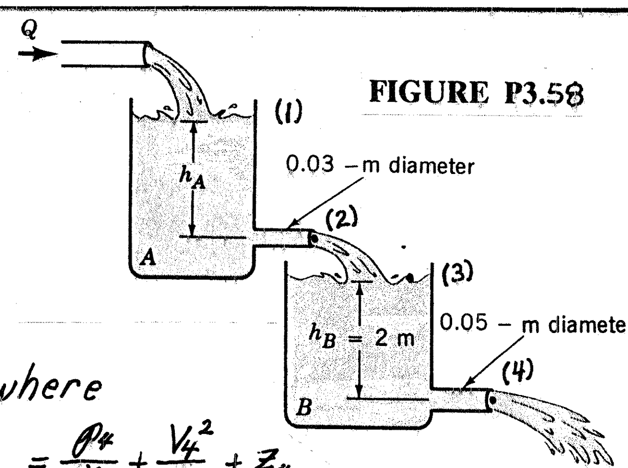


FIGURE P3.58

3.58 Water flows steadily through the large tanks shown in Fig. P3.58. Determine the water depth, h_A.

For steady flow, $Q_2 = Q_4$ where

$Q_4 = A_4 V_4$ with $\frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 = \frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4$

where $p_3 = p_4 = 0$ and $V_3 = 0$

Thus,

$$V_4 = \sqrt{2g(z_3 - z_4)} = \sqrt{2\left(9.81\,\frac{m}{s^2}\right)(2m)} = 6.26\,\frac{m}{s}$$

or

$$Q_4 = \frac{\pi}{4}(0.05m)^2\left(6.26\,\frac{m}{s}\right) = 0.0123\,\frac{m^3}{s}$$

Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = p_2 = 0 \text{ and } V_1 = 0$$

so that

$$V_2 = \sqrt{2g\,h_A}$$

Thus,

$$A_2 V_2 = Q_4 \quad \text{or} \quad \frac{\pi}{4}(0.03m)^2\sqrt{2\left(9.81\,\frac{m}{s^2}\right)h_A} = 0.0123\,\frac{m^3}{s}$$

or

$$h_A = \underline{\underline{15.4\,m}}$$

3.59

3.59 Air at 80 °F and 14.7 psia flows into the tank shown in Fig. P3.59. Determine the flowrate in ft³/s, lb/s, and slugs/s. Assume incompressible flow.

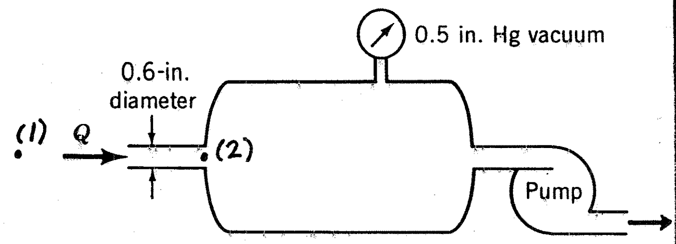


FIGURE P3.59

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2,\ p_1 = 0,\ V_1 = 0$$

Thus,

$$V_2 = \sqrt{-2g\frac{p_2}{\gamma}} = \sqrt{-2\frac{p_2}{\rho}}$$

where $\rho = \frac{p}{RT} = \frac{\left(14.7\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{\left(1716\frac{ft\cdot lb}{slugs\cdot{}^\circ R}\right)(460+80)^\circ R} = 2.28\times10^{-3}\frac{slugs}{ft^3}$

Hence, with $p_2 = -\gamma_{Hg}h = -\left(847\frac{lb}{ft^3}\right)\left(\frac{0.5}{12}ft\right) = -35.3\frac{lb}{ft^2}$

$$V_2 = \left[-2\frac{\left(-35.3\frac{lb}{ft^2}\right)}{2.28\times10^{-3}\frac{slugs}{ft^3}}\right]^{1/2} = 176\frac{ft}{s}$$

Thus,

$$Q = A_2V_2 = \frac{\pi}{4}\left(\frac{0.6}{12}ft\right)^2\left(176\frac{ft}{s}\right) = \underline{\underline{0.346\frac{ft^3}{s}}}$$

$$\dot{m} = \rho Q = \left(2.28\times10^{-3}\frac{slugs}{ft^3}\right)\left(0.346\frac{ft^3}{s}\right) = \underline{\underline{7.89\times10^{-4}\frac{slugs}{s}}}$$

and

$$g\dot{m} = \left(32.2\frac{ft}{s^2}\right)\left(7.89\times10^{-4}\frac{slugs}{ft^3}\right) = \underline{\underline{0.0254\frac{lb}{s}}}$$

3.60

3.60 Water flows from a large tank as shown in Fig. P3.60. Atmospheric pressure is 14.5 psia and the vapor pressure is 1.60 psia. If viscous effects are neglected, at what height, h, will cavitation begin? To avoid cavitation, should the value of D_1 be increased or decreased? To avoid cavitation, should the value of D_2 be increased or decreased? Explain.

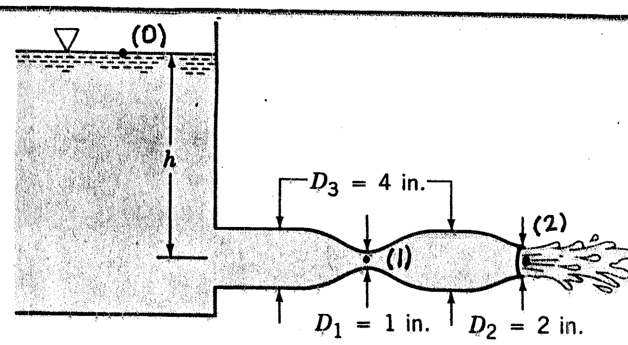


FIGURE P3. 60

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1$$ where $p_0 = 14.5\,psia$, $p_1 = 1.60\,psia$, $z_0 = h$, $z_1 = 0$, and $V_0 = 0$

Thus,

$$h = \frac{p_1 - p_0}{\gamma} + \frac{V_1^2}{2g} \qquad (1)$$

However,

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

where

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ with $p_0 = p_2$ and $z_2 = 0$

Thus,

$$\frac{V_2^2}{2g} = h$$

so that

$$\frac{V_1^2}{2g} = \frac{\left(\frac{D_2}{D_1}\right)^4 V_2^2}{2g} = \left(\frac{D_2}{D_1}\right)^4 h \qquad (2)$$

Combine Eqs. (1) and (2) to obtain

$$h = \frac{p_1 - p_0}{\gamma} + \left(\frac{D_2}{D_1}\right)^4 h$$

or

$$h = \frac{p_0 - p_1}{\gamma\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right]} = \frac{(14.5 - 1.60)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)}{62.4\frac{lb}{ft^3}\left[\left(\frac{2\,in.}{1\,in.}\right)^4 - 1\right]} = \underline{\underline{1.98\,ft}} \qquad (3)$$

From Eq. (3) it is seen that h increases in increasing D_1 and decreasing D_2. Thus, to avoid cavitation (i.e. to have h small enough) <u>D_1 should be increased and D_2 decreased.</u>

3.61

3.61 Water flows into the sink shown in Fig. P3.61 at a rate of 2 gal/min. If the drain is closed, the water will eventually flow through the overflow drain holes rather than over the edge of the sink. How many 0.4-in.-diameter drain holes are needed to ensure that the water does not overflow the sink? Neglect viscous effects.

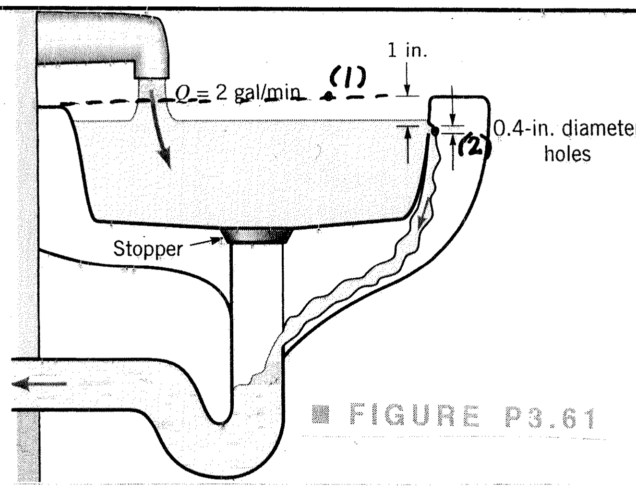


FIGURE P3.61

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\,,\quad \text{where } p_1=0,\ V_1=0,\text{ and } z_2=0,\ p_2=0$$

Thus,

$$z_1=\frac{V_2^2}{2g}\quad \text{or}\quad V_2=\sqrt{2gz_1}=\left[2\left(32.2\,\frac{ft}{s}\right)\left(\frac{1+0.2}{12}\,ft\right)\right]^{1/2}=2.54\,\frac{ft}{s}$$

Also,

$$Q=n\,A_2V_2=n\,C_c\,\frac{\pi}{4}d_2^2\,V_2\,,$$ where n = number of holes required, $d_2=0.4\text{ in.}$, and C_c = contraction coef. $=0.61$ (see Fig. 3.14)

Thus, with

$$Q=2\,\frac{gal}{min}\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{231\text{ in.}^3}{1\text{ gal}}\right)\left(\frac{1\text{ ft}^3}{1728\text{ in.}^3}\right)=4.46\times10^{-3}\,\frac{ft^3}{s}\,,$$

$$n=\frac{4Q}{\pi C_c d_2^2 V_2}=\frac{4\,(4.46\times10^{-3}\,ft^3/s)}{\pi(0.61)\left(\frac{0.4}{12}\right)^2 ft^2\,(2.54\,ft/s)}=3.30$$

Thus, $\underline{\underline{4}}$ holes are needed.

3.62

3.62 What pressure, p_1, is needed to produce a flowrate of 0.09 ft³/s from the tank shown in Fig. P3.62?

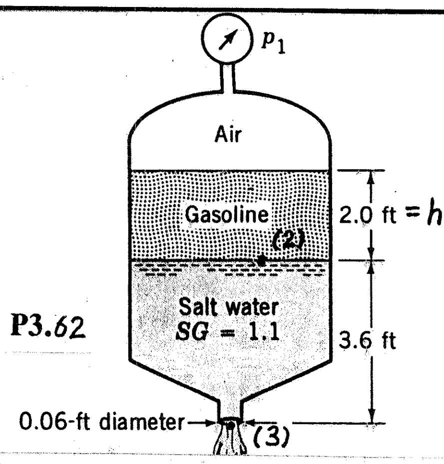


FIGURE P3.62

$$\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3$$ where $p_2 = p_1 + \gamma_o h$, $p_3 = 0$

$z_2 = 3.6\ ft$, $z_3 = 0$

and $V_2 = 0$

Thus,

$$\frac{p_1+\gamma_o h}{\gamma}+z_2=\frac{V_3^2}{2g}$$

where $Q = A_3 V_3 = \frac{\pi}{4} D_3^2 V_3$

or

$$V_3 = \frac{4Q}{\pi D_3^2} = \frac{4\left(0.09\frac{ft^3}{s}\right)}{\pi(0.06\ ft)^2} = 31.8\ \frac{ft}{s}$$

Thus,

$$p_1 = \gamma\left(\frac{V_3^2}{2g}-z_2\right)-\gamma_o h = \left(1.1\left(62.4\frac{lb}{ft^3}\right)\right)\left[\frac{\left(31.8\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} - 3.6\ ft\right]$$

$$- 42.5\frac{lb}{ft^3}(2.0\ ft)$$

or

$$p_1 = 746\ \frac{lb}{ft^2} = \underline{\underline{5.18\ psi}}$$

3.63

3.63 Water flows from the tank shown in Fig. P3.63. If viscous effects are negligible determine the value of h in terms of H and the specific gravity, SG, of the manometer fluid.

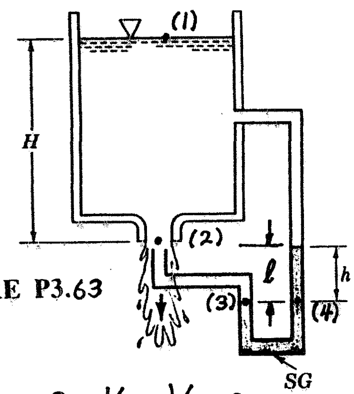


FIGURE P3.63

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = 0$, $V_1 = V_2 = 0$ and $z_1 - z_2 = H$

Thus,

$$\frac{p_2}{\gamma} = H \qquad (1)$$

But, $p_3 = p_2 + \gamma \ell = p_4 = p_1 + \gamma(H + \ell - h) + SG\,\gamma h$

or

$$p_2 = \gamma(H - h + SG\,h) \qquad (2)$$

Combine Eqns. (1) and (2) to give:

$$H = (H + (SG-1)h)$$

or

$$(SG-1)h = 0$$

Thus, if $SG \neq 1$, then $\underline{\underline{h = 0 \text{ for any } SG}}$

3.64

3.64 Water is siphoned from the tank shown in Fig. P3.64. Determine the flowrate from the tank and the pressures at points (1), (2), and (3) if viscous effects are negligible.

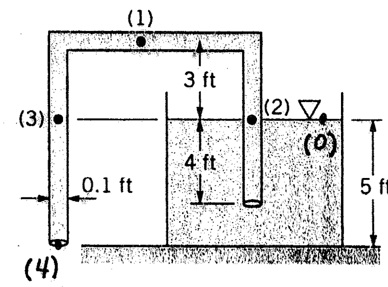


FIGURE P3.64

From the Bernoulli equation,

$$p_0 + \frac{1}{2}\rho V_0^2 + \gamma z_0 = p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4, \quad \text{where } p_0 = p_4 = 0,\ V_0 = 0,\ z_0 = 5\text{ft}, \text{ and } z_4 = 0$$

Thus,

$$\gamma z_0 = \frac{1}{2}\rho V_4^2, \text{ or } V_4 = \sqrt{2\gamma z_0/\rho} = \sqrt{2 g z_0} = \sqrt{2(32.2\tfrac{\text{ft}}{\text{s}^2})(5\text{ft})} = 17.94\tfrac{\text{ft}}{\text{s}}$$

Hence,

$$Q = A_4 V_4 = \frac{\pi}{4}(0.1\text{ft})^2(17.94\tfrac{\text{ft}}{\text{s}}) = \underline{\underline{0.141\tfrac{\text{ft}^3}{\text{s}}}}$$

For p_1: $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4$, which with $p_4 = 0$, $z_4 = 0$, $z_1 = 8\text{ft}$, and $V_1 = V_4$ (since $A_1 = A_4$) becomes

$$p_1 = -\gamma z_1 = -(62.4\text{ lb/ft}^3)(8\text{ft}) = \underline{\underline{-499\tfrac{\text{lb}}{\text{ft}^2}}}$$

For p_3: $p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4$, which with $p_4 = 0$, $z_4 = 0$, $z_3 = 5\text{ft}$, and $V_3 = V_4$ (since $A_3 = A_4$) becomes

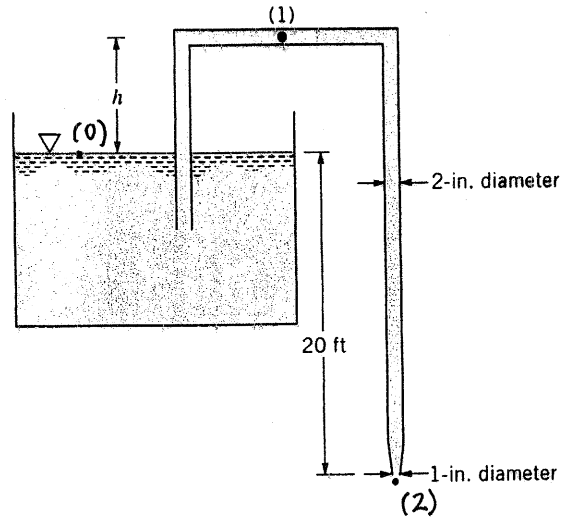
$$p_3 = -\gamma z_3 = -(62.4\text{lb/ft}^3)(5\text{ ft}) = \underline{\underline{-312\text{ lb/ft}^2}}$$

For p_2: Since $z_2 = z_3$ and $V_2 = V_3$ it follows that

$$p_2 = p_3 = \underline{\underline{-312\text{ lb/ft}^2}}$$

3.65

3.65 Water is siphoned from a large open tank through a 2-in.-diameter hose and discharged into the atmosphere (at standard atmospheric pressure) through a nozzle with a tip diameter of 1 in. as shown in Fig. P3.65. Determine the height h so that the pressure at point (1) is equal to 8.0 psi (abs). Assume that viscous effects are negligible.



■ FIGURE P3.65

From the Bernoulli equation,

$$p_0 + \frac{1}{2}\rho V_0^2 + z_0 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \text{; where } p_0 = p_2,\ V_0 = 0,\ z_0 = 0, \text{ and } z_2 = -20\text{ft}$$

Thus,

$$0 = \frac{1}{2}\rho V_2^2 + \rho g z_2 \text{, or } V_2 = \sqrt{-2gz_2} = \sqrt{-2(32.2\tfrac{\text{ft}}{\text{s}^2})(-20\text{ft})} = 35.9\tfrac{\text{ft}}{\text{s}}$$

Also, since $A_1V_1 = A_2V_2$ it follows that

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{1\text{in.}}{2\text{in.}}\right)^2 (35.9\tfrac{\text{ft}}{\text{s}}) = 8.97\tfrac{\text{ft}}{\text{s}}$$

Therefore, with

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \text{ and } z_1 = h,\ z_2 = -20\text{ft},\ p_2 = 14.7\text{psia},\ p_1 = 8\text{ psia}$$

h can be determined from

$$8\tfrac{\text{lb}}{\text{in.}^2}(144\tfrac{\text{in.}^2}{\text{ft}^2}) + \tfrac{1}{2}(1.94\tfrac{\text{slugs}}{\text{ft}^3})(8.97\tfrac{\text{ft}}{\text{s}})^2 + 62.4\tfrac{\text{lb}}{\text{ft}^3}h$$

$$= 14.7\tfrac{\text{lb}}{\text{in.}^2}(144\tfrac{\text{in.}^2}{\text{ft}^2}) + \tfrac{1}{2}(1.94\tfrac{\text{slugs}}{\text{ft}^3})(35.9\tfrac{\text{ft}}{\text{s}})^2 + 62.4\tfrac{\text{lb}}{\text{ft}^3}(-20\text{ft})$$

Thus,

$$h = \underline{\underline{14.2\text{ ft}}}$$

Note: This result could be obtained by using the Bernoulli equation between points (0) and (1) rather than (1) and (2).

3.66

3.66 Determine the manometer reading, h, for the flow shown in Fig. P3.66.

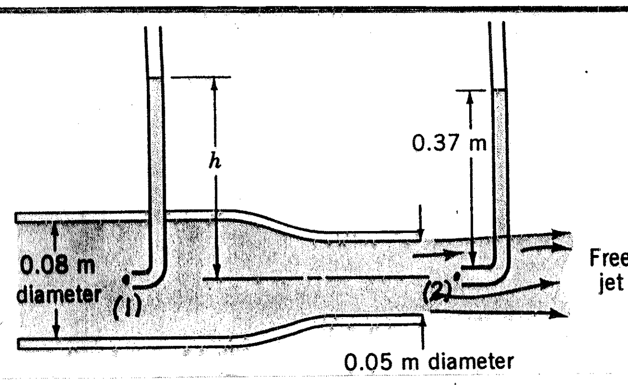


FIGURE P3.66

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $z_1 = z_2$, $V_1 = 0$, and $V_2 = 0$

Thus,

$p_1 = p_2$

However, $p_1 = \gamma h$ and $p_2 = \gamma (0.37 m)$ so that

$h = \underline{\underline{0.37\ m}}$

3.67

3.67 The specific gravity of the manometer fluid shown in Fig. P3.67 is 1.07. Determine the volume flowrate, Q, if the flow is inviscid and incompressible and the flowing fluid is **(a)** water, **(b)** gasoline, or **(c)** air at standard conditions.

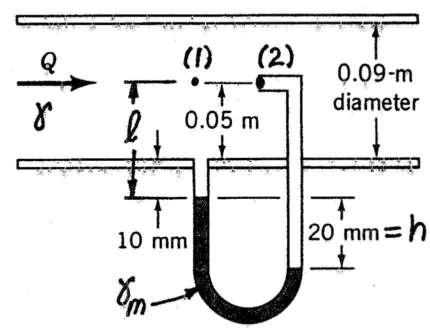


FIGURE P3.67

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $z_1 = z_2$ and $V_2 = 0$

Thus,

$$V_1 = \sqrt{2g\,\frac{(p_2 - p_1)}{\gamma}} \qquad (1)$$

But

$$p_1 + \gamma \ell + \gamma_m h = p_2 + \gamma(\ell + h)$$

or

$p_2 - p_1 = (\gamma_m - \gamma)h$ so that Eq. (1) becomes

$$V_1 = \sqrt{2g\,\frac{(\gamma_m - \gamma)}{\gamma}h} = \sqrt{2\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1.07\left(9.8 \times 10^3\,\frac{N}{m^3}\right)}{\gamma} - 1\right)(0.02\,m)}$$

Thus,

$$Q = A_1 V_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4}(0.09\,m)^2 \sqrt{2(9.81)\left(\frac{10.49 \times 10^3}{\gamma} - 1\right)(0.02)}$$

or

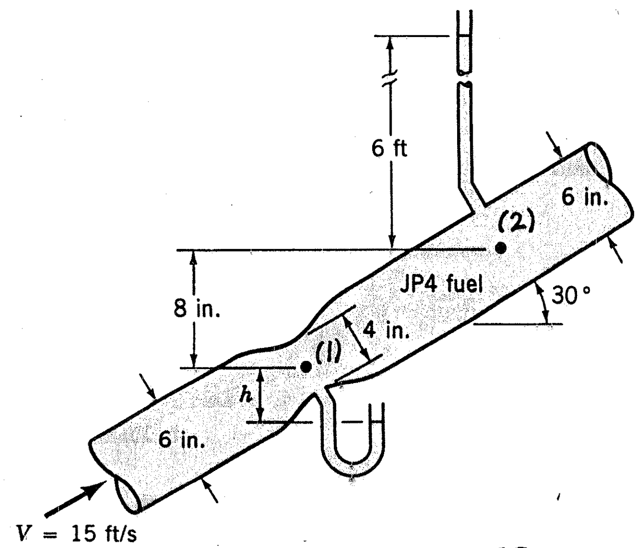
$$Q = 3.99 \times 10^{-3} \sqrt{\frac{10.49}{\gamma} - 1}\ \frac{m^3}{s} \quad \text{where } \gamma \sim \frac{kN}{m^3}$$

For the given fluids this gives:

	fluid	$\gamma, \frac{kN}{m^3}$	$Q, \frac{m^3}{s}$
(a)	water	9.80	1.06×10^{-3}
(b)	gasoline	6.67	3.02×10^{-3}
(c)	air	12×10^{-3}	0.118

3.68

3.68 JP-4 fuel ($SG = 0.77$) flows through the Venturi meter shown in Fig. P3.68 with a velocity of 15 ft/s in the 6-in. pipe. If viscous effects are negligible, determine the elevation, h, of the fuel in the open tube connected to the throat of the Venturi meter.



■ FIGURE P3.68

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = 0,\ z_2 = \frac{8}{12}\text{ ft}, \quad (1)$$

and $V_2 = 15\ \text{ft/s}$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{6\text{ in.}}{4\text{ in.}}\right)^2 \left(15\ \frac{\text{ft}}{\text{s}}\right) = 33.75\ \frac{\text{ft}}{\text{s}}$$

Thus, with $\frac{p_2}{\gamma} = 6\text{ ft}$ Eq. (1) becomes

$$\frac{p_1}{\gamma} + \frac{(33.75\ \frac{\text{ft}}{\text{s}})^2}{2(32.2\ \frac{\text{ft}}{\text{s}^2})} = 6\text{ ft} + \frac{(15\ \frac{\text{ft}}{\text{s}})^2}{2(32.2\ \frac{\text{ft}}{\text{s}^2})} + \frac{8}{12}\text{ ft}$$

or

$$\frac{p_1}{\gamma} = -7.53\text{ ft}$$

But $\frac{p_1}{\gamma} = -h$ so that $\underline{\underline{h = 7.53\text{ ft}}}$

3.69

3.69 Repeat Problem 3.68 if the flowing fluid is water rather than JP-4 fuel.

Note from the solution to Problem 3.68 that the value of γ is not needed. Thus, <u>h = 7.53 ft for either water or JP-4 fuel.</u>

3.70

3.70 Air at standard conditions flows through the cylindrical drying stack shown in Fig. P3.70. If viscous effects are negligible and the inclined water-filled manometer reading is 20 mm as indicated, determine the flowrate.

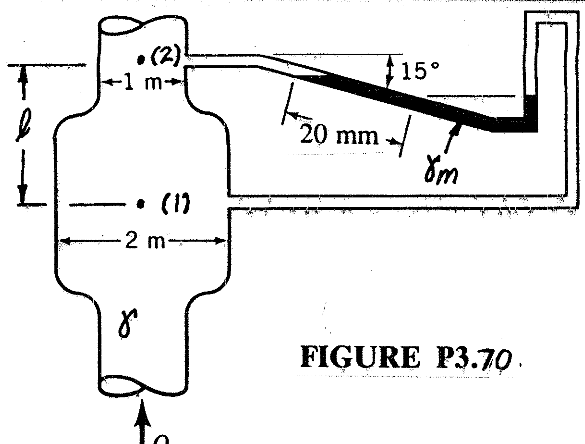


FIGURE P3.70

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

where $z_1=0$, $z_2=\ell$ and

$$V_2=\frac{A_1}{A_2}V_1=\left(\frac{D_1}{D_2}\right)^2 V_1=\left(\frac{2\,m}{1\,m}\right)^2 V_1=4V_1$$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{(4V_1)^2}{2g}+\ell$$

or

$$\frac{15V_1^2}{2g}=\frac{p_1-p_2}{\gamma}-\ell \qquad (1)$$

However, $p_2+\gamma\ell_2+\gamma_m h=p_1-\gamma(\ell-h-\ell_2)$ where $h=(20\,mm)\sin 15$

or $$\frac{p_1-p_2}{\gamma}=\left(\frac{\gamma_m}{\gamma}-1\right)h+\ell \qquad (2)$$

By combining Eqs.(1) and (2)

$$\frac{15V_1^2}{2g}=\left(\frac{\gamma_m}{\gamma}-1\right)h$$

or

$$V_1=\sqrt{\frac{2g\left(\frac{\gamma_m}{\gamma}-1\right)h}{15}}=\sqrt{\frac{2\left(9.81\frac{m}{s^2}\right)\left(\frac{9.80\times10^3\frac{N}{m^3}}{12.0\frac{N}{m^3}}-1\right)(0.02\sin 15)}{15}}$$

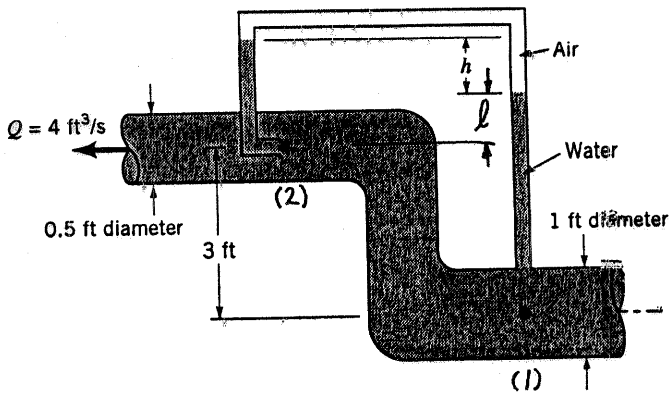
$$=2.35\ \frac{m}{s}$$

Thus,

$$Q=A_1V_1=\frac{\pi}{4}D_1^2V_1=\frac{\pi}{4}(2m)^2\left(2.35\frac{m}{s}\right)=\underline{\underline{7.38\ \frac{m^3}{s}}}$$

3.71

3.71 Water, considered an inviscid, incompressible fluid, flows steadily as shown in Fig. P3.71. Determine h.



■ FIGURE P3.71

$$p_1 + \gamma z_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \gamma z_2 + \tfrac{1}{2}\rho V_2^2$$

where $z_1 = 0$, $z_2 = 3\,ft$, $V_2 = 0$, and $V_1 = \frac{Q}{A_1} = \frac{4\,\frac{ft^3}{s}}{\frac{\pi}{4}(1\,ft)^2} = 5.09\,\frac{ft}{s}$

Thus,

$$p_1 + \tfrac{1}{2}\left(1.94\,\tfrac{slugs}{ft^3}\right)\left(5.09\,\tfrac{ft}{s}\right)^2 = p_2 + 62.4\,\tfrac{lb}{ft^3}\,(3\,ft)$$

or

$$p_1 - p_2 = 162\,\tfrac{lb}{ft^2} \qquad (1)$$

But from the manometer,

$$p_1 - \gamma(\ell + 3\,ft) + \gamma(h + \ell) = p_2$$

or

$$p_1 - 62.4\,\tfrac{lb}{ft^3}\,(3\,ft) + 62.4\,\tfrac{lb}{ft^3}\,h = p_2$$

Hence,

$p_1 = p_2 + 187 - 62.4h$ which when combined with Eq. (1) gives

$$p_2 + 187 - 62.4h - p_2 = 162$$

or

$$h = \underline{\underline{0.400\,ft}}$$

3.72

3.72 Determine the flowrate through the submerged orifice shown in Fig. P3.72 if the contraction coefficient is $C_c = 0.63$.

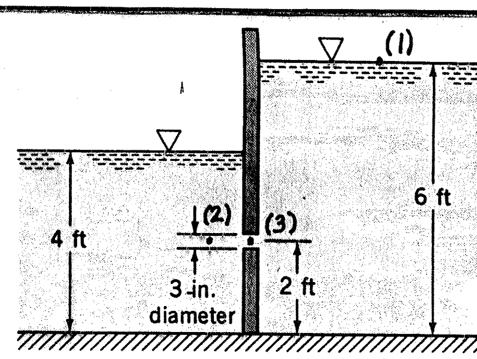


FIGURE P3.72

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ where $p_1 = 0$, $V_1 = 0$, $z_1 = 4\ ft$, $z_2 = 0$, and $\frac{p_2}{\gamma} = 2\ ft$

Thus,

$$4\ ft = 2\ ft + \frac{V_2^2}{2(32.2\ \frac{ft}{s})}$$

or

$$V_2 = 11.34\ \frac{ft}{s}$$

so that

$$Q = A_2 V_2 = C_c A_3 V_2 = (0.63)\frac{\pi}{4}\left(\frac{3}{12}\ ft\right)^2 (11.34\ \tfrac{ft}{s}) = \underline{\underline{0.351\ \frac{ft^3}{s}}}$$

3.73

3.73 Determine the flowrate through the Venturi meter shown in Fig. P3.73 if ideal conditions exist.

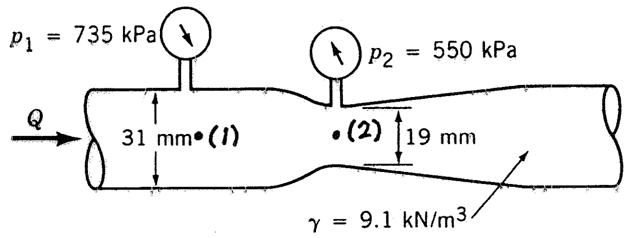


FIGURE P3.73

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ where $z_1=z_2$ and $A_1V_1=A_2V_2$

or

$$V_1=\frac{A_2}{A_1}V_2=\left(\frac{D_2}{D_1}\right)^2V_2$$

Thus,

$$\frac{p_1}{\gamma}+\frac{\left(\frac{D_2}{D_1}\right)^4V_2^2}{2g}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}$$

or

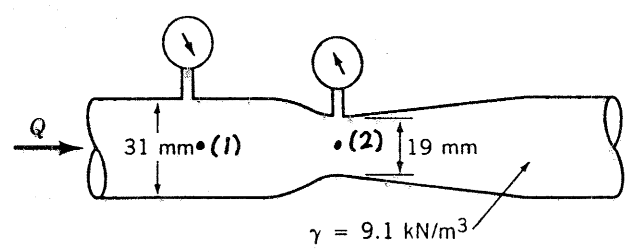
$$V_2=\sqrt{\frac{2g\frac{(p_1-p_2)}{\gamma}}{1-\left(\frac{D_2}{D_1}\right)^4}}=\sqrt{\frac{2(9.81\frac{m}{s^2})\frac{(735-550)kPa}{(9.1\frac{kN}{m^3})}}{1-\left(\frac{19\ mm}{31\ mm}\right)^4}}=21.5\frac{m}{s}$$

so that

$$Q=A_2V_2=\frac{\pi}{4}D_2^2V_2=\frac{\pi}{4}(0.019m)^2(21.5\tfrac{m}{s})=\underline{\underline{6.10\times10^{-3}\frac{m^3}{s}}}$$

3.74

3.74 For what flowrate through the Venturi meter of Prob. 3.73 will cavitation begin if $p_1 = 275$ kPa gage, atmospheric pressure is 101 kPa (abs), and the vapor pressure is 3.6 kPa (abs)?



$$(1) \quad \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = z_2$, $p_2 = 3.6\ kPa(abs)$
and $p_1 = (275 + 101)\ kPa(abs)$
$= 376\ kPa(abs)$

Thus, with $A_1V_1 = A_2V_2$

or

$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$ Eq. (1) becomes

$$V_2 = \sqrt{\frac{2g\left(\frac{p_1 - p_2}{\gamma}\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}} = \left[\frac{2(9.81\frac{m}{s})\frac{(376 - 3.6)kPa}{9.1\ kN/m^3}}{1 - \left(\frac{19\ mm}{31\ mm}\right)^4}\right]^{1/2}$$

or

$V_2 = 30.6\ \frac{m}{s}$

Thus,

$$Q = A_2V_2 = \frac{\pi}{4}D_2^2 V_2 = \frac{\pi}{4}(0.019\ m)^2(30.6\ \frac{m}{s}) = \underline{\underline{8.68 \times 10^{-3}\ \frac{m^3}{s}}}$$

3.75

3.75 What diameter orifice hole, d, is needed if under ideal conditions the flowrate through the orifice meter of Fig. P3.75 is to be 30 gal/min of seawater with $p_1 - p_2 = 2.37\ \text{lb/in.}^2$? The contraction coefficient is assumed to be 0.63.

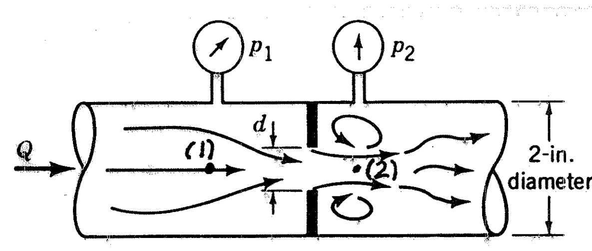


FIGURE P3.75

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $z_1 = z_2$, $C_c = 0.63$, and $p_1 - p_2 = 2.37\,psi$

With

$$Q = \left(30\,\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{231\,in.^3}{1\,gal}\right)\left(\frac{1\,ft^3}{1728\,in^3}\right) = 0.0668\,\frac{ft^3}{s} \quad \text{and } \gamma = 64.0\,\frac{lb}{ft^3}$$

it follows that

$$V_1 = \frac{Q}{A_1} = \frac{0.0668\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2} = 3.06\,\frac{ft}{s}$$

Thus, Eq(1) gives

$$V_2 = \sqrt{V_1^2 + 2g\left(\frac{p_1 - p_2}{\gamma}\right)} = \sqrt{\left(3.06\,\frac{ft}{s}\right)^2 + 2\left(32.2\,\frac{ft}{s^2}\right)\left(\frac{2.37 \times 144\,\frac{lb}{ft^2}}{64.0\,\frac{lb}{ft^3}}\right)}$$

or

$$V_2 = 18.8\,\frac{ft}{s}$$

Thus, since

$$Q = A_2 V_2 = C_c \frac{\pi}{4} d^2 V_2 \quad \text{it follows that}$$

$$d = \left[\frac{4Q}{\pi C_c V_2}\right]^{1/2} = \left[\frac{4 \times 0.0668\,\frac{ft^3}{s}}{\pi\,(0.63)\left(18.8\,\frac{ft}{s}\right)}\right]^{1/2} = 0.0847\,ft = \underline{\underline{1.016\ in.}}$$

3.76

3.76 An ancient device for measuring time is shown in Fig. P3.76. The axisymmetric vessel is shaped so that the water level falls at a constant rate. Determine the shape of the vessel, $R = R(z)$, if the water level is to decrease at a rate of 0.10 m/hr and the drain hole is 5.0 mm in diameter. The device is to operate for 12 hr without needing refilling. Make a scale drawing of the shape of the vessel.

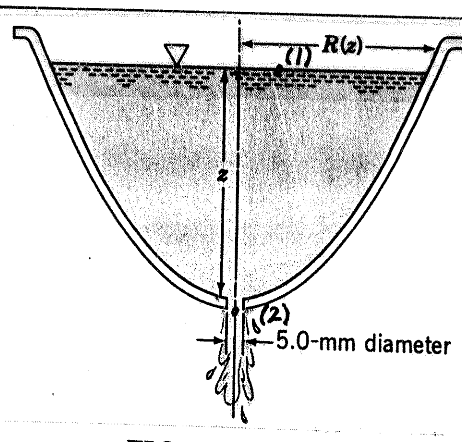


FIGURE P3.76

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ if the flow is assumed to be quasi-steady.

Also, $p_1 = 0$, $p_2 = 0$, $z_1 = z$, and $z_2 = 0$

Thus,

$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} + z$$ which, if $V_1 \ll V_2$ (i.e. $R \gg 5.0mm$), becomes

$$V_2 = \sqrt{2gz}$$

Since $A_1V_1 = A_2V_2$ and $V_1 = \left|\frac{dz}{dt}\right| = 0.1\frac{m}{hr}\left(\frac{1\,hr}{3{,}600s}\right) = 2.78 \times 10^{-5}\frac{m}{s}$

we obtain

$$\pi R^2 \left(2.78 \times 10^{-5} \tfrac{m}{s}\right) = \frac{\pi}{4}(0.005m)^2 \sqrt{2\left(9.81\tfrac{m}{s^2}\right) z},$$

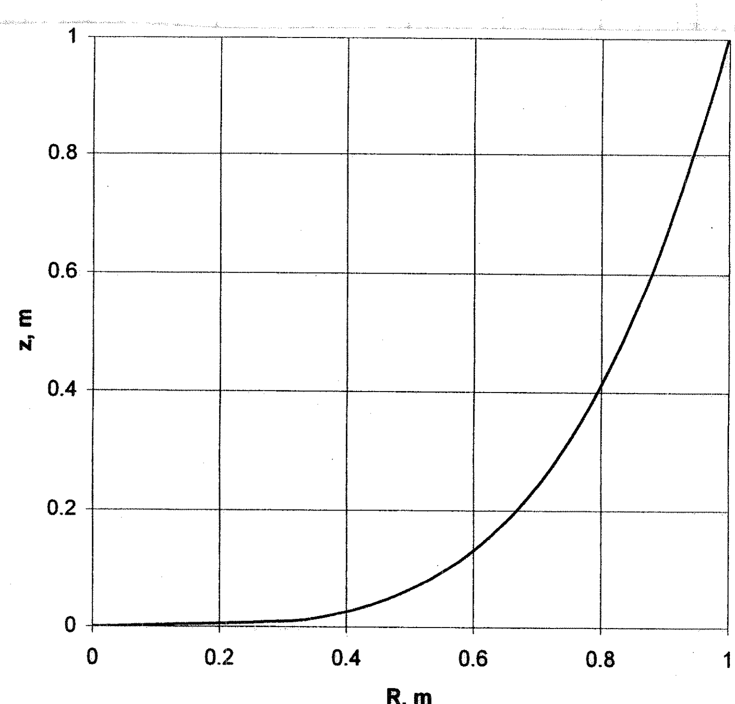
where R and z are ~m

Thus,

$$R = 0.998\, z^{1/4}$$

or

z, m	R, m
0	0
0.02	0.375
0.05	0.472
0.12	0.587
0.22	0.683
0.32	0.751
0.42	0.803
0.52	0.847
0.62	0.886
0.72	0.919
0.82	0.950
0.92	0.977
1.02	1.003
1.12	1.027
1.22	1.049



3.77

3.77 A long water trough of triangular cross section is formed from two planks as is shown in Fig. P3.77. A gap of 0.1 in. remains at the junction of the two planks. If the water depth initially was 2 ft, how long a time does it take for the water depth to reduce to 1 ft.?

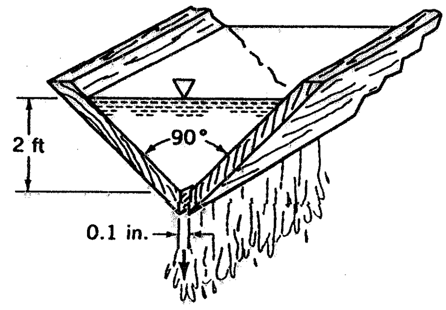
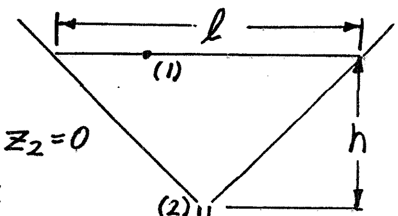


FIGURE P3.77

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad (1)$$



where $p_1=0$, $p_2=0$, $z_1=h$, and $z_2=0$

Also $V_1A_1=V_2A_2$ or since $\ell \gg w$ it follows that $V_1 \ll V_2$, where $V_1=-\frac{dh}{dt}$

Thus, Eq.(1) gives

$V_2=\sqrt{2gh}$ so that

$-A_1\frac{dh}{dt}=A_2\sqrt{2gh}$ with $A_1=b\ell=2bh$ and $A_2=bw$ where b is the tank length.

Thus,

$$-2bh\frac{dh}{dt}=bw\sqrt{2gh}$$

or

$\sqrt{h}\,dh=-w\sqrt{\frac{g}{2}}\,dt$ which can be integrated to give

$$\int_{h_i=2}^{h_f=1} h^{\frac{1}{2}}dh=-w\sqrt{\frac{g}{2}}\int_{t_i=0}^{t_f}dt$$

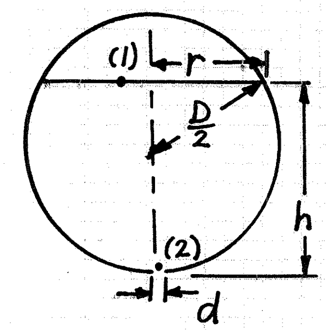
or

$$t_f=\frac{2}{3w}\sqrt{\frac{2}{g}}\left[h_i^{3/2}-h_f^{3/2}\right]=\frac{2}{3\left(\frac{0.1}{12}\right)ft}\sqrt{\frac{2}{32.2\frac{ft}{s^2}}}\left[2^{3/2}-1^{3/2}\right]ft^{3/2}$$

$$=\underline{\underline{36.5\ s}}$$

3.78*

3.78* A spherical tank of diameter D has a drain hole of diameter d at its bottom. A vent at the top of the tank maintains atmospheric pressure within the tank. The flow is quasisteady and inviscid and the tank is full of water initially. Determine the water depth as a function of time, $h = h(t)$, and plot graphs of $h(t)$ for tank diameters of 1, 5, 10, and 20 ft if $d = 1$ in.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = h$, $z_2 = 0$ and $V_1 = -\frac{dh}{dt} << V_2$ if $r >> d$

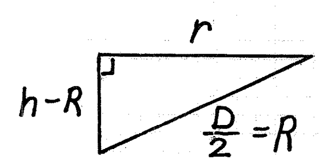
Thus,

$V_2 = \sqrt{2gh}$ which when combined with $A_1V_1 = A_2V_2$ gives

$$-A_1\frac{dh}{dt} = A_2\sqrt{2gh} \quad \text{or} \quad -\pi r^2\frac{dh}{dt} = \frac{\pi}{4}d^2\sqrt{2gh} \qquad (1)$$

where $R^2 = r^2 + (h-R)^2$

with $R = \frac{D}{2}$ = radius of tank



Thus, $r = \sqrt{R^2-(h-R)^2}$ so that Eq. (1) becomes

$$-[R^2-(h-R)^2]\frac{dh}{dt} = \frac{d^2}{4}\sqrt{2gh}$$

or

$$(h^{3/2} - 2Rh^{1/2})\,dh = \frac{d^2\sqrt{2g}}{4}\,dt$$

which can be integrated from the initial time and depth ($t=0$, $h=2R$) to an arbitrary time and depth (t,h) as

$$\int_{2R}^{h}(h^{3/2} - 2Rh^{1/2})\,dh = \frac{d^2\sqrt{2g}}{4}\int_0^t dt$$

or

$$\frac{2}{5}\left(h^{5/2}-(2R)^{5/2}\right) - \frac{4}{3}R\left(h^{3/2}-(2R)^{3/2}\right) = \frac{d^2\sqrt{2g}}{4}\,t \qquad (2)$$

Use $d = \frac{1}{12}$ ft and $g = 32.2\ \frac{ft}{s^2}$ and plot $h = h(t)$ for values of $R = 0.5$, 2.5, 5, and 10 ft

Note: It is easier to solve Eq. (2) as $t = t(h)$ rather than $h = h(t)$

Note: The time taken to empty the tank, t_e, is obtained from Eq. (2) with $h = 0$ as

$$t_e = \frac{64\,R^{5/2}}{15\,d^2\sqrt{g}}$$

(con't)

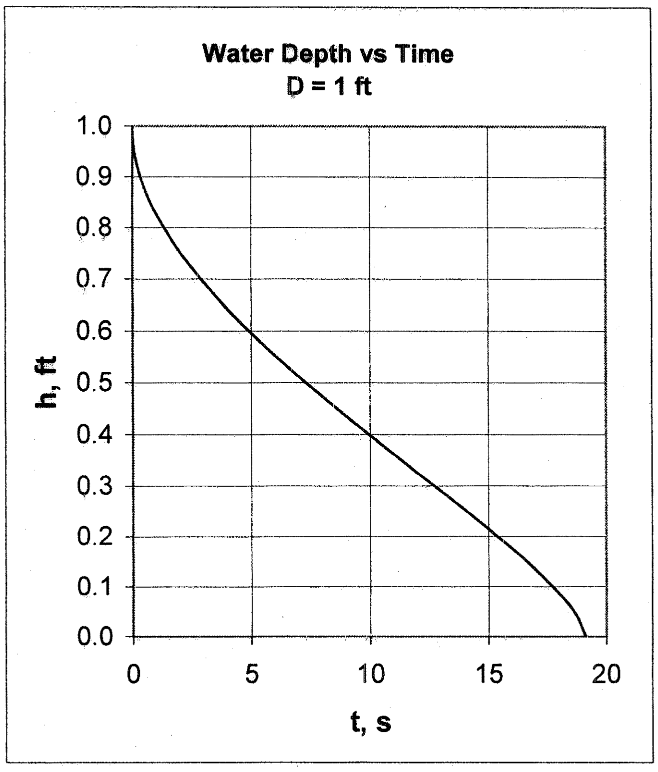
3.78* (con't.)

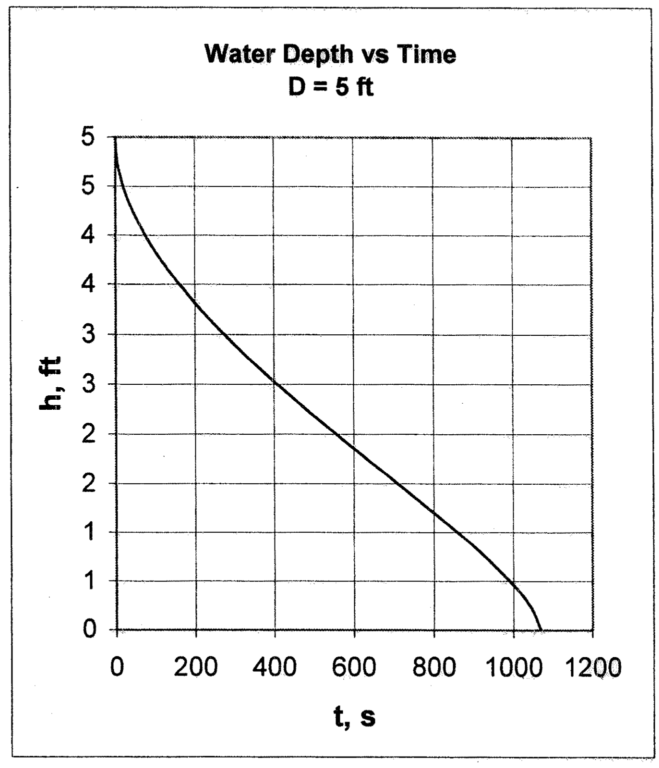
Results of an EXCEL Program to calculate h(t) from Eqn. (2):

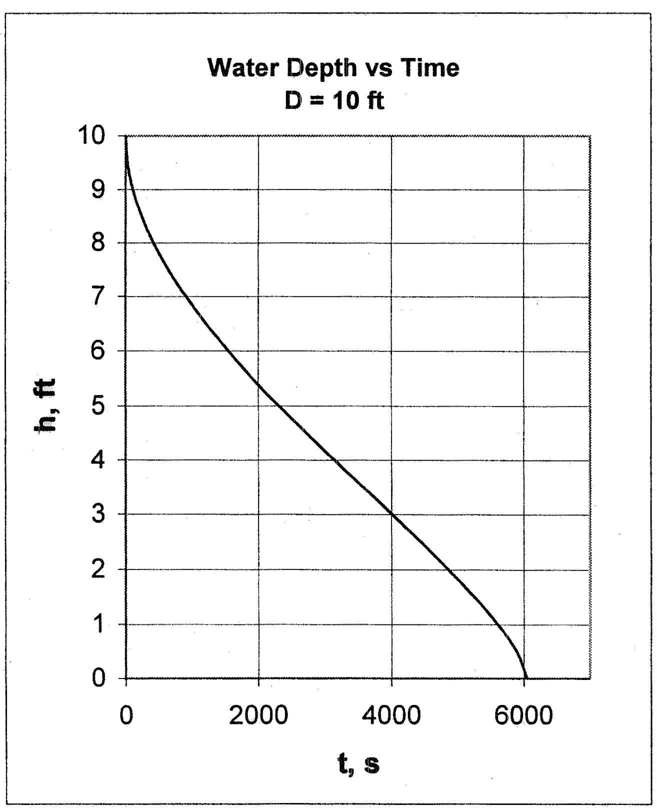
D = 1 ft		D = 5 ft		D = 10 ft		D = 20 ft	
t, s	h, ft	t, s	h, ft	t, s	h, ft	t, s	h, ft
0.00	1.000	0	5.000	0	10.00	0	20
0.09	0.950	5	4.750	28	9.50	158	19
0.35	0.900	19	4.500	110	9.00	620	18
0.77	0.850	43	4.250	242	8.50	1370	17
1.34	0.800	75	4.000	422	8.00	2390	16
2.05	0.750	114	3.750	647	7.50	3661	15
2.89	0.700	161	3.500	913	7.00	5163	14
3.84	0.650	215	3.250	1216	6.50	6876	13
4.91	0.600	274	3.000	1552	6.00	8778	12
6.06	0.550	339	2.750	1917	5.50	10846	11
7.30	0.500	408	2.500	2308	5.00	13055	10
8.60	0.450	481	2.250	2718	4.50	15376	9
9.94	0.400	556	2.000	3143	4.00	17782	8
11.31	0.350	632	1.750	3577	3.50	20237	7
12.69	0.300	710	1.500	4014	3.00	22706	6
14.06	0.250	786	1.250	4445	2.50	25144	5
15.37	0.200	859	1.000	4862	2.00	27502	4
16.61	0.150	929	0.750	5253	1.50	29714	3
17.72	0.100	990	0.500	5603	1.00	31695	2
18.62	0.050	1041	0.250	5889	0.50	33311	1
19.14	0.000	1070	0.000	6053	0.00	34239	0

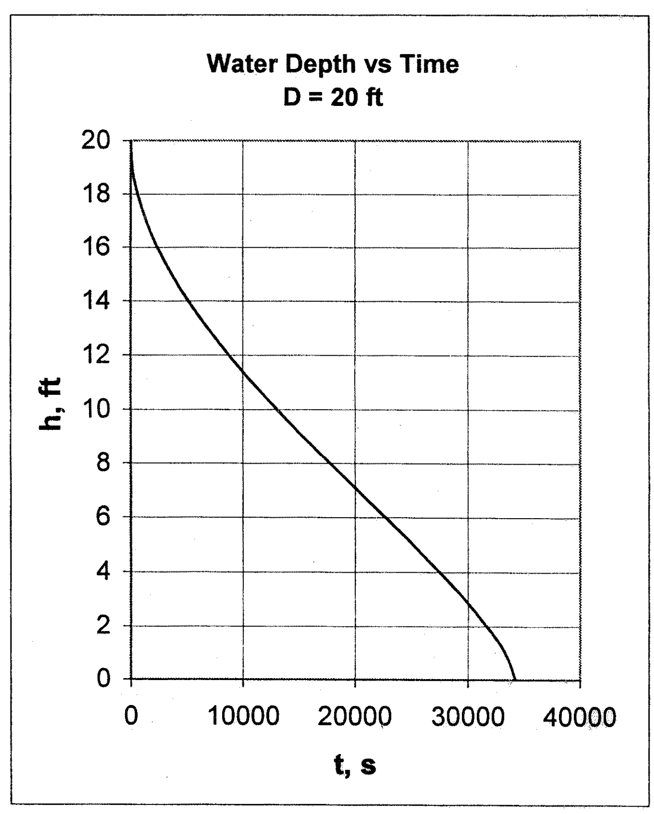
See next page for graphs of above results.

3.78* (con't)


Water Depth vs Time
D = 1 ft
h, ft
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
t, s

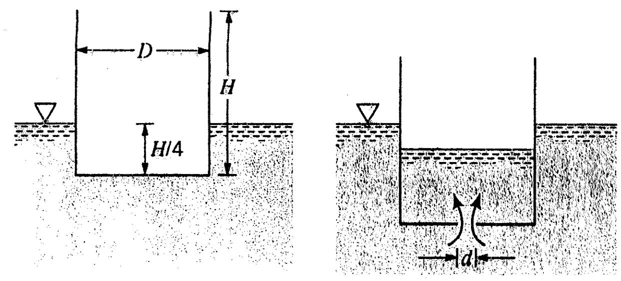

Water Depth vs Time
D = 5 ft
h, ft
5
5
4
4
3
3
2
2
1
1
0
0
200
400
600
800
1000
1200
t, s


Water Depth vs Time
D = 10 ft
h, ft
10
9
8
7
6
5
4
3
2
1
0
0
2000
4000
6000
t, s


Water Depth vs Time
D = 20 ft
h, ft
20
18
16
14
12
10
8
6
4
2
0
0
10000
20000
30000
40000
t, s

3.79

3.79 A round, thin-walled tank of diameter D and height H floats as shown in Fig. P3.79*a* when empty. Suddenly a small hole of diameter d appears in the bottom of the tank, and the tank slowly fills with water as shown in Fig. P3.79*b*. Determine the time it takes for the tank to sink. List any assumptions.



(a) (b)

■ FIGURE P3.79

When the tank is at rest, floating with no hole in the bottom, the weight of the tank equals the weight of the displaced fluid. Thus,

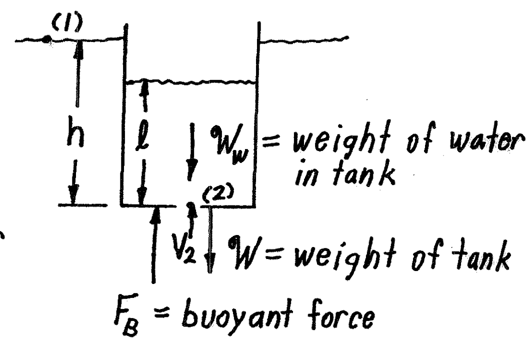
$\mathcal{W}$ = tank weight $= \gamma \mathcal{V} = \gamma \frac{\pi}{4} D^2 \left(\frac{H}{4}\right)$, or $\mathcal{W} = \frac{\pi}{16} \gamma H D^2$

If the tank sinks slowly, then the drag of the surrounding water and the acceleration of the tank are negligible. Hence,

$\Sigma F_z = m a_z$ or $\Sigma F_z = 0$

or

$F_B - \mathcal{W} - \mathcal{W}_w = 0$



Thus, $\gamma \frac{\pi}{4} D^2 h - \frac{\pi}{16} \gamma D^2 H - \gamma \frac{\pi}{4} D^2 \ell = 0$

which gives $h - \ell = \frac{H}{4}$, i.e., the water level in the tank is always a distance $\frac{H}{4}$ below the free surface outside the tank.

As the tank sinks, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$, where $p_1 = 0$, $p_2 = \gamma \ell$, $V_1 = 0$, $z_1 = 0$, and $z_2 = -h$

Thus, $0 = \frac{\gamma \ell}{\gamma} + \frac{V_2^2}{2g} - h$

or $V_2 = \sqrt{2g(h-\ell)} = \sqrt{2g\left(\frac{H}{4}\right)} = \sqrt{\frac{gH}{2}}$

But $Q_2 = A_2 V_2 = \frac{\pi}{4} D^2 \frac{dh}{dt}$, where $\frac{dh}{dt}$ is the rate that the tank sinks.

Hence,

$\frac{\pi}{4} d^2 \sqrt{\frac{gH}{2}} = \frac{\pi}{4} D^2 \frac{dh}{dt}$, or $\frac{dh}{dt} = \left(\frac{d}{D}\right)^2 \sqrt{\frac{gH}{2}}$

so that

$$\int_{h=H/4}^{h=H} dh = \left(\frac{d}{D}\right)^2 \sqrt{\frac{gH}{2}} \int_{t=0}^{t=t_s} dt$$

Hence, t_s = time to sink $= \frac{3\sqrt{2}}{4} \left(\frac{D}{d}\right)^2 \sqrt{\frac{H}{g}}$

*3.80

***3.80** The surface area, A, of the pond shown in Fig. P3.80 varies with the water depth, h, as shown in the table. At time $t = 0$ a valve is opened and the pond is allowed to drain through a pipe of diameter D. If viscous effects are negligible and quasisteady conditions are assumed, plot the water depth as a function of time from when the valve is opened ($t = 0$) until the pond is drained for pipe diameters of $D = 0.5, 1.0, 1.5, 2.0, 2.5,$ and 3.0 ft. Assume $h = 18$ ft at $t = 0$.

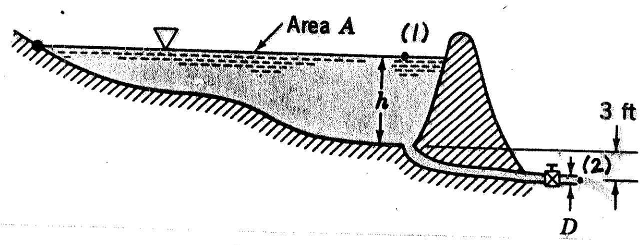


FIGURE P3.80

h (ft)	A [acres (1 acre = 43,560 ft^2)]
0	0
2	0.3
4	0.5
6	0.8
8	0.9
10	1.1
12	1.5
14	1.8
16	2.4
18	2.8

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $p_1 = 0$, $p_2 = 0$, $z_1 = h$, $z_2 = -3\,ft$
and $V_1 = -\frac{dh}{dt} \ll V_2$

Thus, $V_2 = \sqrt{2g(h+3)}$ which when combined with $A_1 V_1 = A_2 V_2$ gives

$-A_1 \frac{dh}{dt} = \frac{\pi}{4} D^2 \sqrt{2g(h+3)}$ where $A_1 = A_1(h)$ as given.

This can be rearranged and integrated to give

$$\int_{18\,ft}^{h} A_1 \frac{dh}{\sqrt{h+3}} = -\frac{\pi}{4}\sqrt{2g}\,D^2 \int_0^t dt = -\frac{\pi}{4} D^2 \sqrt{2g}\, t = -\frac{\pi}{4} D^2 \sqrt{2 \times 32.2}\, t$$

or

$$t = \frac{0.159}{D^2} \int_h^{18} A_1 \frac{dh}{\sqrt{h+3}} \text{, where } t \sim s,\ A_1 \sim ft^2, \text{ and } h \sim ft \qquad (1)$$

Note: It is easier to determine t as a function of h rather than h as a function of t

Note: $t \sim D^{-2}$

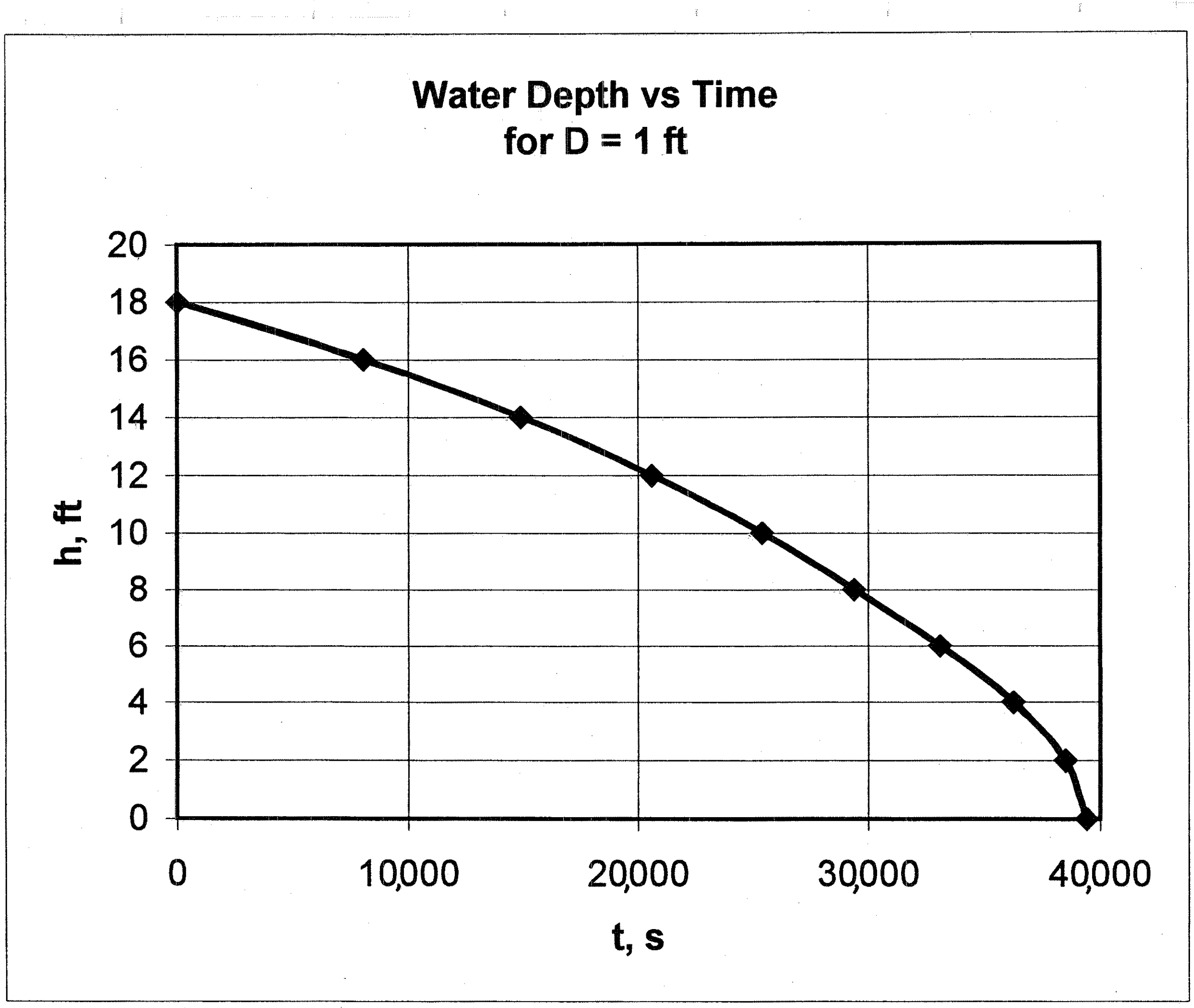
(con't)

3.80 (con't)

An EXCEL Program using a trapezoidal integration approzimation was used to calculate the results shown below.

			D = 0.5 ft	D = 1.0 ft	D = 1.5 ft	D = 2.0 ft	D = 2.5 ft	D = 3.0 ft
h, ft	A, acres	A, ft^2	t, s	t, s	t, s	t, s	t, s	t, s
18	2.8	121968	0	0	0	0	0	0
16	2.4	104544	32181	8045	3576	2011	1287	894
14	1.8	78408	59530	14882	6614	3721	2381	1654
12	1.5	65340	82354	20589	9150	5147	3294	2288
10	1.1	47916	101536	25384	11282	6346	4061	2820
8	0.9	39204	117506	29377	13056	7344	4700	3264
6	0.8	34848	132412	33103	14712	8276	5296	3678
4	0.5	21780	145035	36259	16115	9065	5801	4029
2	0.3	13068	153988	38497	17110	9624	6160	4277
0	0	0	157704	39426	17523	9857	6308	4381

The graph for D = 1 ft is shown below. The shape of the curve is the same for any D.



3.81

3.81 Water flows through a horizontal branching pipe as shown in Fig. P3.81. Determine the pressure at section (3).

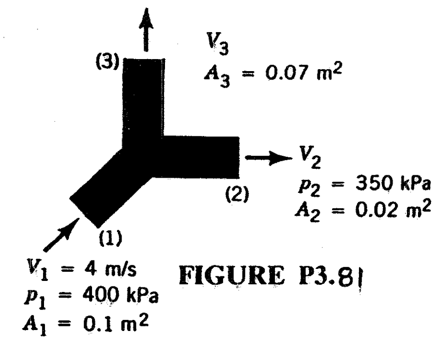


FIGURE P3.81

$Q_1 = Q_2 + Q_3$ or $V_3 = \dfrac{Q_1 - Q_2}{A_3}$ where $Q_1 = A_1 V_1 = 0.1 m^2 (4\frac{m}{s}) = 0.4 \frac{m^3}{s}$

Also $Q_2 = A_2 V_2$ where $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $z_1 = z_2$

Thus,

$$\frac{400 kPa}{9.80 \frac{kN}{m^3}} + \frac{(4\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} = \frac{350 kPa}{9.80\frac{kN}{m^3}} + \frac{V_2^2}{2(9.81\frac{m}{s^2})}$$

or

$V_2 = 10.78 \frac{m}{s}$

Thus,

$$V_3 = \frac{0.4\frac{m^3}{s} - 0.02 m^2 (10.78\frac{m}{s})}{0.07\ m^2} = 2.63 \frac{m}{s}$$

Then from $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ with $z_1 = z_3$ we obtain

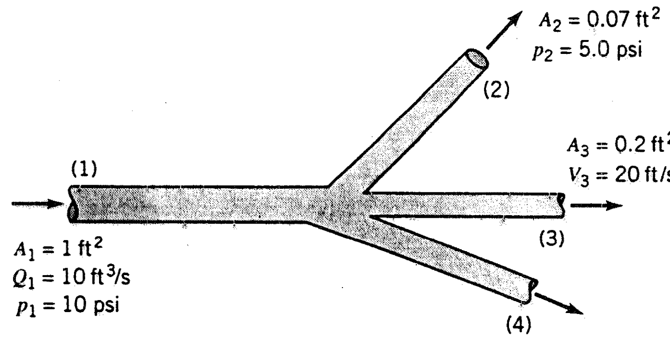
$$p_3 = p_1 + \frac{\gamma}{2g}(V_1^2 - V_3^2) = 400 kPa + \frac{9.80 \frac{kN}{m^3}}{2(9.81\frac{m}{s^2})}(4^2 - 2.63^2)\frac{m^2}{s^2}$$

or

$p_3 = (400 + 4.54)\frac{kN}{m^2} = \underline{\underline{404.5\ kPa}}$

3.82

3.82 Water flows through the horizontal branching pipe shown in Fig. P3.82 at a rate of 10 ft³/s. If viscous effects are negligible, determine the water speed at section (2), the pressure at section (3), and the flowrate at section (4).



■ FIGURE P3.82

From (1) to (2): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $z_1 = z_2$, $p_1 = 10\,psi$, $p_2 = 5\,psi$, and $V_1 = \frac{Q_1}{A_1}$ or $V_1 = (10\,\frac{ft^3}{s})/(1\,ft^2) = 10\,\frac{ft}{s}$

Thus, with $\gamma = \rho g$

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{(10\,\frac{ft}{s})^2}{2} = \frac{(5\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1.94\,\frac{slugs}{ft^3})} + \frac{V_2^2}{2} \text{ or } \underline{\underline{V_2 = 29.0\,\frac{ft}{s}}}$$

From (1) to (3): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ where $z_1 = z_3$, $p_1 = 10\,psi$, $V_1 = 10\,\frac{ft}{s}$ and $V_3 = 20\,\frac{ft}{s}$

Thus,

$$\frac{(10\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + \frac{(10\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = \frac{p_3}{62.4\,\frac{lb}{ft^3}} + \frac{(20\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$p_3 = 1150\,\frac{lb}{ft^2} = \underline{\underline{7.98\,psi}}$$

Also,

$$Q_4 = Q_1 - Q_2 - Q_3 = Q_1 - A_2V_2 - A_3V_3$$

or

$$Q_4 = 10\,\frac{ft^3}{s} - 0.07\,ft^2(29.0\,\frac{ft}{s}) - 0.2\,ft^2(20\,\frac{ft}{s}) = \underline{\underline{3.97\,\frac{ft^3}{s}}}$$

3.83

3.83 Water flows from a large tank through a large pipe that splits into two smaller pipes as shown in Fig. P3.83. If viscous effects are negligible, determine the flowrate from the tank and the pressure at point (1).

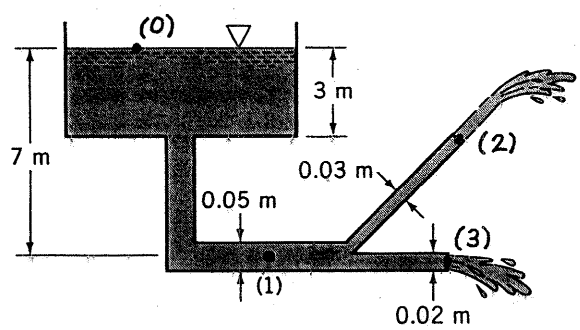


FIGURE P3.83

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_0 = 0,\ p_2 = 0,\ V_0 = 0,\ z_0 = 7m \text{ and } z_2 = 4m$$

Thus,

$$V_2 = \sqrt{2g(z_0 - z_2)} = \sqrt{2(9.81\tfrac{m}{s^2})(7-4)m} = 7.67\,\tfrac{m}{s}$$

Similarly

$$V_3 = \sqrt{2g(z_0 - z_3)} = \sqrt{2(9.81\tfrac{m}{s^2})(7m)} = 11.7\,\tfrac{m}{s}$$

Thus, $Q = Q_2 + Q_3 = \frac{\pi}{4}D_2^2 V_2 + \frac{\pi}{4}D_3^2 V_3^2$

or

$$Q = \frac{\pi}{4}\left[(0.03m)^2(7.67\tfrac{m}{s}) + (0.02m)^2(11.7\tfrac{m}{s})\right] = \underline{\underline{9.10\times10^{-3}\,\tfrac{m^3}{s}}}$$

Also,

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{where } z_1 = 0 \text{ and}$$

or

$$V_1 = \frac{Q}{A_1} = \frac{9.10\times10^{-3}\,\frac{m^3}{s}}{\frac{\pi}{4}(0.05m)^2} = 4.63\,\tfrac{m}{s}$$

$$p_1 = \gamma\left[z_0 - \frac{V_1^2}{2g}\right] = 9.80\times10^3\,\tfrac{N}{m^3}\left[7m - \frac{(4.63\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}\right] = 5.79\times10^4\,\tfrac{N}{m^2}$$

or

$$p_1 = \underline{\underline{57.9\ kPa}}$$

3.84

3.84 Water flows through the horizontal Y-fitting shown in Fig. P3.84. If the flowrate and pressure in pipe (1) are $Q_1 = 2.3\ \text{ft}^3/\text{s}$ and $p_1 = 50\ \text{lb/in.}^2$, determine the pressures, p_2 and p_3, in pipes (2) and (3) under the assumption that the flowrate divides evenly between pipes (2) and (3).

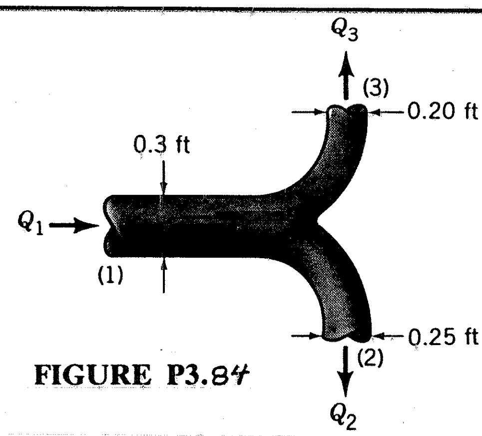


FIGURE P3.84

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2 \text{ and } V_1 = \frac{Q_1}{A_1} \qquad (1)$$

$$V_2 = \frac{Q_2}{A_2} = \frac{0.5\,Q_1}{A_2}$$

Thus, $V_1 = \dfrac{2.3\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.3\,ft)^2} = 32.5\,\frac{ft}{s}$

and

$$V_2 = \frac{(0.5)(2.3\,\frac{ft^3}{s})}{\frac{\pi}{4}(0.25\,ft)^2} = 23.4\,\frac{ft}{s} \quad \text{so that Eq. (1) becomes}$$

$$p_2 = p_1 + \tfrac{1}{2}\rho(V_1^2 - V_2^2) = 50\ psi + \tfrac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left[(32.5)^2 - (23.4)^2\right]\frac{ft^2}{s^2}$$

$$= 50\,psi + \left(493\,\frac{lb}{ft^2}\right)\left(\frac{1\,ft^2}{144\,in.^2}\right) = \underline{\underline{53.4\,psi}}$$

Similarly

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad \text{where } z_1 = z_3 \text{ and } V_3 = \frac{Q_3}{A_3} = \frac{0.5\,Q_1}{A_3}$$

Thus, $V_3 = \dfrac{(0.5)(2.3\,\frac{ft^3}{s})}{\frac{\pi}{4}(0.20\,ft)^2} = 36.6\,\frac{ft}{s}$

so that

$$p_3 = p_1 + \tfrac{1}{2}\rho(V_1^2 - V_3^2) = 50\,psi + \tfrac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)\left[(32.5)^2 - (36.6)^2\right]\frac{ft^2}{s^2}\left(\frac{1\,ft^2}{144\,in.^2}\right)$$

$$= \underline{\underline{48.1\,psi}}$$

3.85

3.85 Water flows from the pipe shown in Fig. P3.85 as a free jet and strikes a circular flat plate. The flow geometry shown is axisymmetrical. Determine the flowrate and the manometer reading, H.

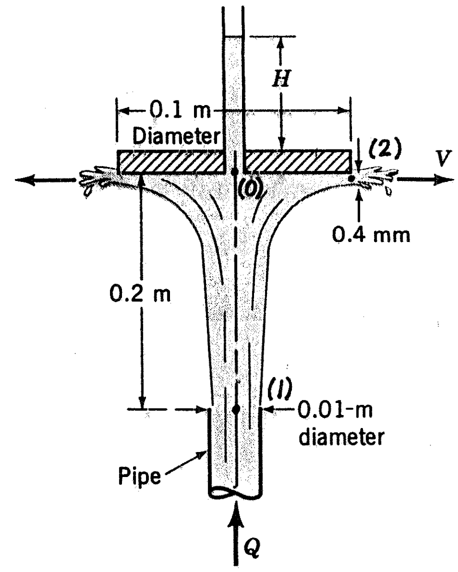


FIGURE P3.85

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$, where $p_1 = 0$, $p_2 = 0$, $z_1 = 0$, and $z_2 = 0.2m$

Thus,

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} + z_2 \quad \text{where} \quad A_1 V_1 = A_2 V_2 = Q \qquad (1)$$

or $V_1 = \frac{A_2}{A_1} V_2 = \frac{\pi D_2 h}{\frac{\pi}{4} D_1^2} V_2 = \frac{4 D_2 h}{D_1^2} V = \frac{4(0.1m)(4\times10^{-4}m)}{(0.01m)^2} V_2 = 1.6 V_2$

Hence, Eq. (1) gives

$(1.60 V_2)^2 = V_2^2 + 2(9.81\frac{m}{s^2})(0.2m)$ or $V_2 = 1.59\frac{m}{s}$
so that

$Q = A_2 V_2 = \pi(0.1m)(4\times10^{-4}m)(1.59\frac{m}{s}) = \underline{\underline{2.00\times10^{-4}\frac{m^3}{s}}}$

Also,

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0$, where $V_0 = 0$, $z_0 = 0.2m$, $V_1 = 1.60 V_2$
or $V_1 = 1.60(1.59\frac{m}{s}) = 2.54\frac{m}{s}$, and $p_1 = 0$

Thus,

$H = \frac{p_0}{\gamma} = \frac{V_1^2}{2g} - z_0 = \frac{(2.54\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} - 0.2m = \underline{\underline{0.129\ m}}$

3.86

3.86 Air flows from a hole of diameter 0.03 m in a flat plate as shown in Fig. P3.86. A circular disk of diameter D is placed a distance h from the lower plate. The pressure in the tank is maintained at 1 kPa. Determine the flowrate as a function of h if viscous effects and elevation changes are assumed negligible and the flow exits radially from the circumference of the circular disk with uniform velocity.

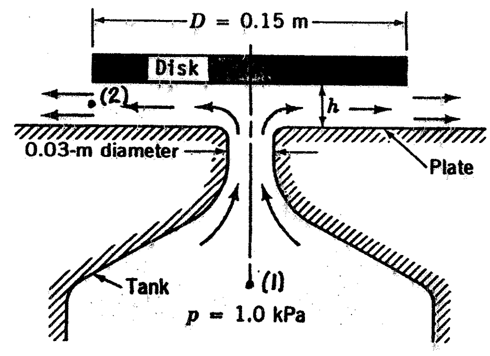


FIGURE P3.86

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } p_0=1\,\frac{kN}{m^2},\ p_2=0,\ z_0\approx z_2,$$

and $V_0=0$

Thus,

$$V_2=\sqrt{\frac{2p_0}{\rho}}=\sqrt{\frac{2\left(1\times 10^3\,\frac{N}{m^2}\right)}{1.23\,\frac{kg}{m^3}}}=40.3\,\frac{m}{s}$$

so that

$$Q=A_2V_2=\pi D_2 h V_2=\pi(0.15\,m)h\left(40.3\,\frac{m}{s}\right)$$

or

$$Q=\underline{\underline{19.0\,h\,\frac{m^3}{s}}}\quad \text{where } h\sim m$$

3.87

3.87 A conical plug is used to regulate the air flow from the pipe shown in Fig. P3.87. The air leaves the edge of the cone with a uniform thickness of 0.02 m. If viscous effects are negligible and the flowrate is 0.50 m³/s, determine the pressure within the pipe.

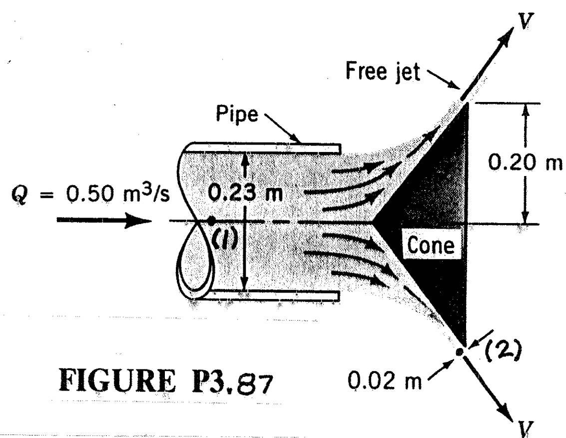


FIGURE P3.87

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad \text{where } z_1 = z_2 \text{ and } p_2 = 0$$

Also,

$$V_1 = \frac{Q}{A_1} = \frac{0.5\,\frac{m^3}{s}}{\frac{\pi}{4}(0.23m)^2} = 12.0\,\frac{m}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{Q}{2\pi R h} = \frac{0.5\,\frac{m^3}{s}}{2\pi(0.2m)(0.02m)} = 19.9\,\frac{m}{s}$$

Thus,

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2}\left(1.23\,\frac{kg}{m^3}\right)(19.9^2 - 12.0^2)\,\frac{m^2}{s^2} = \underline{\underline{155\,\frac{N}{m^2}}}$$

3.88

3.88 An automatic boat bailer is shown in Fig. P3.88. The "bump" on the bottom of the hull is designed to increase the velocity at point (1) to approximately $1.2V_0$, where V_0 is the velocity of the boat through the water. If the pressures at (1) and (2) are essentially equal, what minimum velocity is needed to initiate the bailing action?

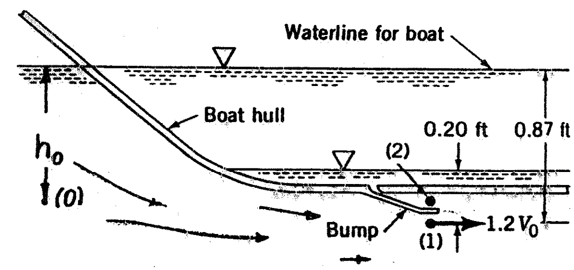


FIGURE P3.88

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{where } p_0 = \gamma h_0,\ z_0 = -h_0, \text{ and } V_1 = 1.2V_0$$

Thus,

$$\frac{\gamma h_0}{\gamma} + \frac{V_0^2}{2g} - h_0 = \frac{p_1}{\gamma} + \frac{(1.2V_0)^2}{2g} + z_1 \quad \text{or} \quad p_1 = \frac{1}{2}\rho V_0^2(1-1.2^2) - \gamma z_1 \quad (1)$$

To initiate bailing $\quad p_2 \approx p_1$ where $p_2 = \gamma h = (62.4\frac{lb}{ft^3})(0.2ft)$

$= 12.5 \frac{lb}{ft^2}$

Thus, with $z_1 = -0.87\ ft$ Eq. (1) becomes

$$12.5 \frac{lb}{ft^2} = \frac{1}{2}(1.94\frac{slugs}{ft^3})(-0.44)V_0^2 - (62.4\frac{lb}{ft^3})(-0.87ft)$$

or

$$V_0 = 9.89 \frac{ft}{s}$$

3.89

3.89 A small card is placed on top of a spool as shown in Fig. P3.89. It is not possible to blow the card off the spool by blowing air through the hole in the center of the spool. The harder one blows, the harder the card "sticks" to the spool. In fact, by blowing hard enough it is possible to keep the card against the spool with the spool turned upside down. (*Note:* It may be necessary to use a thumb tack to prevent the card from sliding from the spool.) Explain this phenomenon.

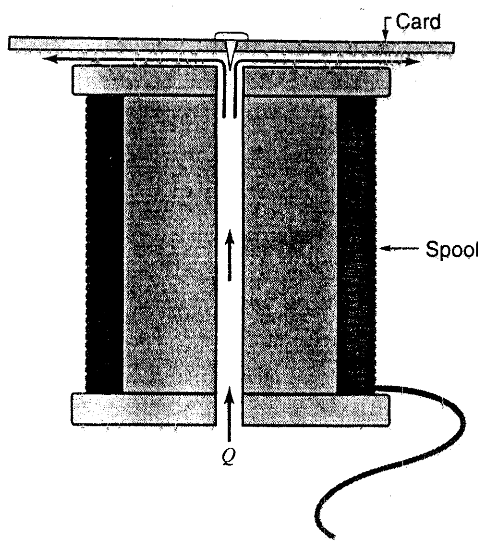
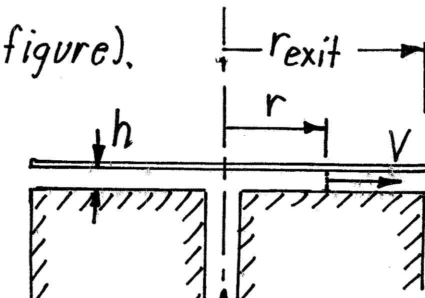


FIGURE P3.89

As the air flows radially outward in the gap between the card and the spool it slows down since the flow area increases with r, the radial distance from the center. That is,

$$Q = 2\pi r h V, \text{ or } V = \frac{Q}{2\pi h r} \quad \text{(see the figure).} \qquad (1)$$



If viscous effects are not important, then

$$\frac{p}{\gamma} + \frac{V^2}{2g} = \text{constant} = \frac{p_{exit}}{\gamma} + \frac{V_{exit}^2}{2g}$$

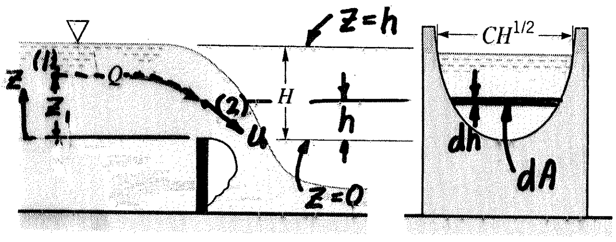
or since $p_{exit} = 0$ (a free jet) it follows that

$$p = \frac{1}{2}\rho\left(V_{exit}^2 - V^2\right), \text{ where from Eq. (1) } V_{exit}^2 - V^2 = \left(\frac{Q}{2\pi h}\right)^2\left[\frac{1}{r_{exit}^2} - \frac{1}{r^2}\right]$$

But $r_{exit} > r$ so that $p < 0$. There is a vacuum within the gap. The card is sucked against the spool. The harder one blows through the spool (larger Q), the larger the vacuum, and the harder the card is held against the spool.

3.90

3.90 Water flows over a weir plate (see **Video V10.7**) which has a parabolic opening as shown in Fig. P3.90. That is, the opening in the weir plate has a width $CH^{1/2}$, where C is a constant. Determine the functional dependence of the flowrate on the head, $Q = Q(H)$.



■ FIGURE P3.90

$Q = \int u \, dA$ where u is a function of h.

That is, from $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ with $\frac{p_1}{\gamma} = H - z_1$, $V_2 \equiv u$

$\frac{p_2}{\gamma} = 0$ ("free jet")

and $z_2 = H - h$

or

$(H - z_1) + \frac{V_1^2}{2g} + z_1 = 0 + \frac{u^2}{2g} + (H - h)$

Thus,

$u = \sqrt{2gh + V_1^2} \approx \sqrt{2gh}$ if V_1 is "small"

Also,

$dA = C\sqrt{z}\, dz$ (i.e. $dA = 0\, dz$ for $z = 0$; $dA = C\sqrt{H}$ for $z = H$) so that

$$Q = \int_{z=0}^{z=H} \sqrt{2g}\sqrt{h}\, C\sqrt{z}\, dz \quad \text{where } h = H - z.$$

Thus, $Q = C\sqrt{2g}\int_0^H \sqrt{zH - z^2}\, dz$, where

$$\int_0^H \sqrt{zH - H^2}\, dz = \frac{1}{2}\left[(z - \tfrac{H}{2})\sqrt{Hz - z^2} + (\tfrac{H}{2})^2 \sin^{-1}\left[(z - \tfrac{H}{2})/(H/2)\right]\right]_{z=0}^{z=H}$$

which reduces to:

$Q = \frac{\pi C}{8}\sqrt{2g}\, H^2$ That is $\underline{\underline{Q \sim H^2}}$

Alternatively, $Q = VA$ where the average velocity is proportional to $\sqrt{H}$ (i.e. $V \sim \sqrt{2gH}$) and the total flow area is proportional to $H^{3/2}$ (i.e. $A \sim H \times (CH^{1/2}) = CH^{3/2}$). Thus,

$Q \sim \sqrt{2gH}\,(CH^{3/2}) = C\sqrt{2g}\, H^2$

That is, $Q \sim H^2$ as obtained above.

3,91

3.91 A weir (see Video V10.7) of trapezoidal cross section is used to measure the flowrate in a channel as shown in Fig. P3.91. If the flowrate is Q_0 when $H = \ell/2$, what flowrate is expected when $H = \ell$?

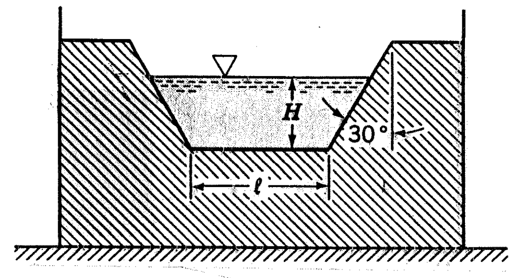


FIGURE P3.91

$Q = AV$ where it is expected that V is a function of the head, H. That is, $V \sim \sqrt{2gH}$

Also, from the geometry $A = \frac{1}{2} H (\ell + \ell_T)$ where $\ell_T = \ell + 2H \tan 30°$

Thus, $A = H(\ell + H \tan 30°)$ so that

$Q = C_1 \sqrt{2g} (\ell + H \tan 30°) H^{3/2}$ where C_1 is a constant

Let Q_0 = flowrate when $H = \frac{\ell}{2}$

and Q_ℓ = flowrate when $H = \ell$

Thus,

$$\frac{Q_0}{Q_\ell} = \frac{C_1\sqrt{2g}\,(\ell + \frac{\ell}{2}\tan 30°)(\frac{\ell}{2})^{3/2}}{C_1\sqrt{2g}\,(\ell + \ell \tan 30°)(\ell)^{3/2}} = \frac{(1 + \frac{1}{2}\tan 30°)}{(1 + \tan 30°)(2^{3/2})} = 0.289$$

or

$$\underline{\underline{Q_\ell = 3.46\, Q_0}}$$

3.92

3.92 Water flows down the sloping ramp shown in Fig. P3.92 with negligible viscous effects. The flow is uniform at sections (1) and (2). For the conditions given show that three solutions for the downstream depth, h_2, are obtained by use of the Bernoulli and continuity equations. However, show that only two of these solutions are realistic. Determine these values.

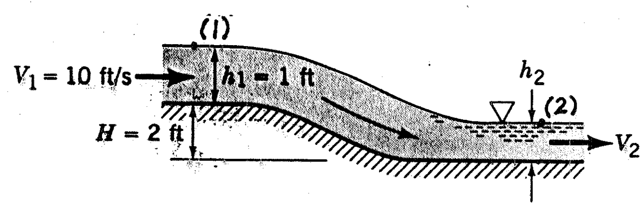


FIGURE P3.92

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 3\text{ ft}$, and $z_2 = h_2$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{(1\text{ft})(10\frac{\text{ft}}{\text{s}})}{h_2} = \frac{10}{h_2}$$

Thus, Eq. (1) becomes

$$\frac{(10\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})} + 3\text{ ft} = \frac{(\frac{10}{h_2})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})} + h_2$$

or

$$64.4 h_2^3 - 293 h_2^2 + 100 = 0$$

By using a root finding program the three roots to this cubic equation are found to be:

$h_2 = 0.630\text{ ft}$

$h_2 = 4.48\text{ ft}$

or

$h_2 =$ a negative root

Clearly it is not possible (physically) to have $h_2 < 0$. Thus, $\underline{h_2 = 0.630\text{ ft}}$ or $\underline{h_2 = 4.48\text{ ft}}$

3.93

3.93 Water flows under the sluice gate shown in Fig. P3.93. Determine the flowrate if the gate is 4.6 ft wide.

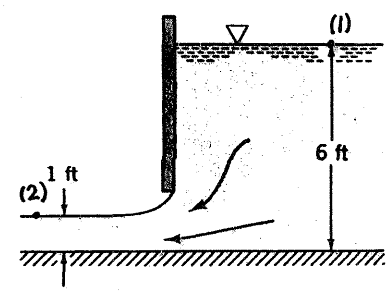


FIGURE P3.93

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where} \quad p_1 = 0,\ p_2 = 0,\ z_1 = 6\text{ ft} \quad \text{and} \quad z_2 = 1\text{ ft} \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{6\text{ ft}}{1\text{ ft}} V_1 = 6 V_1$$

Thus, Eq. (1) becomes

$$[6^2 - 1] V_1^2 = 2\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)(6-1)\text{ft} \quad \text{or} \quad V_1 = 3.03 \frac{\text{ft}}{\text{s}}$$

Hence,

$$Q = A_1 V_1 = (6\text{ft})(4.6\text{ft})\left(3.03 \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{83.6 \frac{\text{ft}^3}{\text{s}}}}$$

3.94

3.94 Water flows in a rectangular channel that is 2.0 m wide as shown in Fig. P3.94. The upstream depth is 70 mm. The water surface rises 40 mm as it passes over a portion where the channel bottom rises 10 mm. If viscous effects are negligible, what is the flowrate?

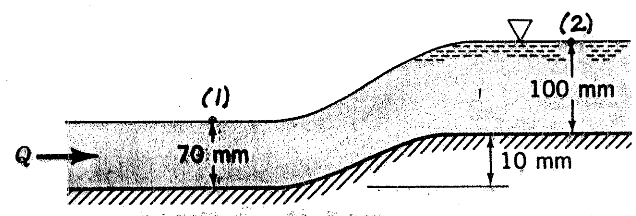


FIGURE P3.94

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ p_2 = 0,\ z_1 = 0.07m, \quad (1)$$

and $z_2 = (0.01 + 0.10)m = 0.11m$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{0.07m}{0.10m} V_1 = 0.7 V_1$$

Thus, Eq. (1) becomes

$$[1 - 0.7^2] V_1^2 = 2(9.81 \tfrac{m}{s^2})(0.11 - 0.07)m \quad \text{or } V_1 = 1.24 \tfrac{m}{s}$$

Hence,

$$Q = A_1 V_1 = (0.07m)(2.0m)(1.24 \tfrac{m}{s}) = \underline{\underline{0.174 \tfrac{m^3}{s}}}$$

3.95

3.95 Water flows under the inclined sluice gate shown in Fig. P3.95. Determine the flowrate if the gate is 8 ft wide.

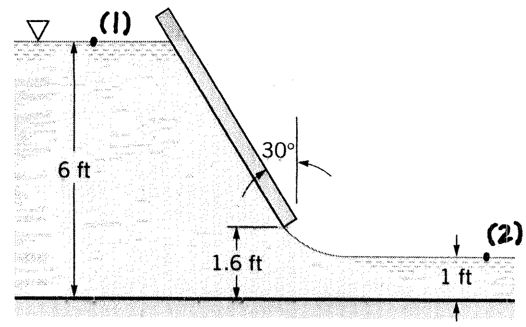


FIGURE P3.95

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $p_1 = 0$, $p_2 = 0$, $z_1 = 6\,ft$, and $z_2 = 1\,ft$

Thus,

$$\frac{V_1^2}{2g} + 6\,ft = \frac{V_2^2}{2g} + 1\,ft \qquad (1)$$

But $A_1 V_1 = A_2 V_2$, or

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{6\,ft}{1\,ft} V_1 = 6\,V_1$$

Hence, Eq. (1) becomes

$$\frac{V_1^2}{2g} + 6\,ft = \frac{(6)^2 V_1^2}{2g} + 1\,ft$$

or

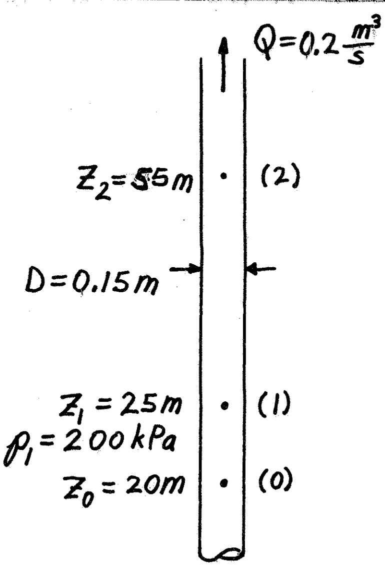
$$[6^2 - 1] V_1^2 = 2\left(32.2\,\frac{ft}{s^2}\right)(6-1)\,ft \quad \text{or} \quad V_1 = 3.03\,\frac{ft}{s}$$

Hence,

$$Q = A_1 V_1 = 6\,ft\,(8\,ft)\left(3.03\,\frac{ft}{s}\right) = \underline{\underline{145\,\frac{ft^3}{s}}}$$

3.96

3.96 Water flows in a vertical pipe of 0.15-m diameter at a rate of 0.2 m³/s and a pressure of 200 kPa at an elevation of 25 m. Determine the velocity head and pressure head at elevations of 20 and 55 m.



$$V = \frac{Q}{A} = \frac{0.2\,\frac{m^3}{s}}{\frac{\pi}{4}\,(0.15m)^2} = 11.3\,\frac{m}{s} = V_0 = V_2$$

At point (0):

$$\frac{V_0^2}{2g} = \frac{(11.3\,\frac{m}{s})^2}{2\,(9.81\,\frac{m}{s^2})} = \underline{\underline{6.51\,m}}$$

and

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{or} \quad \frac{p_0}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_0$$

or

$$\frac{p_0}{\gamma} = \frac{200\,\frac{kN}{m^2}}{9.80\,\frac{kN}{m^3}} + (25-20)m = \underline{\underline{25.4\,m}}$$

Similarly at point (2):

$$\frac{V_0^2}{2g} = \frac{V_2^2}{2g} = \underline{\underline{6.51\,m}}$$

and

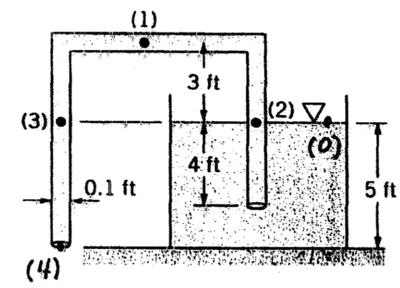
$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \quad \text{or} \quad \frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_2$$

or

$$\frac{p_2}{\gamma} = \frac{200\,\frac{kN}{m^2}}{9.80\,\frac{kN}{m^3}} + (25-55)m = \underline{\underline{-9.59\,m}}$$

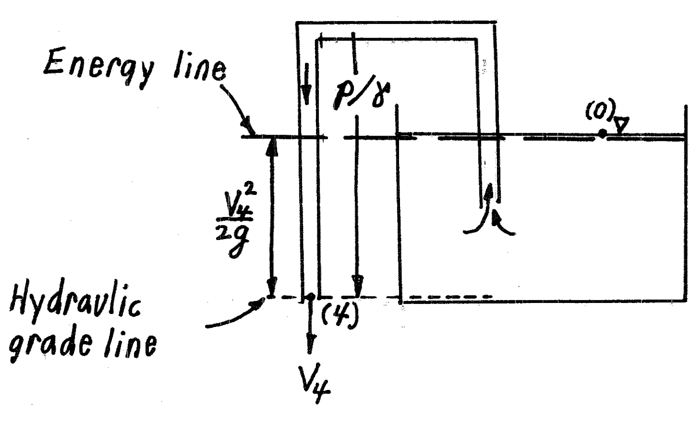
3.97

3.97 Draw the energy line and the hydraulic grade line for the flow shown in Problem 3.64.



■ FIGURE P3.64

For inviscid flow with no pumps or turbines, the energy line is horizontal at the elevation of the free surface of the tank. The hydraulic grade line is one velocity head, $V^2/2g$, below the energy line. Since $V_4 = \sqrt{2g(z_0 - z_4)}$ it follows that the hydraulic grade line is $V_4^2/2g = (z_0 - z_4) = 5$ ft below the free surface at the exit of the pipe. Also, since the pipe is a constant diameter, the velocity is constant throught the pipe. Hence, the hydraulic grade line is horizontal, 5 ft below the free surface. Note that since the pipe is above the hydraulic grade line, the pressure throughout the pipe is less than atmospheric.



3.98

3.98 Draw the energy line and the hydraulic grade line for the flow of Problem 3.60.

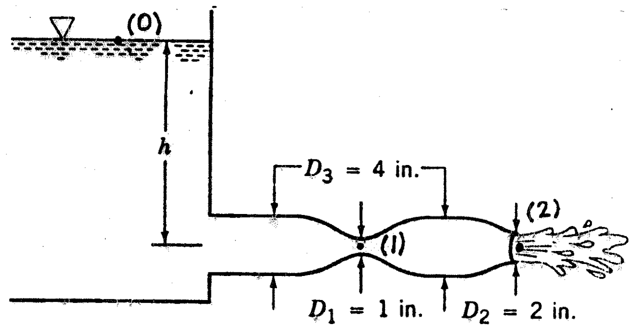


FIGURE P3.60

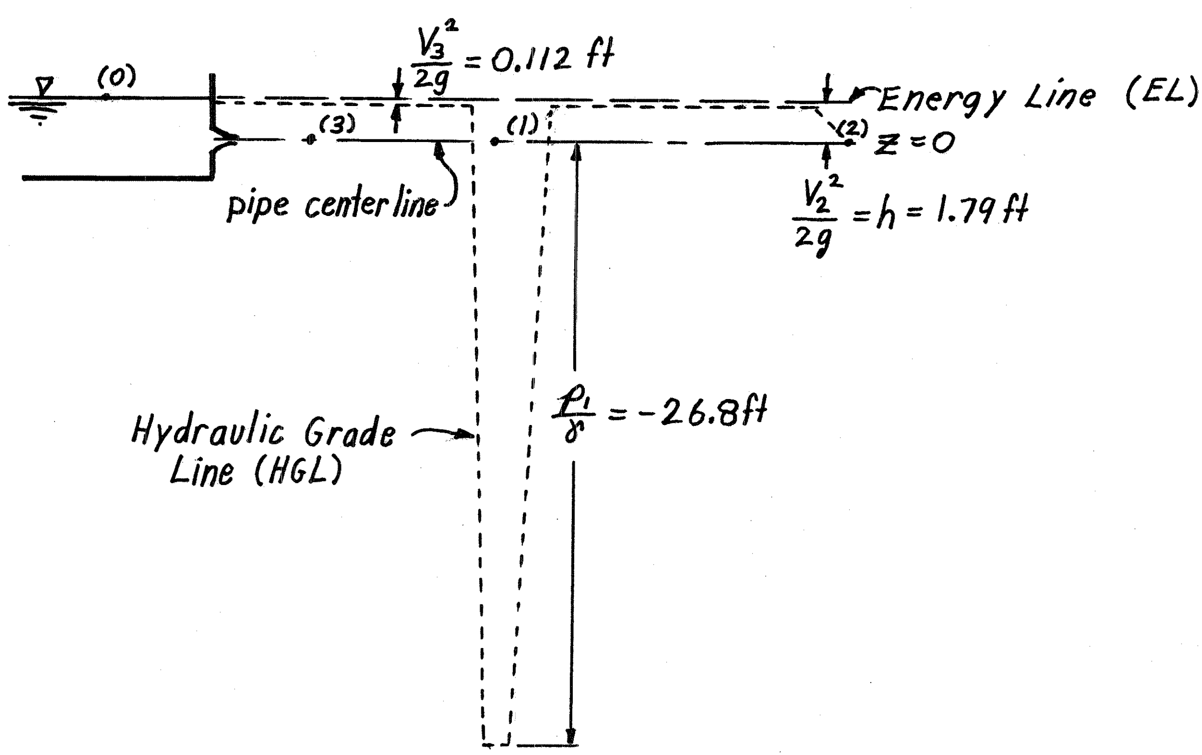
For inviscid flow with no pumps or turbines, the energy line is horizontal, a distance h above the outlet. From Problem 3.60 we obtain $h = 1.79$ ft.

The hydraulic grade line is $\frac{V^2}{2g}$ below the energy line, starting at the free surface where $V_0 = 0$ and ending at the pipe exit where $p_2 = 0$ and $\frac{V_2^2}{2g} = h$. At point (1) the pressure head is $p_1/\gamma = (2.88 - 14.5)\frac{lb}{in.^2}\left(\frac{144\,in.^2}{ft^2}\right)/62.4\frac{lb}{ft^3} = -26.8$ ft, and $z_1 = 0$.

In the 4 in. pipe $V_3 = A_2 V_2 / A_3 = \left(\frac{D_2}{D_3}\right)^2 V_2$ so that

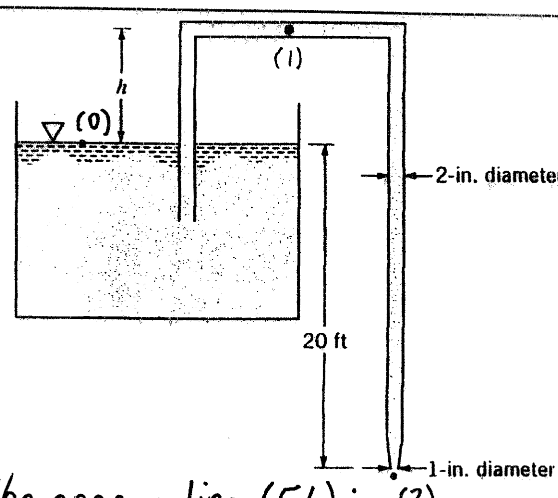
$$\frac{V_3^2}{2g} = \left(\frac{D_2}{D_3}\right)^4 \frac{V_2^2}{2g} = \left(\frac{D_2}{D_3}\right)^4 h = \left(\frac{2}{4}\right)^4 (1.79\,ft) = 0.112\ ft$$

The corresponding EL and HGL are drawn to scale below.



3.99

3.99 Draw the energy line and hydraulic grade line for the flow shown in Problem 3.65.



For inviscid flow with no pumps or turbines, the energy line (EL) is horizontal, at an elevation of the free surface. The hydraulic grade line (HGL) is one velocity head, $V^2/2g$, lower. Since $V_2 = \sqrt{2g(20\text{ ft})}$ or $V_2^2/2g = 20\text{ft}$, it follows that the HGL at the nozzle exit is at the same elevation as the exit. Also, within the constant diameter hose $A_1V_1 = A_2V_2$, or

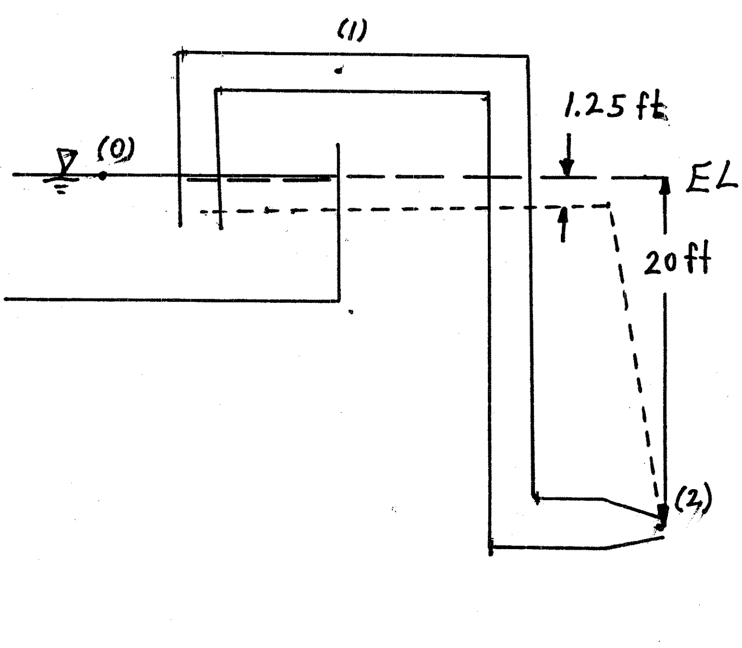
$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{1\text{in.}}{2\text{in.}}\right)^2 V_2 = \frac{1}{4}V_2$$

Thus, within the hose (

$$V_1^2/2g = \left(\tfrac{1}{4}\right)^2 V_2^2/2g = \left(\tfrac{1}{4}\right)^2(20\text{ ft}) = 1.25\text{ ft}$$

so that within the hose the HGL is 1.25 ft below the EL all along the hose.

The EL and HGL are shown in the figure below, with the exit nozzle rotated horizontally for ease of showing the HGL.



Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

3.100*

3.100* Water flows up the ramp shown in Fig. P3.100 with negligible viscous losses. The upstream depth and velocity are maintained at $h_1 = 0.3$ m and $V_1 = 6$ m/s. Plot a graph of the downstream depth, h_2, as a function of the ramp height, H, for $0 \le H \le 2$ m. Note that for each value of H there are three solutions, not all of which are realistic.

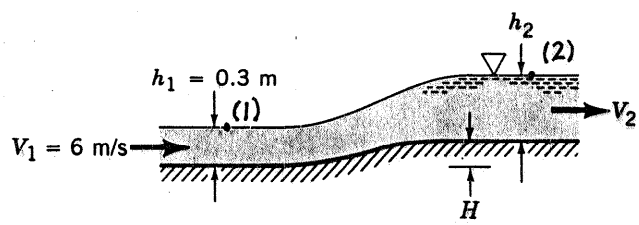


FIGURE P3.100

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ p_2 = 0,\ z_1 = 0.3m, \text{ and } z_2 = H + h_2 \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{(0.3m)(6\frac{m}{s})}{h_2} = \frac{1.8}{h_2} \quad \text{where } h_2 \sim m$$

Thus, Eq. (1) becomes

$$\frac{V_1^2}{2g} + 0.3\,m = \frac{\left(\frac{1.8}{h_2}\right)^2}{2g} + (H + h_2) \quad \text{or with } V_1 = 6\frac{m}{s},$$

$$(6\tfrac{m}{s})^2 + 2(9.81\tfrac{m}{s^2})(0.3 - H - h_2)m = \left(\frac{1.8}{h_2}\right)^2 \frac{m^2}{s^2}$$

which can be written as:

$$h_2^3 - (2.135 - H)h_2^2 + 0.1651 = 0 \qquad (2)$$

For $0 \le H \le 2\,m$ solve Eq. (2) for h_2

Rather than solving a cubic equation for h_2 (give H), one can directly solve for H (given h_2). From Eq. (2):

$$H = 2.135 - h_2 - \frac{0.1651}{h_2^2} \qquad (3)$$

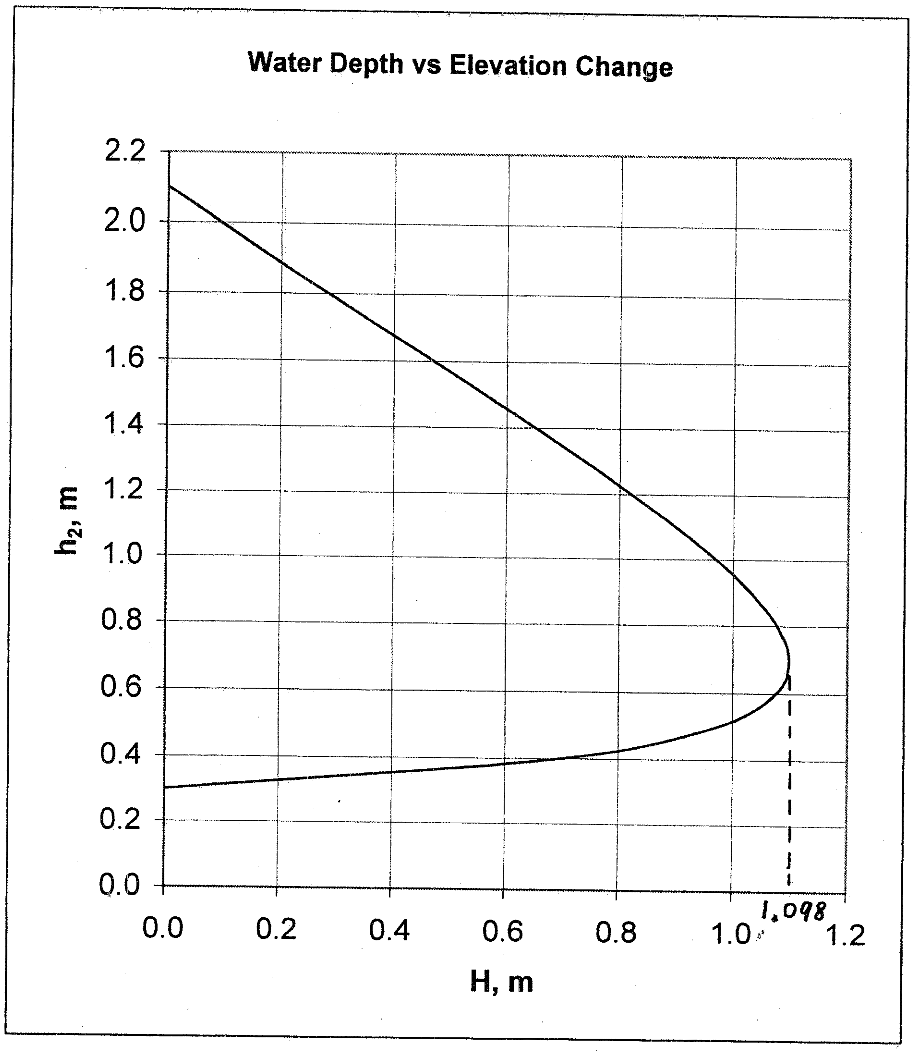
A graph of Eq. (2) or (3) is given on the following page.

(con't)

3.100* (con't)

The results of an EXCEL Program to calculate H for given values of h_2 are shown below.

h_2, m	H, m
0.3	0.001
0.4	0.703
0.5	0.975
0.6	1.076
0.7	1.098
0.8	1.077
0.9	1.031
1.0	0.970
1.1	0.899
1.2	0.820
1.3	0.737
1.4	0.651
1.5	0.562
1.6	0.471
1.7	0.378
1.8	0.284
1.9	0.189
2.0	0.094
2.1	-0.002



For $H \geq 1.098$ m there are no real, positive roots of Eq. (2). That is, for the given upstream conditions ($V_1 = 6 \frac{m}{s}$ and $h_1 = 0.3m$) we must have $H < 1.098$ m. It would not be possible to have the flow go up a ramp of greater height than this without increasing either V_1 and/or h_1. The two possible water depths for a given H are plotted above.

3.101

3.101 Pressure Distribution between Two Circular Plates

Objective: According to the Bernoulli equation, a change in velocity can cause a change in pressure. Also, for an incompressible flow, a change in flow area causes a change in velocity. The purpose of this experiment is to determine the pressure distribution caused by air flowing radially outward in the gap between two closely spaced flat plates as shown in Fig. P3.101.

Equipment: Air supply with a flow meter; two circular flat plates with static pressure taps at various radial locations from the center of the plates; spacers to maintain a gap of height b between the plates; manometer; barometer; thermometer.

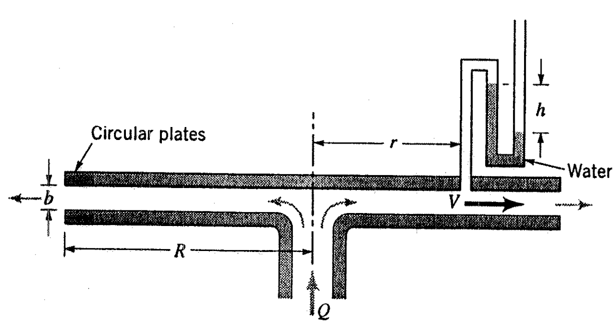
Experimental Procedure: Measure the radius, R, of the plates and the gap width, b, between them. Adjust the air supply to provide the desired, constant flowrate, Q, through the inlet pipe and the gap between the flat plates. Attach the manometer to the static pressure tap located a radial distance r from the center of the plates and record the manometer reading, h. Repeat the pressure measurements (for the same Q) at different radial locations. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings to obtain the experimentally determined pressure distribution, $p = p(r)$, within the gap. That is, $p = -\gamma_m h$, where γ_m is the specific weight of the manometer fluid. Also use the Bernoulli equation ($p/\gamma + V^2/2g =$ constant) and the continuity equation ($AV =$ constant, where $A = 2\pi rb$) to determine the theoretical pressure distribution within the gap between the plates. Note that the flow at the edge of the plates ($r = R$) is a free jet ($p = 0$). Also note that an increase in r causes an increase in A, a decrease in V, and an increase in p.

Graph: Plot the experimentally measured pressure head, p/γ, in feet of air as ordinates and radial location, r, as abscissas.

Results: On the same graph, plot the theoretical pressure head distribution as a function of radial location.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.101

(con't)

3.101 (con't)

Solution for Problem 3.101: Pressure Distribuition between Two Circular Plates

Q, ft^3/s	R, in.	b, in.	H_{atm}, in. Hg	T, deg F	γ_{H20}, lb/ft^3
0.879	5.0	0.125	29.09	83	62.4

		Experiment	Theory	
r, in.	h, in.	p/γ, ft	V, ft/s	p/γ, ft
0.7	-9.05	-663.75	220.8	-740.7
1.0	-6.02	-441.52	161.2	-387.2
1.5	-2.02	-148.15	107.4	-163.1
2.0	-0.96	-70.41	80.6	-84.7
2.5	-0.48	-35.20	64.5	-48.4
3.0	-0.24	-17.60	53.7	-28.7
3.5	-0.13	-9.53	46.0	-16.8
4.0	-0.03	-2.20	40.3	-9.1
4.5	-0.01	-0.73	35.8	-3.8
5.0	0.00	0.00	32.2	0.0

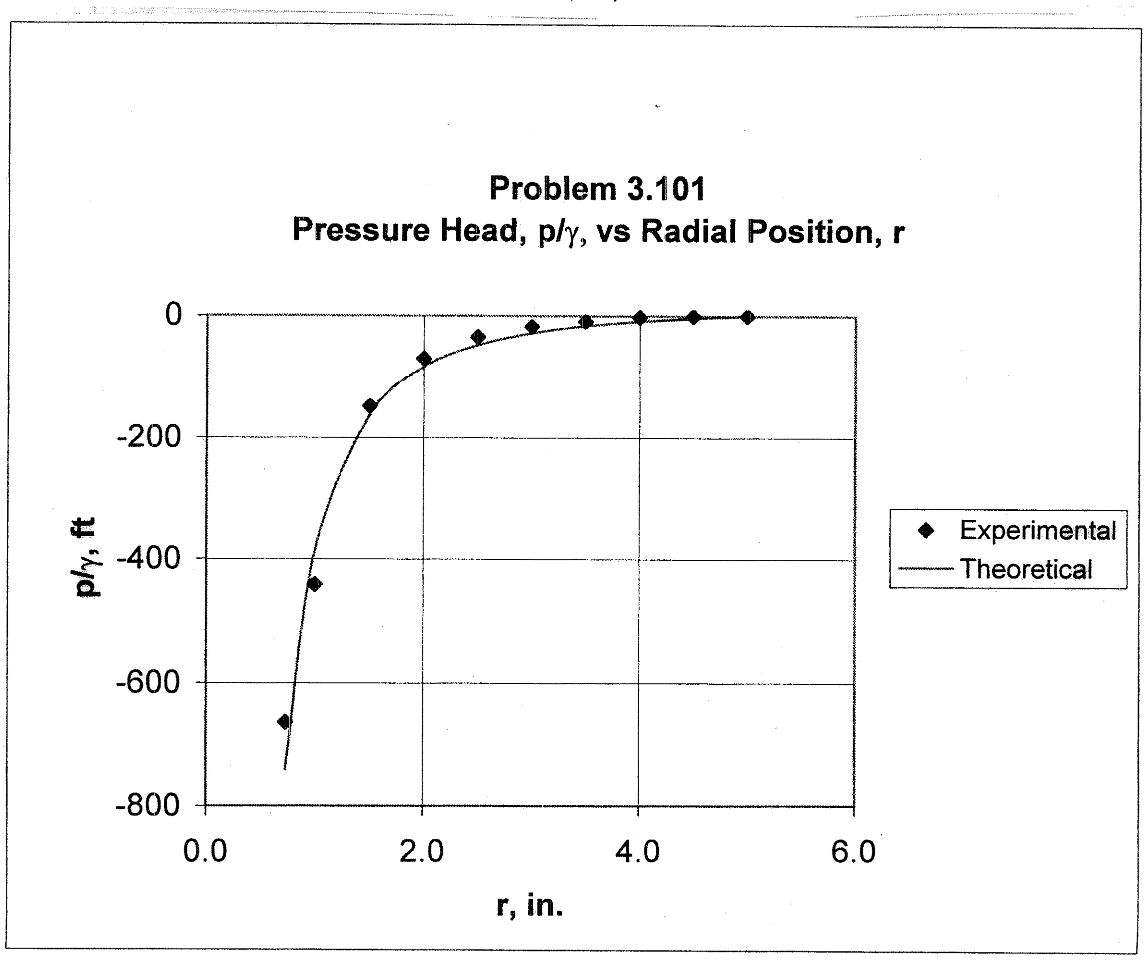
$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.09/12ft) = 2053 lb/ft^2
R = 1716 ft lb/slug deg R
T = 83 + 460 = 543 deg R

Thus, ρ = 0.00220 slug/ft^3 and $\gamma = \rho*g$ = 0.00220*32.2 = 0.0709 lb/ft^3

$p/\gamma = \gamma_{H20}*h/\gamma$

$V = Q/(2\pi rb)$ = 0.879 ft^s/(2*3.1415*(0.125/12)ft*r)



3.102

3.102 Calibration of a Nozzle Flow Meter

Objective: As shown in Section 3.6.3 of the text, the volumetric flowrate, Q, of a given fluid through a nozzle flow meter is proportional to the square root of the pressure drop across the meter. Thus, $Q = Kh^{1/2}$, where K is the meter calibration constant and h is the manometer reading that measures the pressure drop across the meter (see Fig. P3.102). The purpose of this experiment is to determine the value of K for a given nozzle flow meter.

Equipment: Pipe with a nozzle flow meter; variable speed fan; exit nozzle to produce a uniform jet of air; Pitot static tube; manometers; barometer; thermometer.

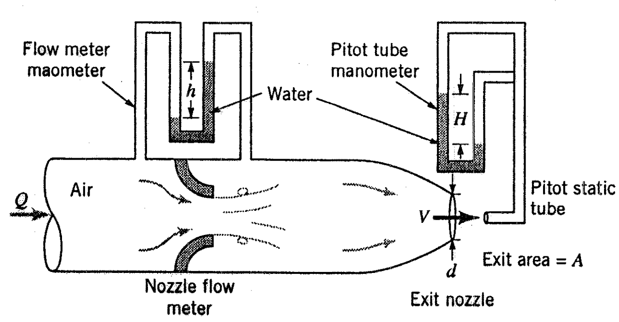
Experimental Procedure: Adjust the fan speed control to give the desired flowrate, Q. Record the flow meter manometer reading, h, and the Pitot tube manometer reading, H. Repeat the measurements for various fan settings (i.e., flowrates). Record the nozzle exit diameter, d. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated from the perfect gal law.

Calculations: For each fan setting determine the flowrate, $Q = VA$, where V and A are the air velocity at the exit and the nozzle exit area, respectively. The velocity, V, can be determined by using the Bernoulli equation and the Pitot tube manometer data, H (see Equation 3.16).

Graph: Plot flowrate, Q, as ordinates and flow meter manometer reading, h, as abscissas on a log-log graph. Draw the best-fit straight line with a slope of ½ through the data.

Results: Use your data to determine the calibration constant, K, in the flow meter equation $Q = Kh^{1/2}$.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.102

(con't)

Solution for Problem 3.102: Calibration of a Nozzle Flow Meter

d, in.	H_{atm}, in. Hg	T, deg F
1.169	29.01	75

h, in.	H, in.	Δp, lb/ft^2	V, ft/s	Q, ft^3/s
11.6	5.6	29.1	162	1.20
11.1	5.4	28.1	159	1.18
10.7	5.2	27.0	156	1.16
10.1	4.9	25.5	151	1.13
9.6	4.7	24.4	148	1.10
8.8	4.3	22.4	142	1.06
7.9	3.9	20.3	135	1.00
7.2	3.6	18.7	130	0.97
6.1	3.1	16.1	120	0.90
5.4	2.7	14.0	112	0.84
4.5	2.3	12.0	104	0.77
3.8	2.0	10.4	97	0.72
2.9	1.5	7.8	84	0.62
2.1	1.1	5.7	72	0.53
1.0	0.6	3.1	53	0.39

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.01/12 ft) = 2048 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

Thus, ρ = 0.00223 slug/ft^3

$V = (2*\Delta p/\rho)^{1/2}$

Q = AV where

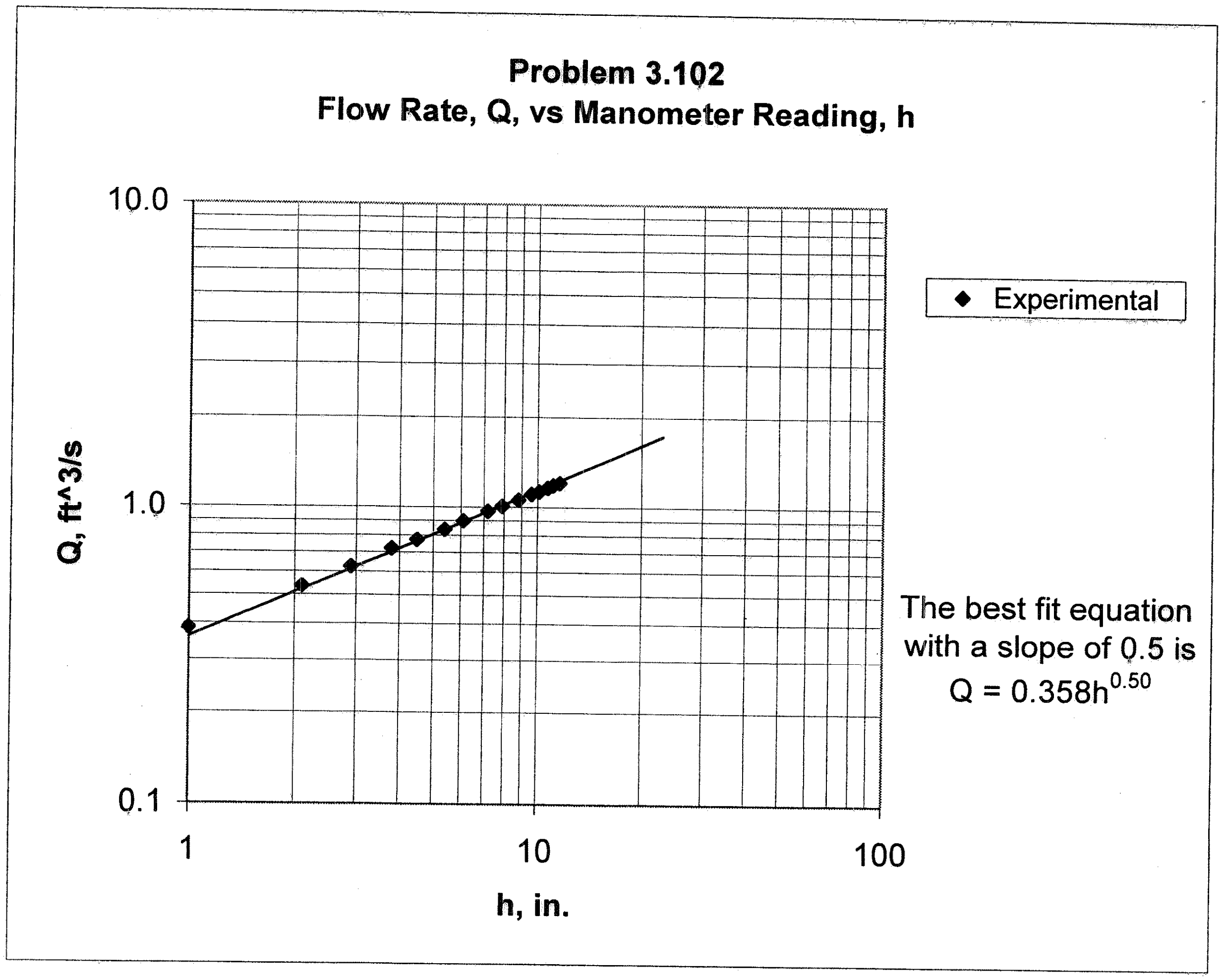
$A = \pi d^2/4$ = π*(1.169/12 ft)^2/4 = 7.45E-3 ft^2

From the graph, $Q = K h^{1/2} = 0.358\ h^{1/2}$ where Q is in ft^3/s and h is in in.

Thus, K = <u>0.358 $ft^3/(s*in.^{1/2})$</u>

(con't)

3.102 (con't)


Problem 3.102
Flow Rate, Q, vs Manometer Reading, h
10.0
1.0
0.1
Q, ft^3/s
1
10
100
h, in.
Experimental
The best fit equation
with a slope of 0.5 is
Q = 0.358h^0.50

3.103

3.103 Pressure Distribution in a Two-Dimensional Channel

Objective: According to the Bernoulli equation, a change in velocity can cause a change in pressure. Also, for an incompressible flow, a change in flow area causes a change in velocity. The purpose of this experiment is to determine the pressure distribution caused by air flowing within a two-dimensional, variable area channel as shown in Fig. P3.103.

Equipment: Air supply with a flow meter; two-dimensional channel with one curved side and one flat side; static pressure taps at various locations along both walls of the channel; ruler; manometer; barometer; thermometer.

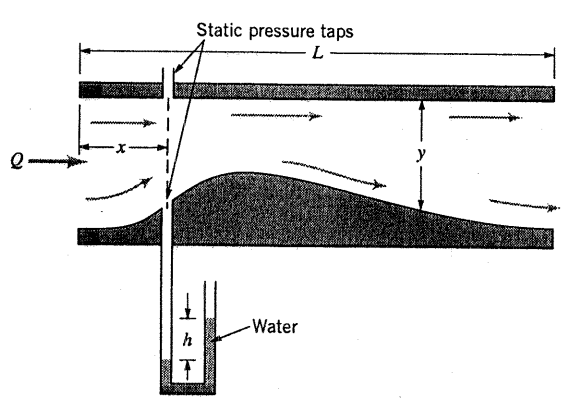
Experimental Procedure: Measure the constant width, b, of the channel and the channel height, y, as a function of distance, x, along the channel. Adjust the air supply to provide the desired, constant flowrate, Q, through the channel. Attach the manometer to the static pressure tap located a distance, x, from the origin and record the manometer reading, h. Repeat the pressure measurements (for the same Q) at various locations on both the flat and the curved sides of the channel. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings, h, to calculate the pressure within the channel, $p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. Convert this pressure into the pressure head, p/γ, where $\gamma = g\rho$ is the specific weight of air. Also use the Bernoulli equation ($p/\gamma + V^2/2g$ = constant) and the continuity equation ($AV = Q$, where $A = yb$) to determine the theoretical pressure distribution within the channel. Note that the air leaves the end of the channel ($x = L$) as a free jet ($p = 0$).

Graph: Plot the experimentally determined pressure head, p/γ, as ordinates and the distance along the channel, x, as abscissas. There will be two curves—one for the curved side of the channel and another for the flat side.

Results: On the same graph, plot the theoretical pressure distribution within the channel.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.103

(con't)

3.103 (con't)

Solution for Problem 3.103: Pressure Distribution in a Two-Dimensional Channel

b, in.	Q, ft^3/s	H_{atm}, in. Hg	T, deg F	L, in.
2.0	1.32	28.96	71	21.75

x, in.	y, in.	h, in. flat side	h, in. curved side	Experimental p/γ, ft flat side	Experimental p/γ, ft curved side	Theory p/γ, ft
0.75	2.00	0.28	0.31	20.2	22.3	0.0
2.50	2.00	0.21	0.37	15.1	26.6	0.0
4.00	1.28	-0.42	0.03	-30.2	2.3	-50.5
4.63	1.05	-0.77	-1.63	-55.5	-117.4	-92.2
5.38	1.05	-1.01	-1.05	-72.7	-75.6	-92.2
8.14	1.29	-0.63	-0.62	-45.4	-44.7	-49.2
10.75	1.54	-0.32	-0.31	-23.0	-22.3	-24.1
13.25	1.77	-0.15	-0.15	-10.8	-10.8	-9.7
15.78	2.00	-0.05	0.00	-3.6	0.0	0.0
21.75	2.00	0.00	0.00	0.0	0.0	0.0

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} \cdot H_{atm}$ = 847 lb/ft^3*(28.96/12 ft) = 2044 lb/ft^2

R = 1716 ft lb/slug deg R

T = 71 + 460 = 531 deg R

Thus, ρ = 0.00224 slug/ft^3 and γ = ρ*g = 0.00224 slug/ft^3*(32.2 ft/s^2) = 0.0722 lb/ft^3

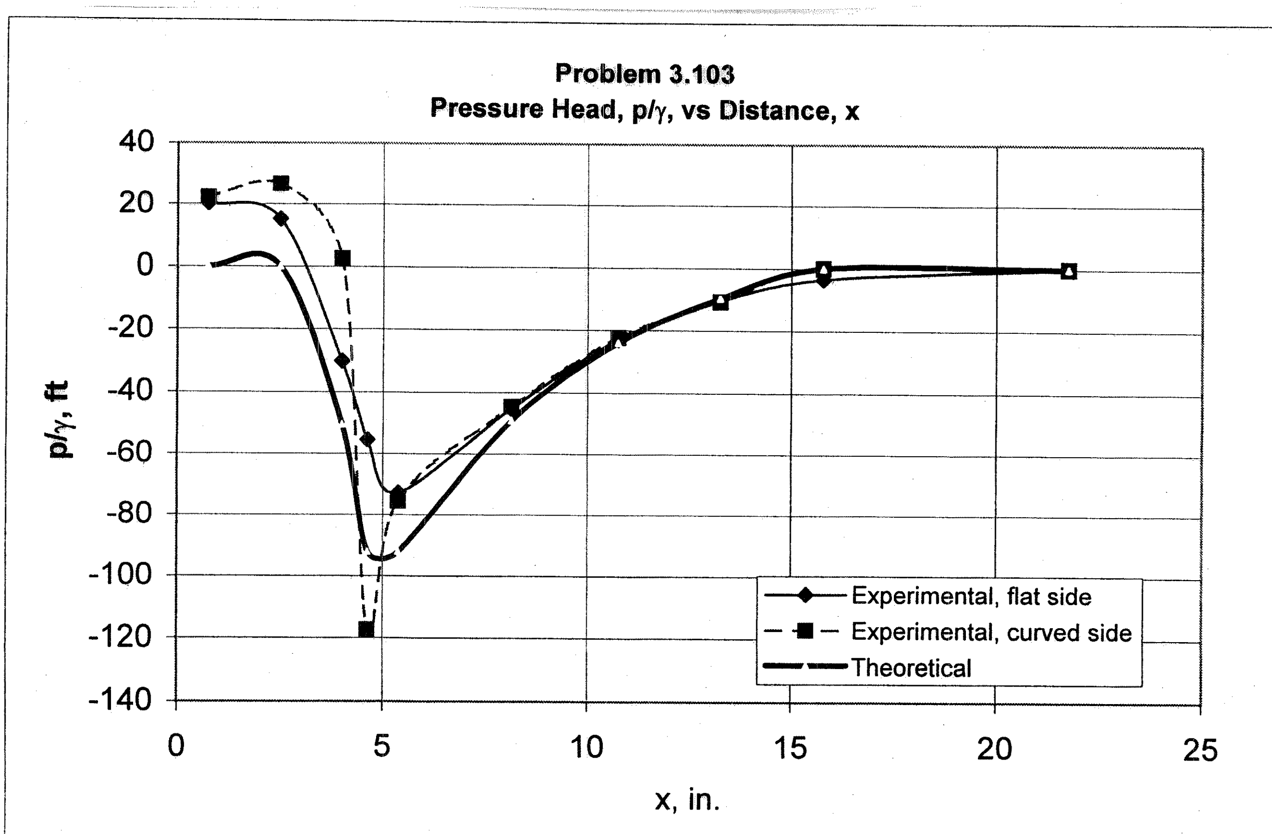
$p/\gamma = \gamma_{H2O} * h/\gamma$

Theoretical:

$p/\gamma = V_{exit}^2/2g - V^2/2g$ where

V = Q/A = Q/(b*y) and

$V_{exit} = Q/A_{exit}$ = (1.32 ft^3/s/)*(2 *2 /144 ft^2) = 47.5 ft/s



3.104

3.104 Sluice Gate Flowrate

Objective: The flowrate of water under a sluice gate as shown in Fig. P3.104 is a function of the water depths upstream and downstream of the gate. The purpose of this experiment is to compare the theoretical flowrate with the experimentally determined flowrate.

Equipment: Flow channel with pump and control valve to provide the desired flowrate in the channel; sluice gate; point gage to measure water depth; float; stop watch.

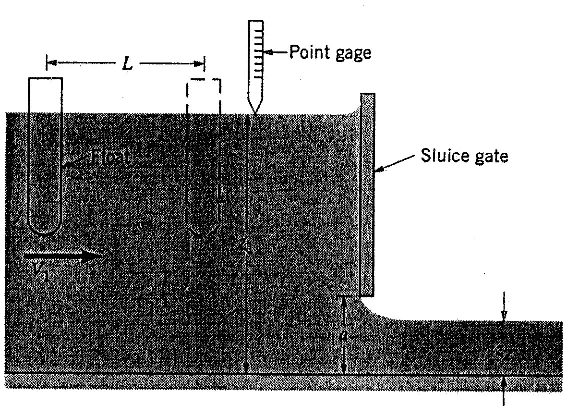
Experimental Procedure: Adjust the vertical position of the sluice gate so that the bottom of the gate is the desired distance, a, above the channel bottom. Measure the width, b, of the channel (which is equal to the width of the gate). Turn on the pump and adjust the control valve to produce the desired water depth upstream of the sluice gate. Insert a float into the water upstream of the gate and measure the water velocity, V_1, by recording the time, t, it takes the float to travel a distance L. That is, $V_1 = L/t$. Use a point gage to measure the water depth, z_1, upstream of the gate. Adjust the control valve to produce various water depths upstream of the gate and repeat the measurements.

Calculations: For each water depth used, determine the flowrate, Q, under the sluice gate by using the continuity equation $Q = A_1 V_1 = b\, z_1 V_1$. Use the Bernoulli and continuity equations to determine the theoretical flowrate under the sluice gate (see Equation 3.21). For these calculations assume that the water depth downstream of the gate, z_2, remains at 61% of the distance between the channel bottom and the bottom of the gate. That is, $z_2 = 0.61a$.

Graph: Plot the experimentally determined flowrate, Q, as ordinates and the water depth, z_1, upstream of the gate as abscissas.

Results: On the same graph, plot the theoretical flowrate as a function of water depth upstream of the gate.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P3.104

(con't)

3.104 (Con't)

Solution for problem 3.104: Sluice Gate Flowrate

a, in.	b, in.	L, ft	z_2, ft
1.2	6.0	4.0	0.061

		Experimental		Theoretical
z_1, ft	t, s	V_1, ft/s	Q, ft^3/s	Q, ft^3/s
0.183	4.2	0.952	0.087	0.091
0.267	5.0	0.800	0.107	0.114
0.343	5.2	0.769	0.132	0.132
0.453	6.2	0.645	0.146	0.155
0.569	6.4	0.625	0.178	0.175
0.725	7.0	0.571	0.207	0.200
0.877	8.6	0.465	0.204	0.222

Experimental:

$V_1 = L/t$

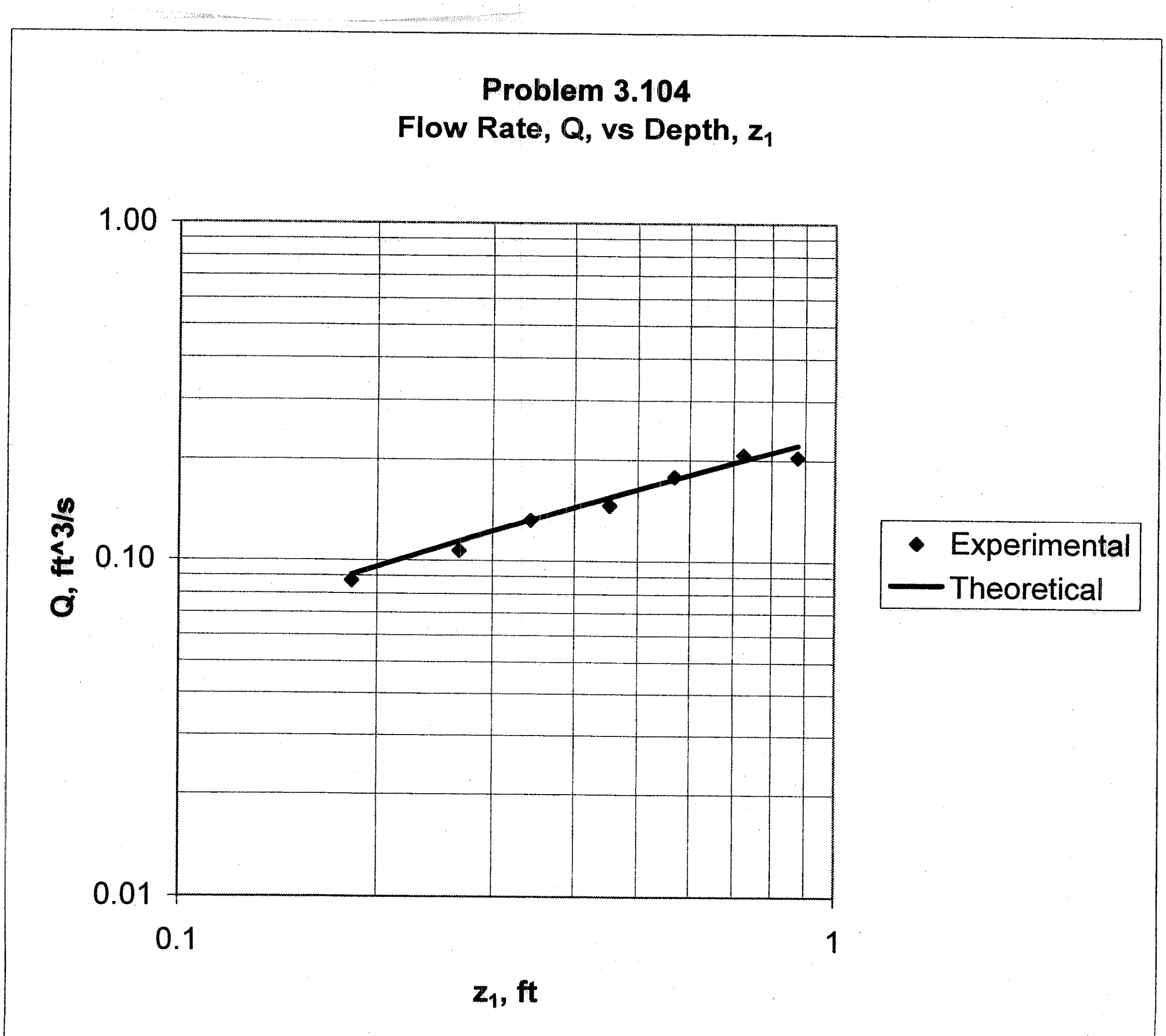
$Q = V_1 b z_1$

Theoretical:

$Q = b*z_2^{3/2}*(2*g)^{1/2}*[((z_1/z_2) - 1)/(1 - (z_2/z_1)^2)]^{1/2}$

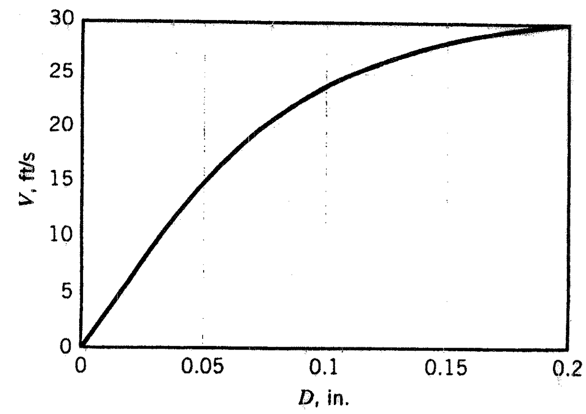
where

$z_2 = 0.61*a$



3.105

3.105 (See **"Incorrect raindrop shape,"** Section 3.2.) The speed, V, at which a raindrop falls is a function of its diameter, D, as shown in Fig. P3.105. For what sized raindrop will the stagnation pressure be equal to half the internal pressure caused by surface tension. Recall from **Section 1.9** that the pressure inside a drop is $\Delta p = 4\sigma/D$ greater than the surrounding pressure, where σ is the surface tension.



■ **FIGURE P3.105**

Determine diameter D for which

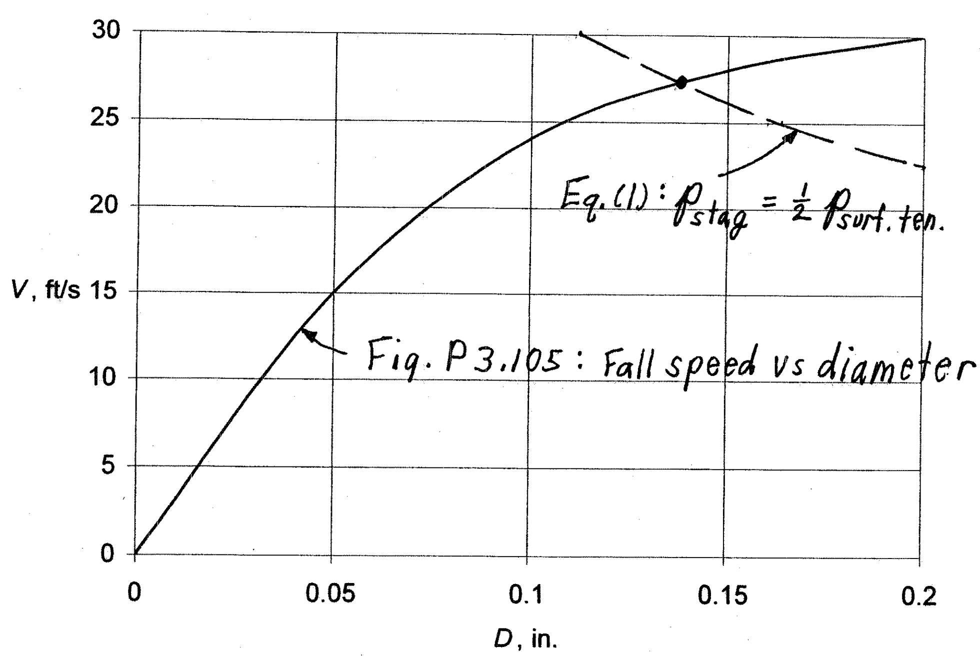
$\frac{1}{2}\rho V^2 = \frac{1}{2}[4\sigma/D]$, or

$\frac{1}{2}(0.00238 \frac{slugs}{ft^3})V^2 = \frac{1}{2}[4(5.03 \times 10^{-3} \frac{lb}{ft})/D]$

or $D = 8.45/V^2$, where $D \sim ft$ and $V \sim ft/s$

or $D = 101/V^2$, where $D \sim in.$ and $V \sim ft/s$ (1)

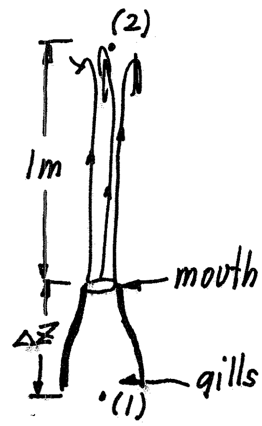
Thus, there are 2 unknowns, D and V, and 2 equations, Eq. (1) and Fig. P3.105. The solution is given by the intersection of these two D-V graphs as shown below.



Thus, $D = \underline{\underline{0.14 \text{ in.} = 3.6 \text{ mm}}}$

3.106

3.106 (See "**Armed with a water jet for hunting,**" Section 3.4.) Determine the pressure needed in the gills of an archerfish if it can shoot a jet of water 1 m vertically upward. Assume steady, inviscid flow.



From the Bernoulli equation,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Assume $V_1 \approx 0$ (large gills), $\Delta z \ll 1m$ (small fish), $p_2 = 0$ (free jet), and $V_2 = 0$ (top of vertical water jet).

Thus,

$$\frac{p_1}{\gamma} = z_2 - z_1 \quad \text{or} \quad p_1 = \gamma(z_2 - z_1) = 9.80 \times 10^3 \frac{N}{m^3}(1m) = 9.80 \times 10^3 \frac{N}{m^2} = \underline{\underline{9.80\,kPa}}$$

3.107

3.107 (See "**Pressurized eyes,**" Section 3.5.) Determine the air velocity needed to produce a stagnation pressure equal to 10 mm of mercury.

$$\frac{1}{2}\rho V^2 = p_{stag} = 10\text{ mm of mercury} = \gamma_{Hg} h, \text{ where } \gamma_{Hg} = 133 \times 10^3 \frac{N}{m^3}$$

Thus,

$$\frac{1}{2}\left(1.23 \frac{kg}{m^3}\right)V^2 = 10mm\left(\frac{1m}{1000mm}\right)\left(133 \times 10^3 \frac{N}{m^3}\right)$$

or

$$V = \underline{\underline{46.5\ m/s}}$$

3.108

3.108 (See "**Bugged and plugged Pitot tubes,**" Section 3.5.) A airplane's Pitot tube used to indicated airspeed is partially plugged by an insect nest so that it measures 60% of the stagnation pressure rather than the actual stagnation pressure. If the airspeed indicator indicates that the plane is flying 150 mph, what is the actual airspeed?

When unplugged the air speed indicator would register a pressure difference of

$\Delta p = \frac{1}{2}\rho V^2 = \frac{1}{2}\rho(150\,mph)^2$

at 150 mph.

However, when plugged and the reading indicates 150 mph, the actual speed would be

$\Delta p = \frac{1}{2}\rho(150\,mph)^2 = 0.60\left[\frac{1}{2}\rho V^2\right]$

or

$V = \underline{\underline{194\,mph}}$