Negative Absolute Pressures : CAVITATION

Under certain situations BE predicts -ve abslolute pressures.

$$\Delta\left(\frac{P}{\rho}+gz+\frac{V^2}{2}\right)=-\mathcal{F} \qquad \text{(no mechanical work)}.$$

$$\Rightarrow P_2 = \underbrace{P_1}_{①} - \underbrace{\rho\left[g\Delta z+\frac{\Delta(V^2)}{2}+\mathcal{F}\right]}_{②} \qquad ① < ②$$

$= -ve$

Gases:
- -ve absolute pressure has no physical meaning
- It indicates wrong assumptions - Equations for incompressible flow must be used.

Liquids:
- -ve absolute pressures may be predicted under certain conditions indicating unstable flow conditions.
- It is known that when

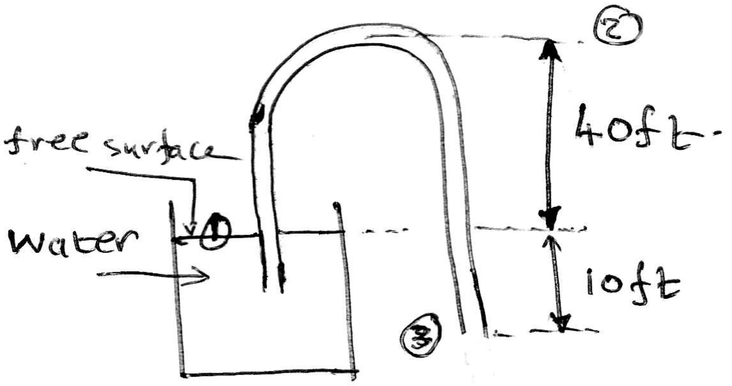
  " $P_{\text{on liquid}} \leq$ Vapor pressure of liquid"

  the liquid will boil (forms bubbles) giving rise to two phase flow (liq + Vap) which has higher values of $\mathcal{F}$ than for 1 phase flow.

∴ when $P_{\text{calculated on liquid}} <$ Vap pressure

This means that the flow as considered is impossible from physical point of view, and the actual flow has higher friction effect.

EX: Siphon Flow:



Required: P_2 ?

Given or assumed:

- $\mathcal{F} = 0$
- $V_1 \simeq 0$ (large cross section)
- $P_1 = P_3 = P_{atmosphere}$
- $\rho = 62.3 \ lbm/ft^3$

Solution:

Apply BE between two points so that P_2 is at one of these Points.

Take Points 1 and 2

$$\frac{P_1 - P_2}{\rho} + g(z_1 - z_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) = 0.$$

$$\Rightarrow \quad P_2 = P_1 - \rho\left[\frac{V_2^2}{2} + g(z_2 - z_1)\right] \quad ; \quad V_1 \simeq 0$$

$P_1 = P_{atm}$

$= 14.7$ psia (lbf/in^2)

$g = 32.2 \ ft/s^2$

$(z_2 - z_1) = (40 - 0)$ ft $\qquad z_1$ is reference

$= 40$ ft

$\rho = 62.3 \ lbm/ft^3$

V_2 is still unknown. To find it we apply BE between other two points and make use of continuity equation.

Apply BE between ① and ③

$P_1 = P_3 \qquad V_1 \simeq 0$

$$g(z_1 - z_3) = + \frac{V_3^2}{2}$$

$$(z_1 - z_3) = + 10\,ft = h$$

$V_3 = V_2$ Continuity equation.

$$\therefore V_2 = \sqrt{2gh}$$

$$= \sqrt{2 \times 32.2 \frac{ft}{s^2} \times 10\,ft}$$

$$= 25.3 \; ft/s$$

$$\therefore P_2 = 14.7 \frac{lbf}{in^2} - 62.3 \frac{lbm}{ft^3} \left[\frac{(25.3\,ft/s)^2}{2} + 32.2 \frac{ft}{s^2} \times 40\,ft \right]$$

$$\times \underbrace{\frac{1\,lbf.s^2}{32.2\,lbm\,ft} \times \frac{1\,ft^2}{144\,in^2}}_{\text{conversion}}$$

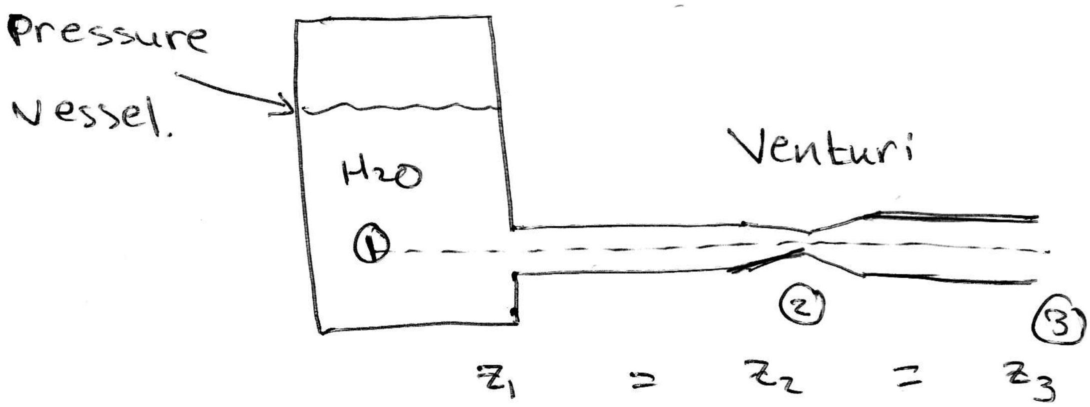
$$= 14.7 - 21.6$$

$= -6.9 \; \frac{lbf}{in^2}$ ⇒ impossible ⇒ siphon will not flow.

Maximum height for water siphon = 34 ft of H_2O

Q: Think of other examples.

Ex: Horizontal Flow



Given:

$P_1 = 10$ psig

$A_2 / A_3 = 0.5$

$P_3 = P_{atm} = 14.7$ psia

Assumptions:

$\mathcal{F} = 0$

$V_1 \simeq 0$ (Large Cross section)

Required:

P_2

Solution:

Apply BE between 1 & 2

$$\frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} = 0 \quad ; \quad \mathcal{F} = 0 \quad \Delta z = 0$$

$$\therefore P_2 = P_1 - \rho \frac{V_2^2}{2} \quad ; \quad V_1 \simeq 0$$

Need to know V_2

P_1 and ρ are given

Apply BE between ① and ③

$$V_3 = \sqrt{\frac{2(P_1 - P_3)}{\rho}}$$

$(P_1 - P_3) = P_1$, gauge
$= 10$ psig

$$= \sqrt{\frac{2 \times 10 \ \text{lbf/in}^2}{62.3 \ \text{lbm/ft}^3} \times \underbrace{\frac{32.2 \ \text{lbm ft}}{\text{lbf} \cdot s^2} \times \frac{144 \ \text{in}^2}{1 \ \text{ft}^2}}_{\text{conversion.}}}$$

$$= 38.6 \ \text{ft/s}$$

CE $\Rightarrow$ $V_2 = V_3 \cdot \frac{A_3}{A_2}$ $\qquad \frac{A_2}{A_3} = 0.5$ (given)

$$\therefore V_2 = 38.6 \ \frac{\text{ft}}{s} \times \frac{1}{0.5}$$

$$= 77.2 \ \text{ft/s}$$

$$\therefore P_2 = (10 + 14.7) \ \frac{\text{lbf}}{\text{in}^2} - \frac{62.3 \ \text{lbm}}{\text{ft}^3} \times \frac{(77.2 \ \text{ft/s})^2}{2} \underbrace{\times \frac{\text{lbf } s^2}{32.2 \ \text{lbm ft}} \times \frac{1 \ \text{ft}^2}{144 \ \text{in}^2}}_{\text{conversion.}}$$

$$= -15.4 \ \text{psia} \quad \text{(unreal)}$$

<u>What is happening?</u>

Between ① and ②

Velocity increases at ②
Pressure decreases at ② $\rightarrow$ liquid may boil if P_2 as calculated is -ve

- → Vapor bubbles formed at ② travel to higher pressure region after venturi and then collapse
- → collapse can cause a pressure pulse. (In pumps the drop in pressure at entry can cause damage to pump).
- → Phenomena of local boiling is called CAVITATION