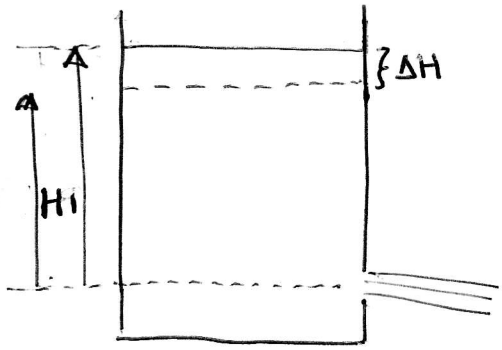
Bernoulli's Equation for Unsteady Flow:

Tank Drainage:

Considering the drainage of a tank in the case where the change in height in the tank is taking into account.

Unsteady state mass balance



Rate of Accumulation of mass in tank = Rate of Mass IN − Rate of Mass OUT

Density is constant

Chang of content of mass in tank with time $= \dfrac{dm}{dt}$; $m = \rho V_{tank}$.

$= \dfrac{d(\rho V)}{dt}$; $V_{tank} = H \cdot A$

A = cross section of tank (constant).

$= \rho A \dfrac{dH}{dt}$; ρ constant.

In the case considered

Rate of mass IN $= 0$ (no flow IN).

Rate of mass OUT $= \rho\, C_D \cdot A_o \sqrt{2gH}$ (Toricelli's equation)

$$\therefore \rho A \frac{dH}{dt} = 0 - \rho C_D A_o \sqrt{2gH}$$

$$\frac{dH}{dt} = - \frac{C_D A_o}{A} \sqrt{2g} \cdot \sqrt{H}$$

To find the time for the liquid height to fall from H_1 to H, we integrate:

$$\int_0^t dt = -\frac{A}{C_D \cdot A_0\sqrt{2g}} \int_{H_1}^{H} \frac{dH}{\sqrt{H}} \qquad \text{(any height } H\text{)}$$

$$t = +\frac{A}{C_D A_0 \sqrt{2g}} \left[\frac{\sqrt{H_1}}{1/2} - \frac{\sqrt{H}}{1/2}\right]$$

Rearrange

$$\boxed{t = \frac{A}{C_D A_0}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H}\right]}$$