7.1

7.1 The Reynolds number, $\rho VD/\mu$, is a very important parameter in fluid mechanics. Verify that the Reynolds number is dimensionless, using both the *FLT* system and the *MLT* system for basic dimensions, and determine its value for ethyl alcohol flowing at a velocity of 3 m/s through a 2-in.-diameter pipe.

$$\text{Reynolds number} = \frac{\rho V D}{\mu} \doteq \frac{(FL^{-4}T^{2})(LT^{-1})(L)}{FL^{-2}T} \doteq \underline{\underline{F^0 L^0 T^0}}$$

$$\doteq \frac{(ML^{-3})(LT^{-1})(L)}{ML^{-1}T^{-1}} \doteq \underline{\underline{M^0 L^0 T^0}}$$

For ethyl alcohol, $\mu = 1.19 \times 10^{-3} \ \frac{N \cdot s}{m^2}$ and $\rho = 789 \ \frac{kg}{m^3}$.

Thus,

$$\frac{\rho V D}{\mu} = \frac{\left(789 \ \frac{kg}{m^3}\right)\left(3 \ \frac{m}{s}\right)\left(\frac{2}{12} \ ft\right)\left(0.3048 \ \frac{m}{ft}\right)}{1.19 \times 10^{-3} \ \frac{N \cdot s}{m^2}}$$

$$= \underline{\underline{1.01 \times 10^5}}$$

7.2

7.2 What are the dimensions of acceleration of gravity, density, dynamic viscosity, kinematic viscosity, specific weight, and speed of sound in (a) the *FLT* system, and (b) the *MLT* system? Compare your results with those given in Table 1.1 in Chapter 1.

g = acceleration of gravity = $\frac{\text{velocity}}{\text{time}} \doteq \frac{L}{T^2}$

ρ = density = $\frac{\text{mass}}{\text{unit volume}} \doteq \frac{M}{L^3} \doteq \frac{FT^2}{L^4}$ (since $F \doteq MLT^{-2}$)

μ = dynamic viscosity = $\frac{\text{stress}}{\text{velocity gradient}} \doteq \frac{FL^{-2}}{T^{-1}} \doteq \frac{M}{LT}$

ν = kinematic viscosity = $\frac{\text{dynamic viscosity}}{\text{density}} \doteq \frac{FL^{-2}T}{FT^2L^{-4}} \doteq \frac{L^2}{T}$

γ = specific weight = $\frac{\text{weight}}{\text{unit volume}} \doteq \frac{F}{L^3} \doteq \frac{(MLT^{-2})}{L^3} \doteq \frac{MT^{-2}}{L^2}$

c = speed of sound = $\frac{\text{length}}{\text{time}} = \frac{L}{T}$

Thus,

(a) in the FLT system,

$g \doteq \underline{\underline{LT^{-2}}}$

$\rho \doteq \underline{\underline{FL^{-4}T^2}}$

$\mu \doteq \underline{\underline{FL^{-2}T}}$

$\nu \doteq \underline{\underline{L^2T^{-1}}}$

$\gamma \doteq \underline{\underline{FL^{-3}}}$

$c \doteq \underline{\underline{LT^{-1}}}$

(b) in the MLT system,

$g \doteq \underline{\underline{LT^{-2}}}$

$\rho \doteq \underline{\underline{ML^{-3}}}$

$\mu \doteq \underline{\underline{ML^{-1}T^{-1}}}$

$\nu \doteq \underline{\underline{L^2T^{-1}}}$

$\gamma \doteq \underline{\underline{ML^{-2}T^{-2}}}$

$c \doteq \underline{\underline{LT^{-1}}}$

7.3

7.3 For the flow of a thin film of a liquid with a depth h and a free surface, two important dimensionless parameters are the Froude number, $V/\sqrt{gh}$, and the Weber number, $\rho V^2 h/\sigma$. Determine the value of these two parameters for glycerin (at 20 °C) flowing with a velocity of 0.7 m/s at a depth of 3 mm.

$$\frac{V}{\sqrt{gh}} = \frac{0.7\ \frac{m}{s}}{\sqrt{\left(9.81\ \frac{m}{s^2}\right)(0.003\ m)}} = \underline{\underline{4.08}}$$

$$\frac{\rho V^2 h}{\sigma} = \frac{\left(1260\ \frac{kg}{m^3}\right)\left(0.7\ \frac{m}{s}\right)^2(0.003\ m)}{6.33\times 10^{-2}\ \frac{N}{m}} = \underline{\underline{29.3}}$$

7.4

7.4 The Mach number for a body moving through a fluid with velocity V is defined as V/c, where c is the speed of sound in the fluid. This dimensionless parameter is usually considered to be important in fluid dynamics problems when its value exceeds 0.3. What would be the velocity of a body at a Mach number of 0.3 if the fluid is: **(a)** air at standard atmospheric pressure and 20 °C, and **(b)** water at the same temperature and pressure?

(a) $\frac{V}{c} = 0.3$

For air at 20°C, $c = 343.3\ \frac{m}{s}$ (Table B.4 in Appendix B) so that

$$V = 0.3\left(343.3\ \frac{m}{s}\right) = \underline{\underline{103\ \frac{m}{s}}}$$

(b) For water at 20°C, $c = 1481\ \frac{m}{s}$ (Table B.2 in Appendix B) so that

$$V = 0.3\left(1481\ \frac{m}{s}\right) = \underline{\underline{444\ \frac{m}{s}}}$$

7.5

7.5 At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity, V, in the larger pipe, and the fluid density, ρ, and viscosity, μ. Use D_1, V, and μ as repeating variables to determine a suitable set of dimensionless parameters. Why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

$$\Delta p = f(D_1, D_2, V, \rho, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D_1 \doteq L$ $\quad D_2 \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $6-3=3$ dimensionless parameters required. Use D_1, V, and μ as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D_1^a V^b \mu^c$$

and

$$(FL^{-2})(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$-2 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=1$, $b=-1$, $c=-1$, and therefore

$$\Pi_1 = \frac{\Delta p\, D_1}{V\mu}$$

Check dimensions using MLT system:

$$\frac{\Delta p\, D_1}{V\mu} \doteq \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

For Π_2:

$$\Pi_2 = D_2\, D_1^a V^b \mu^c$$

$$L\,(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{D_2}{D_1}$$

(con't)

7.5 (Con't)

Π_2 is obviously dimensionless.

For Π_3:

$$\Pi_3 = \rho D_1^a V^b \mu^c$$

$$(FL^{-4}T^2)(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-4 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$

$$2 - b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 1$, $b = 1$, $c = -1$ and therefore

$$\Pi_3 = \frac{\rho D_1 V}{\mu}$$

Check dimensions using MLT system:

$$\frac{\rho D_1 V}{\mu} \doteq \frac{(ML^{-3})(L)(LT^{-1})}{ML^{-1}T^{-1}} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Thus,

$$\frac{\Delta p\, D_1}{V\mu} = \phi\left(\frac{D_2}{D_1}, \frac{\rho D_1 V}{\mu}\right)$$

From the continuity equation,

$$V \frac{\pi}{4} D_1^2 = V_s \frac{\pi}{4} D_2^2$$

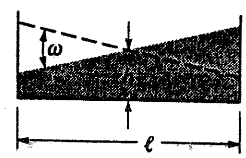
where V_s is the velocity in the smaller pipe. Since

$$V_s = \left(\frac{D_1}{D_2}\right)^2 V$$

V_s is not independent of D_1, D_2, and V and therefore should not be included as an independent variable.

7.6

7.6 Water sloshes back and forth in a tank as shown in Fig. P7.6. The frequency of sloshing, ω, is assumed to be a function of the acceleration of gravity, g, the average depth of the water, h, and the length of the tank, ℓ. Develop a suitable set of dimensionless parameters for this problem using g and ℓ as repeating variables.



■ FIGURE P7.6

$$\omega = f(g, h, \ell)$$

$$\omega \doteq T^{-1} \quad g \doteq LT^{-2} \quad h \doteq L \quad \ell \doteq L$$

From the pi theorem, $4-2=2$ dimensionless parameters required. Use g and ℓ as repeating variables, Thus,

$$\Pi_1 = \omega g^a \ell^b$$

and

$$(T^{-1})(LT^{-2})^a (L)^b \doteq L^0 T^0$$

so that

$$a + b = 0 \qquad \text{(for } L)$$

$$-1 - 2a = 0 \qquad \text{(for } T)$$

It follows that $a = -1/2$, $b = 1/2$, and therefore

$$\Pi_1 = \omega \sqrt{\frac{\ell}{g}}$$

Check dimensions:

$$\omega \sqrt{\frac{\ell}{g}} \doteq \frac{1}{T}\sqrt{\frac{L}{LT^{-2}}} \doteq L^0 T^0 \quad \therefore \text{ OK}$$

For Π_2:

$$\Pi_2 = h g^a \ell^b$$

$$L (LT^{-2})^a (L)^b \doteq L^0 T^0$$

$$1 + a + b = 0 \qquad \text{(for } L)$$

$$-2a = 0 \qquad \text{(for } T)$$

It follows that $a = 0$, $b = -1$, and therefore

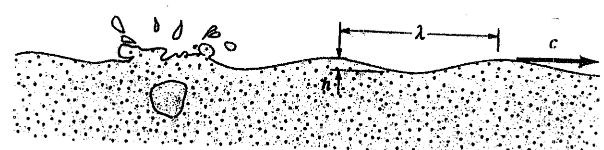
$$\Pi_2 = \frac{h}{\ell}$$

and Π_2 is obviously dimensionless. Thus,

$$\omega \sqrt{\frac{\ell}{g}} = \phi\left(\frac{h}{\ell}\right)$$

7.7

7.7 When a small pebble is dropped into a liquid, small waves travel outward as shown in Fig. P7.7. The speed of these waves, c, is assumed to be a function of the liquid density, ρ, the wavelength, λ, the wave height, h, and the surface tension of the liquid, σ. Use h, ρ, and σ as repeating variables to determine a suitable set of pi terms that could be used to describe this problem.



■ FIGURE P7.7

$$c = f(\rho, \lambda, h, \sigma)$$

$c \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \lambda \doteq L$ $\quad h \doteq L$ $\quad \sigma \doteq FL^{-1}$

From the pi theorem, $5-3 = 2$ pi terms required. Use h, ρ, and σ as repeating variables. Thus,

$$\Pi_1 = c\, h^a \rho^b \sigma^c$$

and

$$(LT^{-1})(L)^a(FL^{-4}T^2)^b(FL^{-1})^c \doteq F^0L^0T^0$$

so that

$$b + c = 0 \quad \text{(for } F\text{)}$$
$$1 + a - 4b - c = 0 \quad \text{(for } L\text{)}$$
$$-1 + 2b = 0 \quad \text{(for } T\text{)}$$

It follows that $a = \frac{1}{2}$, $b = \frac{1}{2}$, $c = -\frac{1}{2}$, and therefore

$$\Pi_1 = c\, h^{1/2}\rho^{1/2}\sigma^{-1/2} = c\sqrt{\frac{\rho h}{\sigma}}$$

Check dimensions:

$$c\sqrt{\frac{\rho h}{\sigma}} \doteq (LT^{-1})\left[\frac{(FL^{-4}T^2)(L)}{(FL^{-1})}\right]^{1/2} \doteq F^0L^0T^0 \;\therefore\; \text{OK}$$

For Π_2:

$$\Pi_2 = \lambda\, h^a \rho^b \sigma^c$$

and

$$(L)(L)^a(FL^{-4}T^2)^b(FL^{-1})^c \doteq F^0L^0T^0$$

so that

$$b + c = 0 \quad \text{(for } F\text{)}$$
$$1 + a - 4b - c = 0 \quad \text{(for } L\text{)}$$
$$2b = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, so that

$$\Pi_2 = \frac{\lambda}{h}$$

which is obviously dimensionless. Thus,

$$c\sqrt{\frac{\rho h}{\sigma}} = \phi\left(\frac{\lambda}{h}\right)$$

7.8

7.8 Assume that the flowrate, Q, of a gas from a smokestack is a function of the density of the ambient air, ρ_a, the density of the gas, ρ_g, within the stack, the acceleration of gravity, g, and the height and diameter of the stack, h and d, respectively. Use ρ_a, d, and g as repeating variables to develop a set of pi terms that could be used to describe this problem.

$$Q = f(\rho_a, \rho_g, g, h, d)$$

$Q \doteq L^3T^{-1}$ $\rho_a \doteq ML^{-3}$ $\rho_g \doteq ML^{-3}$ $g \doteq LT^{-2}$ $h \doteq L$ $d \doteq L$

From the pi theorem, $6-3 = 3$ pi terms required. Use ρ_a, d, and g as repeating variables. Thus,

$$\Pi_1 = Q\, \rho_a^a\, d^b\, g^c$$

and

$$(L^3T^{-1})(ML^{-3})^a(L)^b(LT^{-2})^c \doteq M^0L^0T^0$$

so that

$$a = 0 \qquad \text{(for } M\text{)}$$

$$3 - 3a + b + c = 0 \qquad \text{(for } L\text{)}$$

$$-1 - 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 0$, $b = -\frac{5}{2}$, $c = -\frac{1}{2}$, and therefore

$$\Pi_1 = \frac{Q}{d^{\frac{5}{2}}\, g^{\frac{1}{2}}}$$

Check dimensions using FLT system:

$$\frac{Q}{d^{5/2}g^{1/2}} \doteq \frac{L^3T^{-1}}{(L)^{5/2}(LT^{-2})^{1/2}} \doteq F^0L^0T^0 \qquad \therefore \text{OK}$$

(con't)

7.8 (Con't)

For Π_2:

$$\Pi_2 = \rho_g \rho_a^a d^b g^c$$

$$(ML^{-3})(ML^{-3})^a (L)^b (LT^{-2})^c \doteq M^0 L^0 T^0$$

$$1 + a = 0 \qquad \text{(for } M\text{)}$$

$$-3 - 3a + b + c = 0 \qquad \text{(for } L\text{)}$$

$$-2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{\rho_g}{\rho_a}$$

which is obviously dimensionless.

For Π_3:

$$\Pi_3 = h \rho_a^a d^b g^c$$

$$(L)(ML^{-3})^a (L)^b (LT^{-2})^c \doteq M^0 L^0 T^0$$

$$a = 0 \qquad \text{(for } M\text{)}$$

$$1 - 3a + b + c = 0 \qquad \text{(for } L\text{)}$$

$$-2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 0$, $b = -1$, $c = 0$, and therefore

$$\Pi_3 = \frac{h}{d}$$

which is obviously dimensionless.

Thus,

$$\frac{Q}{d^{5/2} g^{1/2}} = \phi\left(\frac{\rho_g}{\rho_a}, \frac{h}{d}\right)$$

7.9

7.9 The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \omega \doteq T^{-1}$ $\quad Q \doteq L^{3}T^{-1}$

From the pi theorem, 5-3 = 2 pi terms required. Use D, ρ, and ω as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D^a \rho^b \omega^c$$

and so that

$$(FL^{-2})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$$1 + b = 0 \quad \text{(for } F)$$
$$-2 + a - 4b = 0 \quad \text{(for } L)$$
$$2b - c = 0 \quad \text{(for } T)$$

It follows that $a = -2$, $b = -1$, $c = -2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2\rho\omega^2} \doteq \frac{ML^{-1}T^{-2}}{(L)^2(ML^{-3})(T^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = Q\, D^a \rho^b \omega^c$$

$$(L^3T^{-1})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$$b = 0 \quad \text{(for } F)$$
$$3 + a - 4b = 0 \quad \text{(for } L)$$
$$-1 + 2b - c = 0 \quad \text{(for } T)$$

It follows that $a = -3$, $b = 0$, $c = -1$, and therefore

$$\Pi_2 = \frac{Q}{D^3\omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3\omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Thus,

$$\frac{\Delta p}{D^2\rho\omega^2} = \phi\left(\frac{Q}{D^3\omega}\right)$$

7.10

7.10 The drag, $\mathcal{D}$, on a washer shaped plate placed normal to a stream of fluid can be expressed as

$$\mathcal{D} = f(d_1, d_2, V, \mu, \rho)$$

where d_1 is the outer diameter, d_2 the inner diameter, V the fluid velocity, μ the fluid viscosity, and ρ the fluid density. Some experiments are to be performed in a wind tunnel to determine the drag. What dimensionless parameters would you use to organize these data?

$\mathcal{D} \doteq F \quad d_1 \doteq L \quad d_2 \doteq L \quad V \doteq LT^{-1} \quad \mu \doteq FL^{-2}T \quad \rho \doteq FL^{-4}T^2$

From the pi theorem, $6-3=3$ pi terms required. Use d_1, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \mathcal{D}\, d_1^a V^b \rho^c$$

and

$$(F)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F)$$
$$a + b - 4c = 0 \qquad \text{(for } L)$$
$$-b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a=-2$, $b=-2$, $c=-1$, and therefore

$$\Pi_1 = \frac{\mathcal{D}}{d_1^2 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{\mathcal{D}}{d_1^2 V^2 \rho} \doteq \frac{MLT^{-2}}{(L)^2(LT^{-1})^2(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2:

$$\Pi_2 = d_2\, d_1^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F)$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L)$$
$$-b + 2c = 0 \qquad \text{(for } T)$$

(con't)

7.10 (con't)

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{d_2}{d_1}$$

which is obviously dimensionless.

For Π_3:

$$\Pi_3 = \mu \, d_1^a V^b \rho^c$$

$$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \quad \text{(for } F\text{)}$$
$$-2 + a + b - 4c = 0 \quad \text{(for } L\text{)}$$
$$1 - b + 2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = -1$, $c = -1$, and therefore

$$\Pi_3 = \frac{\mu}{d_1 V \rho}$$

Check dimensions using MLT system:

$$\frac{\mu}{d_1 V\rho} \doteq \frac{ML^{-1}T^{-1}}{(L)(LT^{-1})(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Thus,

$$\frac{\mathcal{D}}{d_1^2 V^2 \rho} = \phi\left(\frac{d_2}{d_1}, \frac{\mu}{d_1 V\rho}\right) \qquad (1)$$

Since $\frac{\rho V d_1}{\mu}$ is a standard dimensionless parameter (Reynolds number), Eq. (1) would more commonly be expressed as

$$\frac{\mathcal{D}}{d_1^2 V^2 \rho} = \phi\left(\frac{d_2}{d_1}, \frac{\rho V d_1}{\mu}\right) \qquad (2)$$

As far as dimensional analysis is concerned, Eqs. (1) and (2) are equivalent.

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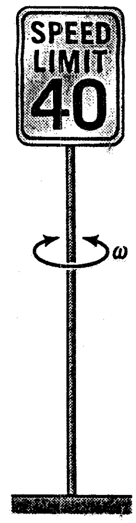
10% Reduction
8 1/2 x 11 trim size

7.11

7.11 Under certain conditions, wind blowing past a rectangular speed limit sign can cause the sign to oscillate with a frequency ω. (See Fig. P7.11 and Video V9.6.) Assume that ω is a function of the sign width, b, sign height, h, wind velocity, V, air density, ρ, and an elastic constant, k, for the supporting pole. The constant, k, has dimensions of FL. Develop a suitable set of pi terms for this problem.



■ FIGURE P7.11

$$\omega = f(b, h, V, \rho, k)$$

$\omega \doteq T^{-1}$ $\quad b \doteq L$ $\quad h \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad k \doteq FL$

From the pi theorem $6-3=3$ pi terms required. Use b, V, and ρ as repeating variables. Thus,

$$\Pi_1 = \omega b^a V^b \rho^c$$

and

$$(T^{-1})(L)^a(LT^{-1})^b(FL^{-4}T^{2})^c \doteq F^0L^0T^0$$

so that

$$c = 0 \qquad \text{(for } F)$$
$$a + b - 4c = 0 \qquad \text{(for } L)$$
$$-1 - b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a = 1$, $b = -1$, $c = 0$, and therefore

$$\Pi_1 = \frac{\omega b}{V}$$

Check dimensions:

$$\frac{\omega b}{V} \doteq \frac{(T^{-1})(L)}{(LT^{-1})} \doteq L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = h\, b^a V^b \rho^c$$

$$(L)(L)^a(LT^{-1})^b(FL^{-4}T^{2})^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F)$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L)$$
$$-b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_2 = \frac{h}{b}$$

which is obviously dimensionless. (con't)

7.11 (Con't)

For Π_3:

$$\Pi_3 = k\, b^a V^b \rho^c$$

$$(FL)(L)^a (LT^{-1})^b (FL^{-4}T^2)^c \doteq F^0 L^0 T^0$$

$$1 + c = 0 \quad \text{(for } F\text{)}$$

$$1 + a + b - 4c = 0 \quad \text{(for } L\text{)}$$

$$-b + 2c = 0 \quad \text{(for } T\text{)}$$

It follows that $a = -3$, $b = -2$, $c = -1$, and therefore

$$\Pi_3 = \frac{k}{b^3 V^2 \rho}$$

Check dimensions using MLT system:

$$\frac{k}{b^3 V^2 \rho} \doteq \frac{M L^2 T^{-2}}{(L^3)(LT^{-1})^2 (ML^{-3})} \doteq M^0 L^0 T^0 \quad \therefore \text{OK}$$

Thus,

$$\frac{\omega b}{V} = \phi\left(\frac{h}{b}, \frac{k}{b^3 V^2 \rho}\right)$$

7.12

7.12 The velocity, V, of a spherical particle falling slowly in a viscous liquid can be expressed as

$$V = f(d, \mu, \gamma, \gamma_s)$$

where d is the particle diameter, μ the liquid viscosity, and γ and γ_s the specific weight of the liquid and particle, respectively. Develop a set of dimensionless parameters that can be used to investigate this problem.

$$V \doteq LT^{-1} \quad d \doteq L \quad \mu \doteq FL^{-2}T \quad \gamma \doteq FL^{-3} \quad \gamma_s \doteq FL^{-3}$$

From the pi theorem, 5-3 = 2 pi terms required. Use d, μ, and γ as repeating variables. Thus,

$$\Pi_1 = V d^a \mu^b \gamma^c$$

and

$$(LT^{-1})(L)(FL^{-2}T)^b (FL^{-3})^c \doteq F^0 L^0 T^0$$

so that

$$b + c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a - 2b - 3c = 0 \qquad \text{(for } L\text{)}$$
$$-1 + b = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -2$, $b = 1$, $c = -1$, and therefore

$$\Pi_1 = \frac{V\mu}{d^2\gamma}$$

Check dimensions using MLT system:

$$\frac{V\mu}{d^2\gamma} \doteq \frac{(LT^{-1})(ML^{-1}T^{-1})}{(L)^2(ML^{-2}T^{-2})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2:

$$\Pi_2 = \gamma_s d^a \mu^b \gamma^c$$

$$(FL^{-3})(L)^a (FL^{-2}T)^b (FL^{-3})^c \doteq F^0L^0T^0$$

$$1 + b + c = 0 \qquad \text{(for } F\text{)}$$
$$-3 + a - 2b - 3c = 0 \qquad \text{(for } L\text{)}$$
$$b = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 0$, $b = 0$, $c = -1$, and therefore

$$\Pi_2 = \frac{\gamma_s}{\gamma}$$

which is obviously dimensionless.

Thus,

$$\frac{V\mu}{d^2\gamma} = \phi\left(\frac{\gamma_s}{\gamma}\right)$$

7.13

7.13 Because of surface tension, it is possible, with care, to support an object heavier than water on the water surface as shown in Fig. P7.13. (See Video V1.5.) The maximum thickness, h, of a square of material that can be supported is assumed to be a function of the length of the side of the square, ℓ, the density of the material, ρ, the acceleration of gravity, g, and the surface tension of the liquid, σ. Develop a suitable set of dimensionless parameters for this problem.

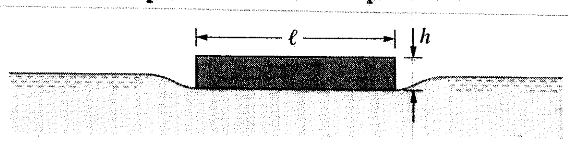


FIGURE P7.13

$$h = f(\ell, \rho, g, \sigma)$$

$h \doteq L \quad \ell \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad g \doteq LT^{-2} \quad \sigma \doteq FL^{-1}$

From the pi theorem, $5 - 3 = 2$ pi terms required. Use ℓ, g, and ρ as repeating variables. Thus,

$$\Pi_1 = h\,\ell^{a} g^{b} \rho^{c}$$

and so that

$$(L)(L)^{a}(LT^{-2})^{b}(FL^{-4}T^{2})^{c} \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F)$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L)$$
$$-2b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a = -1$, $b = 0$, $c = 0$, and therefore

$$\Pi_1 = \frac{h}{\ell}$$

which is obviously dimensionless.

For Π_2:

$$\Pi_2 = \sigma\,\ell^{a} g^{b} \rho^{c}$$

$$(FL^{-1})(L)^{a}(LT^{-2})^{b}(FL^{-4}T^{2})^{c} = F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F)$$
$$-1 + a + b - 4c = 0 \qquad \text{(for } L)$$
$$-2b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a = -2$, $b = -1$, $c = -1$, and therefore

$$\Pi_2 = \frac{\sigma}{\ell^2 g \rho}$$

Check dimensions using MLT system:

$$\frac{\sigma}{\ell^2 g \rho} \doteq \frac{(MT^{-2})}{(L^2)(LT^{-2})(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

Thus,

$$\frac{h}{\ell} = \phi\left(\frac{\sigma}{\ell^2 g \rho}\right)$$

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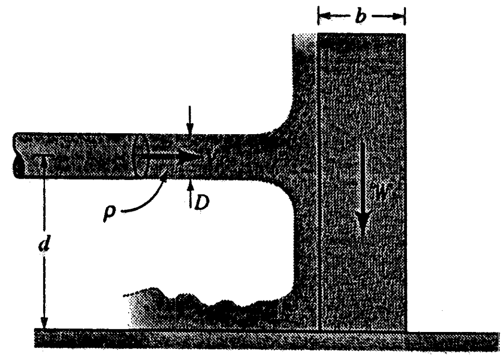
7.14

7.14 As shown in Fig. P7.14 and Video V5.4, a jet of liquid directed against a block can tip over the block. Assume that the velocity, V, needed to tip over the block is a function of the fluid density, ρ, the diameter of the jet, D, the weight of the block, $\mathcal{W}$, the width of the block, b, and the distance, d, between the jet and the bottom of the block. **(a)** Determine a set of dimensionless parameters for this problem. Form the dimensionless parameters by inspection. **(b)** Use the momentum equation to determine an equation for V in terms of the other variables. **(c)** Compare the results of parts **(a)** and **(b)**.



■ FIGURE P7.14

(a) $$V = f(\rho, D, \mathcal{W}, b, d)$$

$$V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad D \doteq L \quad \mathcal{W} \doteq F \quad b \doteq L \quad d \doteq L$$

From the pi theorem, $6-3=3$ pi terms required.
By inspection for Π_1 (containing V)

$$\Pi_1 = VD\sqrt{\frac{\rho}{\mathcal{W}}} \doteq (LT^{-1})(L)\left(\sqrt{\frac{FL^{-4}T^{2}}{F}}\right) \doteq F^0L^0T^0$$

Check using MLT:

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} \doteq (LT^{-1})(L)\left(\sqrt{\frac{ML^{-3}}{MLT^{-2}}}\right) \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 let

$$\Pi_2 = \frac{b}{d}$$

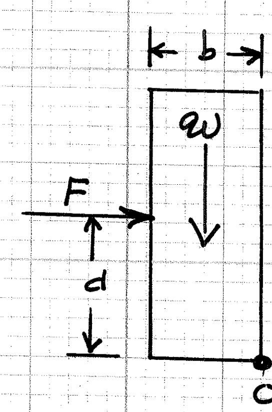
and for Π_3

$$\Pi_3 = \frac{d}{D}$$

and both Π_2 and Π_3 are obviously dimensionless.

Thus,

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$



(b) For impending tipping around O

$$\Sigma M_0 = 0$$

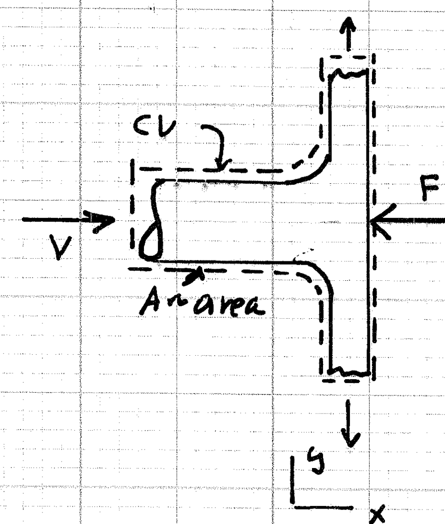
so that

$$Fd = \mathcal{W}\left(\frac{b}{2}\right) \qquad (1)$$

(con't)

7.14 (con't)

From momentum considerations using the CV shown



$(\overset{+}{\rightarrow})$ $\int \rho u \vec{V} \cdot \hat{n} \, dA = \sum F_x$

$$\rho V^2 A = F$$

Thus, from Eq. (1)

$$(\rho V^2 A)(d) = \mathcal{W}\left(\frac{b}{2}\right)$$

so that

$$V = \sqrt{\frac{\mathcal{W}(b)}{2\rho A d}}$$

and with $A = \pi/4 \, D^2$

$$V = \sqrt{\frac{2\mathcal{W} b}{\pi \rho d D^2}} \qquad (2)$$

(c) From part (a)

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}} \, \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$

Eq. (2) can be written as

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}} \left(\sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}\right) \qquad (3)$$

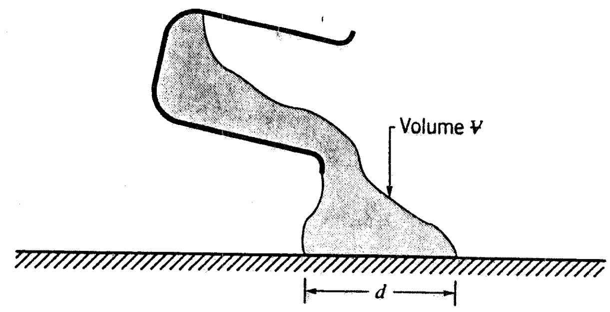
It follows by comparing Eqs. (2) and (3) that

$$\phi\left(\frac{b}{d}, \frac{d}{D}\right) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}$$

so that $\phi\left(\frac{b}{d}, \frac{d}{D}\right)$ is actually independent of $\frac{d}{D}$.

7.15

7.15 A viscous fluid is poured onto a horizontal plate as shown in Fig. P7.15. Assume that the time, t, required for the fluid to flow a certain distance, d, along the plate is a function of the volume of fluid poured, $\forall$, acceleration of gravity, g, fluid density, ρ, and fluid viscosity, μ. Determine an appropriate set of pi terms to describe this process. Form the pi terms by inspection.



■ FIGURE P7.15

$$t = f(d, \forall, g, \rho, \mu)$$

$t \doteq T \quad d \doteq L \quad \forall \doteq L^3 \quad g \doteq LT^{-2} \quad \rho \doteq FL^{-4}T^2 \quad \mu \doteq FL^{-2}T$

From the pi theorem $6-3=3$ pi terms required.

By inspection, for Π_1 (containing t):

$$\Pi_1 = t\sqrt{\frac{g}{d}} \doteq \frac{(T)(LT^{-2})^{1/2}}{(L)^{1/2}} \doteq F^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2 (containing $\forall$):

$$\Pi_2 = \frac{\forall}{d^3} \doteq \frac{(L^3)}{(L)^3} \doteq F^0L^0T^0 \quad \therefore \text{ok}$$

For Π_3 (containing ρ and μ):

$$\Pi_3 = \frac{\rho\sqrt{g}\,d^{3/2}}{\mu} \doteq \frac{(FL^{-4}T^2)(LT^{-2})^{1/2}(L)^{3/2}}{FL^{-2}T} \doteq F^0L^0T^0$$

Check using MLT system:

$$\frac{\rho\sqrt{g}\,d^{3/2}}{\mu} \doteq \frac{(ML^{-3})(LT^{-2})^{1/2}(L)^{3/2}}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

Thus,

$$t\sqrt{\frac{g}{d}} = \phi\left(\frac{\forall}{d^3}, \frac{\rho\sqrt{g}\,d^{3/2}}{\mu}\right)$$

7.16

7.16 Assume that the drag, $\mathfrak{D}$, on an aircraft flying at supersonic speeds is a function of its velocity, V, fluid density, ρ, speed of sound, c, and a series of lengths, $\ell_1, \ldots, \ell_i$, which describe the geometry of the aircraft. Develop a set of pi terms that could be used to investigate experimentally how the drag is affected by the various factors listed. Form the pi terms by inspection.

$$\mathfrak{D} = f(V, \rho, c, \ell_1, \ldots \ell_i)$$

$\mathfrak{D} \doteq F \quad V = LT^{-1} \quad \rho \doteq FL^{-4}T^2 \quad c \doteq LT^{-1}$ all lengths, $\ell_i \doteq L$

From the pi theorem, $(4+i)-3 = 1+i$ pi terms required, where i is the number of length terms ($i = 1, 2, 3$, etc.).

By inspection, for Π_1 (containing $\mathfrak{D}$):

$$\Pi_1 = \frac{\mathfrak{D}}{\rho V^2 \ell_1^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2(L)^2} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\mathfrak{D}}{\rho V^2 \ell_1^2} \doteq \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} \doteq M^0 L^0 T^0 \quad \therefore \text{OK}$$

For Π_2 (containing c):

$$\Pi_2 = \frac{c}{V} \quad \text{or} \quad \frac{V}{c}$$

and both are obviously dimensionless.

For all other pi terms containing ℓ_i

$$\Pi_i = \frac{\ell_i}{\ell_1}$$

and these terms involving the ℓ's are obviously dimensionless.

Thus,

$$\frac{\mathfrak{D}}{\rho V^2 \ell_1^2} = \phi\left(\frac{V}{c}, \frac{\ell_i}{\ell_1}\right)$$

where $\frac{\ell_i}{\ell_1}$ is a series of pi terms, $\frac{\ell_2}{\ell_1}, \frac{\ell_3}{\ell_1}$, etc.

7.17

7.17 A cone and plate viscometer consists of a cone with a very small angle α which rotates above a flat surface as shown in Fig. P7.17. The torque, $\mathcal{T}$, required to rotate the cone at an angular velocity, ω, is a function of the radius, R, the cone angle, α, and the fluid viscosity, μ, in addition to ω. With the aid of dimensional analysis, determine how the torque will change if both the viscosity and angular velocity are doubled.

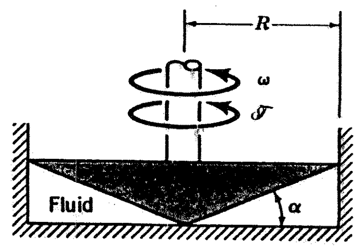


FIGURE P7.17

$$\mathcal{T} = f(R, \alpha, \mu, \omega)$$

$\mathcal{T} \doteq FL \qquad R \doteq L \qquad \alpha \doteq F^0L^0T^0 \qquad \mu \doteq FL^{-2}T \qquad \omega \doteq T^{-1}$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing $\mathcal{T}$):

$$\Pi_1 = \frac{\mathcal{T}}{\mu\omega R^3} \doteq \frac{FL}{(FL^{-2}T)(T^{-1})(L)^3} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\mathcal{T}}{\mu\omega R^3} \doteq \frac{ML^2T^{-2}}{(ML^{-1}T^{-1})(T^{-1})(L)^3} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

The angle, α, can be used as Π_2 since it is dimensionless.

Thus,

$$\frac{\mathcal{T}}{\mu\omega R^3} = \phi(\alpha)$$

or

$$\mathcal{T} = \mu\omega R^3\phi(\alpha)$$

It follows that if both μ and ω are doubled $\mathcal{T}$ will increase by a factor of 4.

7.18

7.18 The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, \ell, V, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \ell \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p\, D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\Delta p\, D}{\mu V} \doteq \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} \doteq M^0 L^0 T^0 \quad \therefore \text{ok}$$

For Π_2 (containing ℓ):

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p\, D}{\mu V} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

From the statement of the problem, $\Delta p \propto \ell$ so that Eq. (1) must be of the form

$$\frac{\Delta p\, D}{\mu V} = K \frac{\ell}{D}$$

Where K is some constant. It thus follows that

$$\underline{\underline{\Delta p \propto \frac{1}{D^2}}}$$

for a given velocity.

7.19

7.19 One type of viscometer consists of an open reservoir with a small diameter tube at the bottom as illustrated in Fig. P7.19. To measure viscosity the system is filled with the liquid of interest and the time required for the liquid level to fall from level H_i to H_f is determined. Use dimensional analysis to obtain a relationship between the viscosity, μ, and the draining time, τ. Assume that the other variables involved are the initial head, H_i, the final head, H_f, the tube diameter, D, and the specific weight of the liquid, γ.

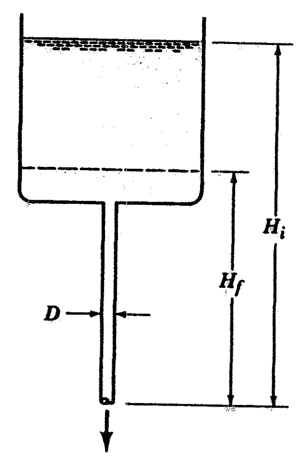


FIGURE P7.19

$$\tau = f(D, H_i, H_f, \mu, \gamma)$$

$\tau \doteq T \quad D \doteq L \quad H_i \doteq L \quad H_f \doteq L \quad \mu \doteq FL^{-2}T \quad \gamma \doteq FL^{-3}$

From the pi theorem, $6-3=3$ pi terms required.

By inspection, for Π_1 (containing τ):

$$\Pi_1 = \frac{\tau\gamma D}{\mu} \doteq \frac{(T)(FL^{-3})(L)}{FL^{-2}T} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\tau\gamma D}{\mu} \doteq \frac{(T)(ML^{-2}T^{-2})(L)}{ML^{-1}T^{-1}} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2 (containing H_i):

$$\Pi_2 = \frac{H_i}{D}$$

which is obviously dimensionless. Similarly,

$$\Pi_3 = \frac{H_f}{D}$$

Thus,

$$\frac{\tau\gamma D}{\mu} = \phi\left(\frac{H_i}{D}, \frac{H_f}{D}\right)$$

and for a fixed geometry (including H_i and H_f)

$$\frac{\tau\gamma D}{\mu} = K \qquad (1)$$

where K is a constant, depending on H_i/D and H_f/D.

From Eq. (1)

$$\mu = \frac{\gamma D}{K}\tau$$

so that

$$\underline{\underline{\mu = K_1 \gamma \tau}}$$

where $K_1 = D/K$ and K_1 is a constant for a fixed geometry.

7.20

7.20 A cylinder with a diameter, D, floats upright in a liquid as shown in Fig. P7.20. When the cylinder is displaced slightly along its vertical axis it will oscillate about its equilibrium position with a frequency, ω. Assume that this frequency is a function of the diameter, D, the mass of the cylinder, m, and the specific weight, γ, of the liquid. Determine, with the aid of dimensional analysis, how the frequency is related to these variables. If the mass of the cylinder were increased, would the frequency increase or decrease?

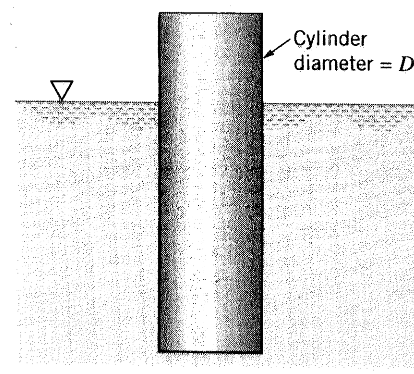


FIGURE P7.20

$$\omega = f(D, m, \gamma)$$

$\omega \doteq T^{-1}$ $D \doteq L$ $m \doteq FL^{-1}T^{2}$ $\gamma \doteq FL^{-3}$

From the pi theorem, 4-3 = 1 pi term required.

By inspection:

$$\Pi_1 = \frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{FL^{-1}T^{2}}{FL^{-3}}} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{M}{ML^{-2}T^{-2}}} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Since there is only 1 pi term, it follows that

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} = C$$

where C is a constant. Thus,

$$\omega = CD\sqrt{\frac{\gamma}{m}}$$

From this result it follows that if m is increased ω will decrease.

7.21*

*7.21 The pressure drop, Δp, over a certain length of horizontal pipe is assumed to be a function of the velocity, V, of the fluid in the pipe, the pipe diameter, D, and the fluid density and viscosity, ρ and μ. (a) Show that this flow can be described in dimensionless form as a "pressure coefficient," $C_p = \Delta p/(0.5\,\rho V^2)$ that depends on the Reynolds number, $\text{Re} = \rho VD/\mu$. (b) The following data were obtained in an experiment involving a fluid with $\rho = 2$ slugs/ft³, $\mu = 2 \times 10^{-3}$ lb · s/ft², and $D = 0.1$ ft. Plot a dimensionless graph and use a power law equation to determine the functional relationship between the pressure coefficient and the Reynolds number.

V, ft/s	Δp, lb/ft²
3	192
11	704
17	1088
20	1280

(c) What are the limitations on the applicability of your equation obtained in part (b)?

(a) $$\Delta p = f(V, D, \rho, \mu)$$

$$\Delta p \doteq FL^{-2} \quad V \doteq LT^{-1} \quad D \doteq L \quad \rho \doteq FL^{-4}T^2 \quad \mu \doteq FL^{-2}T$$

From the pi theorem, 5 − 3 = 2 pi terms required.
By inspection for Π_1,

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0 \quad \therefore \text{OK}$$

Check using MLT system:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = \frac{\rho VD}{\mu} \doteq \frac{(FL^{-4}T^2)(LT^{-1})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0 \quad \therefore \text{OK}$$

Check using MLT system:

$$\frac{\rho VD}{\mu} \doteq \frac{(ML^{-3})(LT^{-1})(L)}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Thus,

$$\frac{\Delta p}{\rho V^2} = \tilde{\phi}\left(\frac{\rho VD}{\mu}\right)$$

Since $\tilde{\phi}$ is an unknown function, a factor of 0.5 can be included in Π_1 (if desired) so that

$$\frac{\Delta p}{0.5\rho V^2} = \phi\left(\frac{\rho VD}{\mu}\right)$$

Thus, $$C_p = \phi(Re)$$

where C_p is the pressure coefficient and Re the Reynolds number.

(con't)

7.21* (con't)

(b) Using the data given,

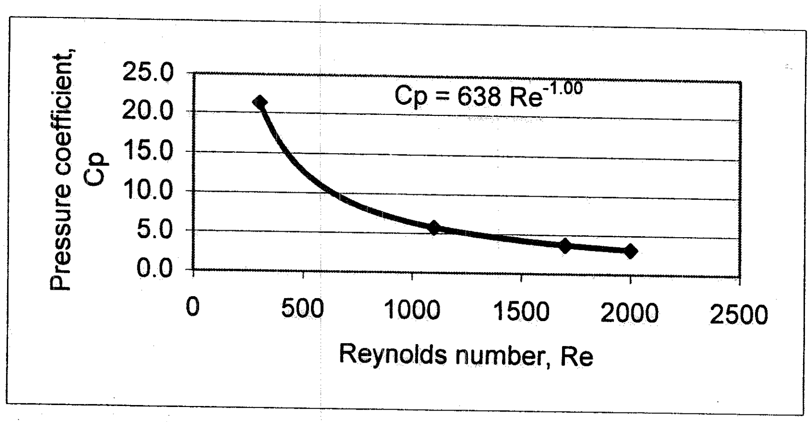
$$C_p = \frac{\Delta p}{0.5\rho V^2} = \frac{\Delta p}{(0.5)\left(2\ \frac{slugs}{ft^3}\right)V^2} = \frac{\Delta p}{V^2}$$

and

$$R_e = \frac{\rho V D}{\mu} = \frac{2\left(\frac{slugs}{ft^3}\right)(V)(0.1ft)}{2\times10^{-3}\ \frac{lb\cdot s}{ft^2}} = 100V$$

Tabulated values for C_p and Re and a plot of the data are shown below.

V, ft/s	Δ p, psf	Re	Cp
3	192	300	21.3
11	704	1100	5.82
17	1090	1700	3.77
20	1280	2000	3.20



The power law relationship is

$$C_p = \frac{638}{R_e} \qquad (1)$$

(c) Based on the variables used and the given data, the empirical relationship, Eq. (1), would only be applicable in the Reynolds number range

$$300 \le R_e \le 2000$$

Note: Although the equation might be valid outside this range, results should not be extrapolated beyond the range of data used.

7.22

7.22 The height, h, that a liquid will rise in a capillary tube is a function of the tube diameter, D, the specific weight of the liquid, γ, and the surface tension, σ. Perform a dimensional analysis using both the FLT and MLT systems for basic dimensions. Note: The results should obviously be the same regardless of the system of dimensions used. If your analysis indicates otherwise, go back and check your work giving particular attention to the required number of reference dimensions.

$$h = f(D, \gamma, \sigma)$$

Using FLT system:

$h \doteq L \qquad D \doteq L \qquad \gamma \doteq FL^{-3} \qquad \sigma \doteq FL^{-1}$

From the pi theorem, $4-2 = 2$ pi terms required.

By inspection, for Π_1 (containing h):

$$\Pi_1 = \frac{h}{D}$$

which is obviously dimensionless.

For Π_2 (containing γ and σ):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} \doteq \frac{FL^{-1}}{(FL^{-3})(L)^2} = F^0L^0$$

Thus,

$$\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)$$

Using MLT system:

$h \doteq L \qquad D \doteq L \qquad \gamma \doteq ML^{-2}T^{-2} \qquad \sigma = MT^{-2}$

Although there appears to be 3 reference dimensions, only 2 reference dimensions are actually required (L and MT^{-2}) to describe the variables. By inspection, for Π_1 (see above)

$$\Pi_1 = \frac{h}{D}$$

and for Π_2 (containing γ and σ):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} = \frac{MT^{-2}}{(ML^{-2}T^{-2})(L)^2} = M^0L^0T^0$$

Thus, (as above)

$$\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)$$

7.23

7.23 Assume that the drag on a small sphere placed in a rapidly moving stream of fluid depends on the fluid density but not the fluid viscosity. Use dimensional analysis to determine how the drag is affected if the velocity of the fluid is doubled.

Let: drag $= \mathscr{D} \doteq F$ sphere diameter $= d \doteq L$

fluid velocity $= V \doteq LT^{-1}$ density of fluid $= \rho \doteq FL^{-4}T^2$

Thus,

$$\mathscr{D} = f(d, V, \rho)$$

From the pi theorem, $4-3=1$ pi term required.

By inspection:

$$\Pi_1 = \frac{\mathscr{D}}{\rho V^2 d^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2(L)^2} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\mathscr{D}}{\rho V^2 d^2} \doteq \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} \doteq M^0 L^0 T^0 \quad \therefore \text{ok}$$

Since there is only 1 pi term, it follows that

$$\frac{\mathscr{D}}{\rho V^2 d^2} = C$$

where C is a constant. Thus,

$$\mathscr{D} = C \rho V^2 d^2$$

and if V is doubled $\mathscr{D}$ will increase by a factor of 4.

7.24*

*7.24 The pressure drop across a short hollowed plug placed in a circular tube through which a liquid is flowing (see Fig. P7.24) can be expressed as

$$\Delta p = f(\rho, V, D, d)$$

where ρ is the fluid density, and V is the mean velocity in the tube. Some experimental data obtained with $D = 0.2$ ft, $\rho = 2.0$ slugs/ft³, and $V = 2$ ft/s are given in the following table:

d (ft)	0.06	0.08	0.10	0.15
Δp (lb/ft²)	493.8	156.2	64.0	12.6

Plot the results of these tests, using suitable dimensionless parameters, on log–log graph paper. Use a standard curve-fitting technique to determine a general equation for Δp. What are the limits of applicability of the equation?

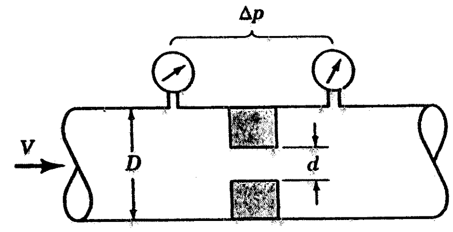


FIGURE P7.24

$\Delta p \doteq FL^{-2}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad V \doteq LT^{-1}$ $\quad D \doteq L$ $\quad d \doteq L$

From the pi theorem, 5−3 = 2 pi terms required. By inspection for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^{2})(LT^{-1})^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing D and d):

$$\Pi_2 = \frac{D}{d}$$

(which is obviously dimensionless). Thus,

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{D}{d}\right)$$

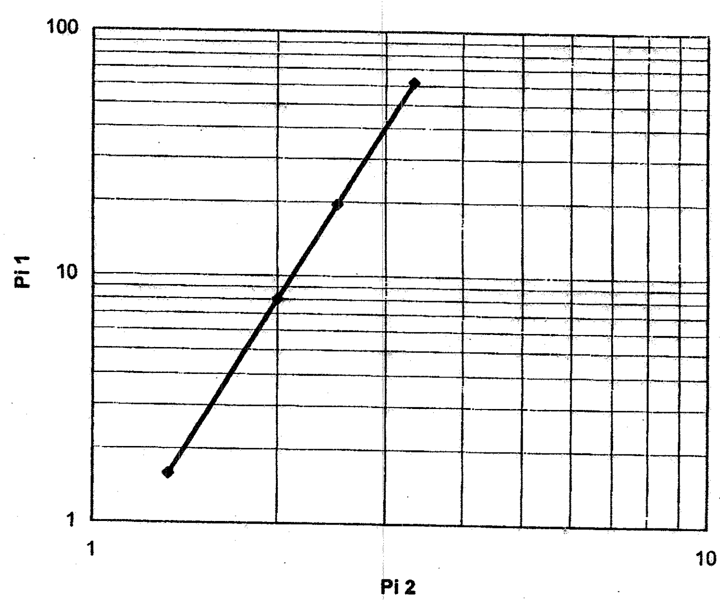
For the data given:

D/d	3.33	2.50	2.00	1.33
$\Delta p/\rho V^2$	61.7	19.5	8.00	1.58

A log-log plot of these data is shown on the following page.

(con't)

7.24* (con't)



Since the data plot as a straight line on a log-log plot, the equation for the data is of the form

$$\Pi_1 = a\,\Pi_2^{\,b}$$

where $\Pi_1 = \Delta p/\rho V^2$ and $\Pi_2 = D/d$. A power law fit of the data gives

$$a = 0.505 \quad \text{and} \quad b = 3.99$$

Thus,

$$\frac{\Delta p}{\rho V^2} = 0.505\left(\frac{D}{d}\right)^{3.99}$$

This equation is applicable over the range of data $1.33 \le \frac{D}{d} \le 3.33$.

7.25

7.25 A liquid flows with a velocity V through a hole in the side of a large tank. Assume that

$$V = f(h, g, \rho, \sigma)$$

where h is the depth of fluid above the hole, g is the acceleration of gravity, ρ the fluid density, and σ the surface tension. The following data were obtained by changing h and measuring V, with a fluid having a density $= 10^3$ kg/m³ and surface tension $= 0.074$ N/m.

V (m/s)	3.13	4.43	5.42	6.25	7.00
h (m)	0.50	1.00	1.50	2.00	2.50

Plot these data by using appropriate dimensionless variables. Could any of the original variables have been omitted?

$V \doteq LT^{-1}$ $\quad h \doteq L$ $\quad g \doteq LT^{-2}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \sigma \doteq FL^{-1}$

From the pi theorem, $5-3 = 2$ pi terms required.

By inspection for Π_1 (containing V):

$$\Pi_1 = \frac{V}{\sqrt{gh}} \doteq \frac{LT^{-1}}{(LT^{-2})^{1/2}(L)^{1/2}} \doteq L^0T^0$$

For Π_2 (containing ρ and σ):

$$\Pi_2 = \frac{\rho g h^2}{\sigma} \doteq \frac{(FL^{-4}T^{2})(LT^{-2})(L)^2}{FL^{-1}} \doteq F^0L^0T^0$$

Check using MLT:

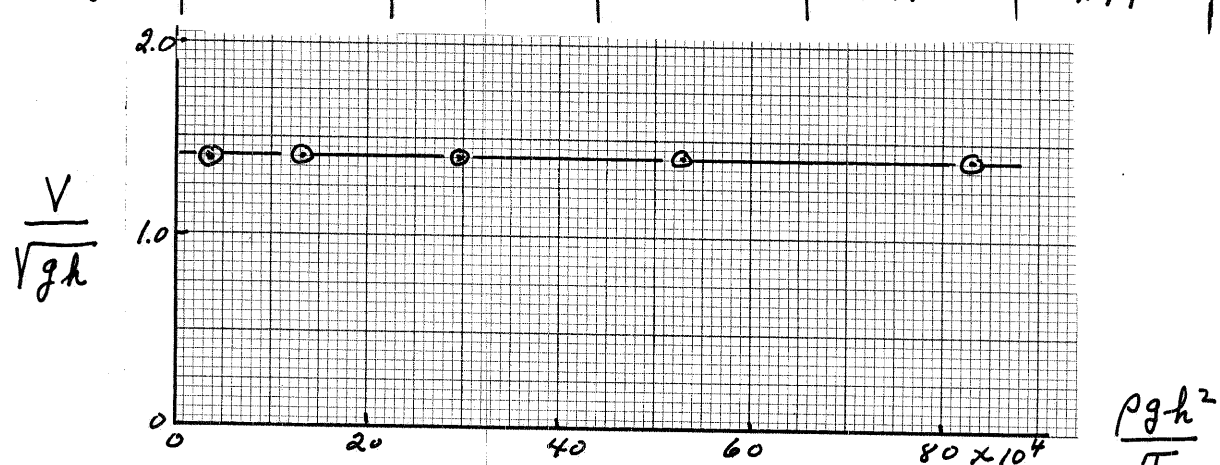
$$\frac{\rho g h^2}{\sigma} \doteq \frac{(ML^{-3})(LT^{-2})(L)^2}{MT^{-2}} \doteq M^0L^0T^0 \;\therefore \text{OK}$$

Thus,

$$\frac{V}{\sqrt{gh}} = \phi\left(\frac{\rho g h^2}{\sigma}\right)$$

For the data given:

$\rho g h^2/\sigma$	3.31×10^4	13.3×10^4	29.8×10^4	53.0×10^4	82.9×10^4
$V/\sqrt{gh}$	1.41	1.41	1.41	1.41	1.41



The graph and table show that $V/\sqrt{gh}$ is independent of $\rho g h^2/\sigma$. Thus, <u>the variables ρ and σ could have been omitted</u>.

7.26

7.26 The time, t, it takes to pour a certain volume of liquid from a cylindrical container depends on several factors, including the viscosity of the liquid. (See Video V1.1.) Assume that for very viscous liquids the time it takes to pour out 2/3 of the initial volume depends on the initial liquid depth, ℓ, the cylinder diameter, D, the liquid viscosity, μ, and the liquid specific weight, γ. The data shown in the following table were obtained in the laboratory. For these tests $\ell = 45$ mm, $D = 67$ mm, and $\gamma = 9.60\ \text{kN/m}^3$. **(a)** Perform a dimensional analysis and based on the data given, determine if variables used for this problem appear to be correct. Explain how you arrived at your answer. **(b)** If possible, determine an equation relating the pouring time and viscosity for the cylinder and liquids used in these tests. If it is not possible, indicate what additional information is needed.

μ (N·s/m²)	11	17	39	61	107
t(s)	15	23	53	83	145

$$t = f(\ell, D, \mu, \gamma)$$

(a) $t \doteq T \quad \ell \doteq L \quad D \doteq L \quad \mu \doteq FL^{-2}T \quad \gamma \doteq FL^{-3}$

From the pi theorem $5-3=2$ pi terms required.

By inspection, for Π_1 (containing t)

$$\Pi_1 = \frac{t\gamma D}{\mu} \doteq \frac{(T)(FL^{-3})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0$$

Check using MLT system:

$$\frac{t\gamma D}{\mu} \doteq \frac{(T)(ML^{-2}T^{-2})(L)}{(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2 (containing ℓ)

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{t\gamma D}{\mu} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

For the data given $\frac{\ell}{D} = \frac{45\text{mm}}{67\text{mm}} = 0.672$ (a constant).

Thus, from Eq. (1) with ℓ/D a constant it follows that $\frac{t\gamma D}{\mu}$ = constant. For the data given:

(con't)

7.26 (con't)

$\frac{t\gamma D}{\mu}$	877	870	874	875	872

Since Π_1 is essentially constant over the range of the experimental data <u>the variables used for the problem appear to be correct.</u>

(b) The average value for Π_1 is 874 so that

$$\frac{t\gamma D}{\mu} = 874$$

and therefore

$$t = \frac{874}{\gamma D}\mu = \frac{874\,\mu}{\left(9.6\times10^3\,\frac{N}{m^3}\right)\left(67\times10^{-3}\,m\right)}$$

$$t = 1.36\,\mu$$

with t in seconds when μ is in units of $N\cdot s/m^2$. Note that this restricted equation is only valid for $\ell/D = 0.672$, $D = 67\,mm$, and $\gamma = 9.60\,kN/m^3$ with 2/3 of the initial volume being poured.

7.27

7.27 The pressure drop per unit length, Δp_ℓ, for the flow of blood through a horizontal small diameter tube is a function of the volume rate of flow, Q, the diameter, D, and the blood viscosity, μ. For a series of tests in which d = 2 mm, and μ = 0.004 N·s/m², the following data were obtained, where the Δp listed was measured over the length, ℓ = 300 mm.

Q (m³/s)	Δp (N/m²)
3.6×10^{-6}	1.1×10^{4}
4.9×10^{-6}	1.5×10^{4}
6.3×10^{-6}	1.9×10^{4}
7.9×10^{-6}	2.4×10^{4}
9.8×10^{-6}	3.0×10^{4}

Perform a dimensional analysis for this problem, and make use of the data given to determine a general relationship between Δp_ℓ and Q (one that is valid for other values of D, ℓ, and μ).

$$\Delta p_\ell = f(Q, D, \mu)$$

$$\Delta p_\ell \doteq FL^{-3} \qquad Q \doteq L^3T^{-1} \qquad D \doteq L \qquad \mu \doteq FL^{-2}T$$

From the pi theorem, 4-3 = 1 pi term required.

By inspection:

$$\Pi_1 = \frac{\Delta p_\ell D^4}{\mu Q} \doteq \frac{(FL^{-3})(L)^4}{(FL^{-2}T)(L^3T^{-1})} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p_\ell D^4}{\mu Q} \doteq \frac{(ML^{-2}T^{-2})(L)^4}{(ML^{-1}T^{-1})(L^3T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

Since there is only 1 pi term, it follows that

$$\frac{\Delta p_\ell D^4}{\mu Q} = C$$

where C is a constant. For the data given

$$\frac{\Delta p_\ell D^4}{\mu Q} = \left(\frac{\Delta p}{0.3\,m}\right)\frac{(0.002\,m)^4}{(0.004\ N\cdot s/m^2)\,Q} = 1.33 \times 10^{-8}\ \frac{\Delta p}{Q}$$

and therefore using the data in the table

$\frac{\Delta p_\ell D^4}{\mu Q}$	40.6	40.7	40.1	40.4	40.7

Thus, the average value for C = 40.5 and

$$\Delta p_\ell = 40.5\ \frac{\mu Q}{D^4}$$

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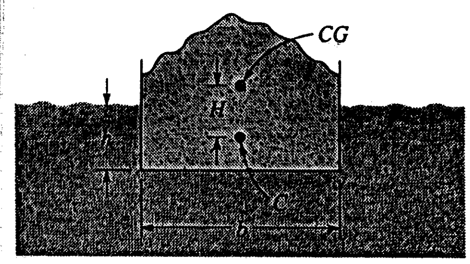
7.28 *

*7.28 As shown in Fig. 2.26, Fig. P7.28, and Video V2.7, a rectangular barge floats in a stable configuration provided the distance between the center of gravity, *CG*, of the object (boat and load) and the center of buoyancy, *C*, is less than a certain amount, *H*. If this distance is greater than *H* the boat will tip over. Assume *H* is a function of the boat's width, *b*, length, ℓ, and draft, *h*. **(a)** Put this relationship into dimensionless form. **(b)** The results of a set of experiments with a model barge with a width of 1.0 m is shown in the table. Plot this data in dimensionless form and determine a power-law equation relating the dimensionless parameters.

ℓ, m	h, m	H, m
2.0	0.10	0.833
4.0	0.10	0.833
2.0	0.20	0.417
4.0	0.20	0.417
2.0	0.35	0.238
4.0	0.35	0.238



■ FIGURE P7.28

(a) $H = f(b, \ell, h)$

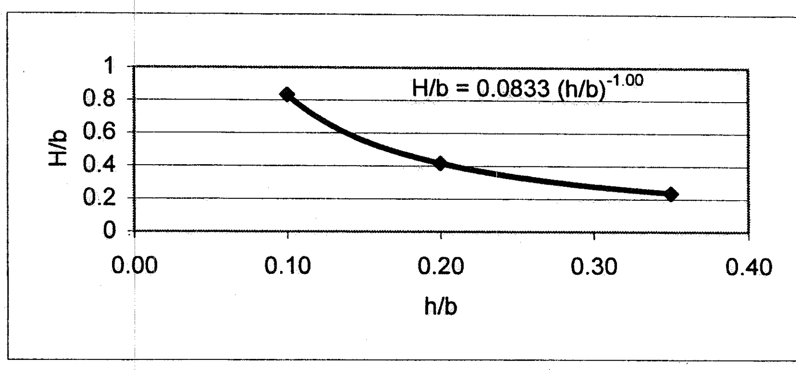
From the pi theorem, 4−1 = 3 pi terms required. By inspection:

$$\frac{H}{b} = \phi\left(\frac{h}{b}, \frac{\ell}{b}\right)$$

All of the pi terms are obviously dimensionless.

(b) For the data given, tabulated values for H/b, h/b, and ℓ/b are shown below.

h/b	H/b	ℓ/b
0.10	0.833	2.0
0.10	0.833	4.0
0.20	0.417	2.0
0.20	0.417	4.0
0.35	0.238	2.0
0.35	0.238	4.0



An inspection of these data reveals that H/b does not depend on ℓ/b, i.e., the same value of H/b is obtained for different values of ℓ/b. Thus,

$$\frac{H}{b} = \phi\left(\frac{h}{b}\right)$$

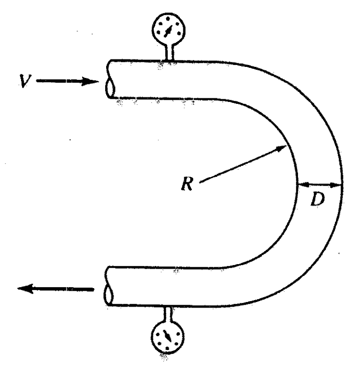
and from the plot of the data, using a power-law equation

$$\frac{H}{b} = 0.0833\left(\frac{h}{b}\right)^{-1.00}$$

7.29

7.29 A fluid flows through the horizontal curved pipe of Fig. P7.29 with a velocity V. The pressure drop, Δp, between the entrance and the exit to the bend is thought to be a function of the velocity, bend radius, R, pipe diameter, D, and fluid density, ρ. The data shown in the following table were obtained in the laboratory. For these tests $\rho = 2.0$ slugs/ft^3, $R = 0.5$ ft, and $D = 0.1$ ft. Perform a dimensional analysis and based on the data given, determine if the variables used for this problem appear to be correct. Explain how you arrived at your answer.

V (ft/s)	2.1	3.0	3.9	5.1
Δp (lb/ft^2)	1.2	1.8	6.0	6.5



■ FIGURE P7.29

$$\Delta p = f(V, R, D, \rho)$$

$\Delta p \doteq FL^{-2}$ $V \doteq LT^{-1}$ $R \doteq L$ $D \doteq L$ $\rho \doteq FL^{-4}T^{2}$

From the pi theorem, $5-3 = 2$ pi terms required.

By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{(FL^{-2})}{(FL^{-4}T^{2})(LT^{-1})^2} \doteq F^0 L^0 T^0$$

Check using MLT system:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{(ML^{-1}T^{-2})}{(ML^{-3})(LT^{-1})^2} \doteq M^0 L^0 T^0 \quad \therefore \text{ok}$$

For Π_2 (containing R and D):

$$\Pi_2 = \frac{D}{R}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{D}{R}\right) \qquad (1)$$

For the data given $\frac{D}{R} = \frac{0.1\text{ ft}}{0.5\text{ ft}} = \frac{1}{5}$ (a constant). Thus, from Eq. (1) with $\frac{D}{R}$ a constant it follows that $\frac{\Delta p}{\rho V^2}$ = constant. However, for the data given:

$\frac{\Delta p}{\rho V^2}$	0.136	0.100	0.197	0.125

Since $\frac{\Delta p}{\rho V^2}$ is not constant, it follows that the variables used for the problem are not correct.

7.30

7.30 The water flowrate, Q, in an open rectangular channel can be measured by placing a plate across the channel as shown in Fig. P7.30. This type of a device is called a *weir*. The height of the water, H, above the weir crest is referred to as the head and can be used to determine the flowrate through the channel. Assume that Q is a function of the head, H, the channel width, b, and the acceleration of gravity, g. Determine a suitable set of dimensionless variables for this problem.

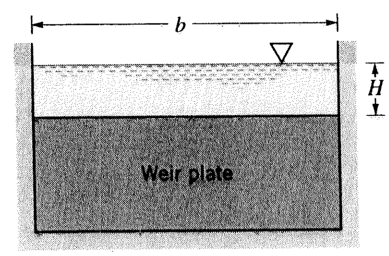


FIGURE P7.30

$$Q = f(H, b, g)$$

$$Q \doteq L^3T^{-1} \quad b \doteq L \quad g \doteq LT^{-2}$$

From the pi theorem, $4-2=2$ pi terms required.

By inspection for Π_1 (containing Q):

$$\Pi_1 = \frac{Q}{H^{5/2}g^{1/2}} \doteq \frac{L^3T^{-1}}{(L)^{5/2}(LT^{-2})^{1/2}} \doteq L^0T^0$$

For Π_2 (containing b):

$$\Pi_2 = \frac{b}{H}$$

which is obviously dimensionless. Thus,

$$\frac{Q}{H^{5/2}g^{1/2}} = \phi\left(\frac{b}{H}\right)$$

7.31

7.31 From theoretical considerations it is known that for the weir described in Problem 7.30 the flowrate, Q, must be directly proportional to the channel width, b. In some laboratory tests it was determined that if $b = 3$ ft and $H = 4$ in., then $Q = 1.96\ \text{ft}^3/\text{s}$. Based on these limited data, determine a general equation for the flowrate over this type of weir.

From Problem 7.30,

$$\frac{Q}{H^{5/2} g^{1/2}} = \phi\left(\frac{b}{H}\right) \qquad (1)$$

Since $Q \propto b$ it follows from Eq. (1) that

$$\frac{Q}{H^{5/2} g^{1/2}} = C\left(\frac{b}{H}\right)$$

where C is a constant. Thus, for the data given

$$C = \frac{Q}{H^{3/2} g^{1/2}\, b} = \frac{1.96\ \frac{ft^3}{s}}{\left(\frac{4}{12}\ ft\right)^{3/2}\left(32.2\ \frac{ft}{s^2}\right)^{1/2}(3\ ft)}$$

$$= 0.598$$

so that the general equation is

$$Q = 0.598\, b\sqrt{g H^3}$$

7.32

7.32 SAE 30 oil at 60 °F is pumped through a 3-ft-diameter pipeline at a rate of 6400 gal/min. A model of this pipeline is to be designed using a 3-in.-diameter pipe and water at 60 °F as the working fluid. To maintain Reynolds number similarity between these two systems, what fluid velocity will be required in the model?

For Reynolds number similarity,

$$\frac{V_m D_m}{\nu_m} = \frac{V D}{\nu}$$

or

$$V_m = \frac{\nu_m}{\nu} \frac{D}{D_m} V \qquad (1)$$

Since,

$$V = \frac{Q}{\text{area}}$$

and

$$Q = \frac{\left(6400 \frac{gal}{min}\right)\left(231 \frac{in.^3}{gal}\right)\left(\frac{1\ ft^3}{1728\ in.^3}\right)}{60 \frac{s}{min}} = 14.3 \frac{ft^3}{s}$$

then

$$V = \frac{14.3 \frac{ft^3}{s}}{\frac{\pi}{4}(3\ ft)^2} = 2.02 \frac{ft}{s}$$

Thus, from Eq. (1)

$$V_m = \frac{\left(1.21 \times 10^{-5} \frac{ft^2}{s}\right)(3\ ft)}{\left(4.5 \times 10^{-3} \frac{ft^2}{s}\right)\left(\frac{3}{12}\ ft\right)}\left(2.02 \frac{ft}{s}\right) = \underline{\underline{6.52 \times 10^{-2} \frac{ft}{s}}}$$

7.33

7.33 Carbon tetrachloride flows with a velocity of 0.30 m/s through a 30-mm-diameter tube. A model of this system is to be developed using standard air as the model fluid. The air velocity is to be 2 m/s. What tube diameter is required for the model if dynamic similarity is to be maintained between model and prototype?

For dynamic similarity, the Reynolds number must be the same for model and prototype. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{V D}{\nu}$$

so that

$$D_m = \frac{\nu_m}{\nu}\frac{V}{V_m} D = \frac{\left(1.46 \times 10^{-5} \frac{m^2}{s}\right)\left(0.30 \frac{m}{s}\right)}{\left(6.03 \times 10^{-7} \frac{m^2}{s}\right)\left(2 \frac{m}{s}\right)} (0.030\, m)$$

$$= 0.109\, m = \underline{\underline{109\, mm}}$$

7.34

7.34 The drag characteristics of a torpedo are to be studied in a water tunnel using a 1:5 scale model. The tunnel operates with freshwater at 20 °C, whereas the prototype torpedo is to be used in seawater at 15.6 °C. To correctly simulate the behavior of the prototype moving with a velocity of 30 m/s, what velocity is required in the water tunnel?

For dynamic similarity, the Reynolds number must be the same for model and prototype. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{VD}{\nu}$$

so that

$$V_m = \frac{\nu_m}{\nu}\frac{D}{D_m}V$$

Since, ν_m (water @ 20°C) = $1.004 \times 10^{-6}\ m^2/s$ (Table B.2), ν (seawater @ 15.6°C) = $1.17 \times 10^{-6}\ m^2/s$ (Table 1.6), and $D/D_m = 5$, it follows that

$$V_m = \frac{\left(1.004 \times 10^{-6}\ \frac{m^2}{s}\right)}{\left(1.17 \times 10^{-6}\ \frac{m^2}{s}\right)}(5)\left(30\ \frac{m}{s}\right) = \underline{\underline{129\ \frac{m}{s}}}$$

7.35

7.35 The flowrate over the spillway of a dam is 27,000 ft³/min. Determine the required flowrate for a 1:25 scale model that is operated in accordance with Froude number similarity.

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

and with $g = g_m$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Since the flowrate, Q, is obtained from the relationship $Q = VA$, where A is an appropriate cross-sectional area, it follows that

$$\frac{Q_m}{Q} = \frac{V_m A_m}{V A} = \sqrt{\frac{\ell_m}{\ell}} \left(\frac{\ell_m}{\ell}\right)^2 = \left(\frac{\ell_m}{\ell}\right)^{\frac{5}{2}}$$

Thus,

$$Q_m = \left(\frac{\ell_m}{\ell}\right)^{\frac{5}{2}} Q = \left(\frac{1}{25}\right)^{5/2} \left(27{,}000 \frac{ft^3}{min}\right)$$

$$= \underline{\underline{8.64 \frac{ft^3}{min}}}$$

7.36

7.36 For a certain fluid flow problem it is known that both the Froude number and the Weber number are important dimensionless parameters. If the problem is to be studied by using a 1:15 scale model, determine the required surface tension scale if the density scale is equal to 1. The model and prototype operate in the same gravitational field.

For dynamic similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}} \qquad \text{(Froude number similarity)}$$

and

$$\frac{\rho_m V_m^2 \ell_m}{\sigma_m} = \frac{\rho V^2 \ell}{\sigma} \qquad \text{(Weber number similarity)}$$

To satisfy Froude number similarity (with $g = g_m$),

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

and therefore for Weber number similarity

$$\frac{\sigma_m}{\sigma} = \frac{\rho_m}{\rho}\left(\frac{V_m}{V}\right)^2 \frac{\ell_m}{\ell} = \frac{\rho_m}{\rho}\left(\frac{\ell_m}{\ell}\right)\frac{\ell_m}{\ell} = \frac{\rho_m}{\rho}\left(\frac{\ell_m}{\ell}\right)^2$$

Thus, with $\ell_m / \ell = 1/15$ and $\rho_m / \rho = 1$,

$$\frac{\sigma_m}{\sigma} = (1)\left(\frac{1}{15}\right)^2 = \underline{\underline{4.44 \times 10^{-3}}}$$

7.38

7.38 If an airplane travels at a speed of 1120 km/hr at an altitude of 15 km, what is the required speed at an altitude of 8 km to satisfy Mach number similarity? Assume the air properties correspond to those for the U.S. standard atmosphere.

For Mach number similarity,

$$\left(\frac{V}{c}\right)_{15\,km} = \left(\frac{V}{c}\right)_{8\,km} \qquad (1)$$

The speed of sound can be calculated from the equation

$$c = \sqrt{kRT} \qquad \text{(Eq. 1.20)}$$

and for air, $k = 1.40$, $R = 286.9\ J/kg \cdot K$.

At 15 km altitude,

$$T = -56.50\,°C + 273.15 = 216.7\ K \qquad \text{(Table C.2)}$$

and at 8 km

$$T = -36.94\,°C + 273.15 = 236.2\ K \qquad \text{(Table C.2)}$$

Thus, at 15 km altitude

$$c_{15\,km} = \sqrt{(1.40)\left(286.9\ \frac{J}{kg \cdot K}\right)(216.7\,K)} = 295\ \frac{m}{s}$$

and at 8 km

$$c_{8\,km} = \sqrt{(1.40)\left(286.9\ \frac{J}{kg \cdot K}\right)(236.2\,K)} = 308\ \frac{m}{s}$$

From Eq. (1)

$$V_{8\,km} = \frac{c_{8\,km}}{c_{15\,km}} V_{15\,km} = \left(\frac{308\ \frac{m}{s}}{295\ \frac{m}{s}}\right)\left(1120\ \frac{km}{hr}\right)$$

$$= \underline{\underline{1170\ \frac{km}{hr}}}$$

7.40

7.40 The lift and drag developed on a hydrofoil are to be determined through wind tunnel tests using standard air. If full scale tests are to be run, what is the required wind tunnel velocity corresponding to a hydrofoil velocity in seawater of 15 mph? Assume Reynolds number similarity is required.

For Reynolds number similarity,

$$\frac{V_m \ell_m}{\nu_m} = \frac{V \ell}{\nu}$$

where ℓ is some characteristic length of the hydrofoil. Thus,

$$V_m = \frac{\nu_m}{\nu} \frac{\ell}{\ell_m} V$$

and with $\ell/\ell_m = 1$ (full scale test)

$$V_m = \frac{\nu_m}{\nu} V = \frac{\left(1.57 \times 10^{-4} \frac{ft^2}{s}\right)}{\left(1.26 \times 10^{-5} \frac{ft^2}{s}\right)} (15 \text{ mph})$$

$$= \underline{\underline{187 \text{ mph}}}$$

7.41

7.41 A 1/50 scale model is to be used in a towing tank to study the water motion near the bottom of a shallow channel as a large barge passes over. (See Video V7.7.) Assume that the model is operated in accordance with the Froude number criteria for dynamic similitude. The prototype barge moves at a typical speed of 15 knots. **(a)** At what speed (in ft/s) should the model be towed? **(b)** Near the bottom of the model channel a small particle is found to move 0.15 ft in one second so that the fluid velocity at that point is approximately 0.15 ft/s. Determine the velocity at the corresponding point in the prototype channel.

(a) For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

Thus,

$$V_m = \sqrt{\frac{1}{50}}\,(15\text{ knots}) = 2.12\text{ knots}$$

From Table A.1 $\quad 1\text{ knot} = \left(0.514\,\frac{m}{s}\right)\left(3.281\,\frac{ft/s}{m/s}\right) = 1.69\,\frac{ft}{s}$

so that

$$V_m = (2.12\text{ knots})\left(1.69\,\frac{ft/s}{\text{knot}}\right) = \underline{\underline{3.58\,\frac{ft}{s}}}$$

(b) Since from Eq. (1)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\frac{1}{50}}$$

so that

$$V = \sqrt{50}\left(0.15\,\frac{ft}{s}\right) = \underline{\underline{1.06\,\frac{ft}{s}}}$$

7.42

7.42 A solid sphere having a diameter d and specific weight γ_s is immersed in a liquid having a specific weight $\gamma_f (\gamma_f > \gamma_s)$ and then released. It is desired to use a model system to determine the maximum height, h, above the liquid surface that the sphere will rise upon release from a depth H. It can be assumed that the important liquid properties are the density, γ_f/g, specific weight, γ_f, and viscosity, μ_f. Establish the model design conditions and the prediction equation, and determine whether the same liquid can be used in both the model and prototype systems.

Assume that $h = f(d, H, \gamma_s, \gamma_f, g, \mu_f)$. Note that by including γ_s, γ_f, and g, both the mass and weight of the fluid and sphere are taken into account. This follows since ρ (density) $= \gamma/g$. It would be incorrect to list γ_f, ρ_f, and g as independent variables. We expect the mass of the sphere to be important since the sphere will have accelerated motion. Since,

$$h \doteq L \quad d \doteq L \quad H \doteq L \quad \gamma_s \doteq FL^{-3} \quad \gamma_f \doteq FL^{-3} \quad g \doteq LT^{-2} \quad \mu_f \doteq FL^{-2}T$$

the pi theorem indicates that $7-3 = 4$ pi terms required. A dimensional analysis yields,

$$\frac{h}{d} = \phi\left(\frac{H}{d}, \frac{\gamma_s}{\gamma_f}, \frac{\mu_f}{\gamma_f}\sqrt{\frac{g}{d^3}}\right)$$

Thus, the model design conditions are

$$\frac{H_m}{d_m} = \frac{H}{d} \qquad \frac{\gamma_{sm}}{\gamma_{fm}} = \frac{\gamma_s}{\gamma_f} \qquad \frac{\mu_{fm}}{\gamma_{fm}}\sqrt{\frac{g_m}{d_m^3}} = \frac{\mu_f}{\gamma_f}\sqrt{\frac{g}{d^3}}$$

and the prediction equation is

$$\frac{h}{d} = \frac{h_m}{d_m}$$

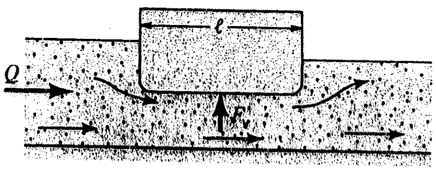
From the last model design condition (with $g = g_m$),

$$\frac{\mu_{fm}}{\mu_f} = \frac{\gamma_{fm}}{\gamma_f}\sqrt{\frac{d_m^3}{d^3}} \qquad (1)$$

Since d_m/d is the length scale, and is presumably not equal to one, Eq. (1) will not be satisfied if the same liquid is used. Thus, the same liquid cannot be used.

7.43

7.43 Water flowing under the obstacle shown in Fig. P7.43 puts a vertical force, F_v, on the obstacle. This force is assumed to be a function of the flowrate, Q, the density of the water, ρ, the acceleration of gravity, g, and a length, ℓ, that characterizes the size of the obstacle. A 1/20 scale model is to be used to predict the vertical force on the prototype. (a) Perform a dimensional analysis for this problem. (b) If the prototype flowrate is 1000 ft³/s, determine the water flowrate for the model if the flows are to be similar. (c) If the model force is measured as $(F_v)_m = 20$ lb, predict the corresponding force on the prototype.



■ FIGURE P7.43

(a) $$F_V = f(Q, \rho, g, \ell)$$

$F_V \doteq F \quad Q \doteq L^3T^{-1} \quad \rho \doteq FL^{-4}T^2 \quad g \doteq LT^{-2} \quad \ell \doteq L$

From the pi theorem, $5-3=2$ pi terms required, and a dimensional analysis yields,

$$\frac{F_V}{\rho g \ell^3} = \phi\left(\frac{Q}{\sqrt{g\ell^5}}\right)$$

(b) For similarity between model and prototype

$$\frac{Q_m}{\sqrt{g_m \ell_m^5}} = \frac{Q}{\sqrt{g\ell^5}}$$

Thus, with $g_m = g$

$$Q_m = \sqrt{\frac{g_m}{g}}\sqrt{\left(\frac{\ell_m}{\ell}\right)^5}\, Q = \sqrt{(1)}\sqrt{\left(\frac{1}{20}\right)^5}\left(1{,}000\ \frac{ft^3}{s}\right)$$

$$= 0.559\ \frac{ft^3}{s}$$

(c) The prediction equation is

$$\frac{F_V}{\rho g \ell^3} = \frac{(F_V)_m}{\rho_m g_m \ell_m^3}$$

so that

$$F_V = \left(\frac{\rho}{\rho_m}\right)\left(\frac{g}{g_m}\right)\left(\frac{\ell}{\ell_m}\right)^3 (F_V)_m$$

and with $g = g_m$, $\rho = \rho_m$

$$F_V = (1)(1)(20)^3(20\ lb) = 1.60 \times 10^5\ lb$$

7.44

7.44 A thin flat plate having a diameter of 0.3 ft is towed through a tank of oil ($\gamma = 53$ lb/ft³) at a velocity of 5 ft/s. The plane of the plate is perpendicular to the direction of motion, and the plate is submerged so that wave action is negligible. Under these conditions the drag on the plate is 1.4 lb. If viscous effects are neglected, predict the drag on a geometrically similar, 2-ft-diameter plate that is towed with a velocity of 3 ft/s through water at 60 °F under conditions similar to those for the smaller plate.

If viscous and wave effects are neglected,

$$\mathscr{D} = f(d, \rho, V)$$

where: $\mathscr{D} \sim$ drag $\doteq F$, $d \sim$ plate diameter $\doteq L$, $\rho \sim$ fluid density $\doteq FL^{-4}T^2$, and $V \sim$ velocity $\doteq LT^{-1}$. From the pi theorem, $4-3 = 1$ pi term required, and a dimensional analysis yields

$$\Pi_1 = \frac{\mathscr{D}}{\rho V^2 d^2}$$

Since there is only one pi term

$$\frac{\mathscr{D}_m}{\rho_m V_m^2 d_m^2} = \frac{\mathscr{D}}{\rho V^2 d^2} = \text{constant}$$

where m refers to the smaller, 0.3-ft-diameter plate.

Thus,

$$\mathscr{D} = \frac{\rho}{\rho_m} \frac{V^2}{V_m^2} \frac{d^2}{d_m^2} \mathscr{D}_m \qquad (1)$$

From the data given:

$\rho = 1.94$ slugs/ft³ ; $d = 2$ ft ; $V = 3$ ft/s

$\rho_m = \dfrac{53 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}$; $d_m = 0.3$ ft ; $V_m = 5$ ft/s ; $\mathscr{D}_m = 1.4$ lb

Therefore, from Eq. (1),

$$\mathscr{D} = \frac{\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)}{\left(\frac{53 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)} \frac{\left(3 \frac{\text{ft}}{\text{s}}\right)^2}{\left(5 \frac{\text{ft}}{\text{s}}\right)^2} \frac{(2 \text{ ft})^2}{(0.3 \text{ ft})^2} (1.4 \text{ lb}) = \underline{\underline{26.4 \text{ lb}}}$$

7.45

7.45 A model is to be used to determine the velocity, V, of liquid flow through a small-diameter passage in a wall separating two pressurized tanks as shown in Fig. P7.45. Prototype characteristics are indicated on the figure. The model is to have a length scale of $\frac{1}{4}$, a viscosity scale of 2.0, and if possible, the pressures are to be the same for model and prototype. Assume that V is a function of $p_1 - p_2$, d, ℓ, and the fluid viscosity, μ. Determine the required dimensions for the model, and the velocity scale.

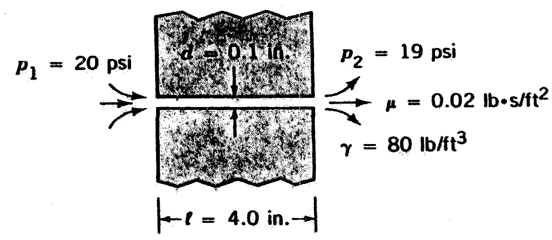


FIGURE P7.45

$$V = f(p_1 - p_2, d, \ell, \mu)$$

$$V \doteq LT^{-1} \quad p_1 - p_2 \doteq FL^{-2} \quad d \doteq L \quad \ell \doteq L \quad \mu \doteq FL^{-2}T$$

From the pi theorem, $5-3=2$ pi terms required, and a dimensional analysis yields

$$\frac{V\mu}{(p_1-p_2)d} = \phi\left(\frac{\ell}{d}\right)$$

Thus, the similarity requirement is

$$\frac{\ell_m}{d_m} = \frac{\ell}{d}$$

and since the length scale is 1/4, it follows that

$$\frac{\ell_m}{\ell} = \frac{d_m}{d} = \frac{1}{4}$$

Therefore,

$$\ell_m = \frac{4.0 \text{ in.}}{4} = \underline{\underline{1.00 \text{ in.}}}, \quad \text{and} \quad d_m = \frac{0.1 \text{ in.}}{4} = \underline{\underline{0.025 \text{ in.}}}$$

The prediction equation is

$$\frac{V\mu}{(p_1-p_2)d} = \frac{V_m \mu_m}{(p_1-p_2)_m d_m}$$

so that

$$\frac{V_m}{V} = \frac{(p_1-p_2)_m}{(p_1-p_2)} \frac{d_m}{d} \frac{\mu}{\mu_m}$$

For $(p_1-p_2)_m = (p_1-p_2)$, $d_m/d = 1/4$, and $\mu_m/\mu = 2.0$, it follows that

$$\frac{V_m}{V} = (1)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \frac{1}{8} = \underline{\underline{0.125}}$$

7.46

7.46 For a certain model study involving a 1:5 scale model it is known that Froude number similarity must be maintained. The possibility of cavitation is also to be investigated, and it is assumed that the cavitation number must be the same for model and prototype. The prototype fluid is water at 30 °C, and the model fluid is water at 70 °C. If the prototype operates at an ambient pressure of 101 kPa (abs), what is the required ambient pressure for the model system?

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

so that (with $g = g_m$)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

For cavitation number similarity,

$$\frac{(p_r - p_v)_m}{\frac{1}{2}\rho_m V_m^2} = \frac{(p_r - p_v)}{\frac{1}{2}\rho V^2}$$

It follows that

$$(p_r - p_v)_m = \frac{\rho_m}{\rho}\frac{V_m^2}{V^2}(p_r - p_v)$$

and making use of Eq. (1)

$$(p_r - p_v)_m = \frac{\rho_m}{\rho}\frac{\ell_m}{\ell}(p_r - p_v) \qquad (2)$$

For water (from Table B.2):

@ 70 °C $\quad \rho_m = 977.8\ kg/m^3$; $p_{vm} = 3.116 \times 10^4\ N/m^2$ (abs)

@ 30 °C $\quad \rho = 995.7\ kg/m^3$; $p_v = 4.243 \times 10^3\ N/m^2$ (abs)

Thus, from Eq. (2)

$$p_{rm} = \frac{\left(977.8\ \frac{kg}{m^3}\right)}{\left(995.7\ \frac{kg}{m^3}\right)}\left(\frac{1}{5}\right)\left(101 \times 10^3\ \frac{N}{m^2} - 4.243 \times 10^3\ \frac{N}{m^2}\right) + 3.116 \times 10^4\ \frac{N}{m^2}$$

$$= \underline{\underline{50.2\ kPa\ (abs)}}$$

7.47

7.47 A thin layer of spherical particles rests on the bottom of a horizontal tube as shown in Fig. P7.47. When an incompressible fluid flows through the tube, it is observed that at some critical velocity the particles will rise and be transported along the tube. A model is to be used to determine this critical velocity. Assume the critical velocity, V_c to be a function of the pipe diameter, D, particle diameter, d, the fluid density, ρ, and viscosity, μ, the density of the particles, ρ_p, and the acceleration of gravity, g. **(a)** Determine the similarity requirements for the model, and the relationship between the critical velocity for model and prototype (the prediction equation). **(b)** For a length scale of $\frac{1}{2}$ and a fluid density scale of 1.0, what will be the critical velocity scale (assuming all similarity requirements are satisfied)?

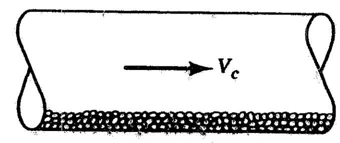


FIGURE P7.47

(a)
$$V_c = f(D, d, \rho, \mu, \rho_p, g)$$

$V_c \doteq LT^{-1}$ $D \doteq L$ $d \doteq L$ $\rho \doteq FL^{-4}T^2$ $\mu \doteq FL^{-2}T$ $\rho_p = FL^{-4}T^2$ $g \doteq LT^{-2}$

From the pi theorem, 7 − 3 = 4 pi terms required, and a dimensional analysis yields

$$\frac{\rho V_c D}{\mu} = \phi\left(\frac{d}{D}, \frac{\rho}{\rho_p}, \frac{g d^3 \rho^2}{\mu^2}\right)$$

Thus, the similarity requirements are

$$\frac{d_m}{D_m} = \frac{d}{D} \qquad \frac{\rho_m}{\rho_{pm}} = \frac{\rho}{\rho_p} \qquad \frac{g_m d_m^3 \rho_m^2}{\mu_m^2} = \frac{g d^3 \rho^2}{\mu^2}$$

The prediction equation is

$$\frac{\rho V_c D}{\mu} = \frac{\rho_m V_{cm} D_m}{\mu_m}$$

(b) If all similarity requirements are satisfied, the prediction equation indicates that

$$\frac{V_{cm}}{V_c} = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{D}{D_m} = (1.0)\left(\frac{\mu_m}{\mu}\right)(2) = 2\frac{\mu_m}{\mu} \qquad (1)$$

From the third similarity requirement (with $g = g_m$),

$$\frac{\mu_m}{\mu} = \sqrt{\left(\frac{d_m}{d}\right)^3\left(\frac{\rho_m}{\rho}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^3(1.0)^2} = \sqrt{\frac{1}{8}}$$

Thus, from Eq. (1)

$$\frac{V_{cm}}{V_c} = 2\sqrt{\frac{1}{8}} = \underline{0.707}$$

7.48

7.48 At a large fish hatchery the fish are reared in open, water-filled tanks. Each tank is approximately square in shape with curved corners, and the walls are smooth. To create motion in the tanks, water is supplied through a pipe at the edge of the tank. The water is drained from the tank through an opening at the center. (See Video V7.3.) A model with a length scale of 1:13 is to be used to determine the velocity, V, at various locations within the tank. Assume that $V = f(\ell, \ell_i, \rho, \mu, g, Q)$ where ℓ is some characteristic length such as the tank width, ℓ_i represents a series of other pertinent lengths, such as inlet pipe diameter, fluid depth, etc., ρ is the fluid density, μ is the fluid viscosity, g is the acceleration of gravity, and Q is the discharge through the tank. (a) Determine a suitable set of dimensionless parameters for this problem and the prediction equation for the velocity. If water is to be used for the model, can all of the similarity requirements be satisfied? Explain and support your answer with the necessary calculations. (b) If the flowrate into the full-sized tank is 250 gpm, determine the required value for the model discharge assuming Froude number similarity. What model depth will correspond to a depth of 32 in. in the full-sized tank?

(a) $$V = f(\ell, \ell_i, \rho, \mu, g, Q)$$

From the pi theorem, 7 − 3 = 4 pi terms required and a dimensional analysis yields

$$\frac{V\ell^2}{Q} = \phi\left(\frac{\ell_i}{\ell}, \frac{Q^2}{\ell^5 g}, \frac{\rho Q}{\ell \mu}\right)$$

Thus, the similarity requirements are

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell} \qquad \frac{Q_m^2}{\ell_m^5 g_m} = \frac{Q^2}{\ell^5 g} \qquad \frac{\rho_m Q_m}{\ell_m \mu_m} = \frac{\rho Q}{\ell \mu}$$

and the prediction equation is

$$\frac{V\ell^2}{Q} = \frac{V_m \ell_m^2}{Q_m}$$

From the last similarity requirement with $\rho_m = \rho$ and $\mu_m = \mu$

$$\frac{Q_m}{Q} = \frac{\rho}{\rho_m}\frac{\mu}{\mu_m}\frac{\ell_m}{\ell} = \frac{\ell_m}{\ell}$$

However, from the second similarity requirement with $g_m = g$

$$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2}$$

Since these two requirements are in conflict it follows that the similarity requirements cannot be satisfied. <u>No</u>.

(con't)

7.48 (Con't)

(b) For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Thus, from the prediction equation

$$\frac{V \ell^2}{Q} = \frac{V_m \ell_m^2}{Q_m}$$

it follows that

$$\frac{Q_m}{Q} = \frac{V_m}{V}\left(\frac{\ell_m}{\ell}\right)^2 = \sqrt{\frac{\ell_m}{\ell}}\left(\frac{\ell_m}{\ell}\right)^2 = \left(\frac{\ell_m}{\ell}\right)^{5/2}$$

so that with $\ell_m/\ell = 1/13$

$$Q_m = \left(\frac{1}{13}\right)^{5/2}(250\ gpm) = \underline{\underline{0.410\ gpm}}$$

Note that this same result can be obtained from the second similarity requirement (which corresponds to Froude number similarity) since

$$\frac{Q_m^2}{\ell_m^5 g_m} = \frac{Q^2}{\ell^5 g}$$

and therefore

$$Q_m = \left(\frac{\ell_m}{\ell}\right)^{5/2} Q$$

Geometric similarity requires that

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell}$$

or

$$\frac{\ell_{im}}{\ell_i} = \frac{\ell_m}{\ell} = \frac{1}{13}$$

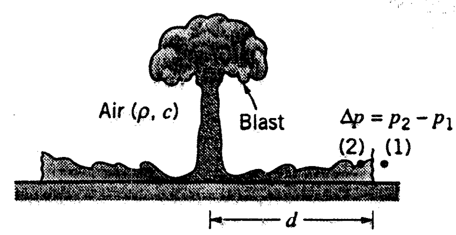
so that all lengths scale as the length scale. Thus,

$$(\text{depth})_{model} = \left(\frac{1}{13}\right)(\text{depth})_{prototype}$$

$$= \left(\frac{1}{13}\right)(32\ in.) = \underline{\underline{2.46\ in.}}$$

7.49

7.49 The pressure rise, Δp, across a blast wave, as shown in Fig. P7.49 and Video V11.5, is assumed to be a function of the amount of energy released in the explosion, E, the air density, ρ, the speed of sound, c, and the distance from the blast, d. **(a)** Put this relationship in dimensionless form. **(b)** Consider two blasts: the prototype blast with energy release E and a model blast with 1/1000th the energy release ($E_m = 0.001\ E$). At what distance from the model blast will the pressure rise be the same as that at a distance of 1 mile from the prototype blast?



■ FIGURE P7.49

(a)
$$\Delta p = f(E, \rho, c, d)$$
$$\Delta p \doteq FL^{-2} \quad E \doteq FL \quad \rho \doteq FL^{-4}T^{2} \quad c \doteq LT^{-1} \quad d \doteq L$$
From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields
$$\frac{\Delta p}{\rho c^2} = \phi\left(\frac{E}{\rho c^2 d^3}\right)$$

(b) For similarity,
$$\frac{E_m}{\rho_m c_m^2 d_m^3} = \frac{E}{\rho c^2 d^3}$$
and with $\rho_m = \rho$, $c_m = c$, it follows that
$$d_m^3 = \frac{E_m}{E} d^3$$
For $E_m/E = 0.001$ and $d = 1\ mi$
$$d_m^3 = (0.001)(1\ mi)^3$$
$$d_m = 0.100\ mi$$
With this similarity requirement satisfied, the prediction equation is
$$\frac{\Delta p_m}{\rho_m c_m^2} = \frac{\Delta p}{\rho c^2}$$
and therefore
$$\Delta p_m = \Delta p$$
at
$$d_m = 0.100\ mi$$

7.50

7.50 The drag, $\mathscr{D}$, on a sphere located in a pipe through which a fluid is flowing is to be determined experimentally (see Fig. P7.50). Assume that the drag is a function of the sphere diameter, d, the pipe diameter, D, the fluid velocity, V, and the fluid density, ρ. **(a)** What dimensionless parameters would you use for this problem? **(b)** Some experiments using water indicate that for $d = 0.2$ in., $D = 0.5$ in., and $V = 2$ ft/s, the drag is 1.5×10^{-3} lb. If possible, estimate the drag on a sphere located in a 2-ft-diameter pipe through which water is flowing with a velocity of 6 ft/s. The sphere diameter is such that geometric similarity is maintained. If it is not possible, explain why not.

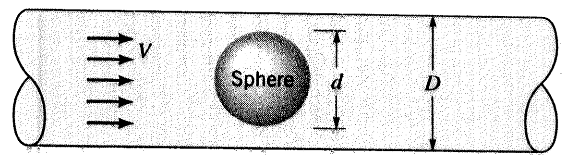


FIGURE P7.50

(a)
$$\mathscr{D} = f(d, D, V, \rho)$$

$$\mathscr{D} \doteq F \quad d \doteq L \quad D \doteq L \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2}$$

From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields

$$\frac{\mathscr{D}}{\rho V^2 D^2} = \phi\left(\frac{d}{D}\right)$$

(b) The similarity requirement is

$$\frac{d_m}{D_m} = \frac{d}{D}$$

so that

$$\frac{0.2 \text{ in.}}{0.5 \text{ in.}} = \frac{d \text{ (ft)}}{2 \text{ ft}}$$

and $d = 0.8$ ft (required diameter).

Thus, the prediction equation is

$$\frac{\mathscr{D}}{\rho V^2 D^2} = \frac{\mathscr{D}_m}{\rho_m V_m^2 D_m^2}$$

So that

$$\mathscr{D} = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2\left(\frac{D}{D_m}\right)^2 \mathscr{D}_m \quad (\text{and with } \rho = \rho_m)$$

$$\mathscr{D} = \left(\frac{6\frac{\text{ft}}{\text{s}}}{2\frac{\text{ft}}{\text{s}}}\right)^2\left(\frac{2 \text{ ft}}{0.5/12 \text{ ft}}\right)^2(1.5 \times 10^{-3} \text{ lb}) = \underline{31.1 \text{ lb}}$$

7.51

7.51 Flow patterns that develop as winds blow past a vehicle, such as a train, are often studied in low-speed environmental (meteorological) wind tunnels. (See Video V7.5.) Typically, the air velocities in these tunnels are in the range of 0.1 m/s to 30 m/s. Consider a cross wind blowing past a train locomotive. Assume that the local wind velocity, V, is a function of the approaching wind velocity (at some distance from the locomotive), U, the locomotive length, ℓ, height, h, and width, b, the air density, ρ, and the air viscosity, μ. (a) Establish the similarity requirements and prediction equation for a model to be used in the wind tunnel to study the air velocity, V, around the locomotive. (b) If the model is to be used for cross winds gusting to $U = 25$ m/s, explain why it is not practical to maintain Reynolds number similarity for a typical length scale 1:50.

(a)
$$V = f(U, \ell, h, b, \rho, \mu)$$

$V \doteq LT^{-1}$ $U \doteq LT^{-1}$ $\ell \doteq L$ $h \doteq L$ $b \doteq L$ $\rho \doteq FL^{-4}T^{2}$ $\mu \doteq FL^{-2}T$

From the pi theorem, $7-3=4$ pi terms required, and a dimensional analysis yields

$$\frac{V}{U} = \phi\left(\frac{\ell}{h}, \frac{b}{h}, \frac{\rho h U}{\mu}\right)$$

Thus, the similarity requirements are

$$\frac{\ell_m}{h_m} = \frac{\ell}{h} \qquad \frac{b_m}{h_m} = \frac{b}{h} \qquad \frac{\rho_m h_m U_m}{\mu_m} = \frac{\rho h U}{\mu}$$

The prediction equation is

$$\frac{V}{U} = \frac{V_m}{U_m}$$

(b) Since the density and viscosity of the air flowing around the train and the air in the wind tunnel would be practically the same ($\rho_m \approx \rho$, $\mu_m \approx \mu$), it follows from the last similarity requirement (which is the Reynolds number) that

$$U_m = \left(\frac{h}{h_m}\right) U$$

Thus, with a length scale of 1:50 and with $U = 25 m/s$

$$U_m = (50)(25 m/s) = 1{,}250 \; m/s$$

This required model velocity is much higher than can be acheived in the wind tunnel and therefore it is not practical to maintain Reynolds number similarity. The required model velocity is too high.

7.52

7.52 An incompressible fluid oscillates harmonically ($V = V_0 \sin \omega t$, where V is the velocity) with a frequency of 10 rad/s in a 4-in.-diameter pipe. A $\frac{1}{4}$ scale model is to be used to determine the pressure difference per unit length, Δp_ℓ (at any instant) along the pipe. Assume that

$$\Delta p_\ell = f(D, V_0, \omega, t, \mu, \rho)$$

where D is the pipe diameter, ω the frequency, t the time, μ the fluid viscosity, and ρ the fluid density. **(a)** Determine the similarity requirements for the model and the prediction equation for Δp_ℓ. **(b)** If the same fluid is used in the model and the prototype, at what frequency should the model operate?

$\Delta p_\ell \doteq FL^{-3}$ $\quad D \doteq L$ $\quad V_0 \doteq LT^{-1}$ $\quad \omega \doteq T^{-1}$ $\quad t \doteq T$ $\quad \mu \doteq FL^{-2}T$ $\quad \rho \doteq FL^{-4}T^2$

From the pi theorem, 7-3 = 4 pi terms required, and a dimensional analysis yields

$$\frac{D\,\Delta p_\ell}{\rho V_0^2} = \phi\left(\frac{V_0 t}{D},\ \omega t,\ \frac{\rho V_0 D}{\mu}\right)$$

(a) Thus, the similarity requirements are

$$\frac{V_{0m} t_m}{D_m} = \frac{V_0 t}{D} \qquad \omega_m t_m = \omega t \qquad \frac{\rho_m V_{0m} D_m}{\mu_m} = \frac{\rho V_0 D}{\mu}$$

and the prediction equation is

$$\frac{D\,\Delta p_\ell}{\rho V_0^2} = \frac{D_m\,\Delta p_{\ell m}}{\rho_m V_{0m}^2}$$

(b) For Reynolds number similarity (the last similarity requirement), with the same fluid in model and prototype,

$$\frac{V_{0m}}{V_0} = \frac{D}{D_m}$$

so that from the first similarity requirement

$$\frac{t_m}{t} = \frac{D_m}{D}\frac{V_0}{V_{0m}} = \left(\frac{D_m}{D}\right)\left(\frac{D_m}{D}\right) = \left(\frac{D_m}{D}\right)^2$$

Thus, to satisfy the remaining similarity requirement

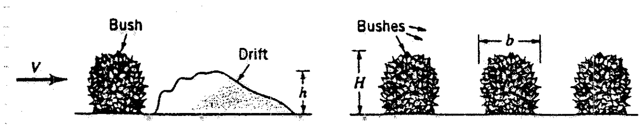
$$\omega_m t_m = \omega t$$

or

$$\omega_m = \frac{t}{t_m}\omega = \left(\frac{D}{D_m}\right)^2 \omega = (4)^2\left(10\ \frac{\text{rad}}{\text{s}}\right) = 160\ \frac{\text{rad}}{\text{s}}$$

7.53

7.53 During a storm, a snow drift is formed behind some bushes as shown in Fig. P7.53 and Video V9.4. Assume that the height of the drift, h, is a function of the number of inches of snow deposited by the storm, d, the height of the bush, H, the width of the bush, b, the wind speed, V, the acceleration of gravity, g, the air density, ρ, the specific weight of the snow, γ_s, and the porosity of the bush, η. Note that porosity is defined as percent open area of the bush. **(a)** Determine a suitable set of dimensionless variables for this problem. **(b)** A storm with 30 mph winds deposits 16 in. of snow having a specific weight of 5.0 lb/ft³. A half-sized scale model bush is to be used to investigate the drifting behind the bush. If the air density is the same for the model and the storm, determine the required specific weight of the model snow, the required wind speed for the model, and the number of inches of model snow to be deposited.



■ FIGURE P7.53

(a)
$$h = f(d, H, b, V, g, \rho, \gamma_s, \eta)$$

$h \doteq L \quad d \doteq L \quad H \doteq L \quad b \doteq L \quad V \doteq LT^{-1} \quad g \doteq LT^{-2}$

$\rho \doteq FL^{-4}T^{2} \quad \gamma_s \doteq FL^{-3} \quad \eta \doteq F^0L^0T^0$

From the pi theorem, $9-3=6$ pi terms required, and a dimensional analysis yields

$$\frac{h}{H} = \phi\left(\frac{d}{H}, \frac{b}{H}, \frac{\rho g}{\gamma_s}, \frac{V}{\sqrt{gH}}, \eta\right)$$

(b) Thus, for similarity between the model and prototype

$$\frac{\rho_m g_m}{\gamma_{sm}} = \frac{\rho g}{\gamma_s}$$

and for $\rho_m = \rho$ and $g_m = g$

$$\gamma_{sm} = \gamma_s = \underline{\underline{5.00 \frac{lb}{ft^3}}}$$

Also,

$$\frac{V_m}{\sqrt{g_m H_m}} = \frac{V}{\sqrt{gH}}$$

so that with $g_m = g$ and $H_m/H = \frac{1}{2}$

$$V_m = \sqrt{\frac{H_m}{H}}\, V = \sqrt{\left(\frac{1}{2}\right)}\,(30\ mph) = \underline{\underline{21.2\ mph}}$$

and

$$\frac{d_m}{H_m} = \frac{d}{H}$$

$$d_m = \left(\frac{H_m}{H}\right) d = \left(\frac{1}{2}\right)(16\ in.) = \underline{\underline{8.00\ in.}}$$

7.54

7.54 As illustrated in Video V7.2, models are commonly used to study the dispersion of a gaseous pollutant from an exhaust stack located near a building complex. Similarity requirements for the pollutant source involve the following independent variables: the stack gas speed, V, the wind speed, U, the density of the atmospheric air, ρ, the difference in densities between the air and the stack gas, $\rho - \rho_s$, the acceleration of gravity, g, the kinematic viscosity of the stack gas, ν_s, and the stack diameter, D. (a) Based on these variables, determine a suitable set of similarity requirements for modeling the pollutant source. (b) For this type of model a typical length scale might be 1:200. If the same fluids were used in model and prototype, would the similarity requirements be satisfied? Explain and support your answer with the necessary calculations.

(a) Since $V \doteq LT^{-1}$ $U \doteq LT^{-1}$ $\rho \doteq FL^{-4}T^{2}$ $\rho - \rho_s \doteq FL^{-4}T^{2}$ $g \doteq LT^{-2}$ $\nu_s \doteq L^2T^{-1}$ $D \doteq L$, it follows from the pi theorem that $7-3 = 4$ pi terms are required. A dimensional analysis yields $\frac{V}{U}$, $\frac{VD}{\nu_s}$, $\frac{V^2}{gD}$, and $\frac{\rho-\rho_s}{\rho}$ as a possible set of pi terms. Thus, the similarity requirements would be:

$$\underline{\underline{\frac{V_m}{U_m} = \frac{V}{U}}} \qquad \underline{\underline{\frac{V_m D_m}{\nu_{sm}} = \frac{VD}{\nu_s}}} \qquad \underline{\underline{\frac{V_m^2}{g_m D_m} = \frac{V^2}{gD}}} \qquad \underline{\underline{\frac{(\rho-\rho_s)_m}{\rho_m} = \frac{(\rho-\rho_s)}{\rho}}}$$

(b) For $\frac{D_m}{D} = \frac{1}{200}$ and $\nu_{sm} = \nu_s$ the second similarity requirement is

$$\frac{V_m}{V} = \frac{\nu_{sm}}{\nu_s}\frac{D}{D_m} = 200 \quad \text{(see above)}$$

However, from the third similarity requirement with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{D_m}{D}} = \sqrt{\frac{1}{200}}$$

This result conflicts with that from the second similarity requirement, and therefore the similarity requirements cannot be satisfied under the stated conditions. <u>No</u>.

7.55

7.55 The drag on a small, completely submerged solid body having a characteristic length of 2.5 mm and moving with a velocity of 10 m/s through water is to be determined with the aid of a model. The length scale is to be 50, which indicates that the model is to be *larger* than the prototype. Investigate the possibility of using either an unpressurized wind tunnel or a water tunnel for this study. Determine the required velocity in both the wind and water tunnels, and the relationship between the model drag and the prototype drag for both systems. Would either type of test facility be suitable for this study?

As demonstrated in Eq. 7.19, for flow around immersed bodies, Reynolds number similarity is required so that

$$\frac{V_m \ell_m}{\nu_m} = \frac{V \ell}{\nu}$$

or

$$V_m = \frac{\nu_m}{\nu} \frac{\ell}{\ell_m} V$$

If model tests are run in unpressurized wind tunnel, then ν_m (standard air) $= 1.46 \times 10^{-5}\ m^2/s$, and ν (water) $= 1.12 \times 10^{-6}\ m^2/s$, so that

$$V_m = \frac{\left(1.46 \times 10^{-5}\ \frac{m^2}{s}\right)}{\left(1.12 \times 10^{-6}\ \frac{m^2}{s}\right)} \left(\frac{1}{50}\right)\left(10\ \frac{m}{s}\right) = \underline{\underline{2.61\ \frac{m}{s}}} \quad \text{(for wind tunnel)}$$

If model tests are run in water tunnel with $\nu_m = \nu$, then

$$V_m = (1)\left(\frac{1}{50}\right)\left(10\ \frac{m}{s}\right) = \underline{\underline{0.200\ \frac{m}{s}}} \quad \text{(for water tunnel)}$$

Since V_m is reasonable in both cases, <u>either the wind tunnel or the water tunnel could be used.</u>

With geometric and dynamic similarity, it follows that

$$\frac{\mathcal{D}}{\rho V^2 \ell^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 \ell_m^2}$$

or

$$\frac{\mathcal{D}}{\mathcal{D}_m} = \frac{\rho}{\rho_m} \frac{V^2}{V_m^2} \frac{\ell^2}{\ell_m^2}$$

Thus, for wind tunnel tests

$$\frac{\mathcal{D}}{\mathcal{D}_m} = \frac{\left(999\ \frac{kg}{m^3}\right)}{\left(1.23\ \frac{kg}{m^3}\right)} \frac{\left(10\ \frac{m}{s}\right)^2}{\left(3.91\ \frac{m}{s}\right)^2} \left(\frac{1}{50}\right)^2 = \underline{\underline{2.13}} \quad \text{(for wind tunnel)}$$

and for water tunnel tests

$$\frac{\mathcal{D}}{\mathcal{D}_m} = (1.0) \frac{\left(10\ \frac{m}{s}\right)^2}{\left(0.300\ \frac{m}{s}\right)^2} \left(\frac{1}{50}\right)^2 = \underline{\underline{0.444}} \quad \text{(for water tunnel)}$$

7.56

7.56 The drag characteristics for a newly designed automobile having a maximum characteristic length of 20 ft are to be determined through a model study. The characteristics at both low speed (approximately 20 mph) and high speed (90 mph) are of interest. For a series of projected model tests an unpressurized wind tunnel that will accommodate a model with a maximum characteristic length of 4 ft is to be used. Determine the range of air velocities that would be required for the wind tunnel if Reynolds number similarity is desired. Are the velocities suitable? Explain.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$V_m = \frac{\mu_m}{\mu} \frac{\rho}{\rho_m} \frac{\ell}{\ell_m} V \qquad (1)$$

Since the wind tunnel is unpressurized, the air properties will be approximately the same for model and prototype. Thus, Eq. (1) reduces to

$$V_m = \frac{\ell}{\ell_m} V$$

and for the data given

$$V_m = \frac{(20\,ft)}{(4\,ft)} V = 5V$$

Therefore, at low speed

$$V_m = 5\,(20\,mph) = 100\,mph$$

and at high speed

$$V_m = 5\,(90\,mph) = 450\,mph$$

so that the model velocity range is 100 mph to 450 mph.

At the high velocity in the wind tunnel, compressibility of the air would start to become an important factor, whereas compressibility is not important for the prototype. Thus, the higher velocity required for the model would not be suitable. No.

7.57

7.57 If the unpressurized wind tunnel of Problem 7.56 were replaced with a tunnel in which the air can be pressurized isothermally to 8 atm (abs), what range of air velocities would be required to maintain Reynolds number similarity for the same prototype velocities given in Problem 7.56? For the pressurized tunnel the maximum characteristic model length that can be accommodated is 2 ft, whereas the maximum characteristic prototype length remains at 20 ft.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$V_m = \frac{\rho}{\rho_m} \frac{\mu_m}{\mu} \frac{\ell}{\ell_m} V \qquad (1)$$

For an ideal gas, $p = \rho R T$, and for isothermal compression

$$\frac{p}{\rho} = \text{constant}$$

Thus,

$$\frac{p_m}{\rho_m} = \frac{p}{\rho}$$

or

$$\frac{\rho}{\rho_m} = \frac{p}{p_m}$$

From Eq. (1) (assuming $\mu_m = \mu$)

$$V_m = \frac{p}{p_m} \frac{\ell}{\ell_m} V$$

where p is atmospheric pressure (pressure at which prototype operates), and p_m is pressure of compressed air in the wind tunnel.
For $p_m = 8p$

$$V_m = \left(\frac{1}{8}\right) \frac{(20 \text{ ft})}{(2 \text{ ft})} V = 1.25 V$$

Thus, at low speed

$$V_m = 1.25 \, (20 \text{ mph}) = 25 \text{ mph}$$

and at high speed

$$V_m = 1.25 \, (90 \text{ mph}) = 112.5 \text{ mph}$$

Therefore, the required model velocity range is
25 mph to 112.5 mph.

7.58

7.58 The drag characteristics of an airplane are to be determined by model tests in a wind tunnel operated at an absolute pressure of 1300 kPa. If the prototype is to cruise in standard air at 385 km/hr, and the corresponding speed of the model is not to differ by more than 20% from this (so that compressibility effects may be ignored), what range of length scales may be used if Reynolds number similarity is to be maintained? Assume the viscosity of air is unaffected by pressure, and the temperature of the air in the tunnel is equal to the temperature of the air in which the airplane will fly.

For Reynolds number similarity,

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

so that

$$\frac{\ell_m}{\ell} = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{V}{V_m} \qquad (1)$$

For an ideal gas, $p = \rho R T$, and with constant temperature,

$$\frac{p}{\rho} = \text{constant}$$

or

$$\frac{p}{p_m} = \frac{\rho}{\rho_m}$$

and Eq. (1) can be written as (with $\mu_m = \mu$)

$$\frac{\ell_m}{\ell} = \frac{p}{p_m}\frac{V}{V_m}$$

For the data given

$$\frac{\ell_m}{\ell} = \frac{(101\text{ kPa})}{(1300\text{ kPa})}\frac{V}{V_m}$$

and with $V_m = (1 \pm 0.2)V$, it follows that

$$\frac{\ell_m}{\ell} = \frac{(101\text{ kPa})}{(1300\text{ kPa})}\frac{1}{(1 \pm 0.2)}$$

Thus, the range of length scales is 0.0647 to 0.0971.

7.59

7.59 Wind blowing past a flag causes it to "flutter in the breeze." The frequency of this fluttering, ω, is assumed to be a function of the wind speed, V, the air density, ρ, the acceleration of gravity, g, the length of the flag, ℓ, and the "area density," ρ_A, (with dimensions of ML^{-2}) of the flag material. It is desired to predict the flutter frequency of a large $\ell = 40$ ft flag in a $V = 30$ ft/s wind. To do this a model flag with $\ell = 4$ ft is to be tested in a wind tunnel. **(a)** Determine the required area density of the model flag material if the large flag has $\rho_A = 0.006$ slugs/ft^2. **(b)** What wind tunnel velocity is required for testing the model? **(c)** If the model flag flutters at 6 Hz, predict the frequency for the large flag.

$$\omega = f(V, \rho, g, \ell, \rho_A)$$

$$\omega \doteq T^{-1} \quad V \doteq LT^{-1} \quad \rho \doteq ML^{-3} \quad g \doteq LT^{-2} \quad \ell \doteq L \quad \rho_A \doteq ML^{-2}$$

From the pi Theorem, $6-3 = 3$ pi terms required, and a dimensional analysis yields

$$\omega\sqrt{\frac{\ell}{g}} = \phi\left(\frac{V}{\sqrt{g\ell}}, \frac{\rho_A}{\rho\ell}\right)$$

(a) For similarity

$$\frac{\rho_{Am}}{\rho_m \ell_m} = \frac{\rho_A}{\rho\ell}$$

and since $\rho_m = \rho$

$$\rho_{Am} = \frac{\ell_m}{\ell}\rho_A = \left(\frac{4\,ft}{40\,ft}\right)\left(0.006\,\frac{slugs}{ft^2}\right) = \underline{\underline{0.0006\,\frac{slugs}{ft^2}}}$$

(b) For similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

and with $g_m = g$

$$V_m = \sqrt{\frac{\ell_m}{\ell}}\,V = \sqrt{\frac{4\,ft}{40\,ft}}\left(30\,\frac{ft}{s}\right) = \underline{\underline{9.49\,\frac{ft}{s}}}$$

(c) With the similarity requirements satisfied the prediction equation is

$$\omega\sqrt{\frac{\ell}{g}} = \omega_m\sqrt{\frac{\ell_m}{g_m}}$$

so that

$$\omega = \sqrt{\frac{g}{g_m}}\sqrt{\frac{\ell_m}{\ell}}\,\omega_m = \sqrt{\frac{4\,ft}{40\,ft}}\,(6\,Hz) = \underline{\underline{1.90\,Hz}}$$

7.60

7.60 River models are used to study many different types of flow situations. (See, for example, Video V7.6.) A certain small river has an average width and depth of 60 ft and 4 ft, respectively, and carries water at a flowrate of 700 ft^3/s. A model is to be designed based on Froude number similarity so that the discharge scale is 1/250. At what depth and flowrate would the model operate?

For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Since the flowrate is $Q = VA$, where A is the appropriate cross sectional area,

$$\frac{Q_m}{Q} = \frac{V_m A_m}{V A} = \sqrt{\frac{\ell_m}{\ell}}\,\frac{A_m}{A}$$

Also,

$$\frac{A_m}{A} = \left(\frac{\ell_m}{\ell}\right)^2$$

so that

$$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2} = \frac{1}{250} \qquad (1)$$

Thus,

$$\frac{\ell_m}{\ell} = 0.110$$

and for a prototype depth of 4 ft the corresponding model depth is

$$\ell_m = (0.110)(4\,ft) = \underline{\underline{0.440\,ft}}$$

The model flowrate is obtained from Eq. (1):

$$Q_m = \left(\frac{1}{250}\right)\left(700\,\frac{ft^3}{s}\right) = \underline{\underline{2.80\,\frac{ft^3}{s}}}$$

7.61

7.61 As winds blow past buildings, complex flow patterns can develop due to various factors such as flow separation and interactions between adjacent buildings. (See Video V7.4.) Assume that the local gage pressure, p, at a particular location on a building is a function of the air density, ρ, the wind speed, V, some characteristic length, ℓ, and all other pertinent lengths, ℓ_i, needed to characterize the geometry of the building or building complex. (a) Determine a suitable set of dimensionless parameters that can be used to study the pressure distribution. (b) An eight-story building that is 100 ft tall is to be modeled in a wind tunnel. If a length scale of 1:300 is to be used, how tall should the model building be? (c) How will a measured pressure in the model be related to the corresponding prototype pressure? Assume the same air density in model and prototype. Based on the assumed variables, does the model wind speed have to be equal to the prototype wind speed? Explain.

(a)
$$p = f(\rho, V, \ell, \ell_i)$$

$p \doteq FL^{-2}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad V \doteq LT^{-1}$ $\quad \ell \doteq L$ $\quad \ell_i \doteq L$

From the pi theorem, 5-3=2 pi terms required, and a dimensional analysis yields

$$\frac{p}{\rho V^2} = \phi\left(\frac{\ell}{\ell_i}\right)$$

(b) For geometric similarity

$$\frac{\ell_m}{\ell_{im}} = \frac{\ell}{\ell_i}$$

so that

$$\frac{\ell_m}{\ell} = \frac{\ell_{im}}{\ell_i}$$

and it follows that all pertinent lengths are scaled with the length scale ℓ_m/ℓ. Thus, with $\ell_m/\ell = 1/300$

$$\text{model height} = \frac{100\text{ ft}}{300} = \underline{0.333\text{ ft}}$$

(c) With geometric similarity satisfied it follows that

$$\frac{p}{\rho V^2} = \frac{p_m}{\rho_m V_m^2}$$

Thus, with $\rho_m = \rho$

$$p = \left(\frac{V}{V_m}\right)^2 p_m$$

With the set of given variables there is no requirement for the velocity scale, V_m/V, so the model wind speed does not have to be equal tho the prototype wind speed. No.

7.62

7.62 A thin rectangular plate is towed through seawater at an average velocity of 5 mph. The plate is held in a vertical position and projects above the undisturbed level of the water to a height z. A 1:4 scale model is to be used to predict the drag on the plate, and the model fluid is also seawater. **(a)** Assuming that Froude number similarity must be maintained, determine the required model velocity. **(b)** What is the required value of z_m/z? **(c)** A measured drag of 1 lb on the model will correspond to what drag on the prototype?

(a) For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

Thus, with $g = g_m$

$$V_m = \sqrt{\frac{\ell_m}{\ell}}\, V = \sqrt{\frac{1}{4}}\,(5\text{ mph}) = \underline{\underline{2.50\text{ mph}}}$$

(b) For geometric similarity,

$$\frac{z_m}{\ell_m} = \frac{z}{\ell}$$

so that

$$\frac{z_m}{z} = \frac{\ell_m}{\ell} = \frac{1}{4} = \underline{\underline{0.250}}$$

(c) The dimensionless drag parameter is

$$\frac{\mathscr{D}}{\rho V^2 \ell^2}$$

and if all similarity requirements are met, then

$$\frac{\mathscr{D}}{\rho V^2 \ell^2} = \frac{\mathscr{D}_m}{\rho_m V_m^2 \ell_m^2}$$

Thus,

$$\mathscr{D} = \frac{\rho}{\rho_m}\,\frac{V^2}{V_m^2}\,\frac{\ell^2}{\ell_m^2}\,\mathscr{D}_m$$

and for the data given

$$\mathscr{D} = (1)(2)^2(4)^2(1\text{ lb}) = \underline{\underline{64.0\text{ lb}}}$$

7.64

7.64 The drag on a sphere moving in a fluid is known to be a function of the sphere diameter, the velocity, and the fluid viscosity and density. Laboratory tests on a 4-in.-diameter sphere were performed in a water tunnel and some model data are plotted in Fig. P7.64. For these tests the viscosity of the water was 2.3×10^{-5} lb·s/ft² and the water density was 1.94 slugs/ft³. Estimate the drag on an 8-ft diameter balloon moving in air at a velocity of 3 ft/s. Assume the air to have a viscosity of 3.7×10^{-7} lb·s/ft² and a density of 2.38×10^{-3} slugs/ft³.

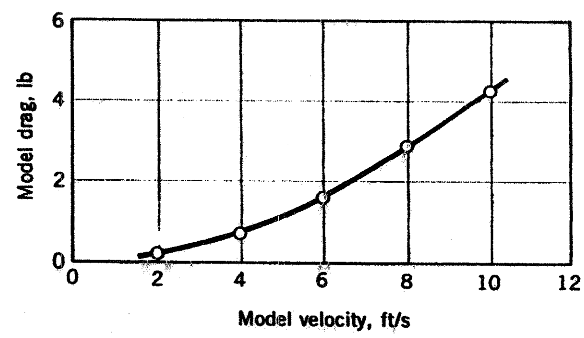


FIGURE P7.64

$$\mathcal{D} = f(d, V, \rho, \mu)$$

where: $\mathcal{D} \sim$ drag $\doteq F$, $d \sim$ sphere diameter $\doteq L$, $V \sim$ velocity $\doteq LT^{-1}$, $\rho \sim$ fluid density $\doteq FL^{-4}T^2$, $\mu \sim$ fluid viscosity $\doteq FL^{-2}T$.

From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \phi\left(\frac{\rho V d}{\mu}\right)$$

Thus, Reynolds number similarity is required so that

$$\frac{\rho_m V_m d_m}{\mu_m} = \frac{\rho V d}{\mu}$$

or

$$V_m = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{d}{d_m}V$$

$$= \frac{\left(2.3 \times 10^{-5}\ \frac{lb\cdot s}{ft^2}\right)\left(2.38 \times 10^{-3}\ \frac{slugs}{ft^3}\right)}{\left(3.7 \times 10^{-7}\ \frac{lb\cdot s}{ft^2}\right)\left(1.94\ \frac{slugs}{ft^3}\right)}\frac{(8\ ft)}{\left(\frac{4}{12}\ ft\right)}\left(3\ \frac{ft}{s}\right)$$

$$= 5.49\ \frac{ft}{s}$$

From the graph, for $V_m = 5.49$ ft/s, $\mathcal{D}_m = 1.30$ lb. Since

$$\frac{\mathcal{D}}{\rho V^2 d^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 d_m^2}$$

or

$$\mathcal{D} = \frac{\rho}{\rho_m}\frac{V^2}{V_m^2}\frac{d^2}{d_m^2}\mathcal{D}_m$$

so that

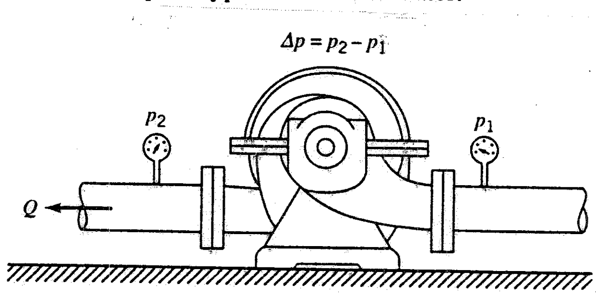
$$\mathcal{D} = \frac{\left(2.38 \times 10^{-3}\ \frac{slugs}{ft^3}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}\frac{\left(3\ \frac{ft}{s}\right)^2}{\left(5.49\ \frac{ft}{s}\right)^2}\frac{(8\ ft)^2}{\left(\frac{4}{12}\ ft\right)^2}(1.30\ lb) = \underline{\underline{0.274\ lb}}$$

7.65

7.65 The pressure rise, Δp, across a centrifugal pump of a given shape (see Fig. P7.65a) can be expressed as

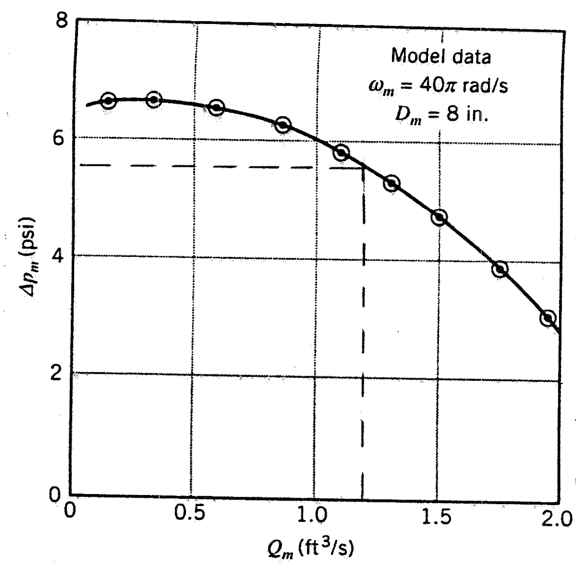
$$\Delta p = f(D, \omega, \rho, Q)$$

where D is the impeller diameter, ω the angular velocity of the impeller, ρ the fluid density, and Q the volume rate of flow through the pump. A model pump having a diameter of 8 in. is tested in the laboratory using water. When operated at an angular velocity of 40π rad/s the model pressure rise as a function of Q is shown in Fig. P7.65b. Use this curve to predict the pressure rise across a geometrically similar pump (prototype) for a prototype flowrate of 6 ft³/s. The prototype has a diameter of 12 in. and operates at an angular velocity of 60π rad/s. The prototype fluid is also water.



Centrifugal pump

(a)



(b)

■ FIGURE P7.65

$$\Delta p = f(D, \omega, \rho, Q)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \omega \doteq T^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad Q \doteq L^3T^{-1}$

From the pi theorem, 5-3 = 2 pi terms required, and a dimensional analysis yields

$$\frac{\Delta p}{\rho \omega^2 D^2} = \phi\left(\frac{Q}{\omega D^3}\right)$$

Thus, the similarity requirement is

$$\frac{Q_m}{\omega_m D_m^3} = \frac{Q}{\omega D^3}$$

so that

$$Q_m = \left(\frac{\omega_m}{\omega}\right)\left(\frac{D_m}{D}\right)^3 Q$$

and for the data given

$$Q_m = \frac{\left(40\pi \frac{rad}{s}\right)}{\left(60\pi \frac{rad}{s}\right)}\left(\frac{8\,in.}{12\,in.}\right)^3\left(6\,\frac{ft^3}{s}\right) = 1.19\,\frac{ft^3}{s}$$

From the graph (Fig. P7.65b) $\Delta p_m = 5.50$ psi for $Q_m = 1.19\,\frac{ft^3}{s}$. Thus,

$$\frac{\Delta p}{\rho \omega^2 D^2} = \frac{\Delta p_m}{\rho_m \omega_m^2 D_m^2}$$

and with $\rho = \rho_m$

$$\Delta p = \left(\frac{\omega}{\omega_m}\right)^2\left(\frac{D}{D_m}\right)^2 \Delta p_m = \frac{\left(60\pi \frac{rad}{s}\right)^2}{\left(40\pi \frac{rad}{s}\right)^2}\left(\frac{12\,in.}{8\,in.}\right)^2 (5.50\,psi)$$

$$= \underline{\underline{27.8\,psi}}$$

7.66

7.66 Start with the two-dimensional continuity equation and the Navier–Stokes equations (Eqs. 7.35, 7.36, and 7.37) and verify the nondimensional forms of these equations (Eqs. 7.38, 7.41, and 7.42).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(Eq. 7.35)}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad \text{(Eq. 7.36)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} - \rho g + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad \text{(Eq. 7.37)}$$

As indicated in Section 7.10 let

$$u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad p^* = \frac{p}{p_0}$$

$$x^* = \frac{x}{\ell} \qquad y^* = \frac{y}{\ell} \qquad t^* = \frac{t}{\tau}$$

The various transformations can be made as follows:

$$\frac{\partial u}{\partial x} = \frac{\partial (Vu^*)}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{V}{\ell}\frac{\partial u^*}{\partial x^*}$$

and similarly,

$$\frac{\partial v}{\partial x} = \frac{V}{\ell}\frac{\partial v^*}{\partial x^*} \qquad \frac{\partial u}{\partial y} = \frac{V}{\ell}\frac{\partial u^*}{\partial y^*} \qquad \frac{\partial v}{\partial y} = \frac{V}{\ell}\frac{\partial v^*}{\partial y^*}$$

Also,

$$\frac{\partial^2 u}{\partial x^2} = \frac{V}{\ell}\frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x} = \frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial x^{*2}}$$

and similarly,

$$\frac{\partial^2 v}{\partial x^2} = \frac{V}{\ell^2}\frac{\partial^2 v^*}{\partial x^{*2}} \qquad \frac{\partial^2 u}{\partial y^2} = \frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial y^{*2}} \qquad \frac{\partial^2 v}{\partial y^2} = \frac{V}{\ell^2}\frac{\partial^2 v^*}{\partial y^{*2}}$$

For the local acceleration,

$$\frac{\partial u}{\partial t} = \frac{\partial (Vu^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{V}{\tau}\frac{\partial u^*}{\partial t^*}$$

and similarly,

$$\frac{\partial v}{\partial t} = \frac{V}{\tau}\frac{\partial v^*}{\partial t^*}$$

(con't)

7.66 (con't)

For the pressure terms,

$$\frac{\partial p}{\partial x} = \frac{\partial\, p_0 p^*}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{p_0}{\ell}\frac{\partial p^*}{\partial x^*}$$

and similarly,

$$\frac{\partial p}{\partial y} = \frac{p_0}{\ell}\frac{\partial p^*}{\partial y^*}$$

Substitution of the various terms, expressed in terms of the dimensionless variables, can be made into the original differential equations (Eqs. 7.35, 7.36, and 7.37) to yield Eqs. 7.38, 7.39, and 7.40. To obtain the final form for Eqs. 7.41 and 7.42 divide each term by $\rho V^2/\ell$.

7.67

7.67 A viscous fluid is contained between wide, parallel plates spaced a distance h apart as shown in Fig. P7.67. The upper plate is fixed, and the bottom plate oscillates harmonically with a velocity amplitude U and frequency ω. The differential equation for the velocity distribution between the plates is

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$

where u is the velocity, t is time, and ρ and μ are fluid density and viscosity, respectively. Rewrite this equation in a suitable nondimensional form using h, U, and ω as reference parameters.

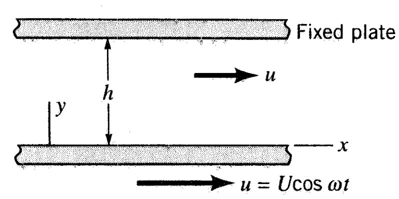


FIGURE P7.67

Let $y^* = \frac{y}{h}$, $u^* = \frac{u}{U}$, and $t^* = \omega t$ so that:

$$\frac{\partial u}{\partial t} = \frac{\partial (U u^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = U \frac{\partial u^*}{\partial t^*}(\omega) = U\omega \frac{\partial u^*}{\partial t^*}$$

$$\frac{\partial u}{\partial y} = \frac{\partial (U u^*)}{\partial y^*}\frac{\partial y^*}{\partial y} = U \frac{\partial u^*}{\partial y^*}\left(\frac{1}{h}\right) = \frac{U}{h}\frac{\partial u^*}{\partial y^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U}{h}\frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \frac{U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Thus, the original differential equation becomes

$$\rho U\omega \frac{\partial u^*}{\partial t^*} = \frac{\mu U}{h^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

or

$$\left[\frac{\rho \omega h^2}{\mu}\right]\frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}}$$

7.68

7.68 The deflection of the cantilever beam of Fig. P7.68 is governed by the differential equation

$$EI\frac{d^2y}{dx^2} = P(x - \ell)$$

where E is the modulus of elasticity and I is the moment of inertia of the beam cross section. The boundary conditions are $y = 0$ at $x = 0$ and $dy/dx = 0$ at $x = 0$. **(a)** Rewrite the equation and boundary conditions in dimensionless form using the beam length, ℓ, as the reference length. **(b) Based on the results of part (a) what are the similarity requirements and the prediction equation for a model to predict deflections?**

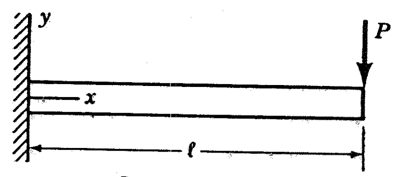


FIGURE P7.68

(a) Let $y^* = \dfrac{y}{\ell}$ and $x^* = \dfrac{x}{\ell}$ so that

$$\frac{dy}{dx} = \frac{d(\ell y^*)}{dx^*}\frac{dx^*}{dx} = \ell\frac{dy^*}{dx^*}\left(\frac{1}{\ell}\right) = \frac{dy^*}{dx^*}$$

and

$$\frac{d^2y}{dx^2} = \frac{d}{dx^*}\left(\frac{dy^*}{dx^*}\right)\frac{dx^*}{dx} = \frac{1}{\ell}\frac{d^2y^*}{dx^{*2}}$$

Thus, the original differential equation becomes

$$\left[\frac{EI}{\ell}\right]\frac{d^2y^*}{dx^{*2}} = P(\ell x^* - \ell)$$

or

$$\frac{d^2y^*}{dx^{*2}} = \left[\frac{P\ell^2}{EI}\right](x^* - 1)$$

and the boundary conditions are

$y^* = 0$ at $x^* = 0$ and $\dfrac{dy^*}{dx^*} = 0$ at $x^* = 0$.

(b) The similarity requirements are

$x_m^* = x^*$ or $\dfrac{x_m}{\ell_m} = \dfrac{x}{\ell}$ and $\dfrac{P_m \ell_m^2}{E_m I_m} = \dfrac{P\ell^2}{EI}$

The prediction equation is

$$y^* = y_m^*$$

or

$$\frac{y}{\ell} = \frac{y_m}{\ell_m}$$

7.69

7.69 A liquid is contained in a pipe that is closed at one end as shown in Fig. P7.69. Initially the liquid is at rest, but if the end is suddenly opened the liquid starts to move. Assume the pressure p_1 remains constant. The differential equation that describes the resulting motion of the liquid is

$$\rho \frac{\partial v_z}{\partial t} = \frac{p_1}{\ell} + \mu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} \right)$$

where v_z is the velocity at any radial location, r, and t is time. Rewrite this equation in dimensionless form using the liquid density, ρ, the viscosity, μ, and the pipe radius, R, as reference parameters.

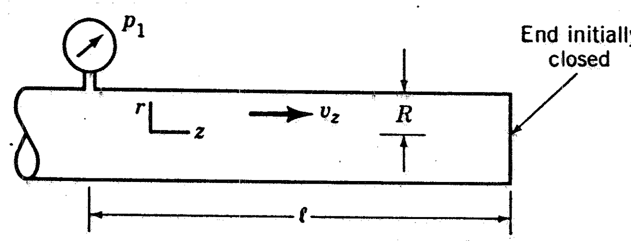


FIGURE P7.69

Let $r^* = \frac{r}{R}$, $t^* = \frac{t}{\tau}$, and $v_z^* = \frac{v_z}{V}$ where τ is some combinations of the parameters ρ, μ, and R having the dimensions of time, and V is some combination of the same parameters having the dimensions of a velocity. Let

$$\tau = \frac{\rho R^2}{\mu} \doteq \frac{(FL^{-4}T^2)(L)^2}{FL^{-2}T} \doteq T$$

and

$$V = \frac{\mu}{\rho R} \doteq \frac{FL^{-2}T}{(FL^{-4}T^2)(L)} \doteq LT^{-1}$$

With these dimensionless variables:

$$\frac{\partial v_z}{\partial t} = \frac{\partial (V v_z^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = V\frac{\partial v_z^*}{\partial t^*}\left(\frac{1}{\tau}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{\mu}{\rho R^2}\right)\frac{\partial v_z^*}{\partial t^*} = \left(\frac{\mu}{\rho}\right)^2 \frac{1}{R^3}\frac{\partial v_z^*}{\partial t^*}$$

$$\frac{\partial v_z}{\partial r} = \frac{\partial (V v_z^*)}{\partial r^*}\frac{\partial r^*}{\partial r} = V\frac{\partial v_z^*}{\partial r^*}\left(\frac{1}{R}\right) = \left(\frac{\mu}{\rho R}\right)\left(\frac{1}{R}\right)\frac{\partial v_z^*}{\partial r^*} = \frac{\mu}{\rho R^2}\frac{\partial v_z^*}{\partial r^*}$$

$$\frac{\partial^2 v_z}{\partial r^2} = \frac{\mu}{\rho R^2}\frac{\partial}{\partial r^*}\left(\frac{\partial v_z^*}{\partial r^*}\right)\frac{\partial r^*}{\partial r} = \frac{\mu}{\rho R^2}\frac{\partial^2 v_z^*}{\partial r^{*2}}\left(\frac{1}{R}\right) = \frac{\mu}{\rho R^3}\frac{\partial^2 v_z^*}{\partial r^{*2}}$$

The original differential equation can now be expressed as

$$\left[\rho\left(\frac{\mu}{\rho}\right)^2\frac{1}{R^3}\right]\frac{\partial v_z^*}{\partial t^*} = \frac{p_1}{\ell} + \left[\mu\left(\frac{\mu}{\rho R^3}\right)\right]\left(\frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_z^*}{\partial r^*}\right)$$

or

$$\frac{\partial v_z^*}{\partial t^*} = \frac{p_1 \rho R^3}{\ell \mu^2} + \frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial v_z^*}{\partial r^*}$$

7.70

7.70 An incompressible fluid is contained between two infinite parallel plates as illustrated in Fig. P7.70. Under the influence of a harmonically varying pressure gradient in the x direction, the fluid oscillates harmonically with a frequency ω. The differential equation describing the fluid motion is

$$\rho \frac{\partial u}{\partial t} = X \cos \omega t + \mu \frac{\partial^2 u}{\partial y^2}$$

where X is the amplitude of the pressure gradient. Express this equation in nondimensional form using h and ω as reference parameters.

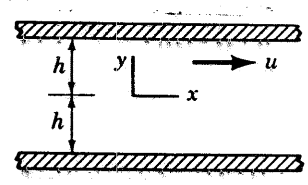


FIGURE P7.70

Let $y^* = \frac{y}{h}$, $t^* = \omega t$, and $u^* = \frac{u}{h\omega}$ so that:

$$\frac{\partial u}{\partial t} = \frac{\partial (h\omega u^*)}{\partial t^*}\frac{\partial t^*}{\partial t} = h\omega \frac{\partial u^*}{\partial t^*}(\omega) = h\omega^2 \frac{\partial u^*}{\partial t^*}$$

$$\frac{\partial u}{\partial y} = \frac{\partial (h\omega u^*)}{\partial y^*}\frac{\partial y^*}{\partial y} = h\omega \frac{\partial u^*}{\partial y^*}\left(\frac{1}{h}\right) = \omega \frac{\partial u^*}{\partial y^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \omega \frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y} = \omega \frac{\partial^2 u^*}{\partial y^{*2}}\left(\frac{1}{h}\right) = \frac{\omega}{h}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The original differential equation can now be expressed as

$$\left[\rho h \omega^2\right] \frac{\partial u^*}{\partial t^*} = X \cos t^* + \left[\frac{\mu \omega}{h}\right] \frac{\partial^2 u^*}{\partial y^{*2}}$$

or

$$\frac{\partial u^*}{\partial t^*} = \left[\frac{X}{\rho h \omega^2}\right] \cos t^* + \left[\frac{\mu}{\rho h^2 \omega}\right] \frac{\partial^2 u^*}{\partial y^{*2}}$$

7.71

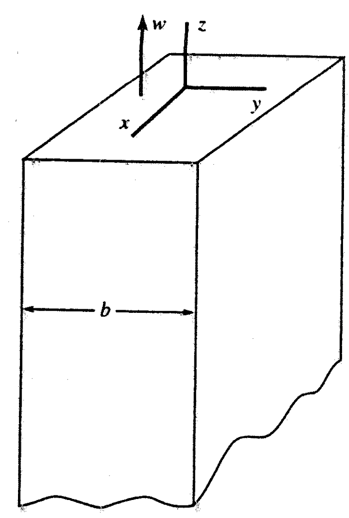
7.71 A viscous fluid flows through a vertical, square channel as shown in Fig. P7.71. The velocity w can be expressed as

$$w = f(x, y, b, \mu, \gamma, V, \partial p/\partial z)$$

where μ is the fluid viscosity, γ the fluid specific weight, V the mean velocity, and $\partial p/\partial z$ the pressure gradient in the z direction. (a) Use dimensional analysis to find a suitable set of dimensionless variables and parameters for this problem. (b) The differential equation governing the fluid motion for this problem is

$$\frac{\partial p}{\partial z} = -\gamma + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Write this equation in a suitable dimensionless form, and show that the similarity requirements obtained from this analysis are the same as those resulting from the dimensional analysis of part (a).



■ FIGURE P7.71

(a) $$w = f\left(x, y, b, \mu, \gamma, V, \frac{\partial p}{\partial z}\right)$$

$w \doteq LT^{-1}$ $x \doteq L$ $y \doteq L$ $b \doteq L$ $\mu \doteq FL^{-2}T$ $\gamma \doteq FL^{-3}$ $V \doteq LT^{-1}$ $\frac{\partial p}{\partial z} \doteq FL^{-3}$

From the pi theorem, $8-3=5$ pi terms required, and a dimensional analysis yields

$$\frac{w}{V} = \phi\left(\frac{x}{b}, \frac{y}{b}, \frac{1}{\gamma}\frac{\partial p}{\partial z}, \frac{\gamma b^2}{\mu V}\right) \qquad (1)$$

(b) Let $w^* = \frac{w}{V}$, $x^* = \frac{x}{b}$, $y^* = \frac{y}{b}$ so that:

$$\frac{\partial w}{\partial x} = \frac{\partial (V w^*)}{\partial x^*}\frac{\partial x^*}{\partial x} = \frac{V}{b}\frac{\partial w^*}{\partial x^*}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{V}{b}\frac{\partial}{\partial x^*}\left(\frac{\partial w^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x} = \frac{V}{b^2}\frac{\partial^2 w^*}{\partial x^{*2}}$$

Similarly, $$\frac{\partial^2 w}{\partial y^2} = \frac{V}{b^2}\frac{\partial^2 w^*}{\partial y^{*2}}$$

The original differential equation can now be expressed as

$$\frac{\partial p}{\partial z} = -\gamma + \left[\frac{\mu V}{b^2}\right]\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right)$$

or

$$\frac{b^2}{\mu V}\frac{\partial p}{\partial z} = -\frac{\gamma b^2}{\mu V} + \left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) \qquad (2)$$

(con't)

7.71 (Con't)

Eq. (2) indicates that

$$w^* = \frac{w}{V} = \phi\left(x^*, y^*, \frac{\gamma b^2}{\mu V}, \frac{b^2}{\mu V}\frac{\partial p}{\partial z}\right) \qquad (3)$$

Although this result does not appear to match the equation obtained by dimensional analysis (Eq. 1), the last two pi terms in Eq.(1) can be combined to yield

$$\left(\frac{1}{\gamma}\frac{\partial p}{\partial z}\right)\left(\frac{\gamma b^2}{\mu V}\right) = \frac{b^2}{\mu V}\frac{\partial p}{\partial z}$$

so that Eq.(1) can also be written as

$$\frac{w}{V} = \phi\left(\frac{x}{b}, \frac{y}{b}, \frac{\gamma b^2}{\mu V}, \frac{b^2}{\mu V}\frac{\partial p}{\partial z}\right) \qquad (4)$$

and this result is the same as that in Eq.(3). Thus, the similarity requirements indicated by Eqs.(3) and (4) are the same.

7.72

7.72 Flow from a Tank

Objective: When the drain hole in the bottom of the tank shown in Fig. P7.72 is opened, the liquid will drain out at a rate which is a function of many parameters. The purpose of this experiment is to measure the liquid depth, h, as a function of time, t, for two geometrically similar tanks and to learn how dimensional analysis can be of use in situations such as this.

Equipment: Two geometrically similar cylindrical tanks; stop watch; thermometer; ruler.

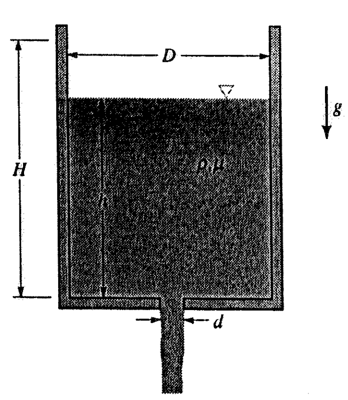
Experimental Procedure: Make appropriate measurements to show that the two tanks are geometrically similar. That is, show that the large tank is twice the size of the small tank (twice the height; twice the diameter; twice the hole diameter in the bottom). Fill the large tank with cold water of a known temperature, T, and determine the water depth, h, in the tank as a function of time, t, after the drain hole is opened. Thus, obtain $h = h(t)$. Note that t ranges from $t = 0$ when $h = H$ (where H is the initial depth of the water), to $t = t_{final}$ then the tank is completely drained ($h = 0$). Repeat the measurements using the small tank with the same temperature water. To ensure geometric similarity, the initial water level in the small tank must be one-half of what it was in the large tank. Repeat the experiment for each tank with hot water. Thus you will have a total of four sets of $h(t)$ data.

Calculations: Assume that the depth, h, of water in the tank is a function of its initial depth, H, the diameter of the tank, D, the diameter of the drain hole in the bottom of the tank, d, the time, t, after the drain is opened, the acceleration of gravity, g, and the fluid density, ρ, and viscosity, μ. Develop a suitable set of dimensionless parameters for this problem using H, g, and ρ as repeating variables. Use t as the dependent parameter. For each of the four conditions tested, calculate the dimensionless time, $tg^{1/2}/H^{1/2}$, as a function of the dimensionless depth, h/H.

Graph: On a single graph, plot the depth, h, as ordinates and time, t, as abscissas for each of the four sets of data.

Results: On another graph, plot the dimensionless water depth, h/H, as a function of dimensionless time, $tg^{1/2}/H^{1/2}$, for each of the four sets of data. Based on your results, comment on the importance of density and viscosity for your experiment and on the usefulness of dimensional analysis.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P7.72

(con't)

7.72 (con't)

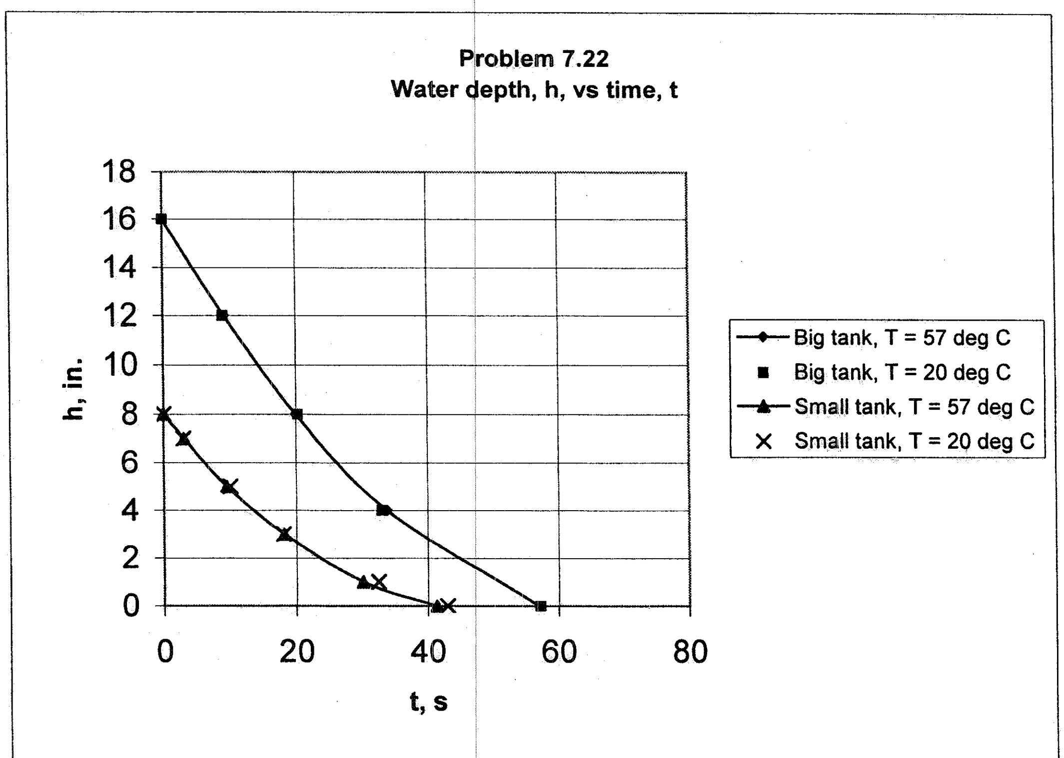
Solution for Problem 7.72: Flow from a Tank

H for big tank, in.	H for small tank, in.
16.0	8.0

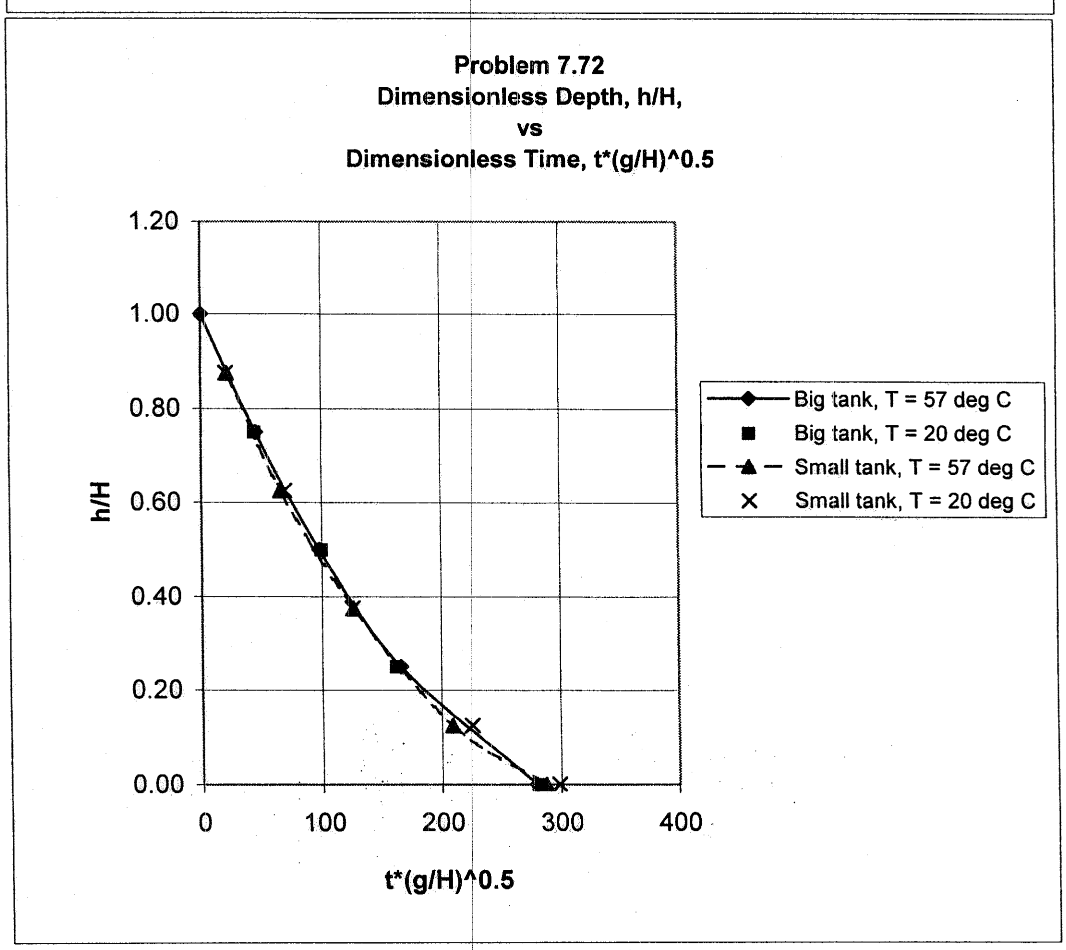
h, in.	t, s	$tg^{1/2}/H^{1/2}$	h/H
Big Tank with T = 57 deg C			
16.0	0.0	0.0	1.000
12.0	9.2	45.2	0.750
8.0	20.0	98.3	0.500
4.0	33.8	166.1	0.250
0.0	57.0	280.1	0.000
Big Tank with T = 20 deg C			
16.0	0.0	0.0	1.000
12.0	9.0	44.2	0.750
8.0	20.3	99.8	0.500
4.0	33.0	162.2	0.250
0.0	57.2	281.1	0.000
Small Tank with T = 57 deg C			
8.0	0.0	0.0	1.000
7.0	3.1	21.5	0.875
5.0	9.5	66.0	0.625
3.0	18.2	126.5	0.375
1.0	30.1	209.2	0.125
0.0	41.4	287.7	0.000
Small Tank with T = 20 deg C			
8.0	0.0	0.0	1.000
7.0	3.0	20.8	0.875
5.0	10.0	69.5	0.625
3.0	18.1	125.8	0.375
1.0	32.5	225.9	0.125
0.0	43.0	298.8	0.000

(con't)

7.72 (con't)



Problem 7.22
Water depth, h, vs time, t
h, in.
t, s
0
2
4
6
8
10
12
14
16
18
0
20
40
60
80
Big tank, T = 57 deg C
Big tank, T = 20 deg C
Small tank, T = 57 deg C
Small tank, T = 20 deg C



Problem 7.72
Dimensionless Depth, h/H,
vs
Dimensionless Time, t*(g/H)^0.5
h/H
t*(g/H)^0.5
0.00
0.20
0.40
0.60
0.80
1.00
1.20
0
100
200
300
400
Big tank, T = 57 deg C
Big tank, T = 20 deg C
Small tank, T = 57 deg C
Small tank, T = 20 deg C

7.73

7.73 Vortex Shedding from a Circular Cylinder

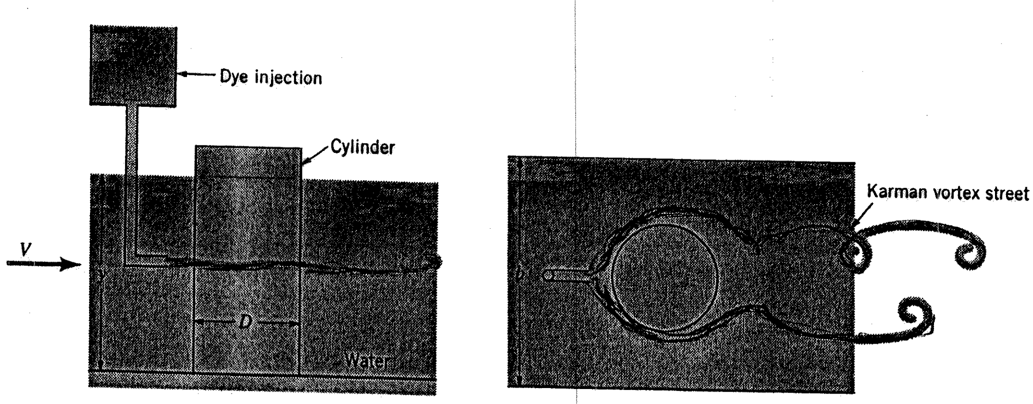
Objective: Under certain conditions, the flow of fluid past a circular cylinder will produce a Karman vortex street behind the cylinder. As shown in Fig. P7.73, this vortex street consists of a set of vortices (swirls) that are shed alternately from opposite sides of the cylinder and then swept downstream with the fluid. The purpose of this experiment is to determine the shedding frequency, ω cycles (vortices) per second, of these vortices as a function of the Reynolds number, Re, and to compare the measured results with published data.

Equipment: Water channel with an adjustable flowrate; flow meter; set of four different diameter cylinders; dye injection system; stopwatch.

Experimental Procedure: Insert a cylinder of diameter D into the holder on the bottom of the water channel. Adjust the control valve and the downstream gate on the channel to produce the desired flowrate, Q, and velocity, V. Make sure that the flow-straightening screens (not shown in the figure) are in place to reduce unwanted turbulence in the flowing water. Measure the width, b, of the channel and the depth, y, of the water in the channel so that the water velocity in the channel, $V = Q/(by)$, can be determined. Carefully adjust the control valve on the dye injection system to inject a thin stream of dye slightly upstream of the cylinder. By viewing down onto the top of the water channel, observe the vortex shedding and measure the time, t, that it takes for N vortices to be shed from the cylinder. For a given velocity, repeat the experiment for different diameter cylinders. Repeat the experiment using different velocities. Measure the water temperature so that the viscosity can be looked up in Table B.1.

Calculations: For each of your data sets calculate the vortex shedding frequency, $\omega = N/t$, which is expressed as vortices (or cycles) per second. Also calculate the dimensionless frequency called the Strouhl number, $St = \omega D/V$, and the Reynolds number, $Re = \rho VD/\mu$.

Graph: On a single graph, plot the vortex shedding frequency, ω, as ordinates and the water velocity, V, as abscissas for each of the four cylinders you tested. On another graph, plot the Strouhl number as ordinates and the Reynolds number as abscissas for each of the four sets of data.



Side view   Top view

■ FIGURE P7.73

(con't)

7.73 (con't)

Results: On your Strouhl number verses Reynolds number graph, plot the results taken from the literature and shown in the following table.

St	Re
0	<50
0.16	100
0.18	150
0.19	200
0.20	300
0.21	400
0.21	600
0.21	800

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

Solution for Problem 7.73: Vortex Shedding from a Circular Cylinder

T, deg F	b, ft
70	0.50

Q, ft^3/s	y, ft	D, ft	N	t, s	ω, cycles/s	V, ft/s	Re	St	Data from Literature Re	St
0.036	0.82	0.0202	10.0	13.2	0.758	0.0878	169	0.174	50	0.00
0.036	0.82	0.0314	10.0	19.9	0.503	0.0878	263	0.180	100	0.16
0.036	0.82	0.0421	10.0	24.5	0.408	0.0878	352	0.196	150	0.18
0.036	0.82	0.0518	10.0	30.1	0.332	0.0878	433	0.196	200	0.19
									300	0.20
									400	0.21
0.062	0.79	0.0202	10.0	6.3	1.587	0.1570	302	0.204	600	0.21
0.062	0.79	0.0314	10.0	9.6	1.042	0.1570	469	0.208	800	0.21
0.062	0.79	0.0421	10.0	12.5	0.800	0.1570	629	0.215		
0.062	0.79	0.0518	10.0	15.1	0.662	0.1570	774	0.219		
0.029	0.86	0.0202	10.0	19.2	0.521	0.0674	130	0.156		
0.029	0.86	0.0314	10.0	28.2	0.355	0.0674	202	0.165		
0.029	0.86	0.0421	10.0	33.1	0.302	0.0674	270	0.189		
0.029	0.86	0.0518	10.0	36.7	0.272	0.0674	333	0.209		
0.018	0.92	0.0202	10.0	31.2	0.321	0.0391	75	0.165		
0.018	0.92	0.0314	10.0	41.3	0.242	0.0391	117	0.194		
0.018	0.92	0.0421	10.0	52.2	0.192	0.0391	157	0.206		
0.018	0.92	0.0518	10.0	65.3	0.153	0.0391	193	0.203		

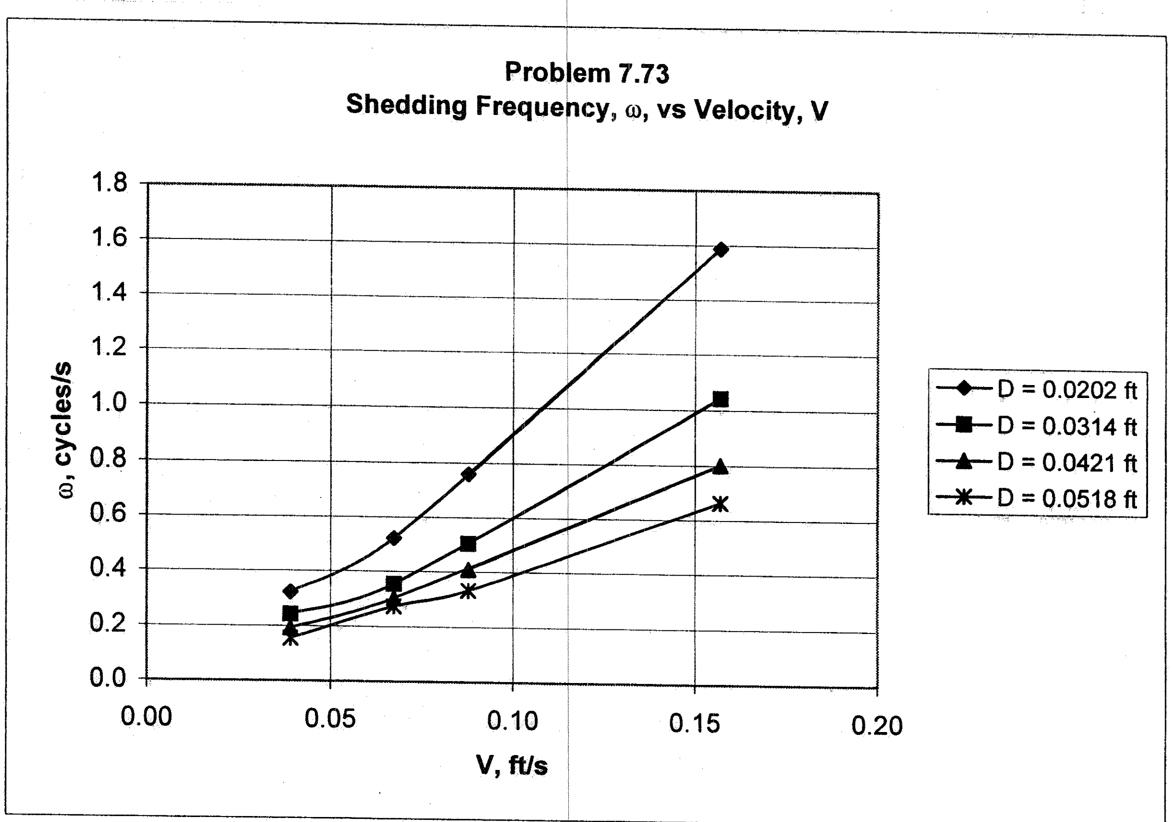
$\omega = N/t$
$V = Q/(by)$

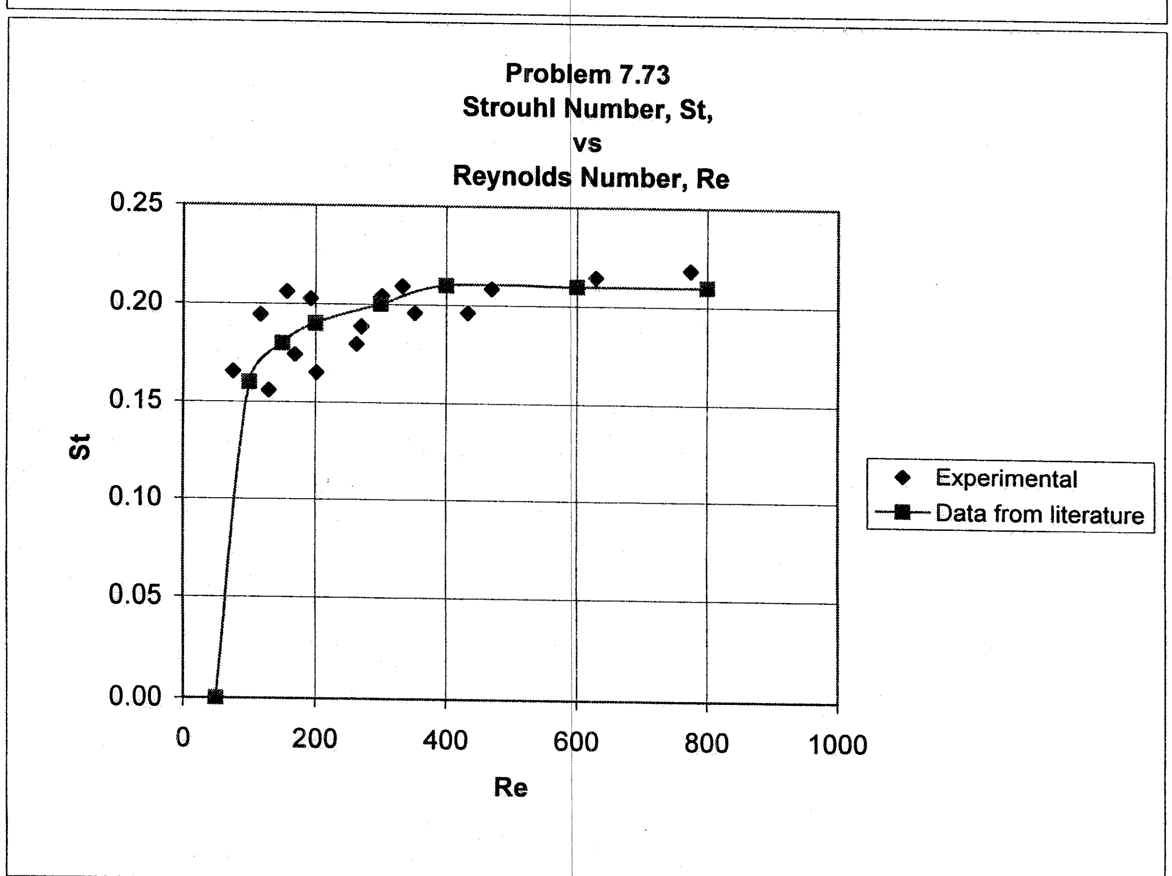
$St = \omega D/V$ and $Re = DV/\nu$, where

ν = 1.052E-5 ft^2/s

(con't)

7.73 (con't)


Problem 7.73
Shedding Frequency, ω, vs Velocity, V
ω, cycles/s
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
0.20
V, ft/s
D = 0.0202 ft
D = 0.0314 ft
D = 0.0421 ft
D = 0.0518 ft


Problem 7.73
Strouhl Number, St,
vs
Reynolds Number, Re
St
0.25
0.20
0.15
0.10
0.05
0.00
0
200
400
600
800
1000
Re
Experimental
Data from literature

7.74

7.74 Head Loss across a Valve

Objective: A valve in a pipeline like that shown in Fig. P7.74 acts like a variable resistor in an electrical circuit. The amount of resistance or head loss across a valve depends on the amount that the valve is open. The purpose of this experiment is to determine the head loss characteristics of a valve by measuring the pressure drop, Δp, across the valve as a function of flowrate, Q, and to learn how dimensional analysis can be of use in situations such as this.

Equipment: Air supply with flow meter; valve connected to a pipe; manometer connected to a static pressure tap upstream of the valve; barometer; thermometer.

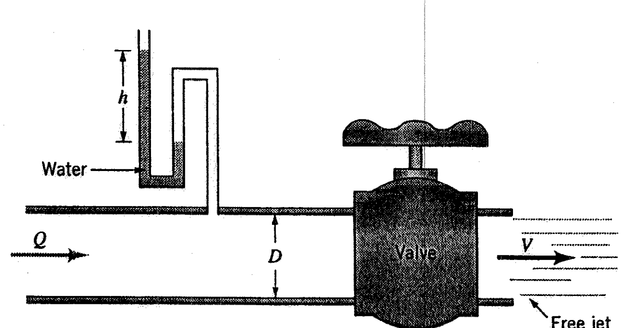
Experimental Procedure: Measure the pipe diameter, D. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Completely close the valve and then open it N turns from its closed position. Adjust the air supply to provide the desired flowrate, Q, of air through the valve. Record the manometer reading, h, so that the pressure drop, Δp, across the valve can be determined. Repeat the measurements for various flowrates. Repeat the experiment for various valve settings, N, ranging from barely open to wide open.

Calculations: For each data set calculate the average velocity in the pipe, $V = Q/A$, where $A = \pi D^2/4$ is the pipe area. Also calculate the pressure drop across the valve, $\Delta p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. For each data set also calculate the loss coefficient, K_L, where the head loss is given by $h_L = \Delta p/\gamma = K_L V^2/2g$ and γ is the specific weight of the flowing air.

Graph: On a single graph, plot the pressure drop, Δp, as ordinates and the flowrate, Q, as abscissas for each of the valve settings, N, tested.

Results: On another graph, plot the loss coefficient, K_L, as a function of valve setting, N, for all of the data sets.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P7.74

(con't)

7.74 (con't)

Solution for Problem 7.74: Head Loss across a Valve

D, in.	H_{atm}, in. Hg	T, deg F
0.81	28.7	70

h, in.	Q, ft^3/s	Δp, lb/ft^2	V, ft/s	N	K_L
N = 2 Turns Open Data					
9.20	0.235	47.8	65.7	2	9.95
6.50	0.195	33.8	54.5	2	10.21
5.04	0.169	26.2	47.2	2	10.54
N = 3 Turns Open Data					
9.40	0.479	48.9	133.9	3	2.45
6.33	0.386	32.9	107.9	3	2.54
5.01	0.341	26.1	95.3	3	2.57
3.62	0.289	18.8	80.8	3	2.59
1.92	0.214	10.0	59.8	3	2.50
N = 4 Turns Open Data					
9.35	0.827	48.6	231.1	4	0.816
7.65	0.767	39.8	214.3	4	0.777
6.01	0.691	31.3	193.1	4	0.752
4.32	0.578	22.5	161.5	4	0.772
3.24	0.504	16.8	140.8	4	0.762
2.62	0.456	13.6	127.4	4	0.752
1.85	0.391	9.6	109.3	4	0.723
0.98	0.283	5.1	79.1	4	0.731
N = 5 Turns Open Data					
3.03	0.897	15.8	250.7	5	0.225
2.37	0.799	12.3	223.3	5	0.222
1.79	0.701	9.3	195.9	5	0.218
1.39	0.618	7.2	172.7	5	0.217
0.97	0.517	5.0	144.5	5	0.217
0.64	0.426	3.3	119.0	5	0.211

$\Delta p = \gamma_{H2O} * h$

$K_L = \Delta p/(\rho V^2/2)$ where

$V = Q/A = Q/(\pi * D^2/4)$

and

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(28.7/12 ft) = 2026 lb/ft^2

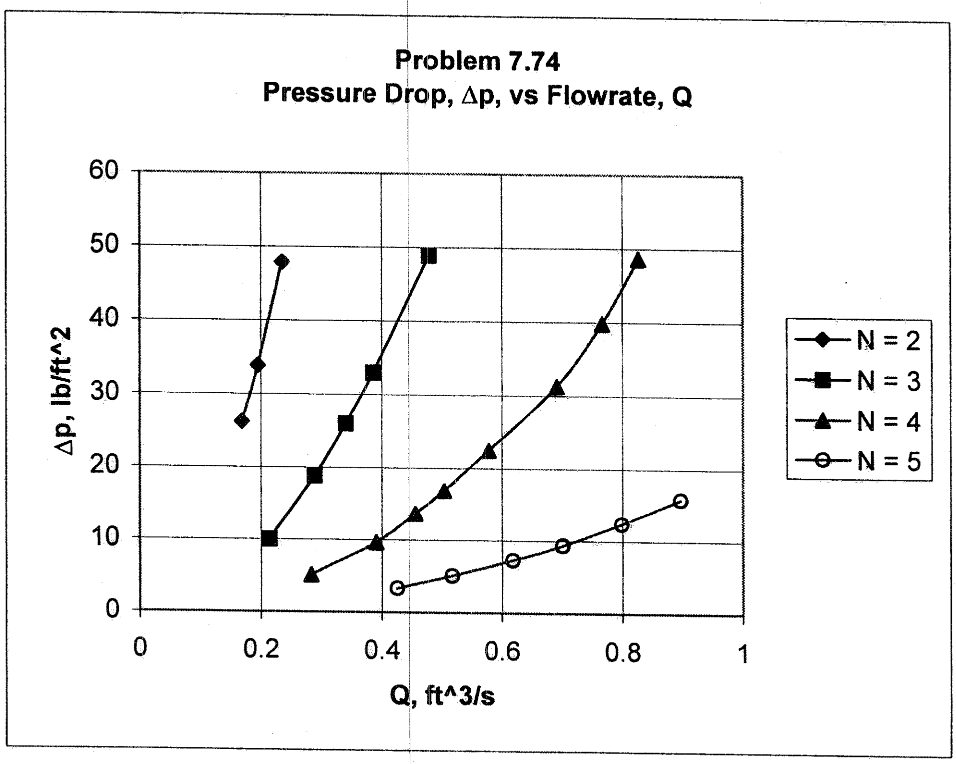
R = 1716 ft lb/slug deg R

T = 70 + 460 = 530 deg R

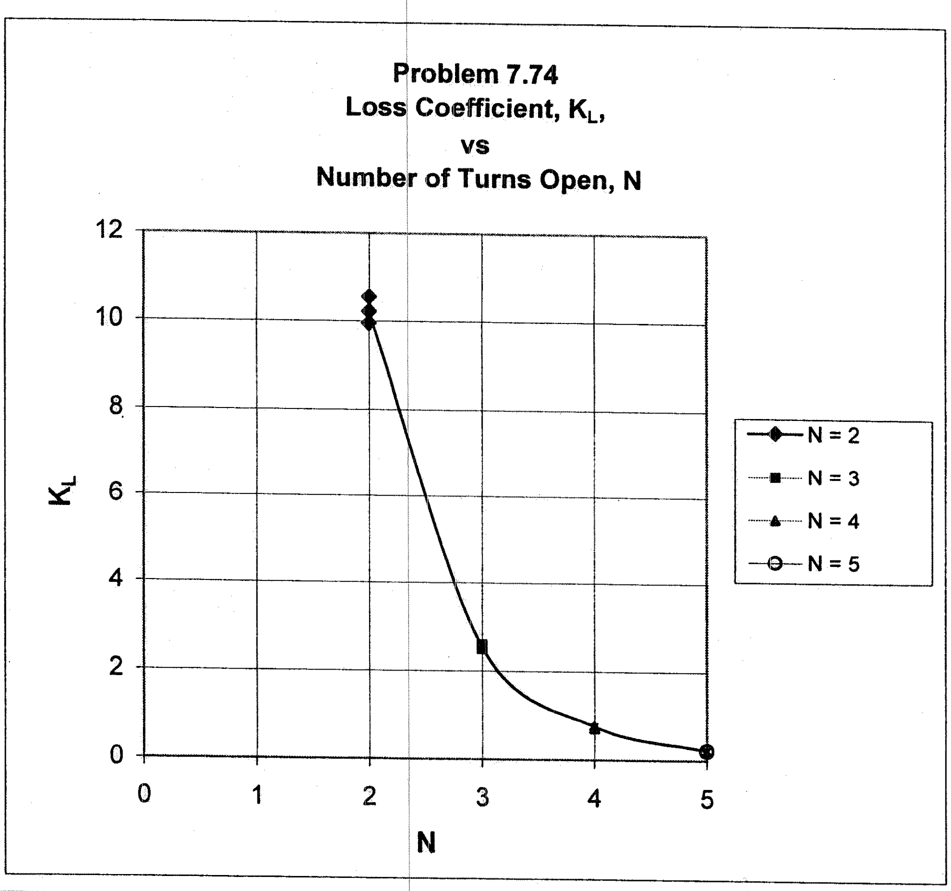
Thus, ρ = 0.00223 slug/ft^3

(con't)

7.74 (con't)



Problem 7.74
Pressure Drop, Δp, vs Flowrate, Q
Δp, lb/ft^2
Q, ft^3/s
N = 2
N = 3
N = 4
N = 5



Problem 7.74
Loss Coefficient, K_L,
vs
Number of Turns Open, N
K_L
N
N = 2
N = 3
N = 4
N = 5

7.75

7.75 Calibration of a Rotameter

Objective: The flowrate, Q, through a rotameter can be determined from the scale reading, SR, which indicates the vertical position of the float within the tapered tube of the rotameter as shown in Fig. P7.75. Clearly, for a given scale reading, the flowrate depends on the density of the flowing fluid. The purpose of this experiment is to calibrate a rotameter so that it can be used for both water and air.

Equipment: Rotameter, air supply with a calibrated flow meter, water supply, weighing scale, stop watch, thermometer, barometer.

Experimental Procedure: Connect the rotameter to the water supply and adjust the flowrate, Q, to the desired value. Record the scale reading, SR, on the rotameter and measure the flowrate by collecting a given weight, W, of water that passes through the rotameter in a given time, t. Repeat for several flow rates.

Connect the rotameter to the air supply and adjust the flowrate to the desired value as indicated by the flow meter. Record the scale reading on the rotameter. Repeat for several flowrates. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For the water portion of the experiment, use the weight, W, and time, t, data to determine the volumetric flowrate, $Q = W/\gamma t$. The equilibrium position of the float is a result of a balance between the fluid drag force on the float, the weight of the float, and the buoyant force on the float. Thus, a typical dimensionless flowrate can be written as $Q/[d(\rho/V g(\rho_f - \rho))^{1/2}]$, where d is the diameter of the float, V is the volume of the float, g is the acceleration of gravity, ρ is the fluid density, and ρ_f is the float density. Determine this dimensionless flowrate for each condition tested.

Graph: On a single graph, plot the flowrate, Q, as ordinates and scale reading, SR, as abscissas for both the water and air data.

Results: On another graph, plot the dimensionless flowrate as a function of scale reading for both the water and air data. Note that the scale reading is a percent of full scale and, hence, is a dimensionless quantity. Based on your results, comment on the usefulness of dimensional analysis.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

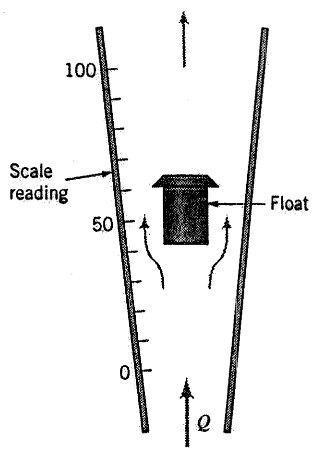


FIGURE P7.75

(con't)

7.75 (con't)

Solution for Problem 7.75: Calibration of a Rotameter

d, in.	**V**, in.^3	ρ_f, slug/ft^3	H_{atm}, in.	T, deg F
1.40	1.50	15.1	29.05	78

Air Flow Data

SR	Q, ft^3/s	(Q/d)[ρ/(**V**g(ρ_f-ρ))]1/2
14.6	0.229	0.142
21.5	0.321	0.200
28.1	0.413	0.257
33.6	0.491	0.305
39.2	0.564	0.351
44.8	0.644	0.400
50.2	0.714	0.444
55.9	0.798	0.496
63.1	0.888	0.552
68.6	0.973	0.605
73.5	1.05	0.653
76.2	1.08	0.671

Water Flow Data

SR	W, lb	t, s	Q, ft^3/s	(Q/d)[ρ/(**V**g(ρ_f-ρ))]1/2
13.1	6.52	19.9	0.0053	0.103
18.5	8.01	17.7	0.0073	0.143
24.2	7.02	10.4	0.0108	0.213
28.2	7.81	10.1	0.0124	0.244
37.1	8.20	8.4	0.0156	0.308
45.7	9.21	7.5	0.0197	0.387
52.6	8.19	5.7	0.0230	0.453

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.05/12 ft) = 2050 lb/ft^2

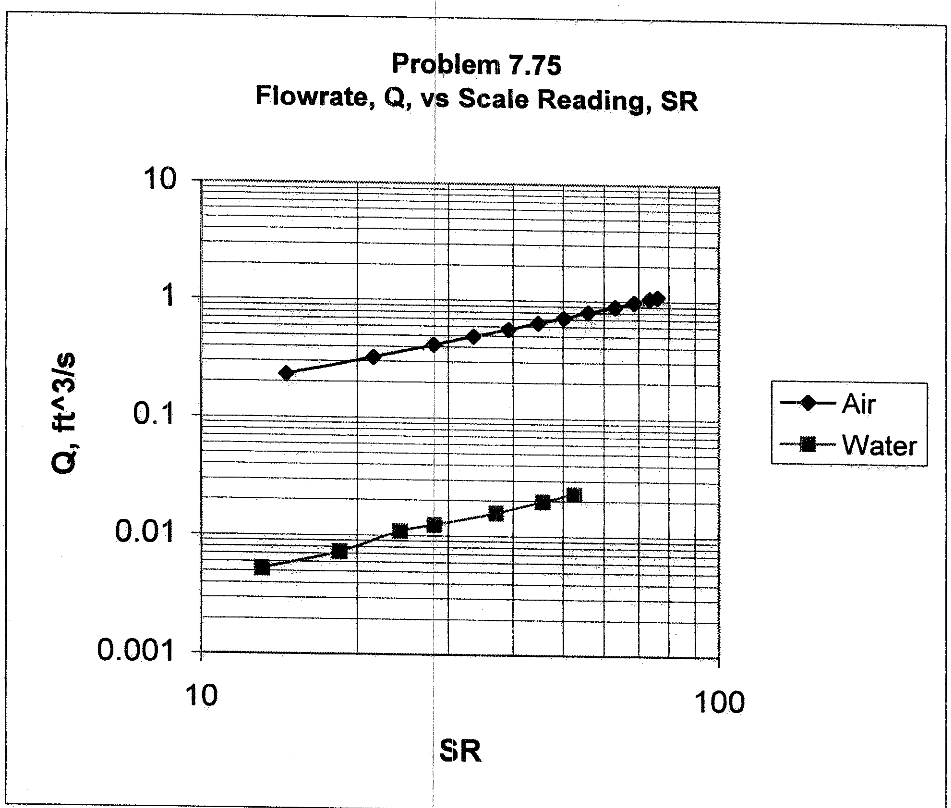
R = 1716 ft lb/slug deg R

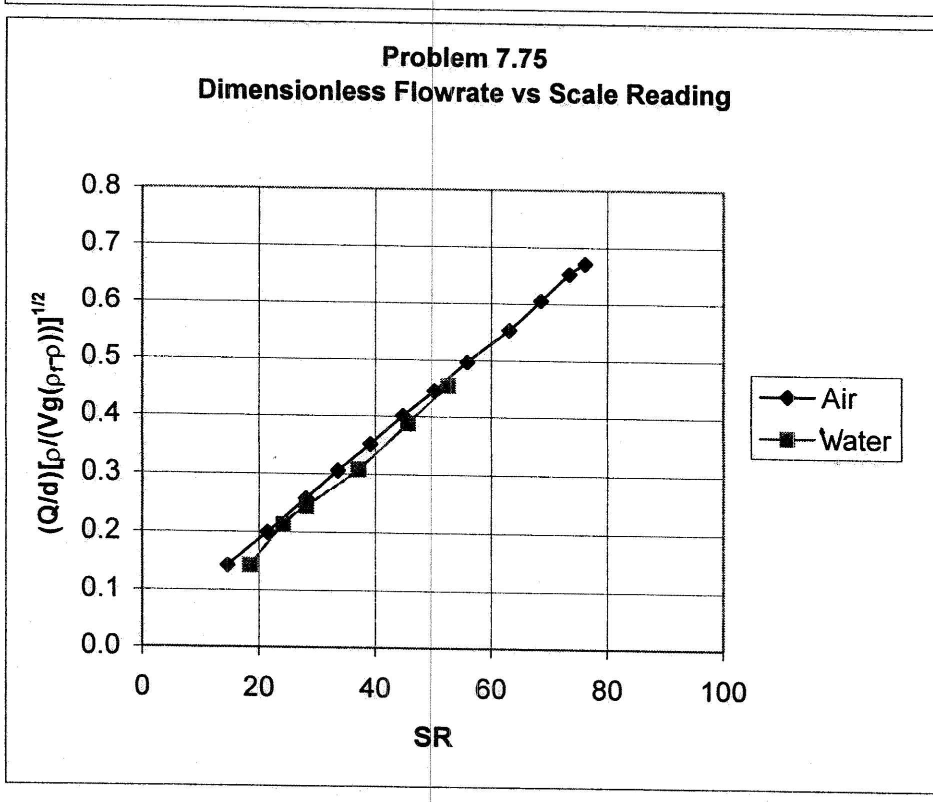
T = 78 + 460 = 538 deg R

Thus, ρ = 0.00222 slug/ft^3

(con't)

7.75 (con't)


Problem 7.75
Flowrate, Q, vs Scale Reading, SR
Q, ft^3/s
10
1
0.1
0.01
0.001
10
100
SR
Air
Water


Problem 7.75
Dimensionless Flowrate vs Scale Reading
(Q/d)[ρ/(Vg(ρf-ρ))]^1/2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
20
40
60
80
100
SR
Air
Water

7.76

7.76 (See "**Modeling parachutes in a water tunnel,**" Section 7.8.1.) Flow characteristics for a 30-ft-diameter prototype parachute are to be determined by tests of a 1-ft-diameter model parachute in a water tunnel. Some data collected with the model parachute indicate a drag of 17 lb when the water velocity is 4 ft/s. Use the model data to predict the drag on the prototype parachute falling through air at 10 ft/s. Assume the drag to be a function of the velocity, V, the fluid density, ρ, and the parachute diameter, D.

$$\mathcal{D} = f(V, \rho, D)$$

$$\mathcal{D} \doteq F \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad D \doteq L$$

From the pi theorem, 4-3=1 pi term required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 D^2} = C$$

where C is a constant. Thus, for similarity between model and prototype

$$\frac{\mathcal{D}}{\rho V^2 D^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 D_m^2}$$

So that

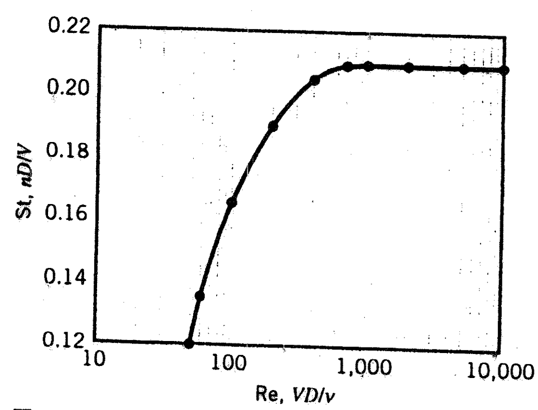
$$\mathcal{D} = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{D}{D_m}\right)^2 \mathcal{D}_m$$

$$= \left(\frac{2.38\times10^{-3}\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}}\right)\left(\frac{10\ \frac{ft}{s}}{4\ \frac{ft}{s}}\right)^2\left(\frac{30\ ft}{1\ ft}\right)^2 (17\ lb)$$

$$= \underline{\underline{117\ lb}}$$

7.77

7.77 (See "Galloping Gertie," Section 7.8.2.) The Tacoma Narrows bridge failure is a dramatic example of the possible serious effects of wind-induced vibrations. As a fluid flows around a body, vortices may be created which are shed periodically creating an oscillating force on the body. If the frequency of the shedding vortices coincides with the natural frequency of the body, large displacements of the body can be induced as was the case with the Tacoma Narrows bridge. To illustrate this type of phenomenon, consider fluid flow past a circular cylinder. Assume the frequency, n, of the shedding vortices behind the cylinder is a function of the cylinder diameter, D, the fluid velocity, V, and the fluid kinematic viscosity, ν. (a) Determine a suitable set of dimensionless variables for this problem. One of the dimensionless variables should be the Strouhal number, nD/V. (b) Some results of experiments in which the shedding frequency of the vortices (in Hz) was measured, using a particular cylinder and Newtonian, incompressible fluid, are shown in Fig. P7.77. Is this a "universal curve" that can be used to predict the shedding frequency for any cylinder placed in any fluid? Explain. (c) A certain structural component in the form of a 1-in.-diameter, 12-ft-long rod acts as a cantilever beam with a natural frequency of 19 Hz. Based on the data in Fig. P7.77, estimate the wind speed that may cause the rod to oscillate at its natural frequency. *Hint:* Use a trial and error solution.



■ FIGURE P7.77

(a) $n = f(D, V, \nu)$

$n \doteq T^{-1} \quad D \doteq L \quad V \doteq LT^{-1} \quad \nu \doteq L^2T^{-1}$

From the pi theorem, $4-2=2$ pi terms required, and a dimensional analysis yields

$$\frac{nD}{V} = \phi\left(\frac{VD}{\nu}\right)$$

(b) Yes. If the variables of part (a) are correct then this is a "universal" or general relationship between the Strouhal number and the Reynolds number. It is valid over the range of Reynolds numbers covered in the experiment.

(con't)

7.77 (con't)

(c) For $n = 19\ Hz$ and $D = \frac{1\ in.}{12\ \frac{in.}{ft}} = \frac{1}{12}\ ft$

$$S_t = \frac{nD}{V} = \frac{(19\ Hz)(\frac{1}{12}\ ft)}{V} \qquad (1)$$

From Fig. P7.77, assume $S_t = 0.21$ and from Eq. (1)

$$0.21 = \frac{(19\ Hz)(\frac{1}{12}\ ft)}{V}$$

so that $V = 7.54\ \frac{ft}{s}$ (5.14 mph)

Check Re: $Re = \frac{VD}{\nu} = \frac{(7.54\ \frac{ft}{s})(\frac{1}{12}\ ft)}{1.57 \times 10^{-4}\ \frac{ft^2}{s}} = 4000$

From Fig. P7.77 at $Re = 4000$, $St = 0.21$ and therefore assumed value of S_t OK.

7.78

7.78 (See "Ice engineering," Section 7.9.3.) A model study is to be developed to determine the force exerted on bridge piers due to floating chunks of ice in a river. The piers of interest have square cross sections. Assume that the force, R, is a function the pier width, b, the depth of the ice, d, the velocity of the ice, V, the acceleration of gravity, g, the density of the ice, ρ_i, and a measure of the strength of the ice, E_i, where E_i has the dimensions FL^{-2}. **(a)** Based on these variables determine a suitable set of dimensionless variables for this problem. **(b)** The prototype conditions of interest include an ice thickness of 12 in. and an ice velocity of 6 ft/s. What model ice thickness and velocity would be required if the length scale is to be 1/10? **(c)** If the model and prototype ice have the same density can the model ice have the same strength properties as that of the prototype ice? Explain.

(a) $$R = f(b, d, V, g, \rho_i, E_i)$$

$R \doteq F \quad b \doteq L \quad d \doteq L \quad V \doteq LT^{-1} \quad g \doteq LT^{-2} \quad \rho_i \doteq FL^{-4}T^2 \quad E_i \doteq FL^{-2}$

From the pi theorem, 7-3 = 4 pi terms required, and a dimensional analysis yields

$$\frac{R}{E_i b^2} = \phi\left(\frac{b}{d}, \frac{V^2}{gd}, \frac{\rho_i V^2}{E_i}\right)$$

(b) For similarity,

$$\frac{b_m}{d_m} = \frac{b}{d} \quad \text{or} \quad \frac{d_m}{d} = \frac{b_m}{b} = \frac{1}{10}$$

so that

$$d_m = \frac{1}{10}(12 \text{ in.}) = \underline{1.20 \text{ in.}}$$

Also,

$$\frac{V_m^2}{g_m d_m} = \frac{V^2}{gd} \qquad (1)$$

and with $g_m = g$

$$V_m = \sqrt{\left(\frac{g_m}{g}\right)\left(\frac{d_m}{d}\right)}\, V = \sqrt{(1)\left(\frac{1}{10}\right)}\left(6\ \frac{ft}{s}\right) = \underline{1.90\ \frac{ft}{s}}$$

(c) For similarity,

$$\frac{\rho_{im} V_m^2}{E_{im}} = \frac{\rho V^2}{E_i}$$

Thus,

$$\frac{E_{im}}{E_i} = \left(\frac{\rho_{im}}{\rho_i}\right)\left(\frac{V_m}{V}\right)^2 = \frac{d_m}{d}$$ since $\rho_{im} = \rho_i$ and

from Eq. (1) $\left(\frac{V_m}{V}\right)^2 = \frac{d_m}{d}$

Since $d_m/d = 1/10$, $E_{im} \neq E_i$ and model ice cannot have same strength properties. No.