10.1

10.1 Water flows in a rectangular channel with a depth of 2 ft and a flowrate of 160 ft³/s. The flow is critical. Determine the channel width.

$$Fr = \frac{V}{\sqrt{gy}} = 1 \quad \text{so that} \quad V = \sqrt{gy} = \sqrt{(32.2\ ft/s^2)(2\ ft)} = 8.02\ ft/s$$

Also,

$$Q = AV = byV \quad \text{or}$$

$$160\ \frac{ft^3}{s} = b\,(2\ ft)\left(8.02\ \frac{ft}{s}\right)$$

Hence, $b = \underline{\underline{9.98\ ft}}$

10.2

10.2 The flowrate per unit width in a wide channel is $q = 2.3\ m^2/s$. Is the flow subcritical or supercritical if the depth is **(a)** 0.2 m, **(b)** 0.8m, or **(c)** 2.5 m?

$$V = \frac{Q}{A} = \frac{qb}{yb} = \frac{q}{y} \quad \text{so that} \quad Fr = \frac{V}{\sqrt{g\,y}} = \frac{q}{y\sqrt{g\,y}} = \frac{q}{\sqrt{g}\,y^{3/2}}$$

or

$$Fr = \frac{2.3\ \frac{m^2}{s}}{\sqrt{9.81\ \frac{m}{s^2}}\ y^{3/2}} = \frac{0.734}{y^{3/2}}, \quad \text{where } y \sim m$$

	y, m	Fr	flow type
a)	0.2	8.21	supercritical
b)	0.8	1.03	supercritical
c)	2.5	0.186	subcritical

10.3

10.3 A rectangular channel 3 m wide carries 10 m³/s at a depth of 2 m. Is the flow subcritical or supercritical? For the same flowrate, what depth will give critical flow?

$$Q = AV \text{ or } V = \frac{Q}{by} = \frac{10\,\frac{m^3}{s}}{(3m)(2m)} = 1.667\,\frac{m}{s}$$

Thus,

$$Fr = \frac{V}{\sqrt{gy}} = \frac{1.667\,\frac{m}{s}}{\left[(9.81\,\frac{m}{s^2})(2m)\right]^{\frac{1}{2}}} = 0.376 < 1$$ The flow is subcritical.

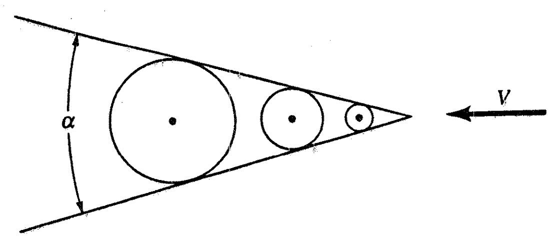
Also,

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}, \text{ where } q = \frac{Q}{b} = \frac{10\,\frac{m^3}{s}}{3m} = 3.33\,\frac{m^2}{s} \text{ so that}$$

$$y_c = \left(\frac{(3.33\,\frac{m^2}{s})^2}{9.81\,\frac{m}{s^2}}\right)^{\frac{1}{3}} = \underline{\underline{1.04\,m}}$$

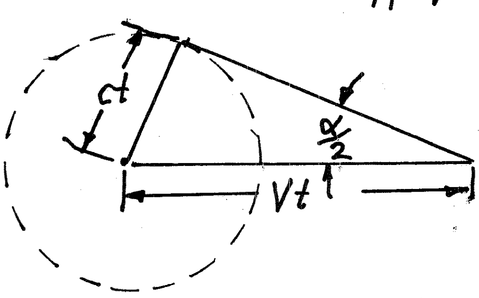
10.4

10.4 Consider waves made by dropping objects (one after another from a fixed location) into a stream of depth y that is moving with speed V as shown in Fig. P10.4 (see Video V9.1). The circular wave crests that are produced travel with speed $c = (gy)^{1/2}$ relative to the moving water. Thus, as the circular waves are washed downstream, their diameters increase and the center of each circle is fixed relative to the moving water. **(a)** Show that if the flow is supercritical, lines tangent to the waves generate a wedge of half-angle $\alpha/2 = \arcsin(1/Fr)$, where $Fr = V/(gy)^{1/2}$ is the Froude number. **(b)** Discuss what happens to the wave pattern when the flow is subcritical, $Fr < 1$.



■ FIGURE P10.4

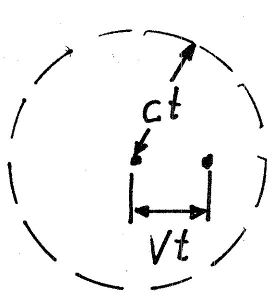
(a) In a time interval of t since the object hit the water (and iniatiated the wave), the center of the wave has been swept downstream a distance Vt and the wave has expanded to be a distance ct from its center. This is shown in the figure below. Note that $Vt > ct$ if $V > c$ (i.e. $Fr > 1$).



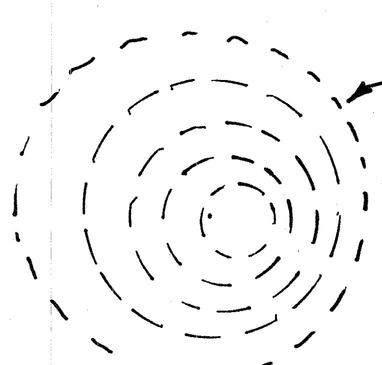
Thus, from the figure, $\sin\frac{\alpha}{2} = \frac{ct}{Vt} = \frac{c}{V} = \frac{\sqrt{gy}}{V} = \frac{1}{Fr}$

or $\frac{\alpha}{2} = \arcsin(1/Fr)$

(b) If $Fr<1$ the above result gives $\sin\frac{\alpha}{2} > 1$, which is impossible. For $Fr<1$ the following wave pattern would result. There is no "wedge" produced.



$Vt < ct$ if $Fr < 1$



10.5

10.5 Waves on the surface of a tank are observed to travel at a speed of 2 m/s. How fast would these waves travel if (a) the tank were in an elevator accelerating downward at a rate of 4 m/s², (b) the tank accelerates horizontally at a rate of 9.81 m/s², (c) the tank were aboard the orbiting Space Shuttle? Explain.

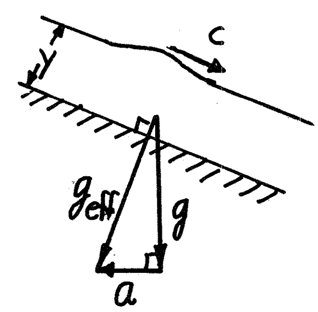
Since $c = \sqrt{gy}$ it follows that the tank depth is

$$y = \frac{c^2}{g} = \frac{(2\,\frac{m}{s})^2}{9.81\,\frac{m}{s^2}} = 0.408\ m$$

(a) If the tank accelerates down with acceleration a, the effective acceleration of gravity is $g_{eff} = g - a = (9.81-4)\frac{m}{s^2} = 5.81\ \frac{m}{s^2}$
Thus,

$$c = \sqrt{g_{eff}\,y} = \sqrt{(5.81\ \tfrac{m}{s^2})(0.408m)} = \underline{\underline{1.54\ \tfrac{m}{s}}}$$

(b) If the tank accelerates horizontally with acceleration a, the effective acceleration is



$$g_{eff} = \sqrt{g^2 + a^2} = \sqrt{9.81^2 + 9.81^2} = 13.87\ \frac{m}{s^2}$$

Thus,

$$c = \sqrt{(13.87\tfrac{m}{s^2})(0.408m)} = \underline{\underline{2.38\ \tfrac{m}{s}}}$$

(c) In orbit $g_{eff} = 0$ (weightless) so $c = \underline{\underline{0}}$

10.6

10.6 In flowing from section (1) to section (2) along an open channel, the water depth decreases by a factor of two and the Froude number changes from a subcritical value of 0.5 to a supercritical value of 3.0. Determine the channel width at (2) if it is 12 ft wide at (1).

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = 0.5, \text{ or } \sqrt{g y_1} = 2.0\, V_1 \qquad (1)$$

and

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = 3.0 \quad \text{where } y_2 = 0.5\, y_1$$

$$\text{Thus, } \frac{V_2}{\sqrt{0.5\, g y_1}} = 3.0, \text{ or } \sqrt{g y_1} = V_2/(3\sqrt{0.5}) \qquad (2)$$

By equating Eq. (1) and (2): $2.0\, V_1 = V_2/(3\sqrt{0.5})$

or

$V_2 = 4.24\, V_1$

However, $Q_1 = Q_2$ or $b_1 y_1 V_1 = b_2 y_2 V_2$ where b = channel width.

Thus, with $b_1 = 12$ ft:

$$(12\text{ ft})\, y_1 (V_1) = b_2 (0.5\, y_1)(4.24\, V_1) \text{ or } b_2 = \frac{12 \text{ ft}}{0.5(4.24)} = \underline{\underline{5.66 \text{ ft}}}$$

10.7

10.7 Observations at a shallow sandy beach show that even though the waves several hundred yards out from the shore are not parallel to the beach, the waves often "break" on the beach nearly parallel to the shore as is indicated in Fig. P10.7. Explain this behavior based on the wave speed $c = (gy)^{1/2}$.

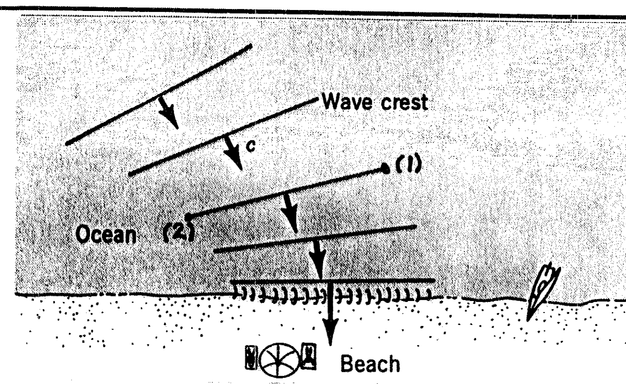


FIGURE P10.7

Since $c = \sqrt{gy}$ it follows that $c_1 > c_2$ because of the fact that $y_1 > y_2$. Therefore, as the waves move, that portion in the deeper water tends to "catch up" with that portion closer to shore in the shallower water. The wave crest tends to become more nearly parallel to the shore line. The waves "break" on the shore as if the wind were blowing normal to the shore.

10.8

10.8 Waves on the surface of a tank containing water are observed to move with a velocity of 1.8 m/s. If the water is replaced by mercury, with all other conditions the same, determine the wave speed expected. Determine the wave speed if the tank were in a laboratory on the surface of Mars where the acceleration of gravity is 37% of that on Earth.

Since $c = \sqrt{gy}$ it follows that the wave speed is independent of the fluid density. Thus, $c_{H_2O} = c_{Hg} = \underline{\underline{1.8\,\frac{m}{s}}}$ on earth.

However, on Mars

$$c_{Mars} = \sqrt{g_{Mars}\,y} = \sqrt{0.37\,g_{Earth}\,y} = 0.608\sqrt{g_{Earth}\,y}$$

$$= 0.608\left(1.8\,\frac{m}{s}\right) = \underline{\underline{1.09\,\frac{m}{s}}} \text{ for water or mercury.}$$

10.9

10.9 Often when an earthquake shifts a segment of the ocean floor, a relatively small amplitude wave of very long wavelength is produced. Such waves go unnoticed as they move across the open ocean; only when they approach the shore do they become dangerous (a tsunami or "tidal wave"). Determine the wave speed if the wavelength, λ, is 6000 ft and the ocean depth is 15,000 ft.

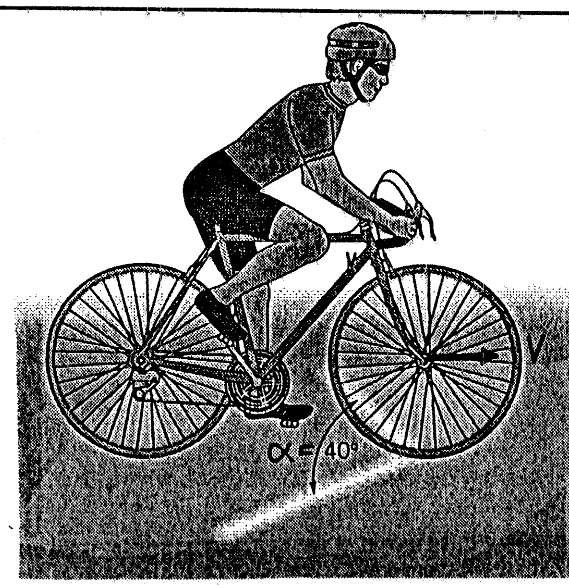
From Eq. 10.4: $c = \left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{\frac{1}{2}}$

or

$$c = \left[\frac{(32.2\,\frac{ft}{s^2})(6000\,ft)}{2\pi}\tanh\left(\frac{2\pi(15{,}000\,ft)}{6000\,ft}\right)\right]^{\frac{1}{2}} = \underline{\underline{175\,\frac{ft}{s}}}$$

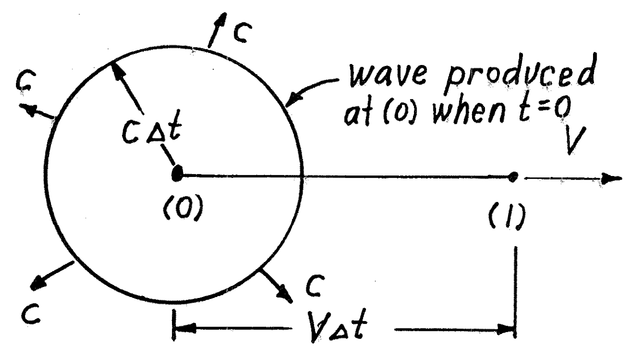
10.10

10.10 A bicyclist rides through a 3-in. deep puddle of water as shown in Video V10.1 and Fig. P10.10 If the angle made by the V-shaped wave pattern produced by the front wheel is observed to be 40 deg, estimate the speed of the bike through the puddle. *Hint:* Make a sketch of the current location of the bike wheel relative to where it was Δt seconds ago. Also indicate on this sketch the current location of the wave that the wheel made Δt seconds ago. Recall that the wave moves radially outward in all directions with speed c relative to the stationary water.

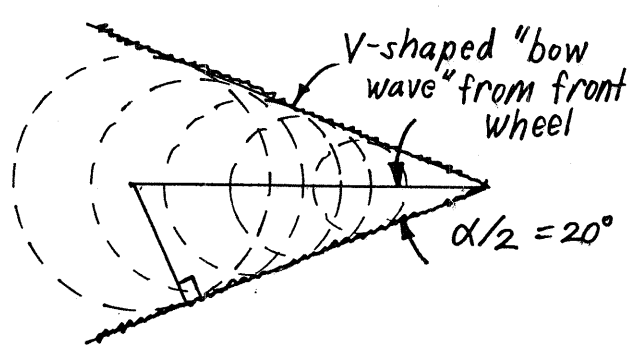


■ FIGURE P10.10

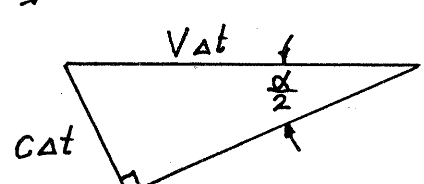
At time $t=0$ the front wheel was at point (0). At the current time, $t=\Delta t$, the wheel has traveled a distance $d=V\Delta t$ and is at point (1). At time $t=\Delta t$, a wave produced by the wheel when it was at (0) will be a distance $c\Delta t$ from (0) as indicated in the figure.



Waves produced at various times (from $t=0$ to $t=\Delta t$) by the front wheel will form a V-shaped wave as shown in the second figure (provided $V>c$; supercritical bike speed).



From the geometry of the figure



$$\sin\frac{\alpha}{2} = \frac{c\Delta t}{V\Delta t}$$

or

$$V = \frac{c}{\sin\frac{\alpha}{2}} \qquad \text{where } c=\sqrt{gy} = \left[32.2\,\frac{ft}{s^2}\left(\frac{3}{12}\,ft\right)\right]^{\frac{1}{2}} = 2.84\,\frac{ft}{s}$$

Thus,

$$V = \frac{2.84\,\frac{ft}{s}}{\sin 20^\circ} = \underline{\underline{8.30\,\frac{ft}{s}}}$$

10.11

10.11 Water flows in a rectangular channel with a flowrate per unit width of $q = 2.5\ \text{m}^2/\text{s}$. Plot the specific energy diagram for this flow. Determine the two possible depths of flow if $E = 2.5$ m.

$$E = y + \frac{q^2}{2g\,y^2} = y + \frac{(2.5\,\frac{m^2}{s})^2}{2(9.81\frac{m}{s^2})y^2} = y + \frac{0.319}{y^2}$$

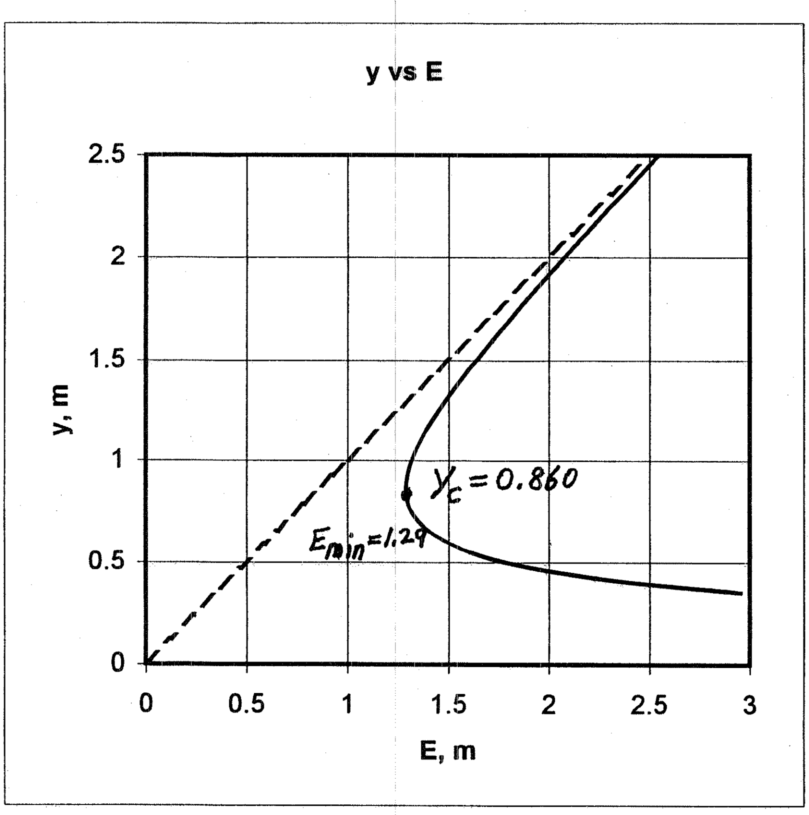
Thus, plot

$$E = y + \frac{0.319}{y^2}\,,\quad \text{where } E \sim m,\ y \sim m$$

Note: $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(2.5\frac{m^2}{s})^2}{9.81\frac{m}{s^2}}\right)^{1/3} = 0.860\,m$

and

$$E_{min} = \frac{3}{2}\,y_c = \frac{3}{2}(0.860\,m) = 1.29\,m$$



For $E = 2.5\,m$, Eq. (1) is $2.5 = y + \frac{0.319}{y^2}$

or $y^3 - 2.5y^2 + 0.319 = 0$

The roots to this equation are $y = 2.45$, 0.338, and -0.335

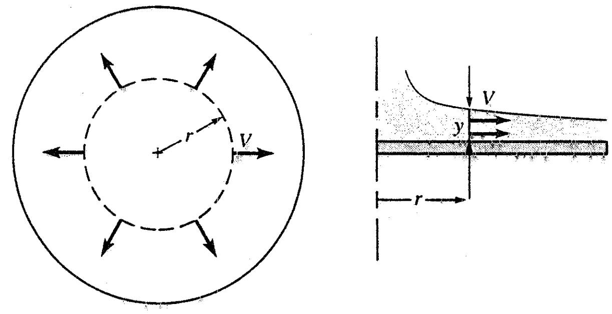
Thus, $\underline{\underline{y = 2.45\ m \text{ or } y = 0.388\ m}}$

10.12

10.12 Water flows radially outward on a horizontal round disk as is shown in Video V10.6 and Fig. P10.12. (a) Show that the specific energy can be written in terms of the flowrate, Q, the radial distance from the axis of symmetry, r, and the fluid depth, y, as

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) For a constant flowrate, sketch the specific energy diagram. Recall Fig. 10.7, but note that for the present case r is a variable. Explain the important characteristics of your sketch. (c) Based on the results of Part (b), show that the water depth increases in the flow direction if the flow is subcritical, but that it decreases in the flow direction if the flow is supercritical.



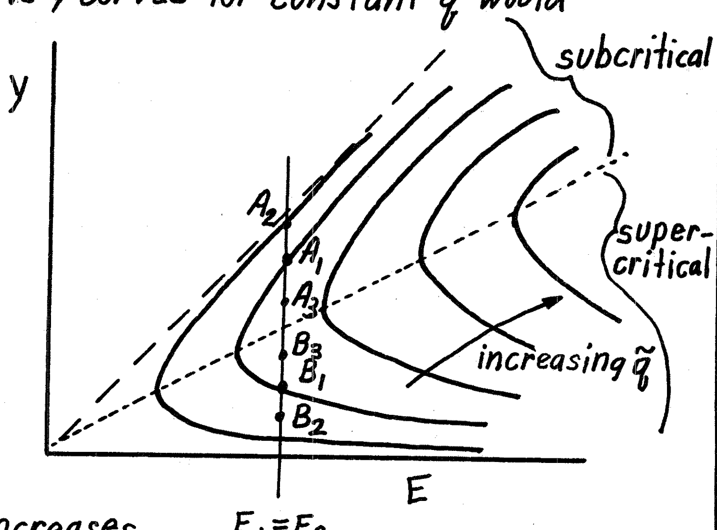
■ FIGURE P10.12

(a) The specific energy is $E = y + \frac{V^2}{2g}$, where $V = \frac{Q}{A} = \frac{Q}{2\pi r y}$

Thus,

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) Let $\tilde{q} \equiv \frac{Q}{2\pi r}$ so that $E = y + \frac{\tilde{q}^2}{2gy^2}$ which is the same as for two dimensional flow with $q = \frac{Q}{b}$ being replaced by $\tilde{q}$. However, for two dimensional flow q is constant; for radial flow $\tilde{q}$ is a variable since r varies. But E vs y curves for constant $\tilde{q}$ would look as shown below (Fig. 10.7).



(c) From the Bernoulli equation $E_1 = E_2$ or $E = \text{constant}$ for this flow. Consider subcritical flow — point A. For outflow r increases so that $\tilde{q}$ decreases. Thus since $E = \text{const.}$, the flow goes from state A_1 to A_2; the depth increases. For subcritical inflow r decreases, $\tilde{q}$ increases, the flow goes from A_1 to A_3, and the depth decreases. For supercritical flow $\dagger$ is true. Thus, outflow increases r, decreases $\tilde{q}$; or from B_1 to B_2 — decreasing depth. Supercritical inflow from B_1 to B_3 — increasing depth.

	subcritical	supercritical
inflow	depth decreases	depth increases
outflow	depth increases	depth decreases

10.13

10.13 Water flows in a rectangular channel with a flowrate per unit width of $q = 25 \text{ ft}^2/\text{s}$. Plot the specific energy diagram for this flow. Determine the two possible depths of flow if $E = 7$ ft.

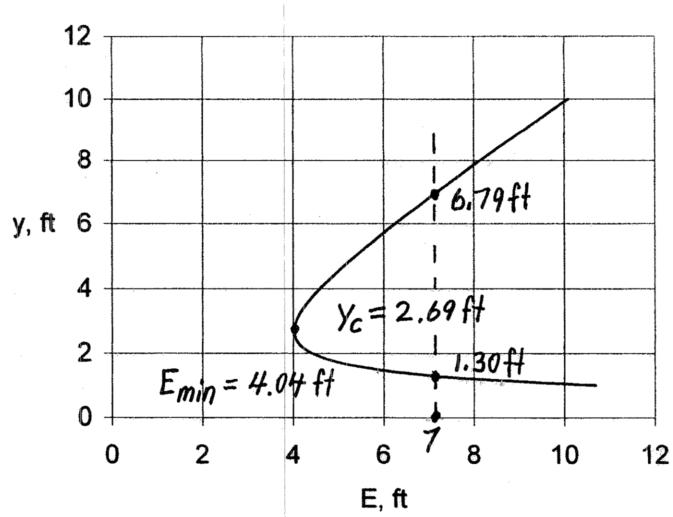
$$E = y + \frac{q^2}{2gy^2} = y + \frac{(25\frac{ft^2}{s})^2}{2(32.2\frac{ft}{s^2})\,y^2} = y + \frac{9.70}{y^2}, \text{ where } y \sim ft,\ E \sim ft$$

(1) Thus, plot $E = y + \frac{9.70}{y^2}$

Note: $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(25\frac{ft^2}{s})^2}{32.2\frac{ft}{s^2}}\right)^{1/3} = 2.69 \text{ ft}$

and

$$E_{min} = \frac{3}{2}y_c = \frac{3}{2}(2.69\,\tfrac{ft}{s}) = 4.04 \text{ ft}$$



For $E = 7$ ft, Eq.(1) is $7 = y + \frac{9.70}{y^2}$

or

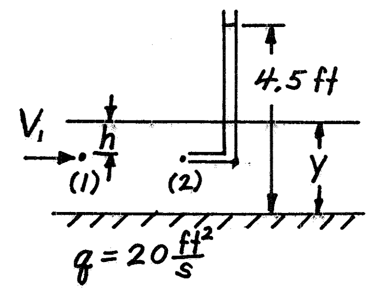
$y^3 - 7y^2 + 9.70 = 0$

The roots of this equation are $y = -1.09$, 1.31, and 6.79

Thus, $\underline{\underline{y = 1.30 \text{ ft or } y = 6.79 \text{ ft}}}$

10.1.4

10.14 Water flows in a rectangular channel at a rate of $q = 20$ cfs/ft. When a Pitot tube is placed in the stream, water in the tube rises to a level of 4.5 ft above the channel bottom. Determine the two possible flow depths in the channel. Illustrate this flow on a specific energy diagram.



$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$, where $z_1 = z_2$,

$V_2 = 0$, $\frac{p_1}{\gamma} = h$, and $\frac{p_2}{\gamma} = 4.5 - (y - h)$

Thus,

$h + \frac{V_1^2}{2g} = 4.5\,ft - y + h$, or $\frac{V_1^2}{2g} = 4.5 - y$

but,

$V_1 = \frac{q}{y} = \frac{20 \frac{ft^2}{s}}{y}$

Hence,

$\frac{\left(\frac{20}{y}\right)^2}{2(32.2 \frac{ft}{s^2})} = 4.5 - y$ or $y^3 - 4.5y^2 + 6.21 = 0$, where $y \sim ft$

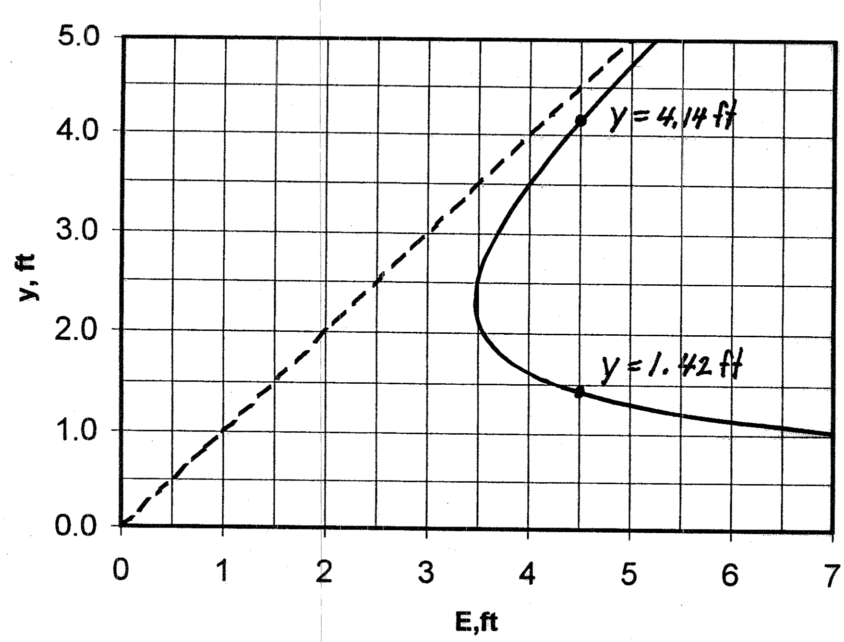
The roots of this equation are $y = 4.14, 1.42$, and -1.06

Thus,

$y = 4.14\,ft$ or $y = 1.42\,ft$

$E = y + \frac{q^2}{2gy^2} = y + \frac{(20 \frac{ft^2}{s})^2}{2(32.2 \frac{ft}{s^2})y^2}$ or $E = y + \frac{6.21}{y^2}$ (1)

The specific energy diagram (plot of Eq. (1)) is shown below.



10.15

10.15 Water flows in a 5-ft-wide rectangular channel with a flowrate of $Q = 30\ \text{ft}^3/\text{s}$ and an upstream depth of $y_1 = 2.5$ ft as is shown in Fig. P10.15. Determine the flow depth and the surface elevation at section (2).

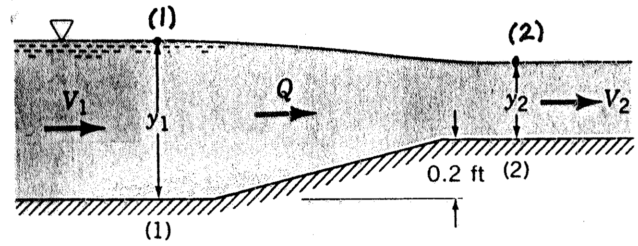


FIGURE P10.15

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2,\ \text{where } p_1=p_2=0,\ z_1=y_1=2\ \text{ft},\ z_2=0.2\ \text{ft}+y_2,$$

$$V_1=\frac{Q}{A_1}=\frac{(30\ \frac{\text{ft}^3}{\text{s}})}{(2\ \text{ft})(5\ \text{ft})}=3\ \frac{\text{ft}}{\text{s}},\ \text{and}\ V_2=\frac{Q}{A_2}=\frac{30\ \frac{\text{ft}^3}{\text{s}}}{(5\ \text{ft})\,y_2}=\frac{6}{y_2}$$

Thus,

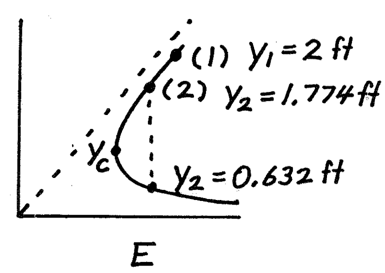
$$\frac{(3\ \frac{\text{ft}}{\text{s}})^2}{2(32.2\ \frac{\text{ft}}{\text{s}^2})}+2\ \text{ft}=\frac{(\frac{6}{y_2}\ \frac{\text{ft}}{\text{s}})^2}{2(32.2\ \frac{\text{ft}}{\text{s}^2})}+0.2\ \text{ft}+y_2$$

or

$y_2^3-1.94y_2^2+0.559=0$ which has roots $y_2=1.774,\ 0.632$, and -0.632

Note: $Fr_1=\dfrac{V_1}{\sqrt{gy_1}}=\dfrac{3\ \frac{\text{ft}}{\text{s}}}{[(32.2\ \frac{\text{ft}}{\text{s}^2})(2\ \text{ft})]^{1/2}}=0.374<1$

If $y_2=0.632$, then $Fr_2>1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $\underline{\underline{y_2=1.774\ \text{ft and } z_2=1.974\ \text{ft}}}$

10.16

10.16 Repeat Problem 10.15 if the upstream depth is $y_1 = 0.5$ ft.

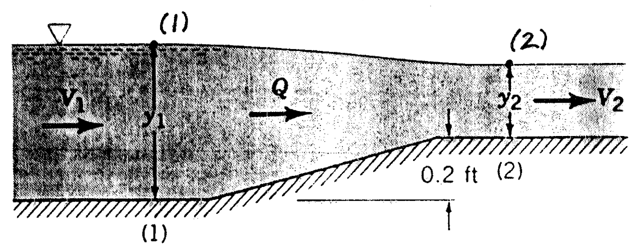


FIGURE P10.16

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 0.5\text{ ft},\ z_2 = 0.2\text{ ft} + y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{30\ \frac{ft^3}{s}}{(0.5ft)(5ft)} = 12\ \frac{ft}{s}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{30\ \frac{ft^3}{s}}{(5ft)y_2} = \frac{6}{y_2}$$

Thus,

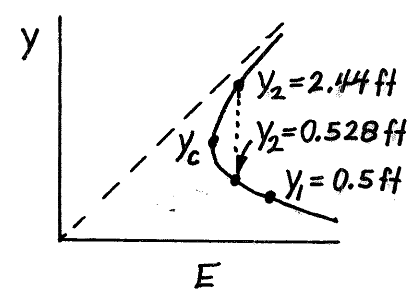
$$\frac{(12\ \frac{ft}{s})^2}{2(32.2\ \frac{ft}{s^2})} + 0.5\text{ ft} = \frac{(\frac{6}{y_2})^2}{2(32.2\ \frac{ft}{s^2})} + 0.2\text{ ft} + y_2$$

or

$y_2^3 - 2.53\,y_2^2 + 0.559 = 0$ which has roots $y_2 = 2.44, 0.528$, and -0.434

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12\ \frac{ft}{s}}{[(32.2\ \frac{ft}{s^2})(0.5\ ft)]^{1/2}} = 2.99 > 1$

If $y_2 = 2.44$ ft, then $Fr_2 < 1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $\underline{\underline{y_2 = 0.528 \text{ ft and } z_2 = 0.728 \text{ ft}}}$

*10.17

10.17* Water flows over the bump in the bottom of the rectangular channel shown in Fig. P10.17 with a flowrate per unit width of $q = 4\ m^2/s$. The channel bottom contour is given by $z_B = 0.2e^{-x^2}$, where z_B and x are in meters. The water depth far upstream of the bump is $y_1 = 2$ m. Plot a graph of the water depth, $y = y(x)$, and the surface elevation, $z = z(x)$, for $-4\ m \le x \le 4$ m. Assume one-dimensional flow.

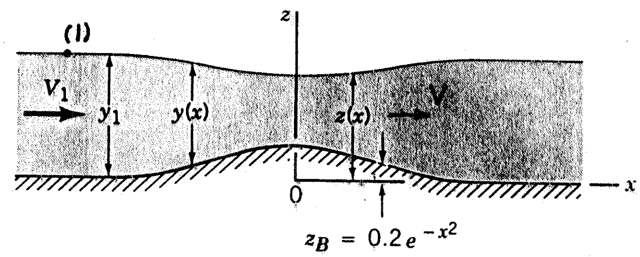


FIGURE P10.17

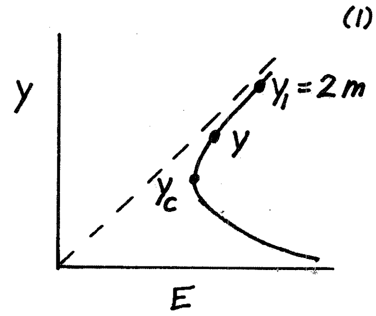
$$\cancel{\frac{p_1}{\gamma}}+\frac{V_1^2}{2g}+z_1=\cancel{\frac{p}{\gamma}}+\frac{V^2}{2g}+z, \text{ where } p_1=p=0,\ z_1=y_1=2m,\ z_2=y+z_B$$

or $z = y + 0.2\,e^{-x^2}$, $V_1 = \frac{q}{y_1} = \frac{4\frac{m^2}{s}}{2m} = 2\frac{m}{s}$, and $V = \frac{q}{y} = \frac{4}{y}$

Thus,
$$\frac{(2\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}+2m=\frac{(\frac{4}{y}\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}+y+0.2\,e^{-x^2}$$

or
$$y^3-(2.20-0.2e^{-x^2})y^2+0.815=0 \text{ where } y\sim m \qquad (1)$$

Solve for y with $-4 \le x \le 4m$



Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2\frac{m}{s}}{[(9.81\frac{m}{s^2})(2m)]^{\frac{1}{2}}} = 0.452 < 1$

Thus, the flow will remain subcritical throughout — the largest root of Eq. (1) will be the correct one.

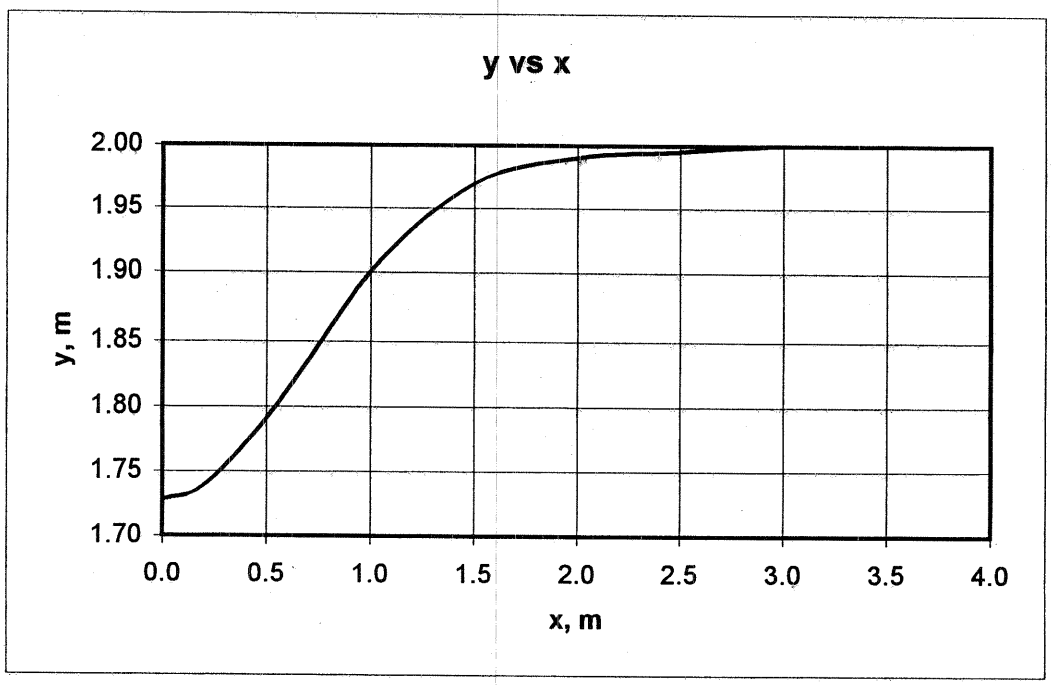
The following results are obtained by solving Eq. (1) for y and then $z = y + 0.2\,e^{-x^2}$ for $-4m \le x \le 4m$.

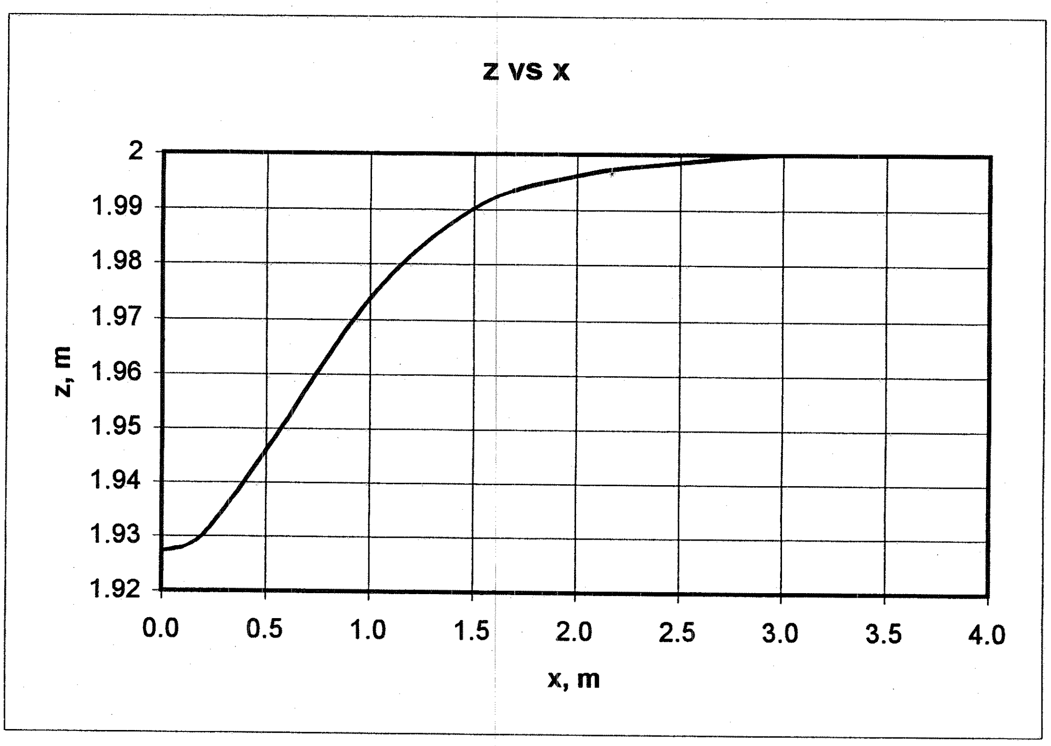
±x, m	y, m	z, m
0.0	1.727	1.927
0.5	1.790	1.946
1.0	1.901	1.974
1.5	1.969	1.990
2.0	1.991	1.994
2.5	1.995	1.995
3.0	1.995	1.995
3.5	1.995	1.995
4.0	1.995	1.995

(con't)

10.17* (con't)

The above results are plotted in the graph below.





*** 10.18**

*10.18 0. 4 m. Repeat Problem 10.17 if the upstream depth is

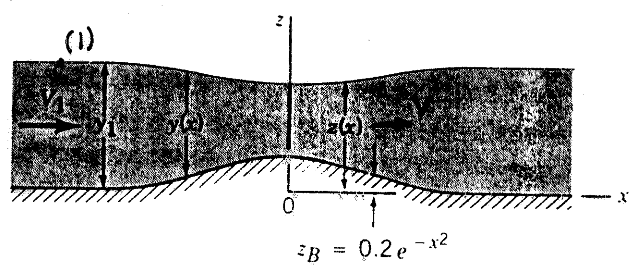


FIGURE P10.18

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p}{\gamma} + \frac{V^2}{2g} + z$, where $p_1 = p = 0$, $z_1 = y_1 = 0.4\,m$, $z_2 = y + z_B$

or $z_2 = y + 0.2e^{-x^2}$, $V_1 = \frac{q}{y_1} = \frac{4\frac{m^2}{s}}{0.4m} = 10\frac{m}{s}$, and $V = \frac{q}{y} = \frac{4}{y}$

Thus, $\frac{(10\frac{m}{s})^2}{2(9.81\frac{m}{s})^2} + 0.4m = \frac{(\frac{4}{y}\frac{m}{s})^2}{2(9.81\frac{m}{s})^2} + y + 0.2e^{-x^2}$

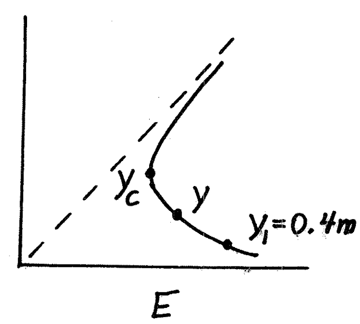
or

$y^3 - (5.50 - 0.2e^{-x^2})y^2 + 0.815 = 0$ where $y \sim m$ (1)

Solve for y with $-4 \leq x \leq 4\,m$

Note: $Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{10\frac{m}{s}}{[(9.81\frac{m}{s^2})(0.4m)]^{1/2}} = 5.05 > 1$

Thus, the flow will remain supercritical throughout — the smallest positive root of Eq.(1) will be the correct one.



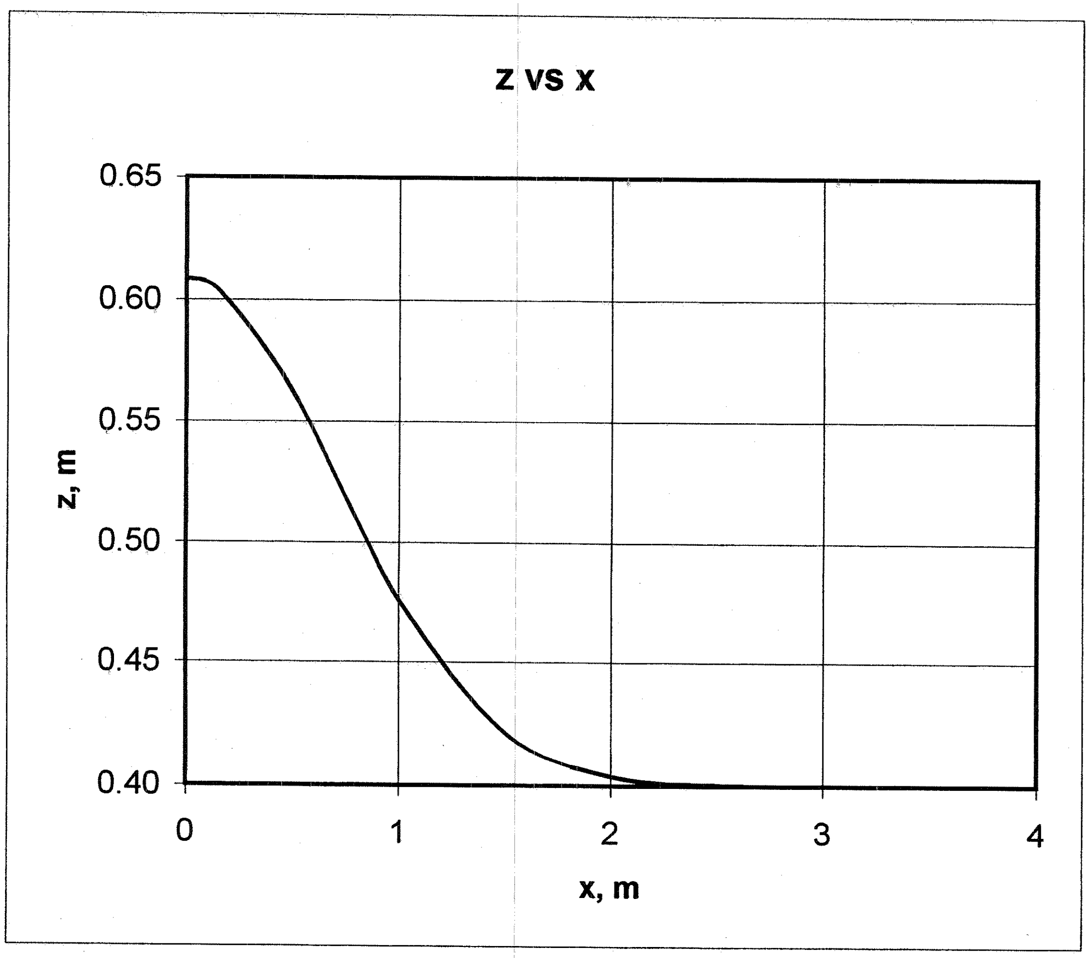
The following results are obtained by solving Eq.(1) for y and then $z = y + 0.2e^{-x^2}$ for $-4m \leq x \leq 4m$.

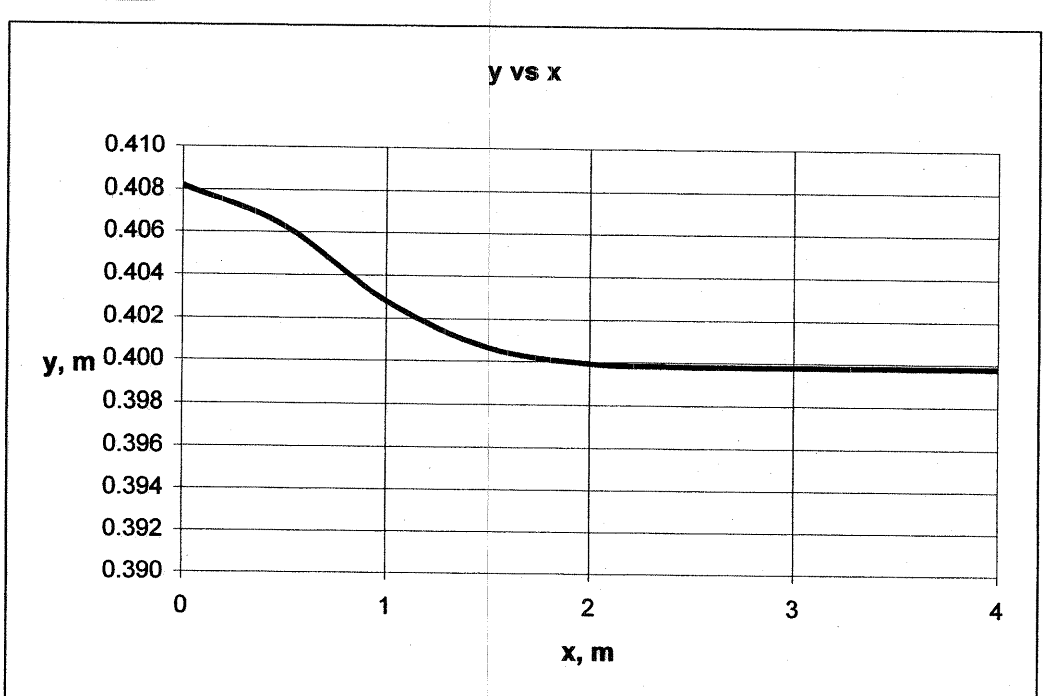
± x, m	y, m	z, m
0.0	0.408	0.608
0.5	0.406	0.562
1.0	0.403	0.476
1.5	0.401	0.422
2.0	0.400	0.404
2.5	0.400	0.400
3.0	0.400	0.400
3.5	0.400	0.400
4.0	0.400	0.400

(con't)

10.18* (con't)

The above results are plotted on the graph below.





10.19

10.19 Water in a rectangular channel flows into a gradual contraction section as is indicated in Fig. P10.19. If the flowrate is $Q = 25\ \text{ft}^3/\text{s}$ and the upstream depth is $y_1 = 2$ ft, determine the downstream depth, y_2.

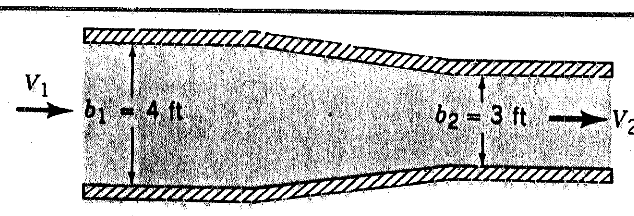
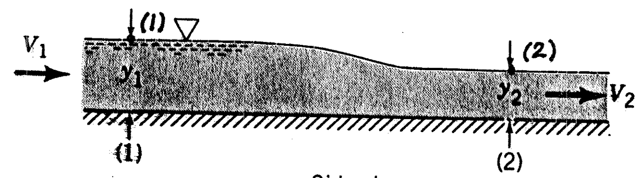


FIGURE P10.19 Top view



Side view

$$\frac{\cancel{p_1}}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{\cancel{p_2}}{\gamma} + \frac{V_2^2}{2g} + z_2 \text{, where } p_1 = p_2 = 0,\ z_1 = y_1 = 2\,\text{ft},\ z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25\,\frac{\text{ft}^3}{\text{s}}}{(4\,\text{ft})(2\,\text{ft})} = 3.13\,\frac{\text{ft}}{\text{s}}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{25\,\frac{\text{ft}^3}{\text{s}}}{(3\,\text{ft})\,y_2} = \frac{8.33}{y_2}$$

Thus,

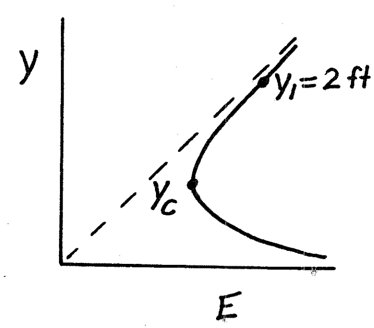
$$\frac{(3.13\,\frac{\text{ft}}{\text{s}})^2}{2(32.2\,\frac{\text{ft}}{\text{s}^2})} + 2\,\text{ft} = \frac{\left(\frac{8.33}{y_2}\right)^2}{2(32.2\,\frac{\text{ft}}{\text{s}^2})} + y_2$$

or

$$y_2^3 - 2.15\,y_2^2 + 1.077 = 0 \text{ which has roots } y_2 = 1.828,\ 0.946, \text{ and } -0.623 \qquad (1)$$

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{3.13\,\frac{\text{ft}}{\text{s}}}{\left[(32.2\,\frac{\text{ft}}{\text{s}^2})(2\,\text{ft})\right]^{\frac{1}{2}}} = 0.390 < 1$

Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 < 1$ also. Thus, it is not possible to have $y_2 = 0.946$



Thus, $\underline{\underline{y_2 = 1.828\,\text{ft}}}$

10.20

10.20 Sketch the specific energy diagram for the flow of Problem 10.19 and indicate its important characteristics. Note that $q_1 \neq q_2$.

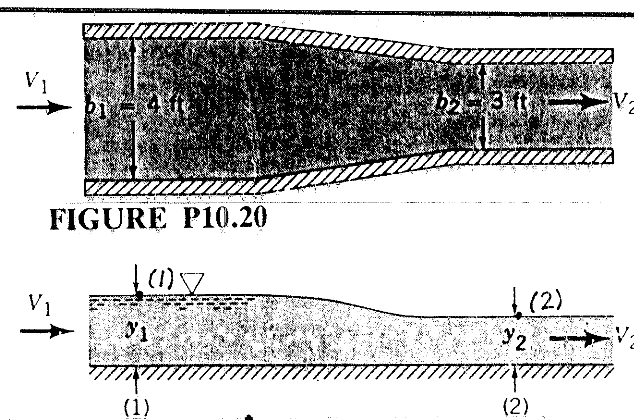


FIGURE P10.20

$$E = y + \frac{q^2}{2gy^2}$$

Thus, for the $b_1 = 4$ ft channel, $q_1 = \frac{Q}{b_1} = \frac{25\,\frac{ft^3}{s}}{4\,ft} = 6.25\,\frac{ft^2}{s}$

or

$$E = y + \frac{(6.25\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2} \quad \text{or} \quad E = y + \frac{0.607}{y^2} \tag{1}$$

For the $b_2 = 3$ ft channel, $q_2 = \frac{Q}{b_2} = \frac{25\,\frac{ft^3}{s}}{3\,ft} = 8.33\,\frac{ft^2}{s}$

or

$$E = y + \frac{(8.33\,\frac{ft^2}{s})^2}{2(32.2\,\frac{ft}{s^2})y^2} \quad \text{or} \quad E = y + \frac{1.077}{y^2} \tag{2}$$

Note: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ so that $y_{c_1} = \left(\frac{q_1^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(6.25\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right)^{\frac{1}{3}} = 1.067\,ft$

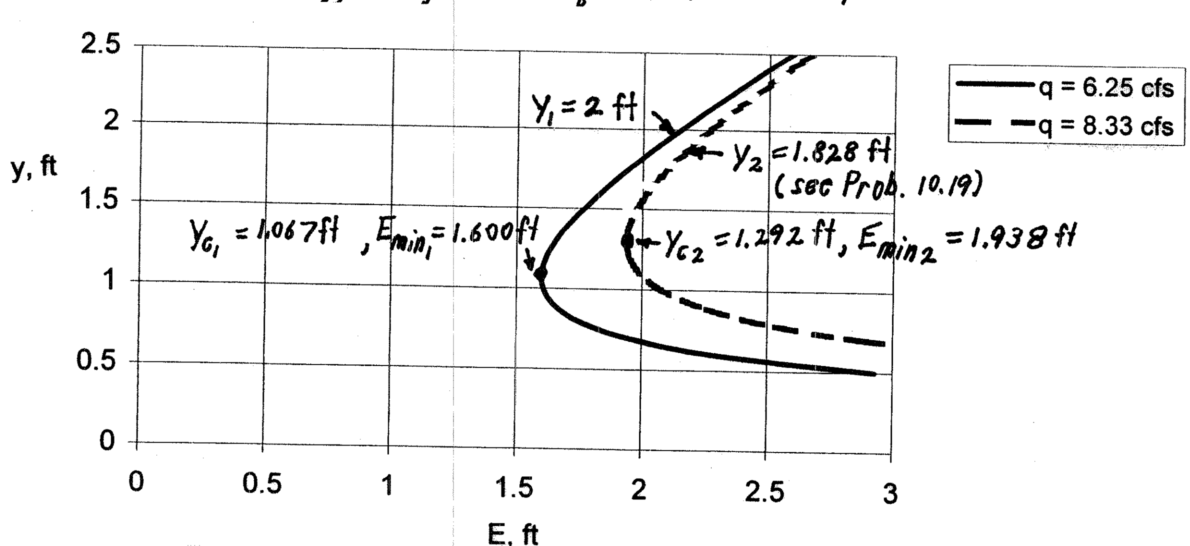
and

$$y_{c_2} = \left(\frac{q_2^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(8.33\,\frac{ft^2}{s})^2}{32.2\,\frac{ft}{s^2}}\right)^{\frac{1}{3}} = 1.292\,ft$$

Also, $E_{min} = \frac{3}{2}y_c$, or $E_{min_1} = \frac{3}{2}(1.067\,ft) = 1.600\,ft$

$$E_{min_2} = \frac{3}{2}(1.292\,ft) = 1.938\,ft$$

The specific energy diagrams (Eqs. (1) and (2) are plotted below:



Note: $E_1 = y_1 + \frac{V_1^2}{2g} = E_2 = y_2 + \frac{V_2^2}{2g} = 2\,ft + \frac{(3.13\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = 2.15\,ft$

10.21

10.21 Repeat Problem 10.19 if the upstream depth is $y_1 = 0.5$ ft. Assume that there are no losses between sections (1) and (2).

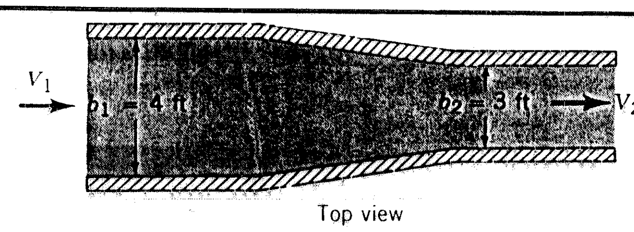
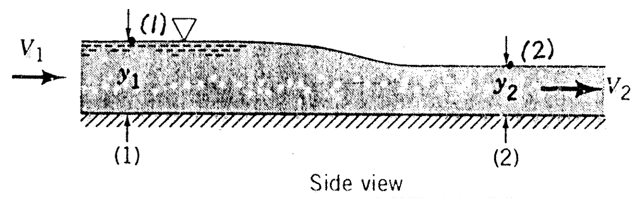


FIGURE P10.21



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1 = 0.5\,ft,\ z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25\,\frac{ft^3}{s}}{(4ft)(0.5ft)} = 12.5\,\frac{ft}{s}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{25\,\frac{ft^3}{s}}{(3ft)\,y_2} = \frac{8.33}{y_2}$$

Thus,

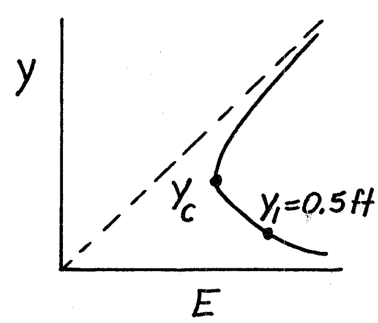
$$\frac{(12.5\,\frac{ft}{s})^2}{2\,(32.2\,\frac{ft}{s^2})} + 0.5\,ft = \frac{\left(\frac{8.33}{y_2}\right)^2}{2\,(32.2\,\frac{ft}{s^2})} + y_2$$

or

$y_2^3 - 2.93\,y_2^2 + 1.077 = 0$ which has roots $y_2 = 2.79,\ 0.694,$ and -0.555

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12.5\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(0.5ft)\right]^{1/2}} = 3.12 > 1$

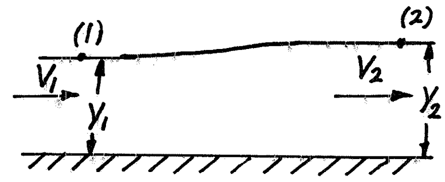
Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 > 1$ also. Thus, it is not possible to have $y_2 = 2.79$ (the subcritical root).



Thus, $y_2 = \underline{\underline{0.694\,ft}}$

10.22

10.22 Water flows in a rectangular channel with a flowrate per unit width of $q = 1.5 \text{ m}^2/\text{s}$ and a depth of 0.5 m at section (1). The head loss between sections (1) and (2) is 0.03 m. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram. Is it possible to have a head loss of 0.06 m? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L, \text{ where } p_1 = p_2 = 0,\ z_1 = 0.5\,m,\ z_2 = y_2, \quad (1)$$

$$V_1 = \frac{q}{y_1} = \frac{1.5\frac{m^2}{s}}{0.5\,m} = 3\frac{m}{s}, \text{ and } V_2 = \frac{q}{y_2} = \frac{1.5\frac{m^2}{s}}{y_2}$$

Thus, with $E = y + \frac{V^2}{2g}$ and $h_L = 0.03\,m$ Eq.(1) is

$$E_1 = E_2 + 0.03$$

Also, $E = y + \frac{q^2}{2g\,y^2} = y + \frac{(1.5\frac{m^2}{s})^2}{2(9.81\frac{m}{s^2})y^2}$

or

$$E = y + \frac{0.1146}{y^2} \quad (2)$$

Eq.(2) is plotted below.

Note: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(1.5\frac{m^2}{s})^2}{9.81\frac{m}{s^2}}\right)^{\frac{1}{3}} = 0.612\ m$

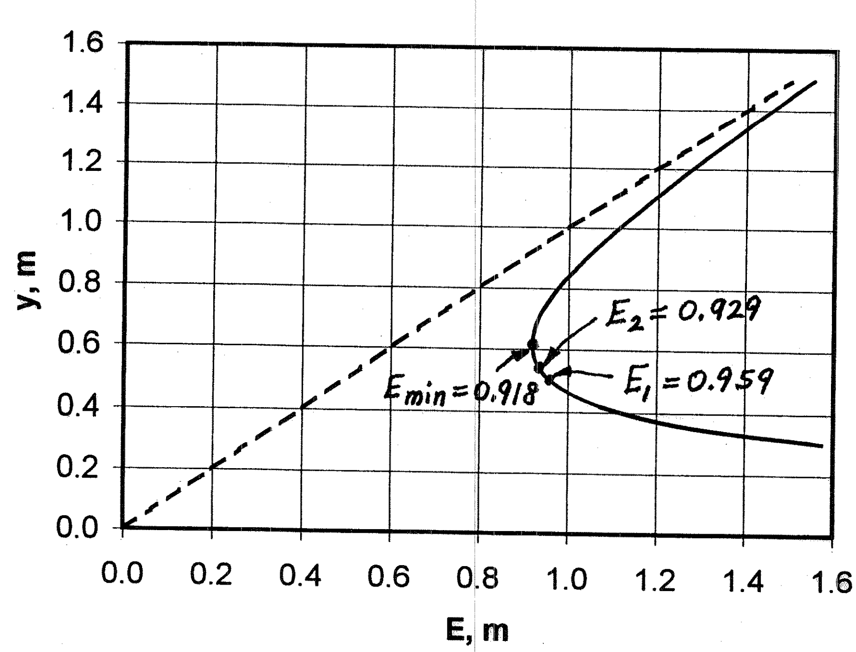
and

$$E_{min} = \frac{3}{2}y_c = \frac{3}{2}(0.612\,m) = 0.918\,m$$

Also,

$$E_1 = y_1 + \frac{q^2}{2g y_1^2} = 0.5 + \frac{(1.5\frac{m^2}{s})}{2(9.81\frac{m}{s^2})(0.5\,m)^2} = 0.959\,m$$

and $E_2 = E_1 - 0.03 = 0.929\,m$

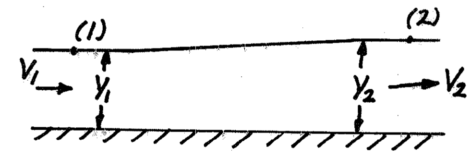


Note: If $h_L = 0.06\,m$ with $E_1 = 0.959\,m$ so that $E_2 = E_1 - 0.06$, then $E_2 = 0.899\ m < E_{min}$

Thus, it is not possible to have $h_L = 0.06$ with the given q and y_1.

10.23

10.23 Water flows in a horizontal rectangular channel with a flowrate per unit width of $q = 10$ ft²/s and a depth of 1.0 ft at the downstream section (2). The head loss between section (1) upstream and section (2) is 0.2 ft. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L, \text{ where } p_1=p_2=0,\ V_2=\frac{q}{y_2}=\frac{10\frac{ft^2}{s}}{1\,ft}=10\frac{ft}{s}, \quad (1)$$

and $y_2 = 1\,ft$

Thus, with $E = y + \frac{V^2}{2g}$

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(10\frac{ft^2}{s})^2}{2(32.2\frac{ft}{s^2})y^2}$$

or

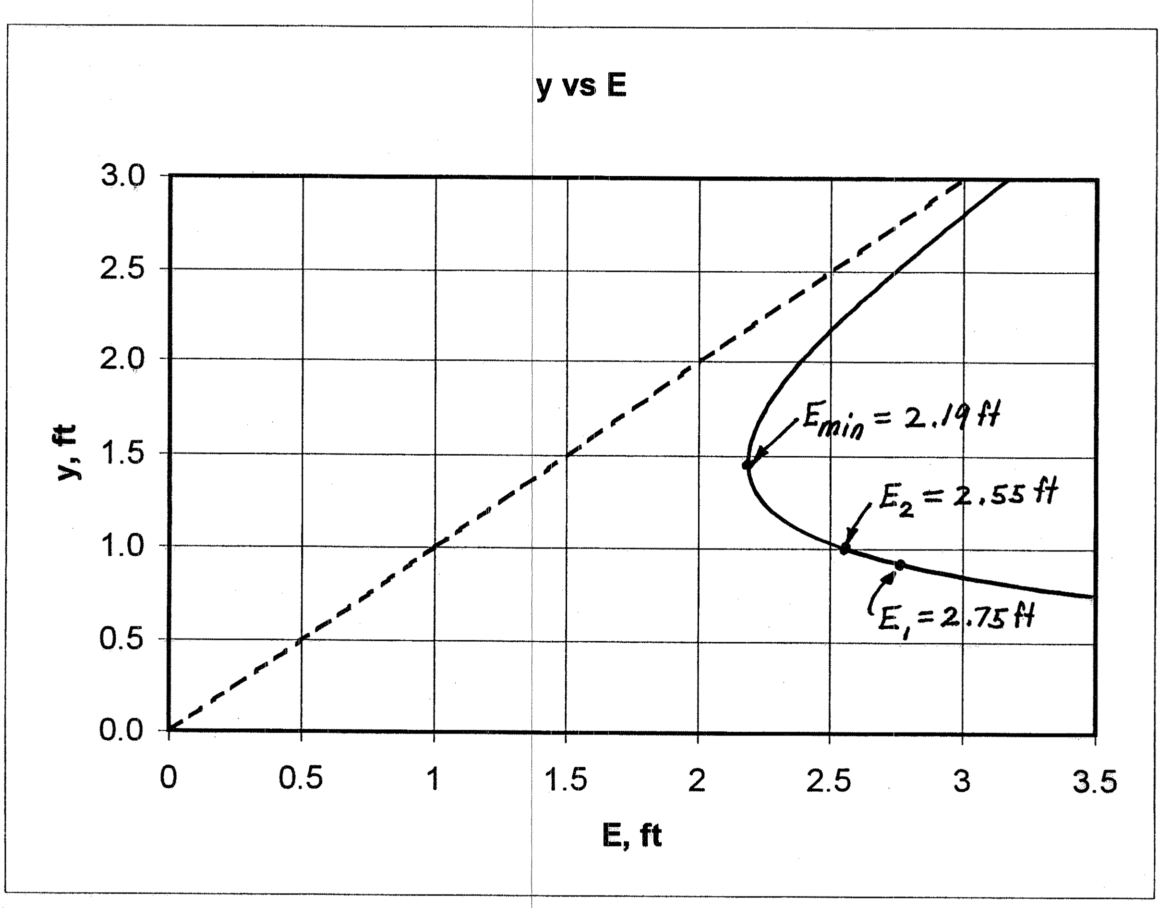
$$E = y + \frac{1.553}{y^2} \text{ where } E \sim ft,\ y \sim ft. \quad (2)$$

and Eq.(1) gives $E_1 = E_2 + h_L = E_2 + 0.2\,ft$

Eq.(2) is plotted below.

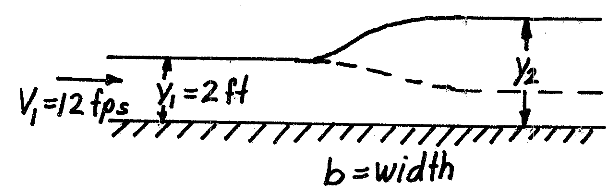
Note: $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(10\frac{ft^2}{s})^2}{32.2\frac{ft}{s^2}}\right)^{1/3} = 1.459\,ft$, $E_{min} = \frac{3}{2}y_c = \frac{3}{2}(1.459\,ft) = 2.19\,ft$,

$$E_2 = y_2 + \frac{1.553}{y_2^2} = 1 + \frac{1.553}{1^2} = 2.55\,ft, \text{ and } E_1 = E_2 + h_L = 2.75\,ft$$



10.24

10.24 Water flows in a horizontal, rectangular channel with an initial depth of 2 ft and initial velocity of 12 ft/s. Determine the depth downstream if losses are negligible. Note that there may be more than one solution. Repeat the problem if the initial depth remains the same, but the initial velocity is 6 ft/s.



$E_1 = E_2$, or $y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$ (1)

but $Q_1 = Q_2$ or $V_1 b y_1 = V_2 b y_2$

so that

$$V_2 = \frac{V_1 y_1}{y_2} = \frac{(12\,\frac{ft}{s})(2\,ft)}{y_2} = \frac{24}{y_2}\,\frac{ft}{s}\text{, where } y_2 \sim ft$$

Thus, Eq. (1) becomes

$$2\,ft + \frac{(12\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})} = y_2 + \frac{(\frac{24}{y_2}\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$y_2^3 - 4.24\,y_2^2 + 8.94 = 0$ which has 3 roots; one negative (no physical meaning), one is $\underline{\underline{y_2 = 2\,ft}}$ (no change in depth), and $\underline{\underline{y_2 = 3.51\,ft}}$ (an increase in depth).

If $V_1 = 6\,\frac{ft}{s}$, then $V_2 = \frac{(6\,\frac{ft}{s})(2\,ft)}{y_2} = \frac{12}{y_2}$

and Eq. (1) becomes

$$2 + \frac{6^2}{2(32.2)} = y_2 + \frac{(\frac{12}{y_2})^2}{2(32.2)}$$

or

$y_2^3 - 2.56\,y_2^2 + 2.24 = 0$ The positive real roots are $\underline{\underline{y_2 = 2\,ft}}$, or $\underline{\underline{y_2 = 1.38\,ft}}$ (a decrease in depth)

10.25

10.25 A smooth transition section connects two rectangular channels as shown in Fig. P10.25. The channel width increases from 6.0 to 7.0 ft and the water surface elevation is the same in each channel. If the upstream depth of flow is 3.0 ft, determine h, the amount the channel bed needs to be raised across the transition section to maintain the same surface elevation.

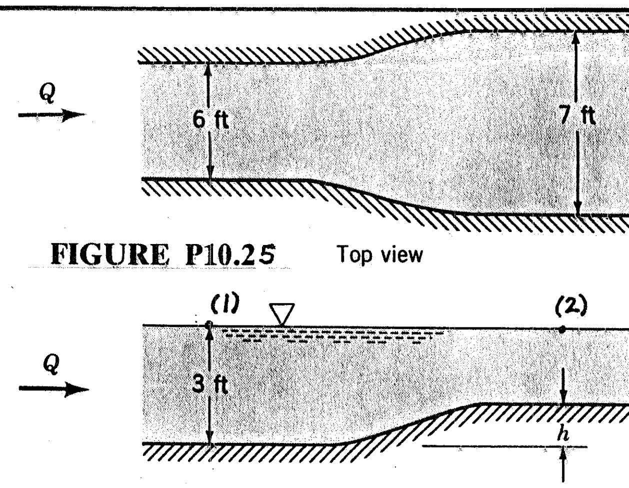


FIGURE P10.25

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\text{, where } p_1=p_2=0 \text{ and } z_1=z_2$$

Thus, $V_1=V_2$ or

$$\frac{Q}{A_1}=\frac{Q}{A_2} \quad \text{Hence, } A_1=A_2 \text{ or } (6\,ft)(3\,ft)=(7\,ft)(3\,ft-h)$$

or

$$h=\underline{\underline{0.429\ ft}}$$

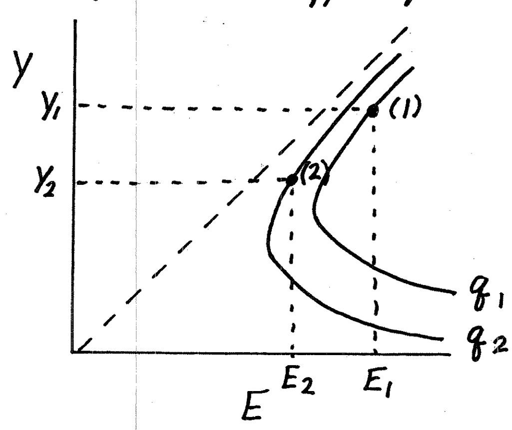
Note: $q_1=\frac{Q}{b_1}=\frac{Q}{6}$ and $q_2=\frac{Q}{b_2}=\frac{Q}{7}<q_1$

and

$$E_1=y_1+\frac{V_1^2}{2g} \text{ and } E_2=y_2+\frac{V_2^2}{2g}$$ Thus, since $V_1=V_2$ it follows that

$$E_1-E_2=y_1-y_2$$

The corresponding specific energy diagram is as indicated below:



10.26

10.26 Water flows over a bump of height $h = h(x)$ on the bottom of a wide rectangular channel as is indicated in Fig. P10.26. If energy losses are negligible, show that the slope of the water surface is given by $dy/dx = -(dh/dx)/[1 - (V^2/gy)]$, where $V = V(x)$ and $y = y(x)$ are the local velocity and depth of flow. Comment on the sign (i.e., <0, $=0$, or >0) of dy/dx relative to the sign of dh/dx.

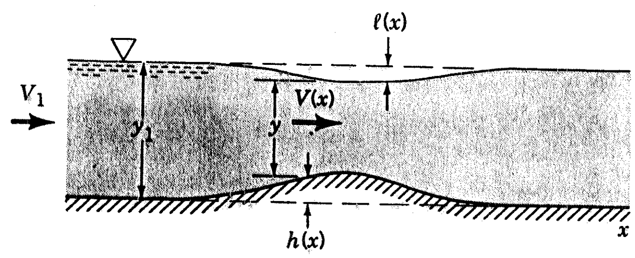


FIGURE P10.26

For any two points on the free surface:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0,\ z_1 = y_1, \text{ and } z_2 = h + y_2$$

Thus, $\frac{V^2}{2g} + h + y = \text{constant}$ so that by differentiating

$$\frac{2V\frac{dV}{dx}}{2g} + \frac{dh}{dx} + \frac{dy}{dx} = 0 \qquad (1)$$

Also, for conservation of mass

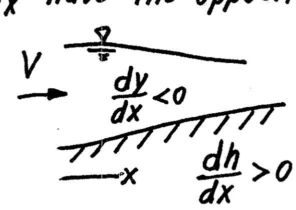
$$V_1 y_1 = Vy \text{ or } V\frac{dy}{dx} + y\frac{dV}{dx} = 0 \text{ or } \frac{dV}{dx} = -\frac{V}{y}\frac{dy}{dx} \qquad (2)$$

Combine Eqs. (1) and (2):

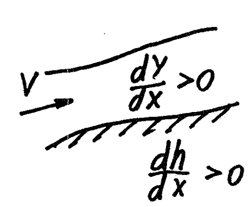
$$\frac{V}{g}\left(-\frac{V}{y}\frac{dy}{dx}\right) + \frac{dh}{dx} + \frac{dy}{dx} = 0, \text{ or } \underline{\underline{\frac{dy}{dx} = \frac{-\left(\frac{dh}{dx}\right)}{\left(1 - \left(\frac{V^2}{gy}\right)\right)}}}$$

Note: If $Fr = \frac{V}{\sqrt{gy}} < 1$, then $\frac{dh}{dx}$ and $\frac{dy}{dx}$ have the opposite sign

If $Fr > 1$, then $\frac{dh}{dx}$ and $\frac{dy}{dx}$ have the same sign.



$Fr < 1$



$Fr > 1$

10.27

10.27 Integrate the differential equation obtained in Problem 10.26 to determine the "draw-down" distance, $\ell = \ell(x)$, indicated in Fig. P10.26. Comment on your results.

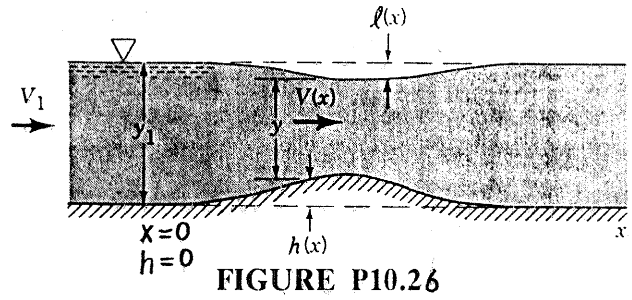


FIGURE P10.26

From Problem 10.26:

$$\frac{dy}{dx} = \frac{-(\frac{dh}{dx})}{(1-(\frac{V^2}{gy}))}, \quad \text{where } V_1 y_1 = Vy, \text{ or } V = \frac{V_1 y_1}{y}$$

Thus, $\frac{V^2}{gy} = \frac{(\frac{V_1 y_1}{y})^2}{gy} = \frac{V_1^2 y_1^2}{gy^3}$ so that

$$\frac{dy}{dx} = \frac{-(\frac{dh}{dx})}{[1-\frac{V_1^2 y_1^2}{gy^3}]}, \text{ or } [1-(\frac{V_1^2 y_1^2}{g})y^{-3}]dy = -(\frac{dh}{dx})dx$$

Integrate from $y = y_1$ and $x=0$, with $\frac{dh}{dx}$ a given function of x.

$$\int_{y=y_1}^{y}[1-(\frac{V_1^2 y_1^2}{g})y^{-3}]dy = -\int_{x=0}^{x}(\frac{dh}{dx})dx = -\int_{h=0}^{h} dh = -h$$

or

$$\left[y-(\frac{V_1^2 y_1^2}{g})(-\frac{1}{2})y^{-2}\right]_{y_1}^{y} = -h \quad \text{Thus, } y+(\frac{V_1^2 y_1^2}{2g})y^{-2} - y_1 - \frac{V_1^2}{2g} = -h$$

or

$$y^3 - (y_1 + \frac{V_1^2}{2g} - h)y^2 + (\frac{V_1^2 y_1^2}{2g}) = 0 \qquad (1)$$

Obtain $y = y(x)$ from Eq. (1) and then $\ell = \ell(x)$ from $y_1 = h + y + \ell$ or $\ell = y_1 - h - y$

Note: Eq. (1) is nothing more than the Bernoulli equation:

$$\frac{V_1^2}{2g} + y_1 = \frac{V^2}{2g} + y + h \quad \text{with } V = \frac{V_1 y_1}{y} \text{ so that}$$

$$\frac{V_1^2}{2g} + y_1 = \frac{(\frac{V_1 y_1}{y})^2}{2g} + y + h \quad \text{which simplifies to Eq. (1).}$$

10.28

10.28 Determine the minimum depth in a 3-m-wide rectangular channel if the flow is to be subcritical with a flowrate of $Q = 60\ m^3/s$.

$$V = \frac{Q}{A} = \frac{60\ \frac{m^3}{s}}{(3m)\, y} = \frac{20}{y}, \text{ where } V \sim \frac{m}{s} \text{ when } y = \text{depth} \sim m$$

Also,

$$Fr = \frac{V}{\sqrt{gy}} = \frac{\left(\frac{20}{y}\ \frac{m}{s}\right)}{\left[\left(9.81\ \frac{m}{s^2}\right) y\right]^{1/2}} = \frac{6.39}{y^{3/2}}$$

Note: As y decreases, Fr increases

Thus, to have $Fr < 1$ we must have $\frac{6.39}{y^{3/2}} < 1$, or

$$y > (6.39)^{2/3} = \underline{\underline{3.44m}}$$

10.29

10.29 Fluid properties such as viscosity or density do not appear in the Manning equation (Eq. 10.20). Does this mean that this equation is valid for any open-channel flow such as that involving mercury, water, oil, or molasses? Explain.

The Manning equation, $Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, was "derived" specifically for water. It is not in dimensionless form and cannot be use without alteration (i.e. different n values; different dependance on R_h; etc) for other fluids.

10.30

10.30 The following data are taken from measurements on a river: $A = 200 \text{ ft}^2$, $P = 80 \text{ ft}$, and $S_0 = 0.015 \text{ ft}/50 \text{ ft}$. Determine the average shear stress on the wetted perimeter of this channel.

$\tau_w = \gamma R_h S_o$, where $R_h = \frac{A}{P} = \frac{200 \text{ ft}^2}{80 \text{ ft}} = 2.50 \text{ ft}$

and

$$S_o = \frac{0.015 \text{ ft}}{50 \text{ ft}} = 0.00030$$

Thus,

$$\tau_w = 62.4 \frac{\text{lb}}{\text{ft}^3} (2.50 \text{ ft})(0.00030) = \underline{\underline{0.0468 \frac{\text{lb}}{\text{ft}^2}}}$$

10.31

10.31 Water flows along the curbing of a street as is shown in Fig. P10.31. If the slope of the street is $S_0 = 0.01$ would you expect the flow to be laminar or turbulent? Explain.

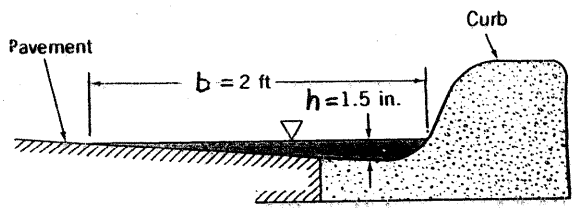


FIGURE P10.31

$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1 (assuming finished concrete) $n = 0.012$ (1)

Also, $R_h = \frac{A}{P}$ where $A \approx \frac{1}{2} bh = \frac{1}{2}(2\,ft)(\frac{1.5}{12}\,ft) = 0.125\,ft^2$
and $P \approx 2\,ft$

Thus,
$R_h \approx \frac{0.125\,ft^2}{2\,ft} = 0.0625\,ft$ so that from Eq. (1)

$$V = \frac{1.49}{0.012}(0.0625)^{2/3}(0.01)^{1/2} = 1.96\,\frac{ft}{s}$$

Therefore,

$$Re = \frac{VR_h}{\nu} = \frac{(1.96\,\frac{ft}{s})(0.0625\,ft)}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 10{,}100$$

Is the flow laminar because the Reynolds number is less than 12,500, the approximate critical value for open channel flow?

Approximate values for laminar or turbulent channel flow are as follows: laminar if $Re_h < 500$; turbulent if $Re_h > 12{,}500$. With $Re_h = 10{,}100$ for the present case it is not clear if the flow is laminar or turbulent — <u>probably turbulent.</u>

10.32

10.32 The following data are obtained for a particular reach of the Provo River in Utah: A = 183 ft^2, free-surface width = 55 ft, average depth = 3.3 ft, R_h = 3.22 ft, V = 6.56 ft/s, length of reach = 116 ft, and elevation drop of reach = 1.04 ft. Determine the **(a)** average shear stress on the wetted perimeter, **(b)** the Manning coefficient, n, and **(c)** the Froude number of the flow.

a) $\tau_w = \gamma R_h S_o$, where $S_o = \frac{1.04\,ft}{116\,ft} = 0.00897$

Thus, $\tau_w = (62.4\,\frac{lb}{ft^3})(3.22\,ft)(0.00897) = \underline{\underline{1.80\,\frac{lb}{ft^2}}}$

b) $Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2} = AV$, where $K = 1.49$

Thus, $n = \frac{1.49\, R_h^{2/3} S_o^{1/2}}{V} = \frac{(1.49)(3.22)^{2/3}(0.00897)^{1/2}}{6.56} = \underline{\underline{0.0469}}$

c) $Fr = \frac{V}{\sqrt{gy}} = \frac{6.56\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s})(3.3\,ft)\right]^{1/2}} = \underline{\underline{0.636}}$ < 1 (subcritical)

10.33

10.33 By what percent is the flowrate reduced in the rectangular channel shown in Fig. P10.33 because of the addition of the thin center board? All surfaces are of the same material.

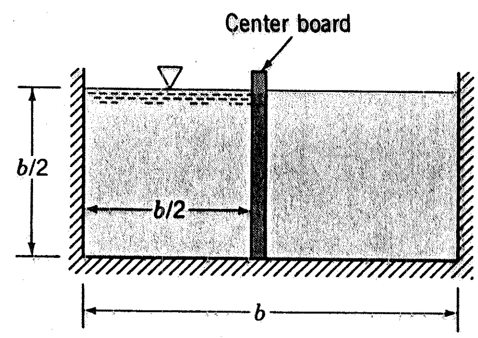


FIGURE P10.33

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$$

Without the centerboard $A = b\left(\frac{b}{2}\right) = \frac{b^2}{2}$, $R_h = \frac{A}{P} = \frac{\frac{b^2}{2}}{2b} = \frac{b}{4}$

or

$$Q_{without} = \frac{K}{n}\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3} S_o^{1/2} \qquad (1)$$

With the centerboard $Q_{with} = 2Q_2$, where $A_2 = \left(\frac{b}{2}\right)^2$,

$$R_{h_2} = \frac{A_2}{P_2} = \frac{\left(\frac{b}{2}\right)^2}{3\left(\frac{b}{2}\right)} = \frac{b}{6}$$

or

$$Q_{with} = 2\frac{K}{n}\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3} S_o^{1/2} \qquad (2)$$

Divide Eq. (2) by Eq. (1) to obtain $\frac{Q_{with}}{Q_{without}} = \frac{2\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3}}{\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3}} = 0.763$

a $100 - 76.3 = \underline{\underline{23.7\ \%\ \text{reduction}}}$

10.34

10.34 An old, rough-surfaced, 2-m-diameter concrete pipe with a Manning coefficient of 0.025 carries water at a rate of 5.0 m³/s when it is half full. It is to be replaced by a new pipe with a Manning coefficient of 0.012 that is also to flow half full at the same flowrate. Determine the diameter of the new pipe.

$$Q_{old} = \frac{\kappa}{n_{old}} A_{old} R_{h_{old}}^{2/3} \sqrt{S_{o_{old}}} \qquad (1)$$

and

$$Q_{new} = \frac{\kappa}{n_{new}} A_{new} R_{h_{new}}^{2/3} \sqrt{S_{o_{new}}} \qquad (2)$$

where $Q_{old} = Q_{new}$ and $S_{o_{old}} = S_{o_{new}}$

Thus, by equating Eqs. (1) and (2),

$$\frac{A_{old} R_{h_{old}}^{2/3}}{n_{old}} = \frac{A_{new} R_{h_{new}}^{2/3}}{n_{new}} \qquad (3)$$

But for a half full pipe, $A = \frac{\pi}{8} D^2$ and $R_h = \frac{A}{P} = \frac{\frac{\pi}{8} D^2}{\frac{\pi}{2} D} = \frac{D}{4}$

Thus,

$A R_h^{2/3} = \frac{\pi}{8} D^2 \left(\frac{D}{4}\right)^{2/3}$ so that Eq. (3) becomes

$$\frac{\frac{\pi}{8} D_{old}^2 \left(\frac{D_{old}}{4}\right)^{2/3}}{n_{old}} = \frac{\frac{\pi}{8} D_{new}^2 \left(\frac{D_{new}}{4}\right)^{2/3}}{n_{new}}$$

or

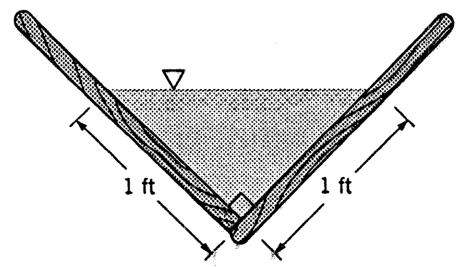
$$\frac{D_{old}^{8/3}}{n_{old}} = \frac{D_{new}^{8/3}}{n_{new}}$$

Thus,

$$D_{new} = \left(\frac{n_{new}}{n_{old}}\right)^{3/8} D_{old} = \left(\frac{0.012}{0.025}\right)^{3/8} (2\ m) = \underline{\underline{1.52\ m}}$$

10.35

10.35 The great Kings River flume in Fresno County, California, was used from 1890 to 1923 to carry logs from an elevation of 4500 ft where trees were cut to an elevation of 300 ft at the railhead. The flume was 54 miles long, constructed of wood, and had a V-cross section as indicated in Fig. P10.35. It is claimed that logs would travel the length of the flume in 15 hours. Do you agree with this claim? Provide appropriate calculations to support your answer.



■ FIGURE P10.35

ℓ = distance traveled $= V_{log}\, t$. Thus,

$$V_{log} = \frac{\ell}{t} = \frac{(54\,mi)(5280\,ft/mi)}{(15\,hr)(3600\,s/hr)} = 5.28\,\frac{ft}{s}$$

Determine the average water velocity, V, and compare it with this V_{log}.

$V = \frac{K}{n} R_h^{2/3} \sqrt{S_o}$, where $K = 1.49$, $A = \frac{1}{2}(1\,ft^2) = 0.5\,ft^2$, $P = 2\,ft$

so that $R_h = \frac{A}{P} = \frac{0.5\,ft^2}{2\,ft} = 0.25\,ft$

Also,

$$S_o = \frac{\Delta z}{\ell} = \frac{(4500-300)\,ft}{(54\,mi)(5280\,ft/mi)} = 0.0147$$

Thus, with $n = 0.012$ (see Table 10.1, planed wood),

$$V = \frac{1.49}{0.012}(0.25)^{2/3}\sqrt{0.0147} = 5.97\,\frac{ft}{s}$$

Note: V is slightly larger than V_{log}. Thus, the claim appears to be correct. <u>Yes.</u>

10.36

10.36 Water flows in a river with a speed of 3 ft/s. The river is a clean, straight natural channel, 400 ft wide with a nearly uniform 3-ft depth. Is the slope of this river greater than or less than the average slope of the Mississippi River which drops a distance of 1470 ft in its 2350-mi length? Support your answer with appropriate calculations.

(1) $V = \frac{K}{n} R_h^{2/3} \sqrt{S_o}$, where $K = 1.49$,

$V = 3 \text{ ft/s}$, $y = 3 \text{ ft}$, $b = 400 \text{ ft}$, $A = by = 1200 \text{ ft}^2$, $P = b + 2y = 406 \text{ ft}$

Thus,

$$R_h = \frac{A}{P} = \frac{1200 \text{ ft}^2}{406 \text{ ft}} = 2.96 \text{ ft}$$

Also, from Table 10.1, $n = 0.03$ so that from Eq. (1):

$$3 = \frac{1.49}{0.03} (2.96)^{2/3} \sqrt{S_o}$$

or

$$S_o = 0.000858$$

The average Mississippi slope is

$$S_{o\,Miss} = \frac{1470 \text{ ft}}{2350 \text{ mi} \left(5280 \frac{\text{ft}}{\text{mi}}\right)} = 0.000118 < 0.000858$$

The unknown river has a <u>greater</u> slope.

10.37

10.37 At a particular location the cross section of the Columbia River is as indicated in Fig. P10.37. If on a day without wind it takes 5 min to float 0.5 mi along the river, which drops 0.46 ft in that distance, determine the value of the Manning coefficient, n.

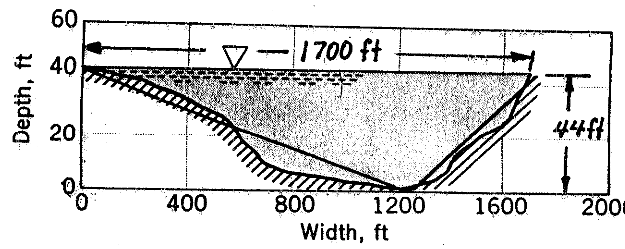


FIGURE P10.37

From the given data, $V = \dfrac{(0.5\,mi)(5280\,\frac{ft}{mi})}{(5\,min)(60\,\frac{s}{min})} = 8.8\,\frac{ft}{s}$.

From the Manning equation,

$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \dfrac{0.46\,ft}{(0.5\,mi)(5280\,\frac{ft}{mi})} = 0.000174$, (1)

and $R_h = \frac{A}{P}$.

Approximate A and P from the figure as

$A \approx \frac{1}{2} b y = \frac{1}{2}(1700\,ft)(44\,ft) = 37{,}400\,ft^2$

and

$P \approx 1800\,ft$ Thus, $R_h \approx \dfrac{37{,}400\,ft^2}{1800\,ft} = 20.8\,ft$

Hence, from Eq. (1):

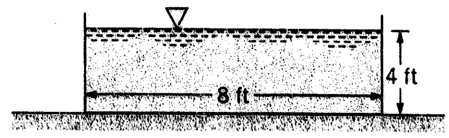
$8.8 = \frac{1.49}{n}(20.8)^{2/3}(0.000174)^{1/2}$

or

$n = \underline{\underline{0.0169}}$

10.38

10.38 The rectangular irrigation canal shown in Fig. P10.38 carries 100 ft^3/s during normal operating conditions when the water depth is 4 ft as shown. Determine the flowrate during a drought when the water depth is only 1 ft.



■ FIGURE P10.38

$$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_o} \qquad (1)$$

Under normal conditions $A = (4\,ft)(8\,ft) = 32\,ft^2$, $P = 8\,ft + 2(4\,ft) = 16\,ft$, and $R_h = A/P = 32\,ft^2/16\,ft = 2\,ft$

Thus, from Eq. (1)

$100 = \frac{\kappa}{n}(32)(2^{2/3})\sqrt{S_o}$

or

$$\frac{\kappa}{n}\sqrt{S_o} = 1.97 \qquad (2)$$

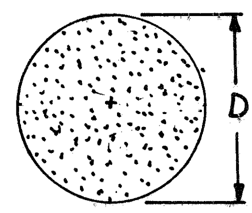
During a drought, $A = (1\,ft)(8\,ft) = 8\,ft^2$, $P = 8\,ft + 2(1\,ft) = 10\,ft$, and $R_h = A/P = 8\,ft^2/10\,ft = 0.8\,ft$

Hence, from Eqs. (1) and (2),

$$Q = \frac{\kappa}{n}\sqrt{S_o}\, A R_h^{2/3} = 1.97(8)(0.8)^{2/3} = \underline{\underline{13.6\,ft^3/s}}$$

10.39

10.39 Rainwater runoff from a 200-ft by 500-ft parking lot is to drain through a circular concrete pipe that is laid on a slope of 3 ft/mi. Determine the pipe diameter if it is to be full with a steady rainfall of 1.5 in./hr.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \frac{3\text{ ft}}{5280\text{ ft}} = 0.000568$, (1)

$A = \frac{\pi}{4} D^2$ and $R_h = \frac{A}{P} = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$

From Table 10.1, $n = 0.012$

Also, $Q = A_{lot}\, r$, where r = rainfall rate $= 1.5 \frac{\text{in.}}{\text{hr}}$

Thus,

$$Q = (200\text{ft})(500\text{ft})(1.5 \tfrac{\text{in.}}{\text{hr}})(\tfrac{1}{12} \tfrac{\text{ft}}{\text{in.}})(\tfrac{1\text{ hr}}{3600\text{s}}) = 3.47 \tfrac{\text{ft}^3}{\text{s}}$$

Hence, from Eq. (1):

$$3.47 = \frac{1.49}{0.012} \left(\frac{\pi}{4} D^2\right) \left(\frac{D}{4}\right)^{2/3} (0.000568)^{1/2}$$

or

$D = \underline{\underline{1.64\text{ft}}}$

10.40

10.40 To prevent weeds from growing in a clean earthen-lined canal, it is recommended that the velocity be no less than 2.5 ft/s. For the symmetrical canal shown in Fig. P10.40, determine the minimum slope needed.

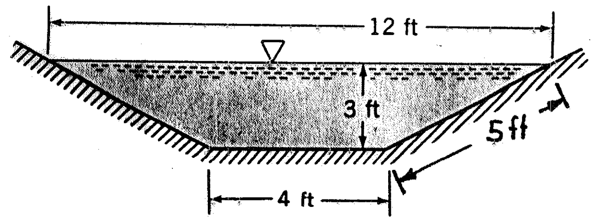


FIGURE P10.40

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \qquad (1)$$

$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2$ and $P = 4\,ft + 2(5\,ft) = 14\,ft$

Thus, $R_h = \frac{24\,ft^2}{14\,ft} = 1.714\,ft$

From Table 10.1, $n = 0.022$ so that Eq.(1) gives (with $V = 2.5\,\frac{ft}{s}$)

$2.5 = \frac{1.49}{0.022}(1.714)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.000664}}$

10.41

10.41 The smooth concrete-lined symmetrical channel shown in Video V10.3 and Fig. P10.40 carries water from the silt-laden Colorado River. If the velocity must be 4.0 ft/s to prevent the silt from settling out (and eventually clogging the channel), determine the minimum slope needed.

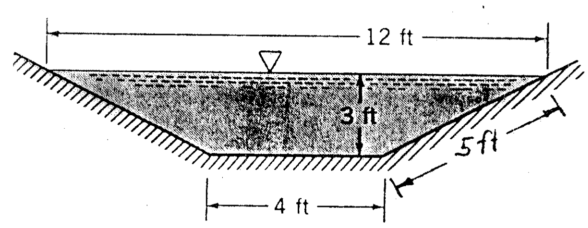


FIGURE P10.40

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \qquad (1)$$

$A = \frac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2$ and $P = 4\,ft + 2(5\,ft) = 14\,ft$

Thus, $R_h = \frac{24\,ft^2}{14\,ft} = 1.714\,ft$

From Table 10.1, $n = 0.012$ so that Eq.(1) gives (with $V = 4\,\frac{ft}{s}$)

$4.0 = \frac{1.49}{0.012}(1.714)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.000505}}$

10.42

10.42 The symmetrical channel shown in Fig. P10.40 is dug in sandy loam soil with $n = 0.020$. For such surface material it is recommended that to prevent scouring of the surface the average velocity be no more than 1.75 ft/s. Determine the maximum slope allowed.

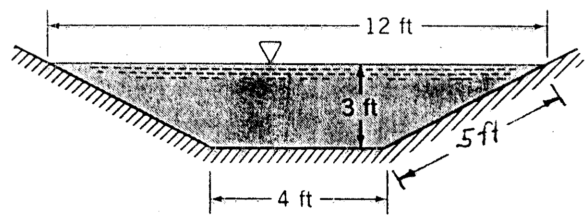


FIGURE P10.40

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and } R_h = \frac{A}{P} \qquad (1)$$

$$A = \tfrac{1}{2}(4\,ft + 12\,ft)(3\,ft) = 24\,ft^2 \text{ and } P = 4\,ft + 2(5\,ft) = 14\,ft$$

Thus, $R_h = \dfrac{24\,ft^2}{14\,ft} = 1.714\,ft$

With $n = 0.020$ and $V = 1.75\,\frac{ft}{s}$ Eq.(1) gives

$$1.75 = \frac{1.49}{0.020}(1.714)^{2/3} S_o^{1/2} \text{ or } S_o = \underline{\underline{0.000269}}$$

10.43

10.43 The flowrate in the clay-lined channel ($n = 0.025$) shown in Fig. P10.43 is to be 300 ft³/s. To prevent erosion of the sides, the velocity must not exceed 5 ft/s. For this maximum velocity, determine the width of the bottom, b, and the slope, S_0.

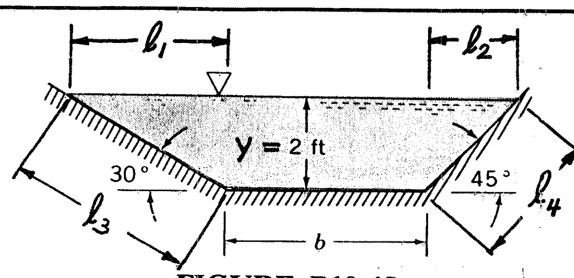


FIGURE P10.43

$$V = \frac{Q}{A}, \text{ where } A = \tfrac{1}{2}[b + (b + \ell_1 + \ell_2)]y \text{ with } \ell_1 = \frac{2\,ft}{\tan 30^\circ} = 3.46\,ft \qquad (1)$$

and $\ell_2 = \dfrac{2\,ft}{\tan 45^\circ} = 2\,ft$

$$\text{Thus, } 5\,\frac{ft}{s} = \frac{300\,\frac{ft^3}{s}}{\frac{1}{2}[b + (b + 3.46\,ft + 2\,ft)](2\,ft)}, \text{ or } b = \underline{\underline{27.3\,ft}}$$

Also, $V = \dfrac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1, $n = 0.025$ (2)

From Eq.(1), $A = \tfrac{1}{2}[2(27.3\,ft) + 3.46\,ft + 2\,ft](2\,ft) = 60.0\,ft^2$

Also,

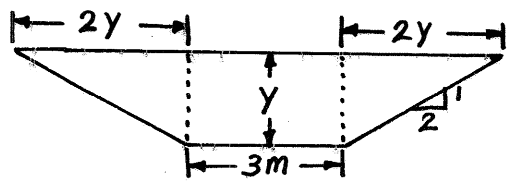
$$P = b + \ell_3 + \ell_4 = 27.3\,ft + \frac{2\,ft}{\sin 30^\circ} + \frac{2\,ft}{\sin 45^\circ} = 34.1\,ft$$

Thus, $R_h = \dfrac{A}{P} = \dfrac{60.0\,ft^2}{34.1\,ft} = 1.76\,ft$ so that Eq.(2) becomes

$$5 = \frac{1.49}{0.025}(1.76)^{2/3} S_o^{1/2}, \text{ or } S_o = \underline{\underline{0.00331}}$$

10.44

10.44 A trapezoidal channel with a bottom width of 3.0 m and sides with a slope of 2:1 (horizontal:vertical) is lined with fine gravel (n = 0.020) and is to carry 10 m³/s. Can this channel be built with a slope of S_0 = 0.00010 if it is necessary to keep the velocity below 0.75 m/s to prevent scouring of the bottom? Explain.



Determine V with $Q = 10\,\frac{m^3}{s}$ and $S_0 = 0.00010$.

$$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_0}\ , \text{ where } A = \tfrac{1}{2} y\left[3 + (3+4y)\right] = 2y^2 + 3y \qquad (0)$$

and $R_h = \frac{A}{P}$, with $P = 3 + 2(\sqrt{5}\,y)$

Thus,

$$10 = \frac{1}{0.02}(2y^2+3y)\left[\frac{2y^2+3y}{3+2\sqrt{5}\,y}\right]^{2/3}(0.0001)^{1/2}$$

or

$$20 = \frac{(2y^2+3y)^{5/3}}{(3+2\sqrt{5}\,y)^{2/3}} \quad \text{which can be written as}$$

$$2y^2 + 3y - 6.03\,(3+2\sqrt{5}\,y)^{0.4} = 0 \qquad (1)$$

A standard root-finding computer program gives the solution to Eq. (1) as $y = 2.25\,m$

Hence, from Eq. (0) $A = 2(2.25)^2 + 3(2.25) = 16.9\ m^2$

so that

$$V = \frac{Q}{A} = \frac{10\,\frac{m^3}{s}}{16.9\ m^2} = 0.592\ \frac{m}{s}$$

Thus, $V < 0.75\,\frac{m}{s}$ so that <u>scouring will not occur.</u>

10.45

10.45 Water flows in a 2-m-diameter finished concrete pipe so that it is completely full and the pressure is constant all along the pipe. If the slope is $S_0 = 0.005$, determine the flowrate by using open-channel flow methods. Compare this result with that obtained by using pipe flow methods of Chapter 8.

For open channel flow $Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1$

Also, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(2m)^2 = 3.14\,m^2$ and $P = \pi D = 6.28\,m$ so that

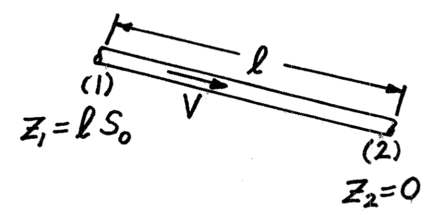
$$R_h = \frac{A}{P} = \frac{3.14\,m^2}{6.28\,m} = 0.5\,m$$

Hence, with $n = 0.012$ for finished concrete (see Table 10.1)

$$Q = \frac{1}{0.012}(3.14)(0.5)^{2/3}(0.005)^{1/2} = \underline{\underline{11.7\,\frac{m^3}{s}}} \text{ (open channel)}$$

For pipe flow with constant pressure:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}$$



where $p_1 = p_2$ and $V_1 = V_2$

Thus, with $z_1 - z_2 = \ell S_o$,

$$\ell S_o = f\frac{\ell}{D}\frac{V^2}{2g}$$

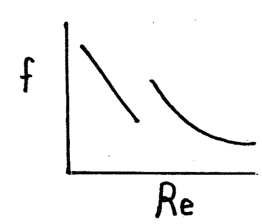
or

$$fV^2 = 2gDS_o = 2(9.81\tfrac{m}{s^2})(2m)(0.005) \quad \text{Thus, } fV^2 = 0.196 \qquad (1)$$

From Table 8.1, for smooth concrete $\frac{\varepsilon}{D} = 0.3\times10^{-3}\,m/2\,m = 1.5\times10^{-4}$

Also, $Re = \frac{VD}{\nu} = \frac{V(2m)}{1.12\times10^{-6}\frac{m^2}{s}} = 1.79\times10^6 V$ (2)

and from the Moody chart (Fig. 8.20):



(3)

Solve Eqs. (1), (2), and (3) for f, V, Re:

Assume $f = 0.015$ so that from Eq. (1)

$$V = \left[\frac{0.196}{0.015}\right]^{1/2} = 3.61\,\frac{m}{s}$$

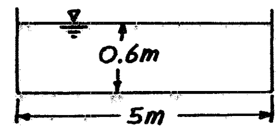
or

$Re = 1.79\times10^6(3.61) = 6.46\times10^6$ Thus, from Eq. (3) (Moody chart) $f = 0.013 \neq 0.015$. Assume $f = 0.013$, or $V = \left[\frac{0.196}{0.013}\right]^{1/2} = 3.88\,\frac{m}{s}$ so that $Re = 1.79\times10^6(3.88) = 6.95\times10^6$ Thus, from Eq. (3) $f = 0.013$ (checks with the assumed value) Hence, $V = 3.88\,\frac{m}{s}$ or

$$Q = AV = \frac{\pi}{4}(2m)^2(3.88\tfrac{m}{s}) = \underline{\underline{12.2\,\frac{m^3}{s}}} \text{ (pipe flow)} \approx 11.7\,\frac{m^3}{s} \text{ (open channel flow)}$$

10.46

10.46 The depth downstream of a sluice gate in a rectangular wooden channel of width 5 m is 0.60 m. If the flowrate is 18 m³/s, determine the channel slope needed to maintain this depth. Will the depth increase or decrease in the flow direction if the slope is (**a**) 0.02; (**b**) 0.01?



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1$ and from Table 10.1, $n = 0.012$ (1)

Also $A = (5m)(0.6m) = 3\,m^2$, $P = 5m + 2(0.6m) = 6.2\,m$

so that $R_h = \frac{A}{P} = \frac{3\,m^2}{6.2\,m} = 0.484\,m$

Hence, from Eq. (1):

$18 = \frac{1}{0.012}(3)(0.484)^{2/3} S_o^{1/2}$ or $S_o = \underline{\underline{0.0136}}$

With $S_o = 0.02 > 0.0136$ the velocity will increase and the water will become less than 0.6 m deep.

With $S_o = 0.01 < 0.0136$ the velocity will decrease and the water will become greater than 0.6 m deep.

10.47

10.47 Water flows in a weedy earthen channel at a rate of 30 m³/s. What flowrate can be expected if the weeds are removed and the depth remains constant?

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$ Let $(\ )_{nw}$ denote no weeds; $(\ )_w$ denote with weeds.

Thus, since $A_w = A_{nw}$, $R_{h_w} = R_{h_{nw}}$ and $S_{ow} = S_{onw}$ it follows that

$$\frac{Q_w}{Q_{nw}} = \frac{\frac{K}{n_w} A_w R_{hw}^{2/3} S_{ow}^{1/2}}{\frac{K}{n_{nw}} A_{nw} R_{h_{nw}}^{2/3} S_{o_{nw}}^{1/2}} = \frac{n_{nw}}{n_w}$$

From Table 10.1 $n_w = 0.030$, $n_{nw} = 0.022$

or

$$Q_{nw} = \frac{n_w}{n_{nw}} Q_w = \frac{0.030}{0.022}\left(30 \frac{m^3}{s}\right) = \underline{\underline{40.9 \frac{m^3}{s}}}$$

10.48

10.48 Overnight a thin layer of ice forms on the surface of a 40-ft-wide river that is essentially of rectangular cross-sectional shape. Under these conditions the flow depth is 3 ft. During the following day the sun melts the ice cover. Determine the new depth if the flowrate remains the same and the surface roughness of the ice is essentially the same as that for the bottom and sides of the river.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}$$

Let $(\ )_i$ denote conditions with the ice cover and $(\ )_n$ with no ice cover. Thus,

$A_i = (40\,\text{ft})(3\,\text{ft}) = 120\,\text{ft}^2$, $P_i = 2(40\,\text{ft}) + 2(3\,\text{ft}) = 86\,\text{ft}$,

and $R_{h_i} = A_i / P_i = 120\,\text{ft}^2 / 86\,\text{ft} = 1.395\,\text{ft}$

Also,

$A_n = 40y$, $P_n = 40 + 2y$, and $R_{h_n} = A_n / P_n = 40y/(40+2y)$

Hence, since $Q_i = Q_n$ it follows that

$$\frac{K}{n_i} A_i R_{h_i}^{2/3} \sqrt{S_{o_i}} = \frac{K}{n_n} A_n R_{h_n}^{2/3} \sqrt{S_{on}}$$

so that with $n_i = n_n$ and $S_{o_i} = S_{on}$ this becomes

$$A_i R_{h_i}^{2/3} = A_n R_{h_n}^{2/3}$$

Hence,

$$120(1.395)^{2/3} = 40y\left(\frac{40y}{40+2y}\right)^{2/3}$$

or

$$3.75 = y\left(\frac{40y}{40+2y}\right)^{2/3} \qquad (1)$$

A standard root-finding computer program gives the solution to Eq. (1) as

$y = \underline{\underline{2.31\,\text{ft}}}$

10.49

10.49 A round concrete storm sewer pipe used to carry rainfall runoff from a parking lot is designed to be half full when the rainfall rate is a steady 1 in./hr. Will this pipe be able to handle the flow from a 2-in./hr rainfall without water backing up into the parking lot? Support your answer with appropriate calculations.

$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_o}$

Let $(\ )_1$ denote conditions when the pipe is half full and $(\ )_2$ when the pipe is full.

Thus, $A_1 = \frac{\pi}{8} D^2$, $R_{h_1} = A_1/P_1 = (\frac{\pi}{8} D^2)/(\frac{\pi}{2} D) = D/4$

and $A_2 = \frac{\pi}{4} D^2$, $R_{h_2} = A_2/P_2 = (\frac{\pi}{4} D^2)/(\pi D) = D/4$

Also, $S_{o_1} = S_{o_2}$ and $n_1 = n_2$

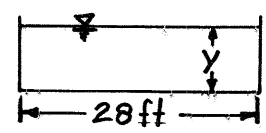
Therefore,

$$\frac{Q_1}{Q_2} = \frac{\frac{\kappa}{n_1} A_1 R_{h_1}^{2/3} \sqrt{S_{o_1}}}{\frac{\kappa}{n_2} A_2 R_{h_2}^{2/3} \sqrt{S_{o_2}}} = \frac{A_1 R_{h_1}^{2/3}}{A_2 R_{h_2}^{2/3}} = \frac{(\frac{\pi}{8} D^2)(\frac{D}{4})^{2/3}}{(\frac{\pi}{4} D^2)(\frac{D}{4})^{2/3}} = \frac{1}{2}$$

That is, $Q_2 = 2Q_1$. The full pipe can carry twice that of the half-full pipe. It can carry the 2 in./hr rainfall.

10.50

10.50 A rectangular unfinished concrete channel of 28-ft-width is laid on a slope of 8 ft/mi. Determine the flow depth and Froude number of the flow if the flowrate is 400 ft³/s.



$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49,\ S_o = \frac{8\text{ ft}}{5280\text{ ft}} = 0.001515, \text{ and}$$

from Table 10.1 $n = 0.014$

Also, $A = 28y$ and $P = 2y + 28$ so that $R_h = \frac{A}{P} = \frac{28y}{2y+28}$

Thus, $$400 = \frac{1.49}{0.014}\left(\frac{28y}{2y+28}\right)^{2/3}(28y)(0.001515)^{1/2}$$

or

$$0.594 = \frac{y^{5/3}}{(y+14)^{2/3}}$$

Hence, $$0.458(y+14) - y^{5/2} = 0 \qquad (1)$$

The solution to Eq. (1) is $\underline{\underline{y = 2.23\text{ ft}}}$

Thus,

$$V = \frac{Q}{A} = \frac{400\frac{\text{ft}^3}{\text{s}}}{(28\text{ ft})(2.23\text{ ft})} = 6.41\,\frac{\text{ft}}{\text{s}}$$

so that

$$Fr = \frac{V}{\sqrt{gy}} = \frac{6.41\frac{\text{ft}}{\text{s}}}{\left[(32.2\frac{\text{ft}}{\text{s}^2})(2.23\text{ ft})\right]^{1/2}} = \underline{\underline{0.756}}$$

10.51

10.51 A 10-ft-wide rectangular channel is built to bypass a dam so that fish can swim upstream during their migration. During normal conditions when the water depth is 4 ft, the water velocity is 5 ft/s. Determine the velocity during a flood when the water depth is 8 ft.

Let $(\ )_n$ and $(\ )_f$ denote normal and flood conditions, respectively. Thus,

(1) $V_n = \frac{\kappa}{n_n} R_{h_n}^{2/3} \sqrt{S_{on}}$ and

(2) $V_f = \frac{\kappa}{n_f} R_{h_f}^{2/3} \sqrt{S_{of}}$

where $n_n = n_f$, $S_{on} = S_{of}$ and

$A_n = 10\,ft(4\,ft) = 40\,ft^2$, $A_f = 10\,ft(8\,ft) = 80\,ft^2$

$P_n = 10\,ft + 2(4\,ft) = 18\,ft$, $P_f = 10\,ft + 2(8\,ft) = 26\,ft$

Thus, $R_{h_n} = \frac{A_n}{P_n} = \frac{40\,ft^2}{18\,ft} = 2.22\,ft$

and $R_{h_f} = \frac{A_f}{P_f} = \frac{80\,ft^2}{26\,ft} = 3.08\,ft$

Hence, divide Eq(2) by Eq(1) to obtain:

$$\frac{V_f}{V_n} = \left(\frac{R_{hf}}{R_{hn}}\right)^{2/3} = \left(\frac{3.08\,ft}{2.22\,ft}\right)^{2/3} = 1.24$$

so that

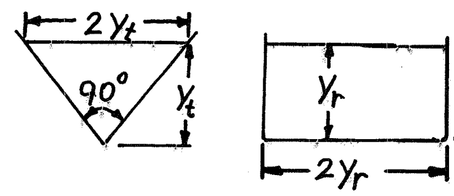
$$V_f = 1.24\,V_n = 1.24\left(5\,\frac{ft}{s}\right) = \underline{\underline{6.22\,\frac{ft}{s}}}$$

10.52

10.52 An engineer is to design a channel lined with planed wood to carry water at a flowrate of 2 m³/s on a slope of 10 m/800 m. The channel cross section can be either a 90° triangle or a rectangle with a cross section twice as wide as its depth. Which would require less wood and by what percent?

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2} \qquad (1)$$

Let $(\ )_t$ denote the triangular cross-section and $(\ )_r$ denote the rectangular cross-section



Thus, $Q_r = Q_t = 2\frac{m^3}{s}$, $S_{or} = S_{ot} = \frac{10}{800}$

and $n_r = n_t$ so that Eq. (1) gives

$$A_r R_{h_r}^{2/3} = A_t R_{h_t}^{2/3}, \text{ where } R_h = \frac{A}{P} \qquad (2)$$

Hence,

$A_r = 2y_r^2$, $P_r = 4y_r$ so that $R_{h_r} = \frac{2y_r^2}{4y_r} = \frac{1}{2}y_r$

Also,

$A_t = \frac{1}{2}(2y_t)y_t = y_t^2$, $P_t = 2(\sqrt{2}\,y_t)$ so that $R_{h_t} = \frac{y_t}{2\sqrt{2}}$

Thus, from Eq. (2):

$2y_r^2\left(\frac{1}{2}y_r\right)^{2/3} = y_t^2\left(\frac{1}{2\sqrt{2}}y_t\right)^{2/3}$, or $y_r = 0.707\, y_t$

The amount of wood is proportional to the wetted perimeter, P.

Since

$$\frac{P_t}{P_r} = \frac{2\sqrt{2}\,y_t}{4y_r} = \frac{2\sqrt{2}\,y_t}{4(0.707)y_t} = 1.00$$

the triangle requires the same amount of wood as the rectangle

10.53

10.53 Water flows in a channel with an equilateral triangle cross section as is shown in Fig. P10.53. Let Q_{full} denote the flowrate when $y = h$. By what percent is Q_{full} less than Q when $y = h - \delta y$, where $\delta y \ll h$? That is, placing a lid on this channel reduces the flowrate by what percent?

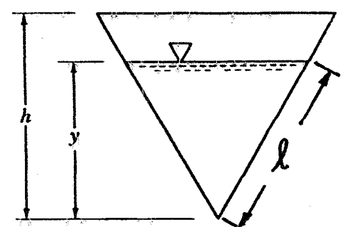


FIGURE P10.53

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2} \qquad (1)$$

Let $(\ )_\ell$ denote conditions when $\delta y = 0$ (i.e., with the lid)
and $(\ )_{n\ell}$ with $\delta y \ll h$ (i.e., no lid)

Thus, with $\left(\frac{K}{n} S_o^{1/2}\right)_\ell = \left(\frac{K}{n} S_o^{1/2}\right)_{n\ell}$ Eq. (1) gives

$$\frac{Q_{n\ell}}{Q_\ell} = \frac{(A R_h^{2/3})_{n\ell}}{(A R_h^{2/3})_\ell}, \text{ where}$$

$A_{n\ell} = A_\ell$ and $R_h = \frac{A}{P}$

Hence, $$\frac{Q_{n\ell}}{Q_\ell} = \frac{A_{n\ell}\left(\frac{A_{n\ell}}{P_{n\ell}}\right)^{2/3}}{A_\ell\left(\frac{A_\ell}{P_\ell}\right)^{2/3}} = \left(\frac{P_\ell}{P_{n\ell}}\right)^{2/3}$$

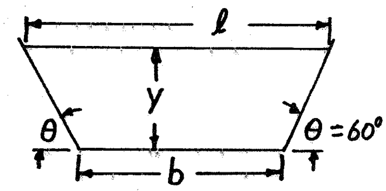
where $P_{n\ell} = 2\ell$ and $P_\ell = 3\ell$

Thus, $$\frac{Q_{n\ell}}{Q_\ell} = \left(\frac{3\ell}{2\ell}\right)^{2/3} = \left(\frac{3}{2}\right)^{2/3} = 1.31, \text{ or } \frac{Q_\ell}{Q_{n\ell}} = \frac{1}{1.31} = 0.763$$

The lid reduces the flowrate by $100 - 76.3 = \underline{\underline{23.7\%}}$

10.54

10.54 Show that for a trapezoidal channel with 60° angle sides, the best hydraulic cross section (i.e., minimum area, A, for a given flowrate) is as shown in Fig. E10.8b.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, or with K, n, and S_o constant,

$$\frac{dQ}{dy} = \frac{K}{n} S_o^{1/2}\left[R_h^{2/3}\frac{dA}{dy} + A\left(\tfrac{2}{3}\right)R_h^{-1/3}\frac{dR_h}{dy}\right] \qquad (1)$$

Thus, for a given flowrate $\frac{dQ}{dy} = 0$ and for the minimum area $\frac{dA}{dy} = 0$

Eq. (1) gives

$$\frac{dR_h}{dy} = 0$$

Also, $R_h = \frac{A}{P}$, where $A = \frac{1}{2}y(b+\ell) = \frac{1}{2}y\left(b + \left(b + 2\frac{y}{\tan\theta}\right)\right)$

or $A = y\left(b + \frac{y}{\tan\theta}\right)$, and $P = b + 2\frac{y}{\sin\theta}$ (2)

Write R_h in terms of A and y (for $\theta = 60°$): From Eq. (2)

$$b = \frac{A}{y} - \frac{y}{\tan\theta} \quad \text{so that}$$

$$R_h = \frac{A}{\frac{A}{y} - \frac{y}{\tan\theta} + \frac{2y}{\sin\theta}}$$

Hence, $\frac{dR_h}{dy} = \frac{\partial R_h}{\partial A}\frac{dA}{dy} + \frac{\partial R_h}{\partial y}$; or with $\frac{dA}{dy} = 0$,

$\frac{\partial R_h}{\partial y} = 0$ which gives $\frac{d}{dy}\left[\frac{A}{y} - \frac{y}{\tan\theta} + \frac{2y}{\sin\theta}\right] = 0$

Hence, $-\frac{A}{y^2} - \frac{1}{\tan\theta} + \frac{2}{\sin\theta} = 0$, or $A = \left[\frac{2}{\sin\theta} - \frac{1}{\tan\theta}\right]y^2$ (3)

Combine Eqs. (2) and (3) to give

$$y\left(b + \frac{y}{\tan\theta}\right) = \left[\frac{2}{\sin\theta} - \frac{1}{\tan\theta}\right]y^2$$

or $b = 2\left[\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right]y$, which for $\theta = 60°$ $\left(\sin\theta = \frac{\sqrt{3}}{2},\ \tan\theta = \sqrt{3}\right)$

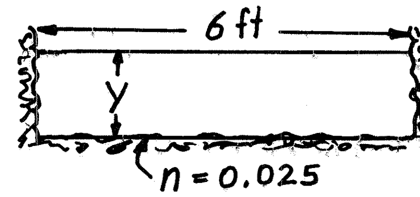
gives

$$b = 2\left[\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right]y$$

or $\underline{\underline{y = \frac{\sqrt{3}}{2}b}}$ (as indicated in Fig. E 10.8b)

10.55

10.55 At what depth will 50 ft^3/s of water flow in a 6-ft-wide rectangular channel lined with rubble masonry set on a slope of 1 ft in 500 ft? Is a hydraulic jump possible under these conditions? Explain.



$$Q = \frac{1.49}{n} A R_h^{2/3} \sqrt{S_0} \quad \text{where}$$

$$A = 6y, \quad R_h = \frac{A}{P} = \frac{6y}{2y+6}, \quad S_0 = \frac{1 \text{ ft}}{500 \text{ ft}}$$

and

$n = 0.025$ (see Table 10.1)

Thus,

$$50 = \frac{1.49}{0.025}(6y)\left[\frac{6y}{2y+6}\right]^{2/3}(0.002)^{1/2}$$

which becomes

$$y^{5/3} = (2y+6)^{2/3}(0.948)$$

~~The trial and error solution to this equation is~~

$$y = \underline{\underline{2.53 \text{ ft}}}$$

Thus, $V = \frac{Q}{A} = \frac{50 \text{ ft}^3/s}{6(2.53)\text{ ft}^2} = 3.29 \text{ ft/s}$

so that

$$Fr = \frac{V}{\sqrt{gy}} = \frac{3.29 \text{ ft/s}}{[(32.2 \text{ ft/s}^2)(2.53 \text{ ft})]^{1/2}} = 0.365$$

Since $Fr < 1$ it is <u>not possible</u> to have a hydraulic jump.

10.56

10.56 Water flows in the symmetrical, unfinished concrete trapezoidal channel shown in Fig. P10.56 at a rate of 120 ft^3/s. The slope is 4.2 ft/2000 ft. Determine the number of cubic yards of concrete needed to line each 1000 ft of the channel.

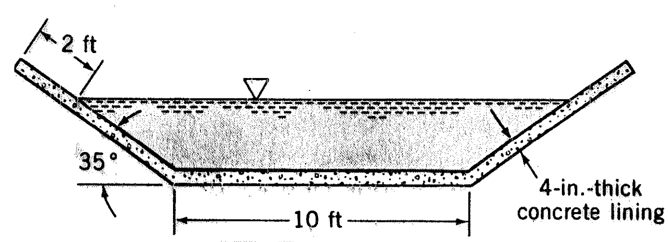
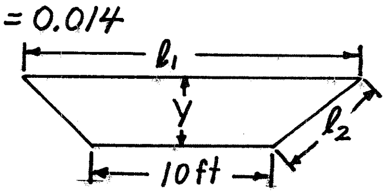


FIGURE P10.56

$$Q = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2}, \text{ where from Table 10.1 } n = 0.014 \qquad (1)$$

Also,



$$A = \tfrac{1}{2}(\ell_1 + 10\,ft)y = \tfrac{1}{2}(20\,ft + 2.86y)y$$

or

$$A = (10 + 1.43y)y$$

$$\ell_1 = 10\,ft + 2\frac{y}{\tan 35°}$$

$$R_h = \frac{A}{P} = \frac{A}{(10\,ft + 2\ell_2)}$$

$$\ell_2 = \frac{y}{\sin 35°} = 1.74y$$

or $$R_h = \frac{(10+1.43y)y}{(10+3.48y)}$$

Hence, with $\kappa = 1.49$ Eq. (1) becomes

$$120 = \frac{1.49}{0.014}(10+1.43y)y\left[\frac{(10+1.43y)y}{(10+3.48y)}\right]^{2/3}\left(\frac{4.2}{2000}\right)^{1/2}$$

or

$$14{,}890 = \frac{(10y+1.43y^2)^5}{(10+3.48y)^2}, \text{ or } \; 122(10+3.48y) - (10y+1.43y^2)^{5/2} = 0 \qquad (2)$$

Using a standard root-finding technique, the solution to Eq. (1) is obtained as $y = 1.664\,ft$. Thus,

$$\forall = \text{volume of concrete per } 1{,}000\,ft$$
$$= (P + 4\,ft)(1{,}000\,ft)\left(\tfrac{4}{12}\,ft\right)$$

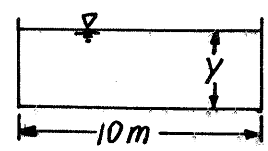
where $P = 10\,ft + 2\ell_2 = 10\,ft + 2(1.74)(1.664\,ft) = 15.8\,ft$

Hence,

$$\forall = (15.8\,ft + 4\,ft)(1{,}000\,ft)\left(\tfrac{4}{12}\,ft\right) = 6{,}600\,ft^3\left(\frac{1\,yd^3}{27\,ft^3}\right) = \underline{\underline{244\,yd^3}}$$

10.57

10.57 Determine the critical depth for a flow of 200 m³/s through a rectangular channel of 10-m width. If the water flows 3.8 m deep, is the flow supercritical? Explain.



$Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2}$ where for critical flow $Fr = 1$ or $V = \sqrt{gy}$

Thus, with $V = \frac{Q}{A} = \frac{200 \frac{m^3}{s}}{10y\, m^2} = \frac{20}{y} \frac{m}{s}$ we have

$\frac{20}{y} = \sqrt{9.81\, y}$ or $\underline{\underline{y = 3.44\, m}}$

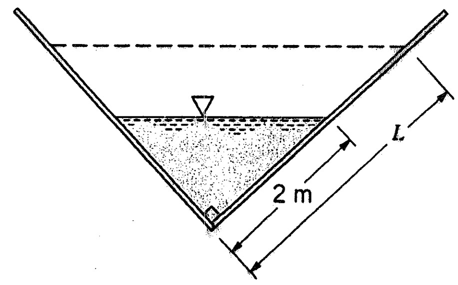
If $y = 3.8\,m$, then $V = \frac{20}{3.8} = 5.26 \frac{m}{s}$ and $Fr = \frac{V}{\sqrt{gy}} = \frac{5.26 \frac{m}{s}}{\left[(9.81 \frac{m}{s^2})(3.8m)\right]^{1/2}}$

$= 0.862$

The flow is $\underline{\underline{\text{subcritical}}}$.

10.58

10.58 When the channel of triangular cross section shown in Fig. P10.58 was new, a flowrate of Q caused the water to reach $L = 2$ m up the side as indicated. After considerable use, the walls of the channel became rougher and the Manning coefficient, n, doubled. Determine the new value of L if the flowrate stayed the same.



■ FIGURE P10.58

$$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_o} \qquad (1)$$

Let $(\ )_o$ and $(\ )_n$ represent the old and new conditions.

Thus, $n_o = 2n_n$, $A_n = \frac{1}{2}(2m)^2 = 2m^2$, $P_n = 4m$, so that

$$R_{h_n} = A_n/P_n = (2m^2)/(4m) = \frac{1}{2}m$$

Also, $A_o = \frac{1}{2}L^2$, $P_o = 2L$, so that $R_{h_o} = A_o/P_o = (\frac{1}{2}L^2)/(2L) = L/4$

Therefore, using Eq. (1) with $Q_o = Q_n$ gives

$$\frac{\kappa}{n_o} A_o R_{h_o}^{2/3} \sqrt{S_{oo}} = \frac{\kappa}{n_n} A_n R_{h_n}^{2/3} \sqrt{S_{on}}$$

or since $S_{on} = S_{oo}$,

$$\frac{1}{n_o} A_o R_{h_o}^{2/3} = \frac{1}{n_n} A_n R_{h_n}^{2/3}$$

By using the above data this becomes

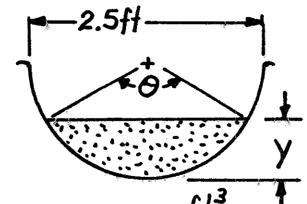
$$\frac{1}{2n_n}\left(\frac{1}{2}L^2\right)\left(\frac{L}{4}\right)^{2/3} = \frac{1}{n_n}(2m^2)\left(\frac{1}{2}m\right)^{2/3} \text{ or } L^{8/3} = 8(2)^{2/3}$$

or

$L = \underline{\underline{2.59\ m}}$

10.59

10.59 A smooth steel water slide at an amusement park is of semicircular cross section with a diameter of 2.5 ft. The slide descends a vertical distance of 35 ft in its 420 ft length. If pumps supply water to the slide at a rate of 6 cfs, determine the depth of flow. Neglect the effects of the curves and bends of the slide.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \frac{35\,ft}{420\,ft} = 0.0833$, $Q = 6.0\,\frac{ft^3}{s}$
and from Table 10.1 $n = 0.012$
Also (see Example 10.5), $A = \frac{D^2}{8}(\theta - \sin\theta)$ and
$R_h = \frac{D(\theta - \sin\theta)}{4\theta}$, where $D = 2.5\,ft$

Thus,

$$Q = \frac{K}{n} S_o^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}\right], \text{ where } \theta \sim rad,$$

or

$$6.0 = \frac{1.49}{0.012}(0.0833)^{1/2} \frac{(2.5)^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}\right]$$

Hence,

$$0.293\,\theta^{2/3} = (\theta - \sin\theta)^{5/3} \qquad 0.0252\,\theta^2 - (\theta - \sin\theta)^5 = 0 \qquad (1)$$

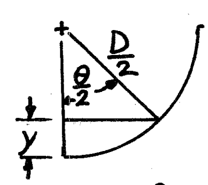
Using a standard root-finding technique gives the solution to Eq. (1) as $\theta = 1.574$ rad.

Thus, $\theta = (1.574\,rad)\left(\frac{180\,deg}{\pi\,rad}\right) = 90.2°$

or since

$y = \frac{D}{2}\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$ it follows that

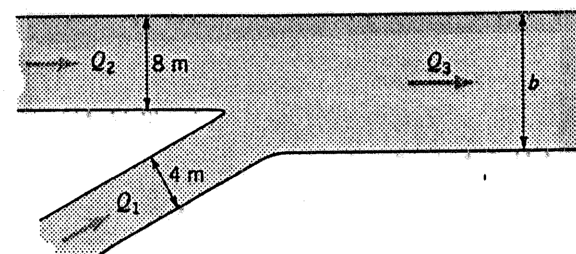
$y = \left(\frac{2.5}{2}\,ft\right)\left(1 - \cos\left(\frac{90.2}{2}\right)\right) = \underline{\underline{0.368\,ft}}$



$y + \frac{D}{2}\cos\frac{\theta}{2} = \frac{D}{2}$

10.60

10.60 Two canals join to form a larger canal as shown in Video V10.2 and Fig. P10.60. Each of the three rectangular canals is lined with the same material and has the same bottom slope. The water depth in each is to be 2 m. Determine the width of the merged canal, b. Explain physically (i.e., without using any equations) why it is expected that the width of the merged canal is less than the combined widths of the two original canals (i.e., $b < 4\text{ m} + 8\text{ m} = 12\text{ m}$).



■ FIGURE P10.60

$Q_3 = Q_1 + Q_2$ where for $i = 1, 2, 3$

$$Q_i = \frac{\kappa}{n_i} A_i R_{h_i}^{2/3} \sqrt{S_{o_i}}$$

Thus,

$$\frac{\kappa}{n_3} A_3 R_{h_3}^{2/3} \sqrt{S_{o_3}} = \frac{\kappa}{n_2} A_2 R_{h_2}^{2/3} \sqrt{S_{o_2}} + \frac{\kappa}{n_1} A_1 R_{h_1}^{2/3} \sqrt{S_{o_1}} \qquad (1)$$

But $n_1 = n_2 = n_3$ and $S_{o_1} = S_{o_2} = S_{o_3}$ so that Eq. (1) becomes

$$A_3 R_{h_3}^{2/3} = A_2 R_{h_2}^{2/3} + A_1 R_{h_1}^{2/3} \qquad (2)$$

where

$A_1 = 2m(4m) = 8m^2$, $P_1 = (2+2+4) = 8m$ so that $R_{h_1} = \frac{A_1}{P_1} = \frac{8m^2}{8m} = 1\,m$

$A_2 = 2m(8m) = 16m^2$, $P_2 = (2+2+8) = 12m$ so that $R_{h_2} = \frac{A_2}{P_2} = \frac{16m^2}{12m} = 1.333m$

and

$A_3 = 2b\,m^2$, $P_3 = (2+2+b) = (b+4)m$ so that $R_{h_3} = \frac{A_3}{P_3} = \frac{2b}{(b+4)}$

Thus, Eq. (2) becomes

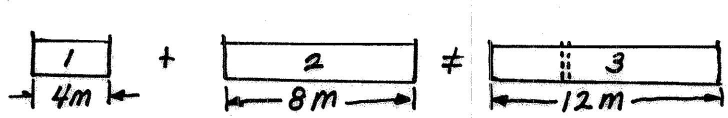
$$(2b)\left[\frac{2b}{(b+4)}\right]^{2/3} = 16\,(1.333)^{2/3} + 8\,(1)^{2/3} = 27.4$$

or

$$b^{5/3} = 8.63\,(b+4)^{2/3} \qquad (3)$$

Using a standard root-finding technique gives the solution to Eq. (3):
$\underline{\underline{b = 10.66\ m}}$

If the two original canals merged to form a 12m wide canal, the water depth would be less than 2m because without the two walls there would be less friction force hold the water back. Thus, to maintain the 2m depth we must have $b < 12m$.



10.61 *

*10.61 Water flows in the painted steel rectangular channel with rounded corners shown in Fig. P10.61. The bottom slope is 1 ft/200 ft. Plot a graph of flowrate as a function of water depth for $0 \le y \le 1$ ft with corner radii of $r = 0, 0.2, 0.4, 0.6, 0.8$, and 1.0 ft.

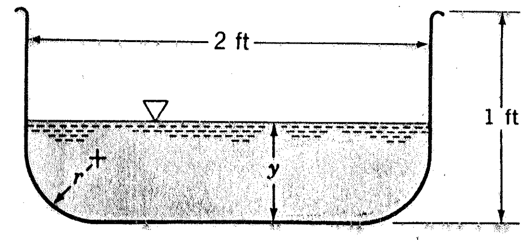
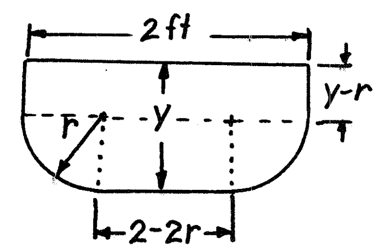


FIGURE P10.61

$Q = \frac{K}{n} A R_h S_o^{1/2}$, where $K = 1.49$, from Table 10.1 $n = 0.014$, and (1)

$S_o = \frac{1\ ft}{200\ ft} = 0.005$



(a) Assume $y \ge r$:

Thus, $A = 2(y-r) + r(2-2r) + \frac{1}{2}\pi r^2$

or $A = 2y - (2 - \frac{\pi}{2})r^2$ (2)

and $P = 2(y-r) + (2-2r) + \pi r$

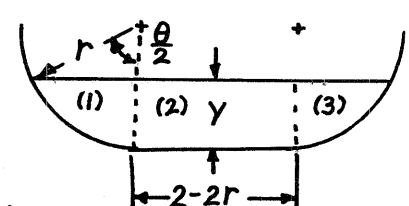
or $P = 2y - (4-\pi)r + 2$ (3)

Hence, with $R_h = \frac{A}{P}$ Eqs. (1), (2), and (3) give

$$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$$

or

$$Q = 7.53 \frac{\left[2y - (2 - \frac{\pi}{2})r^2\right]^{5/3}}{\left[2y - (4-\pi)r + 2\right]^{2/3}}$$ for $r \le y \le 1$, where $r \sim ft$, $y \sim ft$, $Q \sim \frac{ft^3}{s}$ (4)



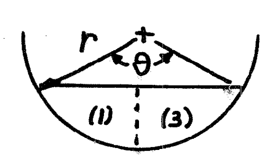
(b) Assume $y \le r$:

Thus, $A = A_1 + A_2 + A_3$

From Example 10.5, with $D = 2r$

$A_1 + A_3 = \frac{(2r)^2}{8}(\theta - \sin\theta)$ where $\theta \sim rad$ and

$\cos\frac{\theta}{2} = \frac{r-y}{r}$



Hence, $A = \frac{r^2}{2}(\theta - \sin\theta) + (2-2r)y$ (5)

Also, $P = 2 - 2r + P_1 + P_3$, where

from Example 10.5, $P_1 + P_3 = \frac{(2r)\theta}{2} = r\theta$

Thus, $P = 2 - 2r + r\theta = 2 + (\theta - 2)r$ (6)

By combing Eqs. (1), (5), and (6) we obtain:

$$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$$

or

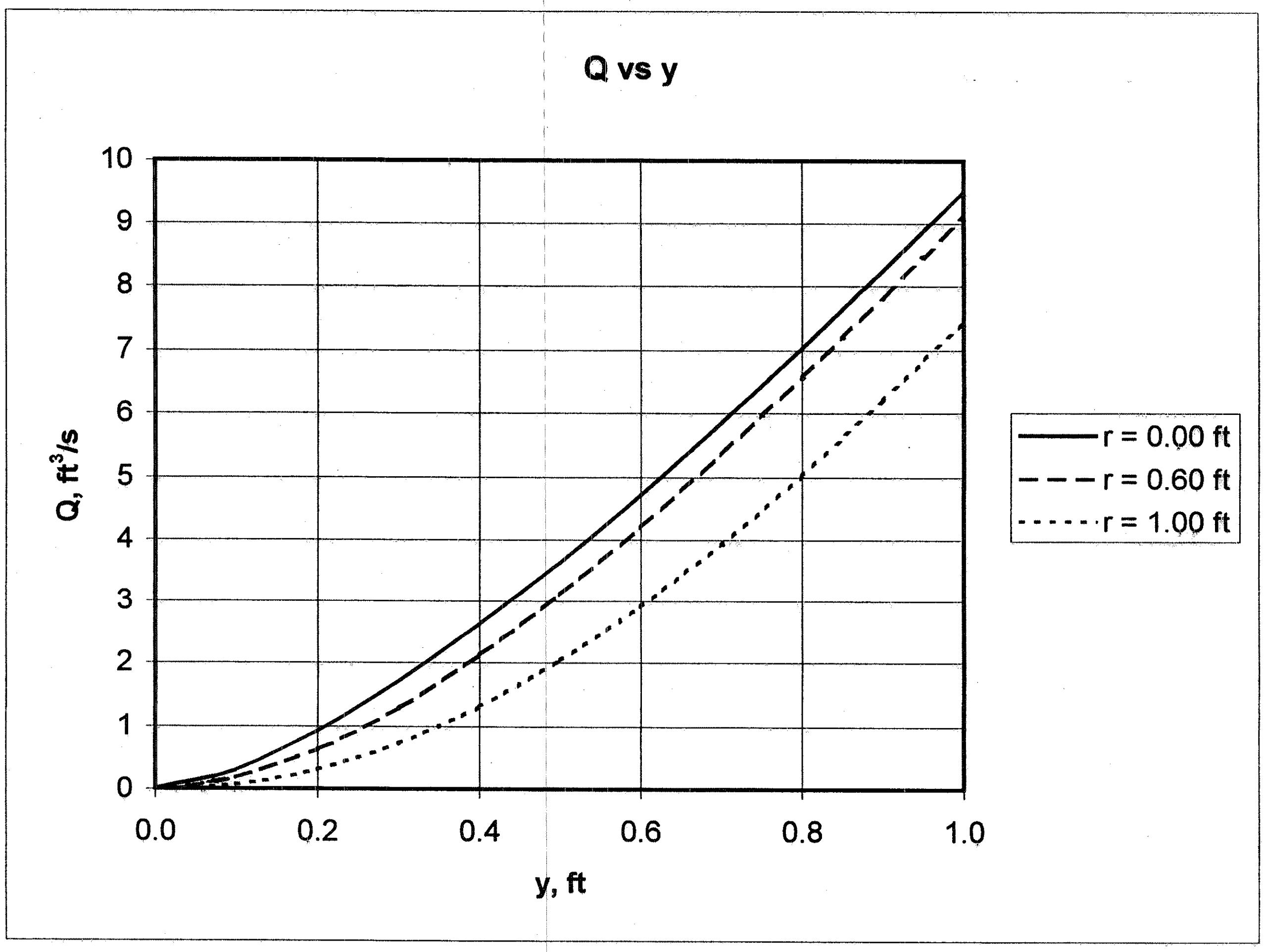
$$Q = 7.53 \frac{\left[\frac{r^2}{2}(\theta - \sin\theta) + (2-2r)y\right]^{5/3}}{\left[2 + (\theta - 2)r\right]^{2/3}}$$ for $0 \le y \le r$, where $r \sim ft$, $y \sim ft$, (7)

$Q \sim \frac{ft^3}{s}$, and $\theta = 2\cos^{-1}\left(\frac{r-y}{r}\right) \sim rad$

(con't)

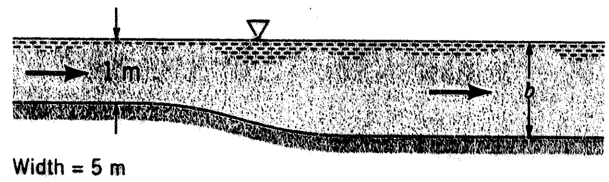
10.61* (con't)

The results, $Q = Q(y)$, are plotted below for $r = 0$, 0.6, and 1 ft.



10.62

10.62 Water flows uniformly at a depth of 1 m in a channel that is 5 m wide as shown in Fig. P10.62. Further downstream the channel cross section changes to that of a square of width and height b. Determine the value of b if the two portions of this channel are made of the same material and are constructed with the same bottom slope.



Width = 5 m

■ FIGURE P10.62

$Q_U = Q_D$, where $(\ )_U$ and $(\ )_D$ denote upstream and downstream conditions.
Thus, since $Q = \frac{\kappa}{n} A\, R_h^{2/3} \sqrt{S_o}$ it follows that

$$\frac{\kappa}{n_U} A_U R_{h_U}^{2/3} \sqrt{S_{oU}} = \frac{\kappa}{n_D} A_D R_{h_D}^{2/3} \sqrt{S_{oD}}$$

Also, $S_{oU} = S_{oD}$ and $n_U = n_D$

Hence,

(1) $A_U R_{h_U}^{2/3} = A_D R_{h_D}^{2/3}$, where $A_U = (1m)(5m) = 5m^2$, $P_U = 2(1m) + 5m = 7m$,

so that $R_{h_U} = A_U / P_U = 5m^2/7m = 0.714m$.

Also, $A_D = b^2$, $P_D = 3b$, so that $R_{h_D} = A_D/P_D = b^2/(3b) = \frac{1}{3}b$

Thus, from Eq. (1):

$(5m^2)(0.714m)^{2/3} = b^2 \left(\frac{1}{3}b\right)^{2/3}$

or

$b = \underline{\underline{2.21\,m}}$

10.63*

10.63* The cross section of a long tunnel carrying water through a mountain is as indicated in Fig. P10.63. Plot a graph of flowrate as a function of water depth, y, for $0 \le y \le 18$ ft. The slope is 2 ft/mi and the surface of the tunnel is rough rock (equivalent to rubble masonry). At what depth is the flowrate maximum? Explain.

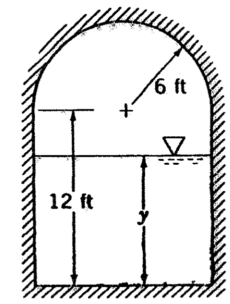


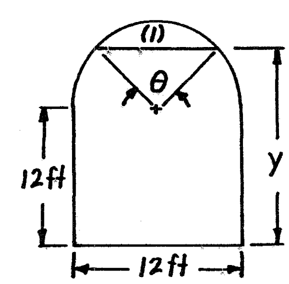
FIGURE P10.63

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \frac{2\,ft}{5280\,ft} = 0.000379$, (1)
and from Table 10.5 $n = 0.025$

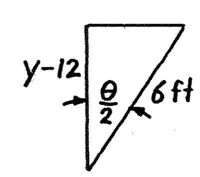
(a) Assume $y \le 12$ ft : Thus, $A = 12y$ and $P = 2y + 12$
so that $R_h = \frac{A}{P} = \frac{12y}{2y+12} = \frac{6y}{y+6}$

Hence,
$Q = \frac{1.49}{0.025}(12y)\left[\frac{6y}{y+6}\right]^{2/3}(0.000379)^{1/2}$
or
$Q = 46.0 \frac{y^{5/3}}{(y+6)^{2/3}}$, for $y \le 12$ where $y \sim ft$, $Q \sim \frac{ft^3}{s}$ (2)

(b) Assume $12 \le y \le 18$ ft :
Thus, $A = (12\,ft)^2 + \frac{\pi}{2}(6\,ft)^2 - A_1$, (3)
where from Example 10.5
$A_1 = \frac{D^2}{8}(\theta - \sin\theta)$, with $\cos\frac{\theta}{2} = \frac{y-12}{6}$



Hence, from Eq. (3)
$A = 201\,ft^2 - \frac{(12\,ft)^2}{8}(\theta - \sin\theta)$
or
$A = 201 - 18(\theta - \sin\theta)$ ft, where $\theta \sim rad$ (4)

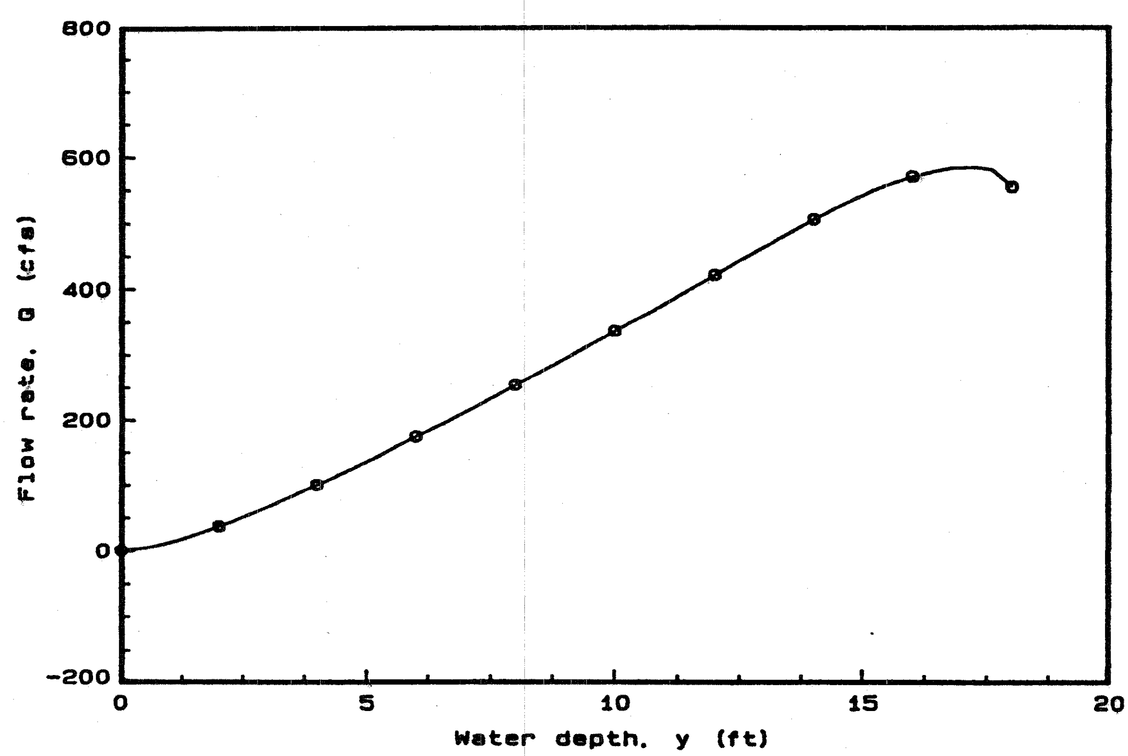


Also,
$P = 3(12\,ft) + (\pi - \theta)(6\,ft) = 36 + 6(\pi - \theta)$ ft (5)
Thus, with $R_h = \frac{A}{P}$ Eqs. (1), (4), and (5) give
$Q = \frac{1.49}{0.025} \frac{[201 - 18(\theta - \sin\theta)]^{5/3}}{[36 + 6(\pi - \theta)]^{2/3}}(0.000379)^{1/2}$
or
$Q = 43.4 \frac{(11.15 - \theta + \sin\theta)^{5/3}}{(9.14 - \theta)^{2/3}}$, for $12 \le y \le 18$ ft, where $\theta \sim rad$, (6)
$Q \sim \frac{ft^3}{s}$, and $\theta = 2\cos^{-1}\left(\frac{y-12}{6}\right)$

For $0 \le y \le 18$ ft calculate $Q = Q(y)$ from either Eq. (2) and Eq. (6),
(con't)

10.63 (con't)

depending on the value of y. The results from this calculation are given below. The maximum flowrate, $Q_{max} = 583\ \frac{ft^3}{s}$, occurs at $y = \underline{\underline{17.1\ ft}}$. For $17.1\,ft \le y \le 18\,ft$, an increase in depth adds only little to the flow area, A, but greatly increases the wetted perimeter, P. Thus, the retarding force is increased considerably.



10.64

10.64 The smooth concrete-lined channel shown in Fig. P10.64 is built on a slope of 2 m/km. Determine the flowrate if the depth is $y = 1.5$ m.

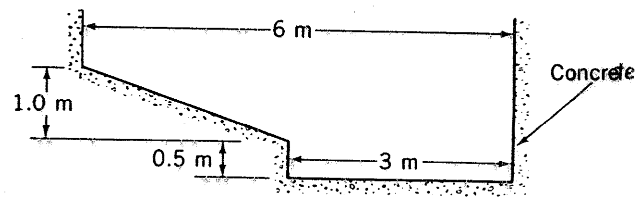


FIGURE P10.64

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1,\ S_o = \frac{2\,m}{1000\,m} = 0.002, \text{ and from Table 10.1} \qquad (1)$$

$n = 0.012$

With $y = 1.5\,m$, $A = (3m)(0.5m) + \frac{1}{2}(3m + 6m)(1.0m) = 6\,m^2$

and

$$P = 1.5\,m + 3\,m + 0.5\,m + (1^2 + 3^2)^{\frac{1}{2}}\,m = 8.16\,m$$

Thus, $R_h = \frac{A}{P} = \frac{6\,m^2}{8.16\,m} = 0.735\,m$, and Eq. (1) gives

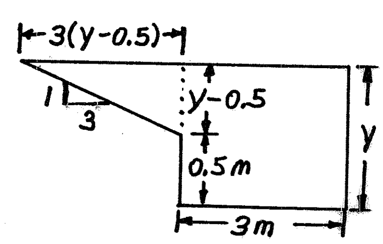
$$Q = \frac{1}{0.012}(6)(0.735)^{2/3}(0.002)^{1/2} = \underline{\underline{18.2\,\frac{m^3}{s}}}$$

10.65

10.65 Determine the flow depth for the channel shown in Fig. P10.64 if the flowrate is 15 m³/s.

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1,\ S_o = \frac{3\,m}{3000\,m} = 0.003, \text{ and from Table 10.1 } n = 0.012$$

Also, $A = 3y + \frac{1}{2}[3(y-0.5)](y-0.5) = \frac{3}{2}y^2 + \frac{3}{2}y + \frac{3}{8}$



and

$$P = y + 3 + 0.5 + \left[(y-\tfrac{1}{2})^2 + 9(y-\tfrac{1}{2})^2\right]^{\frac{1}{2}}$$

$$= y + 3.5 + \sqrt{10}\,(y-0.5) = 4.16\,y + 1.92$$

Hence, with $R_h = \frac{A}{P}$ and $Q = 15\,\frac{m^3}{s}$ we obtain

$$15 = \frac{1}{0.012}(1.5y^2 + 1.5y + 0.375)^{5/3}\frac{1}{(4.16y + 1.92)^{2/3}}(0.003)^{\frac{1}{2}}$$

or

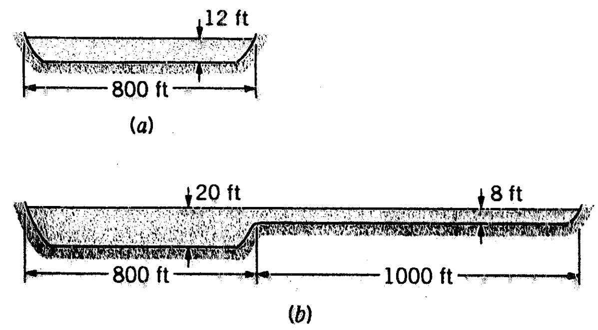
$$2.04(4.16y + 1.92)^{0.4} - 1.5y^2 - 1.5y - 0.375 = 0 \qquad (1)$$

Using a standard root-finding technique, the solution to Eq. (1) is found to be $\underline{\underline{y = 1.22\,m}}$

Note: Since $y < 1.5\,m$ the water does not contact the left vertical wall.

10.66

*10.66 At a given location, under normal conditions a river flows with a Manning coefficient of 0.030 and a cross section as indicated in Fig. P10.66*a*. During flood conditions at this location, the river has a Manning coefficient of 0.040 (because of trees and brush in the floodplain) and a cross section as shown in Fig. P10.66*b*. Determine the ratio of the flowrate during flood conditions to that during normal conditions.



■ FIGURE P10.66

(1) $Q_a = \frac{K}{n_a} A_a R_{h_a}^{2/3} \sqrt{S_{oa}}$, where $A_a = 12\,ft(800\,ft) = 9600\,ft^2$, $P_a = 2(12\,ft) + 800\,ft = 824\,ft$,

so that $R_{h_a} = A_a / P_a = 9600\,ft^2/(824\,ft) = 11.65\,ft$

Similarly,

(2) $Q_b = \frac{K}{n_b} A_b R_{h_b}^{2/3} \sqrt{S_{ob}}$, where $A_b = 20\,ft(800\,ft) + 8\,ft(1000\,ft) = 24{,}000\,ft^2$,

$P_b = 800\,ft + 1000\,ft + 2(20\,ft) = 1840\,ft$ so that

$R_{h_b} = A_b / P_b = 24000\,ft^2/(1840\,ft) = 13.04\,ft$

Thus, from Eqs. (1) and (2), with $S_{oa} = S_{ob}$,

(3) $$\frac{Q_b}{Q_a} = \frac{\frac{K}{n_b} A_b R_{h_b}^{2/3} \sqrt{S_{ob}}}{\frac{K}{n_a} A_a R_{h_a}^{2/3} \sqrt{S_{oa}}} = \frac{n_a A_b R_{h_b}^{2/3}}{n_b A_a R_{h_a}^{2/3}}$$

By using the given and calculated data,

$$\frac{Q_b}{Q_a} = \left(\frac{0.03}{0.04}\right)\left(\frac{24{,}000\,ft^2}{9{,}600\,ft^2}\right)\left(\frac{13.04\,ft}{11.65\,ft}\right)^{2/3} = \underline{\underline{2.02}}$$

10.67

10.67 Repeat Problem 10.64 if the surfaces are smooth concrete as is indicated except for the diagonal surface, which is gravelly with $n = 0.025$.

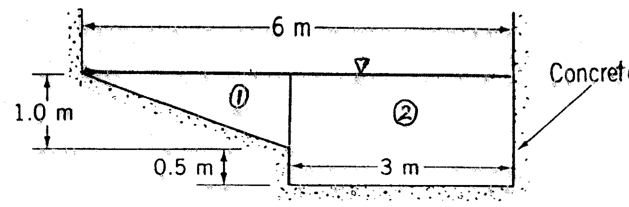


FIGURE P10.64

$Q = Q_1 + Q_2 = \frac{K}{n_1} A_1 R_{h_1}^{2/3} S_o^{1/2} + \frac{K}{n_2} A_2 R_{h_2}^{2/3} S_o^{1/2}$, where $K=1$, $S_o = 0.002$, (1)

$n_1 = 0.025$, and from Table 10.1 $n_2 = 0.012$

Also, $A_1 = \frac{1}{2}(1.0\,m)(3m) = 1.50\,m^2$, $P_1 = (1.0^2 + 3.0^2)^{\frac{1}{2}}\,m = 3.16\,m$

or $R_{h_1} = \frac{A_1}{P_1} = \frac{1.50\,m^2}{3.16\,m} = 0.475\,m$

and

$A_2 = (3m)(1.5m) = 4.5\,m^2$, $P_2 = 0.5m + 3m + 1.5m = 5m$

or $R_{h_2} = \frac{A_2}{P_2} = \frac{4.5\,m^2}{5m} = 0.90\,m$

Hence, from Eq. (1):

$$Q = \frac{1}{0.025}(1.50)(0.475)^{2/3}(0.002)^{1/2} + \frac{1}{0.012}(4.5)(0.90)^{2/3}(0.002)^{1/2}$$

or

$Q = \underline{\underline{17.3\ \frac{m^3}{s}}}$

Note: With all surfaces concrete, $Q = 18.2\ \frac{m^3}{s}$ (see Problem 10.64).

10.68*

10.68* Water flows through the storm sewer shown in Fig. P10.68. The slope of the bottom is 2 m/400 m. Plot a graph of the flowrate as a function of depth for $0 \le y \le 1.7$ m. On the same graph plot the flowrate expected if the entire surface were lined with material similar to that of a clay tile.

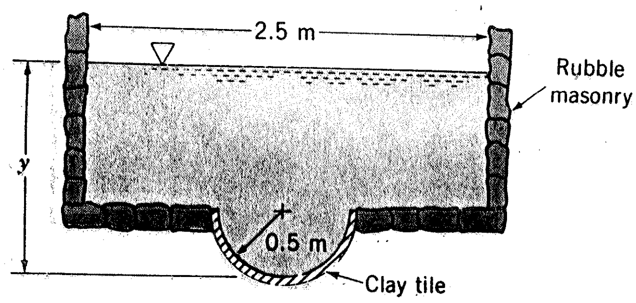


FIGURE P10.68

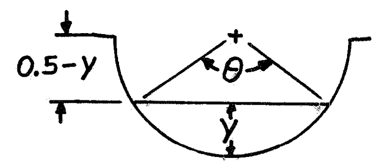
(a) For $0 \le y = 0.5\,m$: The flow is the same as that in a circular pipe. Thus, from Example 10.5 with $D = 1\,m$, $\kappa = 1$, and $n = 0.014$ (Table 10.1):

$$Q = \frac{\kappa}{n} S_o^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} = \frac{1}{0.014}\left(\frac{2\,m}{400\,m}\right)^{1/2} \frac{(1)^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}$$

or

$$Q = 0.251 \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} \frac{m^3}{s}, \text{ where } \theta \sim rad \qquad (1)$$

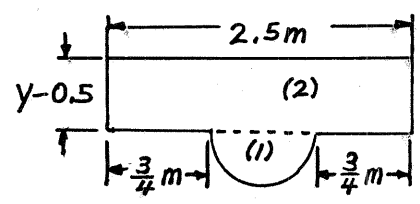
and $\theta = 2\cos^{-1}\left(\frac{0.5 - y}{0.5}\right)$



(b) For $y \ge 0.5\,m$:

$Q = Q_1 + Q_2$, where

$Q_1 = \frac{\kappa}{n_1} A_1 R_{h_1}^{2/3} S_o^{1/2}$, with $n_1 = 0.014$,



$A_1 = \frac{\pi}{2}(0.5\,m)^2 = 0.393\,m^2$, $P_1 = \pi(0.5) = 1.57\,m$ so that

$R_{h_1} = \frac{A_1}{P_1} = \frac{0.393\,m^2}{1.57\,m} = 0.250\ m$

Thus,

$$Q_1 = \frac{1}{0.014}(0.393)(0.250)^{2/3}\left(\frac{2}{400}\right)^{1/2} = 0.787 \frac{m^3}{s}$$

Also,

$$Q_2 = \frac{\kappa}{n_2} A_2 R_{h_2}^{2/3} S_o^{1/2}, \text{ with } n_2 = 0.025 \text{ (see Table 10.1)} \qquad (2)$$

$A_2 = (2.5\,m)(y - 0.5) = 2.5y - 1.25$ and $P_2 = 2(y - 0.5) + 2\left(\frac{3}{4}\right) = 2y + 0.5$

Hence, with $R_{h_2} = \frac{A_2}{P_2}$, Eq. (2) becomes

$$Q_2 = \frac{1}{0.025}(2.5y - 1.25)^{5/3} \frac{1}{(2y + 0.5)^{2/3}} \left(\frac{2}{400}\right)^{1/2} = 13.0 \frac{(y - 0.5)^{5/3}}{(2y + 0.5)^{2/3}}$$

Therefore,

$$Q = 0.787 + 13.0 \frac{(y - 0.5)^{5/3}}{(2y + 0.5)^{2/3}} \frac{m^3}{s} \text{ for } y \ge 0.5\,m \qquad (3)$$

Plot $Q = Q(y)$ for $0 \le y \le 1.7\,m$ using Eqs. (1) and (3).

(con't)

10.68**	(con't)

If the entire surface were lined with material with $n_1 = n_2 = 0.014$, Eqn. (1) would remain valid. The coefficient "13.0" in Eq. (3) would become $13.0\left(\frac{0.025}{0.014}\right) = 23.2$. For this case,

$$Q = 0.787 + 23.2\,\frac{(y-0.5)^{5/3}}{(2y+0.5)^{2/3}}\ \frac{m^3}{s} \text{ for } y \geq 0.5\,m \qquad (4)$$

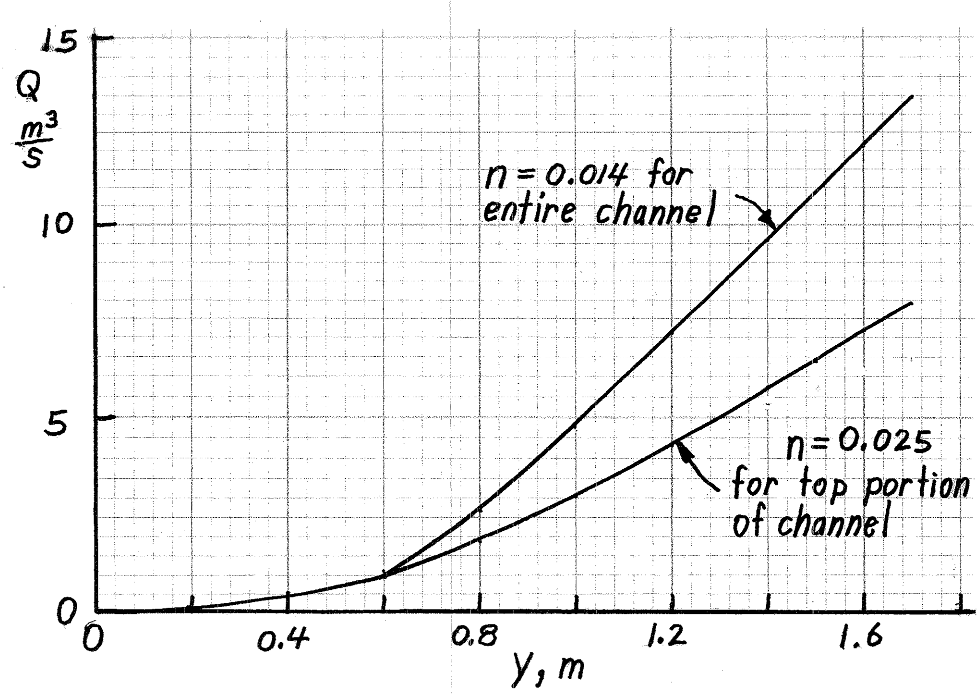
This result is also plotted (i.e. Q from Eq. (1) for $0 \leq y \leq 0.5$, and Q from Eq. (4) for $0.5 < y \leq 1.7\,m$).

With n = 0.025 for part of the channel

y, m	Q, m3/s	y, m	Q, m3/s
0.0	7.552E-11	0.9	2.407E+00
0.1	3.293E-02	1.0	3.010E+00
0.2	1.381E-01	1.1	3.649E+00
0.3	3.089E-01	1.2	4.315E+00
0.4	5.315E-01	1.3	5.003E+00
0.5	7.870E-01	1.4	5.708E+00
0.6	9.837E-01	1.5	6.426E+00
0.7	1.367E+00	1.6	7.157E+00
0.8	1.853E+00	1.7	7.897E+00

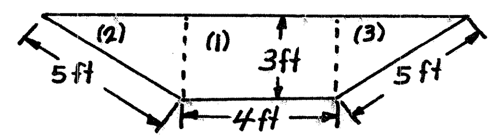
With n = 0.014 for the entire channel

y, m	Q, m3/s	y, m	Q, m3/s
0.0	7.552E-11	0.9	3.678E+00
0.1	3.293E-02	1.0	4.754E+00
0.2	1.381E-01	1.1	5.894E+00
0.3	3.089E-01	1.2	7.083E+00
0.4	5.315E-01	1.3	8.310E+00
0.5	7.870E-01	1.4	9.568E+00
0.6	1.138E+00	1.5	1.085E+01
0.7	1.822E+00	1.6	1.215E+01
0.8	2.689E+00	1.7	1.348E+01



10.69

10.69 Determine the flowrate for the symmetrical channel shown in Fig. P10.40 if the bottom is smooth concrete and the sides are weedy. The bottom slope is $S_0 = 0.001$.



$Q = Q_1 + Q_2 + Q_3 = Q_1 + 2Q_2$, where $Q_i = \frac{K}{n_i} A_i R_{h_i}^{2/3} S_0^{1/2}$ with $K = 1.49$

Also, $A_1 = (3\,ft)(4\,ft) = 12\,ft^2$, $A_2 = \frac{1}{2}(3\,ft)(4\,ft) = 6\,ft^2$, $P_1 = 4\,ft$, and $P_2 = 5\,ft$,

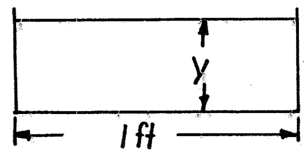
so that $R_{h_1} = \frac{A_1}{P_1} = \frac{12\,ft^2}{4\,ft} = 3\,ft$ and $R_{h_2} = \frac{A_2}{P_2} = \frac{6\,ft^2}{5\,ft} = 1.2\,ft$

Hence, with $n_1 = 0.012$ and $n_2 = 0.030$ (see Table 10.1) we obtain:

$$Q = \frac{1.49}{0.012}(12)(3)^{2/3}(0.001)^{1/2} + (2)\frac{1.49}{0.030}(6)(1.2)^{2/3}(0.001)^{1/2} = \underline{\underline{119\ \frac{ft^3}{s}}}$$

10.70

10.70 Water in a rectangular painted steel channel of width $b = 1$ ft and depth y is to flow at critical conditions, $Fr = 1$. Plot a graph of the critical slope, S_{0c}, as a function of y for $0.05 \text{ ft} \le y \le 5$ ft. What is the maximum slope allowed if critical flow is not to occur regardless of the depth?



$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}$, where $K = 1.49$ and from Table 10.1 $n = 0.014$

Also, $R_h = \frac{A}{P} = \frac{y}{2y+1}$ and with $Fr = \frac{V}{\sqrt{gy}} = 1$, $V = \sqrt{gy}$

Thus,

$$\sqrt{32.2\,y} = \frac{1.49}{0.014}\left(\frac{y}{2y+1}\right)^{2/3} S_{oc}^{1/2} \quad \text{or} \quad S_{oc} = 0.00284\left[\frac{(2y+1)^4}{y}\right]^{1/3} \qquad (1)$$

Equation (1) is plotted below. To determine the minimum critical slope set $\frac{dS_{oc}}{dy} = 0$. That is:

$$\frac{dS_{oc}}{dy} = \left(\tfrac{1}{3}\right)(0.00284)\left[\frac{(2y+1)^4}{y}\right]^{-2/3}\left[\frac{4(2y+1)^3(2)y - (2y+1)^4}{y^2}\right] = 0$$

Thus, $y = \frac{1}{6}$ so that from Eq. (1)

$$S_{oc\,min} = 0.00284\left[\frac{(\frac{2}{6}+1)^4}{\frac{1}{6}}\right]^{1/3} = \underline{\underline{0.00757}}$$

If $S_o < 0.00757$ critical flow cannot occur at any depth.

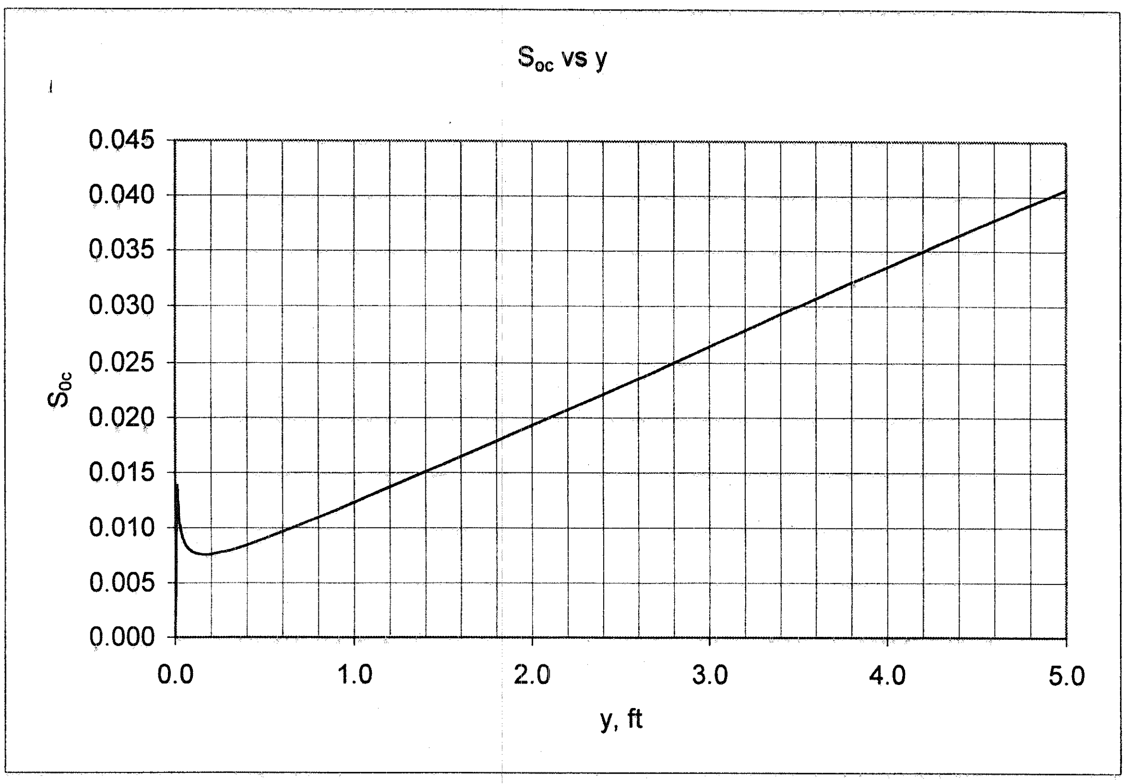
The following values are obtained from Eq. (1). Note that

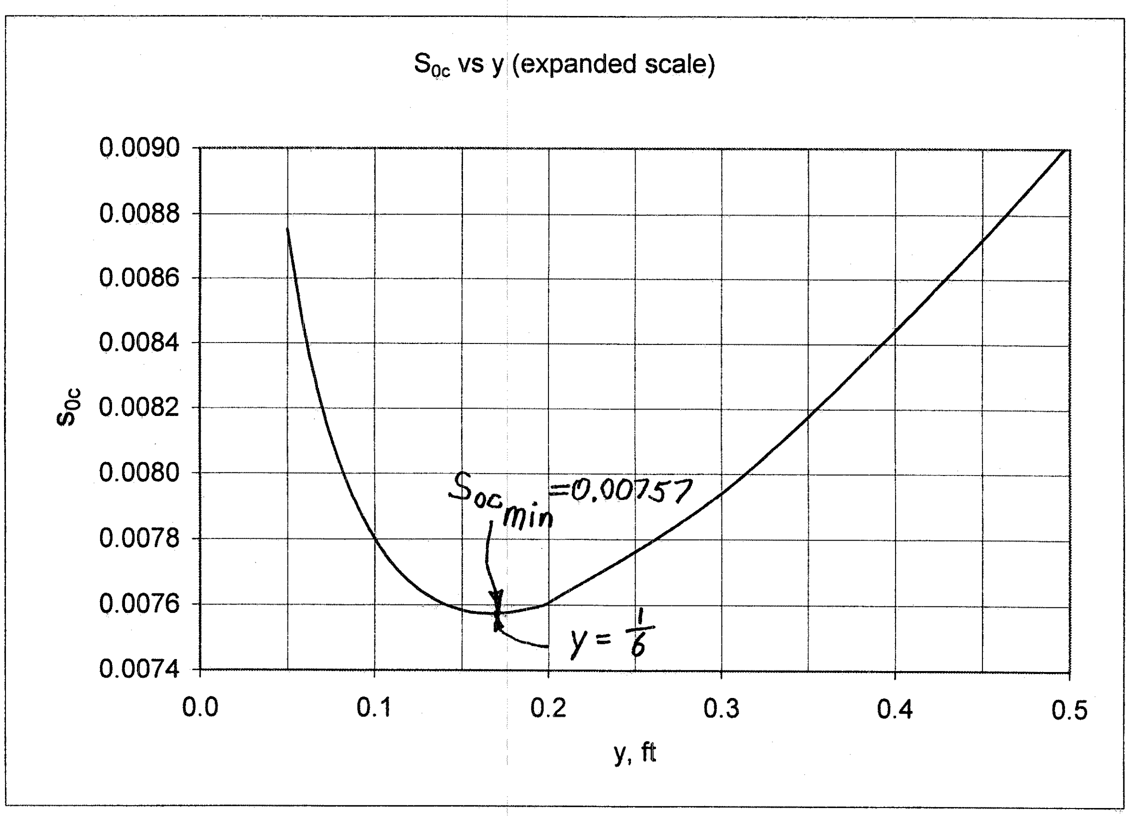
$$\lim_{y\to 0} S_{oc} = 0.00284 \lim_{y\to 0}\left[\frac{(2y+1)^4}{y}\right]^{1/3} = \infty \quad \text{and} \quad \lim_{y\to\infty} S_{oc} = \infty$$

See next page for graphs.

(con't)

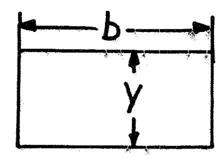
10.70 (con't)


S_{oc} vs y
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
S_{0c}
0.0
1.0
2.0
3.0
4.0
5.0
y, ft


S_{0c} vs y (expanded scale)
0.0090
0.0088
0.0086
0.0084
0.0082
0.0080
0.0078
0.0076
0.0074
s_{0c}
$S_{oc_{min}} = 0.00757$
$y = \frac{1}{6}$
0.0
0.1
0.2
0.3
0.4
0.5
y, ft

10.71

10.71 Water flows in a rectangular channel of width b and depth y with a Froude number of unity. The slope, S_{0c}, of the channel needed to produce this critical flow is a function of y. Show that as $y \to \infty$ the slope becomes proportional to y (i.e., $S_{0c} = C_1 y$, where C_1 is a constant) and that as $y \to 0$ the slope becomes proportional to $y^{-1/3}$ (i.e., $S_{0c} = C_2/y^{1/3}$, where C_2 is a constant). Show that the channel with an aspect ratio of $b/y = 6$ gives the minimum value of S_{0c}.



$V = \frac{\kappa}{n} R_h^{2/3} S_o^{1/2}$ with $Fr = \frac{V}{(gy)^{1/2}} = 1$, or $V = (gy)^{1/2}$

Also, $R_h = \frac{A}{P} = \frac{by}{2y+b}$ Thus,

$(gy)^{1/2} = \frac{\kappa}{n}\left[\frac{by}{2y+b}\right]^{2/3} S_{oc}^{1/2}$, where S_{oc} = critical slope

Thus, $\frac{n^2 g}{\kappa^2} y = \left[\frac{by}{2y+b}\right]^{4/3} S_{oc}$, or $S_{oc} = \left(\frac{n^2 g}{\kappa^2 b^{4/3}}\right)\left[\frac{(2y+b)^4}{y}\right]^{1/3}$ (1)

Hence, as $y \to \infty$, $\frac{(2y+b)^4}{y} \to \frac{16y^4}{y} = 16y^3$ and

$$S_{oc} \to \left(\frac{ng}{\kappa b^{4/3}}\right)(16y^3)^{1/3} = \left(\frac{16^{1/3} ng}{\kappa b^{4/3}}\right) y = C_1 y$$

As $y \to 0$, $\frac{(2y+b)^4}{y} \to \frac{b^4}{y}$ so that

$$S_{oc} \to \left(\frac{ng}{\kappa b^{4/3}}\right)\left(\frac{b^4}{y}\right)^{1/3} = \frac{C_2}{y^{1/3}}$$

To determine the minimum S_{oc}, calculate $\frac{dS_{oc}}{dy} = 0$ from Eq.(1):

$$\frac{dS_{oc}}{dy} = \left(\frac{ng}{\kappa b^{4/3}}\right)\left(\frac{1}{3}\right)\left[\frac{(2y+b)^4}{y}\right]^{-2/3}\left[\frac{4(2y+b)^3(2)y - (2y+b)^4}{y^2}\right] = 0$$

or $(2y+b)^3\left[8y - (2y+b)\right] = 0$

Thus,

$$\underline{\underline{y = \frac{b}{6}}}$$

10.72

10.72 Water flows in a rectangular channel with a bottom slope of 4.2 ft/mi and a head loss of 2.3 ft/mi. At a section where the depth is 5.8 ft and the average velocity 5.9 ft/s, does the flow depth increase or decrease in the direction of flow? Explain.

$$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}, \quad \text{where } S_f = \frac{h_L}{\ell} = \frac{2.3\,ft}{5280\,ft}, \quad S_o = \frac{4.2\,ft}{5280\,ft},$$

$$\text{and } Fr = \frac{V}{(g\,y)^{1/2}} = \frac{5.9\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(5.8\,ft)\right]^{1/2}} = 0.432$$

Thus,

$$\frac{dy}{dx} = \frac{\left(\frac{2.3 - 4.2}{5280}\right)}{1 - (0.432)^2} = -0.000442 < 0$$

The $\underline{\underline{\text{flow depth decreases in flow direction.}}}$

There is less head loss than change in elevation for this subcritical flow. The fluid speeds up and gets shallower.

10.73

10.73 Water flows in the river shown in Fig. P10.73 with a uniform bottom slope. The total head at each section is measured by using Pitot tubes as indicated. Determine the value of dy/dx at a location where the Froude number is 0.357.

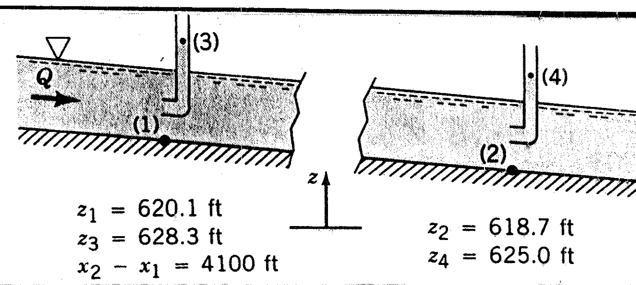


FIGURE P10.73

$$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}, \quad \text{where from the figure } S_f = \frac{h_L}{\ell} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0)\,ft}{4100\,ft}$$

$$\text{or } S_f = 8.05 \times 10^{-4} \text{ and } S_o = \frac{z_1 - z_2}{\ell} = \frac{(620.1 - 618.7)\,ft}{4100\,ft} = 3.41 \times 10^{-4}$$

Thus,

$$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (0.357)^2} = \underline{\underline{0.000532}} \qquad (i.e., 2.81\,\frac{ft}{mi})$$

10.74

10.74 Repeat Problem 10.73 if the Froude number is 2.75.

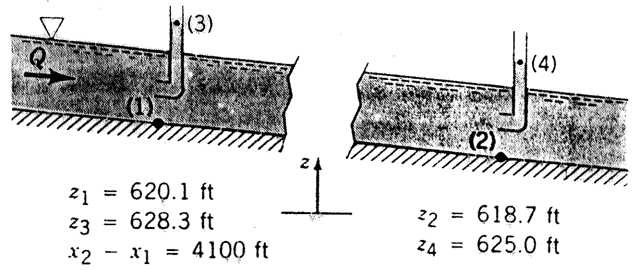


FIGURE P10.73

$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}$, where from the figure $S_f = \frac{h_L}{\ell} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0)\,ft}{4100\,ft}$

or $S_f = 8.05 \times 10^{-4}$ and $S_o = \frac{z_1 - z_2}{\ell} = \frac{(620.1 - 618.7)\,ft}{4100\,ft} = 3.41 \times 10^{-4}$

Thus,

$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (2.75)^2} = \underline{\underline{-7.07 \times 10^{-5}}}$ (i.e., $-0.373\frac{ft}{mi}$)

10.75

10.75 Water flows in a horizontal rectangular channel at a depth of 0.5 ft and a velocity of 8 ft/s. Determine the two possible depths at a location slightly downstream. Viscous effects between the water and the channel surface are negligible.

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{8\ ft/s}{\sqrt{32.2\ ft/s^2 (0.5\ ft)}} = 1.99$$

Thus, with $Fr_1 > 1$ there could be a hydraulic jump with $y_2 > y_1 = 0.5\ ft$. If so, then

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 Fr_1^2}\right] = \frac{1}{2}\left[-1 + \sqrt{1 + 8(1.99)^2}\right] = 2.36$$

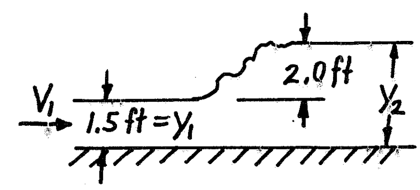
So that

$$y_2 = 2.36\, y_1 = 2.36\,(0.5\ ft) = 1.18\ ft$$

Hence, either $y_2 = 0.5\ ft$ (no jump) or $y_2 = 1.18\ ft$ (with jump)

10.76

10.76 A 2.0-ft standing wave is produced at the bottom of the rectangular channel in an amusement park water ride. If the water depth upstream of the wave is estimated to be 1.5 ft, determine how fast the boat is traveling when it passes through this standing wave (hydraulic jump) for its final "splash."



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1+8Fr_1^2}\right]$$

or $$\left(\frac{2.0\text{ ft} + 1.5\text{ ft}}{1.5\text{ ft}}\right) = \frac{1}{2}\left[-1 + \sqrt{1+8Fr_1^2}\right]$$

Thus, $Fr_1 = 1.97$, or since $Fr_1 = \dfrac{V_1}{\sqrt{g\,y_1}}$

$$V_1 = Fr_1\sqrt{g y_1} = 1.97\sqrt{\left(32.2\,\frac{ft}{s^2}\right)(1.5\,ft)} = \underline{\underline{13.7\,\frac{ft}{s}}}$$

10.77

10.77 The water depths upstream and downstream of a hydraulic jump are 0.3 and 1.2 m, respectively. Determine the upstream velocity and the power dissipated if the channel is 50 m wide.

$\dfrac{y_2}{y_1} = \dfrac{1.2\,m}{0.3\,m} = \dfrac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$ or $Fr_1 = 3.16$ Thus, since $Fr_1 = \dfrac{V_1}{(g\,y_1)^{1/2}}$

it follows that $V_1 = (3.16)\left[\left(9.81\,\frac{m}{s^2}\right)(0.3\,m)\right]^{1/2} = \underline{\underline{5.42\,\frac{m}{s}}}$

The power dissipated is given by

$$\mathcal{P} = \gamma Q h_L, \text{ where } \frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left(1 - \left(\frac{y_1}{y_2}\right)^2\right)$$

or
$$h_L = (0.3\,m)\left[1 - \frac{1.2\,m}{0.3\,m} + \frac{(3.16)^2}{2}\left(1 - \left(\frac{0.3\,m}{1.2\,m}\right)^2\right)\right] = 0.504\,m$$

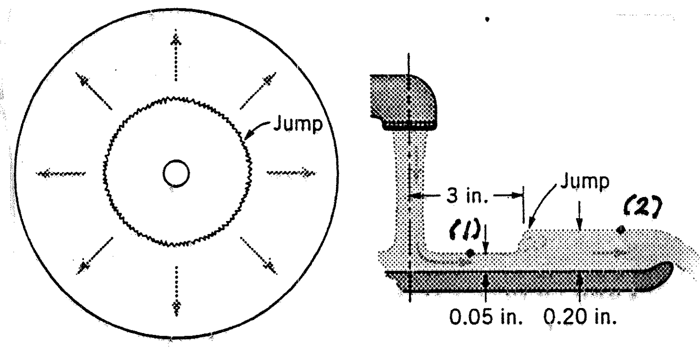
Also, $Q = A_1V_1 = y_1 b V_1 = (0.3\,m)(50\,m)\left(5.42\,\frac{m}{s}\right) = 81.3\,\frac{m^3}{s}$

Thus,

$$\mathcal{P} = \left(9.8\,\frac{kN}{m^3}\right)\left(81.3\,\frac{m^3}{s}\right)(0.504\,m) = 401\,\frac{kN\cdot m}{s} = \underline{\underline{401\,kW}}$$

10.78

10.78 Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in Fig. P10.78 and **Video V10.6**. Consider a situation where a jump forms 3.0 in. from the center of the plate with depths upstream and downstream of the jump of 0.05 in. and 0.20 in., respectively. Determine the flowrate from the faucet.



■ FIGURE P10.78

For a hydraulic jump:

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \quad \text{or}$$

$$\frac{0.20\,in.}{0.05\,in.} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \quad \text{so that} \quad Fr_1 = 3.16 = \frac{V_1}{\sqrt{g y_1}}$$

Thus,

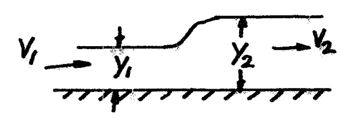
$$V_1 = 3.16\sqrt{32.2\frac{ft}{s^2}\left(0.05/12\right)ft} = 1.16\,\frac{ft}{s}$$

and

$$Q = A_1 V_1 = 2\pi R_1 y_1 V_1 = 2\pi\left(\tfrac{3}{12}\,ft\right)\left(\tfrac{0.05}{12}\,ft\right)\left(1.16\,\tfrac{ft}{s}\right) = \underline{\underline{0.00759\,\tfrac{ft^3}{s}}}$$

10.79

10.79 Show that the Froude number downstream of a hydraulic jump in a rectangular channel is $(y_1/y_2)^{3/2}$ times the Froude number upstream of the jump, where (1) and (2) denote the upstream and downstream conditions, respectively.



$Fr_2 = \frac{V_2}{(g y_2)^{1/2}}$, where $V_1 A_1 = V_2 A_2$, or $V_2 = \frac{A_1}{A_2} V_1 = \frac{y_1}{y_2} V_1$

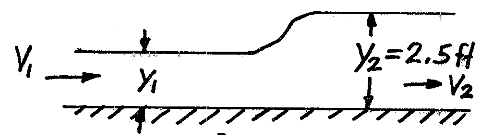
Thus,

$$Fr_2 = \frac{\left(\frac{y_1}{y_2}\right) V_1}{(g y_2)^{1/2}} = \left(\frac{y_1}{y_2}\right)^{3/2} \frac{V_1}{(g y_1)^{1/2}}$$

Hence, $\underline{\underline{Fr_2 = \left(\frac{y_1}{y_2}\right)^{3/2} Fr_1}}$

10.80

10.80 Water flows in a 2-ft-wide rectangular channel at a rate of 10 ft³/s. If the water depth downstream of a hydraulic jump is 2.5 ft, determine **(a)** the water depth upstream of the jump, **(b)** the upstream and downstream Froude numbers, and **(c)** the head loss across the jump.



$Q = 10\,\frac{ft^3}{s}$, $b = \text{width} = 2\,ft$

(a) Use $\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8\,Fr_1^2}\right]$ with $y_2 = 2.5\,ft$ so that

$5 + y_1 = y_1\sqrt{1 + 8\,Fr_1^2}$ Now, with $Fr_1^{\,2} = \frac{V_1^2}{g y_1} = \frac{(Q/(b y_1))^2}{g\,y_1} = \frac{(10/(2y_1))^2}{32.2\,y_1}$,

or $Fr_1^2 = \frac{0.776}{y_1^3}$, we obtain

$5 + y_1 = y_1\left[1 + 8\left(\frac{0.776}{y_1^3}\right)\right]^{\frac{1}{2}}$ By squaring both sides and simplifying we obtain $y_1^2 + 2.5y_1 - 0.621 = 0$ which gives $y_1 = \underline{\underline{0.228\,ft}}$

(b) From the above results

$$Fr_1^2 = \frac{0.776}{(0.228)^3} \quad \text{or} \quad Fr_1 = \underline{\underline{8.09}}$$

Also,

$$V_2 = \frac{Q}{A_2} = \frac{10\,\frac{ft^3}{s}}{(2.5ft)(2ft)} = 2.0\,\frac{ft}{s} \quad \text{so that} \quad Fr_2 = \frac{V_2}{(g y_2)^{1/2}} = \frac{2\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(2.5ft)\right]^{1/2}}$$

or

$Fr_2 = \underline{\underline{0.223}}$

(c) Also,

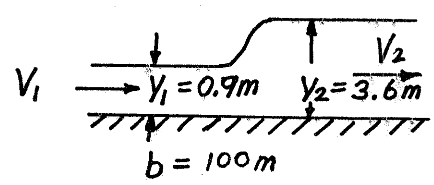
$$h_L = y_1\left[1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2}\left(1 - \left(\frac{y_1}{y_2}\right)^2\right)\right] = 0.228ft\left[1 - \frac{2.5}{0.228} + \frac{(8.10)^2}{2}\left(1 - \left(\frac{0.228}{2.5}\right)^2\right)\right]$$

or

$h_L = \underline{\underline{5.15\,ft}}$

10.81

10.81 A hydraulic jump at the base of a spillway of a dam is such that the depths upstream and downstream of the jump are 0.90 and 3.6 m, respectively (see Video V10.5). If the spillway is 10 m wide, what is the flowrate over the spillway?



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right], \text{ or } \frac{3.6m}{0.9m} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

Hence, $Fr_1 = 3.16$, but $Fr_1 = \frac{V_1}{(g\,y_1)^{1/2}}$ so that

$$V_1 = 3.16\left[(9.81\tfrac{m}{s^2})(0.9m)\right]^{1/2} = 9.39\ \tfrac{m}{s}$$

Thus,

$$Q = A_1V_1 = by_1V_1 = (10.0m)(0.9m)(9.39\tfrac{m}{s}) = \underline{\underline{84.5\ \tfrac{m^3}{s}}}$$

10.82

10.82 Determine the head loss and power dissipated by the hydraulic jump of Problem 10.81.

$$h_L = y_1\left[1-\frac{y_2}{y_1}+\frac{Fr_1^2}{2}\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right], \text{ where from } \frac{y_2}{y_1} = \frac{3.6m}{0.9m} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

Hence, $Fr_1 = 3.16$ so that

$$h_L = (0.9m)\left[1-\frac{3.6m}{0.9m}+\frac{(3.16)^2}{2}\left(1-\left(\frac{0.9m}{3.6m}\right)^2\right)\right] = \underline{\underline{1.51\ m}}$$

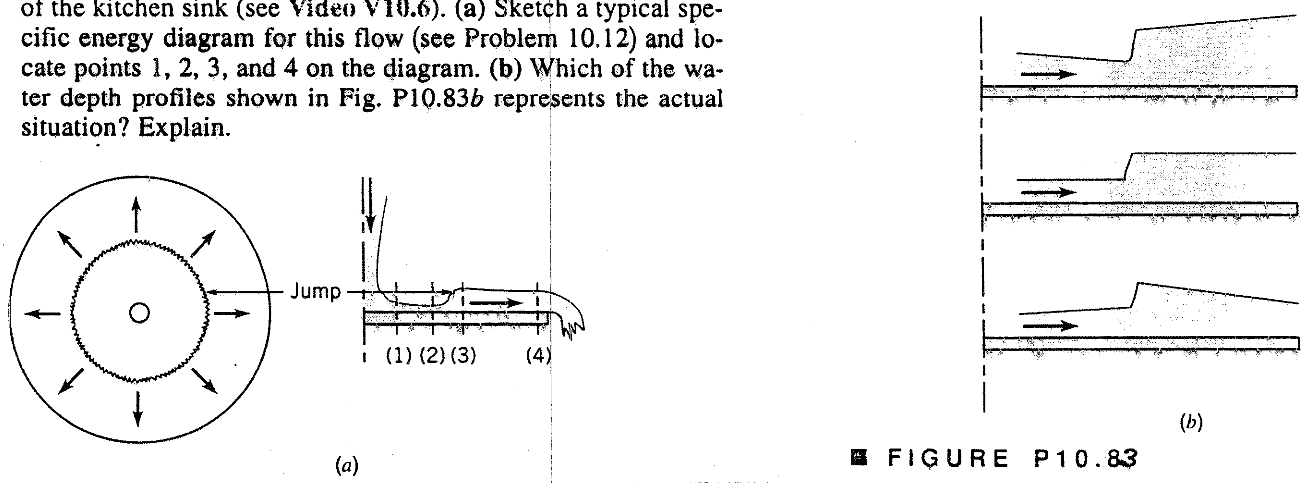
Also, $\mathcal{P} = \gamma Q h_L$, where $V_1 = (gy_1)^{\frac{1}{2}}Fr_1 = \left[(9.81\tfrac{m}{s^2})(0.9m)\right]^{1/2}(3.16) = 9.39\tfrac{m}{s}$

Hence,

$$\mathcal{P} = (9.80\tfrac{kN}{m^3})\left[(0.9m)(100m)(9.39\tfrac{m}{s})\right](1.51\,m) = 12,500\frac{kN\cdot m}{s} = \underline{\underline{12,500kW}}$$

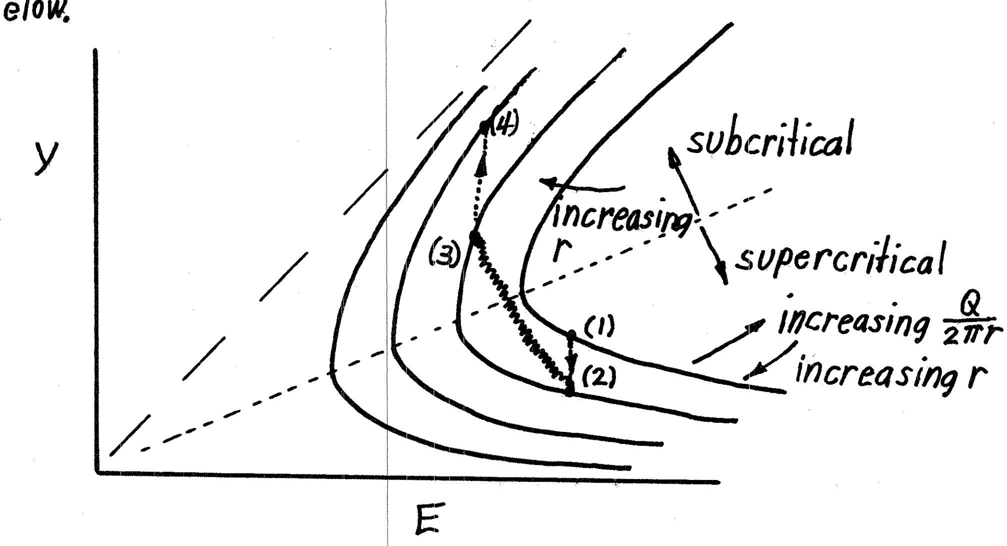
10.83

10.83 Water flowing radially outward along a circular plate forms a circular hydraulic jump as is shown in Fig. P10.83*a*. This is shown easily by holding a dinner plate under the faucet of the kitchen sink (see **Video V10.6**). **(a)** Sketch a typical specific energy diagram for this flow (see Problem 10.12) and locate points 1, 2, 3, and 4 on the diagram. **(b)** Which of the water depth profiles shown in Fig. P10.83*b* represents the actual situation? Explain.



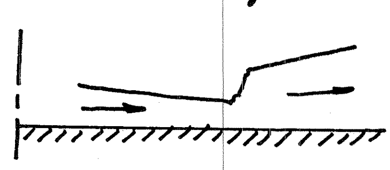
■ FIGURE P10.83

From Problem 10.12 the specific energy diagram for this radial flow is shown below.



Upstream of the jump the flow must be supercritical so (1) and (2) are located. Energy is conserved — $E_1 = E_2$. The depth decreases from (1) to (2). In the jump energy decreases — $E_3 = E_4 < E_2$. The flow is subcritical downstream of the jump and the depth increases. (See the above graph.)

Thus, the flow is like the following:



10.84

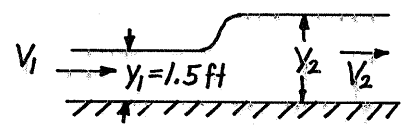
10.84 At a given location in a 12-ft-wide rectangular channel the flowrate is 900 ft^3/s and the depth is 4 ft. Is this location upstream or downstream of the hydraulic jump that occurs in this channel? Explain.

$$V = \frac{Q}{A} = \frac{900\,\frac{ft^3}{s}}{(12\,ft)(4\,ft)} = 18.75\,\frac{ft}{s} \quad \text{so that } Fr = \frac{V}{(gy)^{1/2}} = \frac{18.75\,\frac{ft}{s}}{\left[(32.2\,\frac{ft}{s^2})(4\,ft)\right]^{1/2}} = 1.65$$

Since $Fr > 1$, the location is upstream of the jump.

10.85*

***10.85** A rectangular channel of width b is to carry water at flowrates from $30 \le Q \le 600$ cfs. The water depth upstream of the hydraulic jump that occurs (if one does occur) is to remain 1.5 ft for all cases. Plot the power dissipated in the jump as a function of flowrate for channels of width $b = 10, 20, 30$, and 40 ft.



$$\mathcal{P} = \gamma Q h_L \text{ , where } h_L = y_1\left[1-\left(\frac{y_2}{y_1}\right) + \frac{Fr_1^2}{2}\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right] \quad (1)$$

and

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \text{ , provided } Fr_1 \ge 0 \quad (2)$$

Also, $Fr_1 = \frac{V_1}{(g y_1)^{1/2}}$, where $V_1 = \frac{Q}{A_1} = \frac{Q}{1.5b}$ so that

$$Fr_1 = \frac{\left(\frac{Q}{1.5b}\right)}{\left[(32.2\frac{ft}{s^2})(1.5ft)\right]^{1/2}} = 0.0959\,\frac{Q}{b}$$ Hence, from Eq.(1)

$$h_L = (1.5)\left[1-\left(\frac{y_2}{y_1}\right) + (0.00460)\left(\frac{Q}{b}\right)^2\left(1-\left(\frac{y_1}{y_2}\right)^2\right)\right] \text{ ft, where } b \sim ft,\ Q \sim \frac{ft^3}{s} \quad (3)$$

and from Eq.(2)

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\left(1+0.0736\left(\frac{Q}{b}\right)^2\right)^{1/2}\right] \quad (4)$$

For the given values of plot $\mathcal{P}$ from

$$\mathcal{P} = 62.4 Q h_L \ \frac{ft\cdot lb}{s} \quad \text{for} \quad 30 \le Q \le 600\ \frac{ft\cdot lb}{s} \quad (5)$$

Note: If $Fr_1 < 1$ there is no jump and $\mathcal{P} = 0$. From above, $Fr_1 = 1$ when $Q = \frac{b}{0.0959} = 10.4b$ (6)

Let Q_1 = flowrate when $Fr_1 = 1$. From Eq.(6) we obtain

b, ft	Q_1, $\frac{ft^3}{s}$
10	104
20	208
30	312
40	416

With $b = 10, 20, 30$, or 40 ft calculate and plot $\mathcal{P}$ from:

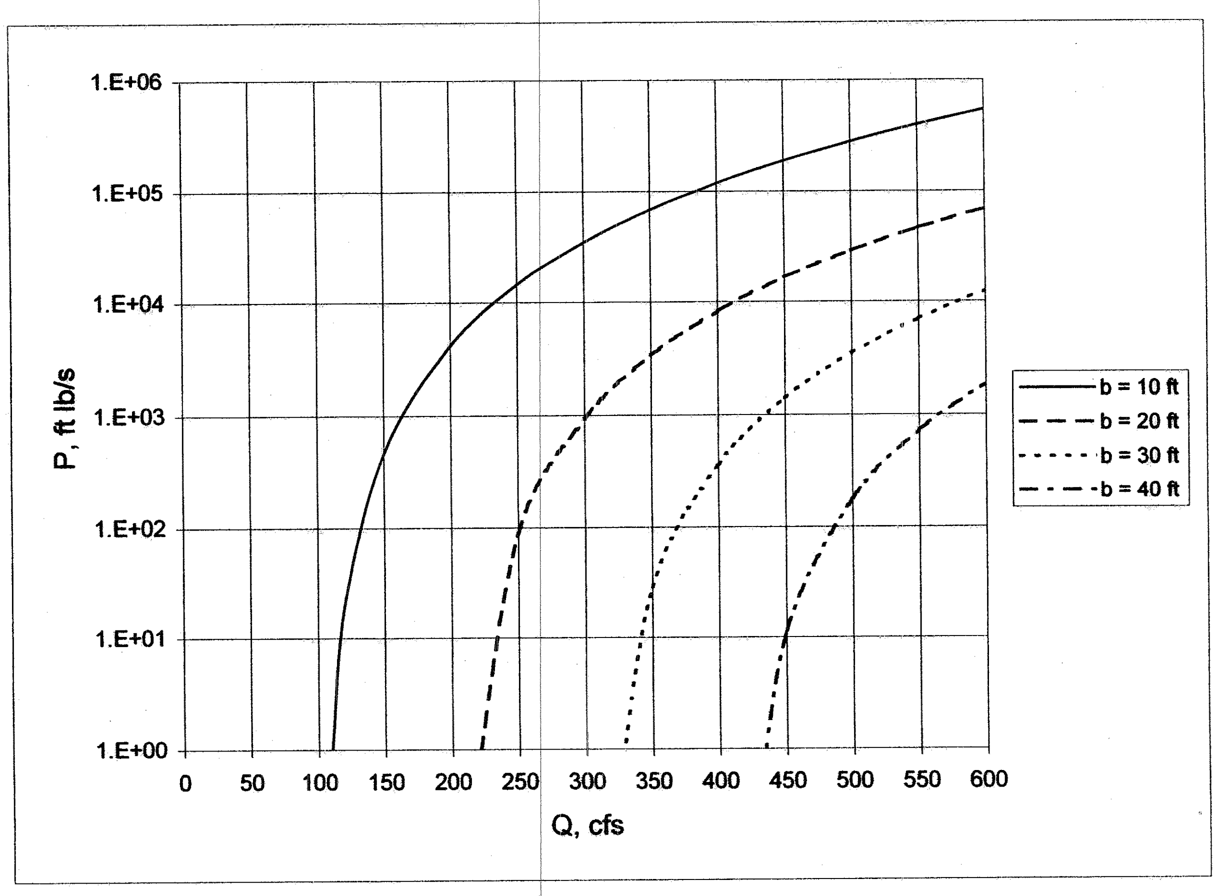
a) $\mathcal{P} = 0$ if $Q < Q_1$

b) $\mathcal{P} = 62.4\, Q h_L \ \frac{ft\cdot lb}{s}$, where obtain h_L from Eq.(3) with $\frac{y_2}{y_1}$ from Eq.(4) if $Q_1 \le Q \le 600\ \frac{ft^3}{s}$

(con't)

10.85* (con't)

The results of the above calculations are plotted below.



10.86

10.86 Water flows in a rectangular channel at a depth of $y = 1$ ft and a velocity of $V = 20$ ft/s. When a gate is suddenly placed across the end of the channel, a wave (a moving hydraulic jump) travels upstream with velocity V_w as is indicated in Fig. P10.86. Determine V_w. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w, the flow appears as a steady hydraulic jump.

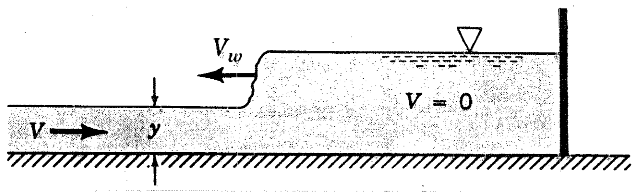
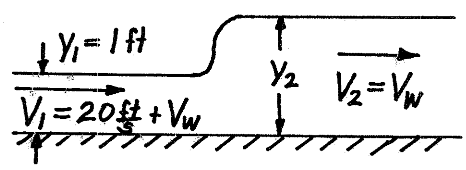


FIGURE P10.86

For an observer moving to the left with speed V_w the flow appears as shown below.

Thus, treat the flow as a jump with



$$Fr_1 = \frac{V_1}{(g y_1)^{1/2}} = \frac{(20+V_w)}{\left[(32.2\frac{ft}{s})(1ft)\right]^{1/2}}$$

or

$$Fr_1 = 0.176\,(20+V_w) \qquad (1)$$

Also, $A_1V_1 = A_2V_2$, or $\dfrac{y_2}{y_1} = \dfrac{V_1}{V_2} = \dfrac{20+V_w}{V_w}$ (2)

and

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$ which when combined with Eqs. (1) and (2) becomes

$$\frac{20+V_w}{V_w} = \frac{1}{2}\left[-1+\sqrt{1+8(0.176)^2(20+V_w)^2}\right]$$

or

$$2(20+V_w)+V_w = V_w\left(1+(0.248)(20+V_w)^2\right)^{\frac{1}{2}}$$

or

$$(40+3V_w)^2 = V_w^2\left[1+(0.248)(20+V_w)^2\right]$$, which can be written as

$$0.248\,V_w^4 + 9.92\,V_w^3 + 91.2\,V_w^2 - 240\,V_w - 1600 = 0 \qquad (3)$$

By using a standard root-finding program, the solution to Eq. (3) is determined to be $\underline{\underline{V_w = 4.36\ ft/s}}$.

10.87

10.87 Water flows in a rectangular channel with velocity $V = 6$ m/s. A gate at the end of the channel is suddenly closed so that a wave (a moving hydraulic jump) travels upstream with velocity $V_w = 2$ m/s as is indicated in Fig. P10.86. Determine the depths ahead of and behind the wave. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w, the flow appears as a steady hydraulic jump.

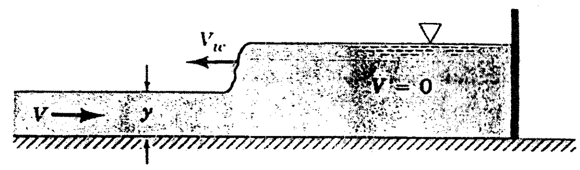
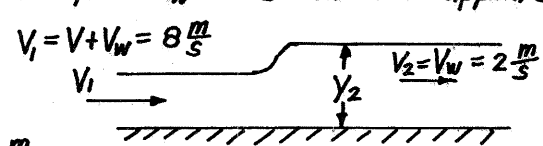


FIGURE P10.87

For an observer moving to the left with speed $V_w = 2\frac{m}{s}$ the flow appears as shown below.



Thus, treat as a jump with

$V_1 = 8\frac{m}{s}$, $V_2 = 2\frac{m}{s}$

Since $A_1 V_1 = A_2 V_2$ or $\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{8\frac{m}{s}}{2\frac{m}{s}} = 4$ it follows that

$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right] = 4$ Hence, $Fr_1 = 3.16$

However, $Fr_1 = \frac{V_1}{(g y_1)^{1/2}}$ so that

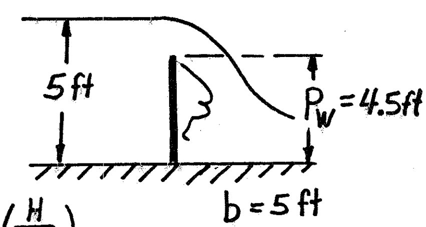
$$y_1 = \frac{V_1^2}{g\,Fr_1^2} = \frac{(8\frac{m}{s})^2}{(9.81\frac{m}{s^2})(3.16)^2} = \underline{\underline{0.652\,m}}$$

and

$y_2 = 4y_1 = 4(0.652\,m) = \underline{\underline{2.61\,m}}$

10.88

10.88 Water flows over a 5-ft-wide, rectangular sharp-crested weir that is $P_w = 4.5$ ft tall. If the depth upstream is 5 ft, determine the flowrate.



$Q = C_{wr} \frac{2}{3}\sqrt{2g}\, b H^{3/2}$, where $C_{wr} = 0.611 + 0.075\left(\frac{H}{P_w}\right)$

with

$H = 5\text{ ft} - 4.5\text{ ft} = 0.5\text{ ft}$

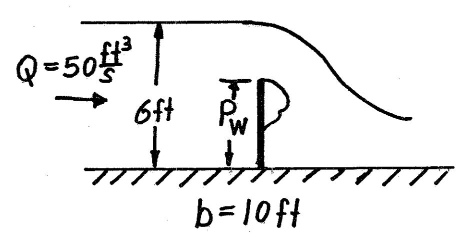
Hence, $C_{wr} = 0.611 + 0.075\left(\frac{0.5\text{ ft}}{4.5\text{ ft}}\right) = 0.619$

and

$Q = (0.619)\left(\frac{2}{3}\right)\left(2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\right)^{1/2}(5\text{ ft})(0.5)^{3/2} = \underline{\underline{5.85\frac{\text{ft}^3}{\text{s}}}}$

10.89

10.89 A rectangular sharp crested weir is used to measure the flowrate in a channel of width 10 ft. It is desired to have the channel flow depth be 6 ft when the flowrate is 50 cfs. Determine the height, P_w, of the weir plate.



$Q = C_{wr}\frac{2}{3}\sqrt{2g}\, b H^{3/2}$, where $H = 6\text{ ft} - P_w$ and

$C_{wr} = 0.611 + 0.075\frac{H}{P_w}$

Thus,

$Q = \left(0.611 + 0.075\left(\frac{6-P_w}{P_w}\right)\right)\left(\frac{2}{3}\right)(2g)^{1/2} b\,(6-P_w)$

or

$50\frac{\text{ft}^3}{\text{s}} = \left(0.611 + 0.075\left(\frac{6-P_w}{P_w}\right)\right)\left(\frac{2}{3}\right)\left(64.4\frac{\text{ft}}{\text{s}^2}\right)^{1/2}(10\text{ ft})(6-P_w)$, where $P_w \sim$ ft

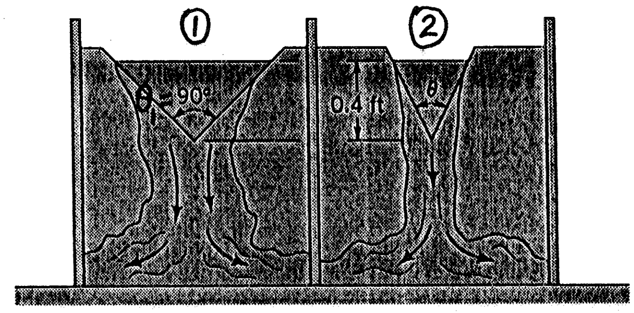
Hence,

$$\left[8.15 + \frac{(6-P_w)}{P_w}\right](6-P_w)^{3/2} - 12.5 = 0 \qquad (1)$$

By using a standard root-finding program, the solution to Eq. (1) is found to be $\underline{\underline{P_w = 4.70\text{ ft}}}$.

10.90

10.90 Water flows from a storage tank, over two triangular weirs, and into two irrigation channels as shown in Video V10.7 and Fig. P10.90. The head for each weir is 0.4 ft and the flowrate in the channel fed by the 90-degree V-notch weir is to be twice the flowrate in the other channel. Determine the angle θ for the second weir.



■ FIGURE P10.90

$$Q = C_{wt}\,\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2} \qquad (1)$$

where

$$\theta_1 = 90^\circ,\ H_1 = H_2 = 0.4\,ft,\ \text{and}\ Q_1 = 2Q_2 \qquad (2)$$

Thus, from Fig. 10.25,

$$C_{wt_1} = 0.590$$

From Eqs.(1) and (2),

$$C_{wt_1}\frac{8}{15}\tan\left(\frac{\theta_1}{2}\right)\sqrt{2g}\,H_1^{5/2} = C_{wt_2}\frac{8}{15}\tan\left(\frac{\theta_2}{2}\right)\sqrt{2g}\,H_2^{5/2} \times 2$$

or

$$0.590\tan 45^\circ = C_{wt_2}\tan\left(\frac{\theta_2}{2}\right)\times 2$$

or

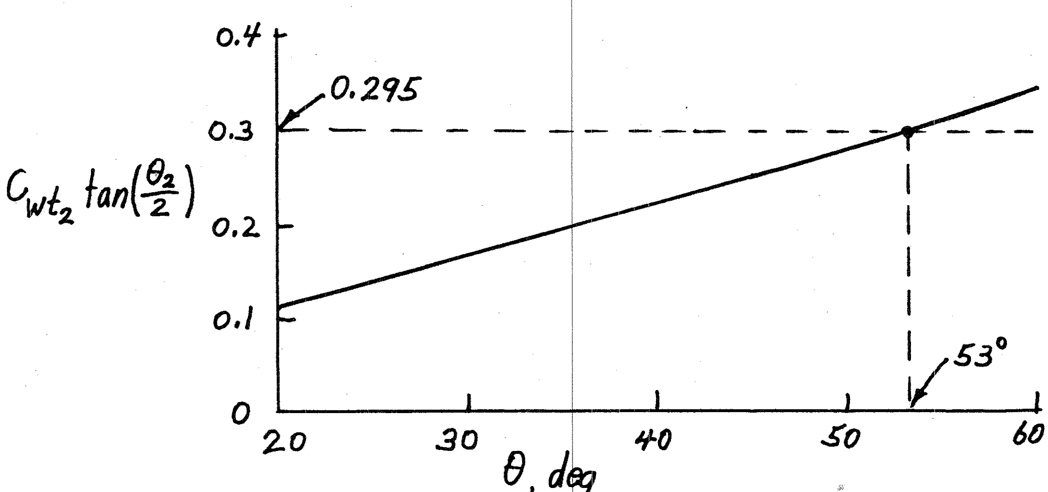
$$C_{wt_2}\tan\left(\frac{\theta_2}{2}\right) = 0.295 \qquad (3)$$

Trial and error solution: Assume $\theta_2 = 20^\circ$. From Fig. 10.16, $C_{wt_2} = 0.626$

Thus, $C_{wt_2}\tan\left(\frac{\theta_2}{2}\right) = 0.626\tan(10^\circ) = 0.110 \neq 0.295$. Thus, $\theta_2 \neq 20^\circ$

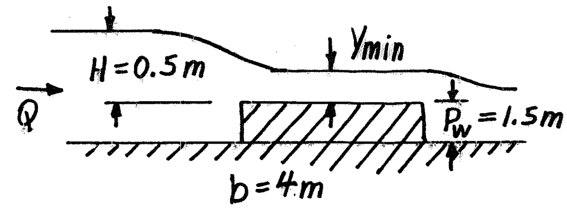
Repeated tries result in the graph below from which we conclude that

$\underline{\underline{\theta_2 = 53^\circ}}$



10.91

10.91 Water flows over a broad-crested weir that has a width of 4 m and a height of $P_w = 1.5$ m. The free-surface well upstream of the weir is at a height of 0.5 m above the surface of the weir. Determine the flowrate in the channel and the minimum depth of the water above the weir block.



$$Q = C_{wb}\, b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}, \text{ where}$$

$$C_{wb} = \frac{0.65}{\left(1+\frac{H}{P_w}\right)^{1/2}} = \frac{0.65}{\left(1+\frac{0.5\,m}{1.5\,m}\right)^{1/2}} = 0.563$$

Thus,

$$Q = (0.563)(4m)\left(9.81\frac{m}{s^2}\right)^{1/2}\left(\frac{2}{3}\right)^{3/2}(0.5m)^{3/2} = \underline{\underline{1.36\ \frac{m^3}{s}}}$$

Also,

$$y_{min} = y_c = \frac{2}{3}H = \left(\frac{2}{3}\right)(0.5m) = \underline{\underline{0.333\ m}}$$

10.92

10.92 A broad-crested weir with a height of 2.0 m is placed across a 6-m-wide channel. Determine the flowrate if the head is 0.30 m.

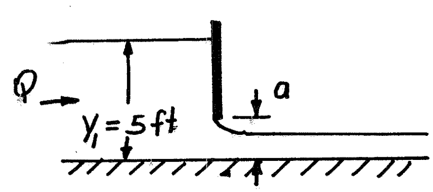
$$Q = C_{wb}\, b \sqrt{g} \left(\tfrac{2}{3}\right)^{3/2} H^{3/2}, \text{ where } H = 0.30\,m \text{ and } C_{wb} = 0.65/(1+H/P_w)^{1/2}$$

Thus, $C_{wb} = 0.65/(1+(0.30m/2.0m))^{1/2} = 0.606$ so that with $b = 6m$,

$$Q = 0.606\,(6m)\sqrt{9.81\,\tfrac{m}{s^2}}\left(\tfrac{2}{3}\right)^{3/2}(0.30m)^{3/2} = \underline{\underline{1.02\,\tfrac{m^3}{s}}}$$

10.93

10.93 Water flows under a sluice gate in a channel of 10-ft width. If the upstream depth remains constant at 5 ft, plot a graph of flowrate as a function of the distance between the gate and channel bottom as the gate is slowly opened. Assume free outflow.

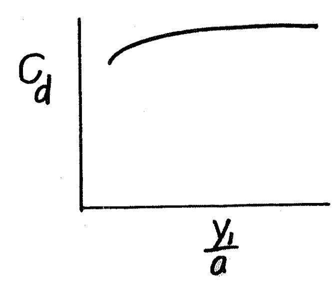


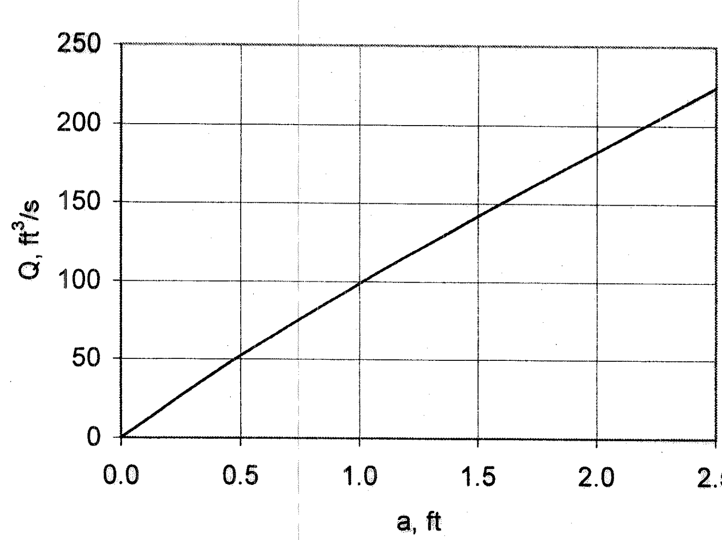
$$Q = q b = b\, C_d\, a \sqrt{2 g y_1}, \text{ where } y_1 = 5\,ft,\ b = 10\,ft, \text{ and } C_d \text{ is from Fig. 10.29}$$

Thus,

$$Q = C_d\,(10ft)\,a\left[2(32.2\tfrac{ft}{s^2})(5\,ft)\right]^{1/2} = 179\, C_d\, a\ \tfrac{ft^3}{s}, \text{ where } a \sim ft$$

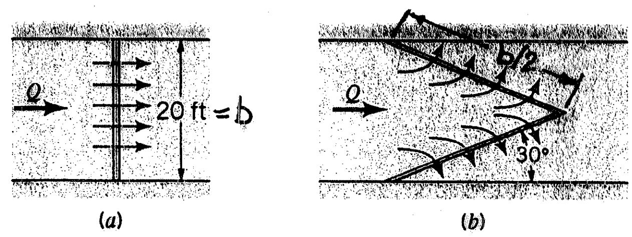
a, ft	$\frac{y_1}{a}$	C_d	Q, $\frac{ft^3}{s}$
0	∞	0.6	0
0.5	10	0.58	51.9
1.0	5	0.55	98.5
1.5	3.33	0.53	142
2.0	2.5	0.51	183
2.5	2	0.50	224





10.94

10.94 (a) The rectangular sharp-crested weir shown in Fig. P10.94*a* is used to maintain a relatively constant depth in the channel upstream of the weir. How much deeper will the water be upstream of the weir during a flood when the flowrate is 45 ft^3/s compared to normal conditions when the flowrate is 30 ft^3/s? Assume the weir coefficient remains constant at $C_{wr} = 0.62$. (b) Repeat the calculations if the weir of part (a) is replaced by a rectangular sharp-crested "duck bill" weir which is oriented at an angle of 30° relative to the channel centerline as shown in Fig. P10.94*b*. The weir coefficient remains the same.



■ FIGURE P10.94

In either case

$$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\;b\,H^{3/2} = 0.62\left(\frac{2}{3}\right)\sqrt{2\,(32.2\ ft/s^2)}\;b\,H^{3/2},$$

or

$$Q = 3.32\,bH^{3/2}, \quad \text{where } Q \sim ft^3/s \text{ when } b \sim ft \text{ and } H \sim ft \qquad (1)$$

(a) From Eq. (1) with $b = 20\ ft$, if $Q = 30\ ft^3/s$ then

$30 = 3.32\,(20)\,H_{30}^{3/2}$, or $H_{30} = 0.589\ ft$

If $Q = 45\ ft^3/s$, then

$45 = 3.32\,(20)\,H_{45}^{3/2}$, or $H_{45} = 0.772\ ft$

Thus, $\Delta H_a = H_{45} - H_{30} = 0.772\ ft - 0.589\ ft = \underline{\underline{0.183\ ft}}$

(b) From Eq. (1) with $b = 2\,(10\ ft)/\sin 30^\circ = 40\ ft$, if $Q = 30\ ft^3/s$ then

$30 = 3.32\,(40)\,H_{30}^{3/2}$, or $H_{30} = 0.371\ ft$

If $Q = 45\ ft^3/s$, then

$45 = 3.32\,(40)\,H_{45}^{3/2}$, or $H_{45} = \underline{\underline{0.486\ ft}}$

Thus, $\Delta H_b = H_{45} - H_{30} = 0.486\ ft - 0.371\ ft = \underline{\underline{0.115\ ft}}$

Note that the "duck bill" weir gives a smaller change in the head than does the "regular" weir.

10.95

10.95 Water flows in a rectangular channel of width $b = 20$ ft at a rate of 100 ft^3/s. The flowrate is to be measured by using either a rectangular weir of height $P_w = 4$ ft or a triangular ($\theta = 90°$) sharp crested weir. Determine the head, H, necessary. If measurement of the head is accurate to only ± 0.04 ft, determine the accuracy of the measured flowrate expected for each of the weirs. Which weir would be the most accurate? Explain.

(a) Rectangular weir:

$Q = \left(0.611 + 0.075\left(\frac{H}{P_w}\right)\right)\left(\frac{2}{3}\right)\sqrt{2g}\, b\, H^{3/2}$, where $P_w = 4$ ft

Thus,

$Q = \left[0.611 + 0.075\left(\frac{H}{4}\right)\right]\left(\frac{2}{3}\right)\left[2\left(32.2\,\frac{ft}{s^2}\right)\right]^{1/2}(20\,ft)\, H^{3/2}$

or

$Q = 107\,(0.611 + 0.0188\,H)\, H^{3/2}$, where $Q \sim \frac{ft^3}{s}$ and $H \sim ft$ (1)

With $Q = 100\,\frac{ft^3}{s}$ this gives $0.935 = (0.611 + 0.0188H)H^{3/2}$

or $(32.5 + H)H^{3/2} - 49.7 = 0$ (2)

By using a standard root-finding program, the solution to Eq. (2) is determined to be

$\underline{\underline{H = 1.294\,ft}}$

(b) Triangular weir:

$Q = C_{wt}\,\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2} = C_{wt}\left(\frac{8}{15}\right)(\tan 45°)\left[2\left(32.2\,\frac{ft}{s^2}\right)\right]^{1/2} H^{5/2}$

or

$Q = 4.28\,C_{wt}\,H^{5/2}\ \frac{ft^3}{s}$, where $H \sim ft$ and C_{wt} is from Fig. 10.25 (2)

For $Q = 100\,\frac{ft^3}{s}$, assume $C_{wt} = 0.58$ so that

$4.28\,(0.58)\,H^{5/2}$, or $\underline{\underline{H = 4.39\,ft}}$ Note: The assumed $C_{wt} = 0.58$ checks (see Fig. 10.25)

Calculate Q for $H = H_{100}$, $H_{100} + 0.04$, and $H_{100} - 0.04$ from Eqs. (1) and (2):

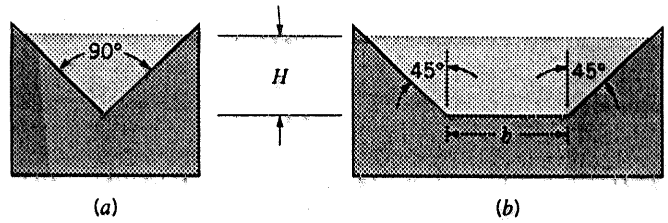
(Rectangular) H, ft	Q, cfs
1.254	95.3
$H_{100} =$ 1.294	100
1.334	104.9

(Triangular) H, ft	Q, cfs
4.35	98.0
$H_{100} =$ 4.39	100
4.43	102.5

With $H \pm 0.04$ ft it is seen that $\underline{\underline{\text{triangular weir is more accurate}}}$ (i.e. smaller variation in Q).

10.96

10.96 Water flows over a triangular weir as shown in Fig. P10.96*a* and Video V10.7. It is proposed that in order to increase the flowrate, Q, for a given head, H, the triangular weir should be changed to a trapezoidal weir as shown in Fig. P10.96*b*. (a) Derive an equation for the flowrate as a function of the head for the trapezoidal weir. Neglect the upstream velocity head and assume the weir coefficient is 0.60, independent of H. (b) Use the equation obtained in part (a) to show that when $b \ll H$ the trapezoidal weir functions as if it were a triangular weir. Similarly, show that when $b \gg H$ the trapezoidal weir functions as if it were a rectangular weir.

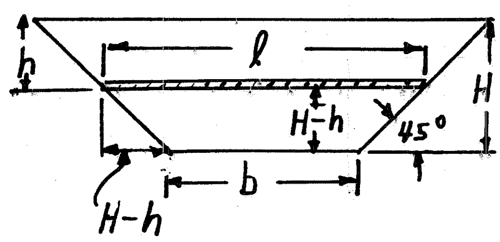


■ FIGURE P10.96

(a) $Q = C_w \int_{h=0}^{h=H} u_2 \, \ell \, dh$, where $u_2 = \sqrt{2gh}$

and

$\ell = b + 2(H-h) = (2H+b) - 2h$



Thus,

$$Q = C_w \int_0^H \sqrt{2gh}\,[(2H+b) - 2h]\,dh$$

$$= C_w\sqrt{2g}\,(2H+b)\int_0^H \sqrt{h}\,dh - C_w\sqrt{2g}\,(2)\int_0^H h^{3/2}\,dh$$

$$= C_w\sqrt{2g}\,(2H+b)\,\frac{2}{3}H^{\frac{3}{2}} - C_w\sqrt{2g}\,\frac{2}{5}H^{5/2}$$

or

(1) $$Q = C_w\left[\frac{2}{3}\sqrt{2g}\,b\,H^{3/2} + \frac{8}{15}\sqrt{2g}\,H^{5/2}\right], \text{ where } C_w = 0.6$$

Note: This equation is simply the sum of Q for a rectangular weir and Q for a triangular weir. That is $Q_{\text{trap}} = Q_{\square} + Q_{\triangledown}$.

(b) From Eq. (1)

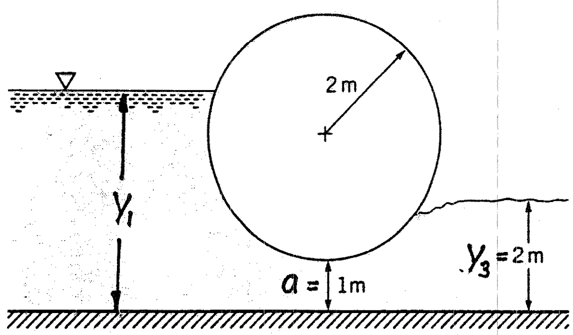
$$Q = C_w\sqrt{2g}\,H^{3/2}\left[\frac{2}{3}b + \frac{8}{15}H\right]$$

Thus, if $b \ll H$, $Q \approx C_w\sqrt{2g}\,H^{\frac{3}{2}}\left[\frac{8}{15}H\right] = C_w\sqrt{2g}\,\frac{8}{15}H^{5/2}$ which is the equation for a triangular wier.

Similarily, if $b \gg H$, $Q \approx C_w\sqrt{2g}\,b\,\frac{2}{3}H^{3/2}$ which is the equation for a rectangular weir.

10.97

10.97 A water-level regulator (not shown) maintains a depth of 2.0 m downstream from a 10-m-wide drum gate as shown in Fig. P10.97. Plot a graph of flowrate, Q, as a function of water depth upstream of the gate, y_1, for $2.0 \le y_1 \le 5.0$ m.



■ FIGURE P10.97

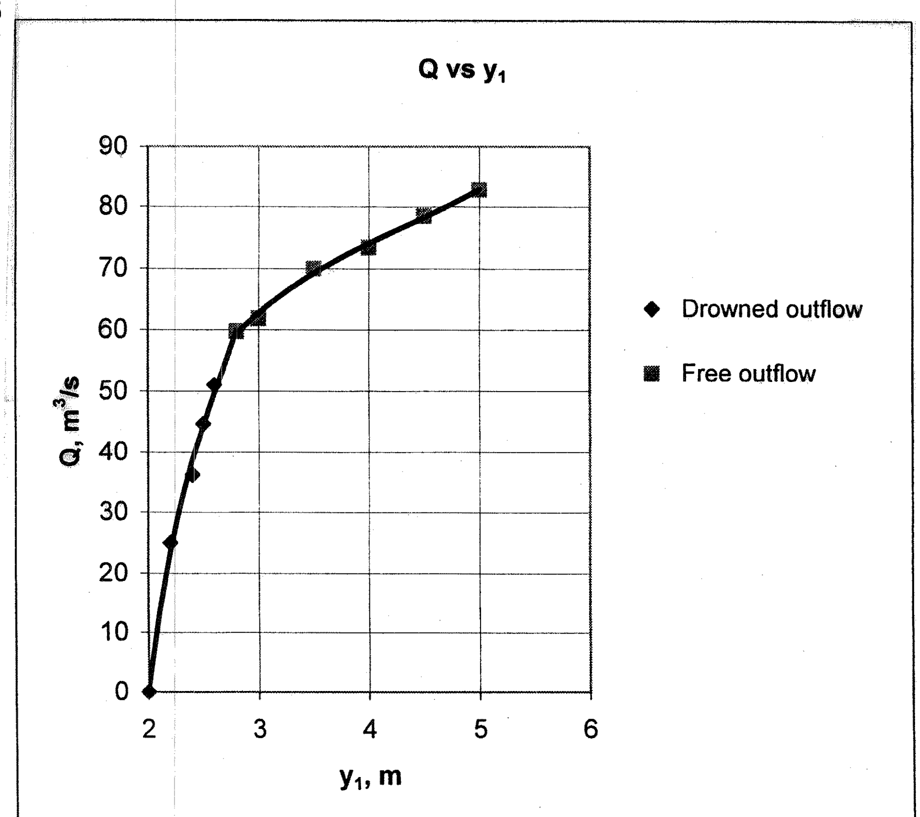
$Q = bq = b\,C_d\,a\sqrt{2gy_1}$ where $a = 1\,m$ and $b = 50\,ft\,(0.3048\,\frac{m}{ft}) = 15.2\,m$

Thus,

$Q = (15.2\,m)\,C_d\,(1\,m)\sqrt{2(9.81\,\frac{m}{s^2})(y_1\,m)} = 67.3\,C_d\sqrt{y_1}\ \frac{m^3}{s}$ where $y_1 \sim m$

Obtain C_d from Fig. 10.29 with $\frac{y_3}{a} = 2$.

y1, m	y1/a	Cd	Q, m^3/s
2.00	2.00	0	0.0
2.20	2.20	0.25	25.0
2.40	2.40	0.35	36.5
2.50	2.50	0.42	44.7
2.60	2.60	0.47	51.0
2.80	2.80	0.53	59.7
3.00	3.00	0.53	61.8
3.50	3.50	0.54	68.0
4.00	4.00	0.545	73.4
4.50	4.50	0.55	78.5
5.00	5.00	0.55	82.8



10.98

10.98 Calibration of a Triangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.98 to calibrate a triangular weir and determine the relationship between flowrate, Q, and weir head, H.

Equipment: Water channel (flume) with a pump and a flow control valve; triangular weir; float; point gage; stop watch.

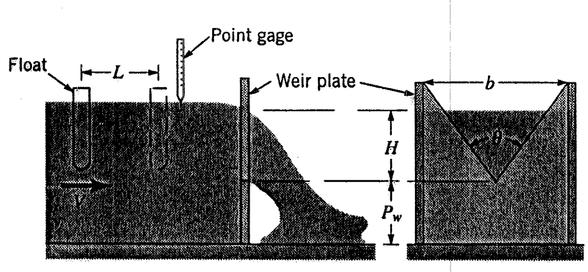
Experimental Procedure: Measure the width, b, of the channel, the distance, P_w, between the channel bottom and the bottom of the V-notch in the weir plate, and the angle, θ, of the V-notch. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q, over the weir. Use the point gage to measure the weir head, H. Insert the float into the water well upstream from the weir and measure the time, t, it takes for the float to travel a known distance, L. Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q, as ordinates and weir head, H, as abscissas. Draw the best-fit line with a slope of 5/2 through the data.

Results: Use the flowrate-weir head data to determine the triangular weir coefficient, C_{wt}, for this weir (see Eq. 10.32). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.98

(con't.)

10.98 (con't)

Solution for Problem 10.98: Calibration of a Triangluar Weir

θ, deg	b, in.	P_w, in.	L, ft
90	6.00	6.55	1.50

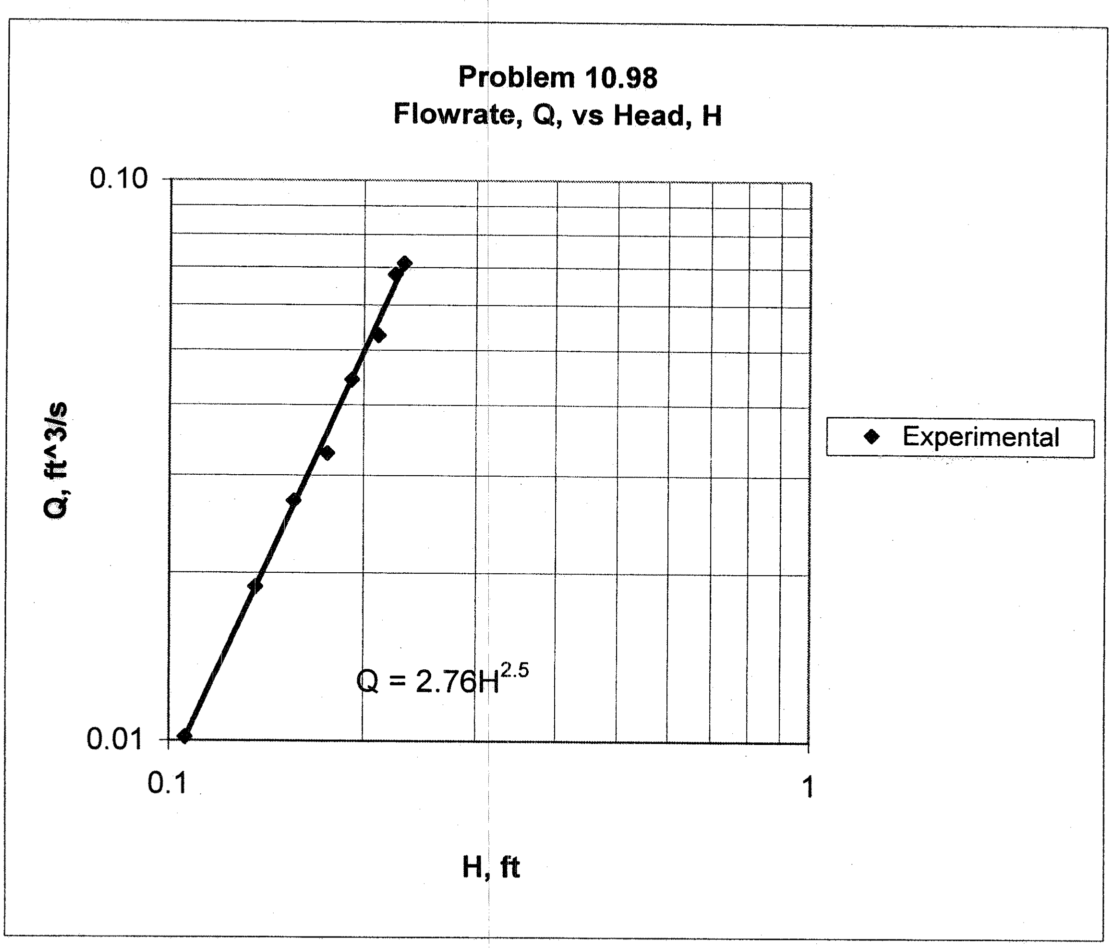
H, ft	t, s	V, ft/s	Q, ft^3/s
0.231	8.2	0.183	0.0711
0.224	8.5	0.176	0.0679
0.211	10.7	0.140	0.0530
0.192	12.5	0.120	0.0443
0.176	16.5	0.091	0.0328
0.156	19.5	0.077	0.0270
0.136	27.1	0.055	0.0189
0.106	48.2	0.031	0.0101
0.091	62.9	0.024	0.0076
0.088	68.1	0.022	0.0070

$Q = VA = V*b(P_w + H)$ where $V = L/t$

$Q = C_{wt}\,(8/15)\tan(\theta/2)\,(2g)^{1/2}\,H^{5/2}$ where from the graph

$Q = 2.76\,H^{2.5}$

Thus, $C_{wt} = (15/8)*2.76/(2*32.2)^{1/2} = \underline{0.645}$



10.99

10.99 Calibration of a Rectangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.99 to calibrate a rectangular weir and determine the relationship between flowrate, Q, and weir head, H.

Equipment: Water channel (flume) with a pump and a flow control valve; rectangular weir; float; point gage; stop watch.

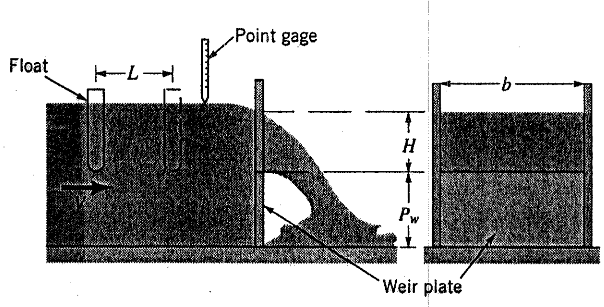
Experimental Procedure: Measure the width, b, of the channel and the distance, P_w, between the channel bottom and the top of the weir plate. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q, over the weir. Use the point gage to measure the weir head, H. Insert the float into the water well upstream from the weir and measure the time, t, it takes for the float to travel a known distance, L. Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q, as ordinates and weir head, H, as abscissas. Draw the best-fit line with a slope of 3/2 through the data.

Results: Use the flowrate-weir head data to determine the rectangular weir coefficient, C_{wr}, for this weir (see Eq. 10.30). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.99

(con't)

10.99 (con't)

Solution for Problem 10.99: Calibration of a Rectangular Weir

b, in.	P_w, in.	L, ft
6.00	6.00	1.40

H, ft	t, s	V, ft/s	Q, ft^3/s
0.254	2.2	0.636	0.240
0.216	2.7	0.519	0.186
0.184	3.0	0.467	0.160
0.162	4.2	0.333	0.110
0.151	4.5	0.311	0.101
0.111	6.6	0.212	0.065
0.060	15.8	0.089	0.025
0.046	23.8	0.059	0.016
0.031	38.4	0.036	0.010

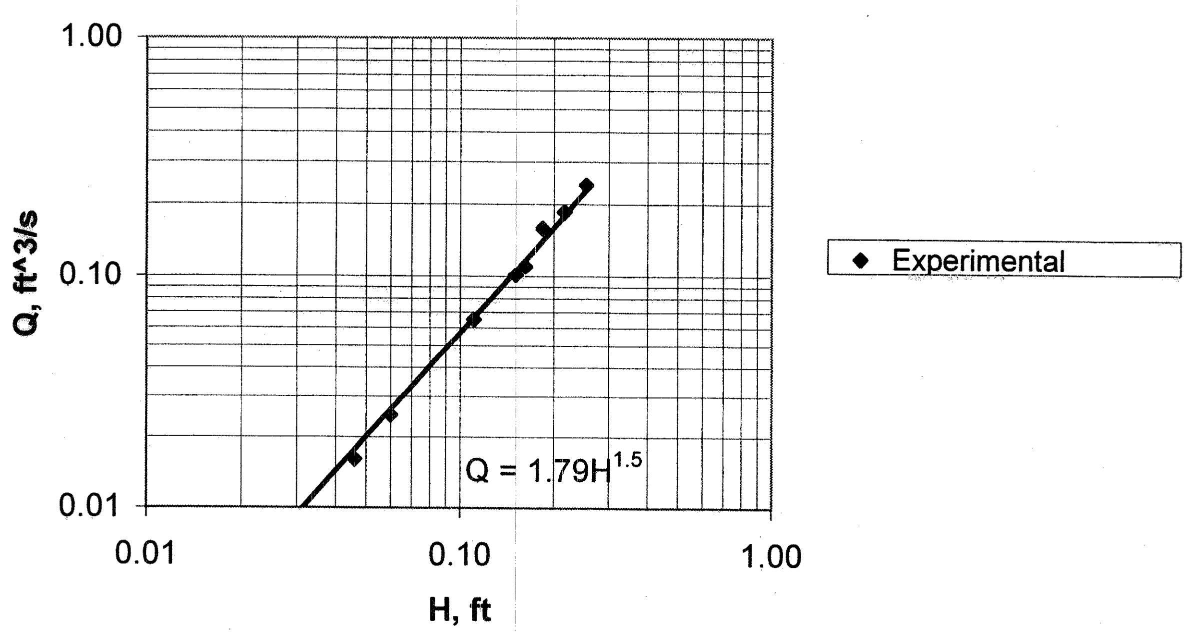
$Q = VA = V*b(P_w + H)$ where $V = L/t$

$Q = C_{wr}\,(2/3)\,(2g)^{1/2}\,H^{3/2}\,b$ where from the graph

$Q = 1.79\,H^{1.5}$

Thus, $C_{wr} = (3/2)*1.79/(0.5*(2*32.2)^{1/2}) = \underline{0.669}$

Problem 10.99
Flowrate, Q, vs Head, H



10.100

10.100 Hydraulic Jump Depth Ratio

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.100 to determine the depth ratio, y_2/y_1, across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1.

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; adjustable tail gate.

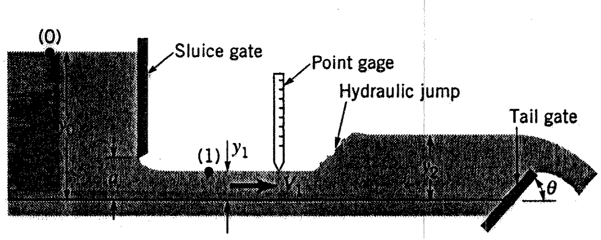
Experimental Procedure: Position the sluice gate so that the distance, a, between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0, upstream of the sluice gate. Carefully adjust the angle, θ, of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0, the depth just upstream from the jump, y_1, and the depth downstream from the jump, y_2. Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1, and Froude number, $\mathrm{Fr}_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump (see Eq. 3.21). Also use the measured depths to determine the depth ratio, y_2/y_1, across the jump.

Graph: Plot the depth ratio, y_2/y_1, as ordinates and Froude number, Fr_1, as abscissas.

Results: On the same graph, plot the theoretical depth ratio as a function of Froude number (see Eq. 10.24).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.100

(con't)

10.100 (con't)

Solution for Problem 10.100: Hydraulic Jump Depth Ratio

y_o, ft	y_1, ft	y_2, ft.	Experimental V_1, ft/s	Experimental Fr_1	Experimental y_2/y_1	Theoretical Fr_1	Theoretical y_2/y_1
0.855	0.055	0.404	7.19	5.40	7.35	1	1.00
0.759	0.055	0.386	6.75	5.07	7.02	2	2.37
0.691	0.055	0.367	6.42	4.82	6.67	3	3.77
0.578	0.055	0.337	5.83	4.38	6.13	4	5.18
0.492	0.055	0.308	5.34	4.01	5.60	5	6.59
0.414	0.055	0.280	4.85	3.65	5.09	6	8.00
0.289	0.055	0.233	3.95	2.97	4.24		
0.248	0.055	0.211	3.62	2.72	3.84		

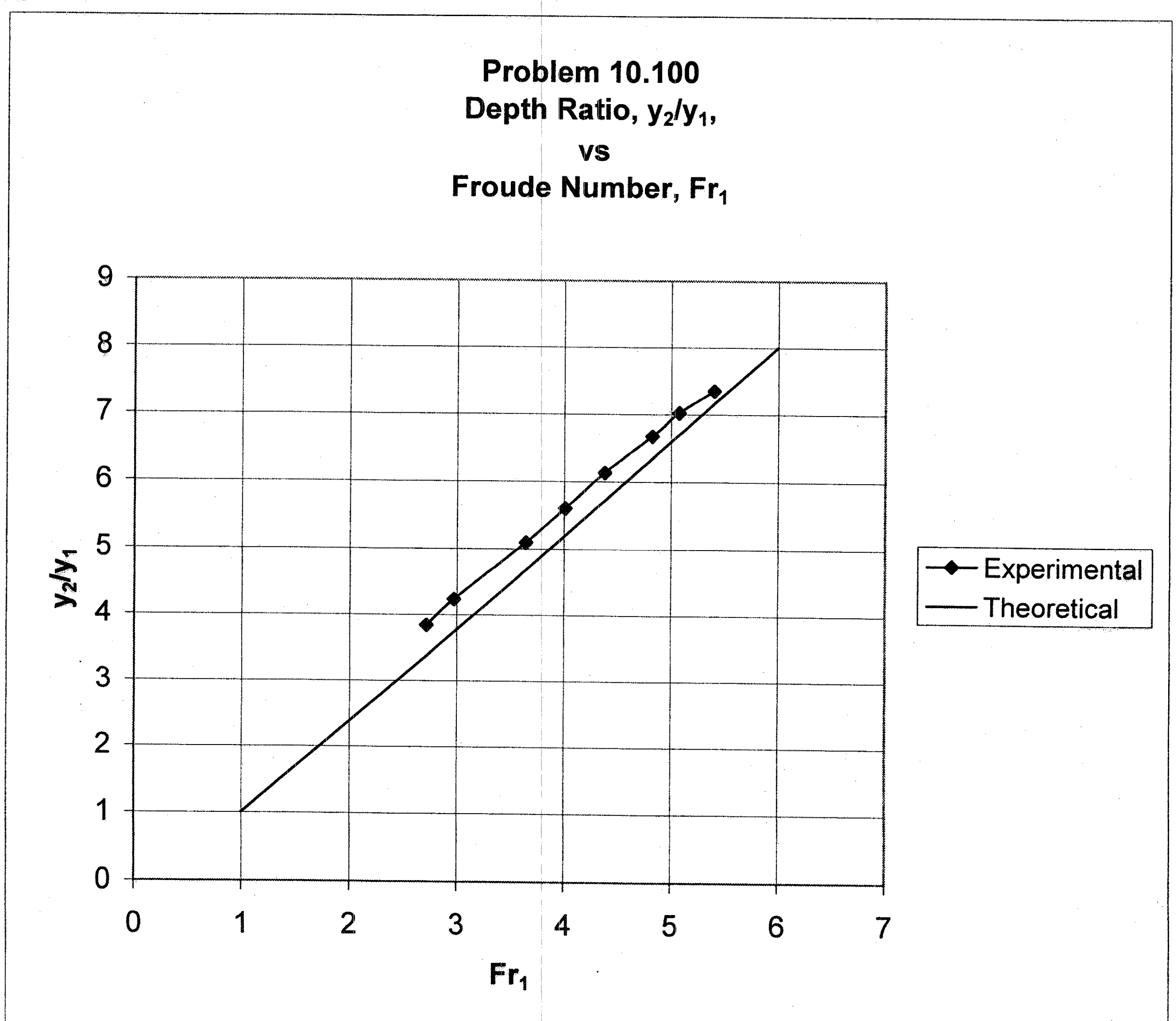
For flow under a sluice gate:

$$V_1 = [2g*(y_o - y_1)/(1 - (y_1/y_o)^2)]^{1/2}$$

Theory:

$$y_2/y_1 = [-1 +(1 +8Fr_1^2)^{1/2}]/2$$

$$Fr_1 = V_1/(gy_1)^{1/2}$$



10.101

10.101 Hydraulic Jump Head Loss

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.101 to determine the head loss ratio, h_L/y_1, across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1.

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; Pitot tubes; adjustable tail gate.

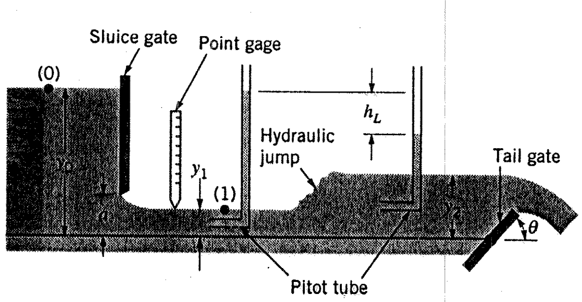
Experimental Procedure: Position the sluice gate so that the distance, a, between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0, upstream of the sluice gate. Carefully adjust the angle, θ, of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0, and the depth just upstream from the jump, y_1. Also measure the head loss, h_L, as the difference in the water elevations in the piezometer tubes connected to the two Pitot tubes located upstream and downstream of the jump. Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1, and the Froude number, $Fr_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump. Also calculate the dimensionless head loss, h_L/y_1, for each data set.

Graph: Plot the dimensionless head loss across the jump, h_L/y_1, as ordinates and the Froude number, Fr_1, as abscissas.

Results: On the same graph, plot the theoretical dimensionless head loss as a function of Froude number (see Eqs. 10.24 and 10.25).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.101

(con't)

10.101 (con't)

Solution for Problem 10.101: Hydraulic Jump Head Loss

				Experimental			Theoretical		
y_0, ft	y_1, ft	y_2, ft.	h_L, ft	V_1, ft/s	Fr_1	h_L/y_1	Fr_1	y_2/y_1	h_L/y_1
0.855	0.055	0.404	0.364	7.19	5.40	6.62	1	1.00	0.00
0.759	0.055	0.386	0.313	6.75	5.07	5.69	2	2.37	0.27
0.691	0.055	0.367	0.271	6.42	4.82	4.93	3	3.77	1.41
0.578	0.055	0.337	0.201	5.83	4.38	3.65	4	5.18	3.52
0.492	0.055	0.308	0.152	5.34	4.01	2.76	5	6.59	6.62
0.414	0.055	0.280	0.117	4.85	3.65	2.13	6	8.00	10.72
0.289	0.055	0.233	0.058	3.95	2.97	1.05			
0.248	0.055	0.211	0.042	3.62	2.72	0.76			

For flow under a sluice gate:

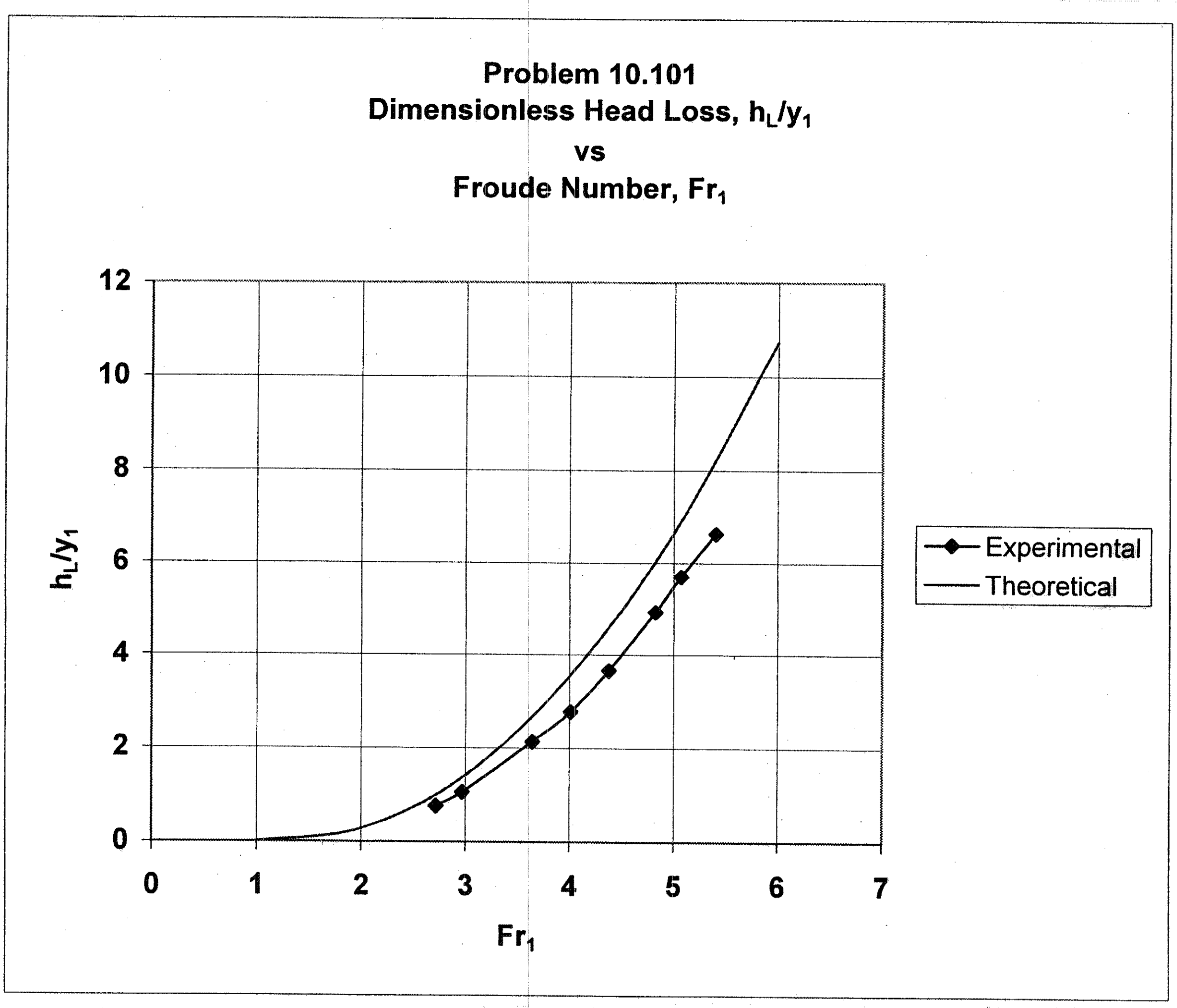
$$V_1 = [2g*(y_0 - y_1)/(1 - (y_1/y_0)^2)]^{1/2}$$

Theory:

$$h_L/y_1 = 1 - (y_2/y_1) + Fr_1^2[1 - (y_1/y_2)^2]/2$$

where

$$y_2/y_1 = [-1 + (1 + 8Fr_1^2)^{1/2}]/2$$



10.102

10.102 (See **"Tsunami, the nonstorm wave,"** Section 10.2.1.) An earthquake causes a shift in the ocean floor that produces a tsunami with a wavelength of 100 km. How fast will this wave travel across the ocean surface if the ocean depth is 3000 m?

$$c = \left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{1/2}, \text{ where } \lambda = 100\,km = 10^5\,m \text{ and } y = 3000\,m.$$

Thus,

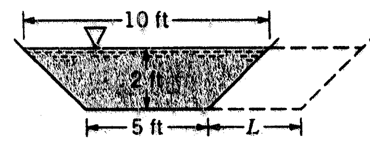
$$c = \left[\frac{9.81\,\frac{m}{s^2}(10^5\,m)}{2\pi}\tanh\left(\frac{2\pi(3000\,m)}{10^5\,m}\right)\right]^{1/2} = 171\,\frac{m}{s}$$

or

$$c = 171\,\frac{m}{s}\left(\frac{3600\,s}{hr}\right)\left(\frac{1\,km}{10^3\,m}\right) = \underline{\underline{616\,\frac{km}{hr}}}$$

10.103

10.103 (See "**Plumbing the Everglades,**" Section 10.4.1.) The canal shown in Fig. P10.103 is to be widened so that it can carry twice the amount of water. Determine the additional width, L, required if all other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same.



■ FIGURE P10.103

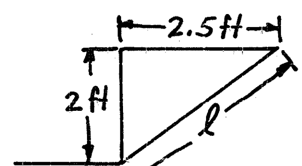
Let $(\ )_o$ denote the original canal and $(\ )_w$ the widened canal. Thus,

(1) $Q_o = \frac{\kappa}{n_o} A_o R_{h_o}^{2/3} \sqrt{S_{o_o}}$ and

(2) $Q_w = \frac{\kappa}{n_w} A_w R_{hw}^{2/3} \sqrt{S_{ow}}$, where $n_o = n_w$ and $S_{o_o} = S_{ow}$

Hence, from Eqs. (1) and (2)

(3) $\frac{Q_w}{Q_o} = \frac{\frac{\kappa}{n_w} A_w R_{hw}^{2/3} \sqrt{S_{ow}}}{\frac{\kappa}{n_o} A_o R_{h_o}^{2/3} \sqrt{S_{o_o}}} = \frac{A_w}{A_o}\left(\frac{R_{hw}}{R_{ho}}\right)^{2/3}$, where $Q_w = 2Q_o$



$\ell = [2.5^2 + 2^2]^{1/2}\ ft = 3.20\ ft$

Also, $A_o = \frac{1}{2}(5\ ft + 10\ ft)(2\ ft) = 15\ ft^2$

$P_o = 5\ ft + 2(3.20\ ft) = 11.4\ ft$, so that

$R_{h_o} = A_o / P_o = 15\ ft^2 / 11.4\ ft = 1.316\ ft$

and

$A_w = \frac{1}{2}\left[(5\ ft + L) + (10\ ft + L)\right](2\ ft) = (15 + 2L)\ ft^2$

$P_w = 5\ ft + L + 2(3.20\ ft) = (11.4 + L)\ ft$, so that

$R_{h_w} = A_w / P_w = (15 + 2L)/(11.4 + L)$

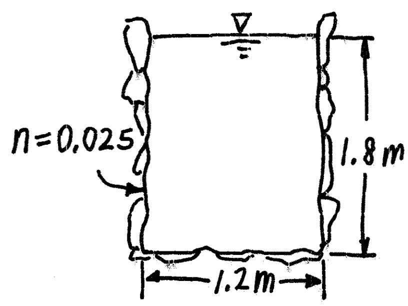
Hence, from Eq. (3) with $\frac{Q_w}{Q_o} = 2$,

(4) $2 = \frac{(15 + 2L)}{15}\left[\frac{(15 + 2L)/(11.4 + L)}{1.316}\right]^{2/3}$, where $L \sim ft$

By using a standard root-finding program, the solution to Eq. (4) is determined to be $\underline{\underline{L = 5.94\ ft}}$

10.104

10.104 (See "Done without a GPS or lasers," Section 10.4.3.) Determine the number of gallons of water delivered per day by a rubble masonary, 1.2-m-wide aqueduct laid on an average slope of 14.6 m per 50 km if the water depth is 1.8 m.



$Q = \frac{\kappa}{n} A R_h^{2/3} \sqrt{S_o}$,

where $A = 1.2\,m\,(1.8\,m) = 2.16\,m^2$ and

$P = 1.2\,m + 2(1.8\,m) = 4.8\,m$ so that

$R_h = A/P = (2.16\,m^2)/(4.8\,m) = 0.450\,m$

Thus, with $\kappa = 1$,

$$Q = \frac{1}{0.025}(2.16\,m^2)(0.450\,m)^{2/3}\left(\frac{14.6\,m}{50\times10^3\,m}\right)^{1/2} = 0.867\,m^3/s$$

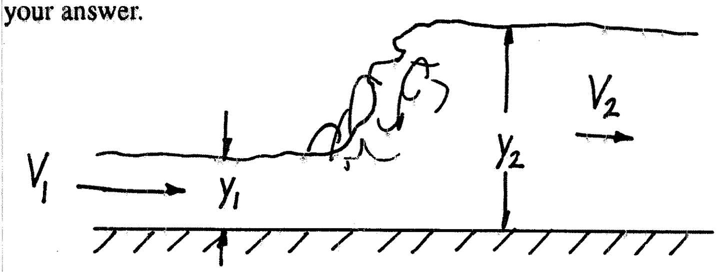
or

$$Q = 0.867\frac{m^3}{s}\left(\frac{3600\,s}{1\,hr}\right)\left(\frac{24\,hr}{1\,day}\right)\left(\frac{1\,ft^3}{0.0283\,m^3}\right)\left(\frac{1728\,in.^3}{1\,ft^3}\right)\left(\frac{1\,gal}{231\,in.^3}\right)$$

$$= \underline{\underline{19.8\times10^6\,gal/day}}$$

10.105

10.105 (See "Grand Canyon rapids building," Section 10.6.1.) During the flood of 1983, a large hydraulic jump formed at "Crystal Hole" rapid on the Colorado River. People rafting the river at that time report "entering the rapid at almost 30 mph, hitting a 20-ft-tall wall of water, and exiting at about 10 mph." Is this information (i.e., upstream and downstream velocities and change in depth) consistent with the principles of a hydraulic jump? Show calculations to support your answer.



Is the given data consistent with a hydraulic jump?

$V_1 = 30\,mph = 44\,ft/s$

$V_2 = 10\,mph = 14.7\,ft/s$

$y_2 - y_1 = 20\,ft$

From conservation of mass: $A_1 V_1 = A_2 V_2$
or $y_1 V_1 = y_2 V_2$ since $b_1 = \text{width} = b_2$

Thus,

$$\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{44\,ft/s}{14.7\,ft/s} = 2.99 \qquad (1)$$

Also, for a hydraulic jump

$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\,Fr_1^2}\right)$ so that $2.99 = \frac{1}{2}\left(-1 + \sqrt{1 + 8\,Fr_1^2}\right)$

or

$Fr_1 = 2.44$

Thus, since $Fr_1 = \frac{V_1}{\sqrt{g y_1}}$ it follows that

$2.44 = \frac{44\,ft/s}{(32.2\,ft/s^2\, y_1)^{1/2}}$ or $y_1 = 10.1\,ft$ so that from Eq. (1),

$y_2 = 2.99\,y_1 = 2.99\,(10.1\,ft) = 30.2\,ft$

Hence, the given data gives $y_2 - y_1 = 30.2\,ft - 10.1\,ft = 20.1\,ft$, which is surprisingly close to the reported depth. Yes, the data is consistent with the principles of a hydraulic jump.