5.4 $\Delta P = \dfrac{\rho\left(V_1^2 - V_2^2\right)}{2} = \dfrac{\rho}{2}\left[\left(10\dfrac{\text{ft}}{\text{s}}\right)^2 - \left(\dfrac{10}{3}\dfrac{\text{ft}}{\text{s}}\right)^2\right] = \rho\dfrac{\left(100\dfrac{\text{ft}^2}{\text{s}^2}\right)}{9/4}$

(a) For water

$$\Delta P = 62.3\frac{\text{lbm}}{\text{ft}^3}\left(\frac{4}{9}\right)\left(100\frac{\text{ft}^2}{\text{s}^2}\right)\cdot\frac{\text{lbf s}^2}{32.2\,\text{lbm ft}}\cdot\frac{\text{ft}^2}{144\,\text{in}^2} = 0.60\ \text{psi} = 4.1\ \text{kPa}$$

(b) For air

$$\Delta P = \text{above answer}\cdot\frac{0.075}{62.3} = 7.2\cdot 10^{-4}\,\text{psi} = 0.0050\ \text{kPa}$$

5.14 $V = \sqrt{2gh}$; $\quad Q = VA_{\text{outlet}} = V_{\substack{\text{tank}\\ \text{surface}}} A_{\text{tank}} = A_{\text{tank}}\left(-\dfrac{dh}{dt}\right)_{\text{tank}}$

$$\left(-\frac{dh}{dt}\right)_{\text{tank}} = \frac{A_{\text{outlet}}}{A_{\text{tank}}}\sqrt{2gh} = 0.01\sqrt{2\cdot 9.81\frac{\text{m}}{\text{s}}\cdot 10\ \text{m}} = 0.14\frac{\text{m}}{\text{s}} = 0.46\frac{\text{ft}}{\text{s}}$$

Some students will say you should use $V = \sqrt{\dfrac{2gh}{1-(A_2/A_1)^2}}$. That multiplies the above answers by 1.00005, which is clearly negligible compared to the uncertainties introduced by the standard Torricelli's assumptions.

5.15 $V_2 = \sqrt{2\left(-\dfrac{\Delta P}{\rho} - g\Delta z\right)}$, but $\Delta P = \rho_{\text{gasoline}}\, g\Delta z$ so that

$$V_2 = \sqrt{2g(-\Delta z)\left(1 - \frac{\rho_{\text{gas}}}{\rho_{\text{w}}}\right)} = \sqrt{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(30\,\text{ft})(1-0.72)} = 23.3\frac{\text{ft}}{\text{s}} = 7.09\frac{\text{m}}{\text{s}}$$

This is the same as Ex. 5.5, with different fluids.

5.16* $V_2 = \sqrt{-\dfrac{\Delta P}{\rho_{\text{w}}}} = \sqrt{\dfrac{P_1 - P_2}{\rho_w}}$

$P_1 = 30\,\text{ft}\cdot g\rho_{\text{water}}$; $\quad P_2 = 10\,\text{ft}\cdot g\rho_{\text{water}} + 20\,\text{ft}\cdot g\rho_{\text{oil}}$

$P_1 - P_2 = 20\,\text{ft}\cdot g\left(\rho_{\text{water}} - \rho_{\text{oil}}\right)$

$$V = \sqrt{2(20\,\text{ft})\cdot\left(32.2\frac{\text{ft}}{s^2}\right)\cdot(1-0.9)} = 11.3\frac{\text{ft}}{\text{s}} = 3.46\frac{\text{m}}{\text{s}}$$

5.17 $\mathcal{F} = -\left(\dfrac{\Delta P}{\rho} + g\Delta z + \dfrac{\Delta V^2}{2}\right)$ Assuming the flow is from left to right

$$\mathcal{F} = \left(-\frac{8\dfrac{\text{lbf}}{\text{in}^2}}{62.3\dfrac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\,\text{in}^2}{\text{ft}^2} + 32.2\frac{\text{ft}}{\text{s}^2} \cdot 20\,\text{ft} + 0\right)$$

$$= \left(-595\frac{\text{ft}}{\text{s}^2} + 644\frac{\text{ft}^2}{\text{s}^2}\right) = -48.6\frac{\text{ft}^2}{\text{s}^2} = -4.51\frac{\text{m}^2}{\text{s}^2}$$

The negative value of the friction heating makes clear that the assumed direction of the flow is incorrect, and that the flow is from right to left.

5.18* $V_2 = \sqrt{2\left(-\dfrac{\Delta P}{\rho} - g\Delta z\right)}$

$$V_2 = \sqrt{2\left(\frac{20\dfrac{\text{lbf}}{\text{in}^2}}{62.3\dfrac{\text{lbm}}{\text{ft}^3}} \cdot \frac{32.2\,\text{lbm ft}}{\text{lbf s}^2} \cdot \frac{144\,\text{in}^2}{\text{ft}^2} + 5\,\text{ft} \cdot 32.2\frac{\text{ft}}{\text{s}^2}\right)} = 57.4\frac{\text{ft}}{\text{s}} = 17.5\frac{\text{m}}{\text{s}}$$

5.34* $V_2 = \sqrt{\dfrac{\dfrac{2(P_1 - P_2)}{\rho} + 2g(z_1 - z_2)}{1 - (A_1 / A_2)^2}}$

$$= \sqrt{\frac{\dfrac{2(7-5)\dfrac{\text{lbf}}{\text{in}^2}}{0.72(62.3)\dfrac{\text{lbm}}{\text{ft}^3}} \cdot \dfrac{32.2\ \text{lbm ft}}{\text{lbf s}^2} \cdot \dfrac{144\ \text{in}^2}{\text{ft}^2} + 2 \cdot 32.2\dfrac{\text{ft}}{\text{s}^2} \cdot 2\ \text{ft}}{1 - (0.5)^4}} = 24.0\frac{\text{ft}}{\text{s}} = 7.33\frac{\text{m}}{\text{s}}$$

$$Q = VA = 24.0\frac{\text{ft}}{s} \cdot \frac{\pi}{4}(0.5\,\text{ft})^2 = 4.72\frac{\text{ft}^3}{\text{s}} = 0.13\frac{\text{m}}{\text{s}}$$

5.36* $V = C_v\sqrt{\dfrac{2(P_1 - P_2)}{\rho\left(1-(A_2/A_1)^2\right)}}$; $\quad P_{\substack{\text{water-mercury}\\ \text{interface}}} = P_2 + \rho_{\text{waater}} g \cdot 2\text{ in}$

$$P_{\substack{\text{mercury-air}\\ \text{interface}}} = P_1 = P_{\substack{\text{mercury-water}\\ \text{interface}}} + \rho_{\text{Hg}} g \cdot 0.1 \text{ in}$$

$$P_1 - P_2 = 2\text{ in }\rho_w g + 0.1\text{ in }\rho_{\text{Hg}} g = \rho_w g \cdot (2 + 1.36)\text{ in}$$

$$V = 1.0\sqrt{\frac{2 \cdot 32.2\dfrac{\text{ft}}{\text{s}^2} \cdot 62.3\dfrac{\text{lbm}}{\text{ft}^3} \cdot \left(\dfrac{3.36}{12}\text{ ft}\right)}{\left(0.075\dfrac{\text{lbm}}{\text{ft}^3}\right)\left(1-0.1^2\right)}} = 123\frac{\text{ft}}{\text{s}} = 37.5\frac{\text{m}}{\text{s}}$$

$$Q = VA = 123\frac{\text{ft}}{\text{s}} \cdot 1\text{ ft}^2 = 123\frac{\text{ft}^3}{\text{s}} = 3.48\frac{\text{m}^3}{\text{s}}$$

5.39 $V_2 = C_v\sqrt{\dfrac{2(-\Delta P)}{\rho\left(1-\left(\dfrac{D}{D_0}\right)^4\right)}}$; $\quad -\Delta P = \dfrac{\rho V_2^2}{2C_v^2}\left(1-\left(\dfrac{D}{D_0}\right)^4\right)$

$$V_2 = \frac{1\text{ ft}}{\text{s}}\left(\frac{\dfrac{\pi}{4}(1\text{ in})^2}{\dfrac{\pi}{4}(0.2\text{in})^2}\right) = 25\frac{\text{ft}}{\text{s}}$$

$$-\Delta P = \frac{\left(55\dfrac{\text{lbm}}{\text{ft}^3}\right)\left(25\dfrac{\text{ft}}{\text{s}}\right)^2\left(1-(0.2)^4\right)}{(2)(0.6)^2} \cdot \frac{\text{lbf s}^2}{32.2\text{ lbm ft}} \cdot \frac{\text{ft}^2}{144\text{in}^2} = 10.3\ \frac{\text{lbf}}{\text{in}^2} = 70.9\text{ kPa}$$

5.41 This is simplest by trial and error. First we guess that $D_2 / D_1 = 0.5$. Then

$$V_2 = V_1 \cdot \left(\frac{D_1}{D_2}\right)^2 = 1\frac{\text{ft}}{\text{s}} \cdot \left(\frac{1}{0.5}\right)^2 = 4\frac{\text{ft}}{\text{s}} \text{ and}$$

$$-\Delta P = \left(1-\left(\frac{D_2}{D_1}\right)^4\right)\frac{\rho}{2} \cdot \left(\frac{V}{C_v}\right)^2$$

$$= \left(1-0.5^4\right)\frac{13.6 \cdot 62.3\dfrac{\text{lbm}}{\text{ft}^3}}{2} \cdot \left(\frac{4\dfrac{\text{ft}}{\text{s}}}{0.62}\right)^2 \cdot \frac{\text{lbf s}^2}{32.2\text{ lbm ft}} \cdot \frac{\text{ft}^2}{144\text{ in}^2} = 3.57\frac{\text{lbf}}{\text{in}^2}$$

This done by Excel as explained in the Excel file given in e learning

5.51 $V_{\text{out}} = \sqrt{2gh}$; $Q_{\text{out}} = A_{\text{exit}} V_{\text{out}} = -A_{\text{tank}} \dfrac{dh}{dt}$

$$A_{\text{tank}} = 50\ \text{ft}^2 + 50\ \text{ft}^2\left(\frac{h}{20\ \text{ft}}\right) = 50\ \text{ft}^2 + 2.5\ \text{ft} \cdot h = a + bh$$

where a and b are constants.

$$\frac{dh}{dt} = \frac{-A_{\text{out}}\sqrt{2gh}}{a+bh}; \qquad -A_{\text{out}}\sqrt{2g}\int_0^{\Delta t} dt = \int_{h_0}^{h} \frac{a\,dh}{\sqrt{h}} + b\int_{h_0}^{h} \sqrt{h}\,dh$$

$$\Delta t = \frac{1}{A_{\text{out}}\sqrt{2g}}\left[\frac{a}{\frac{1}{2}}\sqrt{h} + \frac{b}{\frac{3}{2}} h^{\frac{3}{2}}\right]_{h=0}^{h=h_{\text{top}}}$$

$$= \frac{1}{1\ \text{ft}^2\sqrt{2 \cdot 32.2 \frac{\text{ft}}{\text{s}^2}}}\left[\frac{50\ \text{ft}^2}{\frac{1}{2}}\sqrt{20\ \text{ft}} + \frac{2.5\ \text{ft}}{\frac{3}{2}}(20\ \text{ft})^{\frac{3}{2}}\right] = 74.3\ \text{s}$$