Flow Measurements based on Bernoulli Equation (Variable-Head Meters)

In the following sections we will consider two types of flow meters that make use of the idea of making a narrowing (constriction. i.e. reduce the area) in the flow stream, so that there will be an increase in the kinetic energy (increase in velocity) at the expense of basically injection work (reduction in pressure). This will create a measurable pressure difference over a short distance which can be measured and then used to calculate the velocity through the constriction. This velocity can then be used as an average velocity to calculate the flow through the constriction knowing its area: Q = V A.

Two of the most common types of variable-head meters are the orifice plate and the Venturi meters.

Orifice Meter:

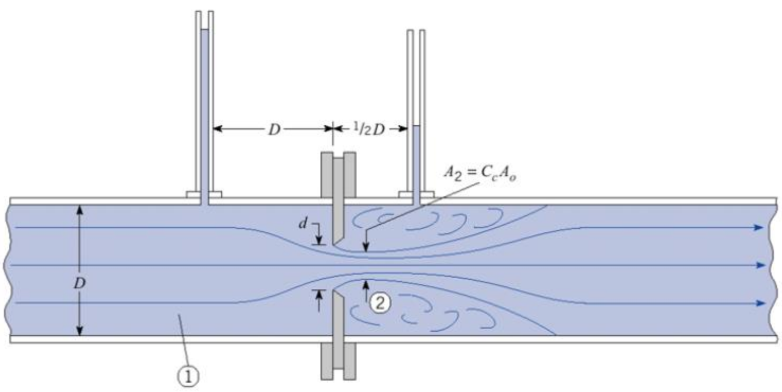
This device is:

- simple
- easy to construct
- relatively cheap
- suitable for small pipe lines (small flow rates)

It consists of a plate placed perpendicular to the flow with a central hole (called orifice). Pressure tapings (openings) are provided upstream and slightly down stream. The pressure taping upstream is placed at a distance equal to the pipe diameter $D \pm .1\,D$ from the orifice, while the downstream taping is placed at a distance equal to $0.5D \pm .05\,D$.

The fluid flows through the orifice as seen in the figure accelerating through it. The fluid leaves the orifice as a jet of slightly smaller diameter then it regains its velocity far from the orifice. Some eddies are formed as seen in the figure at the corners after the plate. The formation of these eddies consumes some energy which we neglect in the initial treatment.

Flow through a sharp-edged pipe orifice.



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The objective is to find the velocity at position 2. Again we apply BE between points 1 and 2 assuming negligible friction and $Z_1 = Z_2$

BE: $\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} = 0$

CE: $V_1 A_1 = V_2 A_2 \quad \rightarrow V_1 = V_2 \quad (\frac{A_2}{A_1})$

Substitute and rearrange and correct for velocity and area, we obtain:

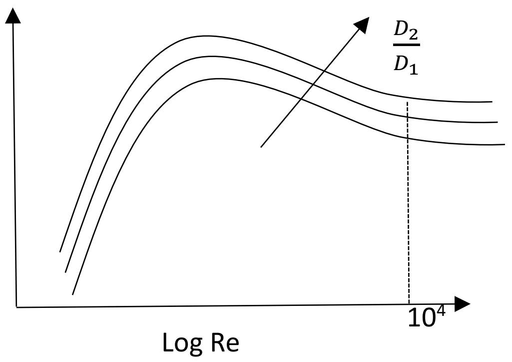
$$V_2 = c_v \sqrt{\frac{2(P_1 - P_2)}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$$

The correction factor $c_v = f\ (\frac{A_2}{A_1}$, Re at orifice i.e. at orifice conditions)

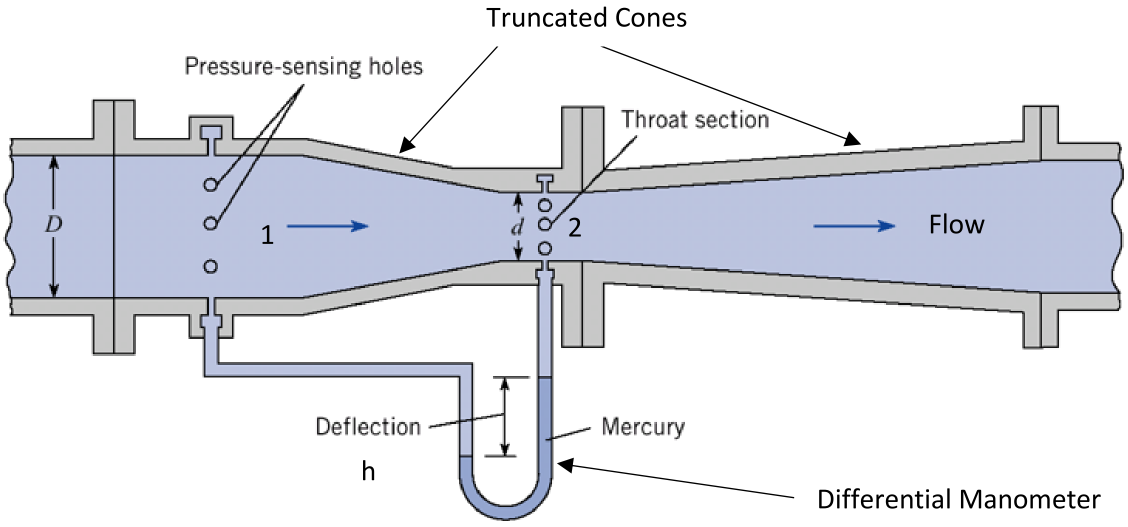
Re: Reynolds number. It is dimensionless.

$$Re = \frac{\rho V D}{\mu}$$

At low Re, c_v increases rapidly to a maximum (0.7 – 0.9) then it falls to a virtually constant value (0.6).



Venturi Meter:



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This is another type of variable-head meter consisting of two truncated cones. The narrowing, which is the place where the two heads of the truncated cones meet, is called the throat of the meter. The section after the throat is called the diffuser.

The BE and the CE can be used to obtain an expression for a velocity which can be used to calculate the flow rate. Using sections 1 and 2 in the figure as the reference points, we obtain the following equations:

BE: $$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} = 0$$

CE: $$V_1 A_1 = V_2 A_2 \quad \rightarrow V_1 = V_2 \quad \left(\frac{A_2}{A_1}\right)$$

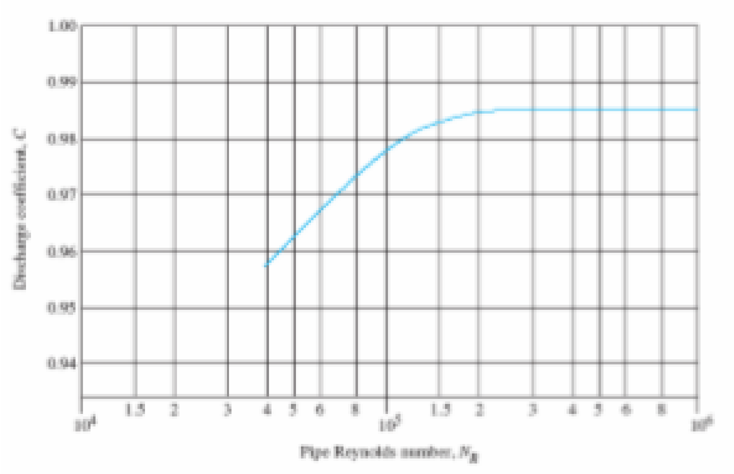
Substitute and rearrange and correct for velocity and area, we obtain:

$$V_2 = c_v \sqrt{\frac{2(P_1 - P_2)}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$$

c_v = discharge coefficient which allows for non-uniform flow velocity distribution and friction effects. This coefficient:

- for a well-designed Venturi meter is equal t0 0.96 – 0.98
- It depends on the ratio $\left(\frac{A_2}{A_1}\right)$

- It depends on the Reynolds number evaluated at the throat conditions.



In theory $Q = V_2 A_2$

In practice $Q_{actual} = Q_{theoretical} \times C_v$

If the pressure difference is measured using a differential type manometer as in this case, then

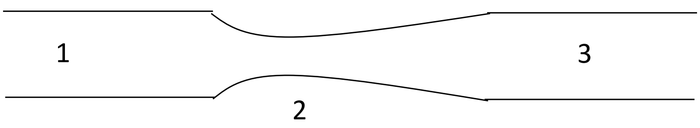
$$(P_1 - P_2) = \Delta\rho g h; \quad \Delta\rho = \rho_m - \rho$$

Therefore

$$V_2 = \sqrt{\frac{2\Delta\rho g h}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$$

Characteristics of Venturi Meter:

- The shape of the Venturi meter is important. The diffuser has a narrow angle (7^o) in order to slow down the recovery of pressure and minimise losses.



$P_1 > P_2$ $\quad$ $P_1 \neq P_3$ (slightly higher)

- It is reliable
- Used for large flow rates
- Difficult to fabricate and fairly expensive
- Requires a fair amount of space