Ch 6: Fluid Friction in Steady One-Dimensional Flow.

Introduction:

In the working form of BE

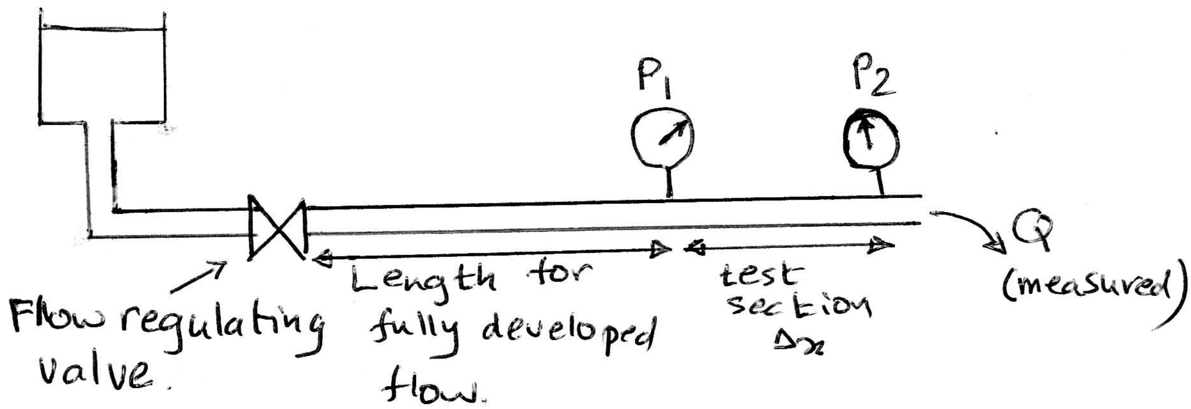
$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = +\hat{W} - \mathcal{F}$$

Q. How is $\mathcal{F}$ evaluated?

A. The form of the relation of $\mathcal{F}$ depends on:
- geometry of system : pipe, duct, channel
- nature of flow : Laminar, turbulent, ---

First let us have a look at what happens to the pressure of a fluid flowing in a pipe for example.

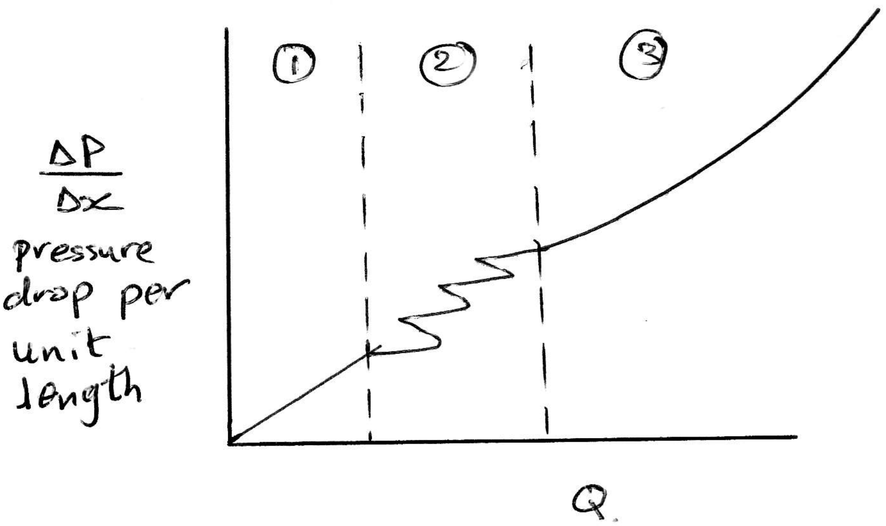
Pressure drop:



BE: $\boxed{\frac{\Delta P}{\rho} = +\mathcal{F}}$ $\quad \Delta P = (P_1 - P_2)$

This tells us that the pressure P_1 drops to P_2. This change in pressure is converted into friction since the cross section is constant and no change in height ($\Delta z = 0$).

For a certain fluid flowing in a certain pipe, it was found experimentally that the pressure drop per unit length is proportional to the flow as follows:



① $\frac{\Delta P}{\Delta x} \propto Q^{1.0}$ (Low Q)

② Results are not stable. They are not reproducible (intermittant flow)

③ $\frac{\Delta P}{\Delta x} \propto Q^{1.8-2.0}$ high Q
1.8: smooth pipes
2.0: rough pipes

Nature of Fluid flow:

The nature of fluid flow might be:

- Laminar : The fluid is flowing as Lamina
- Turbulent: motion in all directions superimposed on the overall motion
- Transition : Laminar or turbulent.

The limits for these types of flows is characterised by a dimensionless number called <u>Reynolds</u> Number defined as:

$$Re = \frac{\text{Inertial force}}{\text{Viscous force}}$$

$$= \frac{\rho V L}{\mu}$$; L: characteristic length. For flow inside a pipe L = D (inside)

Dimensions:

$$\frac{(ML^{-3})(LT^{-1})(L)}{\frac{(MLT^{-2})(L^{-2})}{T^{-1}}} = \frac{MLT^{-2}}{MLT^{-2}}$$

Experimental observations made by Reynolds revealed that there are <u>three</u> types of fluid flow:

① Laminar flow : - Low Q
- Fluid flows as Lamina
- $Re < 2000 - 2200$
- $\frac{\Delta P}{\Delta x} \propto Q^{1.0}$

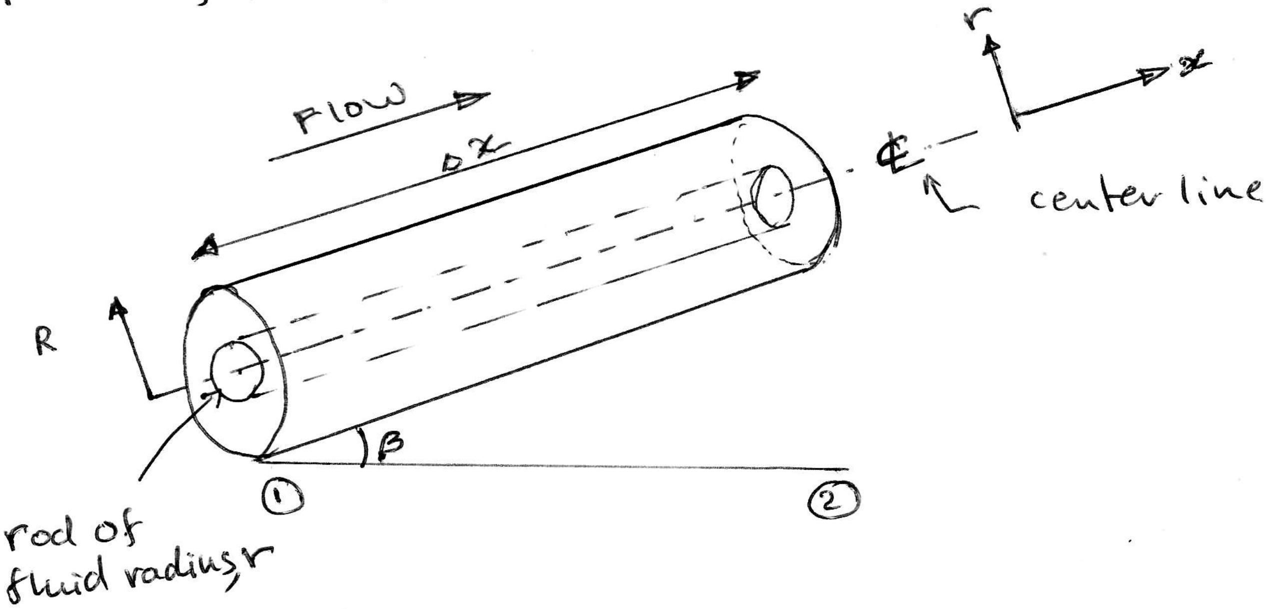
② Unstable Region : - Intermediat flow
- $2000 < Re < 4000$
- Flow is unstable, Laminar or turbulent.

③ Turbulent flow : - High flow rate
- $Re > 4000$
- $\frac{\Delta P}{\Delta x} \propto Q^{1.8-2.0}$
- motion in all directions superimposed on the overall motion.

Laminar Flow in a pipe:

Many quantities can be obtained from simple derivations for laminar flow: Average velocity, Maximum velocity, Friction at the walls, ---

Consider the flow of a fluid in a circular pipe as follows:



Make a force balance on a rod of of radius (r) under the following conditions:

- S.S flow (Laminar)
- ρ, μ are constant
- Newtonian fluid
- Constant cross section
- Location ① is far from entrance (fully developed flow)

In direction x: $\sum F_x = 0$

Net pressure force $= \pi r^2 (P_1 - P_2)$

Gravity force $= -(\pi r^2 \Delta x) \cdot \rho \cdot g \cdot \sin\beta$

Shear force resisting Flow $= -(2\pi r \Delta x) \cdot \tau$

$$\therefore \quad \pi r^2 (P_1 - P_2) - (\pi r^2 \Delta x)\, \rho g \sin\beta - 2\pi r \Delta x \cdot \tau = 0$$

rearrange

$$2\cancel{\pi} r \,\Delta x\, \tau = \cancel{\pi} r^2 (P_1 - P_2) - \cancel{\pi} r^2 \Delta x\, \rho g \sin\beta$$

$$\tau = \frac{r(P_1 - P_2) - r\,\Delta x\, \rho g \sin\beta}{2\,\Delta x}$$

$$= \frac{1}{2} r \left[\frac{(P_1 - P_2) - \rho g\, \Delta x \sin\beta}{\Delta x} \right]$$

$$\boxed{\tau = \frac{1}{2} r \frac{\Delta \mathcal{P}}{\Delta x}}$$; → relation between τ and r

For Newtonian Fluid

$$\tau = -\mu \frac{dv_x}{dr}$$

$$\therefore \boxed{\frac{dv_x}{dr} = -\frac{\Delta \mathcal{P}}{2\mu \Delta x} r}$$; solution → relation between v_x and r

Integrate :

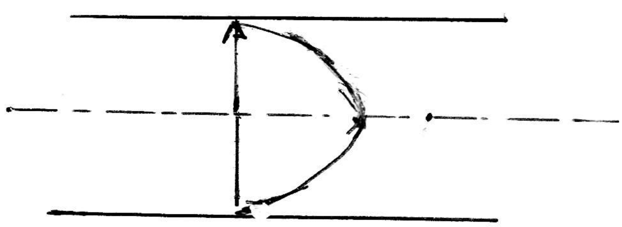
$$v_x = -\frac{\Delta \mathcal{P}}{4\mu \Delta x} r^2 + C$$

at $r = r_0$ $\quad v_x = 0$

$$\therefore C = \frac{\Delta \mathcal{P}}{4\mu \Delta x} r_0^2$$

$$\therefore v_x = -\frac{\Delta \mathcal{P}}{4\mu \Delta x} r^2 + \frac{\Delta \mathcal{P}}{4\mu \Delta x} r_0^2$$

$$\boxed{v_x = \frac{\Delta \mathcal{P} r_0^2}{4\mu \Delta x} \left[1 - \left(\frac{r}{r_0} \right)^2 \right]}$$ parabolic



velocity profile