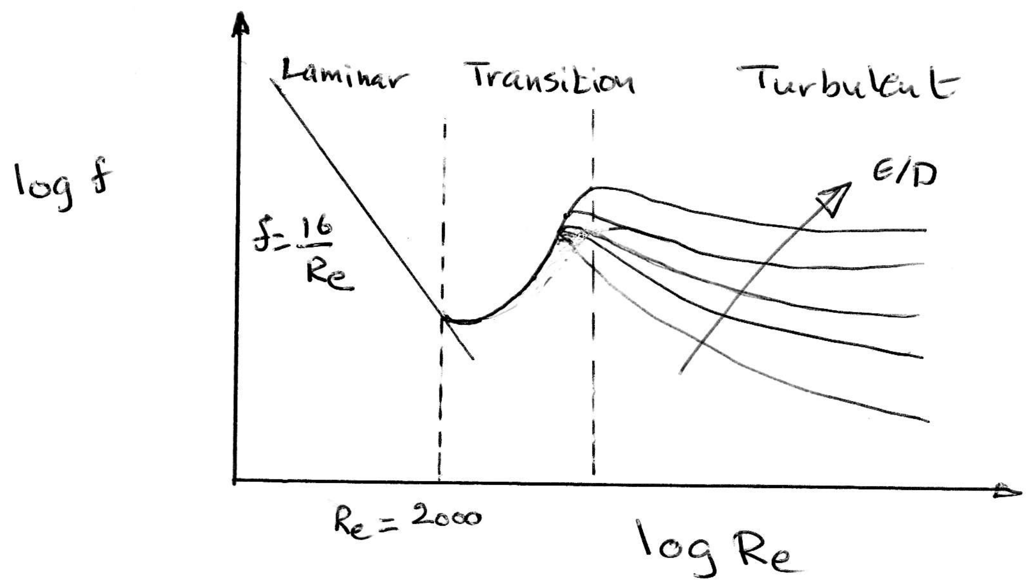
Friction Factor plots:

These are charts relating friction factor "f" to Reynolds number "Re" and relative roughness "$\frac{\epsilon}{D}$" based on Experimental data.

Moody Diagram: (see attached chart and notice the definition used for f)



Correlations for Friction factor:

▲ Laminar Flow : $f = \frac{16}{Re}$

▲ Turbulent Flow: Many correlations are found in literature.

Blasius Equation: (simple correlation for smooth pipes $\frac{\epsilon}{D}$ has negligible effect.

$$f = 0.079 \, Re^{-1/4}$$

used in the range

$$10^4 < Re < 10^5$$

6.12

TABLE 6.2
Values of surface roughness for various materials[†]

Material	Surface roughness ϵ, ft	Surface roughness ϵ, in
Drawn tubing (brass, lead, glass, etc.)	0.000005	0.00006
Commercial steel or wrought iron	0.00015	0.0018
Asphalted cast iron	0.0004	0.0048
Galvanized iron	0.0005	0.006
Cast iron	0.00085	0.010
Wood stave	0.0006–0.003	0.0072–0.036
Concrete	0.001–0.01	0.012–0.12
Riveted steel	0.003–0.03	0.036–0.36

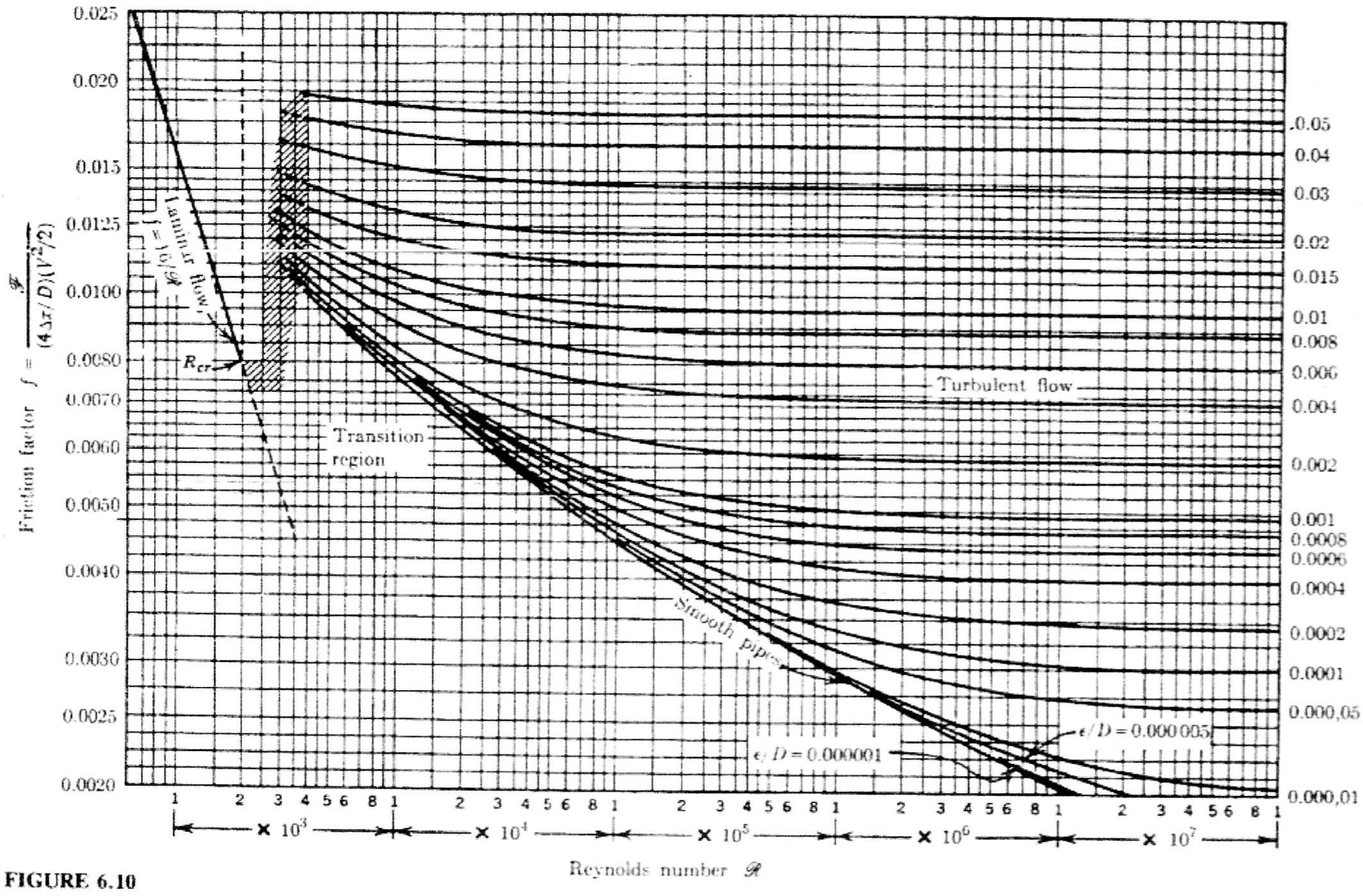


FIGURE 6.10
Friction factor plot for pipes. [From L. W. Moody, "Friction factors for pipeflow," *Trans. ASME 66*: 672 (1944). Reproduced by permission of the publisher.]

- The turbulent and transition region curves on the Moody diagram can be represented with very good approximation by:

$$f = 0.001375\left[1 + \left(20{,}000\frac{\epsilon}{D} + \frac{10^6}{Re}\right)^{1/3}\right]$$

Ex: what is f for $\frac{\epsilon}{D} = 0.0002$ and $Re = 10^5$

$$f = 0.001375\left[1 + \left(20\,000 \times 0.0002 + \frac{10^6}{10^5}\right)^{1/3}\right] = 0.0047$$

From chart $\boxed{f = 0.00475}$ (difference $< 1\%$)

- <u>Colebrook</u> <u>Equation</u>:

$$\frac{1}{\sqrt{f}} = -4\log\left(\frac{\epsilon/D}{3.7} + \frac{1.255}{Re\sqrt{f}}\right)$$

To calculate f a numerical method is used (Trial and error).

$\boxed{f = 0.004751}$

- <u>Wood's Approximation</u>:

In the range of Moody Diagram

$$f = a + b\,Re^{-c}$$

$$a = 0.0235\,(\epsilon/D)^{0.225} + 0.1325\,(\epsilon/D)$$

$$b = 22(\epsilon/D)^{0.44}$$

$$c = 1.62\,(\epsilon/D)^{0.134}$$

Note: For a given pipe roughness a, b and c are constants. This makes the use of this equation easier.

$\boxed{f = 0.00483}$

FLUID FRICTION IN NON CIRCULAR CONDUITS: [Daugherty p 224]

Hydraulic Radius: [convenient means of expressing size and shape of conduit]

$$R_h = \frac{A}{P}$$

A : Cross sectional area available for flow.

P : wetted perimeter.

For example, for a circular pipe flowing full.

$$R_h = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

$\therefore$ $D = 4R_h$

$\therefore$ substitute D by $4R_h$.

$$\therefore \quad R_e = \frac{(4R_h)V\rho}{\mu}$$

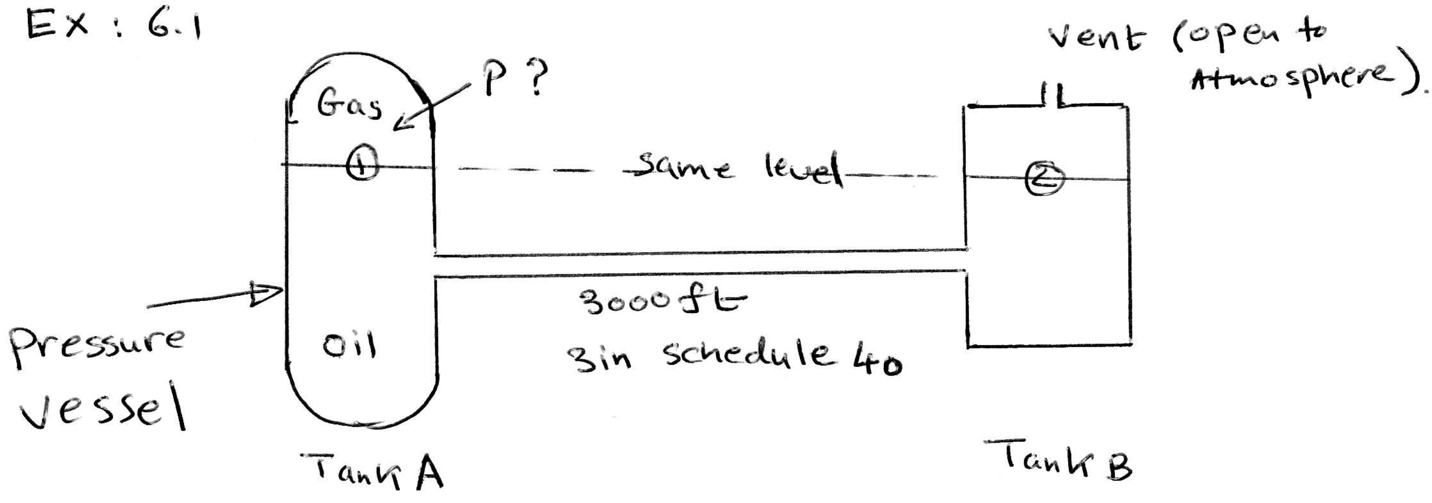
$$f = \frac{\mathcal{F}}{(4\Delta x/4R_h)(V^2/2)} \Rightarrow \mathcal{F} = f\,\frac{4\Delta x}{4R_h}\cdot\frac{V^2}{2}$$

$$\frac{e}{D} = \frac{e}{4R_h}$$

This approach gives good results for turbulent flow. [friction depends on wetted perimeter].

For Laminar flow results are poor, since frictional phenomena depends on viscous effects.

Ex : 6.1



The oil flows from tank A to tank B under the effect of the pressure of the gas in tank A on top of oil surface.

Given:

$Q = 100$ gal/min

$\rho = 58$ lbm/ft^3

$\mu = 100$ cP

ID of pipe = 3in schedule 40.

Required:

P on top of oil

Solution:

From table ID of pipe = 3.068 in

Schedule Number: Related to thickness of pipe. Pipes are manufactured to standards with different sizes having the same "Nominal size" with same OD but different ID. The data is tabulated in tables (Given in e learning)

To find P, we apply BE between ① and ②

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^2 - V_1^2}{2} = -\mathcal{F}$$

$z_2 - z_1 = 0$ (same level)

$V_1^2 - V_2^2 = 0$ (V_1 and V_2 are very small; large tank cross section)

$$\therefore \frac{P_2 - P_1}{\rho} = -\mathcal{F}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho} = +\mathcal{F} \Rightarrow \boxed{P_{1,gauge} = \rho \mathcal{F}} \quad (P_2 = P_{atm}).$$

To find $\mathcal{F}$, we must check on Re first.

$$V_{ave} = \frac{Q}{A} = \frac{100\,gal/min}{(\pi/4)(3.068\,in)^2} \times \underbrace{\frac{144\,in^2}{1\,ft^2} \times \frac{1\,min}{60\,s} \times \frac{1\,ft^3}{7.48\,gal}}_{\text{conversions.}}$$

$$= 4.37\ ft/s \quad (= 1.33\,m/s)$$

$$Re = \frac{\rho V D}{\mu} \quad ; \quad Q = VA \rightarrow V = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}}$$

$$= \frac{4\rho Q}{\pi \mu D}$$

$$= 4 \times \frac{58\,lbm/ft^3 \times 100\,gal/min}{\pi \times 100\,cP \times 3.068\,in} \times \frac{1\,min}{60\,s} \times \frac{1\,ft^3}{7.48\,gal} \times \frac{12\,in}{1\,ft} \times \frac{ft\,s\,cP}{6.72\times10^{-4}\,lbm}$$

$$= 957.73$$

$\therefore$ Flow is laminar.

Applying Hagen Equation.

$$\mathcal{F} = \frac{\Delta P}{\rho} = Q\,\frac{128}{\rho\pi}\,\frac{\mu}{D^4}\cdot\Delta x \qquad (\Delta P = P_1 - P_2)$$

$$\therefore P_{1,gauge} = \frac{100\,gal/min \times 128 \times 100\,cP \times 3000\,ft}{\pi\,(3.068\,in)^4}$$

$$\times \frac{231\,in^3}{gal} \times 2.09\times10^{-5}\,\frac{lbf.s}{cP\,ft^2} \times \frac{1\,min}{60\,s} \times \frac{1\,ft}{12\,in}$$

$$= 93\,\frac{lbf}{in^2}$$

$$= 93\,psig$$

Note: we can use Darcy Equation

$$\Delta P = \frac{4\Delta x f\left(\frac{1}{2}\rho v^2\right)}{D} \quad ; \quad f = \frac{16}{Re}$$

6.17

(continued)

ominal pipe ze, in	Outside diam., in	Schedule no.	Wall thickness, in	Inside diam., in	Cross-sectional area metal, in^2	Inside sectional area, ft^2	Circumference, ft, or surface, ft^2/ft, of length: Outside	Inside	Capacity at 1 ft/s velocity: U.S. gal/min	lb/h water	Weight of pipe, lb/ft
2	2.375	40	0.154	2.067	1.075	0.02330	0.622	0.542	10.45	5,225.0	3.66
		80	0.218	1.939	1.477	0.02050	0.622	0.508	9.20	4,600.0	5.03
		160	0.343	1.689	2.190	0.01556	0.622	0.442	6.97	3,485.0	7.45
2½	2.875	40	0.203	2.469	1.704	0.3322	0.753	0.647	14.92	7,460.0	5.80
		80	0.276	2.323	2.254	0.02942	0.753	0.609	13.20	6,600.0	7.67
		160	0.375	2.125	2.945	0.02463	0.753	0.557	11.07	5,535.0	10.0
3	3.500	40	0.216	3.068	2.228	0.05130	0.917	0.804	23.00	11,500.0	7.58
		80	0.300	2.900	3.016	0.04587	0.917	0.760	20.55	10,275.0	10.3
		160	0.437	2.626	4.205	0.03761	0.917	0.688	16.90	8,450.0	14.3
3½	4.000	40	0.226	3.548	2.680	0.06870	1.047	0.930	30.80	15,400.0	9.11
		80	0.318	3.364	3.678	0.06170	1.047	0.882	27.70	13,850.0	12.5
4	4.500	40	0.237	4.026	3.173	0.08840	1.178	1.055	39.6	19,800.0	10.8
		80	0.337	3.826	4.407	0.07986	1.178	1.002	35.8	17,900.0	15.0
		120	0.437	3.626	5.578	0.07170	1.178	0.950	32.2	16,100.0	19.0
		160	0.531	3.438	6.621	0.06447	1.178	0.901	28.9	14,450.0	22.6
5	5.563	40	0.258	5.047	4.304	0.1390	1.456	1.322	62.3	31,150.0	14.7
		80	0.375	4.813	6.112	0.1263	1.456	1.263	57.7	28,850.0	20.8
		120	0.500	4.563	7.953	0.1136	1.456	1.197	51.0	25,500.0	27.1
		160	0.625	4.313	9.696	0.1015	1.456	1.132	45.5	22,750.0	33.0
6	6.625	40	0.280	6.065	5.584	0.2006	1.734	1.590	90.0	45,000.0	19.0
		80	0.432	5.761	8.405	0.1810	1.734	1.510	81.1	40,500.0	28.6
		120	0.562	5.501	10.71	0.1650	1.734	1.445	73.9	36,950.0	36.4
		160	0.718	5.189	13.32	0.1469	1.734	1.360	65.8	32,900.0	45.3
8	8.625	20	0.250	8.125	6.570	0.3601	2.258	2.130	161.5	80,750.0	22.4
		30	0.277	8.071	7.260	0.3553	2.258	2.115	159.4	79,700.0	24.7
		40	0.322	7.981	8.396	0.3474	2.258	2.090	155.7	77,850.0	28.6
		60	0.406	7.813	10.48	0.3329	2.258	2.050	149.4	74,700.0	35.7
		80	0.500	7.625	12.76	0.3171	2.258	2.000	142.3	71,150.0	43.4
		100	0.593	7.439	14.96	0.3018	2.258	1.947	135.3	67,650.0	50.9
		120	0.718	7.189	17.84	0.2819	2.258	1.883	126.5	63,250.0	60.7
		140	0.812	7.001	19.93	0.2673	2.258	1.835	120.0	60,000.0	67.8
		160	0.906	6.813	21.97	0.2532	2.258	1.787	113.5	56,750.0	74.7
	10.75	20	0.250	10.250	8.24	0.5731	2.814	2.685	257.0	128,500.0	28.1
		30	0.307	10.136	10.07	0.5603	2.814	2.655	252.0	126,000.0	34.3
		40	0.365	10.020	11.90	0.5475	2.814	2.620	246.0	123,000.0	40.5
		60	0.500	9.750	16.10	0.5185	2.814	2.550	233.0	116,500.0	54.8
		80	0.593	9.564	18.92	0.4989	2.814	2.503	224.0	112,000.0	64.4
		100	0.718	9.314	22.63	0.4732	2.814	2.440	212.0	106,000.0	77.0
		120	0.843	9.064	26.24	0.4481	2.814	2.373	201.0	100,500.0	89.2
		140	1.000	8.750	30.63	0.4176	2.814	2.290	188.0	93,750.0	105.0
		160	1.125	8.500	34.02	0.3941	2.814	2.230	177.0	88,500.0	116.0
12	12.75	20	0.250	12.250	9.82	0.8185	3.338	3.31	367.0	183,500.0	33.4
		30	0.330	12.090	12.87	0.7972	3.338	3.17	358.0	179,000.0	43.8
		40	0.406	11.938	15.77	0.7773	3.338	3.13	349.0	174,500.0	53.6
		60	0.562	11.626	21.52	0.7372	3.338	3.05	331.0	165,500.0	73.2
		80	0.687	11.376	26.03	0.7058	3.338	2.98	317.0	158,500.0	88.6
		100	0.843	11.064	31.53	0.6677	3.338	2.90	299.0	149,500.0	108.0
		120	1.000	10.750	36.91	0.6303	3.338	2.82	283.0	141,500.0	126.0
		140	1.125	10.500	41.08	0.6013	3.338	2.75	270.0	135,000.0	140.0
		160	1.312	10.126	47.14	0.5592	3.338	2.66	251.0	125,500.0	161.0
14	14.0	10	0.250	13.500	10.80	0.9940	3.665	3.54	446.0	223,000.0	36.8
		20	0.312	13.376	13.42	0.9750	3.665	3.51	438.0	219,000.0	45.7
		30	0.375	13.250	16.05	0.9575	3.665	3.47	430.0	215,000.0	54.6
		40	0.437	13.126	18.61	0.9397	3.665	3.44	422.0	211,000.0	63.3
		60	0.593	12.814	24.98	0.8956	3.665	3.36	402.0	201,000.0	85.0
		80	0.750	12.500	31.22	0.8522	3.665	3.28	382.0	191,000.0	107.0
		100	0.937	12.126	38.45	0.8020	3.665	3.18	360.0	180,000.0	131.0
		120	1.062	11.876	43.17	0.7693	3.665	3.11	345.0	172,500.0	147.0
		140	1.250	11.500	50.07	0.7213	3.665	3.01	324.0	162,000.0	171.0
		160	1.406	11.188	55.63	0.6827	3.665	2.93	306.0	153,000.0	190.0
16	16.0	10	0.250	15.500	12.37	1.3104	4.189	4.06	587.0	293,500.0	42.1
		20	0.312	15.376	15.38	1.2895	4.189	4.03	578.0	289,000.0	52.3
		30	0.375	15.250	18.41	1.2680	4.189	4.00	568.0	284,000.0	62.6
		40	0.500	15.000	24.35	1.2272	4.189	3.93	550.0	275,000.0	82.8