5.1

5.1 Water enters a conical diffusing passage (see Fig. P5.1) with an average velocity of 10 ft/s. If the entrance cross section area is 1 ft², how large should the diffuser exit area be to reduce the average velocity level to 1 ft/s?

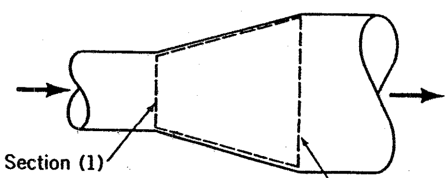


FIGURE P5.1

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$A_1 \overline{V}_1 = A_2 \overline{V}_2$$

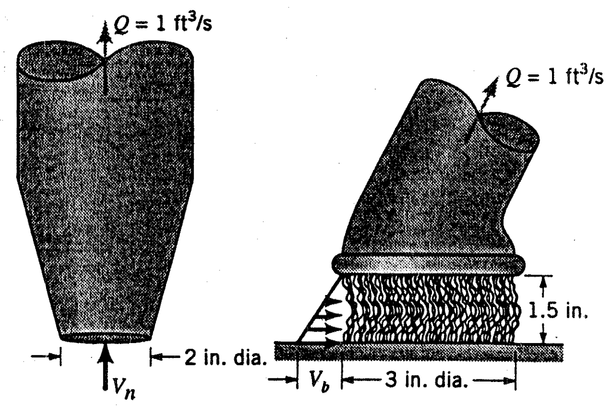
So

$$A_2 = \frac{A_1 \overline{V}_1}{\overline{V}_2} = \frac{(1\ ft^2)\left(10\ \frac{ft}{s}\right)}{\left(1\ \frac{ft}{s}\right)}$$

$$A_2 = \underline{\underline{10\ ft^2}}$$

5.2

5.2 Various types of attachments can be used with the shop vac shown in Video V5.2. Two such attachments are shown in Fig. P5.2—a nozzle and a brush. The flowrate is 1 ft³/s. **(a)** Determine the average velocity through the nozzle entrance, V_n. **(b)** Assume the air enters the brush attachment in a radial direction all around the brush with a velocity profile that varies linearly from 0 to V_b along the length of the bristles as shown in the figure. Determine the value of V_b.



■ FIGURE P5.2

(a) $Q_1 = Q_2$ where $Q_2 = 1\,\frac{ft^3}{s}$

Thus,

$$A_1 V_1 = Q_2 \quad \text{or} \quad V_1 \equiv V_n = \frac{1\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}ft\right)^2}$$

so

$$V_n = \underline{\underline{45.8\,\frac{ft}{s}}}$$

(b) $Q_3 = Q_4$ where $Q_4 = 1\,\frac{ft^3}{s}$ and $Q_3 = \overline{V}_3 A_3$ where

$\overline{V}_3$ = average velocity at (3) $= \frac{1}{2} V_b$ and

$A_3 = \pi D_3 h_3$

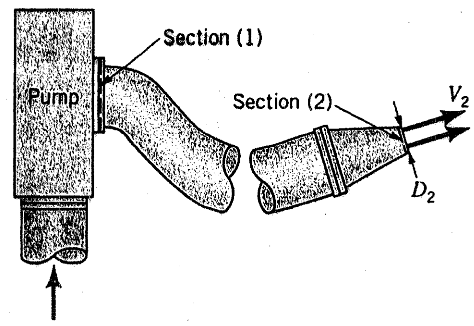
Thus,

$$\frac{1}{2} V_b \left[\pi \left(\frac{3}{12}ft\right)\left(\frac{1.5}{12}ft\right)\right] = 1\,\frac{ft^3}{s}, \text{ or}$$

$$V_b = \underline{\underline{20.4\,\frac{ft}{s}}}$$

5,3

5.3 The pump shown in Fig. P5.3 produces a steady flow of 10 gal/s through the nozzle. Determine the nozzle exit diameter, D_2, if the exit velocity is to be $V_2 = 100$ ft/s.



■ FIGURE P5.3

For steady flow $Q_1 = Q_2$, where $Q_1 = 10\,\frac{gal}{s}\left(231\,\frac{in.^3}{gal}\right)\left(\frac{1\,ft^3}{1728\,in.^3}\right) = 1.337\,\frac{ft^3}{s}$

Thus, with $V_2 = 100\,\frac{ft}{s}$,

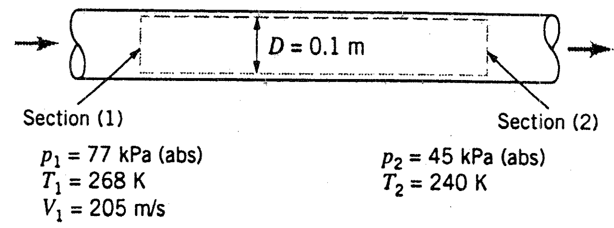
$$1.337\,\frac{ft^3}{s} = A_2 V_2 = \frac{\pi}{4} D_2^2 \left(100\,\frac{ft}{s}\right)$$

or

$$D_2 = 0.130\,ft = \underline{\underline{1.57\,in.}}$$

5.4

5.4 Air flows steadily between two cross sections in a long, straight section of 0.1-m inside-diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.4. If the average air velocity at section (1) is 205 m/s, determine the average air velocity at section (2).



■ FIGURE P5.4

This analysis is similar to the one of Example 5.2. For steady flow between sections (1) and (2)

$$\dot{m}_2 = \dot{m}_1$$

or

$$\rho_2 A_2 \bar{V}_2 = \rho_1 A_1 \bar{V}_1$$

Thus

$$\bar{V}_2 = \frac{\rho_1}{\rho_2}\frac{A_1}{A_2}\bar{V}_1 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.8) to get

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}\frac{T_2}{T_1} \qquad (2)$$

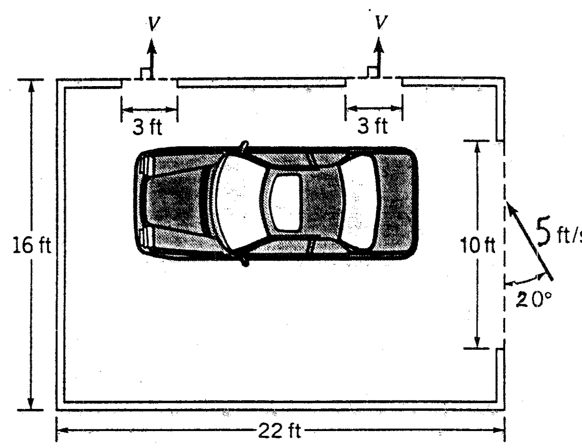
Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\bar{V}_2 = \frac{p_1}{p_2}\frac{T_2}{T_1}\bar{V}_1 = \frac{[77\ kPa(abs)](240K)}{[45\ kPa(abs)](268K)}\left(205\ \frac{m}{s}\right)$$

$$\bar{V}_2 = 314\ \frac{m}{s}$$

5.5

5.5 The wind blows through a 7 ft × 10 ft garage door opening with a speed of 5 ft/s as shown in Fig. P5.5. Determine the average speed, V, of the air through the two 3 ft × 4 ft openings in the windows.



■ FIGURE P5.5

For steady incompressible flow

$$Q_{\substack{garage\\door}} = Q_{window} + Q_{window}$$

or

$$A_{\substack{garage\\door}} V_{\substack{normal\ to\\garage\ door}} = A_{window} V + A_{window} V$$

so the average speed, V, of the air through the two windows is

$$V = \frac{A_{\substack{garage\\door}} V_{normal\ to\ garage\ door}}{2A_{window}} = \frac{(7ft)(10ft)(5\frac{ft}{s})\sin 20^\circ}{2(3ft)(4ft)} = \underline{\underline{4.99\ \frac{ft}{s}}}$$

5.6

5.6 A hydroelectric turbine passes 4 million gal/min through its blades. If the average velocity of the flow in the circular cross-section conduit leading to the turbine is not to exceed 30 ft/s, determine the minimum allowable diameter of the conduit.

For incompressible flow through the conduit and turbine

$$Q_{conduit} = Q_{turbine}$$

Thus

$$A_{conduit} \overline{V}_{conduit} = Q_{turbine}$$

and

$$d_{conduit} = \sqrt{\frac{4}{\pi}\frac{Q_{turbine}}{\overline{V}_{conduit}}} = \sqrt{\frac{(4)(4\times10^6\ \frac{gal}{min})}{\pi\ (30\ \frac{ft}{s})(60\frac{s}{min})(7.48\frac{gal}{ft^3})}}$$

$$d_{conduit} = \underline{\underline{19.5\ ft}}$$

5.7

5.7 Water flows along the centerline of a 50-mm-diameter pipe with an average velocity of 10 m/s and out radially between two large circular disks as shown in Fig. P5.7. The disks are parallel and spaced 10 mm apart. Determine the average velocity of the water at a radius of 300 mm in the space between the disks.

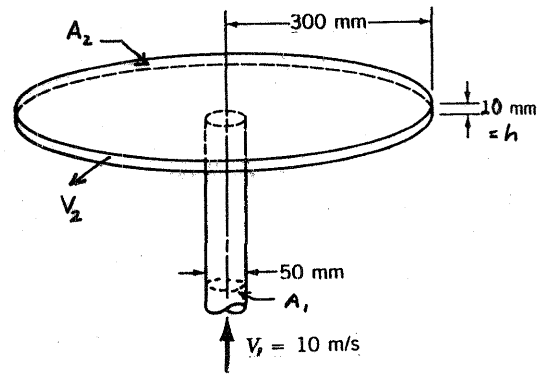


FIGURE P5.7

For steady incompressible flow

$$Q_1 = Q_2$$

or

$$A_1 \overline{V}_1 = A_2 \overline{V}_2$$

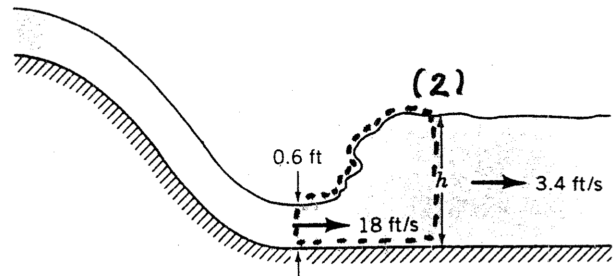
Thus

$$\overline{V}_2 = \frac{A_1 \overline{V}_1}{A_2} = \frac{\pi D_1^2 \, \overline{V}_1}{(4) 2\pi r_2 h} = \frac{(50 \text{ mm})^2 (10 \text{ m/s})}{(4)(2)(300 \text{ mm})(10 \text{ mm})}$$

$$\overline{V}_2 = 1.04 \frac{m}{s}$$

5.8

5.8 A hydraulic jump (see Video V10.5) is in place downstream from a spill-way as indicated in Fig. P5.8. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.



■ FIGURE P5.8

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

Thus

$$\bar{V}_1 h_1 = \bar{V}_2 h_2$$

and

$$h_2 = \frac{\bar{V}_1 h_1}{\bar{V}_2} = \frac{(18\ \frac{ft}{s})(0.6\ ft)}{(3.4\ \frac{ft}{s})} = \underline{\underline{3.18\ ft}}$$

5.9

5.9 A water jet pump (see Fig. P5.9) involves a jet cross section area of 0.01 m^2, and a jet velocity of 30 m/s. The jet is surrounded by entrained water. The total cross section area associated with the jet and entrained streams is 0.075 m^2. These two fluid streams leave the pump thoroughly mixed with an average velocity of 6 m/s through a cross section area of 0.075 m^2. Determine the pumping rate (i.e., the entrained fluid flowrate) involved in liters/s.

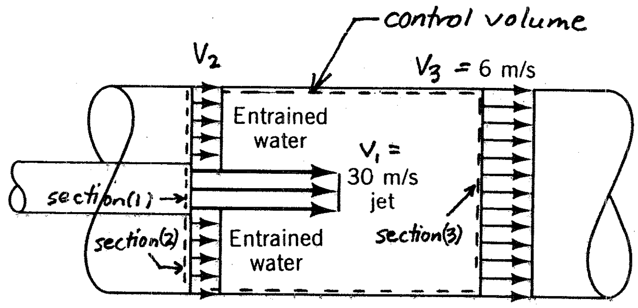


FIGURE P5.9

For steady incompressible flow through the control volume

$$Q_1 + Q_2 = Q_3$$

or

$$\bar{V}_1 A_1 + Q_2 = \bar{V}_3 A_3$$

Thus

$$Q_2 = \bar{V}_3 A_3 - \bar{V}_1 A_1 = \left[\left(6\ \frac{m}{s}\right)\left(0.075\ m^2\right) - \left(30\ \frac{m}{s}\right)\left(0.01\ m^2\right)\right]\left(1000\ \frac{\text{liters}}{m^3}\right)$$

$$Q_2 = 150\ \frac{\text{liters}}{s}$$

5.10

5.10 An evaporative cooling tower (see Fig. P5.10) is used to cool water from 110 to 80°F. Water enters the tower at a rate of 250,000 lbm/hr. Dry air (no water vapor) flows into the tower at a rate of 151,000 lbm/hr. If the rate of wet air flow out of the tower is 156,900 lbm/hr, determine the rate of water evaporation in lbm/hr and the rate of cooled water flow in lbm/hr.

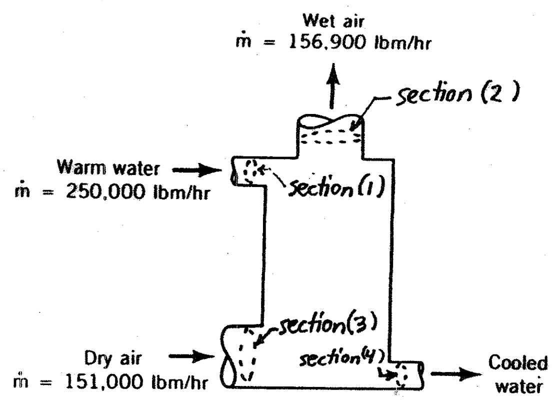


FIGURE P5.10

For steady flow of dry air

$$\dot{m}_3 = \dot{m}_{2,\,dry\ air} \qquad (1)$$

For steady flow of water

$$\dot{m}_1 = \dot{m}_{2,\,water} + \dot{m}_4 \qquad (2)$$

Also

$$\dot{m}_2 = \dot{m}_{2,\,dry\ air} + \dot{m}_{2,\,water} \qquad (3)$$

Combining Eqs. 1 and 3 we get

$$\dot{m}_{2,\,water} = \dot{m}_2 - \dot{m}_3 = \text{rate of water evaporation}$$

So

$$\dot{m}_{2,\,water} = 156,900\ \frac{lbm}{hr} - 151,000\ \frac{lbm}{hr} = \underline{\underline{5900\ \frac{lbm}{hr}}}$$

From Eq. 2 we get

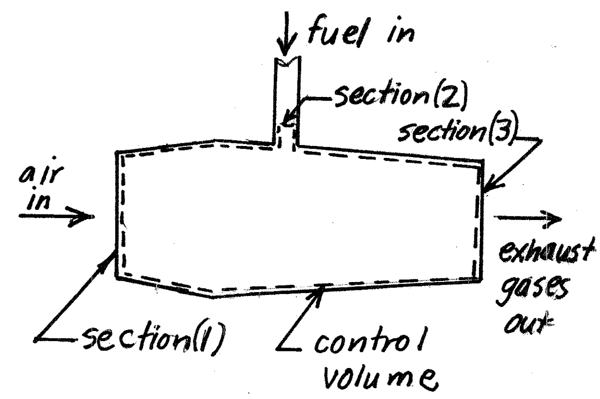
$$\dot{m}_4 = \dot{m}_1 - \dot{m}_{2,\,water} = \text{rate of cooled water flow}$$

or

$$\dot{m}_4 = 250,000\ \frac{lbm}{hr} - 5900\ \frac{lbm}{hr} = \underline{\underline{244,000\ \frac{lbm}{hr}}}$$

5.11

5.11 At cruise conditions, air flows into a jet engine at a steady rate of 65 lbm/s. Fuel enters the engine at a steady rate of 0.60 lbm/s. The average velocity of the exhaust gases is 1500 ft/s relative to the engine. If the engine exhaust effective cross section area is 3.5 ft², estimate the density of the exhaust gases in lbm/ft³.



For steady flow

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

or

$$\rho_3 A_3 \overline{V}_3 = \dot{m}_1 + \dot{m}_2$$

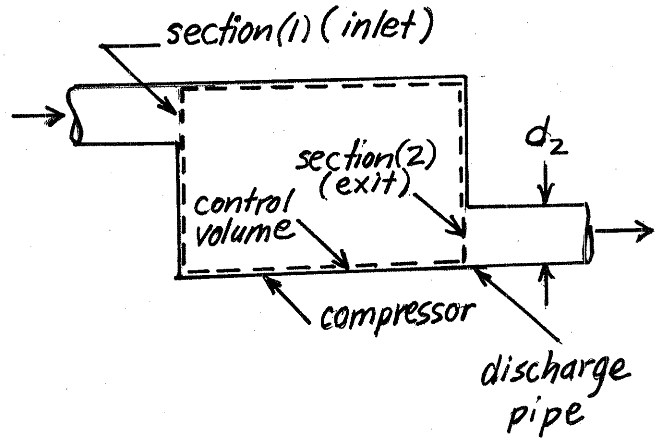
Thus

$$\rho_3 = \frac{\dot{m}_1 + \dot{m}_2}{A_3 \overline{V}_3} = \frac{65 \frac{lbm}{s} + 0.60 \frac{lbm}{s}}{(3.5\ ft^2)(1500 \frac{ft}{s})}$$

$$\rho_3 = \underline{\underline{0.0125}} \frac{lbm}{ft^3}$$

5.12

5.12 Air at standard atmospheric conditions is drawn into a compressor at the steady rate of 30 m^3/min. The compressor pressure ratio, p_{exit}/p_{inlet}, is 10 to 1. Through the compressor p/ρ^n remains constant with $n = 1.4$. If the average velocity in the compressor discharge pipe is not to exceed 30 m/s, calculate the minimum discharge pipe diameter required.



For steady flow

$$\dot{m}_2 = \dot{m}_1$$

or

$$\rho_2 A_2 \overline{V}_2 = \rho_1 Q_1$$

Thus

$$\rho_2 \frac{\pi d_2^2}{4} \overline{V}_2 = \rho_1 Q_1$$

and

$$d_2 = \sqrt{\frac{\rho_1}{\rho_2} \frac{Q_1}{\frac{\pi}{4} \overline{V}_2}}$$

However

$$\frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

so

$$d_2 = \sqrt{\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} \frac{Q_1}{\frac{\pi}{4} \overline{V}_2}} = \sqrt{\left(\frac{1}{10}\right)^{\frac{1}{1.4}} \frac{30 \frac{m^3}{min}}{\frac{\pi}{4}\left(30 \frac{m}{s}\right) 60 \frac{s}{min}}}$$

Finally

$$d_2 = \underline{\underline{0.064 \ m}}$$

5.13

5.13 Two rivers merge to form a larger river as shown in Fig. P5.13. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown. Determine the value of V.

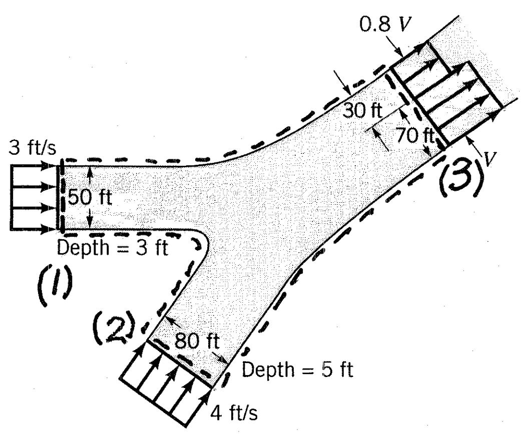


FIGURE P5.13

Use the control volume shown within broken lines in the sketch above. We note that $\dot{m} = \rho A V$ and from the conservation of mass principle we get

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0.8V} + \dot{m}_V$$

Thus

$$\rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0.8V}\, 0.8\, V + \rho A_V V$$

and

$$V = \frac{A_1 V_1 + A_2 V_2}{A_{0.8V}(0.8) + A_V} = \frac{(50\,ft)(3\,ft)(3\,\frac{ft}{s}) + (80\,ft)(5\,ft)(4\,\frac{ft}{s})}{(30\,ft)(6\,ft)(0.8) + (70\,ft)(6\,ft)}$$

$$V = \underline{\underline{3.63}}\ \frac{ft}{s}$$

5.14

5.14 Oil having a specific gravity of 0.9 is pumped as illustrated in Fig. P5.14 with a water jet pump (see Video V3.6). The water volume flowrate is 2 m³/s. The water and oil mixture has an average specific gravity of 0.95. Calculate the rate, in m³/s, at which the pump moves oil.

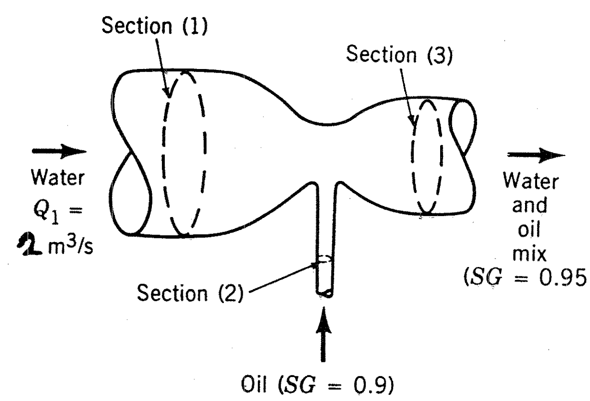


FIGURE P5.14

For steady flow

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

or

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 Q_3 \qquad (1)$$

Also, since the water and oil may be considered incompressible

$$Q_1 + Q_2 = Q_3 \qquad (2)$$

Combining Eqs. 1 and 2 we get

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 (Q_1 + Q_2)$$

or

$$Q_1 + SG_2 Q_2 = SG_3 (Q_1 + Q_2)$$

and

$$Q_2 = \frac{Q_1 (1 - SG_3)}{SG_3 - SG_2}$$

Thus

$$Q_2 = \frac{\left(2 \frac{m^3}{s}\right)(1 - 0.95)}{0.95 - 0.90} = \underline{\underline{2.00 \frac{m^3}{s}}}$$

5.15

5.15 Air at standard conditions enters the compressor shown in Fig. P5.15 at a rate of 10 ft^3/s. It leaves the tank through a 1.2-in.-diameter pipe with a density of 0.0035 $slugs/ft^3$ and a uniform speed of 700 ft/s. **(a)** Determine the rate (slugs/s) at which the mass of air in the tank is increasing or decreasing. **(b)** Determine the average time rate of change of air density within the tank.

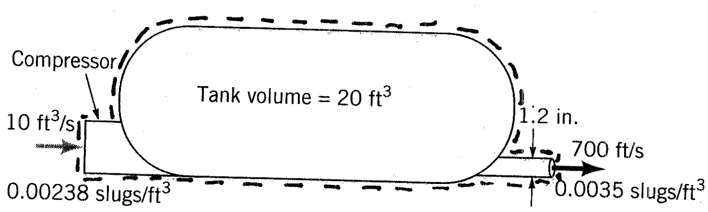


FIGURE P5.15

Use the control volume within the broken lines.

(a) From the conservation of mass principle we get

$$\frac{DM_{sys}}{Dt} = \dot{m}_{in} - \dot{m}_{out} = \rho_{in} Q_{in} - \rho_{out} A_{out} V_{out}$$

$$\frac{DM_{sys}}{Dt} = \left(0.00238 \frac{slug}{ft^3}\right)\left(10 \frac{ft^3}{s}\right) - \left(0.0035 \frac{slug}{ft^3}\right) \frac{\pi (1.2 \text{ in.})^2}{\left(144 \frac{in^2}{ft^2}\right)} \left(700 \frac{ft}{s}\right)$$

$$\frac{DM_{sys}}{Dt} = \underline{\underline{0.00456}} \frac{slug}{s} \quad \text{increasing}$$

(b) $$\frac{DM_{sys}}{Dt} = \frac{D(\rho V_{sys})}{Dt} = V_{sys} \frac{D\rho}{Dt} = 0.00456 \frac{slug}{s}$$

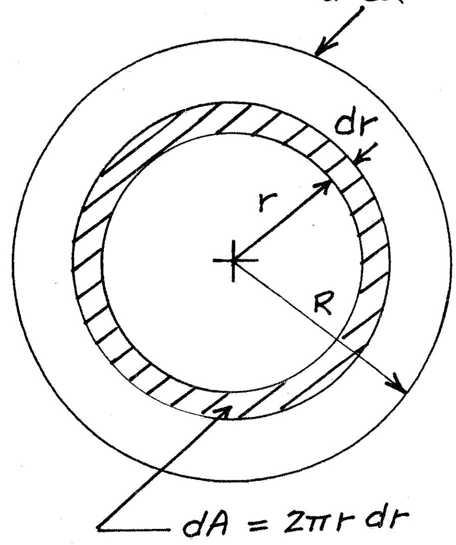
so $$\frac{D\rho}{Dt} = \frac{0.00456 \frac{slug}{s}}{V_{sys}} = \frac{0.00456 \frac{slug}{s}}{20 ft^3} = \underline{\underline{2.28 \times 10^{-4}}} \frac{slug}{ft^3 \, s}$$

5.16

5.16 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c\left(\frac{R-r}{R}\right)^{1/n}\hat{\mathbf{i}}$$

where u_c = centerline velocity, r = local radius, R = pipe radius, and $\hat{\mathbf{i}}$ = unit vector along pipe centerline. Determine the ratio of average velocity, $\bar{u}$, to centerline velocity, u_c, for **(a)** $n = 4$, **(b)** $n = 6$, **(c)** $n = 8$, **(d)** $n = 10$.



For any cross section area

$$\dot{m} = \rho A \bar{u} = \int_A \rho \vec{V}\cdot\hat{n}\, dA$$

Also

$$\vec{V}\cdot\hat{n} = \vec{V}\cdot\hat{i} = u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}}$$

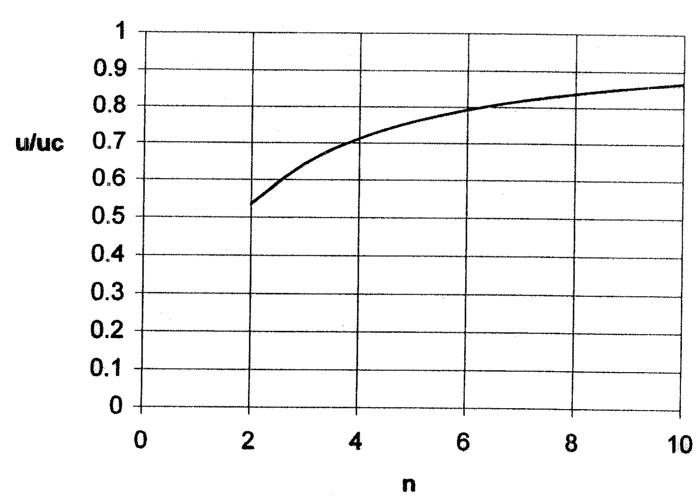
Thus for a uniformly distributed density, ρ, over area A

$$\bar{u} = \frac{\int_0^R u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}} 2\pi r\, dr}{\pi R^2}$$

and

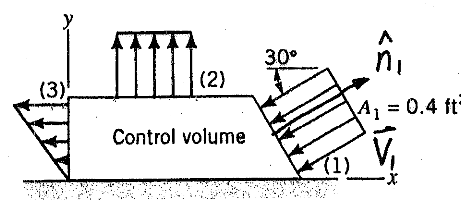
$$\frac{\bar{u}}{u_c} = 2\int_0^R \left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = \frac{2n^2}{2n^2+3n+1}$$

n	$\frac{\bar{u}}{u_c}$
4	0.711
6	0.791
8	0.837
10	0.866



5.17

5.17 Water flows steadily through the control volume shown in Fig. P5.17. The volumetric flowrate across section (3) is 2 ft³/s and the mass flowrate across section (2) is 3 slugs/s.



■ FIGURE P5.17

(a) $\int_{(1)} \gamma \vec{V}\cdot\hat{n}\, dA = $ -weight flowrate across area (1) $= -g\dot{m}_1$ (1)

(Note: $\int_{(1)} \gamma \vec{V}\cdot\hat{n}\, dA < 0$ since $\vec{V}\cdot\hat{n} < 0$ for the inflow area (1))

By conservation of mass, for steady flow,

$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 = \dot{m}_2 + \rho_3 Q_3 = 3\,slugs/s + (1.94\,slugs/ft^3)(2\,ft^3/s)$

or

$\dot{m}_1 = 6.88\,slugs/s$ (2)

Thus, from Eq. (1),

$\int_{(1)} \gamma \vec{V}\cdot\hat{n}\, dA = (-32.2\,ft/s^2)(6.88\,slugs/s) = -222\,(slug\cdot ft/s^2)/s = \underline{\underline{-222\,lb/s}}$

(b) $\int_{(1)} \vec{V}\rho\vec{V}\cdot\hat{n}\, dA = $ momentum flux across area (1)

On (1), $\hat{n}_1 = +\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j}$ and

$\vec{V}_1 = V_1(-\cos 30^\circ\,\hat{i} - \sin 30^\circ\,\hat{j}) = -V_1\hat{n}_1$

where $\rho A_1 V_1 = \dot{m}_1 = 6.88\,\frac{slugs}{s}$ (from Eq. (2))

Thus,

$$V_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{6.88\,\frac{slugs}{s}}{(1.94\,\frac{slugs}{ft^3})(0.4\,ft^2)} = 8.87\,\frac{ft}{s}$$

Hence,

$\int_{(1)} \vec{V}_1\rho\vec{V}_1\cdot\hat{n}_1\, dA = \vec{V}_1\rho(-V_1\hat{n}_1)\cdot\hat{n}_1 A_1 = -\rho V_1 A_1\vec{V}_1 = -\dot{m}_1\vec{V}_1 = +\dot{m}_1 V_1\hat{n}_1$

$= (6.88\,\frac{slugs}{s})(8.87\,\frac{ft}{s})(\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j})$

$= (52.8\,\hat{i} + 30.5\,\hat{j})\,\frac{slug\cdot ft}{s^2}$

$= \underline{\underline{52.8\,\hat{i} + 30.5\,\hat{j}\ lb}}$

5.18

5.18 Air flows steadily in the axial direction in an annulus as indicated in Fig. P5.18. The inner and outer radii of the annulus are 17.5 and 50.8 mm. Axial velocities measured at various radii are listed below. Determine the mass flowrate involved.

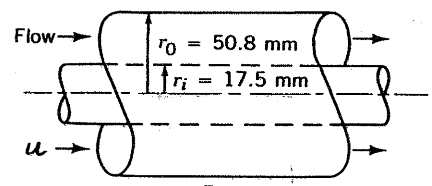


FIGURE P5.18

r (mm)	Axial Velocity (m/s)
17.5	0
18.2	32.0
18.7	33.9
19.3	35.6
20.0	36.8
20.6	37.9
21.9	39.3
23.2	40.4
25.7	41.8
28.2	42.5
30.8	42.8
33.3	42.6
35.8	42.1
38.4	41.4
40.9	40.1
43.5	38.5
46.0	36.0
47.3	34.3
48.6	31.9
49.8	27.4
50.8	0

The mass flowrate is calculated with

$$\dot{m} = \int_{r_i}^{r_o} \rho u \, 2\pi r dr = 2\pi\rho \int_{r_i}^{r_o} u r dr$$

where

$$\rho = 1.23 \frac{kg}{m^3}$$

and $\int_{r_i}^{r_o} u r dr$ is evaluated numerically with the trapezoidal rule with unequal intervals to give

$$\int_{r_i}^{r_o} u r dr = 0.0430 \, m^3/s$$

Thus,

$$\dot{m} = 2\pi\rho \int_{r_i}^{r_o} u r \, dr = 2\pi \left(1.23 \frac{kg}{m^3}\right)\left(0.0430 \frac{m^3}{s}\right) = \underline{\underline{0.332 \frac{kg}{s}}}$$

5.19

5.19 As shown in Fig. P5.19, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity V. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where u is in ft/s and y is in ft. Determine the value of V.

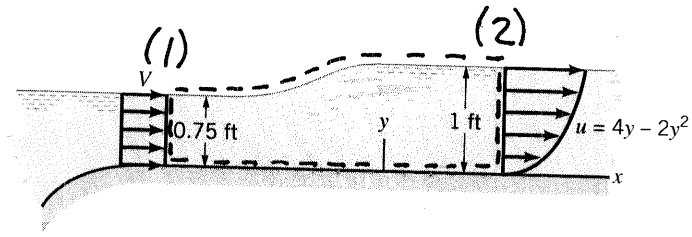


FIGURE P5.19

Use the control volume indicated by the broken lines in the sketch above.

From the conservation of mass principle

$$Q_1 = Q_2$$

$$V_1 A_1 = \int_{A_2} u\,dA \quad \int_0^{1\,ft} (4y - 2y^2)\, b\, dy$$

$$V(0.75\,ft)\,b = 3\left[\frac{4y^2}{2} - \frac{2y^3}{3}\right]_0^{1\,ft} b = \frac{4b}{3}\,\frac{ft^3}{s}$$

$$V = \frac{4}{3(0.75)} = \underline{\underline{1.78}}\ \frac{ft}{s}$$

5.20

5.20 Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.20. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also U. If the x direction velocity profile at section (2) is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at x where the boundary layer thickness is δ.

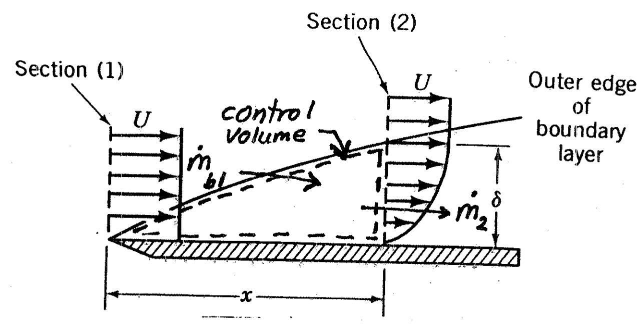


FIGURE P5.20

From the conservation of mass principle applied to the flow through the control volume shown in the figure we have

$$\dot{m}_{bl} = \dot{m}_2 = \int_{A_2} \rho \vec{V} \cdot \hat{n} \, dA$$

For incompressible, flow

$$\rho Q_{bl} = \rho U \ell \delta \int_0^1 \left(\frac{y}{\delta}\right)^{\frac{1}{7}} d\left(\frac{y}{\delta}\right)$$

where

$$\ell = \text{width of the plate}$$

and thus

$$Q_{bl} = \underline{\underline{\frac{7}{8} U \ell \delta}}$$

5.22

5.22 Estimate the time required to fill with water a cone-shaped container (see Fig. P5.22) 5 ft high and 5 ft across at the top if the filling rate is 20 gal/min.

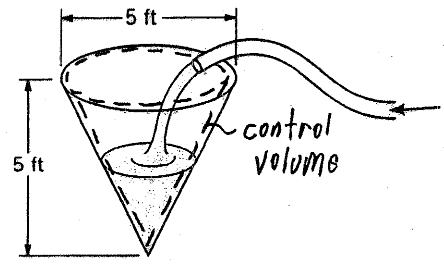


FIGURE P5.22

From application of the conservation of mass principle to the control volume shown in the figure we have

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V}\cdot\hat{n}\, dA = 0$$

For incompressible flow

$$\frac{\partial \forall}{\partial t} - Q = 0$$

or

$$\int_0^{\forall} d\forall = Q\int_0^{t} dt$$

Thus

$$t = \frac{\forall}{Q} = \frac{\pi D^2 h}{12\, Q} = \frac{\pi\,(5\text{ ft})^2(5\text{ ft})\left(1728\,\frac{\text{in.}^3}{\text{ft}^3}\right)}{(12)\left(20\,\frac{\text{gal}}{\text{min}}\right)\left(231\,\frac{\text{in.}^3}{\text{gal}}\right)}$$

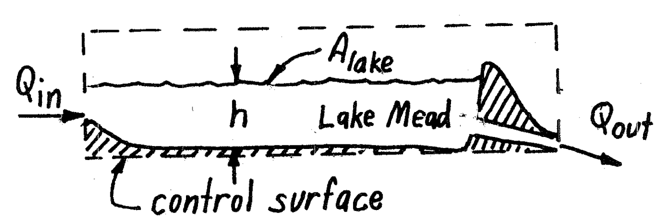
and

$$t = \underline{\underline{12.2\text{ min}}}$$

5.23

5.23 The Hoover Dam backs up the Colorado River and creates Lake Mead, which is approximately 115 miles long and has a surface area of approximately 225 square miles. (See Video V2.3.) If during flood conditions the Colorado River flows into the lake at a rate of 45,000 cfs and the outflow from the dam is 8,000 cfs, how many feet per 24-hour day will the lake level rise?

For the control volume shown:



$$\dot{m}_{in} - \dot{m}_{out} = \frac{\partial}{\partial t}\int_{\substack{cv\\water}} \rho \, d\forall$$

or since $\dot{m} = \rho Q$,

$$Q_{in} - Q_{out} = \frac{\partial}{\partial t}(A_{lake} h) = A_{lake}\frac{dh}{dt}$$

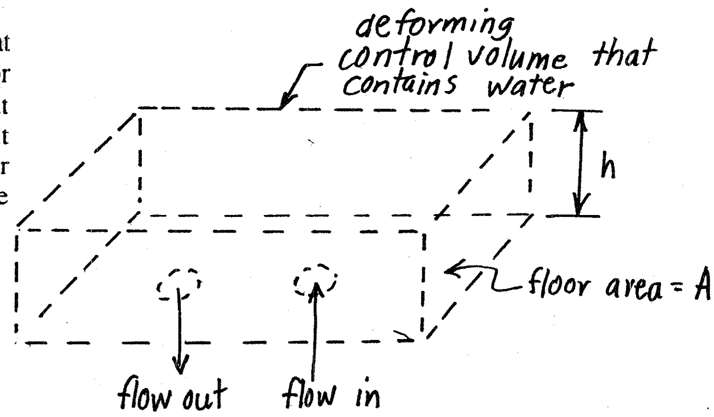
Thus,

$$\frac{dh}{dt} = \frac{Q_{out} - Q_{in}}{A_{lake}} = \frac{(45{,}000 - 8{,}000)\frac{ft^3}{s}}{225\ mi^2\left(5280\frac{ft}{mi}\right)^2} = 5.90 \times 10^{-6}\ \frac{in.}{s}$$

$$= 5.90\times10^{-6}\ \frac{in.}{s}\left(3{,}600\frac{s}{hr}\right)\left(24\frac{hr}{day}\right) = \underline{\underline{0.510\ \frac{ft}{day}}}$$

5.24

5.24 Storm sewer backup causes your basement to flood at the steady rate of 1 in. of depth per hour. The basement floor area is 1500 ft^2. What capacity (gal/min) pump would you rent to **(a)** keep the water accumulated in your basement at a constant level until the storm sewer is blocked off, **(b)** reduce the water accumulation in your basement at a rate of 3 in./hr even while the backup problem exists?



For a deforming control volume that contains the water over the basement floor (see sketch above), the conservation of mass principle (Eq. 5.17) leads to

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}_r \cdot \hat{n} \, dA = 0$$

or for constant fluid density and area (A)

$$A \frac{dh}{dt} - Q_{in} + Q_{out} = 0 \qquad (1)$$

(a) For part a, Eq. 1 leads to

$$Q_{out} = Q_{in}$$

To evaluate Q_{in}, we use Eq. 1 with $Q_{out} = 0$. Thus,

$$Q_{in} = A\frac{dh}{dt} = (1500 \text{ ft}^2)\left(1 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right) = 125 \frac{\text{ft}^3}{\text{hr}}$$

and

$$Q_{out} = \left(125 \frac{\text{ft}^3}{\text{hr}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right) = \underline{\underline{15.6 \frac{\text{gal}}{\text{min}}}}$$

(b) For part b, Eq. 1 yields

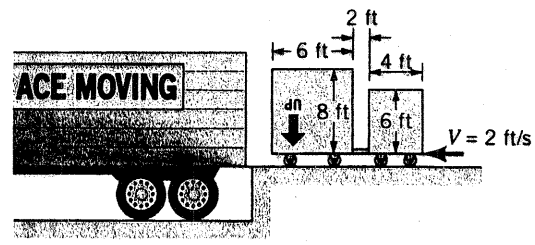
$$Q_{out} = Q_{in} - A\frac{dh}{dt}$$

$$Q_{out} = 15.6 \frac{\text{gal}}{\text{min}} - (1500 \text{ ft}^2)\left(-3 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right)$$

$$Q_{out} = \underline{\underline{62.4 \frac{\text{gal}}{\text{min}}}}$$

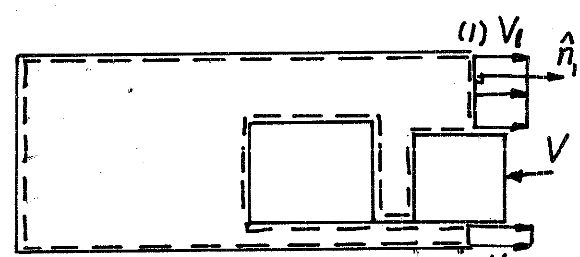
5.25

5.25 Two 8-ft-wide rectangular crates are wheeled into the trailer portion of a semi-truck at a speed of $V = 2$ ft/s as shown in Fig. P5.25. At time $t = 0$ the front of the first crate is 2 ft from the open back of the trailer. Plot a graph of the air flowrate across the open end of the trailer as a function of time for $0 \le t \le 10$ s.



■ FIGURE P5.25

Consider the deforming control volume shown in the sketch. From Eq.(5.17)



— — deforming control vol.

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{W}\cdot\hat{n}\, dA = 0$$

or since $\rho = \text{constant}$

$$\frac{\partial}{\partial t}\int d\forall + \int \vec{W}\cdot\hat{n}\, dA = 0 \quad \text{or} \quad \frac{d\forall_{cv}}{dt} = -\int \vec{W}\cdot\hat{n}\, dA = -V_1 A_1 = -Q_1 \text{, where}$$

Q_1 = airflow rate across end of trailer.

Also,

$\frac{d\forall}{dt}$ = rate at which the volume of air in the trailer changes with time $= -VA_{crate}$, where A_{crate} = cross-sectional area of the crate that is crossing the end of the trailer.

Thus,

$$-VA_{crate} = -Q_1 \quad \text{or} \quad Q_1 = VA_{crate}\text{, where } V = 2\ \text{ft/s} \qquad (1)$$

From the given data

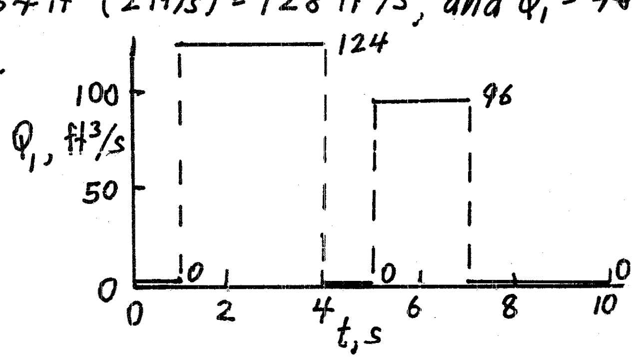
$A_{crate} = 0$ for $0 \le t < 1\,s$; $A_{crate} = (8ft)(8ft) = 64\,ft^2$ for $1s \le t < 4s$;

$A_{crate} = 0$ for $4s \le t < 5s$; $A_{crate} = (6ft)(8ft) = 48\,ft^3$ for $5s \le t \le 7s$

The corresponding flowrates (from Eq. (1)) are

$Q_1 = 0$, $Q_1 = 64\,ft^2(2\,ft/s) = 128\,ft^3/s$, and $Q_1 = 48\,ft^2(2\,ft/s) = 96\,ft^3/s$

as shown below.



5.27

5.27 It takes you 1 min to fill your car's fuel tank with 8.8 gallons of gasoline. What is the approximate average velocity of the gasoline leaving the 0.60-in.-diameter nozzle at this pump?

$$\bar{V}_{nozzle} A_{nozzle} = Q = \frac{(8.8\ gal)}{(1\ min)\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)}$$

and $$A_{nozzle} = \frac{\pi d^2_{nozzle}}{4} = \frac{\pi(0.6\ in.)^2}{(4)\left(12\ \frac{in.}{ft}\right)^2}$$

so $$\bar{V}_{nozzle} = \frac{(8.8)(4)(12)^2}{(\pi)(0.6)^2(7.48)(60)}$$

$$\bar{V}_{nozzle} = \underline{\underline{9.99\ \frac{ft}{s}}}$$

5.28

5.28 Water flows through a horizontal, 180° pipe bend as is illustrated in Fig. P5.28. The flow cross section area is constant at a value of 9000 mm^2. The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.

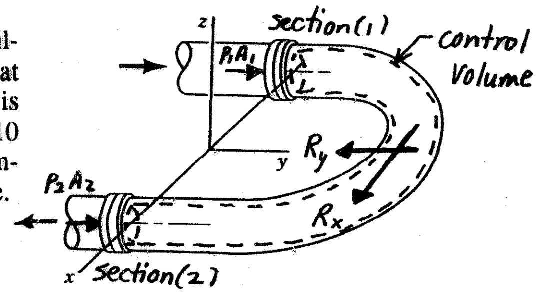


FIGURE P5.28

This analysis is similar to the one of Example 5.11. A fixed, non-deforming control volume that contains the water within the elbow between sections (1) and (2) at an instant is used. The horizontal forces acting on the contents of the control volume in the x and y directions are shown. Application of the x-direction component of the linear momentum equation (Eq. 5.22) leads to

$$R_x = \underline{\underline{0}}$$

Application of the y-direction component of the linear momentum equation yields

$$-v_1 \rho v_1 A_1 - v_2 \rho v_2 A_2 = p_1 A_1 - R_y + p_2 A_2$$

or

$$R_y = \rho A_1 v_1 (v_1 + v_2) + p_1 A_1 + p_2 A_2$$

Thus

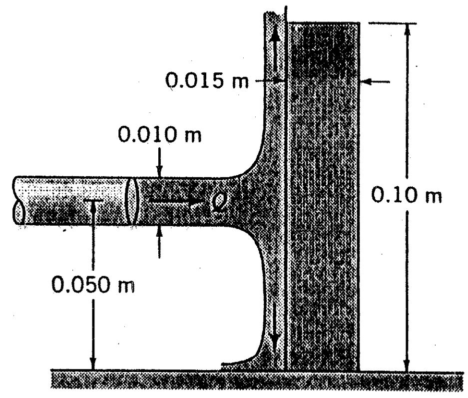
$$R_y = \left(999 \frac{kg}{m^3}\right) \frac{(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2} \left(15 \frac{m}{s}\right)\left(15\frac{m}{s} + 15\frac{m}{s}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right) + \frac{(210\ kPA)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPA}}\right)}$$

$$+ \frac{(165\ kPA)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPA}}\right)}$$

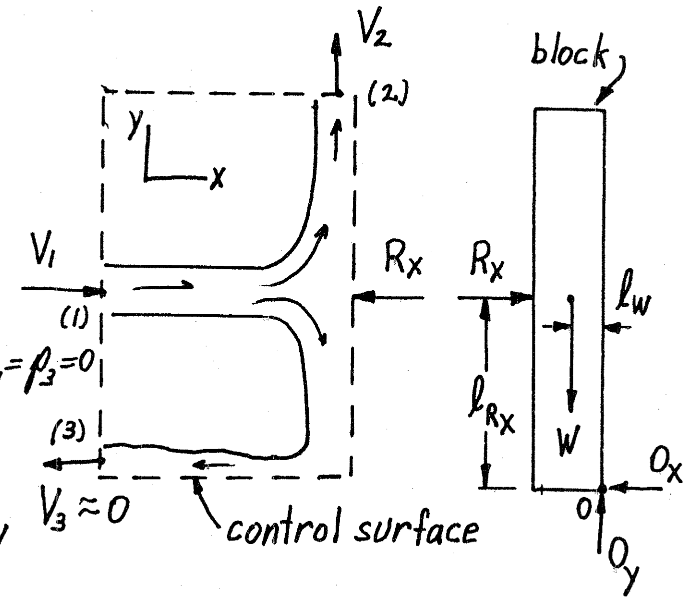
$$R_y = \underline{\underline{7420}}\ N$$

5.29

5.29 A 10-mm diameter jet of water is deflected by a homogeneous rectangular-block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in Video V5.4 and Fig. P5.29 Determine the minimum volume flowrate needed to tip the block.



■ FIGURE P5.29



From the free body diagram of the block when it is ready to tip $\Sigma M_0 = 0$, or

$R_x \ell_{R_x} = W\ell_w$ where R_x is the fore that the water puts on the block.

Thus,

$$R_x = \frac{W\ell_w}{\ell_{Rx}} = \frac{6N\left(\frac{0.015}{2}m\right)}{0.050m} = 0.90N$$

For the control volume shown the x-component of the momentum equation

$$\int_{cs} u\rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_x$$

becomes

$$V_1 \rho(-V_1)A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

$$V_1 = \sqrt{\frac{0.9N}{\left(999\frac{kg}{m^3}\right)\frac{\pi}{4}(0.01m)^2}} = 3.39\frac{m}{s}$$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.01m)^2\left(3.39\frac{m}{s}\right) = \underline{\underline{2.66\times10^{-4}\frac{m^3}{s}}}$$

5.30

5.30 Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.30 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.

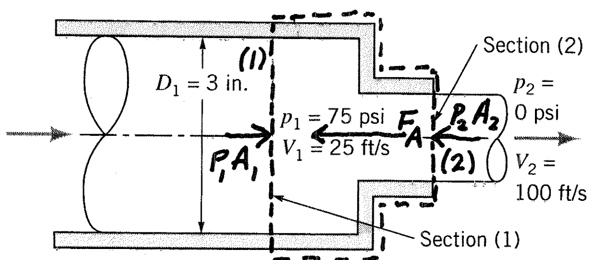


FIGURE P5.30

For this problem we include in the control volume the nozzle as well as the water at an instant between sections (1) and (2) as indicated in the sketch above. The horizontal forces acting on the contents of the control volume are shown in the sketch. Note that the atmospheric forces cancel out and are not shown. Application of the horizontal or x-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = p_1 A_1 - F_A - p_2 A_2 \qquad (1)$$

From the conservation of mass equation (Eq. 5.12) we obtain

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus Eq. (1) may be expressed as

$$\dot{m}(u_2 - u_1) = p_1 A_1 - F_A - p_2 A_2$$

or

$$F_A = p_1 A_1 - p_2 A_2 + \dot{m}(u_2 - u_1) = p_1 \frac{\pi D_1^2}{4} - p_2 \frac{\pi D_2^2}{4} - \rho u_1 \frac{\pi D_1^2}{4}(u_2 - u_1)$$

and

$$F_A = \left(75 \frac{lb}{in^2}\right)\frac{\pi (3\,in.)^2}{4} - 0\,lb - \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right)\frac{\pi (3\,in.)^2}{4\left(144 \frac{in.^2}{ft^2}\right)}\left(100 \frac{ft}{s} - 25 \frac{ft}{s}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)$$

$$F_A = \underline{\underline{352\ lb}}$$

5.31

5.31 Water flows through the 20° reducing bend shown in Fig. P5.31 at a rate of 0.025 m^3/s. The flow is frictionless, gravitational effects are negligible, and the pressure at section (1) is 150 kPa. Determine the x and y components of force required to hold the bend in place.

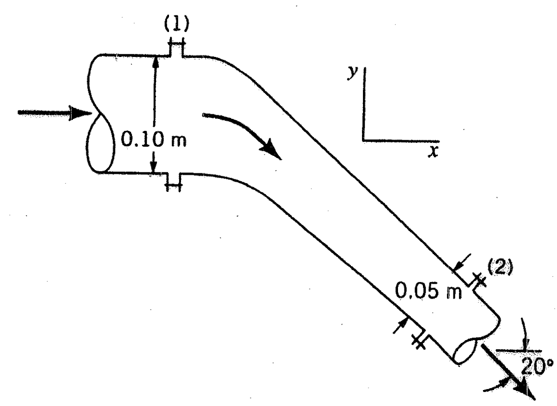
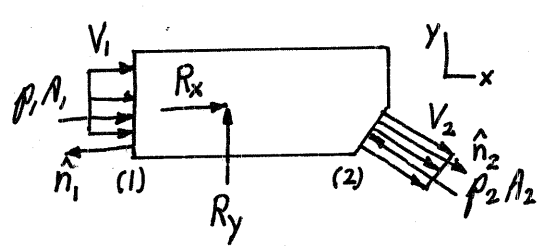


FIGURE P5.31

For the control volume shown the x-component of the momentum equation is



$$\int_{cs} u\rho\vec{V}\cdot\hat{n}\,dA = \Sigma F_x, \text{ or}$$

$$V_1\rho(-V_1)A_1 + (V_2\cos 20°)\rho V_2 A_2 = R_x + p_1A_1 - p_2A_2\cos 20°$$

or

(1) $R_x = p_2A_2\cos 20° - p_1A_1 + (V_2\cos 20° - V_1)\dot{m}$, where $\dot{m} = \dot{m}_1 = \dot{m}_2 = \rho Q = \rho AV$

Also,

$$V_1 = Q/A = (0.025\tfrac{m^3}{s})/(\tfrac{\pi}{4}(0.10m)^2) = 3.18\tfrac{m}{s}$$

and

$$V_2 = Q/A_2 = (0.025\tfrac{m^3}{s})/(\tfrac{\pi}{4}(0.05m)^2) = 12.7\tfrac{m}{s}$$

In addition, from the Bernoulli equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2, \text{ or}$$

$$p_2 = p_1 + \tfrac{1}{2}\rho(V_1^2 - V_2^2) = 150\,kPa + \tfrac{1}{2}(999\tfrac{kg}{m^3})[(3.18\tfrac{m}{s})^2 - (12.7\tfrac{m}{s})^2]$$

$$= 150\times10^3\tfrac{N}{m^2} - 75.5\times10^3\tfrac{kg\cdot m}{s^2}/m^2 = 74.5\,kPa$$

Thus, from Eq. (1),

$$R_x = 74.5\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.05m)^2)\cos 20° - 150\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.1\,m)^2)$$

$$+[(12.7\tfrac{m}{s})\cos 20° - 3.18\tfrac{m}{s}](999\tfrac{kg}{m^3})(0.025\tfrac{m^3}{s}) = \underline{\underline{-822\,N}}$$

Similarly, in the y-direction $\int_{cs} v\rho\vec{V}\cdot\hat{n}\,dA = \Sigma F_y$, or

$$(-V_2\sin 20°)\rho V_2A_2 = p_2A_2\sin 20° + R_y$$

or

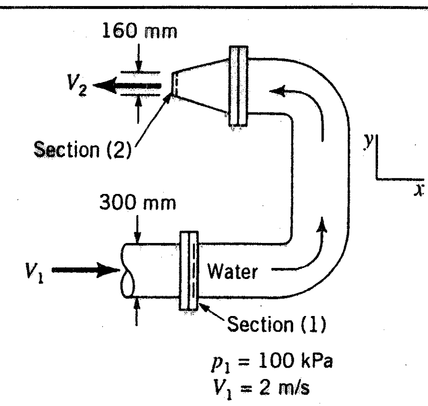
(2) $R_y = -V_2\sin 20°\,\dot{m} - p_2A_2\sin 20°$

$$= -(12.7\tfrac{m}{s})\sin 20°(999\tfrac{kg}{m^3})(0.025\tfrac{m^3}{s}) - 74.5\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.05m)^2)\sin 20°$$

$$= \underline{\underline{-156\,N}}$$

5.32

5.32 Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown in Fig. P5.32 in place. Atmospheric pressure is 100



■ **FIGURE P5.32**

The control volume shown in the sketch above is used. Application of the y direction component of the linear momentum equation yields

$$R_y = \underline{\underline{0}}$$

Application of the x direction linear momentum equation leads to

$$-u_1 \rho u_1 A_1 - u_2 \rho u_2 A_2 = p_1 A_1 - R_x + p_2 A_2$$

From the conservation of mass equation

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus

$$R_x = \rho u_1 A_1 \left(u_1 + u_2\right) + p_1 A_1 + p_2 A_2 = \rho u_1 \frac{\pi D_1^2}{4}\left(u_1 + \frac{D_1^2}{D_2^2} u_1\right) + p_1 \frac{\pi D_1^2}{4} + (0) A_2$$

or

$$R_x = \left(999 \frac{kg}{m^3}\right)\left(2 \frac{m}{s}\right)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left[\left(2 \frac{m}{s}\right) + \frac{(300\ mm)^2}{(160\ mm)^2}\left(2 \frac{m}{s}\right)\right]$$

$$+ (100\ kPa)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left(1000 \frac{N}{m^2 \cdot kPa}\right)$$

and

$$R_x = \underline{\underline{8340\ N}}$$

5.33

5.33 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.33. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.

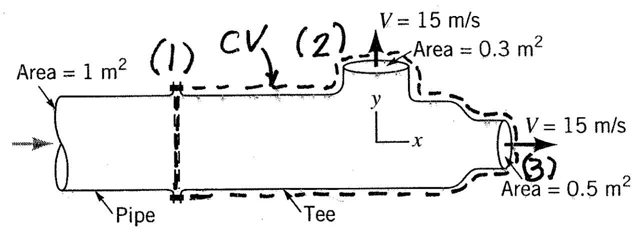


FIGURE P5.33

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1 \rho V_1 A_1 + V_3 \rho V_3 A_3 = P_1 A_1 - P_3 A_3 - P_{atm}(A_1 - A_3) + F_x$$

$$= (P_{1\,gage} + P_{atm})A_1 - (P_{3\,gage} + P_{atm})A_3 - P_{atm}(A_1 - A_3) + F_x$$

$$= P_{1\,gage} A_1 + F_x \qquad (1)$$

To get V_1 we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1 V_1 = A_2 V_2 + A_3 V_3$

so $V_1 = \dfrac{A_2 V_2 + A_3 V_3}{A_1} = \dfrac{(0.3\,m^2)(15\,m/s) + (0.5\,m^2)(15\,m/s)}{1\,m^2} = 12\,m/s$

To estimate $P_{1\,gage}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{P_{1\,gage}}{\rho} + \frac{V_1^2}{2} = \frac{P_{2\,gage}}{\rho} + \frac{V_2^2}{2}$$

$$P_{1\,gage} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[\left(15\,\frac{m}{s}\right)^2 - \left(12\,\frac{m}{s}\right)^2\right]}{2}\left(\frac{1\,N \cdot s^2}{kg \cdot m}\right)$$

$$P_{1\,gage} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq. (1) we get:

$$\left[-\left(12\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(12\,\frac{m}{s}\right)(1\,m^2) + \left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.5\,m^2)\right]\left(\frac{1\,N \cdot s^2}{kg \cdot m}\right) =$$

$$\left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $\underline{\underline{F_x = 72{,}000\,N \leftarrow}}$

For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2 \rho V_2 A_2 = F_y$$

$$\left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.3\,m^2) = \underline{\underline{67{,}400\,N \uparrow}} = F_y$$

5.34

5.34 A converging elbow (see Fig. P5.34) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m³ between sections (1) and (2). The water volume flowrate is 0.4 m³/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place.

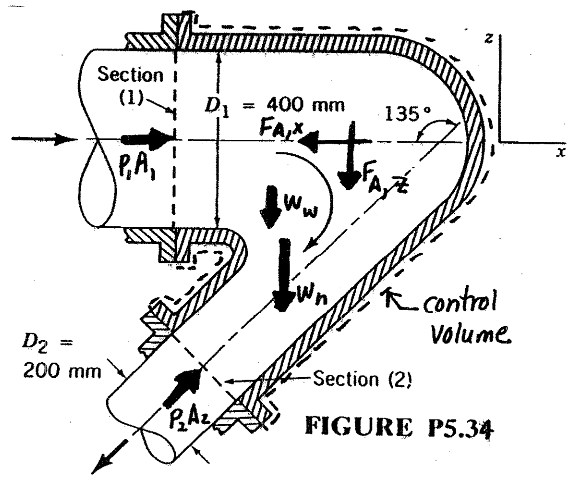


FIGURE P5.34

A control volume that contains the elbow and the water within the elbow between sections (1) and (2) as shown in the sketch above is used. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 - V_2 \cos 45^\circ \rho V_2 A_2 = p_1 A_1 - F_{A,x} + p_2 A_2 \cos 45^\circ$$

From conservation of mass

$$\dot{m} = \rho u_1 A_1 = \rho V_2 A_2 = \rho Q \qquad (1)$$

Thus

$$F_{A,x} = \frac{\rho Q^2}{A_1} + \frac{\rho Q^2}{A_2}\cos 45^\circ + p_1 A_1 + p_2 A_2 \cos 45^\circ = \frac{\rho Q^2}{\frac{\pi D_1^2}{4}} + \frac{\rho Q^2 \cos 45^\circ}{\pi D^2} + p_1 \frac{\pi D_1^2}{4} + p_2 \frac{\pi D_2^2}{4}\cos 45^\circ$$

$$F_{A,x} = \left(999 \frac{kg}{m^3}\right)\left(0.4 \frac{m^3}{s}\right)^2 \frac{1}{\left(\frac{\pi}{4}\right)} \left[\frac{\left(1000 \frac{mm}{m}\right)^2}{(400\ mm)^2} + \frac{\cos 45^\circ \left(1000 \frac{mm}{m}\right)^2}{(200\ mm)^2}\right]\left(\frac{1\ N}{kg \frac{m}{s^2}}\right)$$

$$+ \frac{\pi}{4}\frac{\left(1000 \frac{N}{kPa\ m^2}\right)}{\left(1000 \frac{mm}{m}\right)^2}\left[(150\ kPa)(400\ mm)^2 + (90\ kPa)(200\ mm)^2 \cos 45^\circ\right]$$

$$\underline{\underline{F_{A,x} = 25{,}700\ N}}$$

Application of the vertical or z direction component of the linear momentum equation leads to

$$-V_2 \sin 45^\circ \rho V_2 A_2 = p_2 A_2 \sin 45^\circ - F_{A,z} - W_w - W_e$$

which when combined with Eq. 1 gives

$$F_{A,z} = \frac{\rho Q^2}{A_2}\sin 45^\circ + p_2 A_2 \sin 45^\circ - W_w - W_e = \frac{\rho Q^2}{\frac{\pi D_2^2}{4}}\sin 45^\circ + p_2 \frac{\pi D_2^2}{4}\sin 45^\circ - \rho g \forall_w - m_e g$$

(con't)

5.34 (con't)

$$F_{A,z} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(0.4\,\frac{m^3}{s}\right)^2 \sin 45^\circ \left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{\frac{\pi}{4}\frac{(200\,mm)^2}{\left(1000\,\frac{mm}{m}\right)^2}} + \frac{(90\,kPa)\,\pi\,(200\,mm)^2 \sin 45^\circ}{4\left(1000\,\frac{mm}{m}\right)^2}$$

$$- \left(999\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(0.2\,m^3\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) - (12\,kg)\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)$$

$$F_{A,z} = \underline{\underline{8920\ N}}$$

5.35

5.35 Thrust vector control is a new technique that can be used to greatly improve the maneuverability of military fighter aircraft. It consists of using a set of vanes in the exit of a jet engine to deflect the exhaust gases as shown in Fig. P5.35. **(a)** Determine the pitching moment (the moment tending to rotate the nose of the aircraft up) about the aircraft's mass center (cg) for the conditions indicated in the figure. **(b)** By how much is the thrust (force along the centerline of the aircraft) reduced for the case indicated compared to normal flight when the exhaust is parallel to the centerline?

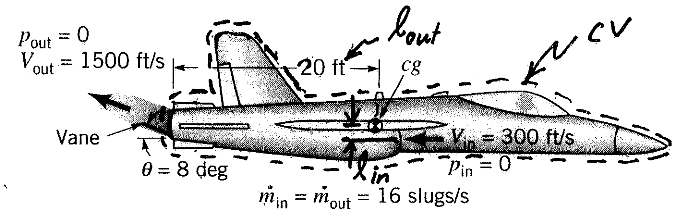


FIGURE P5.35

For part (a) we apply the component of the moment-of-momentum equation that is perpendicular to the plane of the sketch of the aircraft to the contents of the control volume shown to get

$$\ell_{out} V_{out} \sin\theta \, \dot{m}_{out} - \ell_{in} V_{in} \dot{m}_{in} = \text{pitching moment}$$

$$\frac{(20\ ft)(1500\ \frac{ft}{s}) \sin 8^\circ (16\ \frac{slugs}{s})}{1\ \frac{slug \cdot ft}{lb \cdot s^2}} - \frac{\ell_{in} (300\ \frac{ft}{s})(16\ \frac{slugs}{s})}{1\ \frac{slug \cdot ft}{lb \cdot s^2}} = \text{pitching moment}$$

$$66{,}800 - 4800\, \ell_{in} \quad ft \cdot lb = \text{pitching moment}$$

For part (b) we apply the horizontal component of the linear momentum equation to the contents of the control volume to get

$$V_{out} \cos\theta \, \dot{m}_{out} - V_{in} \dot{m}_{in} = \text{thrust}$$

So

$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = V_{out} (\cos 0^\circ - \cos 8^\circ)\, \dot{m}_{out}$$

or

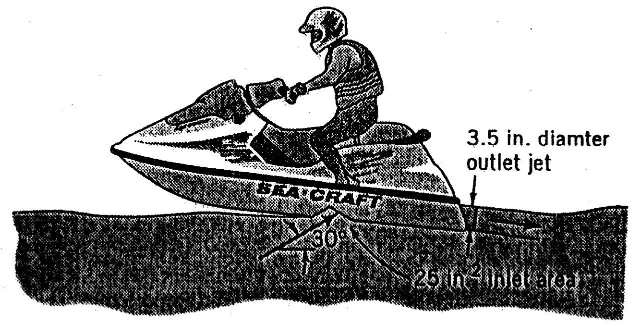
$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = \frac{(1500\ \frac{ft}{s})(\cos 0^\circ - \cos 8^\circ)(16\ \frac{slugs}{s})}{1\ \frac{slug \cdot ft}{lb \cdot s^2}}$$

and

$$\text{thrust}_{\theta=0} - \text{thrust}_{\theta=8^\circ} = \underline{\underline{234\ lb}}$$

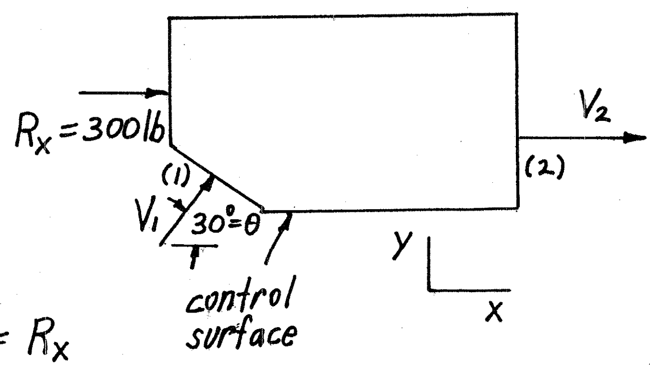
5.36

5.36 The thrust developed to propel the jet ski shown in **Video V9.7** and Fig. P5.36 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300-lb thrust? Assume the inlet and outlet jets of water are free jets.



■ FIGURE P5.36

For the control volume indicated the x-component of the momentum equation



$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_x \text{ becomes}$$

(1) $$(V_1 \cos 30°)\rho(-V_1)A_1 + V_2\rho(+V_2)A_2 = R_x$$

where we have assumed that $p=0$ on the entire control surface and that the exiting water jet is horizontal.

With $\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ Eq. (1) becomes

$$R_x = \dot{m}(V_2 - V_1 \cos\theta) = \rho V_1 A_1 (V_2 - V_1 \cos 30°) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{25 \text{ in.}^2}{\frac{\pi}{4}(3.5 \text{ in.})^2} V_1 = 2.60\, V_1 \qquad (2)$$

By combining Eqs. (1) and (2):

$$R_x = \rho V_1^2 A_1 (2.60 - \cos 30°)$$

or

$$V_1 = \left[\frac{300 \text{ lb}}{\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)\left(\frac{25}{144}\text{ft}^2\right)(2.60 - \cos 30°)}\right]^{1/2} = 22.7 \frac{\text{ft}}{\text{s}}$$

Thus,

$$Q = A_1 V_1 = \left(\frac{25}{144}\text{ft}^2\right)\left(22.7\frac{\text{ft}}{\text{s}}\right) = \underline{\underline{3.94 \frac{\text{ft}^3}{\text{s}}}}$$

5.37

5.37 Water is sprayed radially outward over 180° as indicated in Fig. P5.37. The jet sheet is in the horizontal plane. If the jet velocity at the nozzle exit is 20 ft/s, determine the direction and magnitude of the resultant horizontal anchoring force required to hold the nozzle in place.

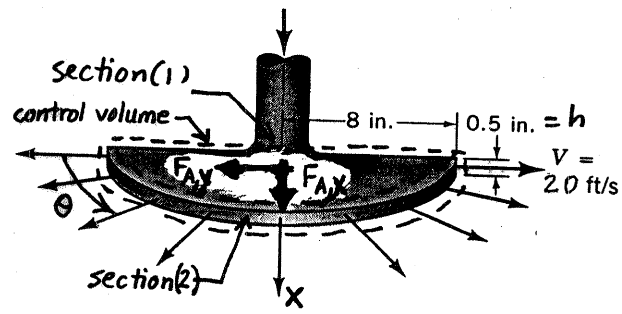


FIGURE P5.37

The control volume includes the nozzle and water between sections (1) and (2) as indicated in the sketch above. Application of the y direction component of the linear momentum equation yields

$$\int_{cs} v \rho \vec{V} \cdot \hat{n}\, dA = -F_{A,y}$$

or

$$F_{A,y} = -\rho \int_0^{\pi} (-V_2 \cos\theta)(V_2)\, h R\, d\theta = \rho h R V_2^2 (\sin\pi - \sin 0)$$

and

$$F_{A,y} = \underline{\underline{0}}$$

Application of the x direction component of the linear momentum equation leads to

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = F_{A,x}$$

or

$$F_{A,x} = \rho \int_0^{\pi} (V_2 \sin\theta)(V_2)\, h R\, d\theta = \rho h R V_2^2 (\cos 0 - \cos\pi)$$

and

$$F_{A,x} = \left(1.94 \frac{slugs}{ft^3}\right) \frac{(0.5\ in.)\ (8\ in.)\ \left(20 \frac{ft}{s}\right)^2 (2)}{\left(12 \frac{in.}{ft}\right)\left(12 \frac{in.}{ft}\right)} \left(1 \frac{lb}{\frac{slug \cdot ft}{s^2}}\right)$$

$$F_{A,x} = \underline{\underline{43\ lb}}$$

5.38

5.38 A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown in Fig. P5.38. A hole at the center of the plate results in a discharge jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force required to hold the plate stationary.

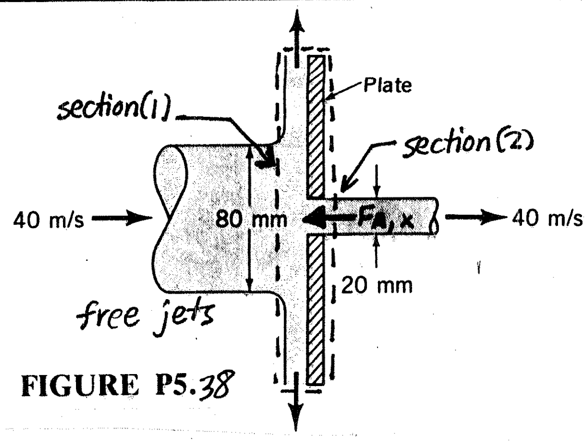


FIGURE P5.38

The control volume contains the plate and flowing air as indicated in the sketch above. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

or

$$F_{A,x} = u_1^2 \rho \frac{\pi D_1^2}{4} - u_2^2 \rho \frac{\pi D_2^2}{4} = u_1^2 \rho \frac{\pi}{4}\left(D_1^2 - D_2^2\right)$$

Thus

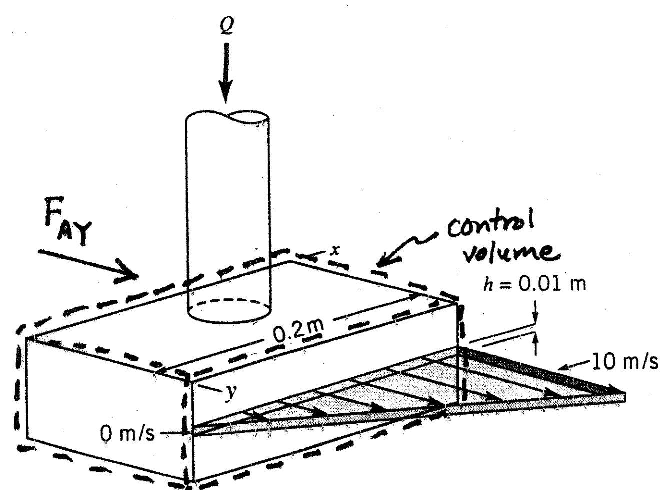
$$F_{A,x} = \left(40 \frac{m}{s}\right)^2 \left(1.23 \frac{kg}{m^3}\right) \frac{\pi}{4}\left[\frac{(80\ mm)^2 - (20\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$F_{A,x} = \underline{\underline{9.27\ N}}$$

5.39

5.39 A sheet of water of uniform thickness ($h = 0.01$ m) flows from the device shown in Fig. P5.39. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the y component of anchoring force necessary to hold this device stationary.



■ FIGURE P5.39

A control volume that contains the box portion of the device and the water in the box as shown in the sketch above is used. Application of the y-direction component of the linear momentum equation yields

$$F_{AY} = \int_{A_{slit}} v \rho \vec{V} \cdot \hat{n} \, dA = \rho \int_0^{0.2} v^2 h \, dx$$

The variation of v with x is linear or

$$v = 50x \ \frac{m}{s}$$

Thus

$$F_{AY} = \rho \int_0^{0.2} (50x)^2 h \, dx = \rho \, (50)^2 h \left. \frac{x^3}{3} \right|_0^{0.2}$$

or

$$F_{AY} = \left(999 \frac{kg}{m^3}\right)\left(50 \frac{1}{s}\right)^2 (0.01 m) \frac{(0.2 m)^3}{3} \left(1 \frac{N \cdot s^2}{kg \cdot m}\right)$$

and

$$F_{AY} = \underline{\underline{66.6 N}}$$

5.40

5.40 A variable mesh screen produces a linear and axisymmetric velocity profile as indicated in Fig. P5.40 in the air flow through a 2-ft-diameter circular cross section duct. The static pressures upstream and downstream of the screen are 0.2 and 0.15 psi and are uniformly distributed over the flow cross section area. Neglecting the force exerted by the duct wall on the flowing air, calculate the screen drag force.

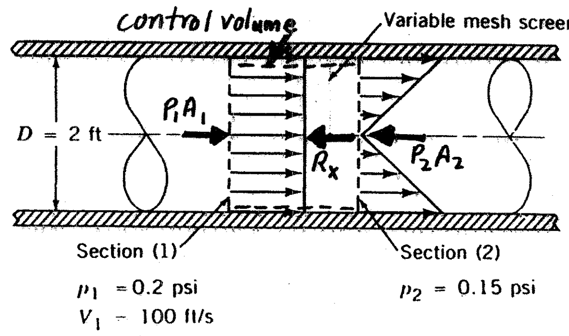


FIGURE P5.40

Application of the axial component of the linear momentum equation to the flow through the control volume shown in the sketch leads to

$$-V_1\rho V_1 A_1 + \int_0^R u_2 \rho u_2 \, 2\pi r\,dr = p_1A_1 - R_x - p_2A_2$$

or

$$R_x = \rho V_1^2 \frac{\pi D_1^2}{4} - 2\pi\rho\int_0^R \left(u_{max}\frac{r}{R}\right)^2 r\,dr + p_1\frac{\pi D_1^2}{4} - p_2\frac{\pi D_2^2}{4} \qquad (1)$$

The value of u_{max} may be obtained from conservation of mass as follows

$$\rho V_1 \frac{\pi D_1^2}{4} = \rho\int_0^R \left(u_{max}\frac{r}{R}\right) 2\pi r\,dr$$

Thus

$$u_{max} = \frac{V_1 D_1^2 R}{(2)(4)\int_0^R r^2 dr} = \frac{3}{2}V_1 = \frac{3}{2}\left(100\frac{ft}{s}\right) = 150\frac{ft}{s}$$

From Eq. 1

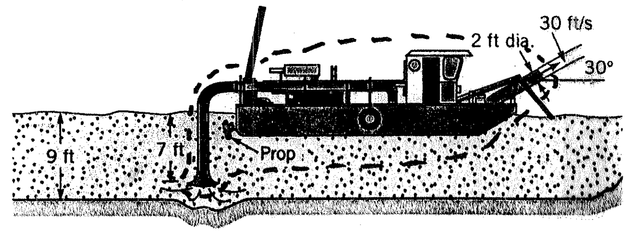
$$R_x = \left(0.00238\frac{slug}{ft^3}\right)\left(100\frac{ft}{s}\right)^2\frac{\pi(2ft)^2}{4}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) - 2\pi\left(0.00238\frac{slug}{ft^3}\right)\left(150\frac{ft}{s}\right)^2\frac{(2ft)^2}{16}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$+\left(0.2\frac{lb}{in.^2}\right)\frac{\pi(2ft)^2}{4}\left(144\frac{in.^2}{ft^2}\right) - \left(0.15\frac{lb}{in.^2}\right)\frac{\pi(2ft)^2}{4}\left(144\frac{in.^2}{ft^2}\right)$$

$$R_x = \underline{\underline{13.3\ lb}}$$

5.41

5.41 The hydraulic dredge shown in Fig. P5.41 is used to dredge sand from a river bottom. Estimate the thrust needed from the propeller to hold the boat stationary. Assume the specific gravity of the sand/water mixture is $SG = 1.2$.



■ FIGURE P5.41

Using the control volume shown by the broken line in the sketch above we use the horizontal or x component of the linear momentum equation to get

$$F_x = \rho A_2 V_2 V_{2x} = \rho_{H_2O} (sg)_2 \frac{\pi d_2^2}{4} V_2 V_2 \cos 30^\circ$$

where section 1 is where flow enters the control volume vertically and section 2 is where flow leaves the control volume at an angle of 30° from the horizontal direction. Note that there is no horizontal direction linear momentum flow at section 1.

$$F_x = \left(1.94 \frac{slugs}{ft^3}\right)(1.4) \frac{\pi (2\ ft)^2}{4} \left(30 \frac{ft}{s}\right)\left(30 \frac{ft}{s}\right) \cos 30^\circ \left(\frac{1\ lb}{\frac{ft\ slug}{s^2}}\right)$$

$$F_x = \underline{\underline{6650\ lb}}$$

5.42

5.42 Water flows vertically upward in a circular cross section pipe as shown in Fig. P5.42. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R-r}{R}\right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe radius, and r = radius from pipe axis. Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

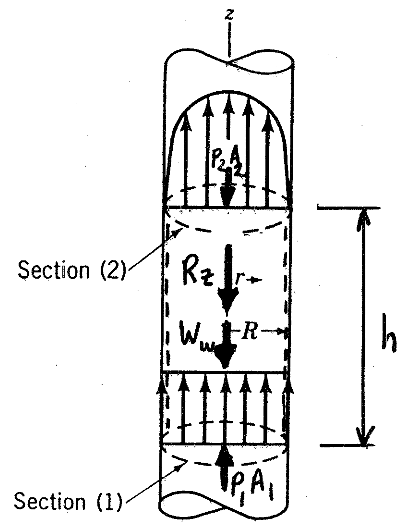


FIGURE P5.42

The analysis for this problem is similar to the one of Example 5.13. The control volume contains the fluid only between sections (1) and (2) as indicated in the sketch. Application of the vertical or z component of the linear momentum equation leads to

$$-w_1 \rho w_1 A_1 + \int_0^R w_2 \rho w_2 \, 2\pi r \, dr = p_1 A_1 - R_z + p_2 A_2 - W_w$$

Thus

$$p_1 - p_2 = \frac{R_z}{A} - \rho w_1^2 + \frac{\rho 2\pi}{A}\int_0^R \left[w_c \left(\frac{R-r}{R}\right)^{\frac{1}{7}}\right]^2 r \, dr + \frac{W_w}{A} \qquad (1)$$

The weight of the water in the control volume may be expressed as

$$W_w = g\rho A h$$

The value of $w_{\mathbb{c}}$ may be obtained from the conservation of mass equation as follows

$$\rho w_1 A_1 = \int_0^R \rho w_c \left(\frac{R-r}{R}\right)^{\frac{1}{7}} 2\pi r \, dr$$

or

$$w_c = \frac{w_1 A_1}{2\pi \int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r \, dr} \qquad (2)$$

To evaluate the integral $\int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r \, dr$ we substitute

$$\alpha = \frac{R-r}{R} \qquad (3)$$

then

$$d\alpha = -\frac{dr}{R} \qquad (4)$$

(con't)

5.42 (con't)

and
$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr = -\int_1^0 \alpha^{\frac{1}{7}}(1-\alpha)R^2\,d\alpha = \frac{49}{120}R^2 \qquad (5)$$

Combining Eqs. 2 and 5 we obtain
$$w_c = \frac{60}{49}w_1$$

Thus from Eq. 1
$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \frac{\rho(2)(60)^2 w_1^2}{R^2(49)^2}\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr + g\rho h \qquad (6)$$

To evaluate the integral $\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr$ we use Eqs. 3 and 4.

Thus
$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr = -\int_1^0 \alpha^{2/7}(1-\alpha)R^2\,d\alpha = \frac{49}{144}R^2$$

and Eq. 6 becomes

$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \rho(1.02)w_1^2 + g\rho h$$

or
$$\underline{\underline{p_1 - p_2 = \frac{R_z}{\pi R^2} + 0.02\rho w_1^2 + g\rho h}}$$

Note that in contrast to the result of Example 5.13, only a very small portion of the pressure drop is due to a change in the momentum flow between sections 1 and 2 in this case.

5.43

5.43 In a laminar pipe flow that is fully developed, the axial velocity profile is parabolic, that is,

$$u = u_c \left[1 - \left(\frac{r}{R}\right)^2\right]$$

as is illustrated in Fig. P5.43. Compare the axial direction momentum flowrate calculated with the average velocity, $\bar{u}$, with the axial direction momentum flowrate calculated with the nonuniform velocity distribution taken into account.

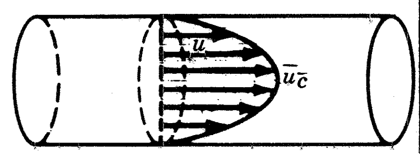


FIGURE P5.43

The axial direction momentum flowrate based on a uniform velocity profile with $u = \bar{u}$ is

$$MF_{x,\,uniform} = \bar{u}\rho\bar{u}A = \rho\bar{u}^2\pi R^2$$

The axial direction momentum flowrate based on the non-uniform parabolic velocity profile is

$$MF_{x,\,non\text{-}uniform} = \int_0^R u\rho u 2\pi r\,dr = \rho u_c^2 2\pi R^2 \int_0^1 \left[1-\left(\frac{r}{R}\right)^2\right]^2 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

$$MF_{x,\,non\text{-}uniform} = \frac{\rho u_c^2 \pi R^2}{3}$$

To obtain a relationship between $\bar{u}$ and $u_{\mathcal{C}}$ we use the conservation of mass equation as follows

$$\rho\bar{u}\pi R^2 = \rho 2\pi R^2 u_c \int_0^R \left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

Thus

$$\bar{u} = \frac{u_c}{2}$$

and

$$MF_{x,\,non\text{-}uniform} = \frac{4}{3}\rho\bar{u}^2\pi R^2 = \frac{4}{3} MF_{x,\,uniform}$$

*5.44

*5.44 For the annulus air flow data of Problem 5.18, calculate the rate of flow of axial direction momentum. How large would the error be if the average axial velocity were used to calculate axial direction momentum flow?

The rate of flow of axial momentum, MF_x, is given by

$$MF_x = \int_{r=R_i}^{R_o} \rho u^2 \, 2\pi r dr = 2\pi\rho\int_{R_i}^{R_o} u^2 r \, dr, \text{ where} \qquad (1)$$

$R_i = 17.5$ mm and $R_o = 50.8$ mm are the inner and outer radii of the annulus and the axial velocity is from the table of problem 5.18.

By using a numerical integration program with the trapezodial rule, the value of the integral is determined to be

$$\int_{R_i}^{R_o} u^2 r \, dr = 1.688 \frac{m^4}{s^2}$$

Hence, from Eq. (1),

$$MF_x = 2\pi(1.23 \tfrac{kg}{m^3})(1.688 \tfrac{m^4}{s^2}) = 13.05 \frac{kg \cdot m}{s^2} = \underline{\underline{13.05 \text{ N}}}$$

The rate of flow of axial momentum based on the average axial velocity, $\overline{V}$, is denoted $\overline{MF_x}$ and given by

$$\overline{MF_x} = \rho \overline{V}^2 A \qquad (2)$$

where

$$\overline{V} = \dot{m} / \rho A$$

From the calculations of problem 5.18, $\dot{m} = 0.332$ kg/s.

Hence, $\overline{V} = (0.332 \text{ kg/s}) / [1.23 \tfrac{kg}{m^3} (\pi)((0.0508 m)^2 - (0.0175 m)^2)] = 37.8$ m/s

Thus, from Eq. (2),

$$\overline{MF_x} = (1.23 \tfrac{kg}{m^3})(37.8 \tfrac{m}{s})^2 (\pi)[(0.0508 m)^2 - (0.0175 m)^2] = 12.56 \text{ N}$$

Therefore, the percent error in approximating the momentum flow by using the average velocity is

$$\left(\frac{\overline{MF_x} - MF_x}{MF_x}\right)(100) = \left(\frac{12.56N - 13.05N}{13.05N}\right)(100) = \underline{\underline{-3.75\%}}$$

5.45

5.45 Consider unsteady flow in the constant diameter, horizontal pipe shown in Fig. P5.45. The velocity is uniform throughout the entire pipe, but it is a function of time: $\mathbf{V} = u(t)\hat{\mathbf{i}}$. Use the x component of the unsteady momentum equation to determine the pressure difference $p_1 - p_2$. Discuss how this result is related to $F_x = ma_x$.

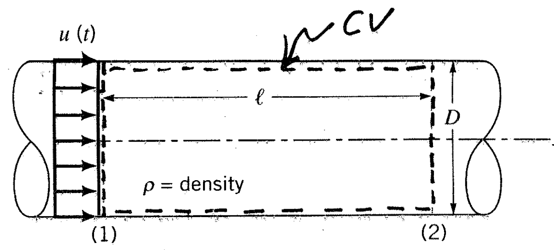


FIGURE P5.45

Using the control volume shown in the sketch and applying the x-component of the unsteady linear momentum equation to the contents of this CV we get

$$\frac{\partial}{\partial t}\int_{cv} \rho u \, d\forall + \int_{cs} \rho u \, \vec{V}\cdot\hat{n}\, dA = \Sigma F_x$$

or

$$\frac{\partial}{\partial t}\left(\rho u \frac{\pi}{4} D^2 \ell\right) - \rho u_1^2 A_1 + \rho u_2^2 A_2 = p_1 A_1 - p_2 A_2 + F_x$$

$\rho u_1^2 A_1 = \rho u_2^2 A_2$ assuming $u_1 = u_2$ at every instant

$F_x = 0$ assuming frictionless flow

Thus,

$$\underbrace{\left(\rho \frac{\pi D^2 \ell}{4}\right)}_{\text{mass in CV}} \underbrace{\left(\frac{\partial u}{\partial t}\right)}_{\text{local acceleration}} = \underbrace{(p_1 - p_2)\frac{\pi D^2}{4}}_{\text{force}}$$

and

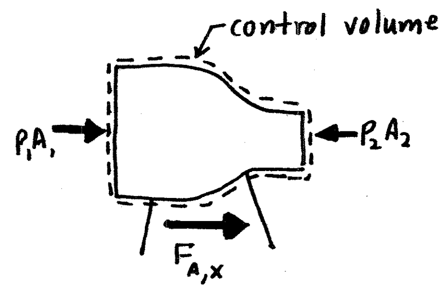
$$p_1 - p_2 = \rho \ell \frac{\partial u}{\partial t}$$

5.46

5.46 A static thrust stand is to be designed for testing a specific jet engine. Knowing the following conditions for a typical test,

intake air velocity = 700 ft/s
exhaust gas velocity = 1640 ft/s
intake cross section area = 10 ft²
intake static pressure = 11.4 psia
intake static temperature = 480 °R
exhaust gas pressure = 0 psi

estimate a nominal thrust to design for.



The analysis for this problem is similar to the one of Example 5.14. A control volume that contains the entire engine and the fluid in the engine as indicated in the sketch is used. Application of the horizontal or x direction component of the linear momentum equation leads to

$$-u_1\rho_1 u_1 A_1 + u_2 \rho_2 u_2 A_2 = p_1 A_1 + F_{A,x}$$

or

$$F_{A,x} = -u_1 \rho_1 u_1 A_1 + u_2 \rho_2 u_2 A_2 - p_1 A_1$$

The conservation of mass principle yields

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

Thus

$$F_{A,x} = \rho_1 u_1 A_1 (u_2 - u_1) - p_1 A_1$$

or since

$$\rho_1 = \frac{p_1}{RT_1}$$

then

$$F_{A,x} = \frac{p_1 u_1 A_1 (u_1 - u_2)}{RT_1} \quad p_1 A_1$$

$$F_{A,x} = \frac{\left(11.4\,\frac{lb}{in.^2}\right)\left(700\,\frac{ft}{s}\right)\left(10\,ft^2\right)\left(1640\,\frac{ft}{s} - 700\,\frac{ft}{s}\right)\left(144\,\frac{in.^2}{ft^2}\right)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(1716\,\frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(480\,{}^\circ R)} - \left(11.4\,\frac{lb}{in.^2} - 14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)\left(10\,ft^2\right)$$

and

$$F_{A,x} = \underline{\underline{17{,}900\ lb}}$$

5.47

5.47 A free jet of fluid strikes a wedge as shown in Fig. P5.47. Of the total flow, a portion is deflected 30°; the remainder is not deflected. The horizontal and vertical components of force needed to hold the wedge stationary are F_H and F_V, respectively. Gravity is negligible, and the fluid speed remains constant. Determine the force ratio, F_H/F_V.

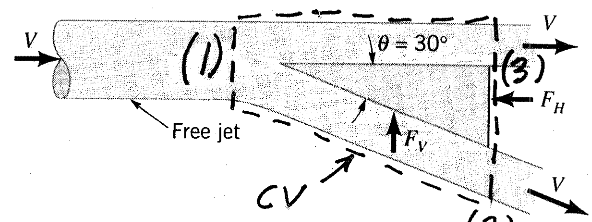


FIGURE P5.47

The horizontal and vertical components of the linear momentum equation are applied to the contents of the control volume shown to get

$$-V_1\rho V_1 A_1 + V_2 \rho V_2 A_2 + V_3 \cos 30^\circ \rho V_3 A_3 = -F_H \quad (1)$$

$$-V_3 \sin 30^\circ \rho V_3 A_3 = F_V \quad (2)$$

However $V_1 = V_2 = V_3 = V$ so eqs. (1) and (2) become

$$V^2\rho\left(A_2 + A_3\cos 30^\circ - A_1\right) = -F_H$$

$$V^2\rho A_3 \sin 30^\circ = -F_V$$

and

$$\frac{F_H}{F_V} = \frac{A_2 + A_3\cos 30^\circ - A_1}{A_3 \sin 30^\circ} \quad (3)$$

From conservation of mass we get

$$Q_1 = Q_2 + Q_3$$

or

$$A_1 V = A_2 V + A_3 V$$

and

$$A_1 = A_2 + A_3 \quad (4)$$

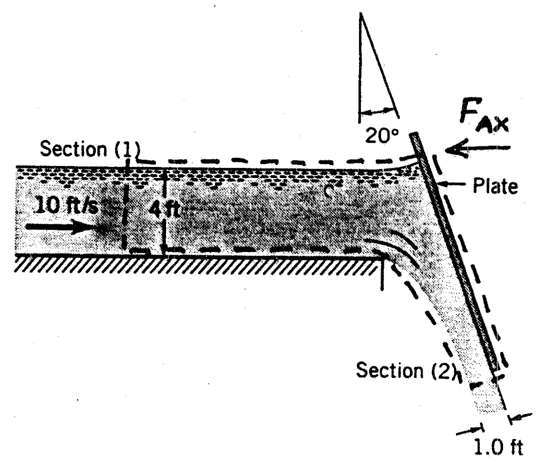
Combining Eqs. (3) and (4) we get

$$\frac{F_H}{F_V} = \frac{A_2 + A_3\cos 30^\circ - A_2 - A_3}{A_3 \sin 30^\circ} = \frac{A_3(\cos 30^\circ - 1)}{A_3 \sin 30^\circ} = \underline{\underline{-0.27}}$$

The negative sign indicates that F_V is down rather than up as shown in the sketch

5.48

5.48 Water flows from a two-dimensional open channel and is diverted by an inclined plate as illustrated in Fig. P5.48. When the velocity at section (1) is 10 ft/s, what horizontal force (per unit width) is required to hold the plate in position? At section (1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.



■ FIGURE P5.48

A control volume that contains most of the plate and the water being turned by the plate as shown in the sketch above is used. Application of the horizontal x-direction component of the linear momentum equation yields

$$-V_1 \rho V_1 A_1 + V_2 \sin 20^\circ \rho V_2 A_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1 A_1 \qquad (1)$$

From conservation of mass we obtain

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{h_1}{h_2} V_1$$

Thus, Eq. 1 becomes for unit width

$$-V_1^2 \rho h_1 + \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20^\circ \rho h_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1^2$$

or

$$F_{Ax} = \frac{1}{2}\gamma_w h_1^2 + V_1^2 \rho h_1 - \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20^\circ \rho h_2$$

Then

$$F_{Ax} = \frac{1}{2}\left(62.4 \frac{lb}{ft^3}\right)(4\,ft)^2 + \left(10\,\frac{ft}{s}\right)^2\left(1.94\,\frac{slugs}{ft^3}\right)\left(1\,\frac{lb \cdot s^2}{slug \cdot ft}\right)(4\,ft)$$
$$- \left[\left(\frac{4\,ft}{1\,ft}\right)\left(10\,\frac{ft}{s}\right)\right]^2 \sin 20^\circ \left(1.94\,\frac{slugs}{ft^3}\right)\left(1\,\frac{lb \cdot s^2}{slug \cdot ft}\right)(1\,ft)$$

and

$$F_{Ax} = \underline{\underline{213}}\ lb$$

5.49

5.49 A horizontal circular cross section jet of air having a diameter of 6 in. strikes a conical deflector as shown in Fig. P5.49. A horizontal anchoring force of 5 lb is required to hold the cone in place. Estimate the nozzle flow rate in ft³/s. The magnitude of the velocity of the air remains constant.

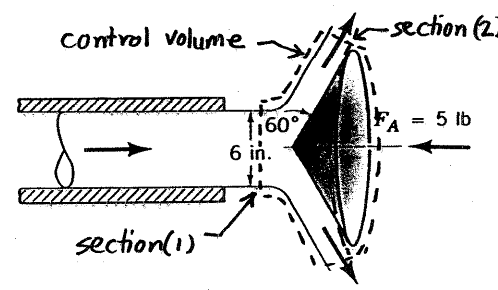


FIGURE P5.49

The control volume shown in the sketch is used. Application of the axial or x-direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

With the conservation of mass principle we can conclude for this incompressible flow that

$$u_1 A_1 = u_2 A_2 = Q$$

Also

$$u_2 = V \cos 60°$$

and

$$u_1 = V = \frac{Q}{A_1}$$

Thus

$$-V\rho Q + V\cos 60° \rho Q = -F_{A,x} = -\frac{Q^2}{A_1}\rho + \frac{Q^2 \cos 60°}{A_1}\rho$$

or

$$Q = \left[\frac{F_{A,x} A_1}{\rho(1-\cos 60°)}\right]^{\frac{1}{2}} = \left[\frac{F_{A,x}\left(\frac{\pi D_1^2}{4}\right)}{\rho(1-\cos 60°)}\right]^{\frac{1}{2}}$$

Thus

$$Q = \left[\frac{(5\ lb)(\pi)(6\ in.)^2}{\left(0.00238 \frac{slugs}{ft^3}\right)(1-\cos 60°)(4)\left(144\ \frac{in.^2}{ft^2}\right)\left(\frac{1\ lb}{slug\ \frac{ft}{s^2}}\right)}\right]^{\frac{1}{2}}$$

and

$$Q = \underline{\underline{28.7}}\ \frac{ft^3}{s}$$

5.50

5.50 A vertical, circular cross-sectional jet of air strikes a conical deflector as indicated in Fig. P5.50. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass (kg) of the deflector. The magnitude of velocity of the air remains constant.

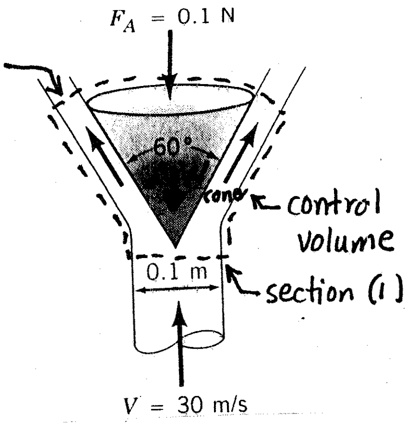


FIGURE P5.50

To determine the mass of the conical deflector we use the stationary, non-deforming control volume shown in the sketch above. Application of the vertical direction component of the linear momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\dot{m}\,(-V_1 + V_2 \cos 30°) = -F_A - W_{cone}$$

or

$$W_{cone} = m_{cone}\, g = \dot{m}\,(V_1 - V_2 \cos 30°) - F_A = \rho A_1 V_1 (V_1 - V_2 \cos 30°) - F_A \quad (1)$$

However

$$V_1 = V_2$$

and

$$A_1 = \frac{\pi D_1^2}{4}$$

Thus Eq. 1 can be expressed as

$$m_{cone} = \rho \frac{\pi D_1^2}{4g} V_1 (V_1 - V_1 \cos 30°) - \frac{F_A}{g}$$

or

$$m_{cone} = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi\,(0.1\,m)^2\left(30\,\frac{m}{s}\right)\left[30\,\frac{m}{s} - \left(30\,\frac{m}{s}\right)\cos 30°\right]}{(4)\left(9.81\,\frac{m}{s^2}\right)} - \frac{0.1\,N}{\left(9.81\,\frac{m}{s^2}\right)\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}$$

and

$$m_{cone} = \underline{\underline{0.108\ kg}}$$

5.51

5.51 Water flows from a large tank into a dish as shown in Fig. P5.51. **(a)** If at the instant shown the tank and the water in it weigh W_1 lb, what is the tension, T_1, in the cable supporting the tank? **(b)** If at the instant shown the dish and the water in it weigh W_2 lb, what is the force, F_2, needed to support the dish?

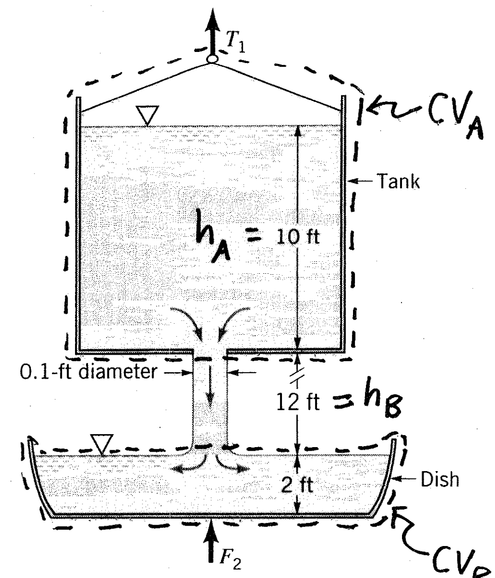


FIGURE P5.51

For part (a) we apply the vertical component of the linear momentum equation to the contents of control volume A, CV_A, to get

$$-V_{out}\,\rho V_{out} A_{out} = T_1 - W_1 \qquad (1)$$

To get value of V_{out} we apply Bernoulli's equation to the flow from the free surface of the water in the tank to the tank outlet to get

$$V_{out} = \sqrt{2gh_A} = \sqrt{(2)(32.2\ \tfrac{ft}{s^2})(10\ ft)} = 25.4\ \tfrac{ft}{s}$$

Then from Eq. (1) we get

$$\frac{-(25.4\ \frac{ft}{s})(1.94\ \frac{slugs}{ft^3})(25.4\ \frac{ft}{s})\ \frac{\pi\,(0.1\ ft)^2}{4}}{1\ \frac{slug \cdot ft}{lb \cdot s^2}} = T_1 - W_1$$

and

$$T_1 = \underline{\underline{W_1 - 9.8\ lb}}$$

For part (b) we apply the vertical component of the linear momentum equation to the contents of CV_B to get

$$V_{\substack{into\\CV_B}}\ \dot{m}_{\substack{into\\CV_B}} = F_2 - W_2 \qquad (2)$$

To get V_{into} CV_B we use Bernoulli's equation between free surface of water in tank to free surface of water in dish to get

$$V_{\substack{into\\CV_B}} = \sqrt{2g(h_A + h_B)} = \sqrt{2(32.2\ \tfrac{ft}{s^2})(10\ ft + 12\ ft)} = 37.6\ \tfrac{ft}{s}$$

For $\dot{m}_{\substack{into\\CV_B}}$ we use from conservation of mass, $\dot{m}_{\substack{into\\CV_B}} = \dot{m}_{\substack{out\ of\\tank}} = \rho V_{out} A_{out}$

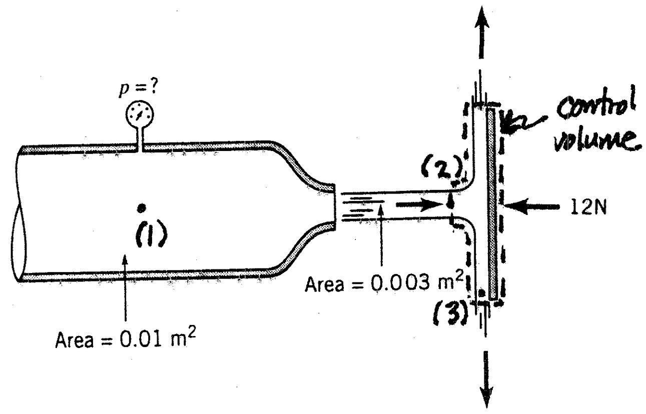
So from Eq. (2) we get

$$(37.6\ \tfrac{ft}{s})(1.94\ \tfrac{slug}{ft^3})(25.4\ \tfrac{ft}{s})\ \frac{\pi\,(0.1\ ft)^2}{4}\left(\frac{1\ lb \cdot s^2}{slug \cdot ft}\right) = F_2 - W_2$$

and $F_2 = \underline{\underline{W_2 + 14.7\ lb}}$

5.52

5.52 Air flows into the atmosphere from a nozzle and strikes a vertical plate as shown in Fig. P5.52. A horizontal force of 12 N is required to hold the plate in place. Determine the reading on the pressure gage. Assume the flow to be incompressible and frictionless.



■ FIGURE P5.52

To determine the static gage pressure at station (1) we first consider the frictionless and incompressible flow of air from (1) to (2). The Bernoulli equation for this flow is

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \overbrace{\cancel{\frac{P_2}{\rho}}}^{0 \text{ gage}} + \frac{V_2^2}{2} \qquad (1)$$

We note that V_1 and V_2 are linked by the continuity (conservation of mass) equation

$$Q_1 = Q_2 \quad \text{or} \quad A_1 V_1 = A_2 V_2 \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{P_1}{\rho} + \frac{\left(\frac{A_2}{A_1} V_2\right)^2}{2} = \frac{V_2^2}{2} \qquad (3)$$

To determine V_2 we use the linear momentum equation for the flow from (2) to (3). For the control volume sketched above the linear momentum principle yields

$$-V_2 \rho V_2 A_2 = -12\,N$$

or

$$V_2 = \sqrt{\frac{(12\,N)}{\rho A_2}} = \sqrt{\frac{12\,N}{\left(1.23\,\frac{kg}{m^3}\right)\left(1\,\frac{N \cdot s^2}{kg \cdot m}\right)\left(0.003\,m^2\right)}}$$

and

$$V_2 = 57\,\frac{m}{s}$$

(con't)

5.52 (con't)

Now, with Eq. 3

$$p_1 = \rho \left[\frac{V_2^2}{2} - \frac{\left(\frac{A_2}{A_1} V_2\right)^2}{2} \right]$$

or

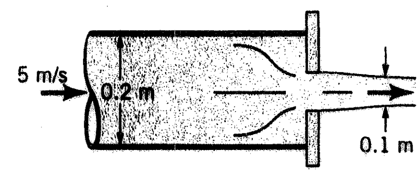
$$p_1 = \left(1.23 \frac{kg}{m^3}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right) \left\{ \frac{\left(57 \frac{m}{s}\right)^2}{2} - \frac{\left[\left(\frac{0.003 \, m^2}{0.01 \, m^2}\right)\left(57 \frac{m}{s}\right)\right]^2}{2} \right\}$$

and

$$p_1 = 1820 \frac{N}{m^2} = 1820 \, Pa = \underline{\underline{1.82}} \, kPa$$

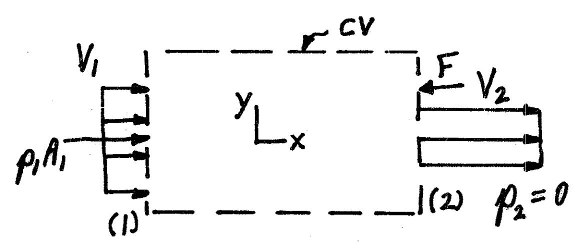
5.53

5.53 The exit plane of a 0.20-m-diameter pipe is partially blocked by a plate with a hole in it that produces a 0.10-m-diameter stream as shown in Fig. P5.53. The water velocity in the pipe is 5 m/s. Gravity and viscous effects are negligible. Determine the force needed to hold the plate against the pipe.



■ FIGURE P5.53

The x-component of the momentum equation for the control volume shown is



$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \sum F_x \text{ , or}$$

$$V_1 \rho(-V_1)A_1 + V_2 \rho V_2 A_2 = p_1 A_1 - F,$$

where F is the force to hold the plate.

Thus,

$$F = p_1 A_1 + \rho V_1^2 A_1 - \rho V_2^2 A_2 = p_1 A_1 + \dot{m}(V_1 - V_2) \qquad (1)$$

where

$$\dot{m} = \rho A_1 V_1 = 999 \frac{kg}{m^3}\left[\frac{\pi}{4}(0.2m)^2\right]\left(5\frac{m}{s}\right) = 157 \frac{kg}{s}$$

Also, $A_1 V_1 = A_2 V_2$ or $V_2 = (A_1/A_2)V_1 = (D_1/D_2)^2 V_1 = (0.2m/0.1m)^2(5m/s) = 20 m/s$

In addition, from the Bernoulli equation,

$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$ so that with $p_2 = 0$,

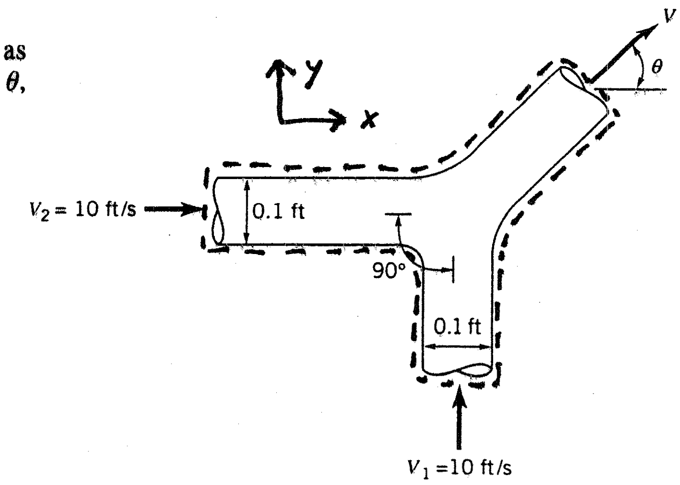
$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2}\left(999 \frac{kg}{m^3}\right)\left[\left(20\frac{m}{s}\right)^2 - \left(5\frac{m}{s}\right)^2\right] = 1.87 \times 10^5 \frac{N}{m^2}$$

Thus, from Eq. (1)

$$F = 1.87 \times 10^5 \frac{N}{m^2}\left[\frac{\pi}{4}(0.2m)^2\right] + 157 \frac{kg}{s}\left(5\frac{m}{s} - 20\frac{m}{s}\right) = \underline{\underline{3{,}520\,N}}$$

5.54

5.54 Two water jets of equal size and speed strike each other as shown in Fig. P5.54. Determine the speed, V, and direction, θ, of the resulting combined jet. Gravity is negligible.



■ FIGURE P5.54

For the control volume shown in the sketch above the linear momentum equation for the x and y directions are, for the x direction

$$-V_2 \rho V_2 A_2 + (V\cos\theta)\rho V A = 0 \qquad (1)$$

and for the y direction

$$-V_1 \rho V_1 A_1 + (V\sin\theta)\rho V A = 0 \qquad (2)$$

Also for conservation of mass we have

$$\rho_1 V_1 A_1 + \rho V_2 A_2 - \rho V A = 0 \qquad (3)$$

From Eqs. 1 and 2 we get

$$\frac{V_2^2 A_2}{V_1^2 A_1} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

so

$$\theta = \cot^{-1}\frac{V_2^2 A_2}{V_1^2 A_1} = \cot^{-1}\left[\frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}\right] = 45^\circ$$

Now, combining Eqs. 2 and 3 we get

$$-V_1^2 A_1 + V\sin\theta\,(V_1A_1 + V_2A_2) = 0$$

or

$$V = \frac{V_1^2 A_1}{\sin\theta\,(V_1A_1+V_2A_2)}$$

$$V = \frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}{(\sin 45^\circ)\left[\left(10\,\frac{ft}{s}\right)\frac{\pi (0.1\,ft)^2}{4} + \left(10\,\frac{ft}{s}\right)\frac{\pi (0.1\,ft)^2}{4}\right]}$$

and

$$V = \underline{\underline{7.07}}\ \frac{ft}{s}$$

5.55

5.55 Assuming frictionless, incompressible, one-dimensional flow of water through the horizontal tee connection sketched in Fig. P5.55, estimate values of the x and y components of the force exerted by the tee on the water. Each pipe has an inside diameter of 1 m.

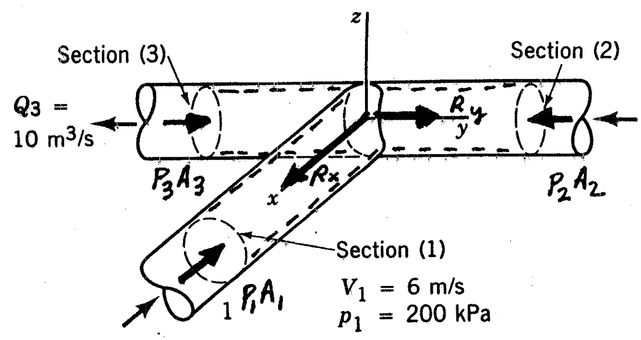


FIGURE P5.55

We can use the x and y components of the linear momentum equation (Eq. 5.22) to determine the x and y components of the reaction force exerted by the water on the tee. For the control volume containing water in the tee, Eq. 5.22 leads to

$$R_x = p_1 A_1 + V_1 \rho Q_1 = p_1 \frac{\pi D_1^2}{4} + V_1 \rho Q_1 \qquad (1)$$

and

$$R_y = p_2 \frac{\pi D_2^2}{4} - p_3 \frac{\pi D_3^2}{4} + V_2 \rho Q_2 - V_3 \rho Q_3 \qquad (2)$$

The reaction forces in Eqs. 1 and 2 are actually exerted by the tee on the water in the control volume. The reaction of the water on the tee is equal in magnitude but opposite in direction.

Conservation of mass (Eq. 5.4) leads to

$$Q_2 = Q_3 - Q_1 = Q_3 - V_1 \frac{\pi D_1^2}{4} = 10\,\frac{m^3}{s} - \left(6\,\frac{m}{s}\right)\frac{\pi (1\,m)^2}{4} = 5.288\,\frac{m^3}{s}$$

Also

$$Q_1 = V_1 \frac{\pi D_1^2}{4} = \left(6\,\frac{m}{s}\right)\frac{\pi (1\,m)^2}{4} = 4.712\,\frac{m^3}{s}$$

Further

$$V_2 = \frac{Q_2}{\frac{\pi D_2^2}{4}} = \frac{\left(5.288\,\frac{m^3}{s}\right)}{\frac{\pi (1\,m)^2}{4}} = 6.733\,\frac{m}{s}$$

and

$$V_3 = \frac{Q_3}{\frac{\pi D_3^2}{4}} = \frac{\left(10\,\frac{m^3}{s}\right)}{\frac{\pi (1\,m)^2}{4}} = 12.73\,\frac{m}{s}$$

(con't)

5.55 (con't)

Because the flow is incompressible and frictionless we assume that Bernoulli's equation (Eq. 5.74) is valid throughout the control volume. Thus

$$p_3 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_3^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(12.73\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_3 = 137\ kPa$$

Also

$$p_2 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(6.733\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_2 = 195.3\ kPa$$

With Eq. 1

$$R_x = \left(200{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(4.712\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) = 185{,}000\ N = 185\ kN$$

and the x-direction component of force exerted by the water on the tee is $\underline{\underline{-185\ kN}}$.

With Eq. 2

$$R_y = \left(195{,}300\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 - \left(137{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)5.2$$

$$+ \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(5.288\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)$$

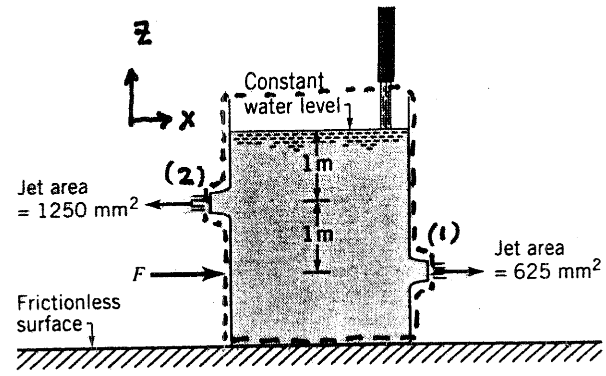
or

$$R_y = -45{,}800\ N = -45.8\ kN$$

and the y-direction component of force exerted by the water on the tee is $\underline{\underline{+45.8}}\ kN$.

5.56

5.56 Water is added to the tank shown in Fig. P5.56 through a vertical pipe to maintain a constant (water) level. The tank is placed on a horizontal plane which has a frictionless surface. Determine the horizontal force, F, required to hold the tank stationary. Neglect all losses.



■ FIGURE P5.56

Applying the x-direction component of the linear momentum equation to the contents of the control volume sketched above we get

$$V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F \qquad (1)$$

Using Bernoulli's equation to describe the frictionless flow from the constant water surface level to the flow leaving at stations (1) and (2) we obtain

$$V_2 = \sqrt{2 g h_2} \qquad (2)$$

and

$$V_1 = \sqrt{2 g h_1} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$F = 2 g h_1 \rho A_1 - 2 g h_2 \rho A_2$$

or

$$F = 2\left(9.81\,\frac{m}{s^2}\right)\left(999\,\frac{kg}{m^3}\right)\left[\frac{(2\,m)(625\,mm^2)}{\left(1000\,\frac{mm}{m}\right)^2} - \frac{(1\,m)(1250\,mm^2)}{\left(1000\,\frac{mm}{m}\right)^2}\right]$$

and

$$F = \underline{\underline{0\ N}}$$

5.57

5.57 An airplane moves forward with a steady airspeed of 250 km/hr. The wings of this plane deflect the air flow around each wing downward by an angle of 10°. Estimate the mass flowrate of the deflected air required over the wing surfaces if the mass of the plane and its contents is 3000 kg. Assume that air leaves the wing at the same speed it entered with and consider momentum change effects only.

This is similar to Example 5.16

To determine the mass flowrate of air required we use a moving control volume like the one of Example 5.16 and Eqs. 2 and 3 of Example 5.16 to get

$$\dot{m}_r = \frac{F_L}{w_{r,2}} = \frac{F_L}{u_{r,1}\,\sin 10^\circ}$$

or

$$\dot{m}_r = \frac{(3000\ \text{kg})(9.81\ \frac{\text{m}}{\text{s}^2})(3600\ \frac{\text{s}}{\text{hr}})}{250{,}000\ \frac{\text{m}}{\text{hr}}\ (\sin 10^\circ)} = \underline{\underline{2440\ \frac{\text{kg}}{\text{s}}}}$$

5.58

5.58 The four devices shown in Fig. P5.58 rest on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.

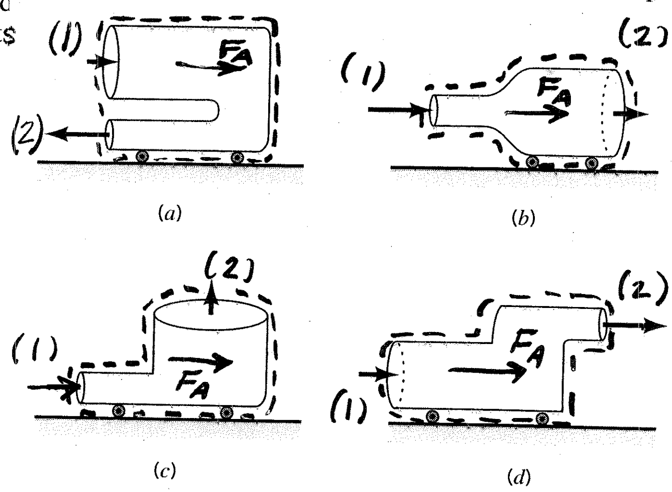


FIGURE P5.58

We apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force F_A. If F_A is in the direction shown in the sketches, motion will be to the left. If F_A is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$, there is no horizontal motion.

For sketch (a)

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F_A$$

Since F_A is to the left, motion is to the right.

For sketch (b)

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$, then F_A is to the left and motion is to the right.

For sketch (c) (note: flow is into CV at (1))

$$-V_1 \rho V_1 A_1 = F_A$$

and F_A is to the left so motion is to the right.

For sketch (d)

$$-V_1 \rho V_1 A + V_2 \rho V_2 A_2 = F_A$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and $V_1 < V_2$

so F_A is to the right and motion is to the left.

5.59

5.59 Water discharges into the atmosphere through the device shown in Fig. P5.59. Determine the x component of force at the flange required to hold the device in place. Neglect the effect of gravity and friction.

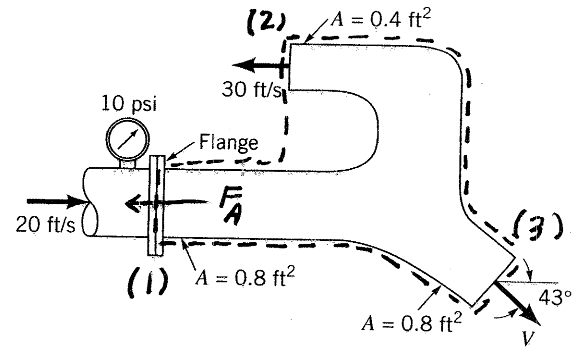


FIGURE P5.59

To calculate the x-direction anchoring force required to hold the device in place, the x-direction component of the linear momentum equation is used on the contents of the control volume shown in the sketch to obtain:

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 + V_3 \cos 43^\circ \rho V_3 A_3 = -F_A + p_1 A_1 \qquad (1)$$

To determine V_3, the conservation of mass equation is used to obtain:

$$Q_1 = Q_2 + Q_3$$

or

$$V_1 A_1 = V_2 A_2 + V_3 A_3$$

and $$\left(20\ \frac{ft}{s}\right)(0.8\ ft^2) = \left(30\ \frac{ft}{s}\right)(0.4\ ft^2) + V_3\,(0.8\ ft^2)$$

so $$V_3 = 5\ \frac{ft}{s}$$

Then from Eq. 1 we get

$$-\frac{\left(20\frac{ft}{s}\right)\left(1.94\frac{slugs}{ft^3}\right)\left(20\frac{ft}{s}\right)(0.8\ ft^2)}{\left(1\ \frac{slug\cdot ft}{lb\cdot s^2}\right)} - \frac{\left(30\frac{ft}{s}\right)\left(1.94\frac{slugs}{ft^3}\right)\left(30\frac{ft}{s}\right)(0.4\ ft^2)}{\left(1\ \frac{slug\cdot ft}{lb\cdot s^2}\right)}$$

$$+\frac{\left(5\frac{ft}{s}\right)(\cos 43^\circ)\left(1.94\frac{slugs}{ft^3}\right)\left(5\frac{ft}{s}\right)(0.8\ ft^2)}{\left(1\ \frac{slug\cdot ft}{lb\cdot s^2}\right)} = -F_A + \left(10\frac{lb}{in^2}\right)\left(144\frac{in^2}{ft^2}\right)(0.8\ ft^2)$$

or

$$F_A = \underline{\underline{2440\ lb}}$$ to the left as shown in the sketch

5.60

5.60 A vertical jet of water leaves a nozzle at a speed of 10 m/s and a diameter of 20 mm. It suspends a plate having a mass of 1.5 kg as indicated in Fig. P5.60. What is the vertical distance h?

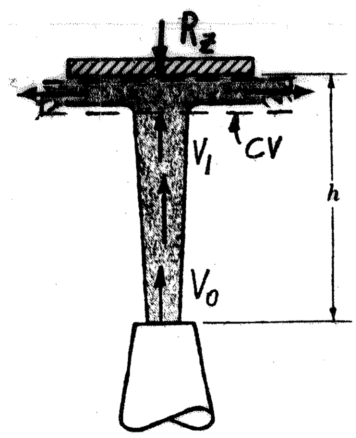


FIGURE P5.60

To determine the vertical distance h we apply the vertical direction component of the linear momentum equation (Eq. 5.22) to the water in the control volume shown in the sketch above. Thus,

$$-R_z - \rho g \forall_{water} = -V_1 \rho A_1 V_1 = -\rho V_1^2 \frac{\pi D_1^2}{4} \qquad (1)$$

The vertical reaction force of the plate on the water is equal in magnitude to the weight of the plate, or

$$R_z = g\, m_{plate} = (9.81 \tfrac{m}{s})(1.5\, kg) = 14.7\ N$$

Also, the weight of the water within the control volume, $\rho g \forall_{water}$, is negligible, and the mass flowrate is

$$\dot{m} = \rho A_1 V_1 = \rho A_0 V_0 = (999 \tfrac{kg}{m^3})\tfrac{\pi}{4}(0.02m)^2(10 \tfrac{m}{s}) = 3.13 \tfrac{kg}{s}$$

Thus, Eq 1 becomes

$$-14.7\ N = -V_1 \dot{m} \quad \text{or} \quad V_1 = \frac{14.7\ N}{3.13\ kg/s} = 4.70 \tfrac{m}{s}$$

From the Bernouli Equation (Eq. 3.7) we have

$$p_0 + \tfrac{1}{2}\rho V_0^2 + \gamma z_0 = p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 \text{, where } p_0 = p_1 = 0$$
$$z_0 = 0,\ z_1 = h$$

Thus,

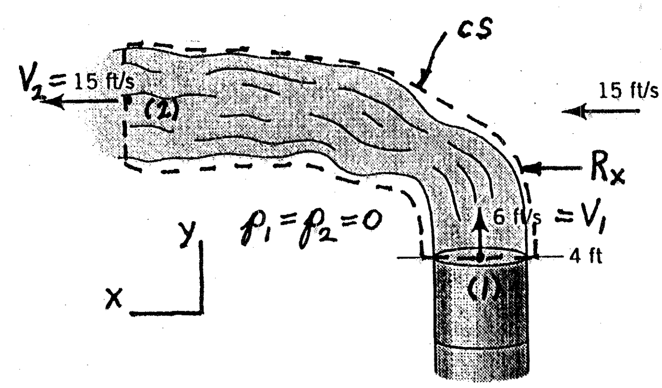
$$\tfrac{1}{2}\rho V_0^2 = \tfrac{1}{2}\rho V_1^2\ h$$

or since $\gamma = \rho g$

$$h = \frac{1}{2g}(V_0^2 - V_1^2) = \frac{1}{2(9.81 \frac{m}{s^2})}(10^2 - 4.70^2)\frac{m^2}{s^2} = \underline{\underline{3.97\ m}}$$

5.61

5.61 Exhaust (assumed to have the properties of standard air) leaves the 4-ft diameter chimney shown in Video V5.3 and Fig. P5.61 with a speed of 6 ft/s. Because of the wind, after a few diameters downstream the exhaust flows in a horizontal direction with the speed of the wind, 15 ft/s. Determine the horizontal component of the force that the blowing wind puts on the exhaust gases.



■ FIGURE P5.61

For the control volume indicated the x-component of the momentum equation

$$\int_{cs} u\,\rho\,\vec{V}\cdot\hat{n}\,dA = \Sigma F_x \text{ becomes}$$

$V_2\,\rho\,V_2\,A_2 = R_x$, where R_x is the net horizontal force that the wind puts on the exhaust gases.

Thus,

$$R_x = \dot{m}_2 V_2 \quad \text{where } \dot{m}_2 = \rho A_2 V_2 = \rho A_1 V_1 \quad (\text{i.e. } \dot{m}_1 = \dot{m}_2)$$

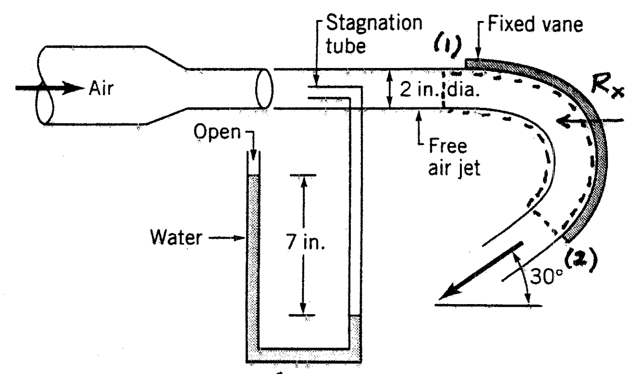
or $\dot{m}_2 = (0.00238\,\frac{slugs}{s})\left[\frac{\pi}{4}(4\,ft)^2\right](6\,\frac{ft}{s}) = 0.179\,\frac{slugs}{s}$

Hence,

$$R_x = 0.179\,\frac{slugs}{s}\,(15\,\frac{ft}{s}) = 2.69\,\frac{slug\cdot ft}{s^2} = \underline{\underline{2.69\,lb}}$$

5.62

5.62 Air discharges from a 2-in.-diameter nozzle and strikes a curved vane, which is in a vertical plane as shown in Fig. P5.62. A stagnation tube connected to a water U-tube manometer is located in the free air jet. Determine the horizontal component of the force that the air jet exerts on the vane. Neglect the weight of the air and all friction.



■ FIGURE P5.62

Note that we ignore the effect of atmospheric pressure on the value of R_x in our solution below and use gage pressures. As indicated in Example 5.10, the atmospheric pressure force may need consideration when identifying reaction forces. For the air flowing through the control volume sketched above, the x-direction component of the linear momentum equation is

$$-V_1 \rho_{air} V_1 A_1 - V_2 \cos 30^\circ \rho_{air} V_2 A_2 = -R_x \tag{1}$$

Application of Bernoulli's equation for the flow from (1) to (2) yields

$$V_2 = V_1 \tag{2}$$

Then, from the conservation of mass principle

$$A_1 V_1 = A_2 V_2 \tag{3}$$

We use the Bernoulli equation again to obtain the following equation for the stagnation tube deceleration

$$\frac{p_1}{\rho_{air}} + \frac{V_1^2}{2} = \frac{p_{stag}}{\rho_{air}} \tag{4}$$

For the manometer, we obtain with the equation of hydrostatics

$$p_{atm} + h_{mano}\,\gamma_{water} - h_{mano}\,\gamma_{air} = p_{stag} \tag{5}$$

With $p_1 = p_{atm}$, we get by combining Eqs. 4 and 5

$$V_1 = \sqrt{2h_{mano}\left(\frac{\gamma_{water}}{\rho_{air}}\right)} \tag{6}$$

(con't)

5.62 (con't)

Combining Eqs. 1,2,3 and 6 we obtain

$$R_x = 2 h_{mano} \left(\frac{\gamma_{water}}{\rho_{air}} \right) \rho_{air} \frac{\pi d_1^2}{4} (1 + \cos 30°)$$

or

$$R_x = 2 \frac{(7\,in.)}{\left(12 \frac{in.}{ft}\right)} \left(62.4 \frac{lb}{ft^3}\right) \frac{\pi\,(2\,in.)^2 (1 + \cos 30°)}{\left(12 \frac{in.}{ft}\right)^2 4}$$

and

$$R_x = \underline{\underline{2.96\ lb}}$$

This is the force exerted by the vane on the flowing air. The force exerted by the flowing air exerts on the vane is equal in magnitude but opposite in direction *(to the right)*

5.65

5.65 A 3-in.-diameter horizontal jet of water strikes a flat plate as indicated in Fig. P5.65. Determine the jet velocity if a 10-lb horizontal force is required to (**a**) hold the plate stationary, (**b**) allow the plate to move at a constant speed of 10 ft/s to the right.

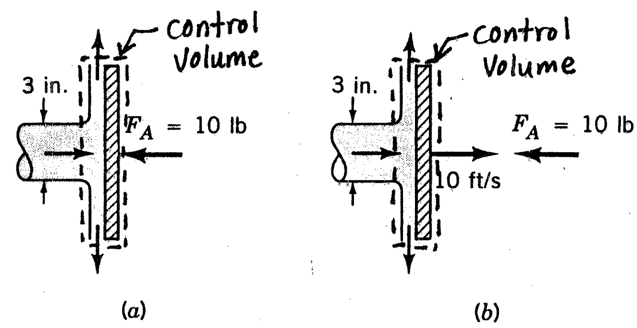


FIGURE P5.65

The control volume shown in the sketch is used. The stationary plate case is considered first. Application of the horizontal or x-direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 = -F_{A,x}$$

or

$$u_1 = \left(\frac{F_{A,x}}{\rho A_1}\right)^{\frac{1}{2}} = \left(\frac{F_{A,x}}{\rho \frac{\pi D_1^2}{4}}\right)^{\frac{1}{2}}$$

Thus

$$u_1 = \left[\frac{(10\text{ lb})}{\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{\pi (3\text{ in.})^2}{4\left(12 \frac{\text{in.}}{\text{ft}}\right)^2} \left(\frac{1\text{ lb}}{\text{slug}\frac{\text{ft}}{\text{s}^2}}\right)}\right]^{\frac{1}{2}}$$

and $u_1 = \underline{\underline{10.2 \frac{\text{ft}}{\text{s}}}}$ stationary plate

When the plate moves to the right with a speed, $\mathcal{U} = 10 \frac{\text{ft}}{\text{s}}$, the x-direction component of the linear momentum equation yields

$$-(u_1 - \mathcal{U})\rho(u_1 - \mathcal{U})A_1 = -F_{A,x}$$

or

$$u_1 - \mathcal{U} = \left(\frac{F_{A,x}}{\rho A_1}\right)^{\frac{1}{2}} = \left(\frac{F_{A,x}}{\rho \frac{\pi D_1^2}{4}}\right)^{\frac{1}{2}}$$

and

$$u_1 = \left(\frac{F_{A,x}}{\rho \frac{\pi D_1^2}{4}}\right)^{\frac{1}{2}} + \mathcal{U} = 10.2 \frac{\text{ft}}{\text{s}} + 10 \frac{\text{ft}}{\text{s}} = \underline{\underline{20.2 \frac{\text{ft}}{\text{s}}}}$$

moving plate

5.66

5.66 A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.66 and Video V5.4. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the x direction component of anchoring force required to (**a**) hold the vane stationary, (**b**) confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.

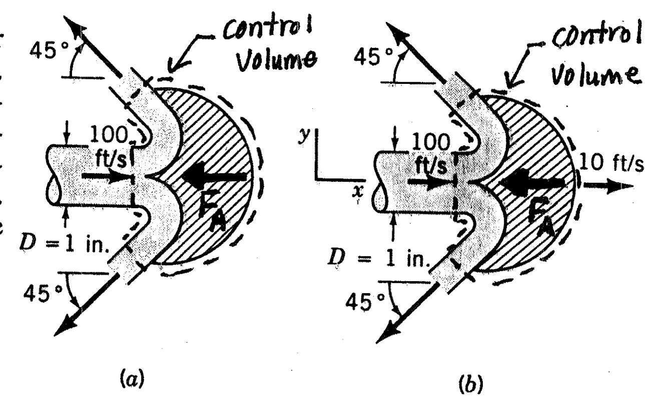


FIGURE P5.66

(a) To determine the x-direction component of anchoring force required to hold the vane stationary we use the stationary control volume shown above and the x-direction component of the linear momentum equation (Eq. 5.22). Thus,

$$F_A = \dot{m}(V_1 + V_2 \cos 45°) = \rho A_1 V_1 (V_1 + V_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} V_1 (V_1 + V_2 \cos 45°)$$

or

$$F_A = \frac{\left(1.94 \frac{slugs}{ft^3}\right) \pi (1 \text{ in.})^2 \left(100 \frac{ft}{s}\right)}{(4)\left(12 \frac{in.}{ft}\right)^2} \left[\left(100 \frac{ft}{s}\right) + \left(100 \frac{ft}{s}\right)\cos 45°\right] \left(\frac{1 \text{ lb}}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$F_A = \underline{\underline{181 \text{ lb}}}$$

(b) To determine the x-direction component of anchoring force required to confine the vane to a constant speed of $10 \frac{ft}{s}$ to the right we use a control volume moving to the right with a speed of $10 \frac{ft}{s}$ and the x-direction component of the linear momentum equation for a translating control volume (Eq. 5.29). Thus,

$$F_A = \rho A_1 W_1 (W_1 + W_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} W_1 (W_1 + W_2 \cos 45°) \qquad (1)$$

We note that

$$W_1 = V_1 - 10 \frac{ft}{s} = 100 \frac{ft}{s} - 10 \frac{ft}{s} = 90 \frac{ft}{s}$$

Thus, Eq. 1 leads to

$$F_A = \left(1.94 \frac{slugs}{ft^3}\right) \frac{\pi (1 \text{ in.})^2}{4 \left(12 \frac{in.}{ft}\right)^2} \left(90 \frac{ft}{s}\right)\left[90 \frac{ft}{s} + \left(90 \frac{ft}{s}\right)\cos 45°\right]\left(\frac{1 \text{ lb}}{slug \cdot \frac{ft}{s^2}}\right)$$

or

$$F_A = \underline{\underline{146 \text{ lb}}}$$

5.67

5.67 How much power is transferred to the moving vane of Problem 5.66?

Power $= F_A V$, where from Problem 5.66 $F_A = 146\,lb$

Thus,

$$\text{Power} = \frac{(146\,lb)(10\,\frac{ft}{s})}{\left(550\,\frac{ft \cdot lb}{s \cdot hp}\right)} = \underline{\underline{2.65\,hp}}$$

5.68

5.68 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.68. The exit cross section area of each of the two nozzles is 0.04 in.^{2} and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.

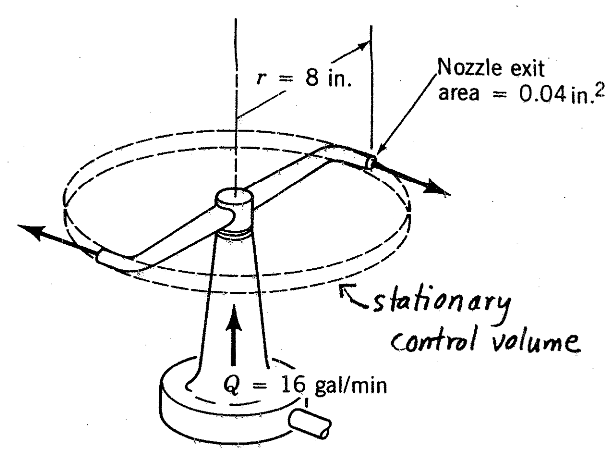


FIGURE P5.68

This is similar to Example 5.17.

(a) To determine the resisting torque required to hold the sprinkler head stationary we use the moment-of-momentum torque equation (Eq. 5.50). Thus,

$$T_{shaft} = \dot{m} r_2 V_{\theta 2} = \rho Q r_2 V_{\theta,2} \qquad (1)$$

For $V_{\theta,2}$ we use

$$V_{\theta 2} = \frac{Q}{2 A_{\substack{nozzle\\exit}}} = \frac{\left(16 \frac{gal}{min}\right)\left(144 \frac{in.^2}{ft^2}\right)}{2\left(0.04\ in.^2\right)\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)}$$

or

$$V_{\theta,2} = 64.17 \frac{ft}{s}$$

With Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94 \frac{slugs}{ft^3}\right)\left(16 \frac{gal}{min}\right)\left(8\ in.\right)\left(64.17 \frac{ft}{s}\right)\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}{\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\left(12 \frac{in.}{ft}\right)}$$

and

$$T_{shaft} = \underline{\underline{2.96\ ft \cdot lb}}$$

(b) To determine the resisting torque associated with a sprinkler speed of 500 $\frac{rev}{min}$ we use Eq. 1 again. However, with rotation we have

$$V_{\theta,2} = W_2 - U_2 \qquad (2)$$

For W_2 we use

$$W_2 = \frac{Q}{2 A_{\substack{nozzle\\exit}}} = \frac{\left(16 \frac{gal}{min}\right)\left(144 \frac{in.^2}{ft^2}\right)}{(2)\left(0.04\ in.^2\right)\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)} = 64.17 \frac{ft}{s}$$

(con't)

5.68 (con't)

For U_2 we use

$$U_2 = r_2\omega = \frac{(8 \text{ in.})\left(500 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)}{\left(12 \frac{in.}{ft}\right)\left(60 \frac{s}{min}\right)} = 34.91 \frac{ft}{s}$$

Thus with Eq. 2 we have

$$V_{\theta,2} = 64.17 \frac{ft}{s} - 34.91 \frac{ft}{s} = 29.26 \frac{ft}{s}$$

and with Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94 \frac{slugs}{ft^3}\right)\left(16 \frac{gal}{min}\right)(8 \text{ in.})\left(29.26 \frac{ft}{s}\right)}{\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\left(12 \frac{in.}{ft}\right)}\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$T_{shaft} = \underline{\underline{1.35 \text{ ft} \cdot \text{lb}}}$$

(c) To determine the angular velocity of the sprinkler if no resisting torque is applied we use the combination of Eqs. 1 and 2 to obtain

$$U_2 = W_2$$

or

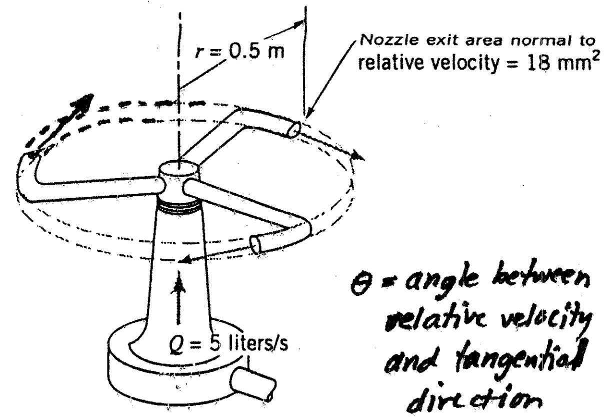
$$\omega = \frac{W_2}{r_2} = \frac{\left(64.17 \frac{ft}{s}\right)\left(12 \frac{in.}{ft}\right)}{(8 \text{ in.})} = 96.3 \frac{rad}{s}$$

The rotor speed, N, is thus

$$N = \left(96.3 \frac{rad}{s}\right)\frac{\left(60 \frac{s}{min}\right)}{\left(2\pi \frac{rad}{rev}\right)} = \underline{\underline{920 \frac{rev}{min}}}$$

5.69

5.69 Five liters/s of water enters the rotor shown in Video V5.5 and Fig. P5.69 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm². How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = 0°$, (b) $\theta = 30°$, (c) $\theta = 60°$?



θ = angle between relative velocity and tangential direction

FIGURE P5.69

To determine the torque required to hold the rotor stationary we use the moment-of-momentum torque equation (Eq. 5.50) to obtain

$$T_{shaft} = \dot{m}\, r_{out} V_{out} \cos\theta \qquad (1)$$

We note that

$$\dot{m} = \rho Q \qquad (2)$$

and

$$V_{out} = \frac{Q}{3 A_{\substack{nozzle\\exit}}} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$T_{shaft} = \frac{\rho Q^2 r_{out} \cos\theta}{3 A_{\substack{nozzle\\exit}}} \qquad (4)$$

To determine the rotor angular velocity associated with zero shaft torque we again use the moment-of-momentum torque equation (Eq. 5.50) to obtain, this time with rotation,

$$T_{shaft} = \dot{m}\, r_{out} (W_{out} \cos\theta - U_{out}) \qquad (5)$$

We note that

$$U_{out} = r_{out}\,\omega \qquad (6)$$

and

$$W_{out} = \frac{Q}{3 A_{\substack{nozzle\\exit}}} \qquad (7)$$

(con't)

5.69 (con't)

Combining Eqs. 2,5,6 and 7 we get

$$T_{shaft} = \rho Q r_{out}\left(\frac{Q}{3A_{\substack{nozzle\\exit}}}\cos\theta - r_{out}\,\omega\right) \qquad (8)$$

(a) For $\theta = 0°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(5\,\frac{liters}{s}\right)^2(0.5\,m)(\cos 0°)\left(1000\,\frac{mm}{m}\right)^2\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{\left(1000\,\frac{liters}{m^3}\right)^2(3)(18\,mm^2)}$$

or

$$T_{shaft} = \underline{\underline{231\ N\cdot m}}$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{Q\cos\theta}{3A_{\substack{nozzle\\exit}}\,r_{out}} = \frac{\left(5\,\frac{liters}{s}\right)(\cos 0°)\left(1000\,\frac{mm}{m}\right)^2}{3(18\,mm^2)\left(1000\,\frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{185\,\frac{rad}{s}}}$$

(b) For $\theta = 30°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(5\,\frac{liters}{s}\right)^2(0.5\,m)(\cos 30°)\left(1000\,\frac{mm}{m}\right)^2\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{\left(1000\,\frac{liters}{m^3}\right)^2(3)(18\,mm^2)}$$

or

$$T_{shaft} = 200\ N\cdot m$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5\,\frac{liters}{s}\right)(\cos 30°)\left(1000\,\frac{mm}{m}\right)^2}{3(18\,mm^2)\left(1000\,\frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{160}}\,\frac{rad}{s}$$

(c) For $\theta = 60°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(5\,\frac{liters}{s}\right)^2(0.5\,m)(\cos 60°)\left(1000\,\frac{mm}{m}\right)^2\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{\left(1000\,\frac{liters}{m^3}\right)^2(3)(18\,mm^2)}$$

or

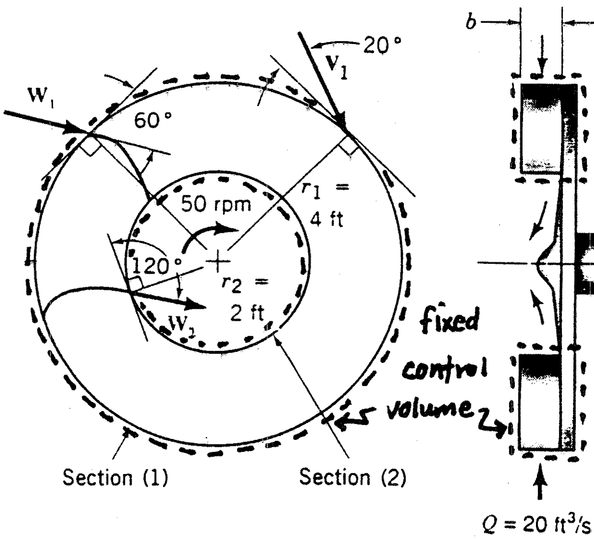
$$T_{shaft} = \underline{\underline{116\ N\cdot m}}$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5\,\frac{liters}{s}\right)(\cos 60°)\left(1000\,\frac{mm}{m}\right)^2}{(3)(18\,mm^2)\left(1000\,\frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{92.5\,\frac{rad}{s}}}$$

5.71

5.71 A water turbine wheel rotates at the rate of 50 rpm in the direction shown in Fig. P5.71. The inner radius, r_2, of the blade row is 2 ft, and the outer radius, r_1, is 4 ft. The absolute velocity vector at the turbine rotor entrance makes an angle of 20° with the tangential direction. The inlet blade angle is 60° relative to the tangential direction. The blade outlet angle is 120°. The flowrate is 20 ft³/s. For the flow tangent to the rotor blade surface at inlet and outlet, determine an appropriate constant blade height, b, and the corresponding power available at the rotor shaft.



■ FIGURE P5.71

since

$$Q = 2\pi r_1 b V_{R,1}$$

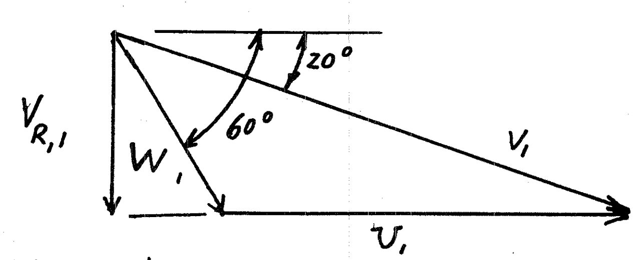
then the blade height, b, is determined with

$$b = \frac{Q}{2\pi r_1 V_{R,1}} \qquad (1)$$

The shaft power, $\dot{W}_{\substack{shaft\\ net\ out}}$, is obtained with the moment-of-momentum power equation (Eq. 5.53). Thus,

$$\dot{W}_{\substack{shaft\\ net\ out}} = \dot{m}\left(U_1 V_{\theta,1} \pm U_2 V_{\theta,2}\right) = \rho Q\left(U_1 V_{\theta,1} \pm U_2 V_{\theta,2}\right) \qquad (2)$$

and the use of "+" or "−" with $U_2 V_{\theta,2}$ depends on whether $V_{\theta,2}$ is opposite to or in the same direction as U_2 respectively. To determine the value of $V_{R,1}$ we use the velocity triangle at section (1). Thus, we have



With the velocity triangle we have

$$\frac{V_{R,1}}{\tan 20^\circ} = \frac{V_{R,1}}{\tan 60^\circ} + U_1 \qquad (3)$$

However

$$U_1 = r_1 \omega$$

(con't)

5.71 (con't)

thus Eq. 3 leads to

$$V_{R,1} = \frac{r_1\omega}{\left(\frac{1}{\tan 20°} - \frac{1}{\tan 60°}\right)} = \frac{(4\,ft)(50\,rpm)(2\pi\,\frac{rad}{rev})}{\left(\frac{1}{\tan 20°} - \frac{1}{\tan 60°}\right)(60\,\frac{s}{min})} = 9.651\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$b = \frac{(20\,\frac{ft^3}{s})}{2\pi(4ft)(9.651\,\frac{ft}{s})} = 0.0825\ ft$$

For the blade velocities in Eq. 2 we get

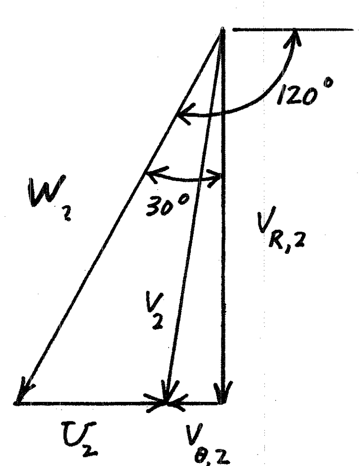
$$U_1 = r_1\omega = \frac{(4\,ft)(50\,rpm)(2\pi\,\frac{rad}{rev})}{(60\,\frac{s}{min})} = 20.94\,\frac{ft}{s}$$

$$U_2 = r_2\omega = \frac{(2ft)(50rpm)(2\pi\,\frac{rad}{rev})}{60\,\frac{s}{min}} = 10.47\,\frac{ft}{s}$$

For $V_{\theta,1}$ we use the velocity triangle at section (1) to obtain

$$V_{\theta,1} = \frac{V_{R,1}}{\tan 20°} = \frac{9.651\,\frac{m}{s}}{\tan 20°} = 26.52\,\frac{ft}{s}$$

For $V_{\theta,2}$ we construct the section (2) velocity triangle sketched below ($V_{\theta,2}$ not to scale)



and we realize that

$$V_{\theta,2} = V_{R,2}\tan 30° - U_2 \qquad (4)$$

From conservation of mass

$$V_{R,2} = V_{R,1}\frac{A_1}{A_2} = V_{R,1}\left(\frac{r_1}{r_2}\right) = \left(9.651\,\frac{ft}{s}\right)\left(\frac{4\,ft}{2\,ft}\right) = 19.3\,\frac{ft}{s}$$

(con't)

5.71 (con't)

so with Eq. 4 we obtain

$$V_{\theta,2} = \left(19.3\ \frac{ft}{s}\right)\tan 30° - 10.47\ \frac{ft}{s} = 0.673\ \frac{ft}{s}$$

Finally with Eq. 2 we obtain

$$\dot{W}_{\substack{shaft\\net\ out}} = \left(1.94\ \frac{slugs}{ft^3}\right)\left(20\ \frac{ft^3}{s}\right)\left[\left(20.94\ \frac{ft}{s}\right)\left(26.52\ \frac{ft}{s}\right)+\left(10.47\ \frac{ft}{s}\right)\left(0.673\ \frac{ft}{s}\right)\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$\dot{W}_{\substack{shaft\\net\ out}} = 2.18\times10^4\ \frac{ft\cdot lb}{s}$$

and

$$\dot{W}_{\substack{shaft\\net\ out}} = \frac{2.18\times10^4\ \frac{ft\cdot lb}{s}}{550\ \frac{ft\cdot lb}{s\cdot hp}} = \underline{\underline{39.6\ hp}}$$

5.72

5.72 An incompressible fluid flows outward through a blower as indicated in Fig. P5.72. The shaft torque involved, T_{shaft}, is estimated with the following relationship:

$$T_{shaft} = \dot{m} r_2 V_{\theta,2}$$

where $\dot{m}$ = mass flowrate through the blower, r_2 = outer radius of blower, and $V_{\theta,2}$ = tangential component of absolute fluid velocity leaving the blower. State the flow conditions that make this formula valid.

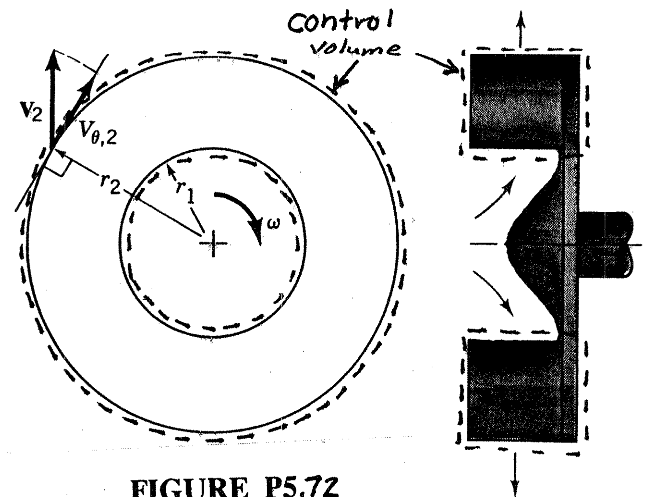


FIGURE P5.72

The flow conditions that make

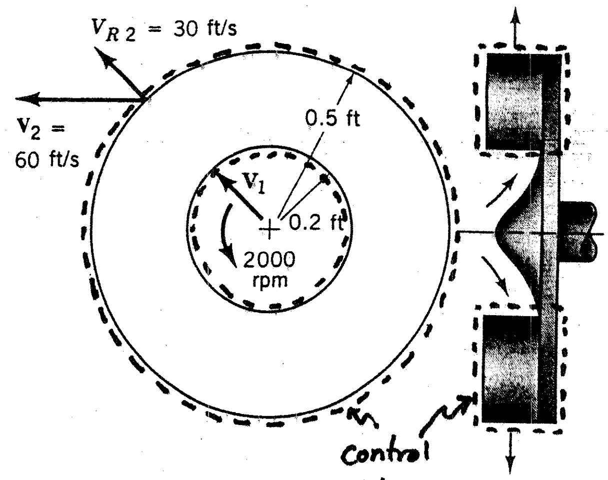
$$T_{shaft} = \dot{m} r_2 V_{\theta,2} \qquad (1)$$

valid may be identified by comparing Eq. 1 with the axial component of Eq. 5.42. These conditions are

a. stationary and non-deforming control volume (see sketch above)
b. steady-in-the-mean flow
c. negligible shear stress torque with respect to axis of rotation
d. $V_{\theta,1} = 0$
e. no torque with respect to axis of rotation due to normal stresses
f. uniform distribution of $V_{\theta,2}$

5.73

5.73 The radial component of velocity of water leaving the centrifugal pump sketched in Fig. P5.73 is 30 ft/s. The magnitude of the absolute velocity at the pump exit is 60 ft/s. The fluid enters the pump rotor radially. Calculate the shaft work required per unit mass flowing through the pump.



■ FIGURE P5.73

The stationary and non-deforming control volume shown in the sketch above is used. To determine the shaft work per unit mass, w_{shaft}, we can use Eq. 5.54. Thus

$$w_{shaft} = U_2 V_{\theta,2} \qquad (1)$$

The blade speed, U_2, can be obtained as follows,

$$U_2 = r_2 \omega = (0.5\ ft)\left(2000\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)\left(\frac{1\ min}{60\ s}\right) = 105\ \frac{ft}{s}$$

The tangential velocity, $V_{\theta,2}$, can be obtained as follows,

$$V_{\theta,2} = \left(V_2^2 - V_{R,2}^2\right)^{\frac{1}{2}} = \left[\left(60\ \frac{ft}{s}\right)^2 - \left(30\ \frac{ft}{s}\right)^2\right]^{\frac{1}{2}} = 52\ \frac{ft}{s}$$

Thus, from Eq. 1

$$w_{shaft} = \left(105\ \frac{ft}{s}\right)\left(52\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) = \underline{\underline{5460}}\ \frac{ft\cdot lb}{slug}$$

5.74

5.74 A fan (see Fig. P5.74) has a bladed rotor of 12-in.-outside diameter and 5-in.-inside diameter and runs at 1725 rpm. The width of each rotor blade is 1 in. from blade inlet to outlet. The volume flowrate is steady at 230 ft³/min and the absolute velocity of the air at blade inlet, V_1, is purely radial. The blade discharge angle is 30° measured with respect to the tangential direction at the outside diameter of the rotor. **(a)** What would be a reasonable blade inlet angle (measured with respect to the tangential direction at the inside diameter of the rotor)? **(b)** Find the power required to run the fan.

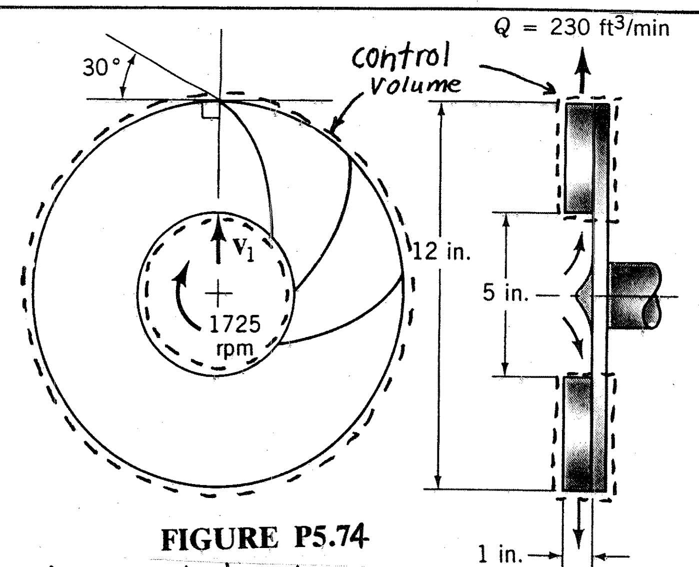
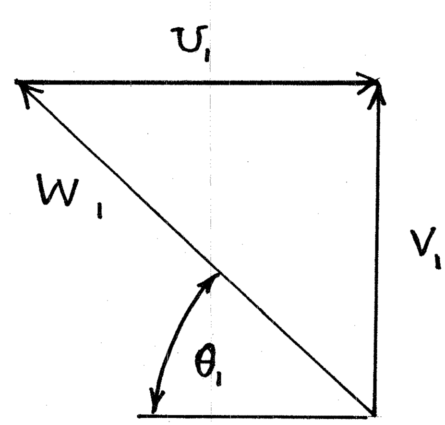


FIGURE P5.74

The stationary and non-deforming control volume shown in the sketch above is used. To determine a reasonable blade inlet angle we assume that the blade should be tangent to the relative velocity at the inlet. The inlet velocity triangle is sketched below.



With the velocity triangle, we conclude that

$$\theta_1 = \tan^{-1}\left(\frac{V_1}{U_1}\right) \quad (1)$$

Now

$$V_1 = \frac{Q}{A_1} = \frac{Q}{2\pi r_1 h_1} = \frac{\left(230\ \frac{ft^3}{min}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{2\pi\,(2.5\ in.)(1\ in.)\left(60\ \frac{s}{min}\right)} = 35.1\ \frac{ft}{s}$$

and

$$U_1 = r_1\omega = \frac{(2.5\ in.)\left(1725\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)}{\left(12\ \frac{in.}{ft}\right)\left(60\ \frac{s}{min}\right)} = 37.6\ \frac{ft}{s}$$

Thus with Eq. 1

$$\theta_1 = \tan^{-1}\left[\frac{\left(35.1\ \frac{ft}{s}\right)}{\left(37.6\ \frac{ft}{s}\right)}\right] = \underline{\underline{43^\circ}}$$

(con't)

5.74 (con't)

The power required, $\dot{W}_{shaft}$, may be obtained with Eq. 5.53. Thus

$$\dot{W}_{shaft} = \dot{m}_2 U_2 V_{\theta,2} \qquad (2)$$

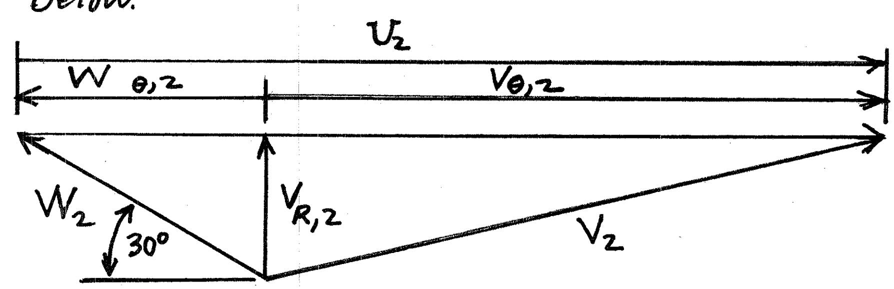
The mass flowrate, $\dot{m}_2$, may be obtained as follows.

$$\dot{m}_2 = \rho Q = \left(2.38 \times 10^{-3}\ \frac{slugs}{ft^3}\right)\left(230\ \frac{ft^3}{min}\right)\left(\frac{1}{60\ \frac{s}{min}}\right) = 9.12 \times 10^{-3}\ \frac{slug}{s}$$

Also

$$U_2 = r_2 \omega = \frac{(6\ in.)\left(1725\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)}{\left(12\ \frac{in.}{ft}\right)\left(60\ \frac{s}{min}\right)} = 90.3\ \frac{ft}{s}$$

The value of $V_{\theta,2}$ may be obtained by considering the velocity triangle for the flow leaving the rotor at section (2). The relative velocity at the rotor exit is considered to be tangent to the blade there. The rotor exit flow velocity triangle is sketched below.



Now

$$V_{\theta,2} = U_2 - W_{\theta,2}$$

and

$$W_{\theta,2} = \frac{V_{R,2}}{\tan 30°} = \frac{\frac{Q}{2\pi r_2 h_2}}{\tan 30°} = \frac{\frac{\left(230\ \frac{ft^3}{min}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{2\pi(6\ in.)(1\ in.)\left(60\ \frac{s}{min}\right)}}{\tan 30°} = 25.4\ \frac{ft}{s}$$

Thus

$$V_{\theta,2} = 90.3\ \frac{ft}{s} - 25.4\ \frac{ft}{s} = 64.9\ \frac{ft}{s}$$

and from Eq. 2

$$\dot{W}_{shaft} = \left(9.12 \times 10^{-3}\ \frac{slug}{s}\right)\left(90.3\ \frac{ft}{s}\right)\left(64.9\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{53.4}}\ \frac{ft \cdot lb}{s}$$

5.75

5.75 An axial flow gasoline pump (see Fig. P5.75) consists of a rotating row of blades (rotor) followed downstream by a stationary row of blades (stator). The gasoline enters the rotor axially (without any angular momentum) with an absolute velocity of 3 m/s. The rotor blade inlet and exit angles are 60° and 45° from the axial direction. The pump annulus passage cross section area is constant. Consider the flow as being tangent to the blades involved. Sketch velocity triangles for flow just upstream and downstream of the rotor and just downstream of the stator where the flow is axial. How much energy is added to each kilogram of gasoline?

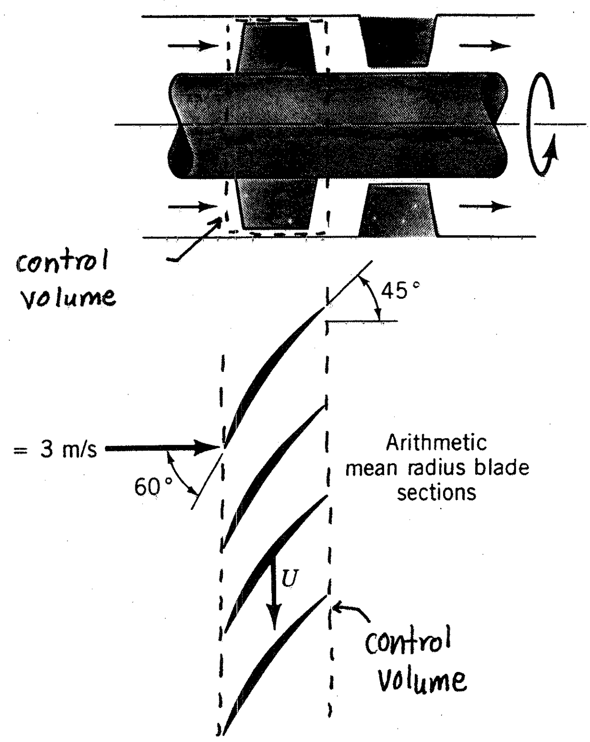
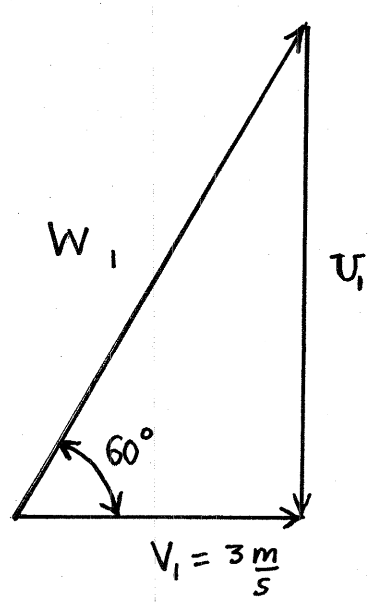


FIGURE P5.75

The velocity triangle for flow just upstream of the rotor is sketched below for the arithmetic mean radius.



With the triangle we conclude that

$$W_1 = \frac{V_1}{\cos 60^\circ} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 60^\circ} = 6\ \frac{m}{s}$$

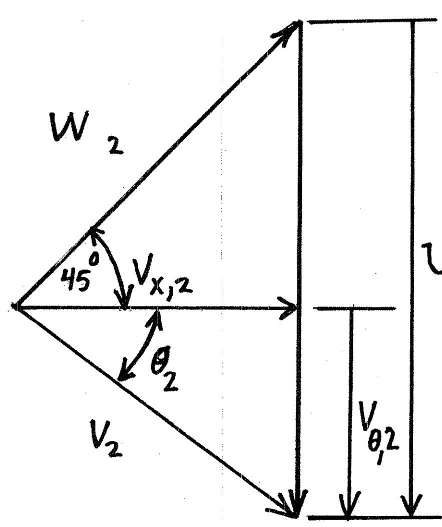
and

$$U_1 = W_1 \sin 60^\circ = \left(6\ \frac{m}{s}\right) \sin 60^\circ = 5.2\ \frac{m}{s}$$

(con't)

5.75 (con't)

The velocity triangle for flow just downstream of the rotor is sketched below for the arithmetic mean radius. For incompressible flow $V_{x,2} = V_1$. For mean radius flow $U_2 = U_1$. Thus for relative flow tangent to the blade we obtain the velocity triangle sketched below.



With the triangle we conclude that

$$V_{\theta,2} = U_2 - W_{\theta,2} = U_2 - V_{x,2}\tan 45^\circ = 5.2\ \frac{m}{s} - \left(3\ \frac{m}{s}\right)\tan 45^\circ = 2.2\ \frac{m}{s}$$

Also

$$\theta_2 = \tan^{-1}\left(\frac{V_{\theta,2}}{V_{x,2}}\right) = \tan^{-1}\left[\frac{\left(2.2\ \frac{m}{s}\right)}{\left(3\ \frac{m}{s}\right)}\right] = 36.2^\circ$$

$$W_2 = \frac{V_{x,2}}{\cos 45^\circ} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 45^\circ} = 4.24\ \frac{m}{s}$$

$$V_2 = \frac{V_{x,2}}{\cos\theta_2} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 36.2^\circ} = 3.72\ \frac{m}{s}$$

Using the stationary and non-deforming control volume shown above in the first sketch of this solution and Eq. 5.54 we can calculate the energy added to each kg of gasoline

$$w_{shaft} = U_2 V_{\theta,2} = \left(5.2\ \frac{m}{s}\right)\left(2.2\ \frac{m}{s}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) = \underline{\underline{11.4}}\ \frac{N\cdot m}{kg}$$

5.76

5.76 A sketch of the arithmetic mean radius blade sections of an axial-flow water turbine stage is shown in Fig. P5.76. The rotor speed is 1000 rpm. (a) Sketch and label velocity triangles for the flow entering and leaving the rotor row. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. Assume flow enters and leaves each blade row at the blade angles shown. (b) Calculate the work per unit mass delivered at the shaft.

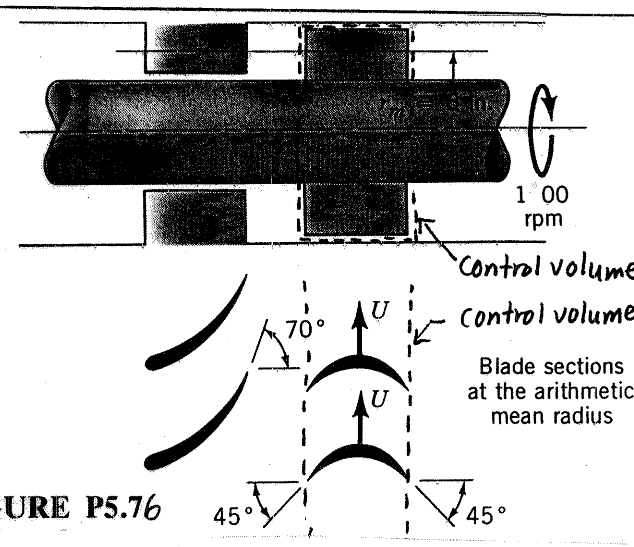
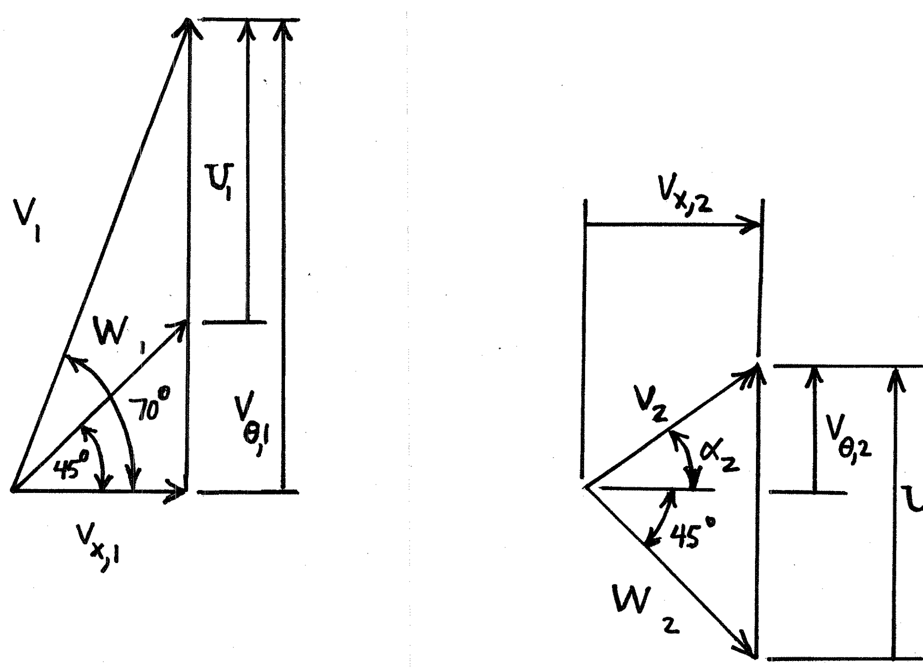


FIGURE P5.76

The velocity triangles for the flow entering and the flow leaving the rotor row at the arithmetic mean radius are sketched below.



At the arithmetic mean radius, the blade velocity, U, is

$$U_1 = U_2 = r_m \omega = \frac{(6\ in.)\ 1000\ \frac{rev}{min}\ \left(2\pi\ \frac{rad}{rev}\right)}{\left(12\ \frac{in.}{ft}\right)\left(60\ \frac{s}{min}\right)} = 52.3\ \frac{ft}{s}$$

With the velocity triangle for the flow entering the rotor we conclude that

$$V_1 \sin 70° = V_{\theta,1} \qquad (1)$$

$$V_1 \cos 70° = V_{x,1} \qquad (2)$$

$$W_1 \sin 45° = V_{\theta,1} - U \qquad (3)$$

$$W_1 \cos 45° = V_{x,1} \qquad (4)$$

(con't)

5.76 (con't)

From the ratio of Eqs. 3 and 4 we obtain

$$\tan 45^\circ = \frac{V_{\theta,1} - U}{V_{x,1}}$$

which when combined with Eqs. 1 and 2 yields

$$\tan 45^\circ = \frac{V_1 \sin 70^\circ - U}{V_1 \cos 70^\circ}$$

or

$$V_1 = \frac{U}{[\sin 70^\circ - (\cos 70^\circ)(\tan 45^\circ)]} = \frac{52.3 \frac{ft}{s}}{[\sin 70^\circ - (\cos 70^\circ)(\tan 45^\circ)]}$$

$$V_1 = 87.6 \frac{ft}{s}$$

Then

$$V_{\theta 1} = V_1 \sin 70^\circ = \left(87.6 \frac{ft}{s}\right) \sin 70^\circ = 82.3 \frac{ft}{s}$$

$$V_{x,1} = V_1 \cos 70^\circ = \left(87.6 \frac{ft}{s}\right) \cos 70^\circ = 29.9 \frac{ft}{s}$$

and

$$W_1 = \frac{V_{x,1}}{\cos 45^\circ} = \frac{\left(29.9 \frac{ft}{s}\right)}{\cos 45^\circ} = 42.4 \frac{ft}{s}$$

With the velocity triangle for the flow leaving the rotor we conclude that

$$W_2 \cos 45^\circ = V_{x,2} \qquad (5)$$

$$V_{\theta,2} = U_2 - W_2 \sin 45^\circ \qquad (6)$$

$$V_2 \sin \alpha_2 = V_{\theta,2} \qquad (7)$$

$$V_2 \cos \alpha_2 = V_{x,2} \qquad (8)$$

From the conservation of mass equation

$$V_{x,1} = V_{x,2} = 29.9 \frac{ft}{s}$$

(con't)

5.76 (con't)

Thus from Eq. 5

$$W_2 = \frac{V_{x,2}}{\cos 45°} = \frac{\left(29.9\ \frac{ft}{s}\right)}{\cos 45°} = 42.4\ \frac{ft}{s}$$

and from Eq. 6

$$V_{\theta,2} = U_2 - W_2 \sin 45° = 52.3\ \frac{ft}{s} - \left(42.4\ \frac{ft}{s}\right)\sin 45° = 22.4\ \frac{ft}{s}$$

The ratio of Eqs. 7 and 8 yields

$$\alpha_2 = \tan^{-1}\left(\frac{V_{\theta,2}}{V_{x,2}}\right) = \tan^{-1}\left[\frac{\left(22.4\ \frac{ft}{s}\right)}{\left(29.9\ \frac{ft}{s}\right)}\right] = 37°$$

and from Eq. 7

$$V_2 = \frac{V_{\theta,2}}{\sin \alpha_2} = \frac{\left(22.4\ \frac{ft}{s}\right)}{\sin(37°)} = 37.2\ \frac{ft}{s}$$

We can use Eq. 5.54 to calculate the work per unit mass delivered at the shaft. Thus

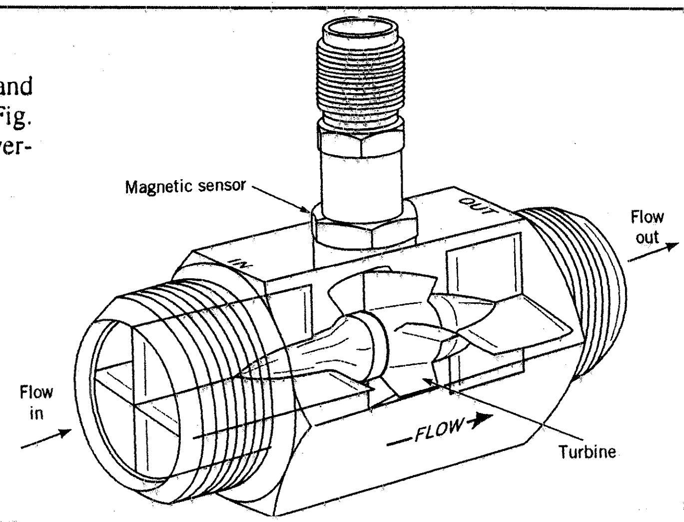
$$w_{shaft} = -U_1 V_{\theta,1} + U_2 V_{\theta,2}$$

$$w_{shaft} = \left[-\left(52.3\ \frac{ft}{s}\right)\left(82.3\ \frac{ft}{s}\right) + \left(52.3\ \frac{ft}{s}\right)\left(22.4\ \frac{ft}{s}\right)\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$w_{shaft} = -\underline{\underline{3130}}\ \frac{ft \cdot lb}{slug}$$

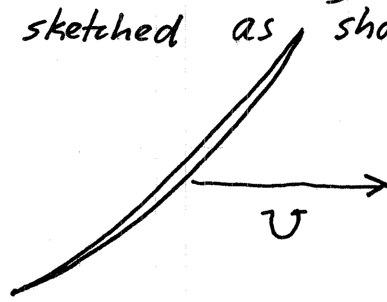
5.77

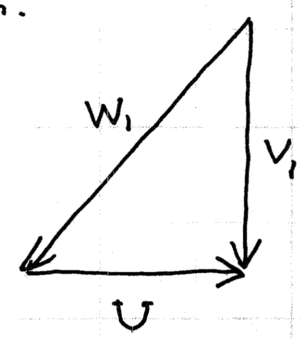
5.77 Sketch the velocity triangles for the flows entering and leaving the rotor of the turbine-type flow meter shown in Fig. P5.77. Show how rotor angular velocity is proportional to average fluid velocity.

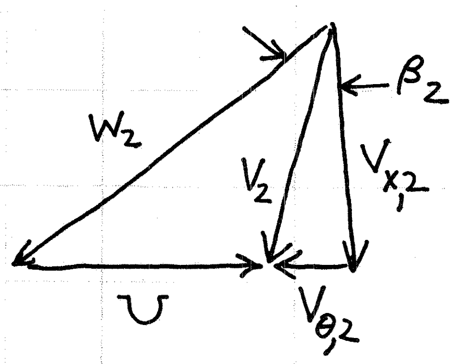


■ FIGURE P5.77 (Courtesy of EG&G Flow Technology, Inc.)

For a section of the turbine blade at radius r, the blade moves tangentially with a velocity $U = r\omega$. The velocity triangles may be sketched as shown.







Using Eq. 5.50 we get

$$T_{shaft} = r_2 V_{\theta,2} \dot{m}_2 = r_2 \left(V_{x,2} \tan\beta_2 - U \right) \dot{m}_2$$

For nearly zero T_{shaft}

$$0 = V_{x,2} \tan\beta_2 - U = V_{x,2} \tan\beta_2 - r\omega$$

so

$$\omega = \frac{V_{x,2} \tan\beta_2}{r}$$

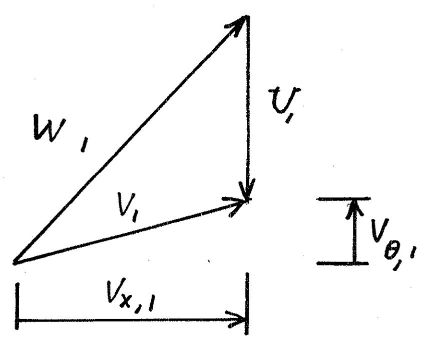
5.78

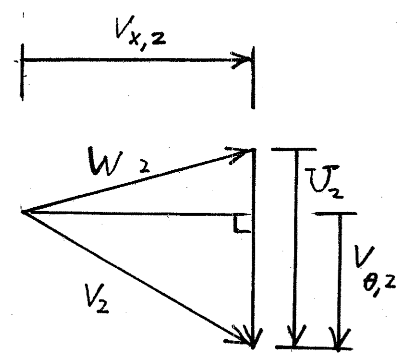
5.78 By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

where V = absolute flow velocity magnitude, W = relative flow velocity magnitude, and U = blade speed.

Any set of velocity triangle for flow through a turbomachine rotor row would give the same result. We use the triangles of Fig. P5.77.





From the inlet flow velocity triangle we get

$$V_{x,1}^2 = V_1^2 - V_{\theta,1}^2 \qquad (1)$$

and

$$V_{x,1}^2 = W_1^2 - (V_{\theta,1} + U_1)^2 = W_1^2 - V_{\theta,1}^2 - 2U_1V_{\theta,1} - U_1^2 \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$U_1 V_{\theta,1} = \frac{W_1^2 - V_1^2 - U_1^2}{2} \qquad (3)$$

From the outlet flow velocity triangle we get

$$V_{x,2}^2 = V_2^2 - V_{\theta,2}^2 \qquad (4)$$

and

$$V_{x,2}^2 = W_2^2 - (U_2 - V_{\theta,2})^2 = W_2^2 - U_2^2 + 2U_2V_{\theta,2} - V_{\theta,2}^2 \qquad (5)$$

(con't)

5.78 (con't)

Combining Eqs. 4 and 5 we obtain

$$U_2 V_{\theta,2} = \frac{V_2^2 - W_2^2 + U_2^2}{2} \quad (6)$$

For the set of velocity triangles

$$w_{\substack{shaft\\net\ in}} = U_1 V_{\theta,1} + U_2 V_{\theta,2} \quad (7)$$

Combining Eqs. 3, 6 and 7 we obtain

$$w_{\substack{shaft\\net\ in}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

*5.79

5.79* Summarized below are air flow data for flow across a low-speed axial flow fan. Calculate the change in rate of flow of axial direction angular momentum across this rotor and evaluate the shaft power input involved. The inner and outer radii of the fan annulus are 142 and 203 mm. The rotor speed is 2400 rpm.

	Upstream of Rotor		Downstream of Rotor	
Radius (mm)	Axial Velocity (m/s)	Absolute Tangential Velocity (m/s)	Axial Velocity (m/s)	Absolute Tangential Velocity (m/s)
142	0	0	0	0
148	32.03	0	32.28	12.64
169	32.03	0	32.37	12.24
173	32.04	0	31.78	11.91
185	32.03	0	31.50	11.35
197	31.09	0	29.64	11.66
203	0	0	0	0
	$V_{x,1}$	$V_{\theta,1}$	$V_{x,2}$	$V_{\theta,2}$

the change in rate of flow of axial direction angular momentum across the rotor, ΔFAM_x, is evaluated with

$$\Delta FAM_x = \int_{r_i}^{r_o} r_2 V_{\theta,2}\,\rho V_{x,2}\,2\pi r_2\,dr_2 - \int_{r_i}^{r_o} r_1 V_{\theta,1}\,\rho V_{x,1}\,2\pi r_1\,dr_1$$

or

$$\Delta FAM_x = 2\pi\rho\left(\int_{r_i}^{r_o} V_{\theta,2} V_{x,2}\, r_2^2\,dr_2 - \int_{r_i}^{r_o} V_{\theta,1} V_{x,1}\, r_1^2\,dr_1\right) \quad (1)$$

where

r_i and r_o are fan annulus inner and outer radii

r_2 and r_1 are local radii at section (2) downstream of fan rotor and section (1) upstream of fan rotor

$V_{\theta,2}$ and $V_{\theta,1}$ are local absolute tangential velocity at sections (2) and (1)

$V_{x,2}$ and $V_{x,1}$ are local axial velocities at sections (2) and (1)

As suggested by Eq. 5.45

$$T_{shaft} = \Delta FAM_x \quad (2)$$

and Eq. 2 is evaluated numerically with a computer program that utilizes the trapezoidal rule with uneven intervals. The result is

$$T_{shaft} = 4.79\ N\cdot m$$

The shaft power input, $\dot{W}_{shaft}$, is evaluated with Eq. 5.47. Thus,

$$\dot{W}_{shaft} = T_{shaft}\,\omega = 4.79\ N\cdot m\left(2400\,\frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(2\pi\,\frac{rad}{rev}\right)$$

$$= 1{,}200\,\frac{N\cdot m}{s} = \underline{\underline{1.20\ kW}}$$

5.80

5.80 Air enters a radial blower with zero angular momentum. It leaves with an absolute tangential velocity, V_θ, of 200 ft/s. The rotor blade speed at rotor exit is 170 ft/s. If the stagnation pressure rise across the rotor is 0.4 psi, calculate the loss of available energy across the rotor and the rotor efficiency.

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$loss = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g(\cancel{z_{in} - z_{out}})^{\nearrow 0,\ neglect} + w_{\substack{shaft\\net\ in}}$$

or

$$loss = \frac{p_{0,in} - p_{0,out}}{\rho} + w_{\substack{shaft\\net\ in}} \qquad (1)$$

The shaft work in, $w_{\substack{shaft\\net\ in}}$ can be obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft\\net\ in}} = U_{out} V_{\theta,out} \qquad (2)$$

Combining Eqs. 1 and 2 leads to

$$loss = \frac{p_{0,in} - p_{0,out}}{\rho} + U_{out} V_{\theta,out}$$

or

$$loss = -\frac{(0.4\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)} + \left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$\underline{\underline{loss = 9800\ \frac{ft \cdot lb}{slug}}}$$

As was done in Example 5.24, we calculate rotor efficiency from

$$\text{rotor efficiency} = \frac{w_{\substack{shaft\\net\ in}} - loss}{w_{\substack{shaft\\net\ in}}} = \frac{U_{out} V_{\theta,out} - loss}{U_{out} V_{\theta,0}}$$

$$\text{rotor efficiency} = \frac{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) - 9800\ \frac{ft \cdot lb}{slug}}{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)} = \underline{\underline{0.71}}$$

5.81

5.81 Water enters a pump impeller radially. It leaves the impeller with a tangential component of absolute velocity of 10 m/s. The impeller exit diameter is 60 mm and the impeller speed is 1800 rpm. If the stagnation pressure rise across the impeller is 45 kPa, determine the loss of available energy across the impeller and the hydraulic efficiency of the pump.

The analysis of Example 5.27 is applicable to solving this problem. Using Eq. 6 of Example 5.27 we obtain

$$loss = U_2 V_{\theta,2} - \frac{\text{actual total pressure rise across impeller}}{\rho}$$

However,

$$U_2 = r_2 \omega = \frac{(60\ mm)}{(2)(1000\ \frac{mm}{m})} \frac{(1800\ \frac{rev}{min})(2\pi\ \frac{rad}{rev})}{(60\ \frac{s}{min})} = 5.66\ \frac{m}{s}$$

Thus

$$loss = (5.66\ \tfrac{m}{s})(10\ \tfrac{m}{s})\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) - \left(45 \times 10^3\ \frac{N}{m^2}\right)\frac{1}{\left(999\ \frac{kg}{m^3}\right)}$$

$$loss = \underline{\underline{11.6}}\ \frac{N \cdot m}{kg}$$

From Eq. 5 of Example 5.27 we obtain

$$\eta = \frac{\frac{\text{actual total pressure rise across impeller}}{\rho}}{U_2 V_{\theta,2}}$$

or

$$\eta = \frac{\left[\frac{(45 \times 10^3\ \frac{N}{m^2})}{(999\ \frac{kg}{m^3})}\right]}{(5.66\ \frac{m}{s})(10\ \frac{m}{s})\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)} = \underline{\underline{0.796}}$$

5.82

5.82 Water enters an axial-flow turbine rotor with an absolute velocity tangential component, V_θ, of 15 ft/s. The corresponding blade velocity, U, is 50 ft/s. The water leaves the rotor blade row with no angular momentum. If the stagnation pressure drop across the turbine is 12 psi, determine the hydraulic efficiency of the turbine.

To determine the efficiency of the turbine we use

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} \qquad (1)$$

The actual work out, $w_{\substack{shaft\\net\ out}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft\\net\ out}} = -w_{\substack{shaft\\net\ in}} = U_{in} V_{\theta,in} \qquad (2)$$

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g(\cancel{z_{in} - z_{out}}) + w_{\substack{shaft\\net\ in}} \qquad (3)$$

neglect

Combining Eqs. 2 and 3 we obtain

$$\text{loss} = \frac{p_{0,in} - p_{0,out}}{\rho} - U_{in} V_{\theta,in} \qquad (4)$$

Combining Eqs. 1, 2 and 4 we obtain

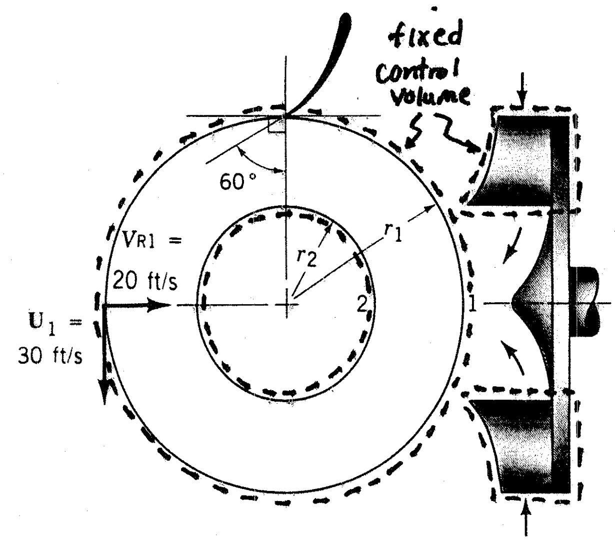
$$\eta = \frac{U_{in} V_{\theta,in}}{U_{in} V_{\theta,in} + \text{loss}} = \frac{U_{in} V_{\theta,in}}{\frac{p_{0,in} - p_{0,out}}{\rho}} = \frac{\left(50\ \frac{ft}{s}\right)\left(15\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)}{\frac{(12\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}}$$

and

$$\eta = \underline{\underline{0.842}}$$

5.83

5.83 An inward flow radial turbine (see Fig. P5.83) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is water and the stagnation pressure drop across the rotor is 16 psi, determine the loss of available energy across the rotor and the hydraulic efficiency involved.



■ FIGURE P5.83

An analysis like the one of Example 5.28 would be appropriate for solving this problem. Since a turbine is involved in this problem, $w_{\substack{shaft\\net\ in}} = -w_{\substack{shaft\\net\ out}}$ and from Eq. 1 of Example 5.28 we can conclude that

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - w_{\substack{shaft\\net\ out}}$$

However from Eq. 5.54 we see that

$$w_{shaft} = w_{\substack{shaft\\net\ in}} = -U_1 V_{\theta,1} = -w_{\substack{shaft\\net\ out}}$$

and thus

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - U_1 V_{\theta,1} \qquad (1)$$

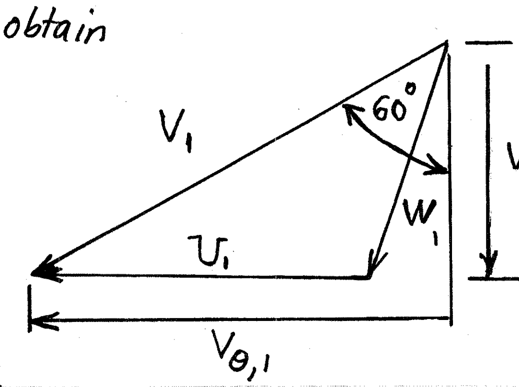
To determine the value of $V_{\theta,1}$ we examine the velocity triangle for the flow entering the rotor that is sketched below.

From the velocity triangle we obtain

$$V_{\theta,1} = V_{R,1} \tan 60°$$

or

$$V_{\theta,1} = \left(20\ \frac{ft}{s}\right) \tan 60° = 34.64\ \frac{ft}{s}$$



(Con't)

5.83 (con't)

From Eq. 1 we obtain

$$loss = \frac{\left(16 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)} - \left(30 \frac{ft}{s}\right)\left(34.64 \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$loss = \underline{\underline{148}} \ \frac{ft \cdot lb}{slug}$$

From Eq. 5.82, we can conclude that

$$w_{\substack{shaft \\ net\ out}} + loss = \frac{\text{stagnation pressure drop across the rotor}}{\rho}$$

or in other words, the stagnation pressure drop across the rotor results in shaft work and loss of available energy.

Thus a meaningful efficiency is

$$\eta = \frac{w_{\substack{shaft \\ net\ out}}}{\left(\frac{\text{stagnation pressure drop across the rotor}}{\rho}\right)}$$

or

$$\eta = \frac{\left(30 \frac{ft}{s}\right)\left(34.64 \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)}{\frac{\left(16 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)}} = \underline{\underline{0.875}}$$

5.84

5.84 An inward flow radial turbine (see Fig. P5.83) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is air and the static pressure drop across the rotor is 0.01 psi, determine the loss of available energy across the rotor and the rotor aerodynamic efficiency.

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82). Thus,

$$\text{loss} = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + \underbrace{g(z_1 - z_2)}_{\text{neglect}} + w_{\substack{\text{shaft}\\ \text{net in}}} \qquad (1)$$

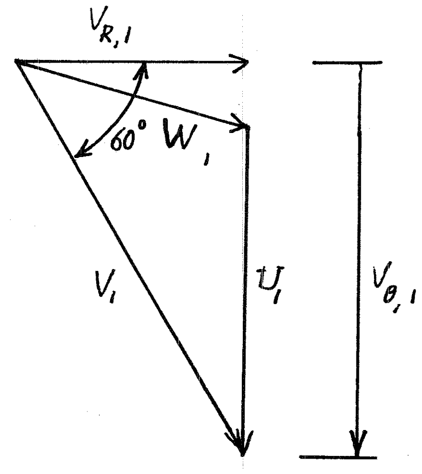
The shaft work, $w_{\substack{\text{shaft}\\ \text{net in}}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{\text{shaft}\\ \text{net in}}} = -U_1 V_{\theta,1} = -w_{\substack{\text{shaft}\\ \text{net out}}} \qquad (2)$$

and combining Eqs. 1 and 2 yields

$$\text{loss} = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} - U_1 V_{\theta,1} \qquad (3)$$

To determine V_1 and $V_{\theta,1}$ we construct the velocity triangle sketched below.



(con't)

5.84 (con't)

With the velocity triangle we conclude that

$$V_1 = \frac{(20\ \frac{ft}{s})}{\cos 60^\circ} = 40\ \frac{ft}{s}$$

and

$$V_{\theta,1} = V_1 \sin 60^\circ = \left(40\ \frac{ft}{s}\right)\sin 60^\circ = 34.64\ \frac{ft}{s}$$

Since the flow leaving the rotor is radial, then

$$V_2 = V_{R,2} = 20\ \frac{ft}{s}$$

From Eq. 3 we obtain

$$\text{loss} = \frac{\left(0.01\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)} + \frac{\left[\left(40\ \frac{ft}{s}\right)^2 - \left(20\ \frac{ft}{s}\right)^2\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)}{2} - \left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$\text{loss} = \underline{\underline{166}}\ \frac{ft\cdot lb}{slug}$$

The efficiency may be obtained with

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} = \frac{U_1 V_{\theta,1}}{U_1 V_{\theta,1} + \text{loss}}$$

or

$$\eta = \frac{\left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(30\ \frac{ft}{s}\right)\left(34.64\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) + 166\ \frac{ft\cdot lb}{slug}} = \underline{\underline{0.86}}$$

5.85

5.85 A 200-m-high waterfall involves steady flow from one large body to another. Determine the temperature rise associated with this flow.

This is like Example 5.22.

To determine the temperature change we use the relationship

$$T_2 - T_1 = \frac{\check{u}_2 - \check{u}_1}{c} \qquad (1)$$

where the specific heat, $c = 1 \frac{Btu}{lbm \cdot {}^\circ R}$. We use the energy equation (Eq. 5.70) to obtain

$$\check{u}_2 - \check{u}_1 = g(z_1 - z_2) \qquad (2)$$

Combining Eqs. 1 and 2 yields

$$T_2 - T_1 = \frac{g(z_1 - z_2)}{c}$$

or

$$T_2 - T_1 = \frac{\left(9.81 \frac{m}{s^2}\right)(200\ m)\left(0.4536 \frac{kg}{lbm}\right)\left(0.5556 \frac{K}{{}^\circ R}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1 \frac{Btu}{lbm \cdot {}^\circ R}\right)\left(1055 \frac{N \cdot m}{Btu}\right)}$$

and

$$T_2 - T_1 = \underline{\underline{0.469\ K}}$$

5-86

5.86 What is the size of the head loss that is needed to raise the temperature of water by 1°F?

This is similar to Example 5.22. From Eq. 5.78 we have

$$\check{u}_{out} - \check{u}_{in} - \cancel{q}_{\substack{net \\ in}} = loss$$

0 assumed

However

$$\check{u}_{out} - \check{u}_{in} = c\,(T_{out} - T_{in})$$

Thus

$$loss = c\,(T_{out} - T_{in})$$

or

$$g h_L = c\,(T_{out} - T_{in})$$

and

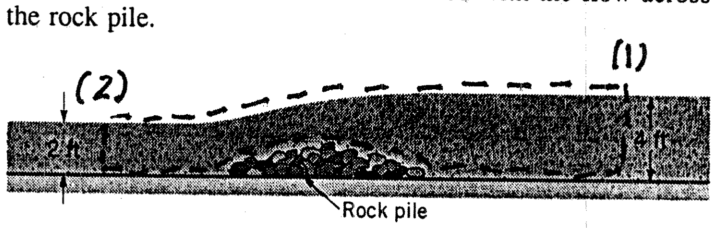
$$h_L = \frac{c}{g}\,(T_{out} - T_{in})$$

$$h_L = \left(1\ \frac{Btu}{lbm\,{}^{\circ}F}\right)(1\,{}^{\circ}F)\left(32.2\ \frac{lbm \cdot ft}{lb \cdot s^2}\right)\left(778\ \frac{ft \cdot lb}{Btu}\right)$$

$$h_L = \underline{\underline{778\ ft}}$$

5.87

5.87 A 100-ft-wide river with a flowrate of 2400 ft³/s flows over a rock pile as shown in Fig. P5.87. Determine the direction of flow and the head loss associated with the flow across the rock pile.



■ FIGURE P5.87

To determine the direction of flow we will assume a direction, use the energy equation (Eq. 5.84) and calculate the head loss. If the head loss is positive, our assumed direction of flow is correct. If the head loss is negative which is not physically possible, our assumed direction of flow is wrong.

So, assuming the flow is from right to left or from point (1) to point (2) in the sketch above, we get using Eq. 5.84

$$\overbrace{\cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\rho}}}^{\text{same pressure}} + \frac{V_1^2}{2g} + z_1 + \cancel{h_s}^{\,0,\text{ no shaft work}} - h_L$$

Now

$$V_1 = \frac{Q}{A_1} = \frac{\left(2400\,\frac{ft^3}{s}\right)}{(4\,ft)(100\,ft)} = 6\,\frac{ft}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{\left(2400\,\frac{ft^3}{s}\right)}{(2\,ft)(100\,ft)} = 12\,\frac{ft}{s}$$

So

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + z_1 - z_2 = \frac{\left(6\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} - \frac{\left(12\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} + 4\,ft - 2\,ft$$

h_L = $\underline{\underline{0.32\ ft}}$ and since h_L is positive, our assumed $\underline{\underline{\text{right to left}}}$ flow is correct

5.88

5.88 If a $\frac{3}{4}$-hp motor is required by a ventilating fan to produce a 24-in. stream of air having a velocity of 40 ft/s as shown in Fig. P5.88, estimate (a) the efficiency of the fan and (b) the thrust of the supporting member on the conduit enclosing the fan.

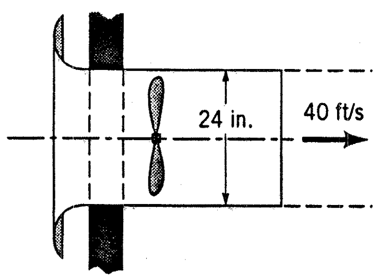


FIGURE P5.88

(a) The solution to this part of the problem is like Example 5.24. We use

$$\eta = \frac{w_{shaft} - loss}{w_{shaft}}$$

to calculate the fan efficiency.

We use the energy equation (Eq. 5.82) for flow through the control volume sketched above to calculate the loss as follows

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\substack{shaft \\ net\ in}} - loss$$

But $\quad p_2 = p_1$ and $z_2 = z_1$; $V_1 \simeq 0$; $w_{\substack{shaft \\ net\ in}} = \dfrac{hp}{\dot{m}}$

Also $\quad \dot{m} = \rho A_2 V_2 = \dfrac{p}{RT}\dfrac{\pi d_2^2}{4} V_2$

So

$$loss = w_{\substack{shaft \\ net\ in}} - \frac{V_2^2}{2} = \frac{hp}{\frac{p}{RT}\left(\frac{\pi d_2^2}{4}\right)V_2} - \frac{V_2^2}{2}$$

$$loss = \frac{\left(\frac{3}{4}\,hp\right)\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}{\left\{\dfrac{\left(14.7\,\frac{lb}{in^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)\pi\left[\dfrac{24\,in.}{\left(12\,\frac{in.}{ft}\right)}\right]^2}{\left(53.3\,\frac{ft\cdot lb}{lbm\cdot {}^\circ R}\right)\left(530\,{}^\circ R\right)\,4}\left(40\,\frac{ft}{s}\right)\right\}} - \frac{\left(40\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{lbm\cdot ft}{lb\cdot s^2}\right)}$$

(con't)

5.88 (con't)

$$\text{loss} = 44\ \frac{ft \cdot lb}{lb_m} - 24.8\ \frac{ft \cdot lb}{lb_m} = 19.2\ \frac{ft \cdot lb}{lb_m}$$

So

$$\eta = \frac{44\ \frac{ft \cdot lb}{lb_m} - 19.2\ \frac{ft \cdot lb}{lb_m}}{44\ \frac{ft \cdot lb}{lb_m}} = \underline{\underline{0.56}}$$

For

(b) We use the horizontal component of the linear momentum equation to evaluate the anchoring force required to hold the fan in place

$$F_{AX} = V_2\,\dot{m}$$

From part (a)

$$\dot{m} = \frac{P}{RT}\frac{\pi d_2^2}{4}V_2 = \frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)\pi\left(\frac{24\ in.}{12\ \frac{in.}{ft}}\right)^2\left(40\ \frac{ft}{s}\right)}{\left(53.3\ \frac{ft \cdot lb}{lb_m\,{}^\circ R}\right)(530\,{}^\circ R)\ 4}$$

$$\dot{m} = 9.41\ \frac{lbm}{s}$$

So

$$F_{AX} = \frac{\left(40\ \frac{ft}{s}\right)\left(9.41\ \frac{lbm}{s}\right)}{\left(32.2\ \frac{lbm \cdot ft}{lb \cdot s^2}\right)} = \underline{\underline{11.7\ lb}}$$

5.89

5.89 Air flows past an object in a pipe of 2-m diameter and exits as a free jet as shown in Fig. P5.89. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m^2, respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. **(a)** Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. **(b)** Determine the force that the air puts on the object.

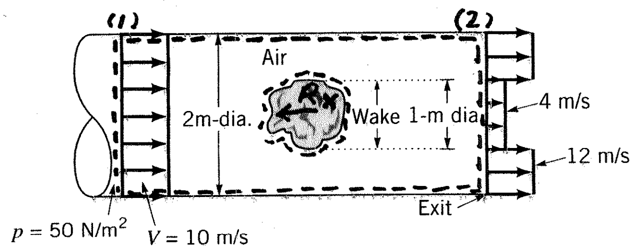


FIGURE P5.89

(a) To determine the loss suffered by a fluid particle as it flows from (1) to a location in the wake at (2) we apply the energy equation (Eq. 5.84) to that particle flow to get:

$$\cancel{\frac{p_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{0} - h_L \qquad (1)$$

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

and

$$h_L = \frac{\left(50\,\frac{N}{m^2}\right)}{\left(12\,\frac{N}{m^3}\right)} + \frac{\left(10\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} - \frac{\left(4\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} = \underline{\underline{8.45\ m}}$$

To determine the head loss associated with the entire flow across the object we use the non-uniform flow energy equation (Eq. 5.89) for flow from (1) to (2) through the control volume shown in the sketch to get:

$$\cancel{\frac{p_2}{\gamma}}^{0} + \frac{\alpha_2 \bar{V}_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{\alpha_1 \bar{V}_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{0} - h_L \qquad (2)$$

From Eq. 5.86 we get:

$$\frac{\alpha \bar{V}^2}{2g} = \frac{\int \frac{V^2}{2g} \rho \vec{V}\cdot\hat{n}\, dA}{\dot{m}} = \frac{\int \frac{V^2}{2g} \rho \vec{V}\cdot\hat{n}\, dA}{\rho \bar{V} A}$$

Eq. (2) becomes

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{\int_{A_2} \frac{V^2}{2g} \rho \vec{V}\cdot\hat{n}\, dA}{(\rho V A)_{4\frac{m}{s}} + (\rho V A)_{12\frac{m}{s}}}$$

(con't)

5.89 (con't)

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{1}{2g}\left[\frac{V_{12\frac{m}{s}}^3 A_{12\frac{m}{s}} + V_{3\frac{m}{s}}^3 A_{3\frac{m}{s}}}{V_{4\frac{m}{s}} A_{4\frac{m}{s}} + V_{12\frac{m}{s}} A_{12\frac{m}{s}}}\right]$$

and

$$h_L = \frac{\left(50\,\frac{N}{m^2}\right)}{\left(12\,\frac{N}{m^3}\right)} + \frac{\left(10\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} - \frac{1}{2\left(9.81\,\frac{m}{s^2}\right)}\left\{\frac{\left(12\,\frac{m}{s}\right)^3 \pi\left[\frac{(2m)^2-(1m)^2}{4}\right] + \left(4\,\frac{m}{s}\right)^3 \frac{\pi(1m)^2}{4}}{\left(4\,\frac{m}{s}\right)\frac{\pi(1m)^2}{4} + \left(12\,\frac{m}{s}\right)\pi\left[\frac{(2m)^2-(1m)^2}{4}\right]}\right\}$$

$$h_L = \underline{\underline{2.58\ m}}$$

(b) To determine the force that the air puts on the object, R_x, we use the horizontal component of the linear momentum equation to get:

$$-\rho V_1^2 A_1 + \rho V_{12\frac{m}{s}}^2 A_{12\frac{m}{s}} + \rho V_{4\frac{m}{s}}^2 A_{4\frac{m}{s}} = p_1 A_1 - R_x$$

and thus

$$R_x = p_1 A_1 + \rho V_1^2 A_1 - \rho\left(V_{12\frac{m}{s}}^2 A_{12\frac{m}{s}} + V_{4\frac{m}{s}}^2 A_{4\frac{m}{s}}\right)$$

So

$$R_x = \left(50\,\frac{N}{m^2}\right)\frac{\pi(2m)^2}{4} + \left(1.23\,\frac{kg}{m^3}\right)\left(10\,\frac{m}{s}\right)^2 \frac{\pi(2m)^2}{4}\left(1\,\frac{N\cdot s^2}{m\cdot kg}\right)$$

$$-1.23\,\frac{kg}{m^3}\left\{\left(12\,\frac{m}{s}\right)^2 \frac{\pi\left[(2m)^2-(1m)^2\right]}{4} + \left(4\,\frac{m}{s}\right)^2 \frac{\pi(1m)^2}{4}\right\}\left(\frac{1\ N\cdot s^2}{m\cdot kg}\right)$$

and

$$R_x = \underline{\underline{110\ N}}$$

5.90

5.90 Oil ($SG = 0.9$) flows downward through a vertical pipe contraction as shown in Fig. P5.90. If the mercury manometer reading, h, is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.

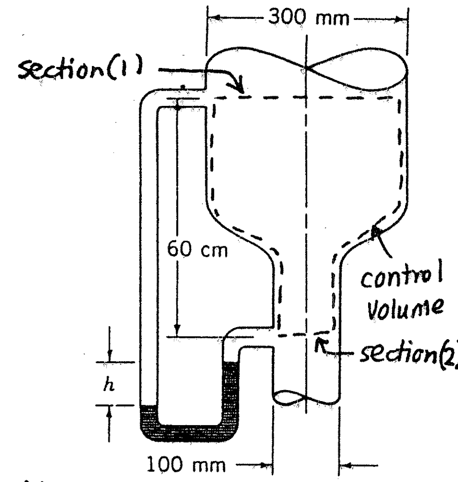


FIGURE P5.90

The volume flowrate may be obtained with

$$Q = A_1 V_1 = A_2 V_2 = \frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

To determine either V_1 or V_2 we apply the energy equation (Eq. 5.82) to the flow between sections (1) and (2). Thus,

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + \overset{0}{\cancel{w_{\substack{shaft \\ net\ in}}}} - \overset{neglect}{\cancel{loss}} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) \qquad (3)$$

To determine $\frac{p_1 - p_2}{\rho}$ we use the manometer equation from section 2.6 to obtain

$$\frac{p_1 - p_2}{\rho} = gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - g(z_1 - z_2) \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$V_2 = \sqrt{\frac{2gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}}$$

or

$$V_2 = \sqrt{\frac{(2)(9.81 \frac{m}{s^2})(0.1\ m)\left(\frac{13.6}{0.9} - 1\right)}{1 - \left(\frac{100\ mm}{300\ mm}\right)^4}} = 5.29 \frac{m}{s}$$

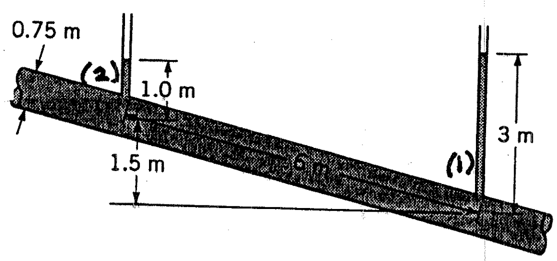
and from Eq. 1 we have

$$Q = \frac{\pi (0.1 m)^2}{4}\left(5.29 \frac{m}{s}\right) = \underline{\underline{0.042 \frac{m^3}{s}}}$$

Actual flowrate would be less than the frictionless value because the loss would be greater than the zero amount used above.

5.91

5.91 An incompressible liquid flows steadily along the pipe shown in Fig. P5.91. Determine the direction of flow and the head loss over the 6-m length of pipe.



■ FIGURE P5.91

Assume flow from (1) to (2) and use the energy equation (Eq. 5.84) to get for the contents of the control volume shown:

$$\frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_s}^{\,0} - h_\ell$$

Thus

$$h_\ell = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + z_1 - z_2 = 3\,m - 1.0\,m - 1.5\,m = \underline{\underline{0.5\,m}}$$

and since $h_\ell > 0$, the assumed direction of flow is correct.

The flow is uphill.

5.92

5.92 A siphon is used to draw water at 70°F from a large container as indicated in Fig. P5.92. The inside diameter of the siphon line is 1 in. and the pipe centerline rises 3 ft above the essentially constant water level in the tank. Show that by varying the length of the siphon below the water level, h, the rate of flow through the siphon can be changed. Assuming frictionless flow, determine the maximum flowrate possible through the siphon. The limiting condition is the occurrence of cavitation in the siphon. Will the actual maximum flow be more or less than the frictionless value? Explain.

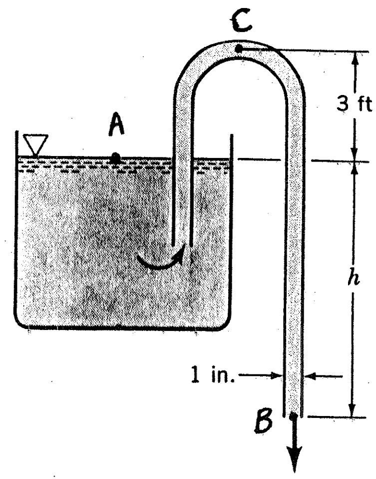


FIGURE P5.92

The flowrate, Q, can be determined with

$$Q = A_B V_B = \frac{\pi D_B^2}{4} V_B \qquad (1)$$

To obtain V_B we apply the energy equation (Eq. 5.82) between points A and B in the sketch above to obtain

$$\cancel{\frac{p_B}{\rho}}^{0} + \frac{V_B^2}{2} + g z_B = \cancel{\frac{p_A}{\rho}}^{0} + \cancel{\frac{V_A^2}{2}}^{0} + g z_A + \cancel{w_{\substack{shaft\\net\ in}}}^{0} - loss \qquad (2)$$

or

$$\frac{V_B^2}{2} = g(z_A - z_B) - loss$$

and

$$V_B = \sqrt{2\left[g(h) - loss\right]} \qquad (3)$$

With Eq. 3 we conclude that as h varies, so does V_B and thus Q. For no loss, the maximum flow will occur when the pressure at point C is just equal to the vapor pressure of water at 0°C. We apply the energy equation (Eq. 5.82) between points A and C to get

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + g z_C = \frac{p_A}{\rho} + \cancel{\frac{V_A^2}{2}}^{0} + g z_A + \cancel{w_{\substack{shaft\\net\ in}}}^{0} - \cancel{loss}^{0} \qquad (4)$$

Using absolute instead of gage pressures we obtain with Eq. 4

$$V_C = \sqrt{2g(z_A - z_C) + \frac{p_A - p_C}{\rho}}$$

or

$$V_C = \sqrt{2\left(9.81\frac{m}{s^2}\right)(-3ft)\left(0.3048\frac{m}{ft}\right) + \frac{\left(101{,}000\frac{N}{m^2} - 1228\frac{N}{m^2}\right)}{\left(999.7\frac{kg}{m^3}\right)\left(1\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 9.048\ \frac{m}{s}$$

(con't)

5.92 (con't)

Since

$$Q = A_c V_c = \frac{\pi D_c^2}{4} V_c$$

we have for the maximum flowrate through the siphon,

$$Q = \frac{\pi (1\ in.)^2}{4\left(144\ \frac{in.^2}{ft^2}\right)}\left(0.3048\ \frac{m}{ft}\right)^2\left(9.048\ \frac{m}{s}\right) = \underline{\underline{4.58\times 10^{-3}\ \frac{m^3}{s}}}$$

With Eqs. 3 and 4 we conclude that any loss would act to lower the value of V in the siphon and thus make the actual maximum flowrate with friction less than the maximum flowrate without friction.

5.93

5.93 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.93. If the friction loss between A and B is $0.8V^2/2$, where V is the velocity of flow in the siphon, determine the flowrate involved.

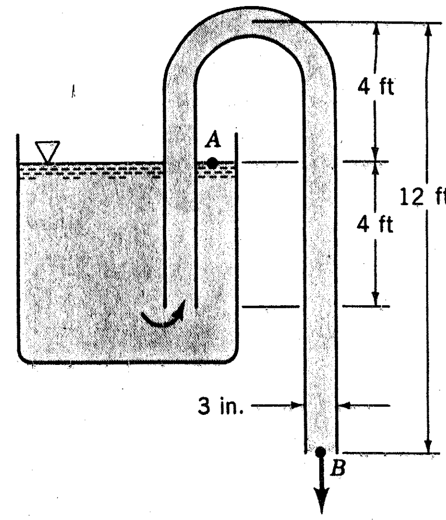


FIGURE P5.93

To determine the flowrate, Q, we use

$$Q = AV = \frac{\pi D^2}{4} V \qquad (1)$$

To obtain V we apply the energy equation (Eq. 5.82) between points A and B in the sketch above. Thus,

$$\cancel{\frac{p_B}{\rho}}^{0} + \frac{V_B^2}{2} + g z_B = \cancel{\frac{p_A}{\rho}}^{0} + \cancel{\frac{V_A^2}{2}}^{0} + g z_A + w_{\substack{shaft \\ net\ in}} - loss$$

or

$$\frac{V^2}{2} + g z_B = g z_A - 0.8 \frac{V^2}{2}$$

Thus

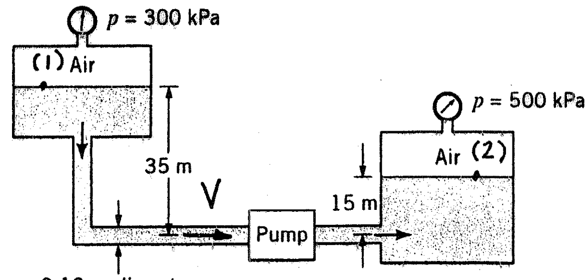
$$V = \sqrt{\frac{g(z_A - z_B)}{0.9}} = \sqrt{\frac{\left(32.2 \frac{ft}{s^2}\right)(8\ ft)}{0.9}} = 16.9 \frac{ft}{s}$$

and with Eq. 1

$$Q = \frac{\pi (3 in.)^2 \left(16.9 \frac{ft}{s}\right)}{4\left(144 \frac{in.^2}{ft^2}\right)} = \underline{\underline{0.830 \frac{ft^3}{s}}}$$

5.94

5.94 Water is pumped steadily through a 0.10-m-diameter pipe from one closed, pressurized tank to another as shown in Fig. P5.94. The pump adds 4.0 kW to the water and the head loss of the flow is 10 m. Determine the velocity of the water leaving the pipe.



■ **FIGURE P5.94**

From the energy equation,

(1) $$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_s-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$, where $z_1=35\,m$, $z_2=15\,m$, $V_1=0$, $V_2=0$, and $h_L=10\,m$.

Also, $$h_s=\frac{\dot{W}_s}{\gamma Q}=\frac{4\times10^3\,\frac{N\cdot m}{s}}{(9.80\times10^3\,\frac{N}{m^3})Q}=0.408/Q$$, where $h_s \sim m$ when $Q \sim m^3/s$.

Thus, Eq. (1) becomes

$$\left(\frac{300\times10^3\,\frac{N}{m^2}}{9.80\times10^3\,\frac{N}{m^3}}\right)+35\,m+\left(\frac{0.408}{Q}\,m\right)-10\,m=\left(\frac{500\times10^3\,\frac{N}{m^2}}{9.80\times10^3\,\frac{N}{m^3}}\right)+15\,m$$

which gives

$$Q=0.0392\,\frac{m^3}{s}=AV$$

Hence,

$$V=\frac{Q}{A}=\frac{0.0392\,\frac{m^3}{s}}{\frac{\pi}{4}(0.1\,m)^2}=\underline{\underline{4.99\,\frac{m}{s}}}$$

5.95

5.95 Water flows through a vertical pipe, as is indicated in Fig. P5.95. Is the flow up or down in the pipe? Explain.

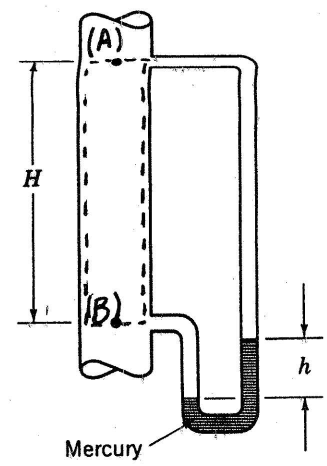


FIGURE P5.95

The control volume shown in the sketch above is used. For steady, incompressible flow downward from (A) to (B) we obtain from Eq. 5.79

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + g z_B = \frac{p_A}{\rho} + \frac{V_A^2}{2} + g z_A - {}_A loss_B$$

From conservation of mass we conclude that

$$V_A = V_B$$

Thus from Eq. 1

$${}_A loss_B = gH + \frac{p_A - p_B}{\rho}$$

However the manometer equation (see Section 2.6) yields

$$\frac{p_A - p_B}{\rho} = g\left[h\left(1 - SG_{Hg}\right) - H\right]$$

and

$${}_A loss_B = gh\left(1 - SG_{Hg}\right)$$

which is a negative quantity since $SG_{Hg} = 13.6$. A negative loss is not physically possible so the flow must be upward from B to A. For upward flow the above analysis leads to

$${}_B loss_A = gh\left(SG_{Hg} - 1\right)$$

which is positive and therefore physically reasonable.

5.96

5.96 A fire hose nozzle is designed to deliver water that will rise 40 m vertically. Calculate the stagnation pressure required at the nozzle inlet if **(a)** no loss is assumed, **(b)** a loss of 30 N · m/kg is assumed.

To determine the stagnation pressure at the nozzle inlet we assume that the stagnation pressure at the nozzle exit is the same as the stagnation pressure at the nozzle inlet and we apply the energy equation (Eq. 5.84) to the flow from the nozzle exit to the maximum elevation of the water flow to get

$$p_0 = \gamma \Delta z + \rho(\text{loss}) \qquad (1)$$

(a) For no loss, Eq. 1 leads to

$$p_0 = \left(9.80 \frac{kN}{m^3}\right)(40\ m) = 392 \frac{kN}{m^2} = \underline{\underline{392\ kPa}}$$

(b) For loss $= 30 \frac{N \cdot m}{kg}$, Eq. 1 yields

$$p_0 = \left(9.80 \frac{kN}{m^3}\right)(40\ m) + \left(999 \frac{kg}{m^3}\right)\left(30 \frac{N \cdot m}{kg}\right)\frac{1}{\left(1000 \frac{N}{kN}\right)} = \underline{\underline{422\ kPa}}$$

5.97

5.97 For the 180° elbow and nozzle flow shown in Fig. P5.97, determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?

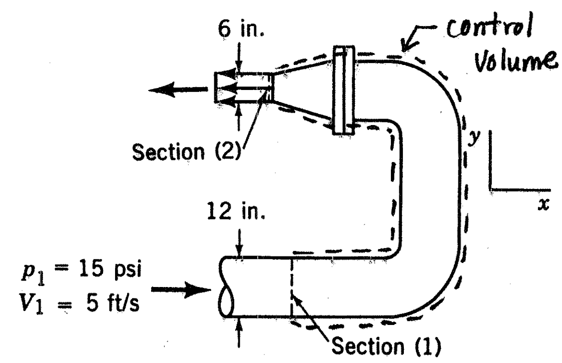


FIGURE P5.97

For solving the first part of this problem, the control volume shown in the sketch above is used. To determine the loss accompanying flow from section 1 to section 2 Eq. 5.79 can be used as follows.

$$ {}_1loss_2 = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) $$

Since x-y coordinates are specified we assume that the flow is horizontal and $z_1 - z_2 = 0$. Also, $p_2 = p_{atm} = 0$ psi.

From the conservation of mass principle we conclude that

$$ V_2 = V_1 \frac{A_1}{A_2} = V_1\left(\frac{D_1^2}{D_2^2}\right) $$

Thus

$$ {}_1loss_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1^2}{D_2^2}\right)^2\right] = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] $$

or

$$ {}_1loss_2 = \frac{\left(15\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)} + \frac{\left(5\,\frac{ft}{s}\right)^2}{2}\left[1 - \left(\frac{12\text{ in.}}{6\text{ in.}}\right)^4\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) $$

$$ {}_1loss_2 = \underline{\underline{926}}\ \frac{ft\cdot lb}{slug} $$

For the second part of this problem we consider the flow of a fluid particle from section 2 to a state of rest, a. Eq. 5.79 leads to

$$ {}_2loss_a = \frac{V_2^2}{2} $$

Note that we have assumed that $p_2 = p_a = p_{atm}$ and $z_2 = z_a$.

Thus

$$ {}_2loss_A = \frac{V_2^2}{2} = \frac{V_1^2}{2}\left(\frac{D_1^2}{D_2^2}\right)^2 = \frac{V_1^2}{2}\left(\frac{D_1}{D_2}\right)^4 = \frac{\left(5\,\frac{ft}{s}\right)^2\left(\frac{12\text{ in.}}{6\text{ in.}}\right)^4\left(1\,\frac{lb}{slug.\frac{ft}{s^2}}\right)}{2} $$

$$ {}_2loss_A = \underline{\underline{200}}\ \frac{ft.lb}{slug} $$

5.98

5.98 An automobile engine will work best when the back pressure at the exhaust manifold, engine block interface is minimized. Show how reduction of losses in the exhaust manifold, piping, and muffler will also reduce the back pressure. How could losses in the exhaust system be reduced? What primarily limits the minimization of exhaust system losses?

We apply the energy equation (Eq. 5.83) to the flow from the engine block, exhaust manifold interface to the exhaust system exit to get

$$P_{in} = P_{out} + \frac{\rho V_{out}^2}{2} - \frac{\rho V_{in}^2}{2} + \rho(loss) \qquad (1)$$

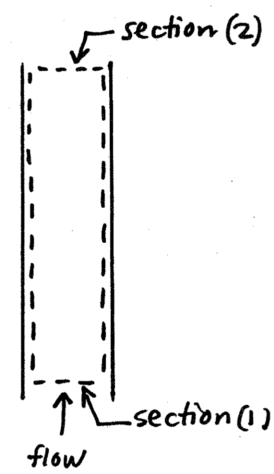
With Eq. 1 we see that reduction of loss in the exhaust system results in a lower value of P_{in} and thus the engine back pressure. Losses in the exhaust system could be reduced by eliminati g major loss components such as the catalytic converter and the muffler as is often done in race cars. However, noise and emissions legislation limits the extent to which this kind of loss reduction can occur in conventional vehicles. Some loss reduction can also occur by configuring the exhaust system piping with few bends and appropriate area distributions. However, requirements often leads to bends and turns in the piping and costs limit the extend of optimizing area distributions.

5.99

5.99 Water flows vertically upward in a circular cross section pipe. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R-r}{R}\right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe inside radius, and, r = radius from pipe axis. Develop an expression for the loss in available energy between sections (1) and (2).



For determining loss we use the energy equation for non-uniform flows, Eq. 5.87. Thus,

$$\text{loss} = \frac{P_1 - P_2}{\rho} + \frac{\alpha_1 \overline{V}_1^2 - \alpha_2 \overline{V}_2^2}{2} + g(z_1 - z_2) \qquad (1)$$

From conservation of mass (Eq. 5.13) we have

$$\overline{V}_1 = \overline{V}_2$$

Also, with Eq. 5.86 for the kinetic energy coefficient, α, we have

$$\alpha_1 = 1.0$$

since the velocity profile at section (1) is uniform. At section (2) we solve Eq. 5.86 (see solution for problem 5.125 (c)) and obtain

$$\alpha_2 = 1.06$$

Thus, Eq. 1 yields

$$\underline{\underline{\text{loss} = \frac{P_1 - P_2}{\rho} - 0.06 \frac{\overline{V}_1^2}{2} + g(z_1 - z_2)}}$$

5.101

5.101 Consider the flow shown in Fig. P5.91. If the flowing fluid is water, determine the axial (along the pipe) and normal (perpendicular to the pipe) components of force that the pipe puts on the fluid in the 6-m section shown.

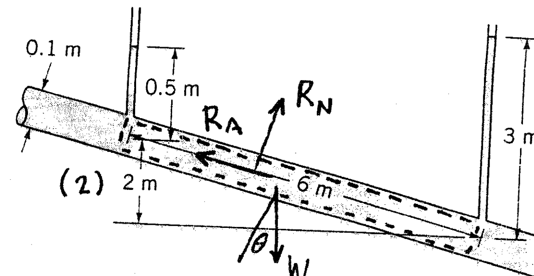


FIGURE P5.91

Using the control volume shown by broken lines we apply the axial and normal components of the linear momentum equation to get:

$\Sigma F_N = 0$ since there is no momentum flow in the normal direction

and $\Sigma F_A = 0$ since the flow is assumed fully developed and the net amount of axial direction momentum flow out of the CV is zero

So

$$R_N - W\cos\theta = 0 \quad \text{or} \quad R_N = W\cos\theta$$

Now $W = \gamma \forall = \gamma A \ell = \gamma \frac{\pi d^2}{4}\ell = \left(9.8\times10^3 \frac{N}{m^3}\right)\frac{\pi (0.1m)^2(6m)}{4} = 462\,N$

and $\theta = \sin^{-1}\frac{2}{6} = 19.5°$

Then $R_N = (462N)(\cos 19.5°) = \underline{\underline{436\,N}}$

For the axial direction

$$P_2 A_2 + R_A + W\sin\theta - P_1 A_1 = 0 \quad \text{or}$$

$$R_A = P_1 A_1 - P_2 A_2 + W\sin\theta = (P_1 - P_2)A + W\sin\theta$$

From the manometer readings

$$P_2 = \gamma h_2 \quad \text{and} \quad P_1 = \gamma h_1$$

thus

$$P_1 - P_2 = \gamma(h_1 - h_2)$$

and

$$R_A = \gamma(h_1 - h_2)A - (W\sin 19.5°)$$

$$R_A = \left(9.8\times10^3 \frac{N}{m^3}\right)(3.0m - 0.5m)\frac{\pi(0.1m)^2}{4} - (462N)(\sin 19.5°)$$

$$R_A = \underline{\underline{38\,N}}$$

5.102

5.102 Water flows steadily down the inclined pipe as indicated in Fig. P5.102. Determine the following: **(a)** The difference in pressure $p_1 - p_2$. **(b)** The loss per unit mass between sections (1) and (2). **(c)** The net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).

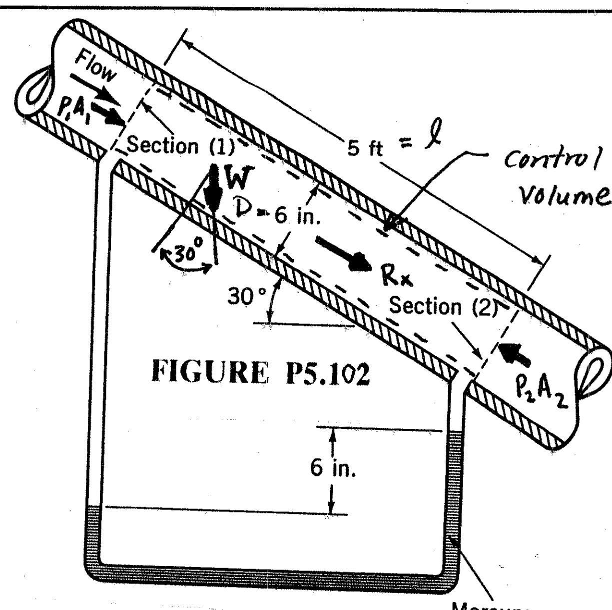


FIGURE P5.102

(a) The difference in pressure, $p_1 - p_2$, may be obtained from the manometer (see Section 2.6) with the fluid statics equation

$$p_1 - p_2 = -\gamma_{H_2O}\left[(5\ ft)\sin 30^\circ + \frac{(6\ in.)}{\left(12\ \frac{in.}{ft}\right)}\right] + \gamma_{Hg}\frac{(6\ in.)}{\left(12\ \frac{in.}{ft}\right)}$$

or

$$p_1 - p_2 = -\left(62.4\ \frac{lb}{ft^3}\right)\left[(5\ ft)\sin 30^\circ + (0.5\ ft)\right] + (13.6)\left(62.4\ \frac{lb}{ft^3}\right)(0.5\ ft) = 237\ \frac{lb}{ft^3}$$

and

$$p_1 - p_2 = 237\ \frac{lb}{ft^3}\ \frac{1}{\left(144\ \frac{in.^2}{ft^2}\right)} = \underline{\underline{1.65\ psi}}$$

(b) The loss per unit mass between sections (1) and (2) may be obtained with Eq. 5.79. Thus

$$loss = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = \left(237\ \frac{lb}{ft^2}\right)\frac{1}{\left(1.94\ \frac{slugs}{ft^3}\right)} + \left(32.2\ \frac{ft}{s^2}\right)(5\ ft)(\sin 30^\circ)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

or

$$loss = \underline{\underline{203\ \frac{ft \cdot lb}{slug}}}$$

(c) The net axial force exerted by the pipe wall on the flowing water may be obtained by using the axial component of the linear momentum equation (Eq. 5.22). Thus for the control volume shown above

$$R_x = -\frac{\pi D^2}{4}(p_1 - p_2) - \gamma\frac{\pi D^2}{4}(\ell)\sin 30^\circ = -\frac{\pi D^2}{4}\left[(p_1 - p_2) + \gamma\ell\sin 30^\circ\right]$$

or

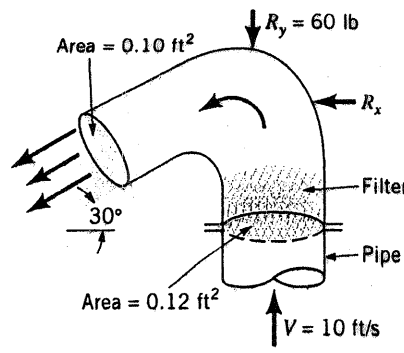
$$R_x = -\frac{\pi}{4}\left(\frac{6\ in.}{12\ \frac{in.}{ft}}\right)^2\left[237\ \frac{lb}{ft^2} + \left(62.4\ \frac{lb}{ft^3}\right)(5\ ft)\sin 30^\circ\right]$$

and

$$R_x = -77.2\ lb = \underline{\underline{77.2\ lb\ \text{opposite to flow direction.}}}$$

5.103

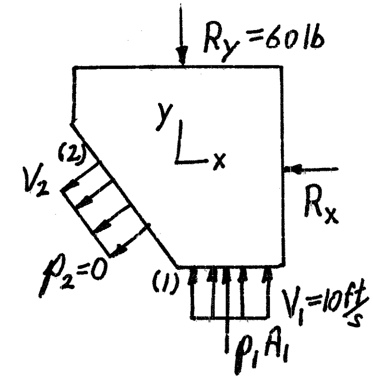
5.103 Water flows steadily in a pipe and exits as a free jet through an end cap that contains a filter as shown in Fig. P5.103. The flow is in a horizontal plane. The axial component, R_y, of the anchoring force needed to keep the end cap stationary is 60 lb. Determine the head loss for the flow through the end cap.



■ FIGURE P5.103

The y-component of the momentum equation,

$\int_{cs} v\,\rho\,\vec{V}\cdot\hat{n}\,dA = \sum F_y$, for the control volume shown is



(1) $V_1\,\rho\,(-V_1)A_1 + (-V_2 \sin 30^\circ)\,\rho\,V_2 A_2 = p_1 A_1 - R_y$

where $V_1 = 10\ \text{ft/s}$ and

$$V_2 = \frac{A_1}{A_2}V_1 = \left(\frac{0.12\ \text{ft}^2}{0.10\ \text{ft}^2}\right)(10\ \text{ft/s}) = 12\ \text{ft/s}$$

Thus, since $\rho A_1 V_1 = \rho A_2 V_2$, Eq. (1) gives

$$p_1 A_1 = R_y - \rho V_1^2 A_1 - \rho V_2^2 \sin 30^\circ A_2 = R_y - \rho A_1 V_1\left[V_1 + V_2 \sin 30^\circ\right]$$

$$= 60\ \text{lb} - \left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)(0.12\ \text{ft}^2)\left(10\ \frac{\text{ft}}{\text{s}}\right)\left[10\ \frac{\text{ft}}{\text{s}} + 12\ \frac{\text{ft}}{\text{s}}\sin 30^\circ\right] = 22.8\ \text{lb}$$

Hence,

$$p_1 = 22.8\ \text{lb}/A_1 = 22.8\ \text{lb}/(0.12\ \text{ft}^2) = 190\ \text{lb/ft}^2$$

From the energy equation for this flow,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} - h_L = \frac{V_2^2}{2g}\ , \text{ or}$$

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2 - V_2^2}{2g} = \frac{190\ \text{lb/ft}^2}{62.4\ \text{lb/ft}^3} + \frac{(10\ \text{ft/s})^2 - (12\ \text{ft/s})^2}{2\,(32.2\ \text{ft/s}^2)} = \underline{\underline{2.36\ \text{ft}}}$$

5.104

5.104 When fluid flows through an abrupt expansion as indicated in Fig. P5.104, the loss in available energy across the expansion, $loss_{ex}$, is often expressed as

$$loss_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

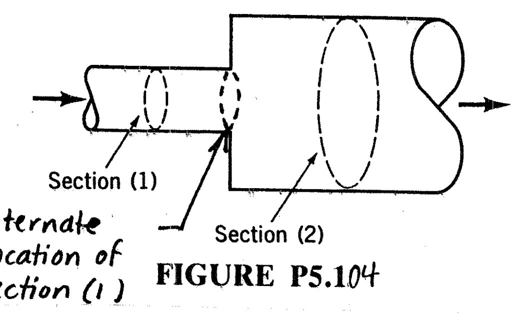


FIGURE P5.104

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$loss_{ex} = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + P_1 A_1 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + P_1 A_2 - P_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, P_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \qquad (4)$$

Combining Eqs. 1 and 4 we obtain

$$loss_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

(con't)

5.104 (con't)

From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$

9.105

5.105 Water flowing in a pipe with a velocity of 25 m/s and a static pressure of 940 kPa is split into two branches as indicated in Fig. P5.105. If the measured static pressure at sections (2) and (3) and the velocity measured at section (3) are as shown in Fig. P5.105, determine the amount of available power lost in the horizontal y connection.

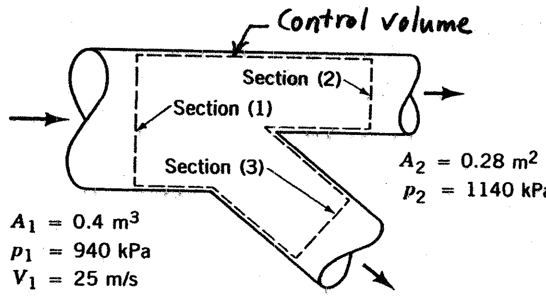


FIGURE P5.105

The control volume shown in the sketch above is used. With Eqs. 5.70 and 5.78 and the conservation of mass principle we can derive for the water that flows from section 1 to section 2 the equation

$$\dot{m}_2 ({}_1 loss_2) = \dot{m}_2 \left(\frac{p_1}{\rho} - \frac{p_2}{\rho} + \frac{V_1^2}{2} - \frac{V_2^2}{2} + g z_1 - g z_2 \right)$$

Similarly

$$\dot{m}_3 ({}_1 loss_3) = \dot{m}_3 \left(\frac{p_1}{\rho} - \frac{p_3}{\rho} + \frac{V_1^2}{2} - \frac{V_3^2}{2} + g z_1 - g z_3 \right)$$

The available power lost in the y-connection is

$$\dot{m}_2 ({}_1 loss_2) + \dot{m}_3 ({}_1 loss_3) = \dot{m}_2 \left(\frac{p_1}{\rho} - \frac{p_2}{\rho} + \frac{V_1^2}{2} - \frac{V_2^2}{2} + g z_1 - g z_2 \right)$$

$$+ \dot{m}_3 \left(\frac{p_1}{\rho} - \frac{p_3}{\rho} + \frac{V_1^2}{2} - \frac{V_3^2}{2} + g z_1 - g z_3 \right)$$

Since the y-connection is horizontal, $z_1 = z_2 = z_3$.

Also

$$\dot{m}_3 = \rho V_3 A_3 = \left(999 \frac{kg}{m^3}\right)\left(30 \frac{m}{s}\right)\left(0.18\ m^2\right) = 5400 \frac{kg}{s}$$

The velocity level at section 2 can be determined with the conservation of mass principle as follows.

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

or

$$\rho V_1 A_1 = \rho V_2 A_2 + \rho V_3 A_3$$

Thus

$$V_2 = \frac{V_1 A_1}{A_2} - \frac{V_3 A_3}{A_2} = \left(25 \frac{m}{s}\right)\frac{(0.4\ m^2)}{(0.28\ m^2)} - \left(30 \frac{m}{s}\right)\frac{(0.18\ m^2)}{(0.28\ m^2)} = 16.4 \frac{m}{s}$$

(con't)

9.105 (con't)

The mass flowrate at section 2 is

$$\dot{m}_2 = \rho V_2 A_2 = \left(999\ \frac{kg}{m^3}\right)\left(16.4\ \frac{m}{s}\right)\left(0.28\ m^2\right) = 4590\ \frac{kg}{s}$$

Thus

$$\dot{m}_2\left(loss_2\right) + \dot{m}_3\left(loss_3\right) = \left(4590\ \frac{kg}{s}\right)\left[\frac{\left(940\times10^3\ \frac{N}{m^2}\right)}{\left(999\ \frac{kg}{m^3}\right)} - \frac{\left(1140\times10^3\ \frac{N}{m^2}\right)}{\left(999\ \frac{kg}{m^3}\right)}\right.$$

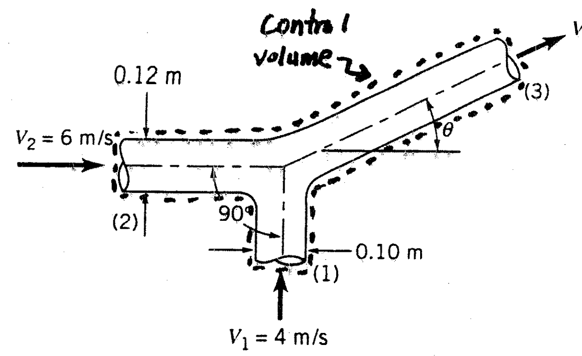
$$\left. + \frac{\left(25\ \frac{m}{s}\right)^2}{2}\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) - \frac{\left(16.4\ \frac{m}{s}\right)^2}{2}\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)\right] + \left(5400\ \frac{kg}{s}\right)\left[\frac{\left(940\times10^3\ \frac{N}{m^2}\right)}{\left(999\ \frac{kg}{m^3}\right)}\right.$$

$$\left. - \frac{\left(570\times10^3\ \frac{N}{m^2}\right)}{\left(999\ \frac{kg}{m^3}\right)} + \frac{\left(25\ \frac{m}{s}\right)^2}{2}\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) - \frac{\left(30\ \frac{m}{s}\right)^2}{2}\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)\right]$$

$$\dot{m}_2\left(loss_2\right) + \dot{m}_3\left(loss_3\right) = \underline{\underline{1.16\times10^6}}\ \frac{N\cdot m}{s} = \underline{\underline{1.16\times10^6}}\ W = \underline{\underline{1.16\ MW}}$$

5.106

5.106 Two water jets collide and form one homogeneous jet as shown in Fig. P5.106. **(a)** Determine the speed, V, and direction, θ, of the combined jet. **(b)** Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



■ FIGURE P5.106

For the water flowing through the control volume sketched above, the x- and y-direction components of the linear momentum equation are

$$-V_2 \rho V_2 A_2 + V_3 \cos\theta \, \rho V_3 A_3 = 0 \qquad (1)$$

and

$$-V_1 \rho V_1 A_1 + V_3 \sin\theta \, \rho V_3 A_3 = 0 \qquad (2)$$

From the conservation of mass principle we get

$$-\rho V_1 A_1 - \rho V_2 A_2 + \rho V_3 A_3 = 0 \qquad (3)$$

Combining Eqs. 1 and 2 we obtain

$$\tan\theta = \frac{V_1^2 A_1}{V_2^2 A_2} = \frac{V_1 \frac{\pi d_1^2}{4}}{V_2 \frac{\pi d_2^2}{4}} = \frac{\left(4\,\frac{m}{s}\right)^2 \frac{\pi (0.1\,m)^2}{4}}{\left(6\,\frac{m}{s}\right)^2 \frac{\pi (0.12\,m)^2}{4}} = 0.3086$$

so

$$\theta = \tan^{-1} 0.3086 = \underline{\underline{17.2^\circ}}$$

Now, combining Eqs. 1 and 3 we get

$$-V_2^2 \rho A_2 + V_3 \cos\theta \left(\rho V_1 A_1 + \rho V_2 A_2\right) = 0$$

or

$$V_3 = \frac{V_2^2 A_2}{\cos\theta \, (V_1 A_1 + V_2 A_2)} = \frac{V_2^2 d_2^2}{\cos\theta \, (V_1 d_1^2 + V_2 d_2^2)}$$

Thus

$$V_3 = \frac{\left(6\,\frac{m}{s}\right)^2 (0.12\,m)^2}{(\cos 17.2^\circ)\left[\left(4\,\frac{m}{s}\right)(0.1\,m)^2 + \left(6\,\frac{m}{s}\right)(0.12\,m)^2\right]}$$

and

$$V_3 = \underline{\underline{4.29\,\frac{m}{s}}}$$

(con't)

5.106 (con't)

To determine the loss of available energy associated with the flow through this control volume we obtain by applying the energy equation (Eq. 5.64)

$$-\left(\check{u}_1+\frac{V_1^2}{2}\right)\dot{m}_1-\left(\check{u}_2+\frac{V_2^2}{2}\right)\dot{m}_2+\left(\check{u}_3+\frac{V_3^2}{2}\right)\dot{m}_3=0 \qquad (4)$$

Also, the conservation of mass equation, Eq. 3, can also be written as

$$-\dot{m}_1-\dot{m}_2+\dot{m}_3=0 \qquad (5)$$

Combining Eqs. 4 and 5, we obtain

$$\dot{m}_1(\check{u}_3-\check{u}_1)+\dot{m}_2(\check{u}_3-\check{u}_2)=\dot{m}_1\left(\frac{V_1^2-V_3^2}{2}\right)+\dot{m}_2\left(\frac{V_2^2-V_3^2}{2}\right) \qquad (6)$$

The left hand side of Eq. 6 represents the rate of available energy loss in this fluid flow. Thus rate of available energy loss is

$$\text{rate of loss}=\rho V_1A_1\left(\frac{V_1^2-V_3^2}{2}\right)+\rho V_2A_2\left(\frac{V_2^2-V_3^2}{2}\right)$$

or

$$\text{rate of loss}=\frac{\rho\pi}{4}\left[d_1^2V_1\left(\frac{V_1^2-V_3^2}{2}\right)+d_2^2V_2\left(\frac{V_2^2-V_3^2}{2}\right)\right]$$

Thus

$$\text{rate of loss}=\frac{\left(999\frac{kg}{m^3}\right)(3.14)\left(1\frac{N\cdot s^2}{kg\cdot m}\right)}{4}\left\{(0.10\,m)^2\left(4\,\frac{m}{s}\right)\left[\frac{\left(4\,\frac{m}{s}\right)^2-\left(4.29\,\frac{m}{s}\right)^2}{2}\right]+(0.12\,m)^2\left(6\,\frac{m}{s}\right)\left[\frac{\left(6\,\frac{m}{s}\right)^2-\left(4.29\,\frac{m}{s}\right)^2}{2}\right]\right\}$$

and

$$\text{rate of loss}=\underline{\underline{558\ \frac{N\cdot m}{s}}}$$

5.107

5.107 The pumper truck shown in Fig. P5.107 is to deliver 1.5 ft^3/s to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in. diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.

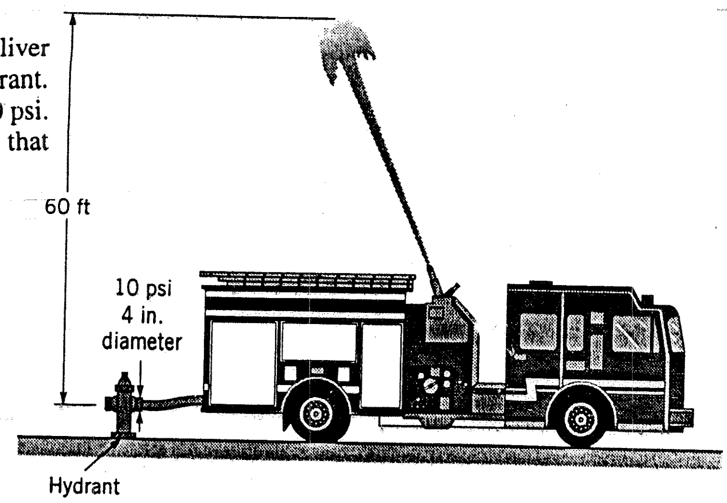


FIGURE P5.107

To solve this problem we first use the energy equation (Eq. 5.84) for flow from the hydrant exit (1) to the maximum desired elevation of 60 ft (2) to get h_s or in this case, the pump head. With the pump head we can get the pump power from Eq. 5.85.

$$\cancelto{0}{\frac{p_2}{\rho}} + \cancelto{0}{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 + h_s - \cancelto{0}{h_L}$$

$$h_s = z_2 - z_1 - \frac{p_1}{\rho} - \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{\left(1.5\,\frac{ft^3}{s}\right)(4)}{\pi\left(\frac{4\text{ in.}}{12\,\frac{\text{in.}}{ft}}\right)^2} = 17.2\,\frac{ft}{s}$$

$$h_s = 60\text{ ft} - \frac{\left(10\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(62.4\,\frac{lb}{ft^3}\right)} - \frac{\left(17.2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)}$$

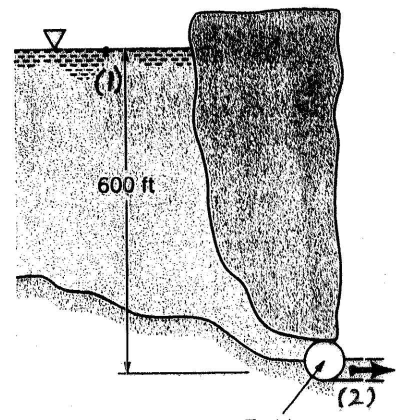
$$h_s = 32.3\text{ ft}$$

$$\dot{W}_{\substack{shaft\\ net\ in}} = \gamma Q h_s = \left(62.4\,\frac{lb}{ft^3}\right)\left(1.5\,\frac{ft^3}{s}\right)\frac{(32.2\text{ ft})}{\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}$$

$$\dot{W}_{\substack{shaft\\ net\ in}} = \underline{\underline{5.48\text{ hp}}}$$

5.108

5.108 The hydroelectric turbine shown in Fig. P5.108 passes 8 million gal/min across a head of 600 ft. What is the maximum amount of power output possible? Why will the actual amount be less?



■ FIGURE P5.108

From the energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = 0$, $p_2 = 0$, and $V_1 = 0$.

Thus,

$$h_s = (z_2 - z_1) + h_L + \frac{V_2^2}{2g}$$

Note: Since this is a turbine, $h_s < 0$. Let $h_T = -h_s$, where $h_T > 0$ and from the above,

$$h_T = (z_1 - z_2) - h_L - \frac{V_2^2}{2g}$$

Also, the power is given by

$$\dot{W}_{turb} = \gamma Q h_T = \gamma Q\left[(z_1 - z_2) - h_L - \frac{V_2^2}{2g}\right]$$

The maximum power would occur if there were no losses ($h_L = 0$) and negligible kinetic energy at the exit ($V_2 \approx 0$; large diameter outlet).

Thus,

$$\dot{W}_{turb\,max} = \gamma Q(z_1 - z_2) = 62.4\,\frac{lb}{ft^3}\left(8\times10^6\,\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{1\,ft^3}{7.48\,gal}\right)(600\,ft)$$

$$= 6.67\times10^8\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{1.21\times10^6\,hp}}$$

5.109

5.109 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.109 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.

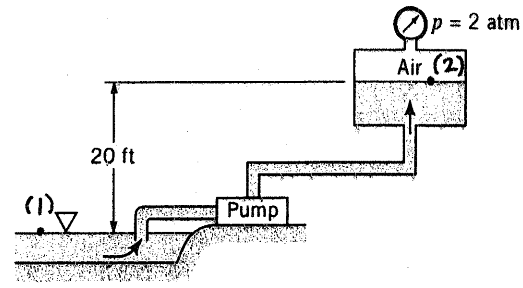


FIGURE P5.109

$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$, where $p_1 = 0$, $z_1 = 0$, $V_1 = 0$, and $z_2 = 20 ft$.

Thus,

(1) $h_p = h_L + \frac{p_2}{\gamma} + z_2$

Also,

$Q = [(1000 gal)/(10 min)]\left(\frac{1 ft^3}{7.48 gal}\right)\left(\frac{1 min}{60 s}\right) = 0.223 \frac{ft^3}{s}$

so that

$h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{(3 hp)(550 \frac{ft \cdot lb/s}{hp})}{(62.4 \frac{lb}{ft^3})(0.223 \frac{ft^3}{s})} = 119 ft$

(a) If $p_2 = 2 atm = 2(14.7 \frac{lb}{in.^2})(144 in.^2/ft^2) = 4{,}230 \frac{lb}{ft^2}$, then from Eq. (1)

$h_p = h_L + \frac{4{,}230 \frac{lb}{ft^2}}{(62.4 \frac{lb}{ft^3})} + 20 ft = h_L + 87.8 ft$

Thus, if

$h_L \leq h_p - 87.8 ft = 119 ft - 87.8 ft = 31.2 ft$ the given pump will work for $p_2 = 2 atm$.

(b) If $p_2 = 3 atm = 6{,}350 \frac{lb}{ft^2}$, then

$h_p = h_L + \frac{6{,}350 \frac{lb}{ft^2}}{(62.4 \frac{lb}{ft^3})} + 20 ft = h_L + 122 ft$

Thus, if this pump is to work

$119 ft = h_L + 122 ft$, or $h_L \leq -3 ft$

Since it is not possible to have $h_L < 0$, the pump will not work for $p_2 = 3 atm$.

5.110

5.110 Water is supplied at 150 ft³/s and 60 psi to a hydraulic turbine through a 3-ft inside diameter inlet pipe as indicated in Fig. P5.110. The turbine discharge pipe has a 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections (1) and (2).

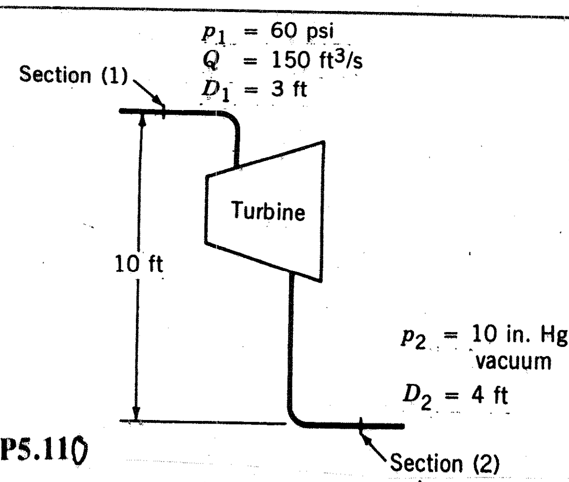


FIGURE P5.110

For flow between sections (1) and (2), Eq. 5.82 leads to

$$\text{power loss} = \rho Q\left[\left(\frac{P_1-P_2}{\rho}\right)+g(z_1-z_2)+\left(\frac{V_1^2-V_2^2}{2}\right)\right]-\dot{W}_{\substack{\text{shaft}\\\text{net out}}} \qquad (1)$$

From given data

$$P_2 = \frac{(-10\text{ in. Hg})}{\left(12\,\frac{\text{in.}}{\text{ft}}\right)}(13.6)\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right) = -708\,\frac{\text{lb}}{\text{ft}^2}$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(4)\left(150\,\frac{\text{ft}^3}{\text{s}}\right)}{\pi\,(3\text{ ft})^2} = 21.22\,\frac{\text{ft}}{\text{s}}$$

From conservation of mass (Eq. 5.13)

$$V_2 = V_1\frac{A_1}{A_2} = V_1\frac{D_1^2}{D_2^2} = \left(21.22\,\frac{\text{ft}}{\text{s}}\right)\frac{(3\text{ ft})^2}{(4\text{ ft})^2} = 11.94\,\frac{\text{ft}}{\text{s}}$$

From Eq. 1

$$\text{power loss} = \frac{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(150\,\frac{\text{ft}^3}{\text{s}}\right)}{\left(550\,\frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}\right)}\left\{\frac{\left(60\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)+\left(708\,\frac{\text{lb}}{\text{ft}^2}\right)}{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)} + \left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)(10\text{ ft})\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right) + \left[\frac{\left(21.22\,\frac{\text{ft}}{\text{s}}\right)^2-\left(11.94\,\frac{\text{ft}}{\text{s}}\right)^2}{2}\right]\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)\right\}$$

$$-2500\text{hp}$$

or

$$\text{power loss} = \underline{\underline{301}}\text{ hp}$$

5.111

5.111 Gasoline ($SG = 0.68$) flows through a pump at $0.12\ m^3/s$ as indicated in Fig. P5.111. The loss between sections (1) and (2) is loss $= h_L g = 0.3\ V_1^2/2$. What will the difference in pressures between sections (1) and (2) be if 20 kW is delivered by the pump to the fluid?

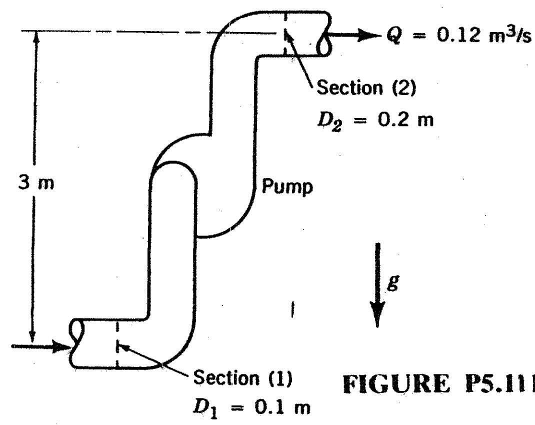


FIGURE P5.111

From Eq. 5.82 we get for the flow from section (1) to section (2)

$$p_1 - p_2 = \rho\left[\frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) - w_{\substack{shaft\\net\ in}} + loss\right] \qquad (1)$$

From the volume flowrate we obtain

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{\left(0.12\ \frac{m^3}{s}\right)^2}{\frac{\pi\,(0.2\,m)^2}{4}} = 3.82\ \frac{m}{s}$$

and from conservation of mass (Eq. 5.13) it follows that

$$V_1 = V_2\,\frac{A_2}{A_1} = V_2\,\frac{D_2^2}{D_1^2} = \left(3.82\ \frac{m}{s}\right)\frac{(0.2\,m)^2}{(0.1\,m)^2} = 15.28\ \frac{m}{s}$$

Also

$$w_{\substack{shaft\\net\ in}} = \frac{\dot{W}_{\substack{shaft\\net\ in}}}{\rho Q} = \frac{\left(20{,}000\ \frac{N\cdot m}{s}\right)}{(0.68)\left(999\ \frac{kg}{m^3}\right)\left(0.12\ \frac{m^3}{s}\right)} = 245.3\ \frac{N\cdot m}{kg}$$

and

$$loss = 0.3\,\frac{V_1^2}{2} = \frac{(0.3)\left(15.28\ \frac{m}{s}\right)^2}{2}\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) = 35.02\ \frac{N\cdot m}{kg}$$

From Eq. 1 then

$$p_1 - p_2 = (0.68)\left(999\ \frac{kg}{m^3}\right)\left\{\left[\frac{\left(3.82\ \frac{m}{s}\right)^2 - \left(15.28\ \frac{m}{s}\right)^2}{2} + \left(9.81\ \frac{m}{s^2}\right)(3\,m)\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) - 245.3\ \frac{N\cdot m}{kg} + 35.02\ \frac{N\cdot m}{kg}\right\}$$

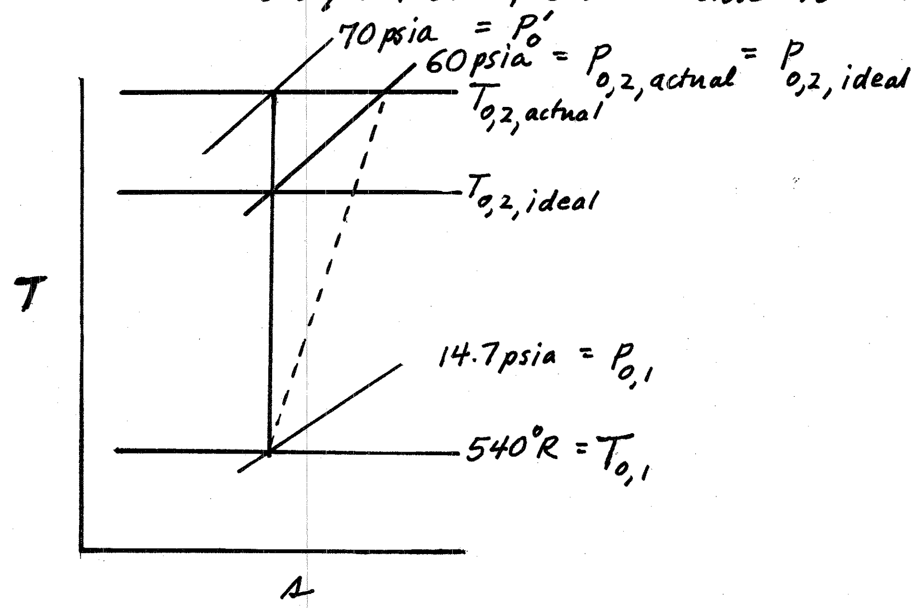
or

$$p_1 - p_2 = \underline{\underline{-197{,}000\ \frac{N}{m^2}}} = \underline{\underline{-197\ kPa}}$$

5.112

5.112 A centrifugal air compressor stage operates between an inlet stagnation pressure of 14.7 psia and an exit stagnation pressure of 60 psia. The inlet stagnation temperature is 80 °F. If the loss of total pressure through the compressor stage associated with irreversible flow phenomena is 10 psi, calculate the actual and ideal stagnation temperature rise through the compressor. Calculate the ratio of ideal to actual temperature rise to obtain efficiency.

We assume that the air compressor operates adiabatically. An ideal compression process is frictionless and adiabatic and thus according to Eq. 5.101, it is a constant entropy or isentropic process. With Eq. 5.101 we also conclude that an actual adiabatic compression process with friction must involve an entropy increase. On temperature - entropy coordinates, the ideal and actual compression processes appear as indicated in the sketch below. Also shown is the 10 psi loss in stagnation pressure due to friction.



We consider the air being compressed to behave as an ideal gas. Then from Eqs. 1.8 and 5.111 we obtain for the ideal processes

$$T_{0,2,\,ideal} = T_{0,1}\left(\frac{p_{0,2,\,ideal}}{p_{0,1}}\right)^{\frac{k-1}{k}} = (540°R)\left(\frac{60\text{ psia}}{14.7\text{ psia}}\right)^{\frac{1.4-1}{1.4}} = 807\ °R$$

and

$$T_{0,2,\,actual} = T_{0,1}\left(\frac{p_0'}{p_{0,1}}\right)^{\frac{1.4-1}{0.4}} = (540°R)\left(\frac{70\text{ psia}}{14.7\text{ psic}}\right)^{\frac{1.4-1}{1.4}} = 843\ °R$$

(con't)

5.112 (Con't)

Then

actual stagnation temperature rise $= T_{0,2,actual} - T_{0,1} = 843^{\circ}R - 540^{\circ}R = \underline{\underline{303^{\circ}R}}$

and

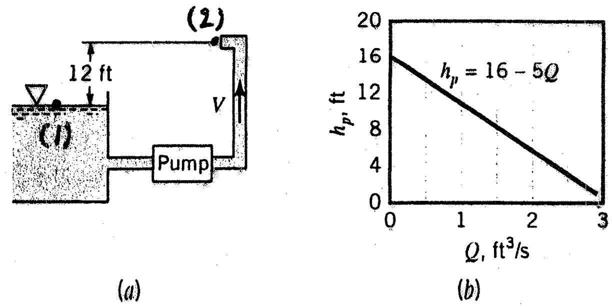
ideal stagnation temperature rise $= T_{0,2,ideal} - T_{0,1} = 807^{\circ}R - 540^{\circ}R = \underline{\underline{267^{\circ}R}}$

Also

$$\text{efficiency} = \frac{T_{0,2,ideal} - T_{0,1}}{T_{0,2,actual} - T_{0,1}} = \frac{267^{\circ}R}{303^{\circ}R} = \underline{\underline{0.88}}$$

5.114

5.114 Water is pumped through a 4-in.-diameter pipe as shown in Fig. P5.114*a*. The pump characteristics (pump head versus flowrate) are given in Fig. P5.114*b*. Determine the flowrate if the head loss in the pipe is $h_L = 8V^2/2g$.



■ **FIGURE P5.114**

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ z_1 = 0,\ z_2 = 12\text{ ft},$$
$$V_1 = 0, \text{ and } V_2 = Q/A_2$$

Thus,

$$h_s - h_L = z_2 + \frac{V_2^2}{2g}, \text{ with}$$

$$h_s = h_p = 16 - 5Q \text{ and } h_L = 8\frac{V_2^2}{2g} = 8\frac{Q^2}{2gA_2^2}$$

Therefore,

$$16 - 5Q - \frac{4Q^2}{gA_2^2} = 12 + \frac{Q^2}{2gA_2^2}$$

or

(1) $$\left(\frac{9}{2gA_2^2}\right)Q^2 + (5)Q - 4 = 0, \text{ where } g \sim \frac{\text{ft}}{\text{s}^2},\ A_2 \sim \text{ft}^2, \text{ and } Q \sim \frac{\text{ft}^3}{\text{s}}$$

Using the given data, Eq. (1) becomes

$$\left[\frac{9}{2(32.2)\left(\frac{\pi}{4}\left(\frac{4}{12}\right)^2\right)^2}\right]Q^2 + 5Q - 4 = 0$$

or

(2) $$18.35\,Q^2 + 5Q - 4 = 0$$

The positive root of Eq. (2) is $\underline{\underline{Q = 0.350 \frac{\text{ft}^3}{\text{s}}}}$

(The negative root of Eq. (2) has no physical meaning.)

5.115

5.115 Water is pumped from the large tank shown in Fig. P5.115. The head loss is known to be equal to $4V^2/2g$ and the pump head is $h_p = 20 - 4Q^2$, where h_p is in ft when Q is in ft^3/s. Determine the flowrate.

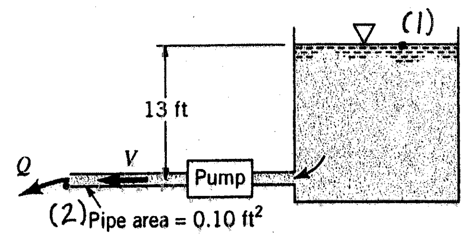


FIGURE P5.115

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ z_1 = 13\text{ ft},\ z_2 = 0,$$
and $V_1 = 0$.

Thus,

(1) $$z_1 + h_p - h_L = \frac{V_2^2}{2g}$$

Also,

$$h_L = 4\frac{V^2}{2g} = 4\frac{V_2^2}{2g} = 4\frac{(Q/A_2)^2}{2g} \quad \text{since } V_2 = \frac{Q}{A_2}$$

Hence, Eq. (1) becomes

$$z_1 + (20 - 4Q^2) - 4\frac{(Q/A_2)^2}{2g} = \frac{(Q/A_2)^2}{2g}$$

or

$$\left[\left(\frac{5}{2g A_2^2}\right) + 4\right] Q^2 = 20 + z_1 \text{ , where } g \sim \frac{\text{ft}}{\text{s}^2},\ A_2 \sim \text{ft}^2, \text{ and } Q \sim \frac{\text{ft}^3}{\text{s}}$$

Thus, with the given data

$$\left[\left(\frac{5}{2(32.2\frac{\text{ft}}{\text{s}^2})(0.1\text{ ft}^2)^2}\right) + 4\right] Q^2 = 20 + 13\text{ ft}$$

or

$$\underline{\underline{Q = 1.67 \frac{\text{ft}^3}{\text{s}}}}$$

5.116

5.116 Water flows by gravity from one lake to another as sketched in Fig. P5.116 at the steady rate of 80 gpm. What is the loss in available energy associated with this flow? If this same amount of loss is associated with pumping the fluid from the lower lake to the higher one at the same flowrate, estimate the amount of pumping power required.

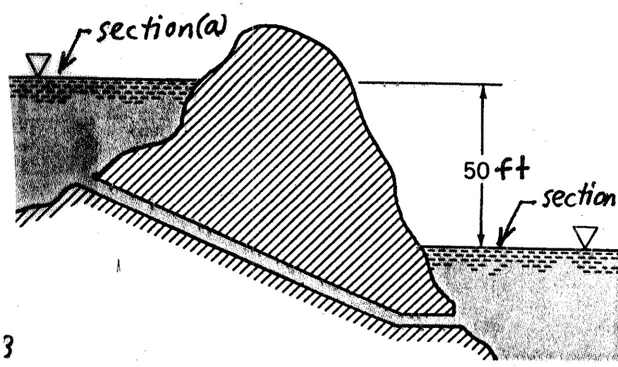


FIGURE P5.116

$$Q = \frac{80 \frac{gal}{min}}{\left(60 \frac{s}{min}\right)\left(7.48 \frac{gal}{ft^3}\right)} = 0.178 \frac{ft^3}{s}$$

For the flow from section (a) to section (b) Eq. 5.82 leads to

$$loss = g(z_a - z_b) = \left(32.2 \frac{ft}{s^2}\right)(50 ft)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{1610}} \frac{ft\ lb}{slug}$$

For pumped flow from section b to section a Eq. 5.82 yields

$$\dot{W}_{\substack{shaft \\ net\ in}} = \rho Q\left[g(z_a - z_b) + loss\right] = \left(1.94 \frac{slugs}{ft^3}\right)\left(0.178 \frac{ft^3}{s}\right)\left[\left(32.2 \frac{ft}{s^2}\right)(50 ft)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) + 1610 \frac{ft \cdot lb}{slug}\right]$$

or

$$\dot{W}_{\substack{shaft \\ net\ in}} = \underline{\underline{1110}} \frac{ft \cdot lb}{s} = \underline{\underline{2.02\ hp}}$$

5.117

5.117 A $\frac{3}{4}$-hp motor is required by an air ventilating fan to produce a 24-in.-diameter stream of air having a uniform speed of 40 ft/s. Determine the aerodynamic efficiency of the fan.

The aerodynamic efficiency of the fan, η, is

$$\eta = \frac{\text{ideal power required}}{\text{actual power required}}$$

The actual shaft power required, $\dot{W}_{actual}$, is 0.75 hp.
The ideal shaft power required, $\dot{W}_{ideal}$, is obtained from Eq. 5.82 for flow without loss across the fan. Thus

$$\dot{W}_{ideal} = \dot{m}\frac{V_{out}^2}{2} = \rho A_{out} V_{out} \frac{V_{out}^2}{2} = \rho \frac{\pi D_{out}^2}{4}\frac{V_{out}^3}{2} = \left(2.38 \times 10^{-3} \frac{slug}{ft^3}\right)\frac{\pi (2 ft)^2}{4} \times \frac{\left(40 \frac{ft}{s}\right)^3}{2}\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\left(\frac{1}{550 \frac{ft \cdot lb}{s \cdot hp}}\right)$$

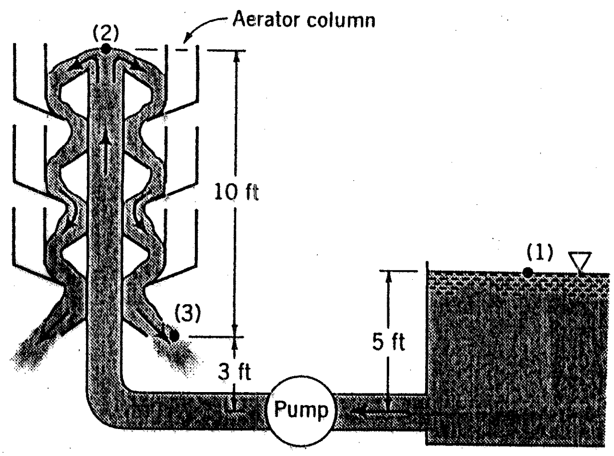
or $\dot{W}_{ideal} = 0.435\ hp$

Then

$$\eta = \frac{0.435\ hp}{0.75\ hp} = \underline{\underline{0.58}}$$

5.118

5.118 Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in **Video V5.8** and **Fig.** P5.118 at a rate of 3.0 ft^3/s. **(a)** Determine the power that the pump adds to the water if the head loss from (1) to (2) where $V_2 = 0$ is 4 ft. **(b)** Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $V_3 = 2$ ft/s.



■ FIGURE P5.118

(a) The energy equation from (1) to (2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with

$p_1 = p_2 = V_1 = V_2 = 0$ gives

$$h_p = h_L + z_2 - z_1 = 4\,ft + (10+3)\,ft - 5\,ft = 12\,ft$$

Thus, the pump power is

$$\dot{W}_s = \gamma Q h_s = 62.4\,\frac{lb}{ft^3}\left(3\,\frac{ft^3}{s}\right)(12\,ft) = 2246\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{4.08\,hp}}$$

(b) The energy equation from (2) to (3)

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_p - h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

with

$p_2 = p_3 = V_2 = h_p = 0$ gives

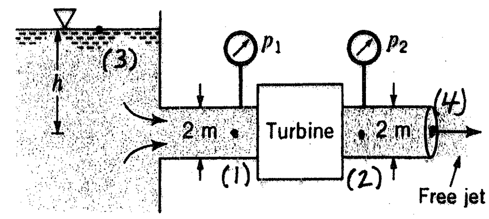
$$h_L = z_2 - z_3 - \frac{V_3^2}{2g} = 13\,ft - 3\,ft - \frac{\left(2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 10\,ft - 0.062\,ft$$

or

$$h_L = \underline{\underline{9.94\,ft}}$$

5.119

5.119 The turbine shown in Fig. P5.119 develops 2500 kW when the water flowrate is 20 m^3/s. The head loss across the turbine from (1) to (2) is negligible, but the head loss for the entire flow is 2.5 m. (a) Determine the pressure difference, $p_1 - p_2$, across the turbine. (b) Determine the elevation h.



■ FIGURE P5.119

(a) The energy equation across the turbine is

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_s-h_{L_{1-2}}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}\ ,\ \text{where } z_1=z_2,\ h_{L_{1-2}}=0,\ V_1=V_2$$

Thus,

$$\frac{p_1}{\gamma}+h_s=\frac{p_2}{\gamma}\quad \text{or}$$

$$p_1-p_2=-\gamma h_s\ ,\ \text{where }\ h_s=\frac{\dot{W}_s}{\gamma Q}$$

Hence,

$$p_1-p_2=-\gamma\left(\frac{\dot{W}_s}{\gamma Q}\right)=-\frac{\dot{W}_s}{Q}=-\left(\frac{-2500\times10^3\ \frac{N\cdot m}{s}}{20\ \frac{m^3}{s}}\right)=125\times10^3\frac{N}{m^2}=\underline{\underline{125\ kPa}}$$

(Note: $\dot{W}_s<0$ because a turbine removes energy from the fluid.)

(b) Also, from (3) to (4)

$$\frac{p_3}{\gamma}+z_3+\frac{V_3^2}{2g}+h_s-h_{L_{3-4}}=\frac{p_4}{\gamma}+z_4+\frac{V_4^2}{2g}\ ,\ \text{where } p_3=p_4=0,\ z_4=0,\ V_3=0,$$
and $z_3=h$

Thus,

$$h+h_s-h_{L_{3-4}}=\frac{V_4^2}{2g}\ ,\ \text{or}$$

$$h=\frac{V_4^2}{2g}+h_{L_{3-4}}-h_s \qquad (1)$$

Also, $V_4=\dfrac{Q}{A_4}=\dfrac{20\frac{m^3}{s}}{\left(\frac{\pi}{4}(2m)^2\right)}=6.37\frac{m}{s}$, $h_{L_{3-4}}=2.5m$, and

$$h_s=\frac{\dot{W}_s}{\gamma Q}=\frac{-2500\times10^3\ \frac{N\cdot m}{s}}{\left(9.83\times10^3\frac{N}{m^3}\right)\left(20\frac{m^3}{s}\right)}=-12.76m$$

Therefore, from Eq. (1)

$$h=\frac{(6.37\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}+2.5m-(-12.76m)=\underline{\underline{17.3\ m}}$$

5.120

5.120 A liquid enters a fluid machine at section (1) and leaves at sections (2) and (3) as shown in Fig. P5.120. The density of the fluid is constant at 2 slugs/ft^3. All of the flow occurs in a horizontal plane and is frictionless and adiabatic. For the above-mentioned and additional conditions indicated in Fig. P5.120, determine the amount of shaft power involved.

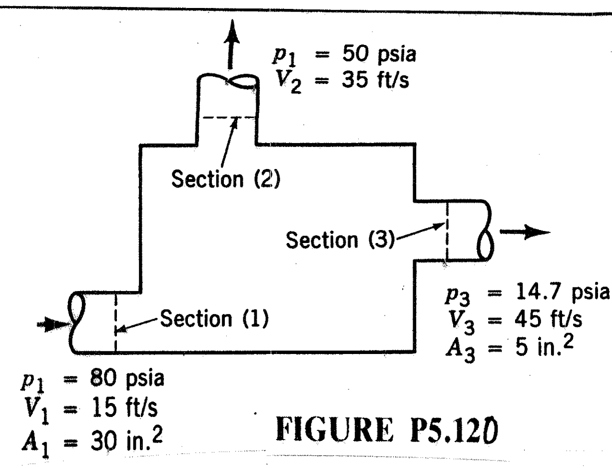


FIGURE P5.120

For the frictionless and adiabatic flow through this fluid machine Eqs. 5.64, 5.65 and 5.76 lead to

$$\dot{W}_{\substack{shaft\\ net\ in}} = \dot{m}_3\left(\frac{p_3}{\rho}+\frac{V_3^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho}+\frac{V_1^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho}+\frac{V_2^2}{2}\right) \qquad (1)$$

since

$$\dot{m}_1\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = (\dot{m}_2+\dot{m}_3)\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = \dot{m}_2(\check{u}_1-\check{u}_2) + \dot{m}_3(\check{u}_1-\check{u}_2) = 0$$

At section (3)

$$\dot{m}_3 = \rho A_3 V_3 = \left(2\ \frac{slugs}{ft^3}\right)\left(\frac{5\ in.^2}{144\ \frac{in.^2}{ft^2}}\right)\left(45\ \frac{ft}{s}\right) = 3.125\ \frac{slugs}{s}$$

At section (1)

$$\dot{m}_1 = \rho A_1 V_1 = \left(2\ \frac{slugs}{ft^3}\right)\left(\frac{30\ in.^2}{144\ \frac{in.^2}{ft^2}}\right)\left(15\ \frac{ft}{s}\right) = 6.25\ \frac{slugs}{s}$$

From conservation of mass

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 6.25\ \frac{slugs}{s} - 3.125\ \frac{slugs}{s} = 3.125\ \frac{slugs}{s}$$

With Eq. 1 we obtain

$$\dot{W}_{\substack{shaft\\ net\ in}} = \Bigg\{\left(3.125\ \frac{slugs}{s}\right)\left[\frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(45\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]$$
$$-\left(6.25\ \frac{slugs}{s}\right)\left[\frac{\left(80\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(15\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]$$
$$+\left(3.125\ \frac{slugs}{s}\right)\left[\frac{\left(50\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(35\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]\Bigg\}\left(\frac{1}{550\ \frac{ft\cdot lb}{s\cdot hp}}\right)$$

or

$$\dot{W}_{\substack{shaft\\ net\ in}} = \underline{\underline{-31.1\ hp}}$$, the net shaft power is out (<0)

5.121

5.121 Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.121. The loss in available energy associated with 2.5 ft³/s being pumped from sections (1) to (2) is $61\overline{V}^2/2$ where $\overline{V}$ is the average velocity of water in the 8-in.-inside diameter piping involved. Determine the amount of shaft power required.

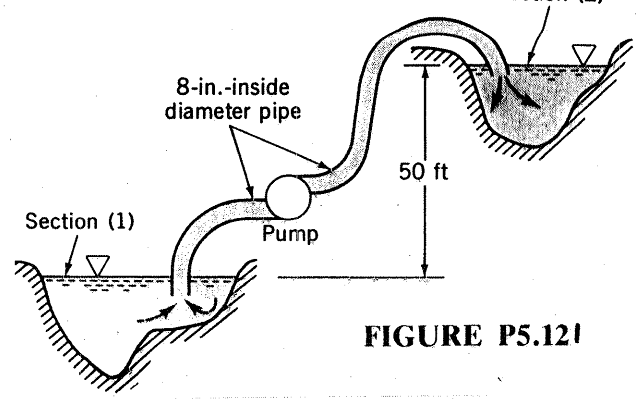


FIGURE P5.121

For the flow from section (1) to section (2) Eq. 5.82 leads to

$$\dot{W}_{\substack{shaft\\net\ in}} = \rho Q\left[g(z_2 - z_1) + loss\right] = \rho Q\left[g(z_2 - z_1) + 61\frac{\overline{V}^2}{2}\right] \quad (1)$$

From the volume flowrate we obtain

$$\overline{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{\left(2.5\ \frac{ft^3}{s}\right)}{\frac{\pi}{4}\left(\frac{8\ in.}{12\ \frac{in.}{ft}}\right)^2} = 7.162\ \frac{ft}{s}$$

Thus, from Eq. 1

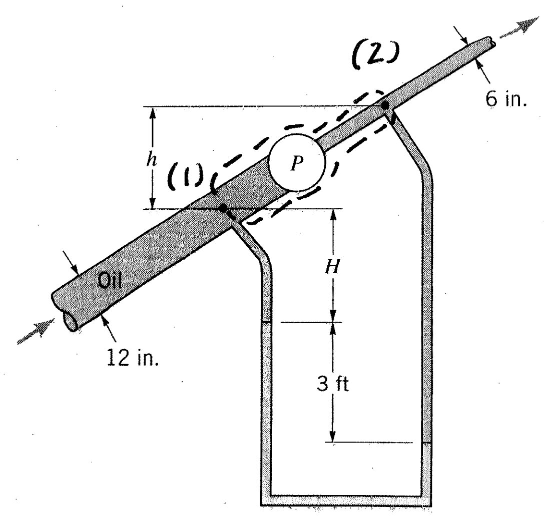
$$\dot{W}_{\substack{shaft\\net\ in}} = \left(1.94\ \frac{slugs}{ft^3}\right)\left(2.5\ \frac{ft^3}{s}\right)\left[\left(32.2\ \frac{ft}{s^2}\right)(50\ ft) + \frac{(61)\left(7.162\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\left(\frac{1}{550\ \frac{ft\cdot lb}{s\cdot hp}}\right)$$

or

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{28\ hp}}$$

5.122

5.122 Oil ($SG = 0.88$) flows in an inclined pipe at a rate of 5 ft^3/s as shown in Fig. P5.122. If the differential reading in the mercury manometer is 3 ft, calculate the power that the pump supplies to the oil if head losses are negligible.



■ FIGURE P5.122

Using the control volume shown and the energy equation (Eq. 5.84) we get:

$$\frac{p_2}{\gamma_{oil}} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + z_1 + h_s - h_L \quad (1)$$

The power supplied by the pump to the oil is, from Eq. 5.85:

$$\dot{W}_{\substack{shaft\\net\ in}} = \gamma_{oil} Q h_s = SG_{oil}\, \gamma_{H_2O}\, Q h_s \quad (2)$$

Since $V = \frac{Q}{A} = \frac{Q}{\frac{\pi d^2}{4}}$ we get

$$V_1 = \frac{5\ \frac{ft^3}{s}}{\frac{\pi}{4}(1\ ft)^2} = 6.37\ \frac{ft}{s} \quad \text{and} \quad V_2 = \frac{5\ \frac{ft^3}{s}}{\frac{\pi}{4}(\frac{1}{2}\ ft)^2} = 25.5\ \frac{ft}{s}$$

From the manometer equation (see section 2.6) we get:

$$p_1 + \gamma_{oil} H + 3\gamma_{Hg} - (3 + H + h)\gamma_{oil} = p_2$$

Thus

$$\frac{p_1}{\gamma_{oil}} + H + 3\frac{\gamma_{Hg}}{\gamma_{oil}} - (3 + H + h) = \frac{p_2}{\gamma_{oil}}$$

or

$$\frac{p_1}{\gamma_{oil}} + 3\frac{\gamma_{Hg}}{\gamma_{oil}} - 3 - h = \frac{p_2}{\gamma_{oil}} \quad (3)$$

Combining Eqs. (1) and (3) we get:

$$\frac{p_1}{\gamma_{oil}} + (3\ ft)\frac{SG_{Hg}}{SG_{oil}} - (3\ ft) - h + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + z_1 + h_s - \cancel{h_L}^{\,0}$$

or

$$h_s = z_2 - z_1 + 3\ ft\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - h + \frac{V_2^2 - V_1^2}{2g} = (3\ ft)\left(\frac{13.6}{0.88} - 1\right) + \frac{\left(25.5\frac{ft}{s}\right)^2 - \left(6.37\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)}$$

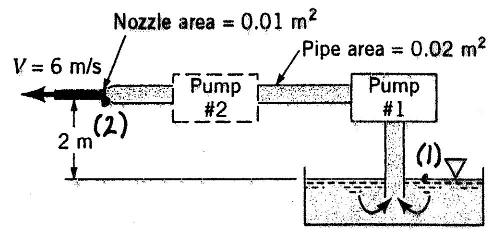
$$h_s = 52.9\ ft$$

Finally from Eq. (2)

$$\dot{W}_{\substack{shaft\\net\ in}} = (0.88)\left(62.4\ \frac{lb}{ft^3}\right)\left(5\ \frac{ft^3}{s}\right)(52.9\ ft) = \underline{\underline{14,500\ \frac{ft \cdot lb}{s}}}$$

5.123

5.123 Water is to be pumped from the large tank shown in Fig. P5.123 with an exit velocity of 6 m/s. It was determined that the original pump (pump 1) that supplies 1 kW of power to the water did not produce the desired velocity. Hence, it is proposed that an additional pump (pump 2) be installed as indicated to increase the flowrate to the desired value. How much power must pump 2 add to the water? The head loss for this flow is $h_L = 250Q^2$, where h_L is in m when Q is in m^3/s.



■ FIGURE P5.123

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0,\ V_1 = 0,\ z_1 = 0,\ z_2 = 2m.$

Thus,

$h_p = h_L + z_2 + \frac{V_2^2}{2g}$, where $V_2 = 6 m/s$ so that $Q = A_2 V_2 = 0.01 m^2 (6\ m/s)$
$= 0.06\ m^3/s$

Note: $h_p = h_{pump1} + h_{pump2}$

Thus, with $h_L = 250\,Q^2 = 250\,(0.06)^2 = 0.90 m$ it follows that

$$h_p = 0.90m + 2m + \frac{(6\ m/s)^2}{2(9.81\ \frac{m}{s^2})} = 4.73m$$

so that

$$\dot{W}_p = \gamma Q h_p = (9.80 \times 10^3 \tfrac{N}{m^3})(0.06 \tfrac{m^3}{s})(4.73m) = 2.78 \times 10^3 \tfrac{N \cdot m}{s} = 2.78\ kW$$

Therefore,

$\dot{W}_p = \dot{W}_{pump1} + \dot{W}_{pump2} = 2.78\ kW$, with $\dot{W}_{pump1} = 1\ kW$

Hence,

$$\dot{W}_{pump2} = 2.78\ kW - 1\ kW = \underline{\underline{1.78\ kW}}$$

5.124

5.124 The velocity profile in a turbulent pipe flow may be approximated with the expression

$$\frac{u}{u_c} = \left(\frac{R-r}{R}\right)^{1/n}$$

where u = local velocity in the axial direction, u_c = centerline velocity in the axial direction, R = pipe inner radius from pipe axis, r = local radius from pipe axis, and n = constant. Determine the kinetic energy coefficient, α, for: **(a)** $n = 5$; **(b)** $n = 6$; **(c)** $n = 7$; **(d)** $n = 8$; **(e)** $n = 9$; **(f)** $n = 10$.

For the kinetic energy coefficient, α, we may use Eq. 5.86. Thus,

$$\alpha = \frac{\int_0^R \frac{u^2}{2}\rho u\, 2\pi r\,dr}{\rho \bar{u}\,\pi R^2 \frac{\bar{u}^2}{2}} = \frac{2\int_0^1 u^3\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)}{\bar{u}^3} = \frac{2u_c^3\int_0^1\left(1-\frac{r}{R}\right)^{\frac{3}{n}}\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)}{\bar{u}^3} \quad (1)$$

For the average velocity, $\bar{u}$, we may use Eq. 5.7. Thus,

$$\bar{u} = \frac{\int_0^R \rho u\, 2\pi r\,dr}{\rho\pi R^2} = 2\int_0^1 u\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) = 2u_c\int_0^1\left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) \quad (2)$$

To facilitate the integrations we make the substitution

$$\beta = 1-\frac{r}{R} \quad (3)$$

Thus,

$$d\beta = -d\left(\frac{r}{R}\right) \quad (4)$$

and Eq. 2 becomes

$$\bar{u} = -2u_c\int_1^0 \beta^{\frac{1}{n}}(1-\beta)\,d\beta = \frac{2n^2}{(n+1)(2n+1)}\,u_c \quad (5)$$

Combining Eqs. 1, 3, 4 and 5 we obtain

$$\alpha = \frac{-2\int_1^0 \beta^{\frac{3}{n}}(1-\beta)\,d\beta}{\left[\frac{2n^2}{(n+1)(2n+1)}\right]^3} = \left[\frac{2n^2}{(3+n)(3+2n)}\right]\left[\frac{(n+1)(2n+1)}{2n^2}\right]^3 \quad (6)$$

(a) For $n = 5$, Eq. 6 yields

$$\alpha = \left\{\frac{2(5)^2}{(3+5)[3+2(5)]}\right\}\left\{\frac{(5+1)[(2)(5)+1]}{2(5)^2}\right\}^3 = \underline{\underline{1.11}}$$

(b) For $n = 6$

$\alpha = \underline{\underline{1.08}}$

(c) For $n = 7$

$\alpha = \underline{\underline{1.06}}$

(d) For $n = 8$

$\alpha = \underline{\underline{1.05}}$

(e) For $n = 9$

$\alpha = \underline{\underline{1.04}}$

(f) For $n = 10$

$\alpha = \underline{\underline{1.03}}$

5.125

5.125 A small fan moves air at a mass flowrate of 0.004 lbm/s. Upstream of the fan, the pipe diameter is 2.5 in., the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1, is equal to 2.0. Downstream of the fan, the pipe diameter is 1 in., the flow is turbulent, the velocity profile is quite flat, and the kinetic energy coefficient, α_2, is equal to 1.08. If the rise in static pressure across the fan is 0.015 psi and the fan shaft draws 0.00024 hp, compare the value of loss calculated: **(a)** assuming uniform velocity distributions; **(b)** considering actual velocity distributions.

(a) For uniform velocity distributions upstream and downstream of the fan, Eq. 5.82 is applicable. Thus,

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g(\underbrace{z_{in} - z_{out}}_{0 \text{ for air}}) + w_{\substack{shaft\\net\ in}} \qquad (1)$$

We obtain the shaft work, $w_{\substack{shaft\\net\ in}}$ from the given shaft power, $\dot{W}_{\substack{shaft\\net\ in}}$, with

$$w_{\substack{shaft\\net\ in}} = \frac{\dot{W}_{\substack{shaft\\net\ in}}}{\dot{m}} = \frac{(0.00024\ hp)\left(550\ \frac{ft\cdot lb}{s\cdot hp}\right)}{0.004\ \frac{lbm}{s}} = 33\ \frac{ft\cdot lb}{lbm}$$

For V_{in} and V_{out} we use Eq. 5.11. Thus,

$$V_{in} = \frac{\dot{m}}{\rho A_{in}} = \frac{\dot{m}}{\rho \frac{\pi D_{in}^2}{4}} = \frac{\left(0.004\ \frac{lbm}{s}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)\frac{\pi(2.5\ in.)^2}{4}} = 1.53\ \frac{ft}{s}$$

and

$$V_{out} = \frac{\dot{m}}{\rho A_{out}} = \frac{\dot{m}}{\rho \frac{\pi D_{out}^2}{4}} = \frac{\left(0.004\ \frac{lbm}{s}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)\frac{\pi(1\ in.)^2}{4}} = 9.57\ \frac{ft}{s}$$

Now from Eq. 1 we obtain

$$\text{loss} = \frac{(-0.015\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38\times10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)} + \left[\frac{\left(1.53\ \frac{ft}{s}\right)^2 - \left(9.57\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} + 33\ \frac{ft\cdot lb}{lbm}$$

or

$$\text{loss} = \underline{\underline{3.43\ \frac{ft\cdot lb}{lbm}}}$$

(b) For non-uniform velocity distributions upstream and downstream of the fan Eq. 5.87 is applicable. Thus

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{\alpha_{in}\overline{V}_{in}^2 - \alpha_{out}\overline{V}_{out}^2}{2} + g(\underbrace{z_{in} - z_{out}}_{0 \text{ for air}}) + w_{\substack{shaft\\net\ in}}$$

or

$$\text{loss} = -28.18\ \frac{ft\cdot lb}{lbm} + \left[\frac{(2)(1.53\quad)^2}{2} - \frac{(1.08)\left(9.57\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} + 33\ \frac{ft\cdot lb}{lbm}$$

and

$$\text{loss} = \underline{\underline{3.36\ \frac{ft\cdot lb}{lbm}}}$$

5.126

5.126 Force from a Jet of Air Deflected by a Flat Plate

Objective: A jet of a fluid striking a flat plate as shown in Fig. P5.126 exerts a force on the plate. It is the equal and opposite force of the plate on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the plate with the experimentally measured force.

Equipment: Air source with an adjustable flowrate and a flow meter; nozzle to produce a uniform air jet; balance beam with an attached flat plate; weights; barometer; thermometer.

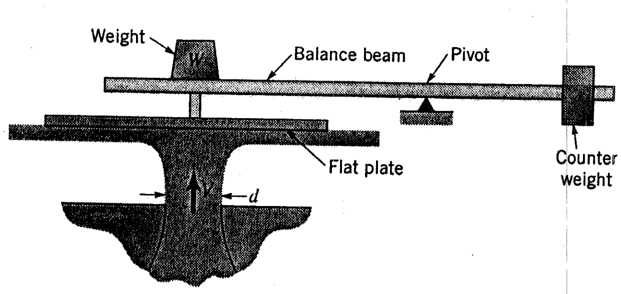
Experimental Procedure: Adjust the counter weight so that the beam is level when there is no mass, m, on the beam and no flow through the nozzle. Measure the diameter, d, of the nozzle outlet. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Place a known mass, m, on the flat plate and adjust the fan speed control to produce the necessary flowrate, Q, to make the balance beam level again. The flowrate is related to the flow meter manometer reading, h, by the equation $Q = 0.358\, h^{1/2}$, where Q is in ft^3/s and h is in inches of water. Repeat the measurements for various masses on the plate.

Calculations: For each flowrate, Q, calculate the weight, $W = mg$, needed to balance the beam and use the continuity equation, $Q = VA$, to determine the velocity, V, at the nozzle exit. Use the momentum equation for this problem, $W = \rho V^2 A$, to determine the theoretical relationship between velocity and weight.

Graph: Plot the experimentally measured force on the plate, W, as ordinates and air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.126

(con't)

5.126 (con't)

Solution for Problem 5.126: Force from a Jet of Air Deflected by a Flat Plate

d, in.	H_{atm}, in. Hg	T, deg F	Q = 0.358 h^0.5, with Q in cfs and h in inches of water
1.174	29.25	70	

		Experimental			Theoretical
m, kg	h, in.	Q, ft^3/s	V, ft/s	m, slug	W, lb	W, lb
0.010	0.54	0.263	35.0	0.00069	0.022	0.021
0.020	1.08	0.372	49.5	0.00137	0.044	0.042
0.030	1.52	0.441	58.7	0.00206	0.066	0.059
0.040	2.18	0.529	70.3	0.00274	0.088	0.084
0.050	2.72	0.590	78.5	0.00343	0.110	0.105
0.060	3.25	0.645	85.8	0.00411	0.132	0.126
0.070	3.81	0.699	92.9	0.00480	0.154	0.147
0.080	4.32	0.744	98.9	0.00548	0.177	0.167
0.090	4.92	0.794	105.6	0.00617	0.199	0.190
0.100	5.46	0.837	111.2	0.00685	0.221	0.211
0.150	8.13	1.021	135.7	0.01028	0.331	0.315
0.200	10.85	1.179	156.8	0.01370	0.441	0.420
0.250	13.72	1.326	176.3	0.01713	0.552	0.531

Experimental:

V = Q/A where

A = $\pi d^2/4$ = π*(1.174/12 ft)^2/4 = 7.52E-3 ft^2

W = mg

Theoretical:

W = $\rho V^2 A$ where

$\rho = p_{atm}/RT$ with

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2

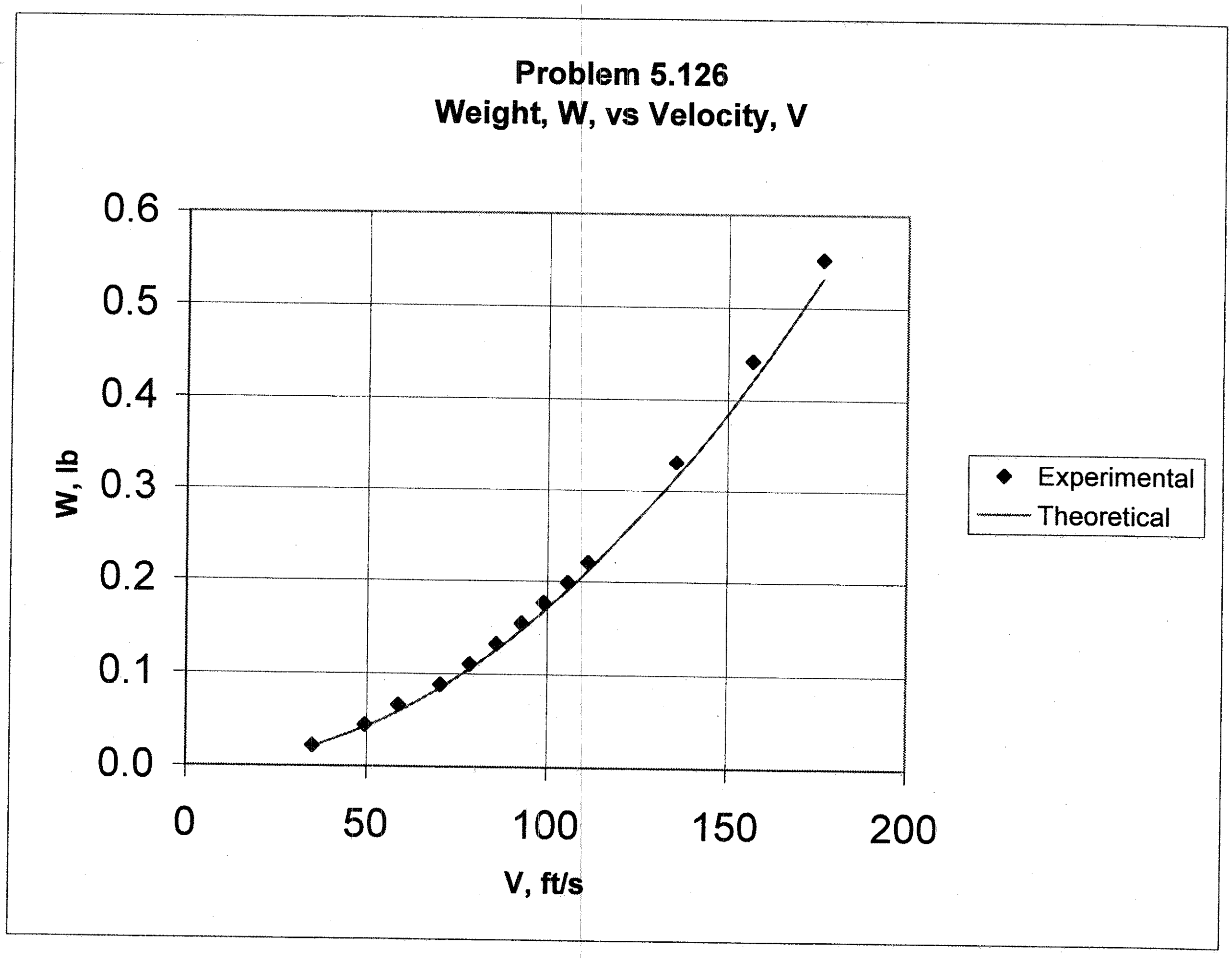
R = 1716 ft lb/slug deg R

T = 70 + 460 = 530 deg R

Thus, ρ = 0.00227 slug/ft^3

(con't)

5.126 (con't)


Problem 5.126
Weight, W, vs Velocity, V
W, lb
V, ft/s
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
50
100
150
200
Experimental
Theoretical

5.127

5.127 Pressure Distribution on a Flat Plate Due to the Deflection of an Air Jet

Objective: In order to deflect a jet of air as shown in Fig. P5.127, the flat plate must push against the air with a sufficient force to change the momentum of the air. This causes an increase in pressure on the plate. The purpose of this experiment is to measure the pressure distribution on the plate and to compare the resultant pressure force to that needed, according to the momentum equation, to deflect the air.

Equipment: Air supply with a flow meter; nozzle to produce a uniform jet of air; circular flat plate with static pressure taps at various radial locations; manometer; barometer; thermometer.

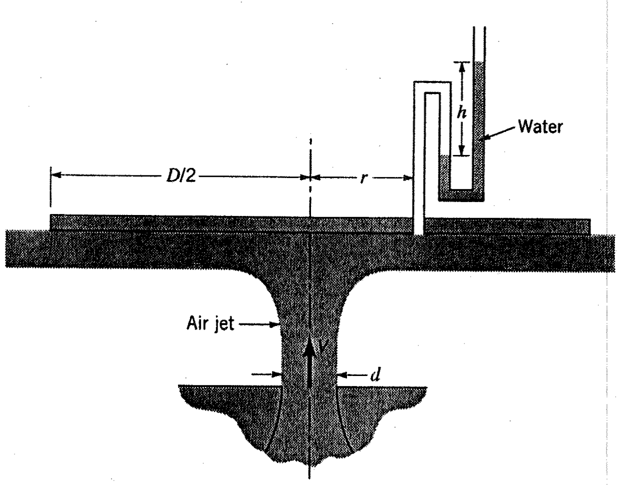
Experimental Procedure: Measure the diameters of the plate, D, and the nozzle exit, d, and the radial locations, r, of the various static pressure taps on the plate. Carefully center the plate over the nozzle exit and adjust the air flowrate, Q, to the desired constant value. Record the static pressure tap manometer readings, h, at various radial locations, r, from the center of the plate. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings, h, to determine the pressure on the plate as a function of location, r. That is, calculate $p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid.

Graph: Plot pressure, p, as ordinates and radial location, r, as abscissas.

Results: Use the experimentally determined pressure distribution to determine the net pressure force, F, that the air jet puts on the plate. That is, numerically or graphically integrate the pressure data to obtain a value for $F = \int p\,dA = \int p\,(2\pi r\,dr)$, where the limits of the integration are over the entire plate, from $r = 0$ to $r = D/2$. Compare this force obtained from the pressure measurements to that obtained from the momentum equation for this flow, $F = \rho V^2 A$, where V and A are the velocity and area of the jet, respectively.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.127

(con't)

5.127 (Con't)

Solution for Problem 5.127: Pressure Distribution on a Flat Plate due to the Deflection of an Air Jet

D, in.	d, in.	H_{atm}, in. Hg	T, deg F	Q. ft^3/s
8.0	1.174	29.25	77	1.41

r, in.	h, in.	p, lb/ft^2	p, lb/in.^2	p*r, lb/in.	i	pr_i+pr_{i+1}	$r_{i+1} - r_i$
0.00	6.62	34.42	0.2391	0.0000	1	0.0834	0.39
0.39	5.92	30.78	0.2138	0.0834	2	0.1701	0.40
0.79	3.04	15.81	0.1098	0.0867	3	0.1114	0.45
1.24	0.55	2.86	0.0199	0.0246	4	0.0355	0.35
1.59	0.19	0.99	0.0069	0.0109	5	0.0205	0.45
2.04	0.13	0.68	0.0047	0.0096	6	0.0174	0.37
2.41	0.09	0.47	0.0033	0.0078	7	0.0130	0.44
2.85	0.05	0.26	0.0018	0.0051	8	0.0086	0.38
3.23	0.03	0.16	0.0011	0.0035	9	0.0035	0.44
3.67	0.00	0.00	0.0000	0.0000			

$p = \gamma_{H2O}$*h

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}$*H_{atm} = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2
R = 1716 ft lb/slug deg R
T = 77 + 460 = 537 deg R

Thus, ρ = 0.00224 slug/ft^3

Using the trapezoidal rule for integration

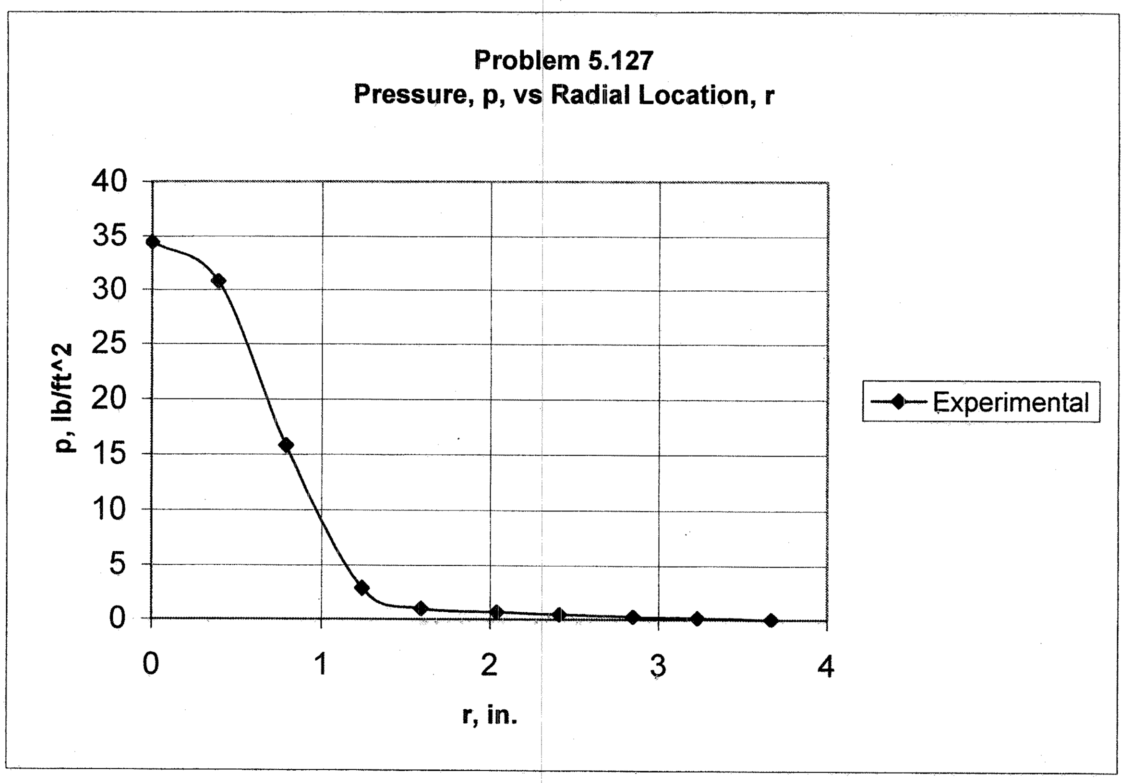
$F_{exp} = 2\pi*0.5*\Sigma_{1\ to\ 9}[(pr_i + pr_{i+1})*(r_{i+1} - r_i)] = 2\pi*0.5*0.189 = \underline{0.594\ lb}$

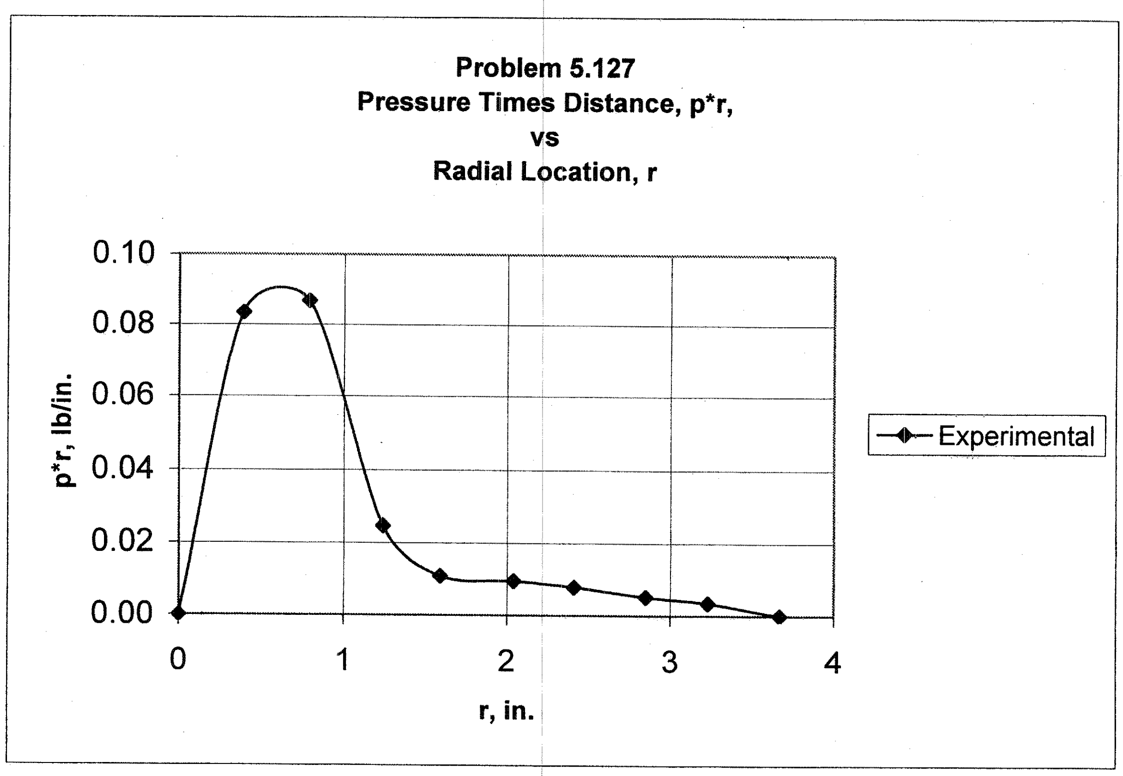
Theory:

$F = \rho V^2 A$ where
$A = \pi d^2/4 = \pi$*(1.174/12 ft)^2/4 = 0.00752 ft^2
V = Q/A = (1.41 ft^3/s)/(0.00752 ft^2) = 188 ft/s
Thus,
F_{th} = 0.00224 slug/ft^3*(188 ft/s)^2*(0.00752 ft^2) = <u>0.595 lb</u>

(con't)

5.127 (con't)


Problem 5.127
Pressure, p, vs Radial Location, r
40
35
30
25
20
15
10
5
0
p, lb/ft^2
0
1
2
3
4
r, in.
Experimental


Problem 5.127
Pressure Times Distance, p*r,
vs
Radial Location, r
0.10
0.08
0.06
0.04
0.02
0.00
p*r, lb/in.
0
1
2
3
4
r, in.
Experimental

5.128

5.128 Force from a Jet of Water Deflected by a Vane

Objective: A jet of a fluid striking a vane as shown in Fig. P5.128 exerts a force on the vane. It is the equal and opposite force of the vane on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the vane with the experimentally measured force.

Equipment: Water source; nozzle to produce a uniform jet of water; vanes to deflect the water jet; weigh tank to collect a known amount of water in a measured time period; stop watch; force balance system.

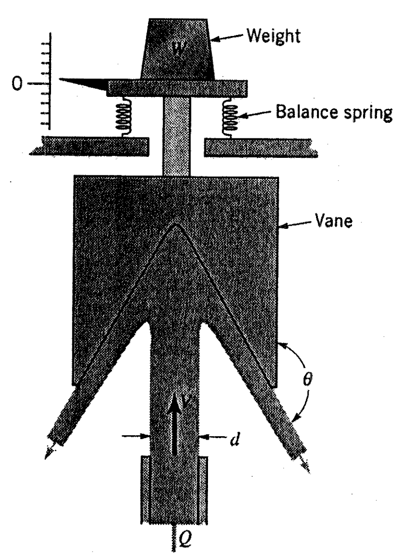
Experimental Procedure: Measure the outlet diameter, d, of the nozzle. Fasten the $\theta = 90$ degree vane to its support and adjust the balance spring to give a zero reading when there is no weight, W, on the platform and no flow through the nozzle. Place a known mass, m, on the platform and adjust the control valve on the pump to provide the necessary flowrate from the nozzle to return the platform to a zero reading. Determine the flowrate by collecting a known weight of water, W_{water}, in the weigh tank during a measured amount of time, t. Repeat the measurements for various masses, m. Repeat the experiment using a $\theta = 180$ degree vane.

Calculations: For each data set, determine the weight, $W = mg$, on the platform and the volume flowrate, $Q = W_{water}/(\gamma t)$, through the nozzle. Determine the exit velocity from the nozzle, V, by using $Q = VA$. Use the momentum equation to determine the theoretical weight that can be supported by the water jet as a function of V and θ.

Graph: For each vane, plot the experimentally determined weight, W, as ordinates and the water velocity, V, as abscissas.

Results: On the same graph plot the theoretical weight as a function of velocity for each vane.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.128

(con't)

5.128 (con't)

Solution for Problem 5.128: Force from a Jet of Water Deflected by a Vane

d, in.
0.40

m, kg	W_{water}, lb	t, s	m, slug	Experimental W, lb	Q, ft^3/s	V, ft/s	Theoretical W, lb
Data for θ = 90 deg:							
0.02	7.71	29.8	0.0014	0.044	0.0041	4.7	0.038
0.07	8.66	18.2	0.0048	0.154	0.0076	8.7	0.129
0.17	8.87	10.1	0.0116	0.375	0.0141	16.1	0.440
0.12	8.92	12.6	0.0082	0.265	0.0113	13.0	0.286
0.22	9.66	10.6	0.0151	0.485	0.0146	16.7	0.474
Data for θ = 180 deg:							
0.05	6.81	24.5	0.0034	0.110	0.0045	5.1	0.088
0.10	9.02	20.8	0.0069	0.221	0.0069	8.0	0.215
0.20	8.84	13.2	0.0137	0.441	0.0107	12.3	0.512
0.25	7.88	10.9	0.0171	0.552	0.0116	13.3	0.597
0.30	8.86	11.1	0.0206	0.662	0.0128	14.7	0.727
0.35	7.97	9.5	0.0240	0.772	0.0134	15.4	0.803
0.40	6.37	7.6	0.0274	0.883	0.0134	15.4	0.802

$W = mg$
$Q = W_{water}/(\gamma * t)$
$V = Q/A$ where

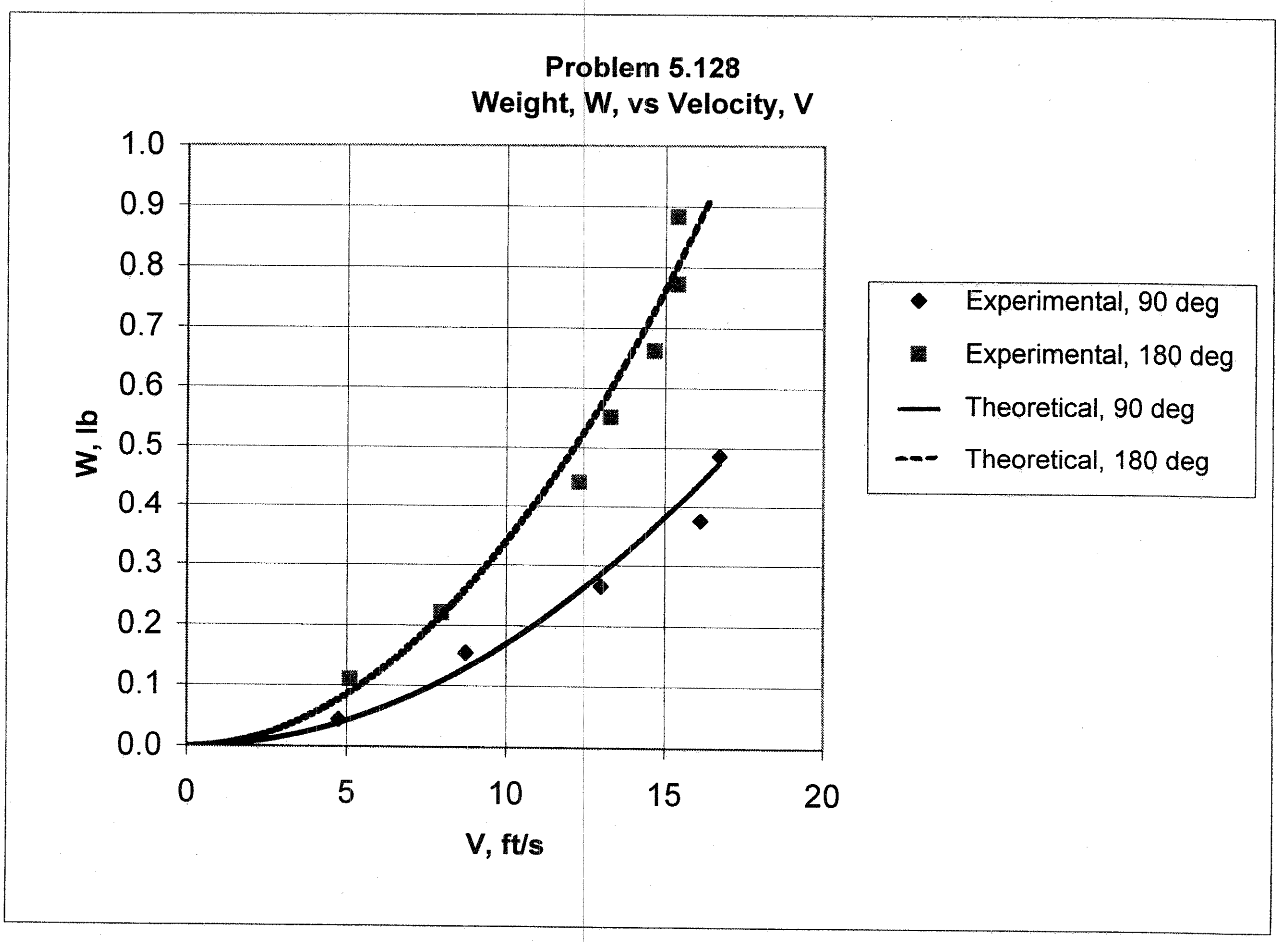
$A = \pi d^2/4 = \pi*(0.40/12\ ft)^2/4 = 0.000873\ ft^2$

Theoretical:

$W = \rho V^2 A$ for $\theta = 90$ deg
and
$W = 2\rho V^2 A$ for $\theta = 180$ deg

(Con't)

5.128 (Con't)


Problem 5.128
Weight, W, vs Velocity, V
W, lb
V, ft/s
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
Experimental, 90 deg
Experimental, 180 deg
Theoretical, 90 deg
Theoretical, 180 deg

5.129

5.129 Force of a Flowing Fluid on a Pipe Elbow

Objective: When a fluid flows through an elbow in a pipe system as shown in Fig. P5.129, the fluid's momentum is changed as the fluid changes direction. Thus, the elbow must put a force on the fluid. Similarly, there must be an external force on the elbow to keep it in place. The purpose of this experiment is to compare the theoretical vertical component of force needed to hold an elbow in place with the experimentally measured force.

Equipment: Variable speed fan; Pitot static tube; air speed indicator; air duct and 90-degree elbow; scale; barometer; thermometer.

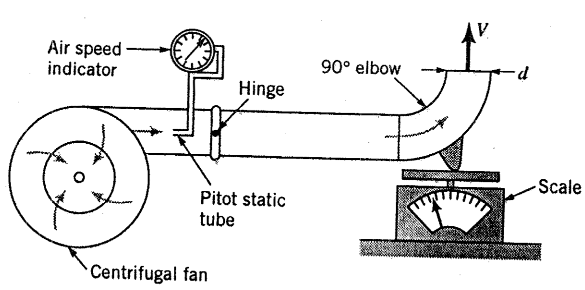
Experimental Procedure: Measure the diameter, d, of the air duct and adjust the scale to read zero when the elbow rests on it and there is no flow through it. Note that the duct is connected to the fan outlet by a pivot mechanism that is essentially friction free. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Adjust the variable speed fan to give the desired flowrate. Record the velocity, V, in the pipe as given by the Pitot static tube which is connected to an air speed indicator that reads directly in feet per minute. Record the force, F, indicated on the scale at this air speed. Repeat the measurements for various air speeds. Obtain data for two types of elbows: (1) a long radius elbow and (2) a mitered elbow (see Figs. 8.30 and 8.31).

Calculations: For a given air speed, V, use the momentum equation to calculate the theoretical vertical force, $F = \rho V^2 A$, needed to hold the elbow stationary.

Graph: Plot the experimentally measured force, F, as ordinates and the air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.129

(Con't)

5.129 (con't)

Solution for Problem 5.129: Force of a Flowing Fluid on a Pipe Elbow

d, in.	H_{atm}, in. Hg	T, deg F
8.0	29.07	73

Experiment

V, ft/min	F, lb	V, ft/s
Long Radius Elbow Data		
0	0	0.0
1200	0.38	20.0
1420	0.51	23.7
1800	0.79	30.0
2160	1.05	36.0
2440	1.38	40.7
2700	1.65	45.0
2900	1.91	48.3
3100	2.19	51.7
3520	2.83	58.7
3750	3.12	62.5
3950	3.38	65.8
Mitered Elbow Data		
1400	0.30	23.3
1780	0.55	29.7
2000	0.74	33.3
2300	1.12	38.3
2630	1.44	43.8
2900	1.72	48.3
3150	2.06	52.5
3360	2.38	56.0
3550	2.62	59.2
3620	2.74	60.3

Theory

V, ft/s	F_{th}, lb
0	0
5.0	0.02
10.0	0.08
15.0	0.18
20.0	0.31
25.0	0.49
30.0	0.70
35.0	0.96
40.0	1.25
45.0	1.58
50.0	1.95
55.0	2.36
60.0	2.81
65.0	3.30

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.07/12ft) = 2052 lb/ft^2

R = 1716 ft lb/slug deg R

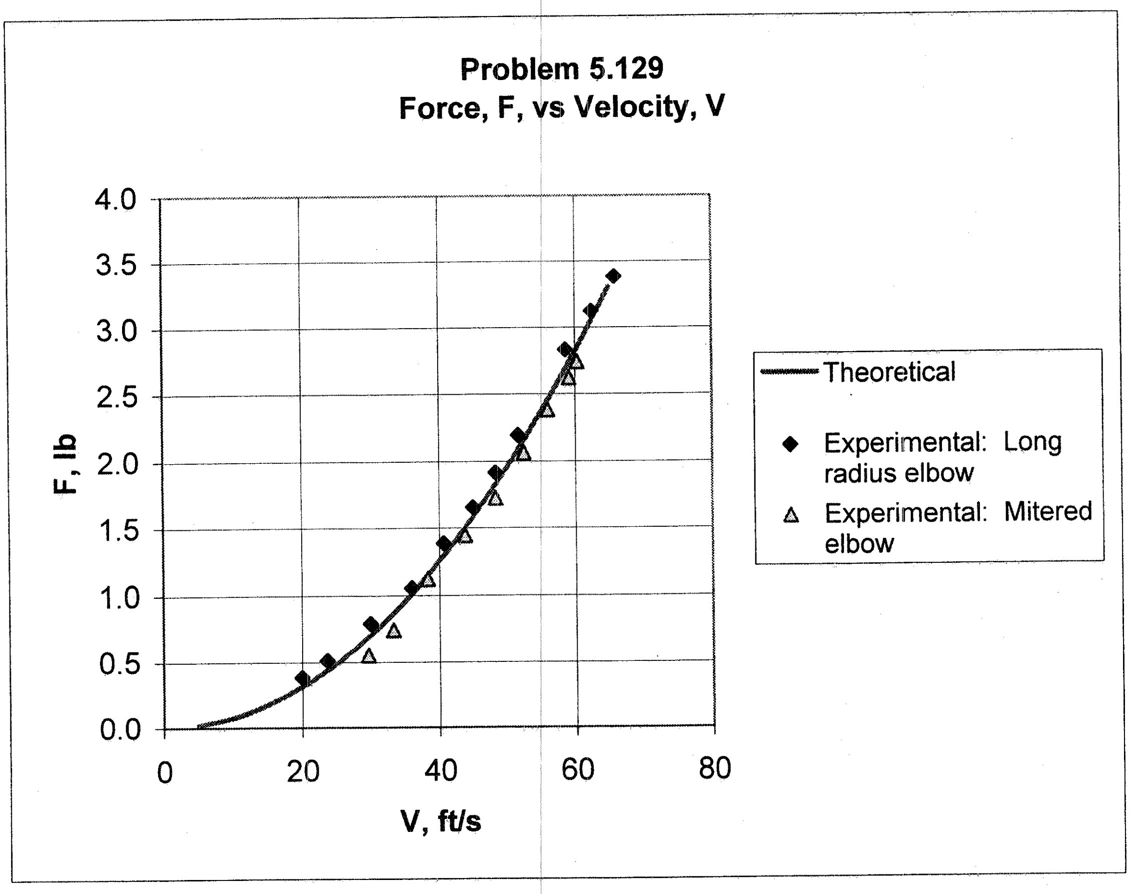
T = 73 + 460 = 533 deg R

Thus, ρ = 0.00224 slug/ft^3

A = πd^2/4 = π*(8/12)^2/4 = 0.349 ft^2

(con't)

5.129 (con't)


Problem 5.129
Force, F, vs Velocity, V
F, lb
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
20
40
60
80
V, ft/s
Theoretical
Experimental: Long radius elbow
Experimental: Mitered elbow

5.130

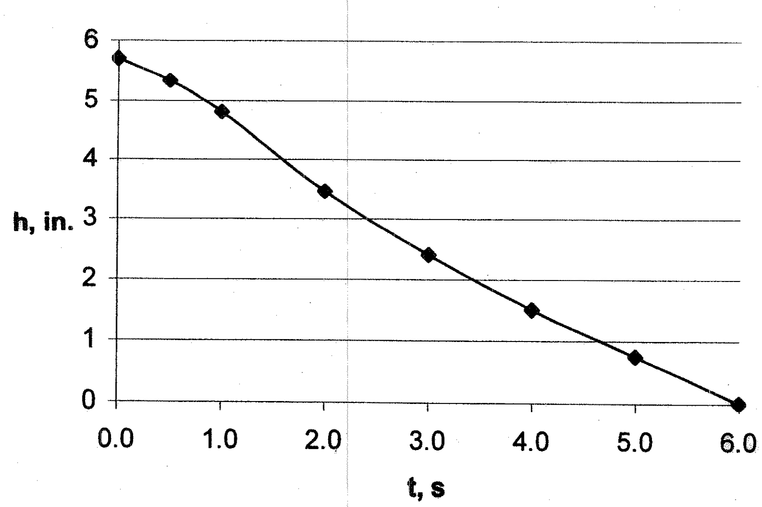
5.130 (See "New 1.6 gpf standards," Section 5.1.2.) When a toilet is flushed, the water depth, h, in the tank as a function of time, t, is as given in the table. The size of the rectangular tank is 19 in. by 7.5 in. **(a)** Determine the volume of water used per flush, gpf. **(b)** Plot the flowrate for $0 \leq t \leq 6$ s.

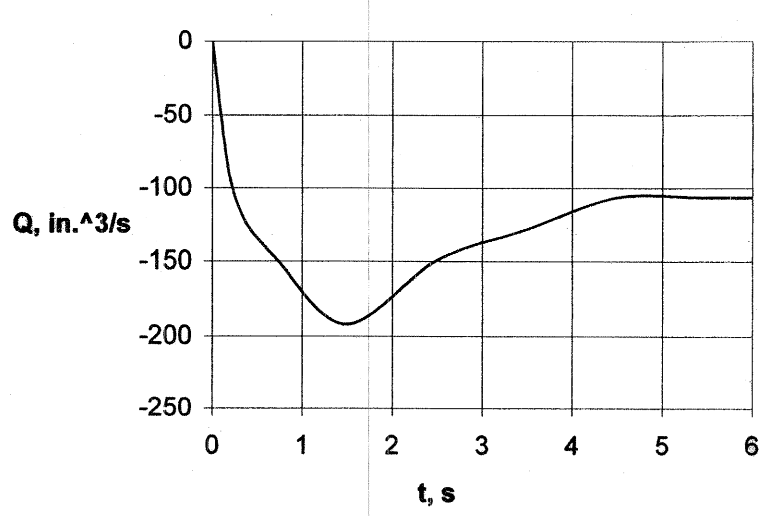
t (s)	h (in.)
0	5.70
0.5	5.33
1.0	4.80
2.0	3.45
3.0	2.40
4.0	1.50
5.0	0.75
6.0	0

(a) Volume of water per flush $= 5.70\,in.\,(19\,in. \times 7.5\,in.) = 812\,in.^3$

$= 812\,in.^3\left(\frac{1\,gal}{231\,in.^3}\right) = \underline{\underline{3.52\,gal.}} = 3.52\,GPF$

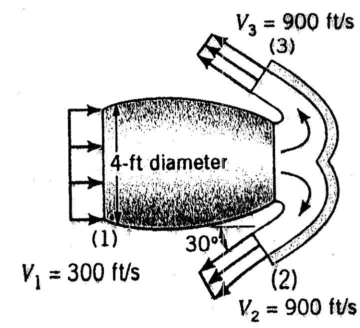
(b) $Q = \frac{d(\text{volume in tank})}{dt} = A_{tank}\frac{dh}{dt}$, where $\frac{dh}{dt}$ is obtained by numerical differentiation of the h vs t data shown below. The resulting Q vs t results are also shown below.



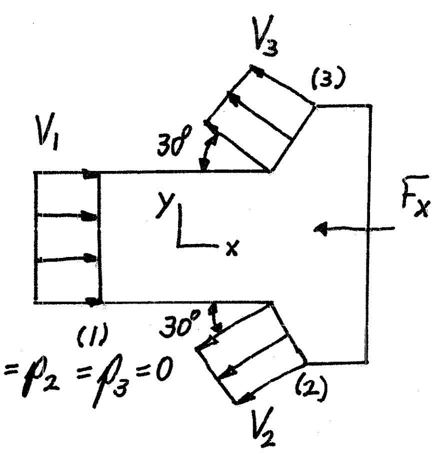


5.131

5.131 (See "**Where the plume goes,**" Section 5.2.2.) Air flows into the jet engine shown in Fig. P5.131 at a rate of 9 slugs/s and a speed of 300 ft/s. Upon landing, the engine exhaust exits through the reverse thrust mechanism with a speed of 900 ft/s in the direction indicated. Determine the reverse thrust applied by the engine to the airplane. Assume the inlet and exit pressures are atmospheric and that the mass flowrate of fuel is negligible compared to the air flowrate through the engine.



■ FIGURE P5.131



The momentum equation (x-component),

$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_x$, for the control volume shown can be written as

$p_1 = p_2 = p_3 = 0$

$$V_1 \rho (-V_1) A_1 + (-V_2 \cos 30^\circ) \rho V_2 A_2 + (-V_3 \cos 30^\circ) \rho V_3 A_3 = -F_x$$

or

$$F_x = (\rho V_1 A_1) V_1 + (\rho V_2 A_2) V_2 \cos 30^\circ + (\rho V_3 A_3) V_3 \cos 30^\circ \qquad (1)$$

But from conservation of mass,

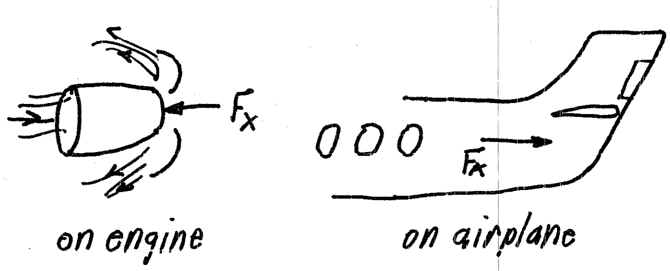
$$\rho V_1 A_1 = \rho V_2 A_2 + \rho V_3 A_3 = \dot{m} = 9\,\text{slugs/s}$$

Also, $V_2 = V_3$, so that Eq. (1) becomes

$$F_x = \dot{m}(V_1 + V_2 \cos 30^\circ) = 9\,\frac{\text{slugs}}{\text{s}}\left(300\,\frac{\text{ft}}{\text{s}} + 900 \cos 30^\circ\,\frac{\text{ft}}{\text{s}}\right)$$

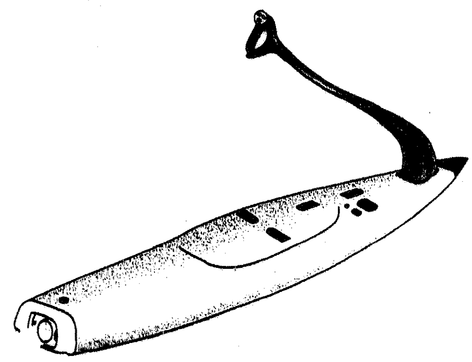
$$= 9710\,\frac{\text{slugs}\cdot\text{ft}}{\text{s}^2} = \underline{\underline{9170\ \text{lb}}}$$

Note direction of F_x on engine and engine on airplane.



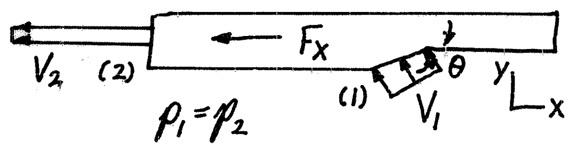
5.132

5.132 (See "**Motorized surfboard,**" Section 5.2.2.) The thrust to propel the powered surfboard shown in Fig. P5.132 is a result of water pumped through the board that exits as a high-speed 2.75-in.-diameter jet. Determine the flowrate and the velocity of the exiting jet if the thrust is to be 300 lb. Neglect the momentum of the water entering the pump.



■ FIGURE P5.132

The x-component of the momentum equation, $\int_{cs} u\rho\vec{V}\cdot\hat{n}\,dA = \sum F_x$, for the control volume shown is



$$(-V_1\cos\theta)\rho(-V_1)A_1 + (-V_2)\rho V_2 A_2 = -F_x$$

or

$$F_x = \rho V_2^2 A_2 - \rho V_1^2 A_1\cos\theta \approx \rho V_2^2 A_2$$ if the momentum of the entering water is neglected.

Thus,

$$300\ lb = \left(1.94\ \frac{slugs}{ft^3}\right)V_2^2\left(\frac{\pi}{4}\left(\frac{2.75}{12}\ ft^2\right)\right)$$

or

$$\underline{\underline{V_2 = 61.2\ \frac{ft}{s}}}$$

and

$$Q = A_2V_2 = \frac{\pi}{4}\left(\frac{2.75}{12}\ ft\right)^2\left(61.2\ \frac{ft}{s}\right) = \underline{\underline{2.52\ \frac{ft^3}{s}}}$$

5.133

5.133 (See **"Bow thrusters,"** Section 5.2.2) The bow thruster on the boat shown in Fig. P5.133 is used to turn the boat. The thruster produces a 1-m-diameter jet of water with a velocity of 10 m/s. Determine the force produced by the thruster. Assume that the inlet and outlet pressures are zero and that the momentum of the water entering the thruster is negligible.

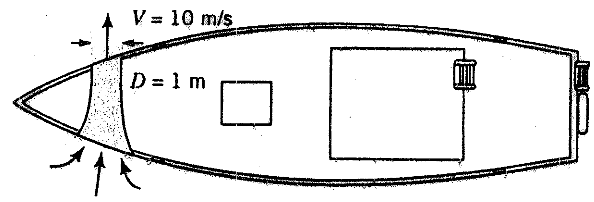
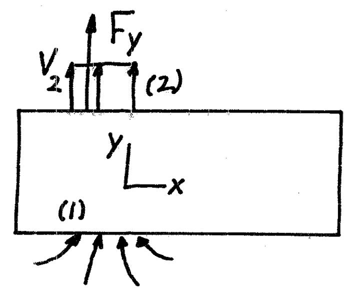


FIGURE P5.133



The y-component of the momentum equation,

$\int_{cs} v \rho \vec{V}\cdot\hat{n}\, dA = \sum F_y$, for the control volume shown is,

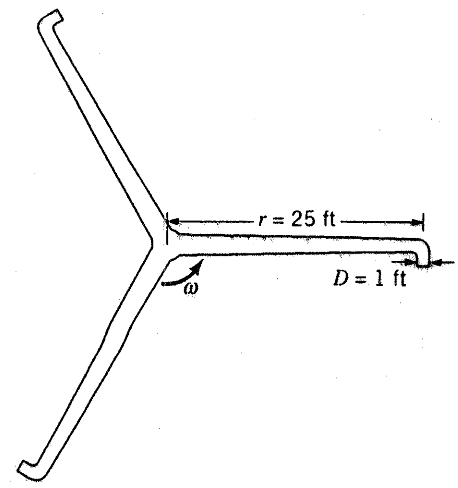
$$\int_{(1)} v \rho \vec{V}\cdot\hat{n}\, dA + V_2 \rho V_2 A_2 = F_y$$

If the momentum of the entering water is negligible the equation becomes

$$F_y = \rho V_2^2 A_2 = 999 \frac{kg}{m^3}\left(10 \frac{m}{s}\right)^2\left(\frac{\pi}{4}(1m)^2\right) = 78{,}500 \frac{kg \cdot m}{s^2} = \underline{\underline{78.5\ kN}}$$

5.134

5.134 (See **"Tailless helicopters,"** Section 5.2.4.) Exhaust gas from a tailless helicopter turbojet engine flows through the three 1-ft-diameter rotor blade nozzles shown in Fig. P5.134 at a rate of 500 ft^3/s. Determine the angular velocity of the rotor if the torque on the rotor is negligible.



■ FIGURE P5.134

As discussed in Example 5.18, the torque on the rotor is given by

(1) $T_{shaft} = -r(W - r\omega)\dot{m}$, where W is the velocity of the exiting fluid relative to the moving exit. Thus, with $T_{torque} = 0$ Eq. (1) gives

(2) $W = r\omega$, where

$$W = \frac{Q}{A} = \frac{500\ \frac{ft^3}{s}}{3\left[\frac{\pi}{4}(1\ ft)^2\right]} = 212\ \frac{ft}{s}$$

Thus, from Eq. (2)

$$\omega = \frac{W}{r} = \frac{212\ \frac{ft}{s}}{25\ ft} = 8.49\ \frac{rad}{s}\left(\frac{1\ rev}{2\pi\ rad}\right) = 1.35\ \frac{rev}{s}\left(\frac{60\ s}{min}\right) = 81.1\ rpm$$

5.135

5.135 (See **"Curtain of air,"** Section 5.3.3.) The fan shown in Fig. P5.135 produces an air curtain to separate a loading dock from a cold storage room. The air curtain is a jet of air 10 ft wide, 0.5 ft thick moving with speed $V = 30$ ft/s. The loss associated with this flow is $loss = K_L V^2/2$, where $K_L = 5$. How much power must the fan supply to the air to produce this flow?

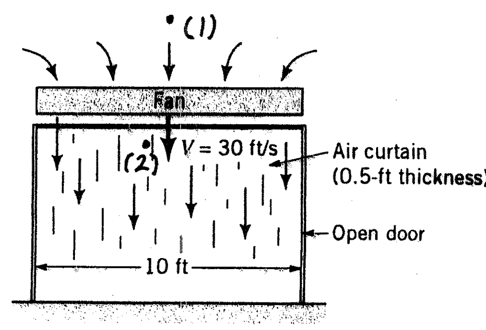


FIGURE P5.135

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g},$$

where

$p_1 \approx p_2 \approx 0,\ z_1 \approx z_2,\ V_1 \approx 0$, and $h_L = \frac{loss}{g} = 5\frac{V_2^2}{2g}$

Thus,

$$h_s = h_L + \frac{V_2^2}{2g} = 5\frac{V_2^2}{2g} + \frac{V_2^2}{2g} = \frac{3V_2^2}{g} = \frac{3\left(30\frac{ft}{s}\right)^2}{\left(32.2\frac{ft}{s^2}\right)} = 83.9\ ft$$

Hence,

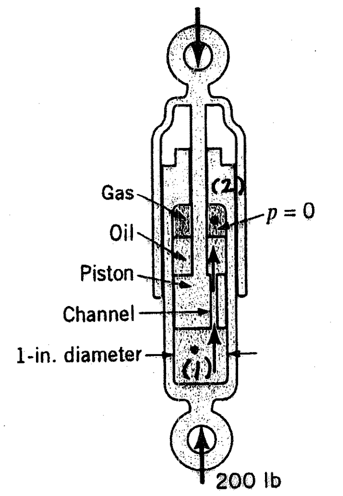
$$\dot{W}_s = \gamma Q h_s = \rho g A_2 V_2 h_s = \left(0.00238\frac{slugs}{ft^3}\right)\left(32.2\frac{ft}{s^2}\right)(10\ ft)(0.5\ ft)\left(30\frac{ft}{s}\right)(83.9\ ft)$$

$$= 964\ \frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{1.75\ hp}}$$

5.136

5.136 (See "Smart shocks," Section 5.3.3.) A 200-lb force applied to the end of the piston of the shock absorber shown in Fig. P5.136 causes the two ends of the shock absorber to move toward each other with a speed of 5 ft/s. Determine the head loss associated with the flow of the oil through the channel. Neglect gravity and any friction force between the piston and cylinder walls.



■ FIGURE P5.136

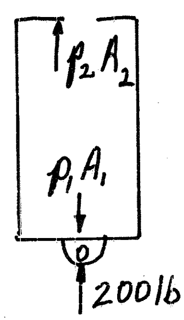
From a force balance on the cylinder

$p_1 A_1 - p_2 A_2 = 200\,lb$

or with $p_2 = 0$,

$p_1 = 200\,lb/A_1 = 200\,lb/\left(\frac{\pi}{4}\left(\frac{1}{12}ft\right)^2\right)$

$= 3.67 \times 10^4 \frac{lb}{ft^2} = 255\,psi$



From the energy equation,

$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$, where

$z_1 \approx z_2$, $V_2 = 0$, $V_1 = 5\,\frac{ft}{s}$, $p_1 = 255\,psi$, and $p_2 = 0$. Assume $\gamma = 50\,\frac{lb}{ft^3}$.

Thus,

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{3.67 \times 10^4 \frac{lb}{ft^2}}{\left(50\,\frac{lb}{ft^3}\right)} + \frac{\left(5\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 734\,ft + 0.388\,ft = \underline{\underline{734\,ft}}$$