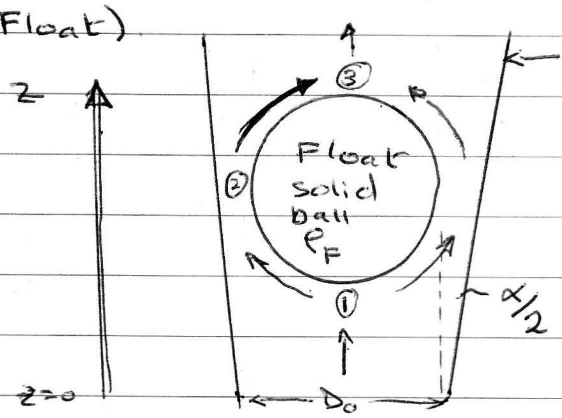
Variable Area Flowmeter – Rotameter.

- widely used to measure low flow rates.
- It uses a fixed pressure difference and a variable geometry (area) which is function of flow rate.
- Flow is linearly proportional to scale reading.

Principle and Mode of operation.

- Fluid flows up a tapered transparent (glass or plastic) tube and supports a floating weight (Float).

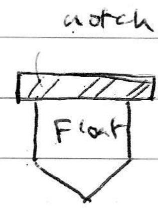


tapered tube with diameter

$$D = D_o + Bz.$$

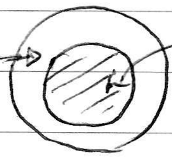
$$B = 2\tan(\alpha/2)$$

Shape of float
- Ball
- cone (more stable)
- Cone with notches → Float spins → more stability.



- Height of float indicates flow rate
- Δp across the float is virtually constant.
- Area between float and tube varies with flow.

Flow area



Float

Theory:

At steady state conditions, the force balance around the ball is

$$\Sigma F = 0 \qquad +ve \downarrow$$

$$0 = F_{gravity} + F_{pressure\ force\ on\ top} - F_{buoyancy} - F_{pressure\ on\ bottom}$$

$$0 = \rho_F g V_F + P_3 A_F - \rho_f g V_F - P_1 A_F$$

$$\therefore V_F (\rho_F - \rho_f) g = A_F (P_1 - P_3)$$

A_F : projected area of float

V_F : vol. of float

ρ_F : Float density

ρ_f : fluid density

(A) $$\boxed{(P_1 - P_3) = \frac{V_F (\rho_F - \rho_f) g}{A_F}}$$ ← constant

Bernoulli's equation:

Between (1) & (2) ; ignoring changes in static head.

$$(P_1 - P_2) = \rho_f \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)$$

using continuity equation $V_1 A_1 = V_2 A_2$

$$(P_1 - P_2) = \rho_f \frac{V_2^2}{2}\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)$$

$$\left(A_2/A_1\right)^2 << 1$$

$$\therefore (P_1 - P_2) = \rho_f \frac{V_2^2}{2}$$

$P_2 \simeq P_3$ (sudden expansion)

(B) $$\therefore \boxed{(P_1 - P_3) = \rho_f \frac{V_2^2}{2}}$$

(A) = (B)

$$\rightarrow \rho_f \frac{V_2^2}{2} = \frac{V_F (\rho_F - \rho_f) g}{A_F}.$$

$$V_2 = \sqrt{\frac{2 V_F g}{A_F} \frac{(\rho_F - \rho_f)}{\rho_f}}$$

Volumetric flow rate

$$Q = C_D \cdot A_2 \cdot V_2$$

$$A_F = \frac{\pi D_o^2}{4} \qquad D_o = D_F$$

$$A_2 = \frac{\pi}{4}\left[D^2 - D_o^2\right]$$

$$= \frac{\pi}{4}\left[(D_o + 2h \tan\alpha/2)^2 - D_o^2\right]$$

$$= \frac{\pi}{4}\left[\cancel{D_o^2} + 4h \tan\alpha/2 \cdot D_o + \underbrace{4h^2 \tan^2\alpha/2}_{\simeq 0} - \cancel{D_o^2}\right]$$

$$\simeq \pi h D_o \tan\alpha/2$$

$$\therefore Q = C_D \cdot \pi h \cancel{D_o} \tan\alpha/2 \sqrt{\frac{2 V_F g}{\left(\frac{\pi D_o^2}{4}\right)} \frac{(\rho_F - \rho_f)}{\rho_f}}$$

$$= C_D h \tan(\alpha/2) \sqrt{8\pi V_F g \left(\frac{\rho_F - \rho_f}{\rho_f}\right)}$$

$$\therefore Q = K \sqrt{\frac{(\rho_F - \rho_f)}{\rho_f}} \cdot h \qquad K = C_D \tan\alpha/2 \sqrt{2\pi V_F g}$$

Mass flow rate

$$m = K \sqrt{\rho_f (\rho_F - \rho_f)}\, h.$$

K = constant from calibration and geometry of tube and float.

Errors are least when $\frac{dm}{d\rho_f} = 0$

$$\rightarrow \boxed{\rho_F = 2\rho_f}$$

Effect of temp or ρ_f on reading:

From calibration h indicates to a certain Q.

$$Q = K\sqrt{\frac{\Delta\rho}{\rho_f}} \cdot h \qquad \Delta\rho = \rho_F - \rho_f.$$

At conditions ① & ② we have for the same h

$$Q_1 = \cancel{K}\sqrt{\left(\frac{\Delta\rho}{\rho_f}\right)_1} \cdot \cancel{h}$$

$$Q_2 = \cancel{K}\sqrt{\left(\frac{\Delta\rho}{\rho_f}\right)_2} \cdot \cancel{h}.$$

$$\therefore \frac{Q_1}{Q_2} = \sqrt{\left(\frac{\Delta\rho}{\rho_f}\right)_1 \Big/ \left(\frac{\Delta\rho}{\rho_f}\right)_2}$$

$$\frac{Q_1}{Q_2} = \sqrt{\left(\frac{\Delta\rho_1}{\Delta\rho_2}\right)\left(\frac{\rho_{f2}}{\rho_{f1}}\right)}$$

$\Delta\rho = \rho_F - \rho_f$.
if $\rho_F \gg \rho_f$
OR $\Delta\rho_1 \simeq \Delta\rho_2$.

$$\therefore \frac{Q_1}{Q_2} = \sqrt{\left(\frac{\rho_{f2}}{\rho_{f1}}\right)}$$

$$\therefore Q_1 = Q_2\left(\frac{\rho_{f2}}{\rho_{f1}}\right)^{1/2}$$

ρ_{f2}, ρ_{f1} could be densities at two different temp.
OR densities of two different fluids.

ex:

h for N_2 $\rightarrow$ $100\ \frac{cm^3}{min.}$; $Q_{helium} = ?$

$$Q_{He} = Q_{N_2}\left(\frac{\rho_{nitrogen}}{\rho_{helium}}\right)^{1/2} \quad ; \quad \frac{\rho_{N_2}}{\rho_{H_2}} = \frac{28}{4}$$

$$\therefore Q_{He} = 100\ \frac{cm^3}{min}\left(\frac{28}{4}\right)^{1/2} = 265\ cm^3/min.$$