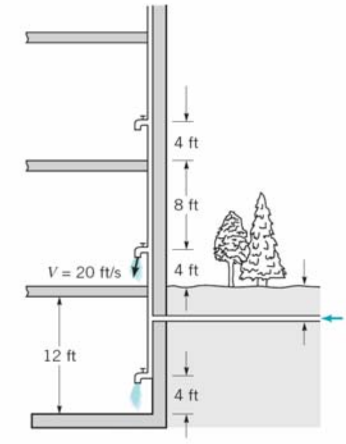
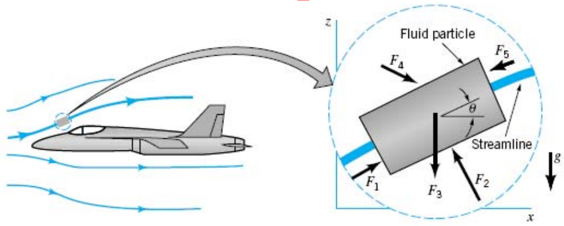
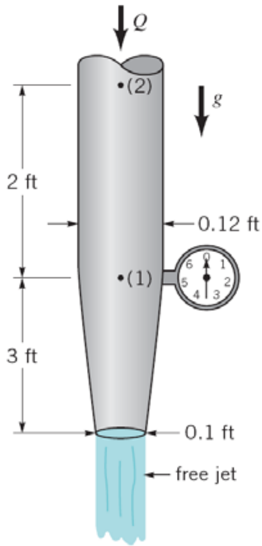


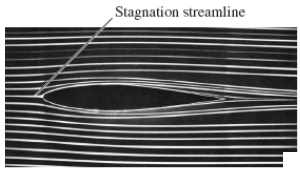
Fluid Mechanics (0905241)

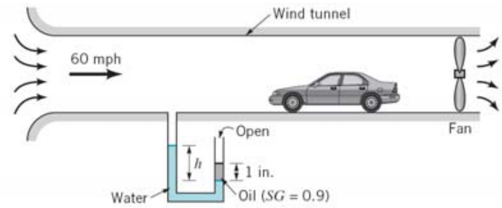
Bernouli's Equation

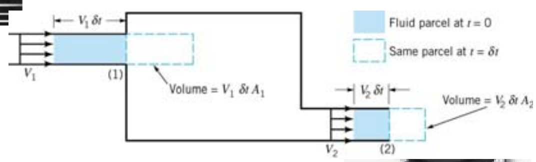




Prof. Zayed Al-Hamamre







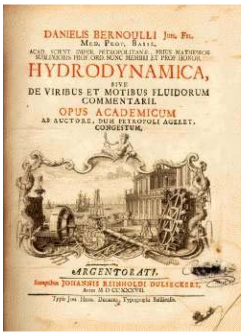
Content

- Introduction
- Bernoulli Equation
- Physical Interpretation
- Limitations on the Use of the Bernoulli Equation
- Application of Bernoulli Equation
- Flow Rate Measurement
- B.E. for Non-uniform Flows
- B.E. for Unsteady flows
- General Form of the Flow Equation

Introduction

Daniel Bernoulli

(1700-1782)



Is one of the most well-known equations of motion in fluid mechanics

Swiss mathematician, son of Johann Bernoulli, who showed that as the velocity of a fluid increases, the pressure decreases, a statement known as the Bernoulli principle. He won the annual prize of the French Academy ten times for work on vibrating strings, ocean tides, and the kinetic theory of gases. For one of these victories, he was ejected from his jealous father's house, as his father had also submitted an entry for the prize. His kinetic theory proposed that the properties of a gas could be explained by the motions of its particles.

- Acceleration of Fluid Particles give Fluid Dynamics
- Newton's Second Law is the Governing Equation
- Applied to an Idealized Flow and Assumes Inviscid Flow
- There are numerous assumptions.

The Bernoulli Equation is Listed in Michael Guillen's book "Five Equations that Changed the World: The Power and Poetry of Mathematics"

Conservation Principle of Energy

➢ Forms of energy, E:

$$E = E_k + E_P + E_U$$

$$E_k : kinetic\ energy; \quad E_k = \frac{1}{2} m u^2$$

$$E_p : potential\ energy; \quad E_p = mgz$$

$$U : Internal\ energy$$

$$U = function(T, P, phase)$$

➢ In this course we study fluids of one phase with moderate pressure changes, Thus:

$$U = function(T)$$

Conservation Principle of Energy

- For flowing streams of fluids there is additional form of energy which is **pressure energy** (PV):
- Pressure energy: work required to introduce fluid within the system for inlet streams or to remove it from system for outlet streams.

- For flowing streams of fluids:

$$\dot{E} = \dot{E}_k + \dot{E}_P + \dot{E}_U + PQ$$

$$= \frac{1}{2}\dot{m}u^2 + \dot{m}gz + \dot{U} + PQ$$

Thus

$$\dot{E} = \frac{1}{2}\dot{m}u^2 + \dot{m}gz + \dot{U} + \frac{\dot{m}P}{\rho}$$

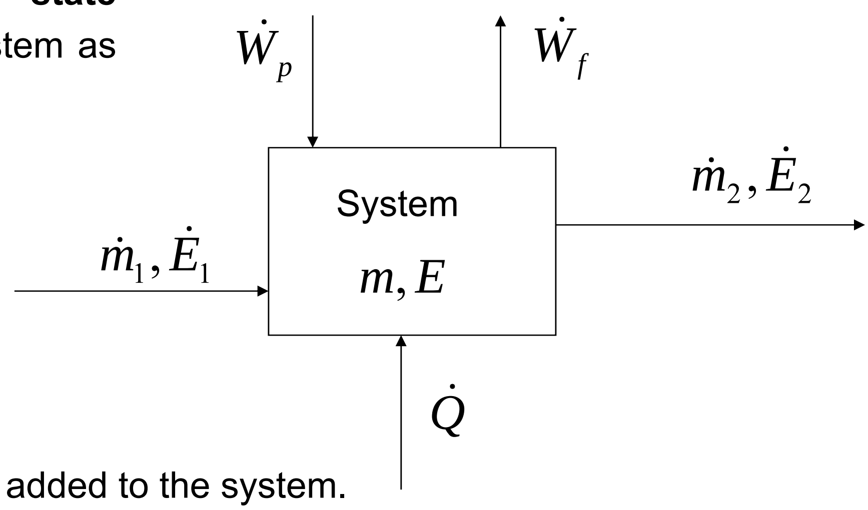
- Pressure energy = PV
- Rate of pressure energy =

$$P\dot{V} \equiv PQ$$

$$Where\, Q = \frac{\dot{m}}{\rho}$$

Conservation Principle of Energy

- Consider **a steady state** process of the open system as shown :



$\dot{Q}$: rate of heat energy added to the system.

$\dot{W}_p$: rate of shaft work done on the system by pump, mixer, piston,… etc.

$\dot{W}_f$: rate of work done by fluid, in the system, to overcome friction(rate energy losses due to friction)

Conservation Principle of Energy

- Apply steady state mass balance:

$$\frac{dm}{dt} = \dot{m}_1 - \dot{m}_2 = 0 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$$

- Apply steady state energy balance:

$$\frac{dE}{dt} = \dot{E}_1 - \dot{E}_2 + \dot{Q} + \dot{W}_p - \dot{W}_f = 0$$

Or

$$\dot{E}_2 - \dot{E}_1 = \dot{Q} + \dot{W}_p - \dot{W}_f$$

- If **adiabatic process**: $\dot{Q} = 0$

$$\dot{E}_2 - \dot{E}_1 = \dot{W}_p - \dot{W}_f$$

But $$\dot{E} = \frac{1}{2}\dot{m}u^2 + \dot{m}gz + \dot{U} + \frac{\dot{m}P}{\rho}$$

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Conservation Principle of Energy

- Then:

$$\left(\frac{1}{2}\dot{m}_2 u_2^2 + \dot{m}_2 g z_2 + \dot{U}_2 + \frac{\dot{m}_2 P_2}{\rho_2}\right) - \left(\frac{1}{2}\dot{m}_1 u_2^2 + \dot{m}_1 g z_1 + \dot{U}_1 + \frac{\dot{m}_1 P_1}{\rho_1}\right) =$$
$$\dot{W}_p - \dot{W}_f$$

- From St.St. MB: $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$\left(\frac{1}{2}\dot{m}u_2^2 + \dot{m}gz_2 + \dot{U}_2 + \frac{\dot{m}P_2}{\rho_2}\right) - \left(\frac{1}{2}\dot{m}u_1^2 + \dot{m}gz_1 + \dot{U}_1 + \frac{\dot{m}P_1}{\rho_1}\right) = \dot{W}_p - \dot{W}_f$$

- Dividing by $\dot{m}$ gives:

$$\left(\frac{1}{2}u_2^2 + gz_2 + \frac{\dot{U}_2}{\dot{m}} + \frac{P_2}{\rho_2}\right) - \left(\frac{1}{2}u_1^2 + gz_1 + \frac{\dot{U}_1}{\dot{m}} + \frac{P_1}{\rho_1}\right) = \frac{\dot{W}_p}{\dot{m}} - \frac{\dot{W}_f}{\dot{m}}$$

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Conservation Principle of Energy

Let: $\frac{\dot{W}_p}{\dot{m}} = w_p$: Shaft work done on the system per unit mass of the flowing fluid; in SI units J/kg.

$\frac{\dot{W}_f}{\dot{m}} = w_f$: Energy losses due to friction per unit mass of the flowing fluid ; in SI units J/kg.

$\frac{\dot{U}}{\dot{m}} = \hat{U}$: Internal energy per unit mass of the flowing fluid ; in SI units J/kg.

and rearrange the equation to become:

$$g(z_2 - z_1) + \frac{1}{2}\left(u_2^2 - u_1^2\right) + \left(\hat{U}_2 - \hat{U}_1\right) + \left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) = w_p - w_f$$

- If **isothermal process** ($T_1 = T_2 = T$) and thus the specific internal energy does not change from 1 to 2 for moderate pressure changes:

Conservation Principle of Energy

$$\Rightarrow \hat{U}_2 - \hat{U}_1 = 0$$

$$g(z_2 - z_1) + \frac{1}{2}\left(u_2^2 - u_1^2\right) + \left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) = w_p - w_f$$

- If incompressible fluid($\rho_1 = \rho_2 = \rho$):

$$g(z_2 - z_1) + \frac{1}{2}\left(u_2^2 - u_1^2\right) + \frac{P_2 - P_1}{\rho} = w_p - w_f$$

Mechanical Energy Balance (MEB)

- Units of each term in MEB:
- SI: m^2/s^2 or J/kg
- British units: ft^2/s^2 or lbf.ft/slug

Conservation Principle of Energy

- Dividing MEB by gravitational acceleration, g, gives head form of energy balance:

$$(z_2 - z_1) + \frac{1}{2g}\left(u_2^2 - u_1^2\right) + \frac{P_2 - P_1}{\rho g} = \frac{w_p}{g} - \frac{w_f}{g}$$

Let:

$\frac{w_p}{g} = h_p$:Total energy head done on the system by pump for example; in SI units m.

$\frac{w_f}{g} = h_f$: Energy head losses due to friction; in SI units m

Head form of (MEB)

$$(z_2 - z_1) + \frac{1}{2g}\left(u_2^2 - u_1^2\right) + \frac{P_2 - P_1}{\rho g} = h_p - (h_f + h_m)$$

Potential head	velocity head	Pressure head	Pump head	Head losses

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Conservation Principle of Energy

- MEB represents transformation between potential, kinetic, pressure, shaft, and friction energy when fluid flows from 1 to 2.

- To overcome energy losses due to friction and to have a fluid in motion(kinetic energy) from 1 to 2, the system must have at least:

- - Potential source i.e. $z_1 > z_2$
- - Pressure source of energy: $P_1 > P_2$

- mechanical source of energy: $w_p > 0$

-Or a combination of these sources.

- If pressure source only available, then the fluid flows from the high pressure zone to low pressure zone.

-Remember the physical fact that energy loss due to friction: $\mathbf{w_f \geq 0}$. This help us sometimes to explore the flow direction.

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Conservation Principle of Energy

- Ideal (inviscid) fluid: fluid of zero viscosity:

$$\tau = \mu \frac{du}{dy} = 0 \Rightarrow$$ ➢Frictionless flow

Ideal fluid does not exist in practice

- If **frictionless flow or ideal fluid** (w_f=0), MEB becomes:

$$g(z_2 - z_1) + \frac{1}{2}(u_2^2 - u_1^2) + \frac{P_2 - P_1}{\rho} = w_p$$

- If energy losses due to friction is low it is safe to assume frictionless flow.

Bernoulli Equation

- If no shaft wok between 1 and 2: (w_p=0), MEB becomes:

$$g(z_2 - z_1) + \frac{1}{2}(u_2^2 - u_1^2) + \frac{P_2 - P_1}{\rho} = 0$$

Bernoulli's equation

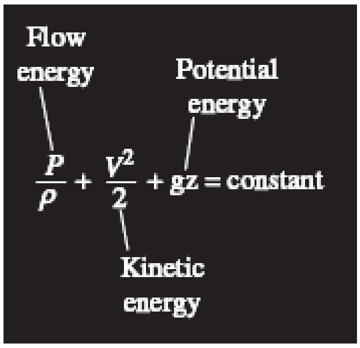
- MEB is derived under the following 4 assumptions: **steady state, adiabatic, isothermal process and incompressible fluid.**

- Two more assumptions are involved to derive Bernoulli's equation: **frictionless flow** and **no shaft work**. Thus Bernoulli's Eq. is special case of MEB with 6 assumptions

Introduction

- The **Bernoulli equation is relation between pressure,** velocity, and elevation,
- It is a special a case of the mechanical energy equation.
- The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant when the Friction effect is negligible.

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$



- As a fluid particle moves, pressure and gravity both do work on the particle:

P is the pressure work term, and γz is the work done by weight.

$1/2\rho V^2$ is the kinetic energy of the particle.

Physical Interpretation:

Bernoulli's Equation can be written in terms of heads:

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \textit{Total Head (constant)} \quad \Rightarrow \quad \frac{\Delta p}{\rho g} + \frac{\Delta V^2}{2g} + \Delta z = 0.0$$

Pressure Head ($\frac{p}{\rho g}$)

Velocity or Dynamic Head ($\frac{V^2}{2g}$)

Elevation Term (z)

Pressure Head: represents the height of a column of fluid that is needed to produce the pressure p.

Velocity Head: represents the vertical distance needed for the fluid to fall freely to reach V.

Elevation Term: related to the potential energy of the particle.

Static, Dynamic, and Stagnation Pressures

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$

P is the **static pressure** represents the actual thermodynamic pressure of the fluid

$\rho V^2/2$ is the **dynamic pressure** represents the pressure rise when the fluid in motion is brought to a stop isentropically

$\rho g z$ is the **hydrostatic pressure**, it accounts for the elevation effects

The sum of the static and dynamic pressures is called the **stagnation pressure**

The sum of the static, dynamic, and hydrostatic pressures is called the **total pressure** ➡ ***The total pressure along a streamline is constant***

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Static, Stagnation, Dynamic, and Total Pressure: Bernoulli Equation

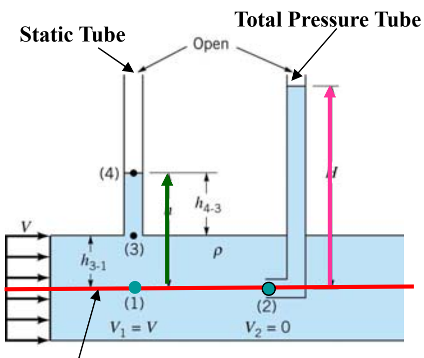
- The sum of the static and dynamic pressures is called the **stagnation pressure**

Following a streamline:

$$p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 = p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1$$

0   0, no elevation   0, no elevation

$$p_2 = p_1 + \frac{1}{2}\rho V_1^2$$



Follow a Streamline from point 1 to 2

Note: $p_2 = \gamma H$ "Total Pressure = Dynamic Pressure

$H > h$ Stagnated + Static Pressure"

$$V_1 = \sqrt{2g(H-h)}$$

- In this way we obtain a measurement of the centerline flow with piezometer tube.

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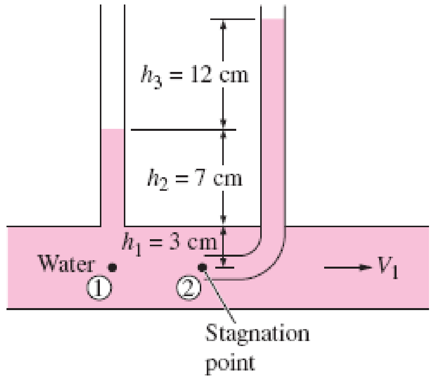
Example

- A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown, to measure static and stagnation (static dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.



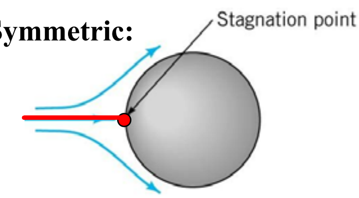
Stagnation Point: Bernoulli Equation

Stagnation point: the point on a stationary body in every flow where V= 0

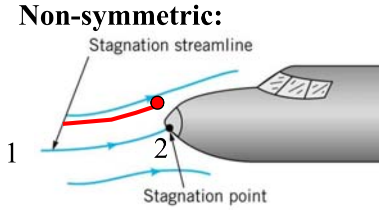
- There is a stagnation point on any stationary body that is placed into a flowing fluid

Stagnation Streamline: The streamline that terminates at the stagnation point.

Symmetric:



Non-symmetric:



If there are no elevation effects, the stagnation pressure is largest pressure obtainable along a streamline: all kinetic energy goes into a pressure rise:

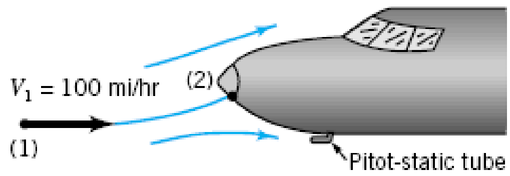
$$p_2 = p_1 + \frac{\rho V_1^2}{2}$$ Stagnation pressure

The largest pressure obtainable along a given streamline.

The blunt body stopping the fluid does not have to be a solid. I could be a static column of fluid

Example

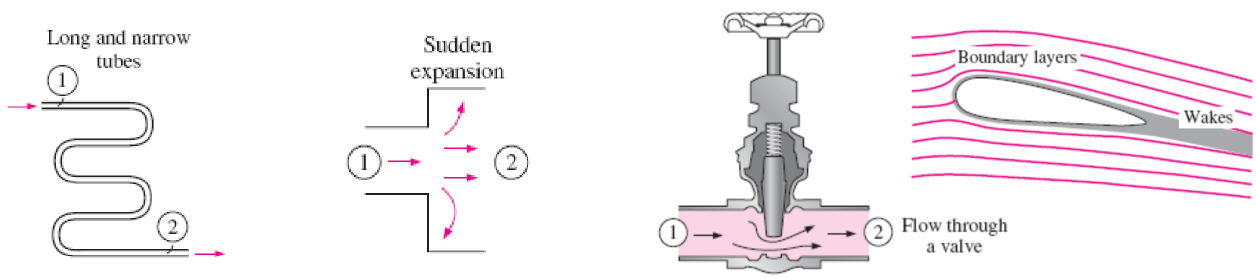
➢ An airplane flies 100 mi/hr at an elevation of 10,000 ft in a standard atmosphere as shown. Determine the pressure at the stagnation point on the nose of the airplane, point 2, and the pressure difference indicated by a Pitot-static probe attached to the fuselage



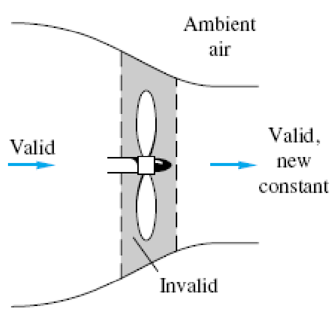
Limitations on the Use of the Bernoulli Equation

➢ **Steady flow**: it should not be used during the transient start-up and shut-down periods, or during periods of change in the flow conditions.

➢ **Frictionless flow**: frictional effects are negligible for short flow sections with large cross sections, especially at low flow velocities. Frictional effects are usually significant

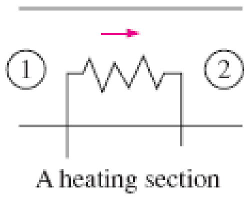


➢ **No shaft work: it** is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles



Limitations on the Use of the Bernoulli Equation

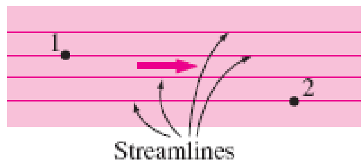
- **Incompressible flow:** This condition is satisfied by liquids and also by gases at Mach numbers less than about 0.3.
- **No heat transfer**



- **Flow along a streamline**

$P/\rho + V^2/2 + gz = C$ is applicable along a streamline

C, is constant *different for different streamlines*



$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

- When a region of the flow is *irrotational (no vorticity* in the flow field, i.e. zero fluid angular velocity) the value of the constant *C remains the same for all* streamlines,

The Bernoulli equation becomes applicable *across streamlines*

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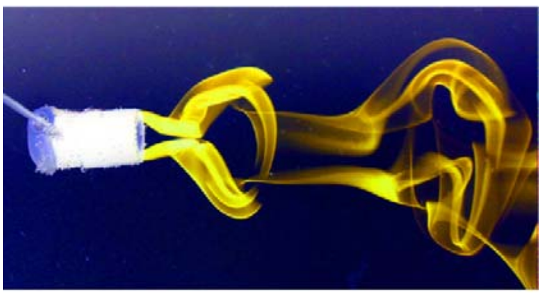
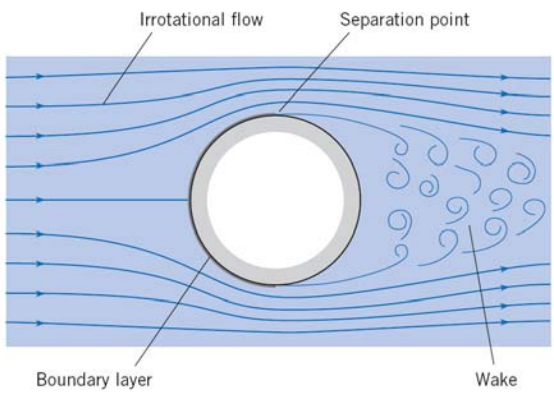
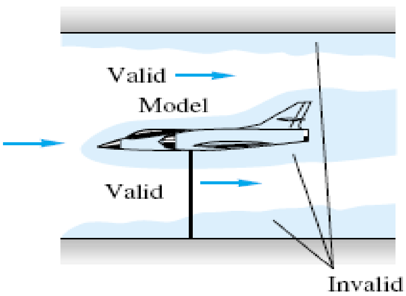
Limitations on the Use of the Bernoulli Equation

Rotational effects







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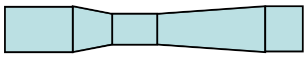
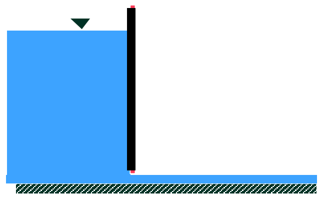
Bernoulli Equation Applications

- Stagnation tube
- Pitot tube
- Free Jets
- Orifice
- Venturi
- Sluice gate
- Sharp-crested weir



Applicable to contracting streamlines (accelerating flow).



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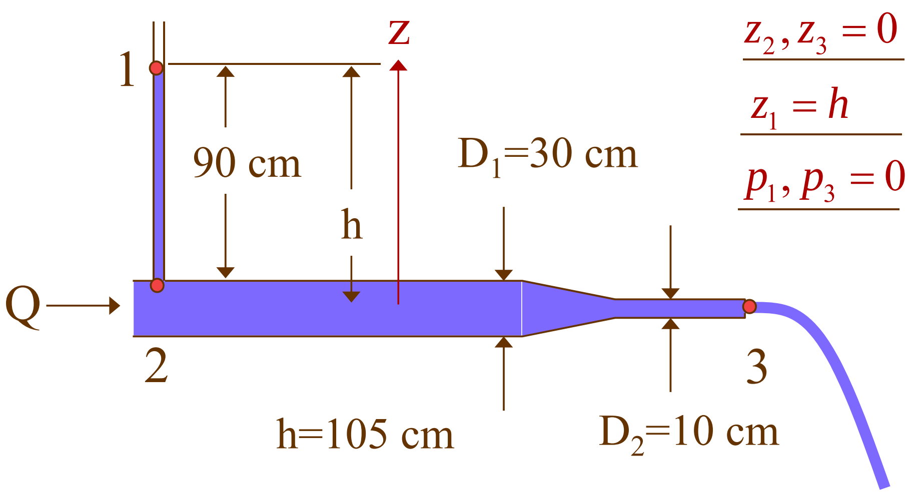
Nozzle Flow Rate: Find Q

Crossing streamlines

Along streamline

Coordinate system

Pressure datum gage pressure



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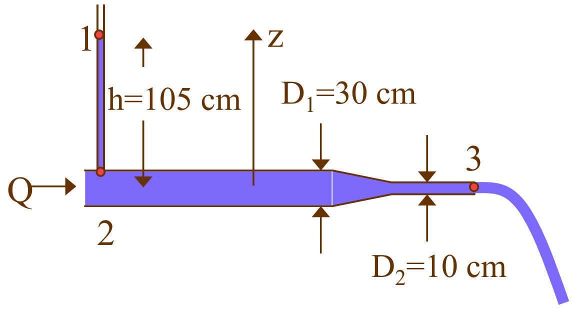
Nozzle Flow Rate: Find Q

Across streamlines (point 1 and point 2)

$$\cancel{\frac{p_1}{\gamma}} + z_1 = \frac{p_2}{\gamma} + \cancel{z_2} \longrightarrow h = \frac{p_2}{\gamma}$$



Now along the streamline

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C_{p''}$$

$$\underbrace{\frac{p_2}{\gamma}}_{h} + \cancel{z_2} + \frac{V_2^2}{2g} = \cancel{\frac{p_3}{\gamma}} + \cancel{z_3} + \frac{V_3^2}{2g}$$

$$h + \frac{V_2^2}{2g} = \frac{V_3^2}{2g}$$

Two unknowns…

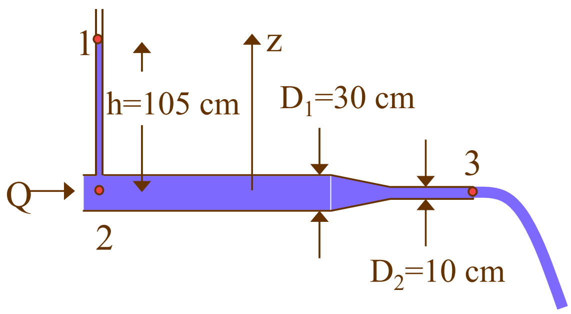
Mass conservation

$$Q = V_2 A_2 = V_3 A_3 \qquad V_2 = \frac{4Q}{\pi d_2^2}$$

Nozzle Flow Rate: Find Q

$$h + \frac{8Q^2}{g\pi^2 d_2^4} = \frac{8Q^2}{g\pi^2 d_3^4}$$

$$h = \frac{8}{g\pi^2}\left[\frac{1}{d_3^4} - \frac{1}{d_2^4}\right] Q^2$$



Free Jets

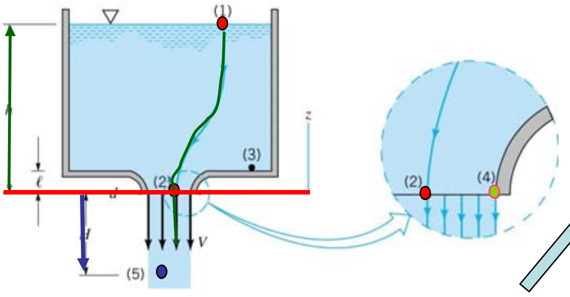
Torricelli's Equation

New form for along a streamline between any two points:

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

If we know 5 of the 6 variable we can solve for the last one.

Free Jets: Case 1



$$\gamma h = \tfrac{1}{2}\rho V^2$$

Torricelli's Equation (1643):

$$V = \sqrt{2\frac{\gamma h}{\rho}} = \sqrt{2gh}$$

As the jet falls:

$$V = \sqrt{2g\,(h + H)}$$

Following the streamline between (1) and (2):

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

0 gage | **0** | **h** | **0 gage** | **V** | **0**

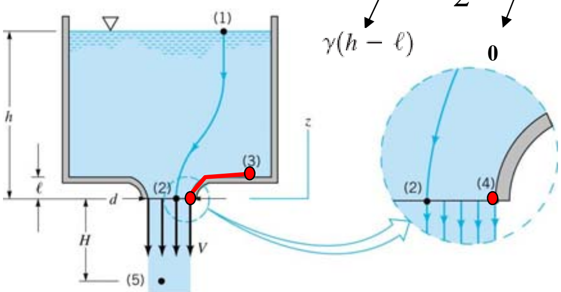
reservoir is large

Free Jets

Free Jets: Case 2

$$p_3 + \frac{1}{2}\rho V_3^{\,2} + \gamma z_3 = p_4 + \frac{1}{2}\rho V_4^{\,2} + \gamma z_4$$

$\gamma(h-\ell)$ | **0** | ℓ | **0 gage** | **V** | **0**



Then,

$$V = \sqrt{2gh}$$

- From physics or dynamics that any object dropped from rest through a distance *h in a vacuum will obtain the speed* $V = \sqrt{2gh}$

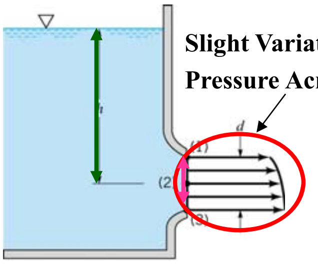
Physical Interpretation:

All the particles potential energy is converted to kinetic energy assuming no viscous dissipation.

The potential head is converted to the velocity head.

Free Jets

Free Jets: Case 3 "Horizontal Nozzle: Smooth Corners"



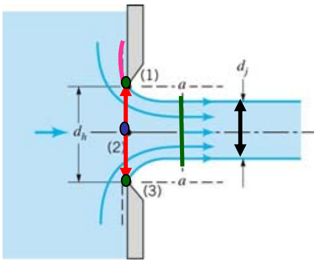
Slight Variation in Velocity due to Pressure Across Outlet

$V_1 < V_2 < V_3$

However, we calculate the average velocity at h (centerline Velocity)**, if h >> d:**

$$V = \sqrt{2\frac{\gamma h}{\rho}} = \sqrt{2gh}$$

Free Jets: Case 4 "Horizontal non smooth Nozzle: Sharp-Edge Corners"



plane of the vena contracta, section a–a.

vena contracta**: The diameter of the jet d_j is less than that of the hole d_h due to the inability of the fluid to turn the 90° corner.**

The pressure at (1) and (3) is zero, and the pressure varies across the hole since the streamlines are curved.

The pressure at the center of the outlet is the greatest.

The assumption of uniform velocity with straight streamlines and constant pressure is not valid at the exit plane.

However, in the jet the pressure at a-a is uniform, we can us Torrecelli's equation if d_j << h.

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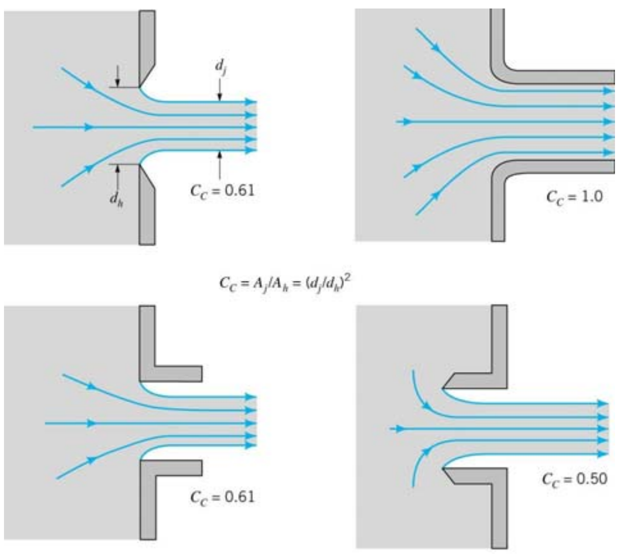
Free Jets

Free Jets: Case 4 "Horizontal Nozzle: Sharp-Edge Corners"

Vena-Contracta Effect and Coefficients for Geometries



$$C_C = A_j/A_h = (d_j/d_h)^2$$

- The velocity calculated is the theoretical value of velocity.
- Unfortunately it will be an over estimate of the real velocity because friction losses have not been taken into account.
- To incorporate friction the ***contraction coefficient*** is used to correct the **theoretical velocity,** $u_{actual} = C_v u_{theoretical}$

  in the range(0.97 - 0.99)

$$A_j = C_c A_h$$

$$V_2 = \sqrt{\frac{2gh}{1 - \frac{A_j^2}{A_1^2}}} \quad \Rightarrow \quad V_2 = C_v\sqrt{\frac{2gh}{1 - \frac{C_c^2 A_h^2}{A_1^2}}}$$

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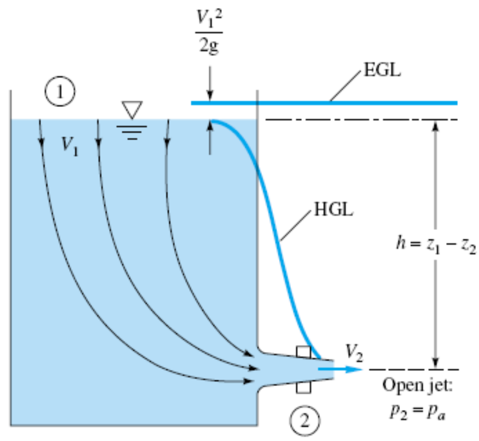
Example

Find a relation between nozzle discharge velocity V^2 and tank free-surface height h

Assume steady frictionless flow.



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Confined Flows

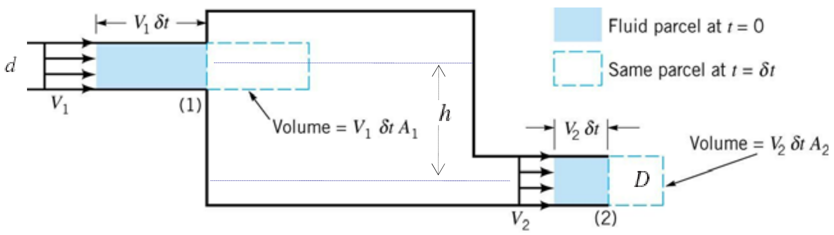
There are some flow where we can-not know the pressure a-priori because the system is confined, i.e. inside pipes and nozzles with changing diameters.

In order to address these flows, we consider both conservation of mass (continuity equation) and Bernoulli's equation.

$$\frac{d}{dt}\int_{CV} \rho \, dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

$$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

Consider flow in and out of a Tank:



The mass flow rate in must equal the mass flow rate out for a steady state flow:

$\dot{m}$ (slugs/s or kg/s) $\quad \dot{m} = \rho Q$ and $Q = VA$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

With constant density, $A_1 V_1 = A_2 V_2$, or $Q_1 = Q_2$

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Confined Flows

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

With the assumptions that $p_1 = p_2 = 0$, $z_1 = h$, and $z_2 = 0$

$$\tfrac{1}{2}V_1^2 + gh = \tfrac{1}{2}V_2^2$$

conservation of mass requires $Q_1 = Q_2$, or $A_1V_1 = A_2V_2$, ➡ $\frac{\pi}{4}D^2V_1 = \frac{\pi}{4}d^2V_2$

Hence, $V_1 = \left(\frac{d}{D}\right)^2 V_2$

$$V_2 = \sqrt{\frac{2gh}{1-(d/D)^4}}$$

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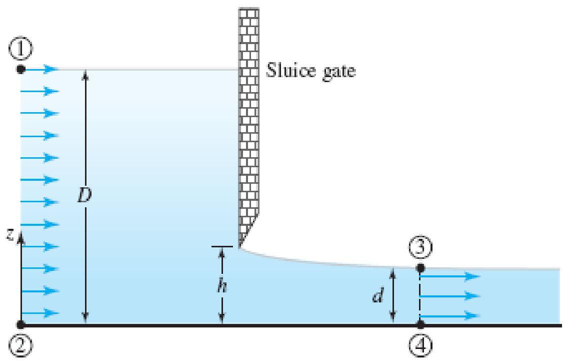
Sluice Gate

- A sluice gate is a simple device that may be used to control and measure the flow of water in an open channel flow such as that in a river, drainage ditch, or irrigation canal

- Find the relationship between the velocities and stream depths upstream and downstream

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_3}{\rho} + \frac{1}{2}V_3^2 + gz_3$$



conservation of mass requires

$$Q = V_1(DW) = V_3(dW),$$

$$V_1 = Q/DW \text{ and } V_3 = Q/dW.$$

$$\frac{p_A}{\rho} + \frac{1}{2}\left(\frac{Q}{DW}\right)^2 + gD = \frac{p_A}{\rho} + \frac{1}{2}\left(\frac{Q}{dW}\right)^2 + gd \quad ➡ \quad Q = dW\sqrt{\frac{2gD(1-d/D)}{1-(d/D)^2}}$$

Keep in mind that $C_C = A_j/A_h = (d_j/d_h)^2$ $\quad u_{actual} = C_v u_{theoretical}$

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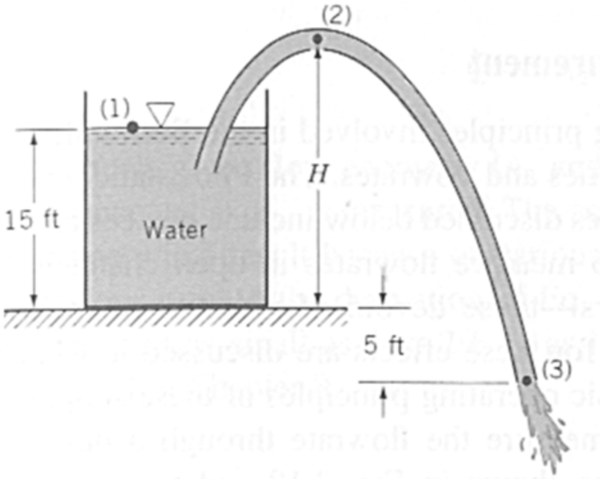
Example

Given: $T_{H_2O} = 60^0 F \quad p_{atm} = 14.7 Psia$

Determine $V_3 = ?$

$H_{max} = ?$



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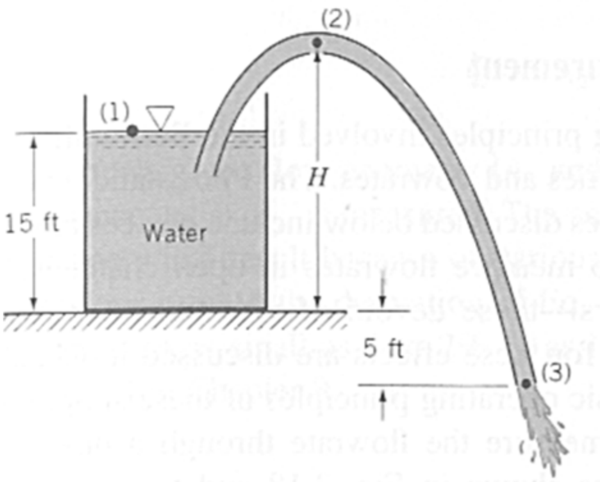
Example contd.



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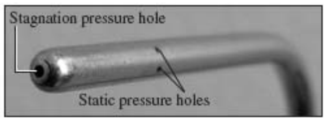
Flow Rate Measurement

Pitot Tube: Speed of Flow

H. De Pitot

(1675-1771)

- Used to measure air speed on airplanes
- Can connect a differential pressure transducer to directly measure $V^2/2g$
- Can be used to measure the flow of water in pipelines



Point measurement!

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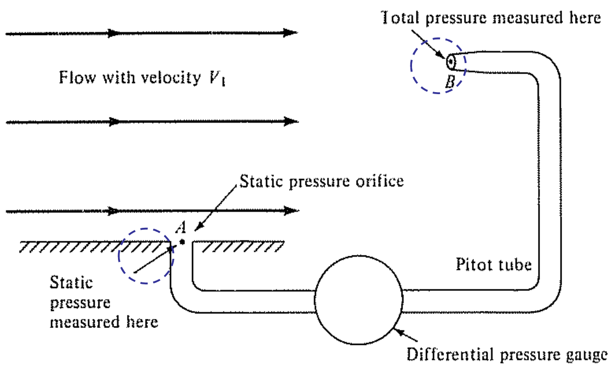
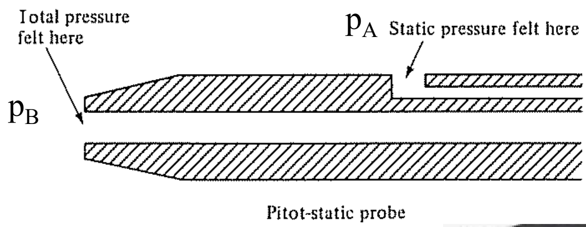
Pitot Tube

- Point A: Static Pressure, p_A
  - Surface is parallel to flow, so only random motion of gas is measured
- Point B: Total Pressure, p_B
  - Aligned parallel to flow, so particles are isentropically decelerated to zero velocity
- A combination of p_A and p_B allows us to measure V_1 at a given point
- Instrument is called a Pitot-static probe





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Pitot Tube

- The *Pitot static tube combines the tubes and they can then be easily connected to a manometer.*
- The holes on the side of the tube connect to one side of a manometer and register the *static pressure,* p_1
- The central hole is connected to the other side of the manometer to register, as before, the *stagnation pressure, p_2*

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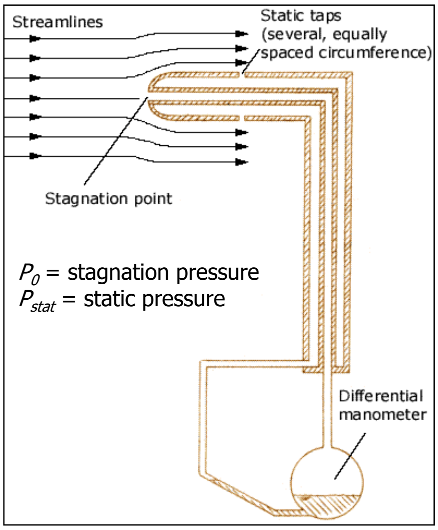
Pitot Tube



$$p_0 = p_{stat} + \frac{1}{2}\rho V^2, (Bernoulli)$$

$$V = \sqrt{2(p_0 - p_{stat})/\rho}$$

$$V = C\sqrt{2(p_0 - p_{stat})/\rho}$$

The tubes sensing static and stagnation pressures are usually combined into one instrument known as pitot static tube.

- Pressure taps sensing **static pressure** (also the reference pressure for this measurement) are placed radially on the probe stem and then combined into one tube leading to the differential manometer (p_{stat}).
- The pressure tap located at the probe tip senses the **stagnation pressure** (p_0).
- Use of the two measured pressures in the Bernoulli equation allows to determine one component of the **flow velocity** at the probe location.
- Special arrangements of the pressure taps (Three-hole, Five-hole, seven-hole Pitot) in conjunction with special calibrations are used two measure all velocity components.
- It is difficult to measure stagnation pressure in real, due to friction. The measured stagnation pressure is always less than the actual one. This is taken care of by an empirical factor C.

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Pitot Tube

We know that $p_2 = p_{stag} = p_1 + \rho \dfrac{u_1^2}{2}$

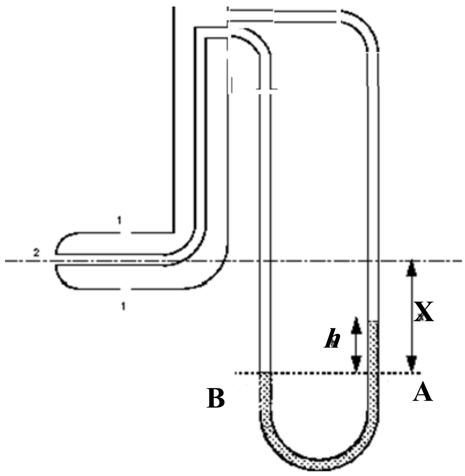
$$p_A = p_2 + \rho g X$$

$$p_B = p_1 + \rho g (X - h) + \rho_{man} g h$$

$$p_A = p_B$$

$$p_2 + \rho g X = p_1 + \rho g (X - h) + \rho_{man} g h$$

$$p_1 + gh(\rho_{man} - \rho) = p_1 + \rho \frac{u_1^2}{2} \Rightarrow u_1 = \sqrt{\frac{2gh[\rho_{man} - \rho]}{\rho}}$$



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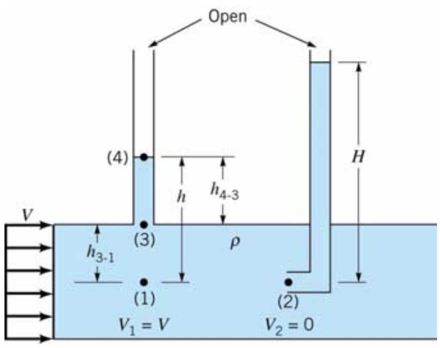
Static pressure

$$\left.\begin{aligned} p_1 &= p_3 + \rho g h_{3-1} \\ p_3 &= \rho g h_{4-3} + p_4 \quad (p_4 \to 0) \end{aligned}\right\}$$

$$\Rightarrow p_1 = \rho g h_{4-3} + \rho g h_{3-1} = \rho g h \quad (static \quad pressure)$$



p[=]Thermodynamic pressure (static pressure)

Hydrostatic pressure

ρgz - It is not actually a pressure, but does represent the change in pressure possible due potential energy variation of the fluid as a result of elevation changes.

Dynamic pressure

$\frac{1}{2}\rho V^2$ at point (2), $V_2 = 0$ (assuming $z_1 = z_2$)

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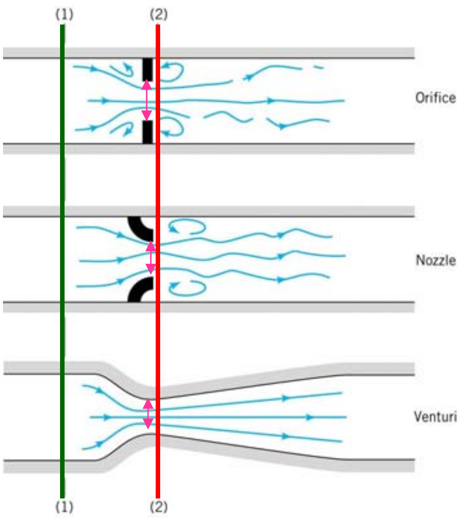
Flow Rate Measurement

Venturi and Orifice Meters



Horizontal Flow:
An increase in velocity results in a decrease in pressure.

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

Assuming conservation of mass:

$$Q = A_1 V_1 = A_2 V_2$$

Substituting we obtain:

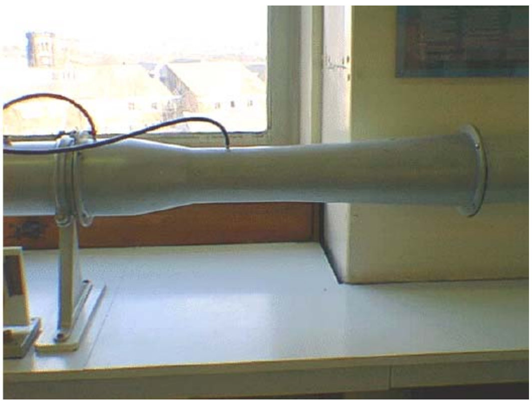
$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

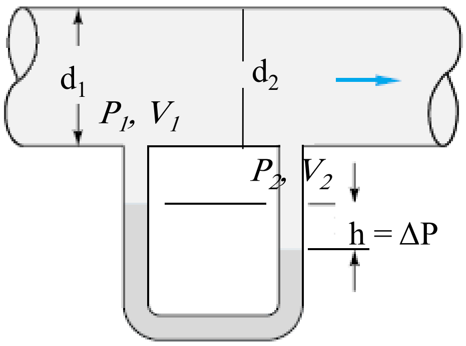
So, if we measure the pressure difference between (1) and (2) we have the flow rate.

- To get the actual discharge taking in to account the losses due to friction, we include a coefficient of discharge

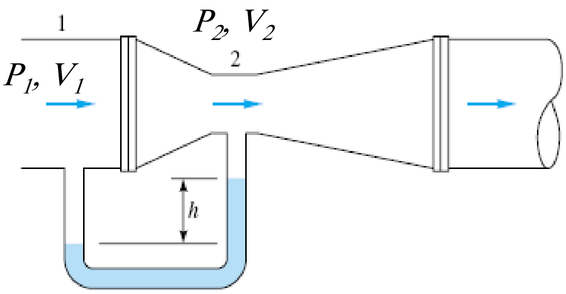
$$Q = C_c A_h \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \frac{C_c^2 A_h^2}{A_1^2})}}$$

Venturi and Orifice Meters





Orifice

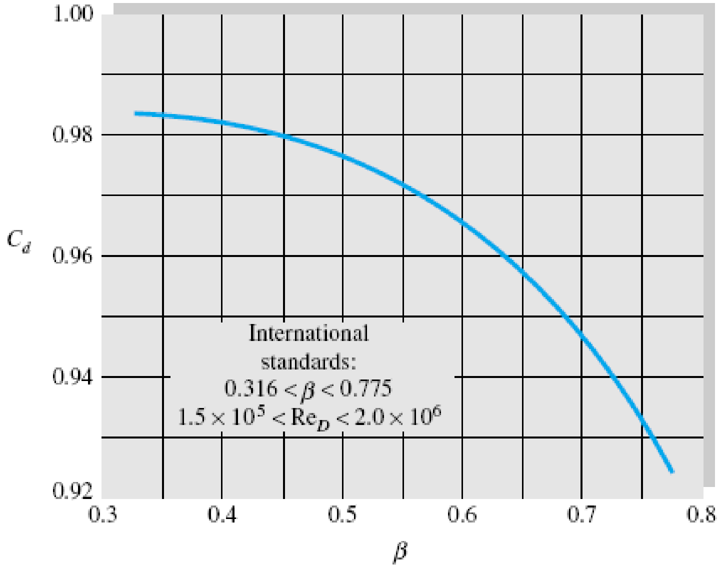


Venturi

If the viscous effect is significant (low *Re*), additional factor should be used (C_v, velocity coefficient) $u_{actual} = C_v u_{theoretical}$ in the range(0.97 - 0.99)

$$Q = C_v C_c A_h \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \frac{C_c^2 A_h^2}{A_1^2})}} \qquad C_d = C_v C_c$$

Venturi and Orifice Meters



$$\beta = \frac{D_2}{D_1}$$

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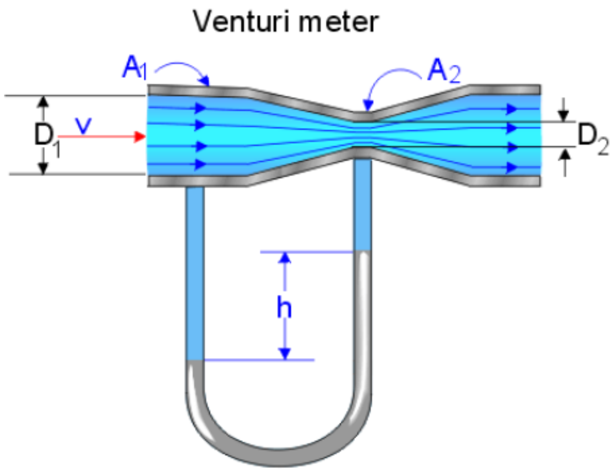
Example

In the given venturi meter, estimate the flow rate of the fluid within the device.



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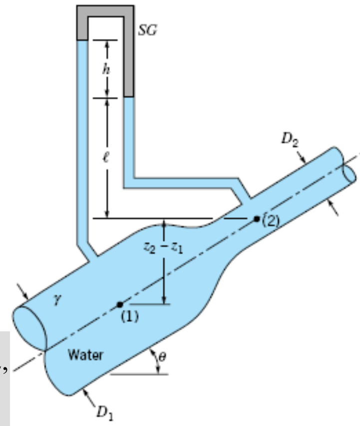
Example Cont.

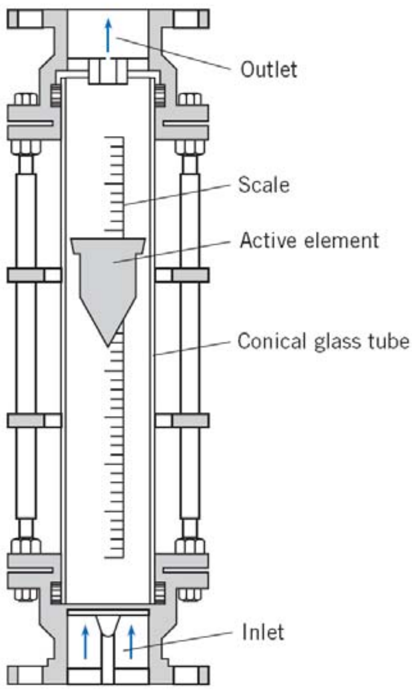


- For inclined venturi meter, the elevation change in the meter is compensated be the elevation change in the manometer legs. Thus, the result is independent of the angel to the vertical of the venturi

Flow Rate Measurement

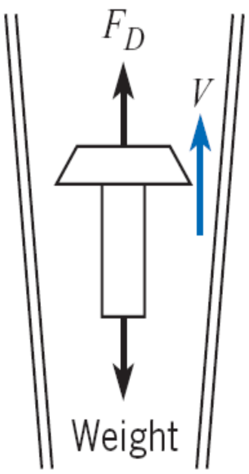
Rotameters

- Used a fixed pressure difference and a variable geometry.
- Fluid flow upward passing around an interior float of different shape.
- At steady state, the ball is being hold by the flow without movement and thus net forces is zero



Flow Rate Measurement

Rotameters



$$F_{gravity} + F_{pressure\ from\ top} = F_{buoyancy} + F_{pressure\ from\ below}$$

$F_{gravity}$: $\frac{\pi}{6} D_o^3 \rho_{ball} g$

$F_{pressure\ from\ top}$: $\frac{\pi}{4} D_o^2 P_{above}$

$F_{buoyancy}$: $\frac{\pi}{6} D_o^3 \rho_{fluid} g$

$F_{pressure\ from\ below}$: $\frac{\pi}{4} D_o^2 P_{below}$

Rotameters

- Re-arranging

$$\frac{\pi}{6} D_o^3 (\rho_{ball} - \rho_{fluid}) g = \frac{\pi}{4} D_o^2 (P_{below} - P_{above})$$

$$P_{below} - P_{above} = \frac{2}{3} D_o (\rho_{ball} - \rho_{fluid}) g$$

- From B.E;

$$\Delta P = \rho_{fluid} \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) = \rho_{fluid} \frac{V_2^2}{2} \left(1 - \frac{A_2^2}{A_1^2} \right)$$

- Since $\frac{A_2^2}{A_1^2} \ll 1$

$$V_2 = \sqrt{\frac{4}{3} D_o g \left(\frac{\rho_{ball} - \rho_{fluid}}{\rho_{fluid}} \right)}$$

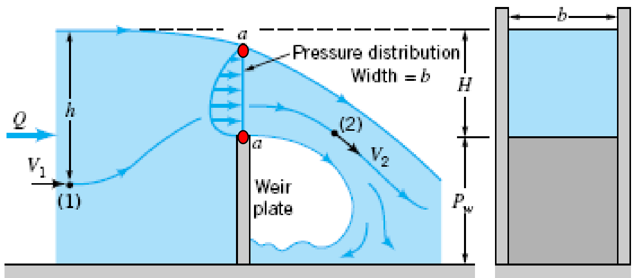
$$\Rightarrow \quad Q = V_2 A_2$$

B.E. for Non-uniform Flows

Flow through a weir.

- Another device used to measure flow in an open channel
- The flow rate of liquid over the top of the weir plate is dependent on the weir height, P_w, b, and the water head above the top of the weir, H

Pressure changes across section a–a

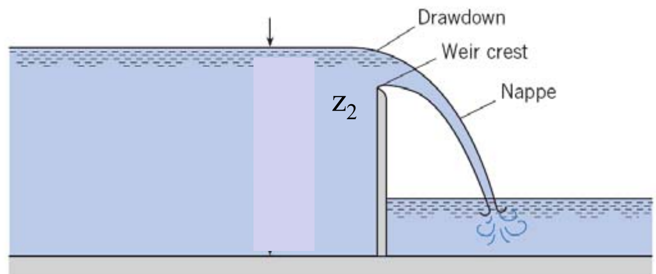


$$V_2^2/2 + gz_2 = V_1^2/2 + gz_1$$

$$V_2 = \sqrt{2gh}$$

$$dQ = V\,W dh$$

$$Q = W\int_0^{h1} \sqrt{2gh}\,dh$$



$$Q = \frac{2W}{3}\sqrt{2g}\,h_1^{3/2} \;\rightarrow\; Q = \frac{2W}{3}\sqrt{2g}\,h_1^{3/2}\,C_d$$

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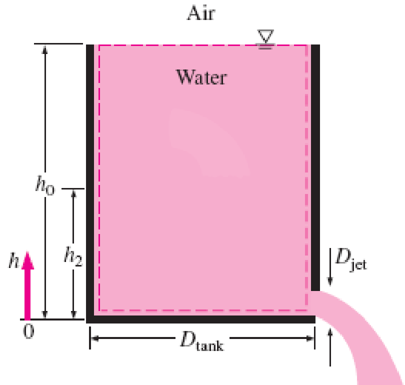
Example

- A 4-ft-high, 3-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams out. The average velocity of the jet is given by *V. Determine* how long it will take for the water level in the tank to drop to 2 ft from the bottom.



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Example cont.

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