Friction Factor Problems:

Flow problems involve six parameters:

D, V_{ave} (or Q), ρ, μ, ε, $\mathcal{F}$ (or head loss $h = \mathcal{F}/g$)

∴ If one of these parameters is unknown, then <u>five</u> parameters must be specified or known in order to calculate the sixth.

For a given flow problems:

ρ, μ and ε are fixed

provided that these parameters are given, the three most common types of problems, depending on what is given and what is required, are:

Type	Given	Find	Solution
1	D, Q	$\mathcal{F}$	Direct
2	$D, \mathcal{F}$	Q	Numerical } since f depends on Q or D
3	$Q, \mathcal{F}$	D	Numerical } since f depends on Q or D

The working equation is

$$\mathcal{F} = f\, 4\left(\frac{\Delta x}{D}\right)\frac{V^2}{2}$$

Solutions using the chart or relation for f:

Type 1: $\rho, \mu, D, Q \rightarrow Re + \varepsilon/D \rightarrow f \rightarrow \mathcal{F}$

Type 2: Estimate $f \rightarrow Q + (\rho, \mu, D, \varepsilon) \rightarrow Re \rightarrow$ New f → f? — Yes → stop; No → using the new f (back to start)

Type 3: Estimate $f \rightarrow D + \rho, \mu, Q, \varepsilon \rightarrow Re \rightarrow$ New f → f? — Yes → stop; No → using the new f (back to start)

Working Equation for pipe flow:

Darcy Equation: $\mathcal{F} = f\, 4 \left(\frac{\Delta x}{D}\right) \cdot \frac{V^2}{2}$

we can write the equation in terms of Energy loss per unit length of pipe:

$$\left(\frac{\mathcal{F}}{\Delta x}\right) = f\, \frac{2V^2}{D}$$

Divide by $g \longrightarrow$ head loss per unit length:

$$h_L = \frac{\mathcal{F}}{\Delta x g} = \frac{2fV^2}{Dg} \quad ; \quad V = \frac{4Q}{\pi D^2}$$

$$= \frac{2f(4Q/\pi D^2)^2}{Dg}$$

Rearrange

$$\boxed{h_L = \left(\frac{32}{g\pi^2}\right) f\, \frac{Q^2}{D^5}} \quad *$$

<u>Laminar flow:</u>

$$f = \frac{16}{Re} \quad ; \quad Re = \frac{4\rho Q}{\pi \mu D}$$

substitute and rearrange (equation *)

$$\boxed{h_L = \left(\frac{128\mu}{\rho g \pi}\right) \frac{Q}{D^4}}$$

<u>Turbulent Flow:</u>

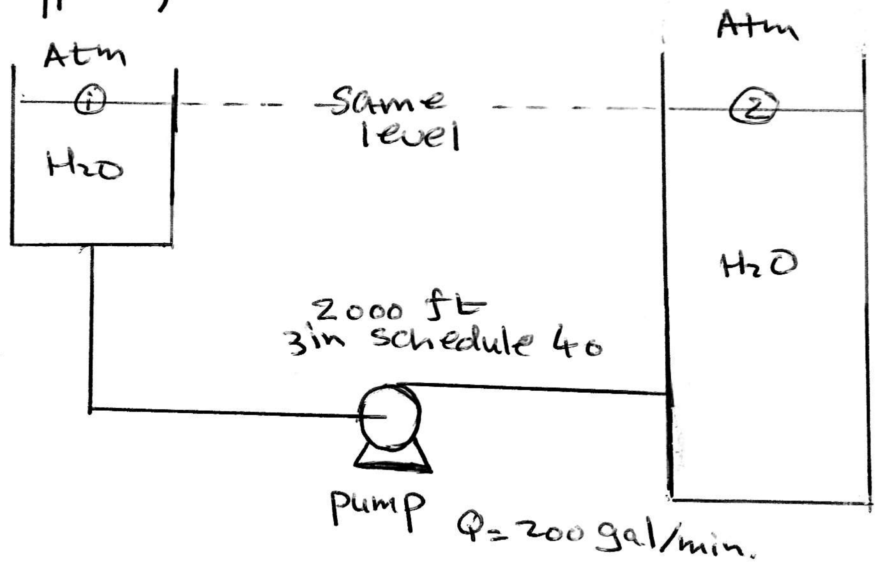
we need to know the dependence of f on Re & $\frac{\epsilon}{D}$

Therefore the solution requires solving equation * and $f(Re, \epsilon/D)$. i.e.

$$h_L = \left(\frac{32}{g\pi^2}\right) f\, \frac{Q^2}{D^5}$$

$$f = f\left(\frac{4\rho Q}{\pi \mu D}, \epsilon/D\right) \quad [\text{or chart}]$$

Ex: 6.8 (Type 1)



<u>Given:</u>

- D, Q, ρ, μ, ε
- ρ and μ are for water
- ε for commercial steel. (ε = 0.0018 from table).

<u>Required:</u>

- pump work / unit mass
- pump power ($\eta = 85\%$)
- pump pressure rise.

<u>Solution:</u>

We apply BE between sections ① & ②.

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = +\hat{W} - \mathcal{F}$$

($\hat{W}$ ← pump work / unit mass)

Left side = 0 $\Delta P = 0$; $\Delta z = 0$; $\Delta V^2 = 0$

$$\therefore \boxed{\hat{W} = \mathcal{F}}$$

Before we calculate $\mathcal{F}$ we must check Re

D : for 3in sch 40   ID = 3.068 in.

$$Re = \frac{4\rho Q}{\pi \mu D}$$

$\rho = 62.3$ lbm/ft^3
$\mu = 1$ cP
$Q = 200$ gal/min

$$= 2.05 \times 10^5$$

∴ Flow is turbulent

From chart $f = 0.0048$ (Eq 6.2 → .0045)

$$\therefore \hat{W} = \mathcal{F} = 4f \frac{\Delta x}{D} \frac{V^2}{2} \quad ; \quad V = \frac{4Q}{\pi D^2}$$

$= 176$ ft. lbf / lbm

$= 524$ J/kg

Power:

$$\text{power} = \frac{\text{work}}{\text{mass}} \times \frac{\text{mass}}{\text{time}}$$

$$= \hat{W} \times \dot{m}$$

$$\dot{m} = \rho Q$$

$$= 27.8 \text{ lbm s}^{-1}$$

$$\therefore \text{Power} = 176 \frac{\text{ft lbf}}{\text{lbm}} \times 27.8 \frac{\text{lbm}}{\text{s}} \times \frac{1 \text{ hp.s}}{550 \text{ ft lbf}}$$

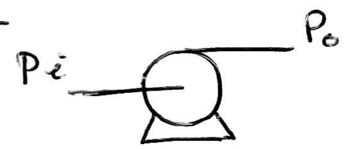
$= 8.9$ hp $= 6.63$ kW

$$\text{Actual power} = \frac{\text{Power}}{\eta}$$

$$= \frac{8.9}{0.85}$$

$= 10.47$ hp.

Pressure rise:



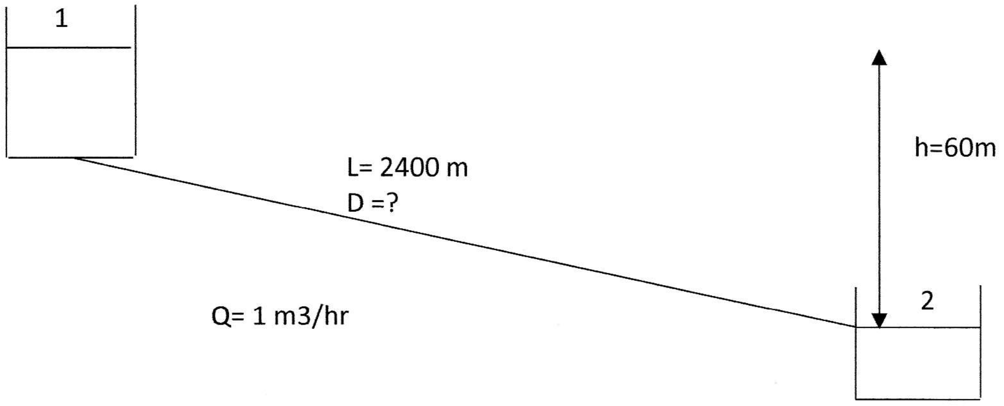
A pump adds energy to a flowing fluid. This energy raise the pressure from $P_i \rightarrow P_o$ ($P_o > P_i$)

Apply BE across pump, neglecting friction

$$\therefore \frac{\Delta P}{\rho} = \hat{W} \quad ; \quad \Delta P (P_o - P_i)$$

$$\therefore \Delta P = \rho \hat{W}$$

$= 76.1$ lbf/in^2



1] Apply BE Between 1 and 2

$g\Delta z = -\mathfrak{F}$; $\Delta z = -h$ → $\mathfrak{F} = gh$

2] Darcy Equation

$$D^5 = \frac{32\,Q^2 L}{gh\pi^2} \cdot f \quad \rightarrow \quad D = K_1 \,.\, f^{0.2} \quad ; \quad K_1 = \left(\frac{32\,Q^2 L}{gh\pi^2}\right)^{0.2}$$

$$Re = \frac{4\rho Q}{\pi\mu} \cdot \frac{1}{D} \quad \rightarrow \quad Re = K_2 \cdot \frac{1}{D} \quad ; \quad K_2 = \frac{4\rho Q}{\pi\mu}$$

Flow Q,m3/hr	1
Density, kg/m3	1000
Viscosity, Pa s	0.001
g, m/s2	9.81
Roughness, mm	0.04572
Height ,m	60

K1= 0.063347

K2= 353.6777

- Laminar: $f = \frac{16}{Re}$
- Turbulent : $f = 0.001375\left[1 + \left(20000\frac{\epsilon}{D} + \frac{10^6}{Re}\right)^{1/3}\right]$

Trial	f	D= K_1 $f^{0.2}$	Re = K_2 . 1/D	ϵ/D	f from eq or Chart	Criteria
1						
2						
3						
4						
5						
6						