# Fluid flow measurements

In the following sections we will consider methods for measuring fluid flow. This is a very important matter in industry. It refers to the ability to measure the velocity, volume flow rate, or mass flow rate of any fluid. Many devices are available for measuring flow. Some measure an average velocity of flow that can be used to calculate volume flow rate by Q=VA. Some provide direct primary measurements, whereas others require calibration or using discharge coefficients to the measured values. Virtually any measuring device can give an accuracy of 5% of the actual flow. Most commercial meters are capable of 2% accuracy or better. Cost usually becomes an important factor when great accuracy is desired. The performance of fluid meters is affected by the nature of fluid (liquid or gas), viscosity, temperature, corrosiveness, electrical conductivity, optical clarity and other factors. Slurries and multiphase require special meters.

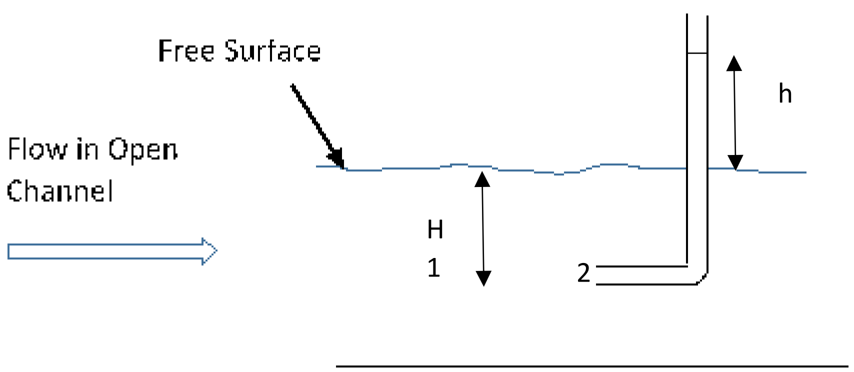
The devices that are still in use in industry which will be considered in the following sections are based on the following assumptions:

- Bernoulli's Equation assumptions
- Negligible friction heating effects (will be corrected for)
- The depend on direct measurement of a pressure difference present or created between two points in the flow stream. (In most cases a constriction is introduced in the flow stream to create a relatively large pressure difference over a short distance that can be easily measured. Constriction means narrowing. The decrease in pressure is translated into an increase in kinetic energy).

**Pitot Tubes:** two types are available: In both types local point velocities can be measured indirectly based o pressure difference measurements as explained below.

**Pitot Tube (Impact tube or stagnation tube)**

This consists of a tube bent into an L shape and placed in the flow stream in such a manner that it is facing the opposite direction of flow as shown in the diagram. The flow of a liquid in an open channel is considered.



Some of the liquid enters the tube and rises in it to a height h above the free surface of the liquid. This is equivalent to converting kinetic energy into potential energy. The velocity at a point below the surface at a distance H (point 1) can be found by applying BE between points 1 and 2. Point 1 is considered far from point 2 so that it is not affected by the presence of the tube. Point 2 is just at the entrance of the tube.

In this case: Friction is negligible

$V_2$ is zero since the fluid in the tube is not moving. It is stagnant. $Z_1 = Z_2$

BE: $$\frac{P_2 - P_1}{\rho} - \frac{V_1^2}{2} = 0$$

$$\left.\begin{array}{l} P_1 = P_{atm} + \rho g H \\ P_2 = P_{atm} + \rho g (H + h) \end{array}\right\} \rightarrow \quad (P_2 - P_1) = \rho g h$$

Substitute and rearrange $\therefore\ V_1 = \sqrt{2gh}$

h is measured → $V_1$ is calculated

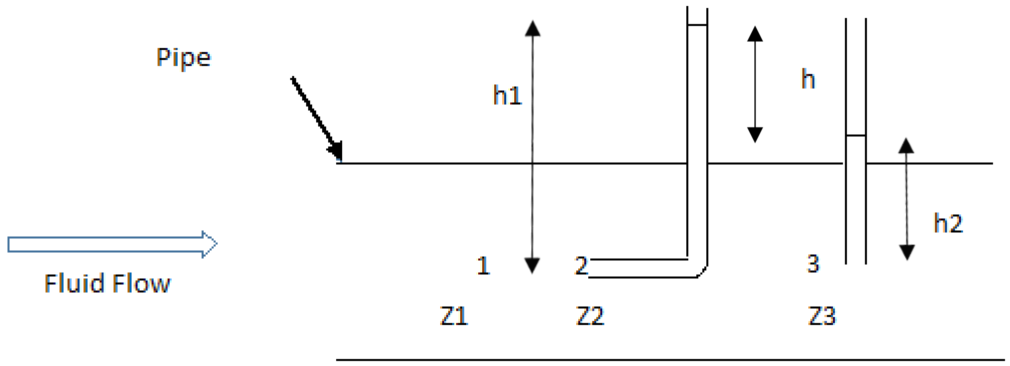
Note that an indirect measure of V is obtained and $V \propto \sqrt{h}$

This device is:

- ✓ Used to measure a local point velocity in a given direction (i.e. velocity at a certain point and in a certain direction)
- ✓ Suitable for flow measurements in open channels or measurement of velocities of boats.

# Pitot Static Tube:

When velocities are high (large h) a different device is used. It is similar to the previous one but with an additional tube that measures static pressure. The L shaped tube is still the same and positioned in the same way as above, and it measures both the effect of static pressure and kinetic energy effect. In both tubes the height of liquid corresponding to the potential head gained in both cases, so that we expect the height in the L shaped tube is higher. The difference in height corresponds to the pressure difference between them which can be measured and can be used to calculate the local point velocity at point 1.
This device is suitable for flow measurements in pipes and in aeroplanes to measure their speeds in air.




To find $V_1$ we apply BE as Follows:

$Z_1=Z_2=Z_3$ $\quad$ $V_2 = 0$ (stagnant fluid) $\quad$ $V_1= V_3$ (same area, no change in velocity)

First apply BE between 1 and 2

$$\frac{P_2 - P_1}{\rho} - \frac{V_1^2}{2} = 0 \quad \rightarrow \quad V_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

Now we need to find $(P_2 - P_1)$

Apply BE between 1 and 3

$$\frac{P_3 - P_1}{\rho} - \frac{V_3^2 - V_1^2}{2} = 0 \quad \rightarrow \quad V_1 = V_3 \;;\; \text{from Continuity equation}$$

$\therefore P_1 = P_3$ and

$$V_1 = \sqrt{\frac{2(P_2 - P_3)}{\rho}}$$

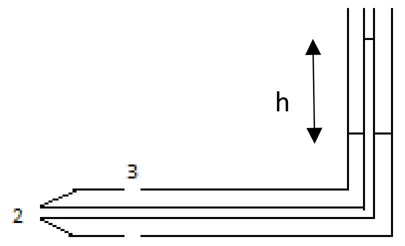
$(P_2 - P_3) = \Delta P$ which can be measured by pressure gauges, OR

$$\left.\begin{aligned} P_2 &= P_{atm} + \rho g h_1 \\ P_3 &= P_{atm} + \rho g h_2 \end{aligned}\right\} \rightarrow (P_2 - P_3) = \rho g h$$

$$\therefore V_1 = \sqrt{2gh}$$

h can be measured.

In practice the two tubes are often combined one inside the other.




The inner tube faces the flow point 2; while the outer tube has holes tangent to the flow point 3. The inner tube measures the total head (static + dynamic); while the outer tube measures the static head or Piezometric head (points P1 or P2).

Notes:

- In BE

  $(\frac{P}{\rho g} + Z + \frac{V^2}{2g}) = constant = Total\ head\ (static + dynamic\ heads)\ [point\ P2]$

  $(\frac{P}{\rho g} + Z) = static\ or\ piezometric\ head\ \ [points\ P1\ , P3]$

  $(\frac{V^2}{2g})$ = Dynamic head

  In effect the Pitot tube function is equivalent to:

  Dynamic head = total head – static head

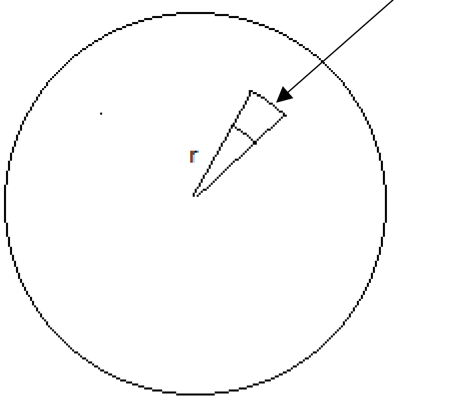
- A pressure gauge connected to the side of a tube measures the Piezometric head

**Volumetric flow rate, Q**

Consider the case of a circular tube, the volume flow rate dQ through an elemental area dA is:

$dA = r\, d\theta\, dr$

dQ = V dA;




V is the local velocity measured by a Pitot tube at a distance r from the center of the tube. It is function of r only and independent of $\theta$ due to symmetry.

$$\therefore Q = \int V\, dA$$

For flow in pipe

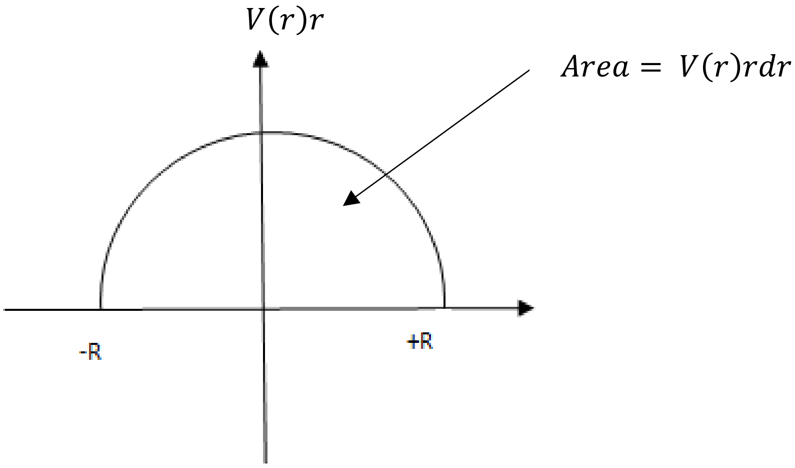
$$Q = \int_0^R \int_0^{2\pi} V(r) r\, d\theta dr$$

$$Q = 2\pi \int_0^R V(r) r dr$$

V(r) is the velocity as function of r.

As an approximation: $Q \cong 2\pi \sum V(r) r \Delta r$

So we make 10 measurements starting at one side of the tube moving to the other end by equal intervals $\Delta r$ passing through the center. Use the approximate equation to find Q. The integral is the area under the curve. The 10 measurements give twice this area since we are going from –R to +R.


$V(r)r$
$Area = V(r)rdr$
-R
+R

**Extra Notes on pitot tubes**

## PRINCIPLE AND WORKING OF PITOT TUBE - DEFINITION

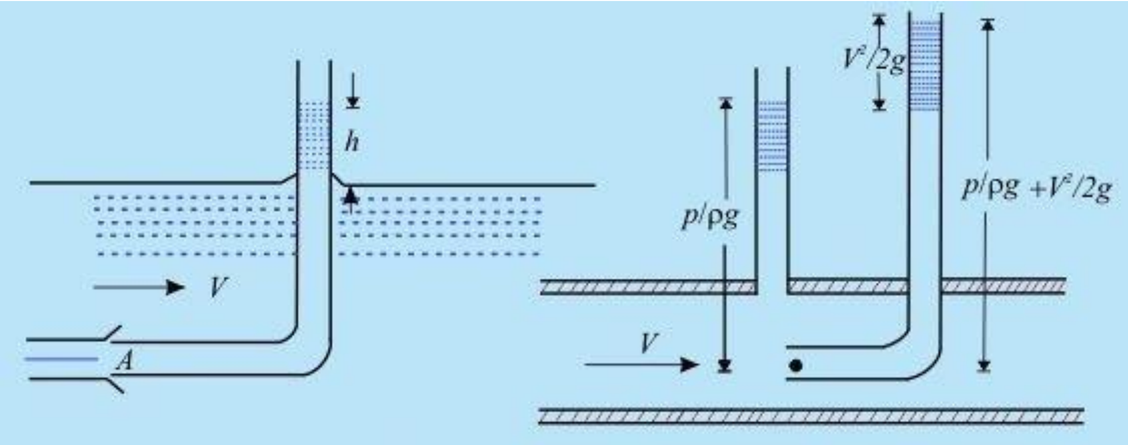



Fig Simple Pitot Tube (a) tube for measuring the Stagnation Pressure (b) Static and Stagnation tubes together

A pitot tube is an open-ended right-angled tube pointing in opposition to the flow of a fluid and used to measure pressure.

Working:

The liquid flows up the tube and when equilibrium is attained, the liquid reaches a height above the free surface of the water stream. Since the static pressure, under this situation, is equal to the hydrostatic pressure due to its depth below the free surface, the difference in level between the liquid in the glass tube and the free surface becomes the measure of dynamic pressure. Therefore, we can write, neglecting friction,

$$(P_0 - P) = \frac{\rho V^2}{2} = \rho g h$$

where $p_0$, $p$ and $V$ are the stagnation pressure, static pressure and velocity respectively at point A (fig. a);

$$\therefore\ V = \sqrt{2gh}$$

Such a tube is known as a Pitot tube and provides one of the most accurate means of measuring the fluid velocity. For an open stream of liquid with a free surface, this single tube is sufficient to determine the velocity. But for a fluid flowing through a closed duct, the Pitot tube measures only the stagnation pressure and so the static pressure must be measured separately. Measurement of static pressure in this case is made at the boundary of the wall (Fig. b). The axis of the tube measuring the static pressure must be perpendicular to the boundary and free from burrs, so that the boundary is smooth and hence the streamlines adjacent to it are not curved. This is done to sense the static pressure only without any part of the dynamic pressure. A Pitot tube is also inserted as shown (Fig. b) to sense the stagnation pressure. The ends of the Pitot tube, measuring the stagnation pressure, and the piezometric tube, measuring the static pressure, may be connected to a suitable differential manometer for the determination of flow velocity and hence the flow rate.

Example: A Pitot tube is inserted in an air flow (at STP) to measure the flow speed. The tube is inserted so that it points upstream into the flow and the pressure sensed by the tube is the stagnation pressure. The static pressure is measured at the same location in the flow, using a wall pressure tap. If the pressure difference is 30 mm of mercury, determine the flow speed.

A Pitot tube inserted in a flow as shown. The flowing fluid is air and the manometer liquid is mercury. Determine the flow speed.

Solution:

Governing equation: BE
Assumptions: (1) Steady flow.
(2) Incompressible flow.
(3) Flow along a streamline.
(4) Frictionless deceleration along stagnation streamline.
Writing Bernoulli's equation along the stagnation streamline (with $\Delta z$=0) yields

$(P_0 - P) = \frac{\rho V^2}{2}$ $P_0$ is the stagnation pressure at the tube opening where the speed has been reduced, without friction, to zero. Solving for $V$ gives

$$V = \sqrt{\frac{2(P_0 - P)}{\rho_{air}}}$$

From the diagram,

$$P_0 - p = \rho_{Hg} g\ h = \rho_{H2O}\ g\ h\ SG_{Hg}$$

and

$$V = \sqrt{\frac{2\ \rho_{H_2O}\ g\ h\ SG_{Hg}}{\rho_{air}}}$$

$$V = \sqrt{2 \text{ x } 1000\ \frac{kg}{m^3} \text{ x } 9.81 \frac{kgm}{s^2} \text{ x } 30\ mm \text{ x } 13.6 \text{ x } \frac{m^3}{123\ kg} \text{ x } \frac{1m}{1000\ mm}}$$

V= 80.8 m/s

At $T$=20$^o C$, the speed of sound in air is 343 m/s. Hence, $M$=0.236 and the assumption of incompressible flow is valid. M is the Mach number = multiples of speed of sound.