7. Momentum Balance

- Extensive property
- It is an abstract quantity like energy
- $\overrightarrow{momentum} = m\vec{v}$ vector quantity.

$$\frac{dm\vec{v}}{dt} = \text{rate of change of momentum}$$

$$= m\vec{a} \quad \text{(constant mass).}$$

$$\therefore \vec{F} = \frac{dm\vec{v}}{dt}$$

$$\vec{F} - m\vec{a} = 0$$

$$(F_x.\vec{i} + F_y\vec{j} + F_z\vec{k}) - m(a_x.\vec{i} + a_y\vec{j} + a_z\vec{k}) = 0$$

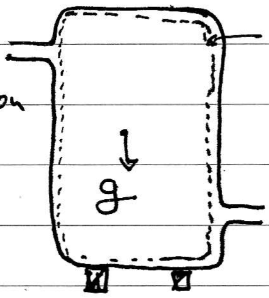
$$(F_x - ma_x)\vec{i} + (F_y - ma_y)\vec{j} + (F_z - ma_z)\vec{k} = 0$$

$$\therefore F_x - ma_x = 0 \; ; \; F_y - ma_y = 0 \quad F_z - ma_z = 0$$

Momentum Balance:

General Balance equation:

Mom. Accumulation = Mom IN − Mom OUT + Production



Mom Accumulation = $d(m\vec{v})_{system}$

$\vec{v}$ uniform.

Mom. IN − Mom. OUT = $\vec{V}_{in} \cdot dm_{in} - \vec{V}_{out} \cdot dm_{out}$

Production. : Newton's Law

$$\vec{F} = \frac{d(m\vec{v})_{syst}}{dt} \longrightarrow d(m\vec{v})_{syst} = \vec{F} \cdot dt.$$

Assuming closed system, the terms of production must be forces.

$$\therefore \quad d(m\vec{V})_{sys} = \vec{V}_{in}\, dm_{in} - \vec{V}_{out}\, dm_{out} + \sum \vec{F}\, dt.$$

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$$\frac{d(m\vec{V})_{sys}}{dt} = \vec{V}_{in}\, \dot{m}_{in} - \vec{V}_{out}\, \dot{m}_{out} + \sum \vec{F}$$

This is Netwon's Law of motion for fluids.
It is a Momentum Balance Equation.

System Boundaries:

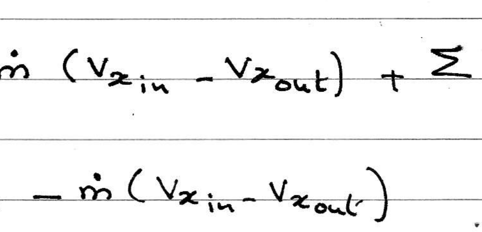
Chosen in a way that one can conveniently calculate all the terms in the balance equation. It should be possible to account for all flows of matter across the boundaries and all external forces.

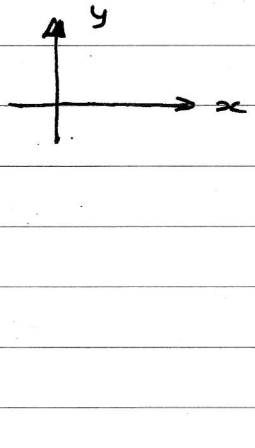
Steady Flow applications:

For any direction, x for example

$$0 = V_{x_{in}} \cdot \dot{m} - V_{x_{out}} \cdot \dot{m} + \sum F_x$$

EX 7.3





$Q_{jet} = 0.01\, m^3 s^{-1}$ $\quad V_{jet} = 30\, ms^{-1}$

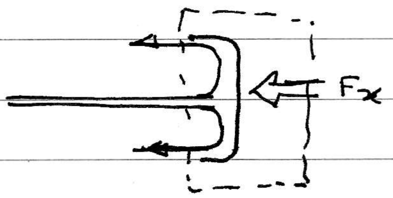
$$0 = \dot{m}\,(V_{x_{in}} - V_{x_{out}}) + \sum F_x$$

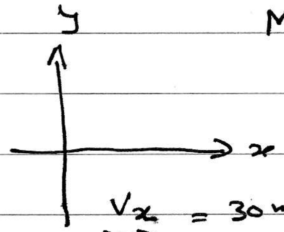
$$\therefore \quad F_x = -\dot{m}\,(V_{x_{in}} - V_{x_{out}})$$

$$= -0.01 m^3 s^{-1} \times 998.2 \frac{kg}{m^3}\ (30\, ms^{-1} - 0)$$

$$= -\ 299.5\, N.$$

EX 7.4.





$V_x = 30\,ms^{-1}$

$V_x = 15\,ms^{-1}$

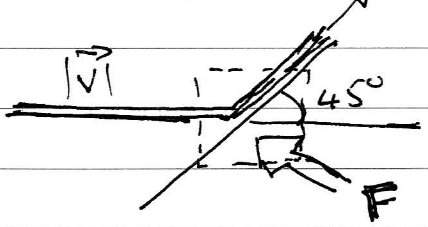
$$F_x = -\dot{m}\left(V_{x_{in}} - V_{x_{out}}\right)$$

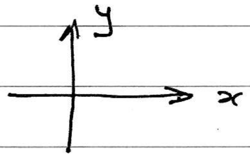
$$= -0.01\,m^3s^{-1} \cdot 998.2\,kgm^{-3}\left(30ms^{-1} - (-15ms^{-1})\right)$$

$$= -449.3\,N.$$

Ex7.5

$\vec{V}$ unchanged. (assumption).





$v_{x\,out} = |V| \cos 45$

$v_{y\,out} = |V| \sin 45$

$$F = \sqrt{F_x^2 + F_y^2}$$

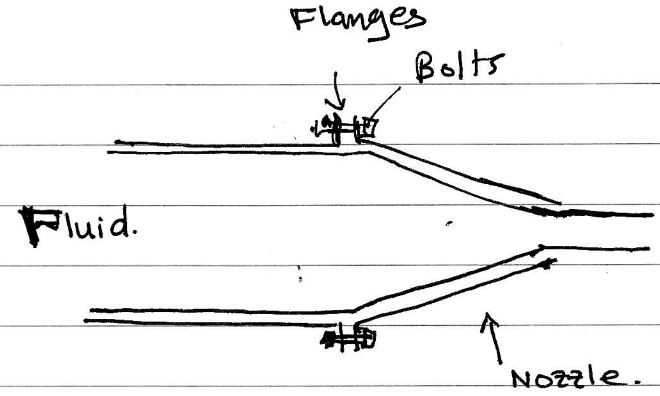
$$F_x = -.01\,m^3s^{-1} \times 998.2\,kgm^{-3}\left(30\,ms^{-1} - 30\cos 45\,ms^{-1}\right) = -87.7N$$

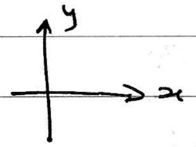
$$F_y = -.01\,m^3s^{-1} \times 998.2\,kgm^{-3}\left(0 - 30\sin 45\,ms^{-1}\right) = +211.6\,N$$

$$\therefore F = \sqrt{(-87.7)^2 + (211.6)^2}$$

$$= 229.1\,N$$

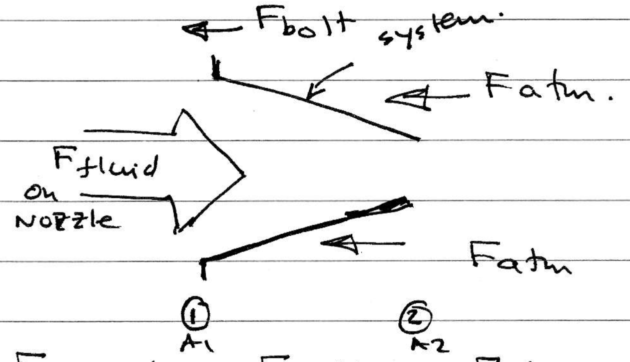
EX.7.6. MB4





$F_{bolt} = ?$

Force balance on Nozzle. $\Sigma F_x = 0$. (System is Nozzle)



$\therefore \quad 0 = F_{fluid} - F_{bolt} - F_{atm}$

$F_{bolt} = F_{fluid} - F_{atm}$. *

$F_{atm} = P_{atm} \cdot A_{xN}$ $\qquad A_{xN}$: x-projected area of nozzle.

$= P_{atm}(A_1 - A_2)$. $\qquad = A_1 - A_2$

$F_{bolt} = F_{fluid} - P_{atm}(A_1 - A_2)$

$F_{fluid\ on\ Nozzle} = F_{nozzle\ on\ fluid}$.

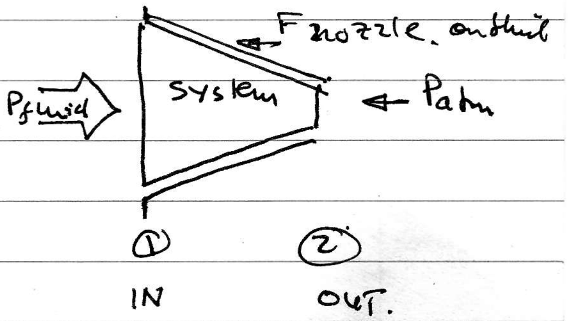
$F_{nozzle\ on\ fluid}$ can be obtained by considering the fluid in the nozzle to be the system.

@ S.S

$0 = \text{mom IN} - \text{mom OUT} + \Sigma F_x$.

$\text{mom. IN} = \dot{m} V_1 = \dot{m} V_2 \cdot \frac{A_2}{A_1}$

$\text{mom OUT} = \dot{m} V_2$



$\therefore \quad 0 = \dot{m} V_2 \left(\frac{A_2}{A_1} - 1 \right) + \Sigma F_x$.

$$\Sigma F_x = -\dot{m} V_2 \left(\frac{A_2}{A_1} - 1\right)$$

But:

$$\Sigma F_x = P_1 . A_1 - P_{atm} . A_2 - F_{nozzle}$$

$$\therefore F_{nozzle \text{ on fluid}} = P_1 . A_1 - P_{atm} . A_2 + \dot{m} V_2 \left(\frac{A_2}{A_1} - 1\right).$$

From *

$$F_{bolt} = P_1 A_1 - \cancel{P_{atm} A_2} + \dot{m} V_2 \left(\frac{A_2}{A_1} - 1\right) - P_{atm} (A_1 - \cancel{A_2})$$

$$= P_1 . A_1 - P_{atm} A_1 + \dot{m} V_2 \left(\frac{A_2}{A_1} - 1\right).$$

$$= P_{1,gauge} . A_1 + \dot{m} V_2 \left(\frac{A_2}{A_1} - 1\right).$$

Given:

$P_{1\,gauge} = 100$ psi

$A_1 = 10 \text{ in}^2$

$A_2 = 1 \text{ in}^2$

$\dot{m} = 43.3 \text{ lbm s}^{-1}$

$V_2 = 100 \text{ ft s}^{-1}$

$$\therefore F_{bolt} = 100 \times 10 \text{ lbf} + 43.3 \frac{\text{lbm}}{\text{s}} \times 100 \frac{\text{ft}}{\text{s}} \left(\frac{1}{10} - 1\right) \times \frac{1 \text{ lbf s}^2}{32.2 \text{ lbm ft}}$$

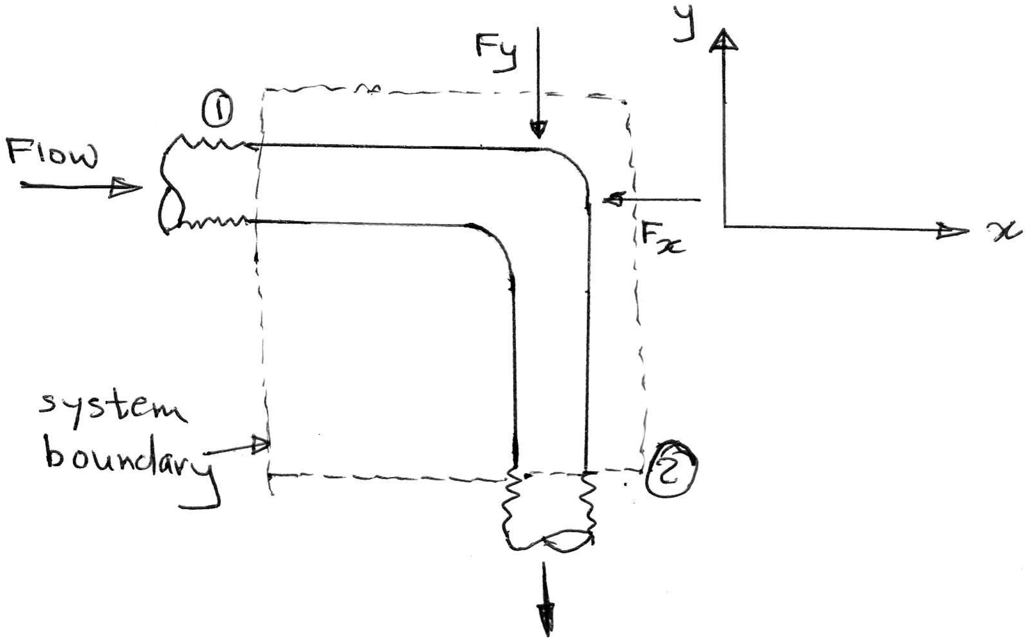
$$= 1000 - 43.3 \times 100 \times \frac{9}{10} \frac{1}{32.2}$$

$$= 1000 - 121$$

$$= -879 \text{ lbf}.$$

EX 7.7

The pipe Bend in the figure below is attached to the rest of the piping system by two flexible hoses, which transmit no forces. Water enters in the $+x$ direction and leaves in the $-y$ direction. The flow rate is 500 kg/s, and the cross-sectional area of the pipe is constant $= 0.1\ m^2$. The pressure throughout the pipe is 200 kPa gauge. Calculate F_x and F_y, the x and y components of the force in the pipe support.



The system is the fluid inside the pipe delimited by the boundary shown on the figure.

x - direction

At steady state:

$$0 = \dot{m}\,(V_{x_{in}} - V_{x_{out}}) + P_1 A_1 + \sum F_x$$

$$-F_x = \dot{m}\,(V_{x_{in}} - V_{x_{out}}) + P_1 A_1$$

y - direction:

At steady state:

$$0 = \dot{m}\,(V_{y_{in}} - V_{y_{out}}) + P_2 A_2 + \sum F_y$$

$$-F_y = \dot{m}\,(v_{y_{in}} - v_{y_{out}}) + P_2 A_2$$

MB7

We need V_x and V_y:

Since cross section is constant, then the velocity is constant.

$$V = \frac{Q}{A} = \frac{\dot{m}/\rho}{A} = \frac{(500\,kg/s)\,/(998.2\,Kg/m^3)}{0.1\,m^2} = 5.01\,m/s$$

$\therefore v_{x_{in}} = +5.01\,m/s$ $\qquad v_{x_{out}} = 0$

$v_{y_{in}} = 0$ $\qquad v_{y_{out}} = -5\,m/s$

$$\therefore -F_x = 500\,\frac{kg}{s}\,(5.01-0)\,\frac{m}{s} + 200\,kPa \times 0.1\,m^2$$

$$= 2505\,Kg(ms^{-2}) + 20{,}000\,(Pa \cdot m^2)\ ; \quad Pa\,m^2 = \frac{N}{\cancel{m^2}}\,\cancel{m^2}, \quad Kg\,ms^{-2} = N$$

$$= (2505 + 20000)\,N$$

$$= \underline{22{,}505\,N}$$

$$-F_y = 500\,\frac{kg}{s}\,(0-(-5.01))\,\frac{m}{s} + 200\,kPa \times 0.1\,m^2$$

$$= (2505 + 20000)\,N$$

$$= \underline{22{,}505\,N}$$