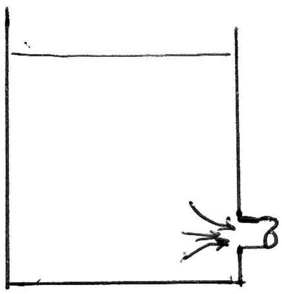
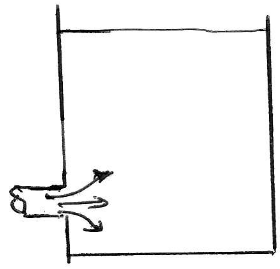
Enlargements and Contractions:

In previous sections we considered flow of fluid in a straight section of a pipe where the flow is fully developed (Laminar or turbulent). In flow problems the fluid flows into the pipe from a tank or leaves the pipe to a tank.



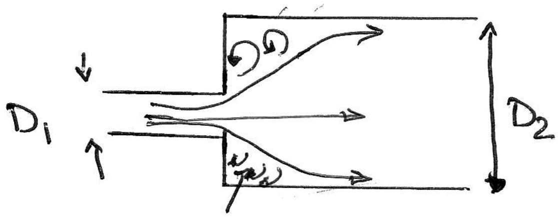
Sudden Contraction.



Sudden Enlargement.

There is a friction loss associated with these changes that must be taken into account. This friction loss is added to the pipe friction.

Sudden Enlargements



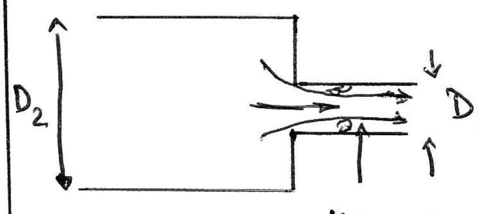
Eddies form in the corners
→ Energy loss. Kinetic Energy is converted into internal energy.

It can be shown that

$$\mathcal{F} = K\frac{V^2}{2}$$

$$= \frac{V^2}{2}\underbrace{\left[1-\left(\frac{D_1}{D_2}\right)^2\right]^2}_{K}$$

Suddent contractions



They result in a vena contracta. The velocity in the vena contracta is higher than the velocity downstream in the pipe.

Some of the energy is lost as friction

It is found that

$\mathcal{F} = K\frac{V^2}{2}$; K is an experimental constant.

The friction in the sudden contraction is called $\mathcal{F}_{entrance}$ (entrance to pipe); while for sudden enlargement it is called $\mathcal{F}_{exit}$ (exit from pipe).

The K values are called resistance coefficients and can be found from the attached figure. However for sudden enlargements K can be found from

$$K = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2$$

when $\frac{D_1}{D_2}$ is small (as is encountered in most cases where the pipes are connected to large tanks), then $\left(\frac{D_1}{D_2}\right)^2$ is $\simeq 0$. In such cases: K

$$K_{enlargement} = 1.0 \Rightarrow \mathcal{F}_{exit} = 1 \times \frac{V^2}{2}$$

$$K_{contraction} = 0.5 \Rightarrow \mathcal{F}_{entrance} = 0.5 \frac{V^2}{2}$$

In both cases V is the average flow velocity in the pipe.

Notes: ① for Laminar Flow; these losses are negligible since kinetic energy is small.

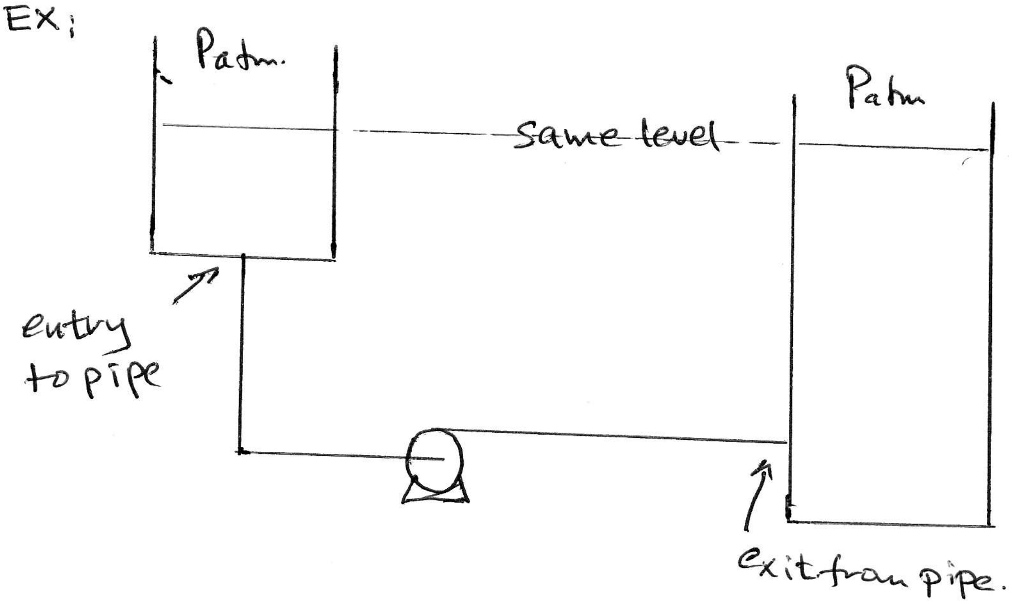
② : In the case of pipes connected in series

V is the larger of the two velocities and $\left(\frac{D_1}{D_2}\right)^2$ cannot be neglected.

③ $\frac{D_1}{D_2} = 1 \Rightarrow$ no change in cross section

$\Rightarrow$ no loss.

EX;



As before

$$0 = +\hat{W} - \mathcal{F}_{tot}.$$

$$\hat{W} = \mathcal{F}_{tot}$$

$$\mathcal{F}_{tot} = \mathcal{F}_{ipe} + \mathcal{F}_{entry} + \mathcal{F}_{exit}.$$

$$= \frac{4 f \Delta x V^2}{2D} + K_{entry} \cdot \frac{V^2}{2} + K_{exit} \cdot \frac{V^2}{2}$$

Entry: $D_{pipe} << D_{tank} \longrightarrow K = 0.5$

Exit: $D_{pipe} << D_{tank} \longrightarrow K = 1.0$

$$\therefore \mathcal{F}_{tot} = \Big[\underbrace{4 f \frac{\Delta x}{D}}_{\text{pipe}} + \underbrace{0.5 + 1.0}_{\text{entry + exit.}} \Big] \frac{V^2}{2}$$

It is obvious that $\mathcal{F}_{tot}$ is largely affected by the length of the pipe.

<u>Effect on ΔP across pump:</u>

BE across pump:

$$\Delta P_{EC} = \rho \, \mathcal{F}_{tot}$$

ΔP_{EC}: is pressure rise taking into account entrance and exit losses.

Error for neglecting EC losses:

$$\frac{\Delta P_{EC} - \Delta P}{\Delta P_{EC}} = \frac{\rho \mathcal{F}_{tot} - \rho \mathcal{F}_{pipe}}{\rho \mathcal{F}_{tot}}$$

$$= \frac{1.5 \frac{V^2}{2}}{\left[4f \frac{\Delta x}{D} \frac{V^2}{2} + 1.5\frac{V^2}{2}\right]}$$

$$= \frac{1.5}{\left[4f \frac{\Delta x}{D} + 1.5\right]}$$

In the previous example

$f = 0.0048$

$\Delta x = 2000$ ft

$D = (3.068/12)$ ft.

$$\therefore \text{Error} = \frac{1.5}{\left[4 \times 0.0048 \times \frac{2000}{\left(\frac{3.068}{12}\right)} + 1.5\right]}$$

$= 0.0098$

$\simeq 0.98\%$ (which is negligible).

However if $\Delta x = 100$ ft, then

Error $= 0.1665$

$\simeq 16.65\%$ Cannot be neglected.

Fitting losses:

Fittings are introduced in pipe networks serving various purposes: Fittings can be: Values, elbows, T-pieces, bends, ----

Fittings introduce additionl head loss (friction) which must be taken in to account.

Head loss through fittings has been studied experimentally and correlations are available allowing us to treat them as one dimensional flow problems. The results have been correlated in two different ways:

$\boxed{1}$ Equivalent lengths [preferred]:

$$\mathcal{F}_{fitting} = \text{Constant specific to the fitting} \times \mathcal{F} \text{ through a pipe of length} = 1 \text{ pipe diameter}$$

$\boxed{2}$ Velocity heads:

$$\mathcal{F}_{fitting} = K \frac{V^2}{2}$$; the constant K is experimental and given tables (see attached table).

Equivalent length:

It is that length of pipe that would give the same head loss as the fitting. It is expressed as a number of pipe diameters (n). The friction due to the fitting is considered to be the same as an additional pipe of length $= nD$, where D is the pipe diameter and n is independent of D.
n found from table.

This method avoids the necessity for giving a different value for every conceivable pipe size.

∴ For the purpose of calculating the total friction $\mathcal{F}_{tot}$; we use a length called the adjusted length in the Darcy equation.

$$\text{Adjuste length} = \text{pipe length} + \sum \text{equivalent lengths}$$

In these cases:

$$\mathcal{F}_{pipe+fittings} = 4f\frac{\Delta x}{D}\frac{V^2}{2} + 4f\frac{\sum L_{eq}}{D}\frac{V^2}{2}$$

$$= 4f.\frac{\text{Adjusted length}}{D}.\frac{V^2}{2}$$

EX:

In previous example, we have 2 globe valves, 1 swing value and 9 90° elbows.

Equivalent length method:

Fitting	Number	n (Table)	L = nD	Leq
Globe valve	2	350	350D	700 D
Swing Valve	1	110	110D	110 D
Elbows	9	32	32D	288 D
				1098 D

∴ Adjusted length $= 2000 + 1098 \times \frac{3.068\,in}{12\,in/ft}$

$= 2000 + 280.7$

$= 2280.7$

$$\mathcal{F} = \frac{4 \times 0.0048 \times 2280.7\,ft}{(3.068/12)\,ft} \times \left(\frac{200\,gal/min}{60s/min \times 7.48\,gal/ft^3}\right)^2 \times \frac{1}{2} \div \left(\frac{\pi \times (3.068/12)^2}{4}\right)^2 \div \frac{lbf\,s^2}{32.2\,lbm\,ft}$$

$= 200.4\ ft\ lbf/lbm$

$\mathcal{F}_{without} = 176\ ft\ lbf/lbm.$

2 velocity head Method.

$$\mathcal{F}_{\text{with fittings}} = \mathcal{F}_{pipe} + \mathcal{F}_{fittings}.$$

$$\mathcal{F}_{pip} = 4f \frac{\Delta x}{D} \frac{V^2}{2}$$

$$\mathcal{F}_{fittings} = (\Sigma K)\frac{V^2}{2}$$

$$\Sigma K = 3 \times 6.3 + 1 \times 2.0 + 9 \times 0.74 = 27.56$$

$$\therefore \mathcal{F}_{fittings} = 4f \frac{\Delta x}{D} \frac{V^2}{2} + (\Sigma K) \frac{V^2}{2}$$

$$= 176 \text{ ft lbf/lbm} + 27.56 \times \left(\frac{200 \text{ gal/min}}{60 \text{ s/min} \times 7.48 \text{ gal/ft}^3} \right)^2 \times \frac{1}{2}$$

$$\div \left(\frac{\pi (3.068/12)^2}{4} \right)^2 \div \frac{\text{lbf s}^2}{32.2 \text{ lbm ft}}$$

$$= 176 + 32.3$$

$$= 208 \text{ ft lbf/lbm}$$

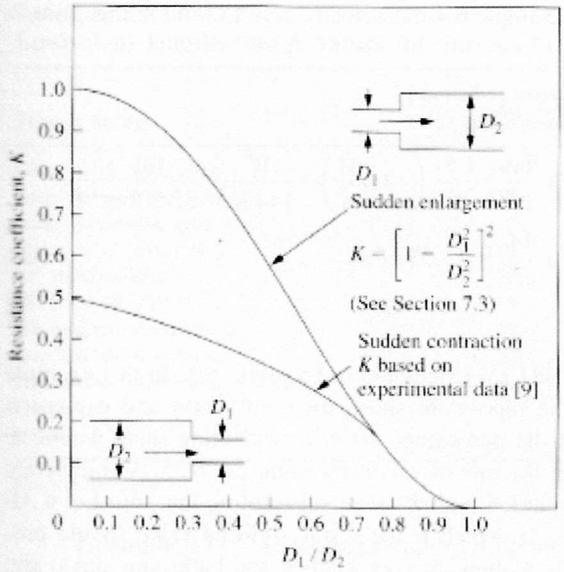


FIGURE 6.16
Resistance due to sudden enlargements and contractions. The resistance coefficient K is defined in Eq. 6.25. (From Crane Technical Paper No. 410, reproduced by permission of the Crane Company.)

TABLE 6.7
Equivalent lengths and K values for various kinds of fitting*

Type of fitting	Equivalent length, L/D, dimensionless	Constant, K, in Eq. 6.25, dimensionless
Globe valve, wide open	350	6.3
Angle valve, wide open	170	3.0
Gate valve, wide open	7	0.13
Check valve, swing type	110	2.0
90° standard elbow	32	0.74
45° standard elbow	15	0.3
90° long-radius elbow	20	0.46
Standard tee, flow-through run	20	0.4
Standard tee, flow-through branch	60	1.3
Coupling	2	0.04
Union	2	0.04