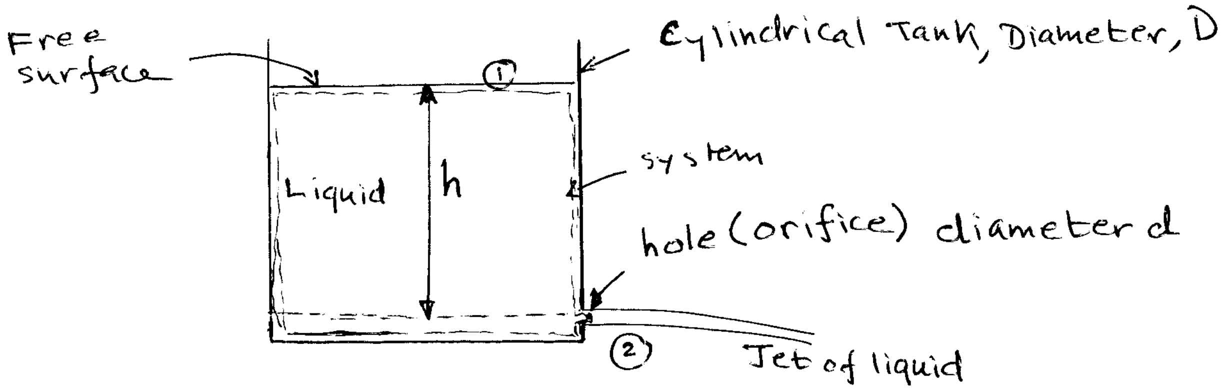
TORICELLI'S EQUATION:

Based on assumption that the flow is frictionless.

Tank Draining:

objective: Find the volume flow rate issuing from a tank with a hole at the side or the bottom of the tank.



Definition of system: Liquid as indicated between level ① free surface (considered as input to system) and plane ② just outside the tank (considered as output from system).

Target: find velocity $V_2 \rightarrow$ multiply by area $A_2 \rightarrow Q$

Solution: Apply Bernoulli's equation to a system in such a way that V_2 is at the outlet of the system.

Bernoulli's Equation:

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = +\hat{W} - \mathcal{F}$$

Assumptions:

- Steady state (or quasi steady state ≡ ie changes in height h, and hence Q are very small (negligible) over short period of time $\rightarrow$ constant h
- $P_1 \simeq P_2 = P_{atm}$
- $\mathcal{F} = 0$ (Frictionless flow).
- No mechanical work

Application of Bernoulli's Equation between ① and ②

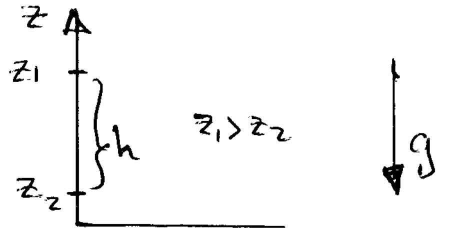
BE $\Rightarrow$ $$\left(\frac{P_2}{\rho}+gz_2+\frac{V_2^2}{2}\right)-\left(\frac{P_1}{\rho}+gz_1+\frac{V_1^2}{2}\right)=0$$

$P_1=P_2 \Rightarrow$ $$g(z_2-z_1)+\left(\frac{V_2^2}{2}-\frac{V_1^2}{2}\right)=0$$

Continuity equation between ① and ②

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad \rho_1=\rho_2$$

$$V_1 = V_2\left(\frac{A_2}{A_1}\right)$$



$$(z_2-z_1)=-h$$

$$\Rightarrow \quad -gh+\frac{V_2^2}{2}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]=0$$

$$\therefore \boxed{V_2=\sqrt{\frac{2gh}{1-\left(\frac{A_2}{A_1}\right)^2}}} \quad *$$

Change in PE is converted into KE

Notes:

1. $A_1 >> A_2 \Rightarrow \left(\frac{A_2}{A_1}\right)^2 \to 0$ This implies that $V_1 \simeq 0$ (very small that it can be neglected)

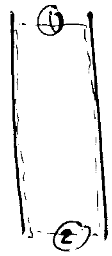
$$\therefore \boxed{V_2=\sqrt{2gh}}$$

TORICELLI'S EQUATION.

It is seen that V_2 in this case is less than V_2 in the general case $*$, since the fluid in that case is moving with measurable kinetic energy down in the tank. This means that the increase in V_2 (KE at ②) is not just PE conversion, there is conversion of KE energy ($\frac{V_1^2}{2}$) in addition to that.

2 $A_1 = A_2$ [vertical pipe open at both ends]

Based on general case $V_2 = \infty$!!!



This means that $\mathcal{F}$ cannot be neglected.

$$PE \Rightarrow \mathcal{F}$$

BE : $g(z_2 - z_1) = -\mathcal{F}$

3 The measured flow rate is less than theoretically calculated. To correct for this, a coefficient called: "coefficient of discharge" is used:

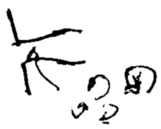
$$C_D = \frac{\text{Actual Discharge}}{\text{Theoretical Discharge}}$$

$$Q_{actual} = C_D \cdot \sqrt{2gh} \cdot A_o$$

A_o: orifice area
: hole area

This difference is due to:

- Actual discharge velocity is less than theoretical.
  This is due to friction at exit and eddying in exit stream



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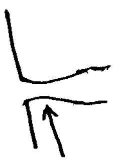
eddies.

$\therefore V_{actual} = V_{theoretical} \times C_v$

C_v = coefficient of velocity (< 1)

- True area of flow is less than orifice area due to the contraction of flow area to a minimum. The fluid is accelerated a little bit, then it slows down



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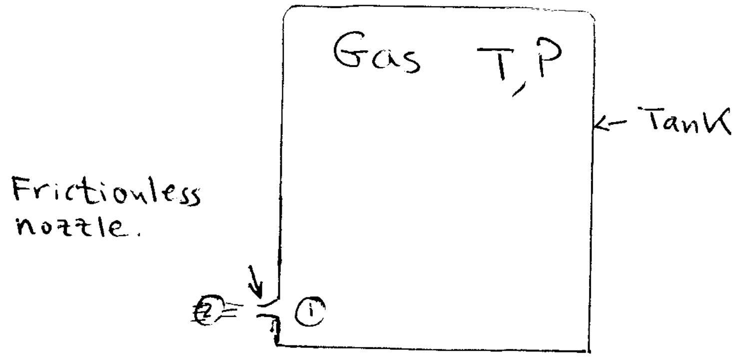
$A_{actual} = A_o \times C_c$

- $\therefore Q_{actual} = C_c \cdot C_v \sqrt{2gh} \cdot A_o$

$$\therefore \boxed{C_D = C_c \cdot C_v}$$

Ex. Flow of a gas



Required: exit velocity of gas

Given: $V_1 \simeq 0$ large tank

$P_2 = P_{atm}$

$z_1 = z_2$

BE: $\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = +\hat{W} - \mathcal{F}$

$\Rightarrow \left(\frac{P_2 - P_1}{\rho}\right) + \left(\frac{V_2^2 - V_1^2}{2}\right) = 0$

$\Rightarrow \frac{P_1 - P_2}{\rho} - \frac{V_2^2}{2} = 0$ $V_1 = 0$, $P_2 = P_{atm}$, ρ constant

$\therefore V_2 = \sqrt{\frac{2(P_1 - P_{atm})}{\rho}}$

For perfect gas $\rho = \frac{P_1 M}{R T_1}$

$\therefore V_2 = \left[\frac{2 R T_1}{P_1 M}(P_1 - P_{atm})\right]^{1/2}$

Analysis:

- choice of P_1, T_1 to calculate ρ is arbitrary
- This equation is reasonable for gas velocities up to approximately 200 m/s
- Bernoulli's equation can be applied to gases when pressure variations are small (less than 10