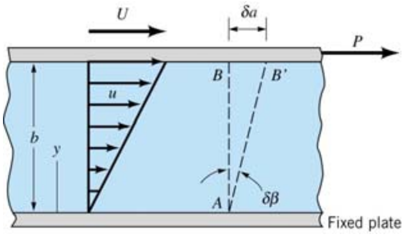
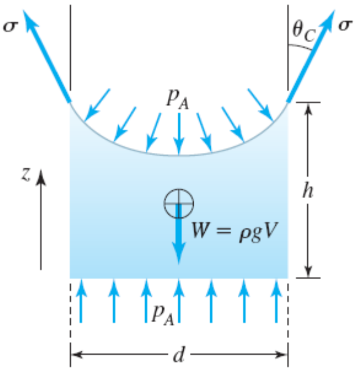
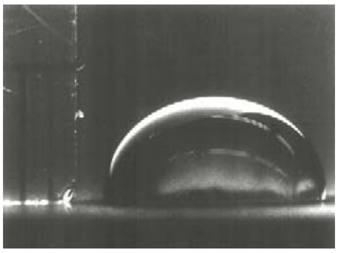
University Of Jordan
Faculty of Engineering and Technology
Department of Chemical Engineering

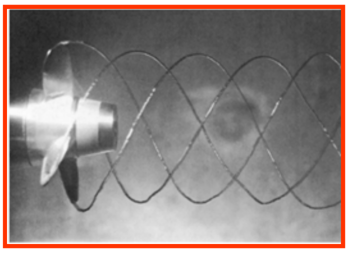


# Fluid Mechanics (0905241)

## Fluid Properties







Prof. Zayed Al-Hamamre

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# Content

- **Fluid Properties:**
  - **The Continuum Hypothesis**
  - **Density and specific gravity**
  - **Vapor pressure**
  - **Cavitations**
  - **Viscosity**
  - **Compressibility**
  - **Surface tension**

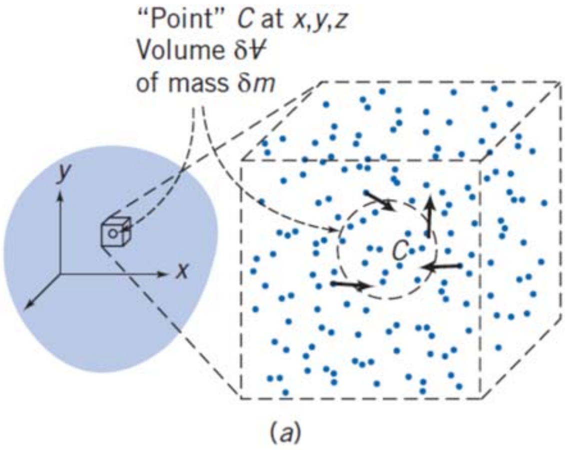
A drop forms when liquid is forced out of a small tube. The shape of the drop is determined by a balance of pressure, gravity, and surface tension forces.

# Introduction

- Any characteristic of a system is called a **property.**
  - Familiar: pressure *P*, temperature *T*, volume *V*, and mass *m*.
  - Less familiar: viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, vapor pressure, surface tension.
- *Intensive* properties are independent of the mass of the system. Examples: temperature, pressure, and density.
- *Extensive* properties are those whose value depends on the size of the system. Examples: Total mass, total volume, and total momentum.
- Extensive properties per unit mass are called **specific properties**. Examples include specific volume $v = V/m$ and specific total energy $e=E/m$.

# The Continuum Hypothesis

- Characteristics of fluid structure
  - Fluids are aggregations of molecules, widely spaced for a gas, closely spaced for a liquid.
  - The distance between molecules is very large compared with the molecular diameter.
  - The molecules are not fixed in a lattice but move about freely relative to each other.
  - Hence, the mass is *not* continuously distributed in space, thus fluid density, or mass per unit volume, has no precise meaning because the number of molecules occupying a given volume continually changes.



# The Continuum Hypothesis

- This effect becomes unimportant if
  - The unit volume is large compared with, say, the cube of the molecular spacing,
  - when the number of molecules within the volume will remain nearly constant in spite of the enormous interchange of particles across the boundaries.
- If, however, the chosen unit volume is too large, there could be a noticeable variation in the bulk aggregation of the particles

- But what is the minimum volume, $\delta V^*$ that a "point" $C$ must be, so that we can talk about continuous fluid properties such as the density at a point?

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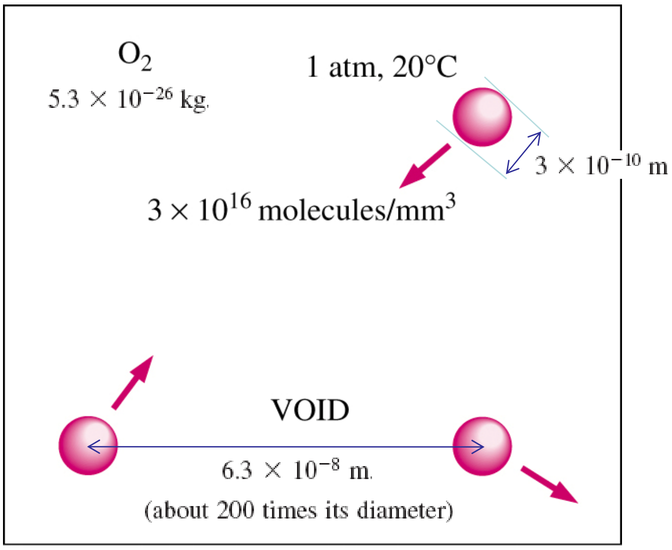
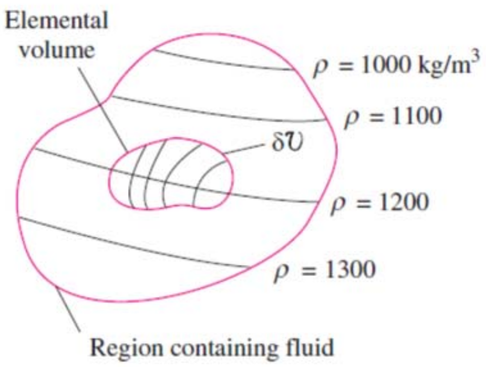
# The Continuum Hypothesis



The distances involved at the molecular level, (a container filled with oxygen at atmospheric conditions)

# The Continuum Hypothesis



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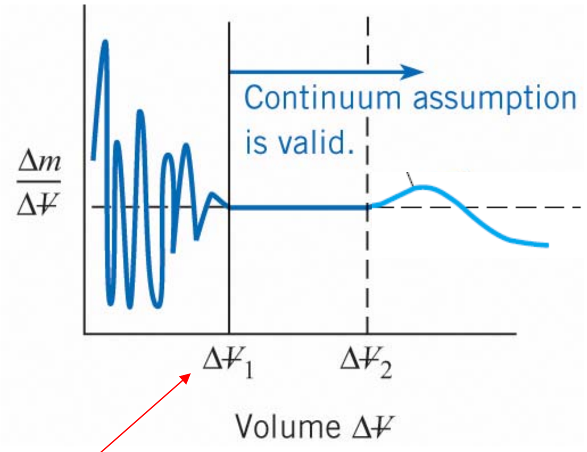

$$\rho = \lim_{\delta\mathcal{V} \to \delta\mathcal{V}^*} \frac{\delta m}{\delta\mathcal{V}}$$

- The limiting volume is about $\delta V^*$ is $10^{-9}$ mm$^3$ for all liquids and for gases at atmospheric pressure.

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# The Continuum Hypothesis

- Under these conditions:
  - We to disregard the atomic nature of the fluid and view it as continuous, homogeneous substance or continuum, rather than as a group of discrete molecules.
  - We thus assume that all the fluid characteristics we are interested in (pressure, velocity, etc.) as point functions vary continuously in space with no jump discontinuities, i.e. we define a fluid particle as a point which moves with the velocity of the fluid at that point

The trajectory of a fluid particle $\vec{x}(t)$ is given by $\frac{d\vec{x}(t)}{dt} = \vec{u}(\vec{x}, t)$

$$\alpha_R(t) = \alpha\,(\vec{x}, t) \quad \Rightarrow \quad \frac{d\alpha_R}{dt} = \frac{\partial \alpha}{\partial t} + \frac{\partial \alpha}{\partial \vec{x}}\left(\frac{d\vec{x}}{dt}\right)$$

# The Continuum Hypothesis

- This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules
- Keep in mind that, the mean free path of gas molecules λ, is inversely proportional to the density of the gas

$$\lambda_2/\lambda_1 = \rho_1/\rho_2 = p_1/p_2$$

- The mean free path in air is approximately $10^{-7}$ m (0.1 µm) at standard conditions. Is a continuum model appropriate to describe the flow of gas in 0.5 µm pores of a filter media?

Since the pore diameter is approximately five times the mean free path, we should be cautious in using results based on the continuum hypothesis in this application

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# The Continuum Hypothesis

- when a fluid flows through the tiny passages in nanotechnology devices, then the spacing between molecules is significant compared to the size of these passageways.

➡ **The continuum assumption is invalid**

- Check the applicability in the case of Vacuum (p = $10^{-6}$ atm)

- At very high vacuums or very high elevations, the mean free path may become large

➡ we should be cautious in using results based on the continuum hypothesis in this application

- The continuum hypothesis is nearly always applicable in liquids………..

Because the distance between liquid molecules is small and relatively constant

## Mass and Weight

- The mass of an object is defined as a measure of its resistance to acceleration, i.e., resistance to a change in velocity.
- For an object of fixed mass, the relationship between acceleration, a, force, F, and mass, M, is given by Newton's second law:

$$F = M \times a$$

- The weight of an object, W, is the magnitude of the force acting on the object due to Earth's gravity field.
- If the acceleration produced by gravity is g, the weight of a mass M is:

$$W = M \times g$$

- Mass and weight depend on the amount of material in an object or system

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## Density and Specific Gravity

Density

$$\rho = \frac{m}{V} \qquad (\text{kg/m}^3)$$

Specific volume

$$v = \frac{V}{m} = \frac{1}{\rho}$$

$V = 12\ \text{m}^3$
$m = 3\ \text{kg}$
⬇
$\rho = 0.25\ \text{kg/m}^3$
$v = \frac{1}{\rho} = 4\ \text{m}^3/\text{kg}$

Density is mass per unit volume; specific volume is volume per unit mass.

Specific gravity: The ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C).

$$\text{SG} = \frac{\rho}{\rho_{\text{H}_2\text{O}}}$$

Specific weight: The weight of a unit volume of a substance.

$$\gamma_s = \rho g \qquad (\text{N/m}^3)$$

The weight of an object, W, is the magnitude of the force acting on the object due to Earth's gravity field

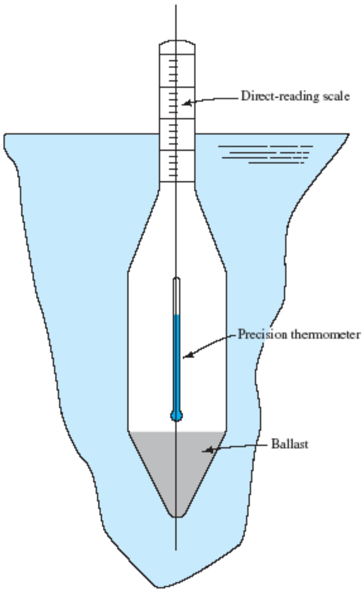
$$W = Mg$$

Specific gravities of some substances at 0°C

| Substance | SG |
|---|---|
| Water | 1.0 |
| Blood | 1.05 |
| Seawater | 1.025 |
| Gasoline | 0.7 |
| Ethyl alcohol | 0.79 |
| Mercury | 13.6 |
| Wood | 0.3–0.9 |
| Gold | 19.2 |
| Bones | 1.7–2.0 |
| Ice | 0.92 |
| Air (at 1 atm) | 0.0013 |

# Density and Specific Gravity

- **Hydrometer** is a common device used to measure density, sg, or deg API, deg Baume.



"Hydrometer with built-in thermometer (thermo-hydrometer)."

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# Density of Ideal Gases

- Equation of state: Any equation that relates the pressure, temperature, and density (or specific volume) of a substance.
- Ideal-gas equation of state: The simplest and best-known equation of state for substances in the gas phase.

$$Pv = RT \qquad \text{or} \qquad P = \rho RT$$

$$R = R_u/M \qquad R_u = 8.314 \text{ kJ/kmol} \cdot \text{K}$$

- $R_u$: The universal gas constant

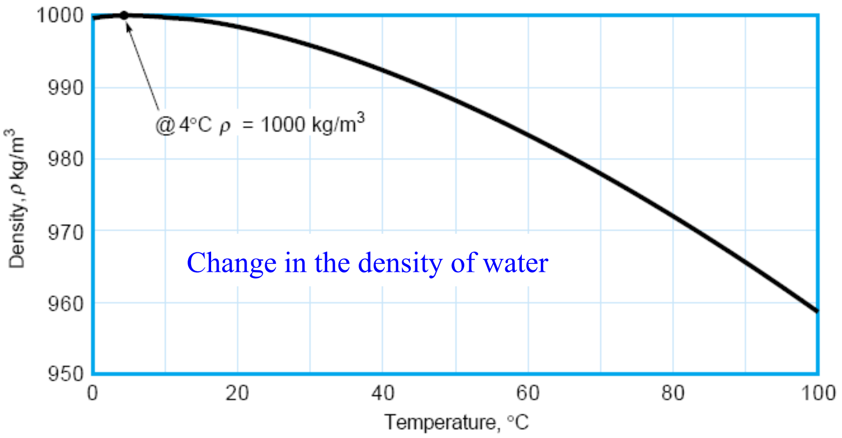
$$PV = mRT \text{ or } PV = NR_uT.$$

- The thermodynamic temperature scale in the SI is the Kelvin scale.
- In the English system, it is the Rankine scale.

$$T(\text{K}) = T(^\circ\text{C}) + 273.15 = T(R)/1.8$$

$$T(\text{R}) = T(^\circ\text{F}) + 459.67 = 1.8\,T(\text{K})$$

# Density



Typical values:

Water = $1000\ kg\, m^{-3}$ , Mercury = $13546\ kg\, m^{-3}$

Air = $1.23\ kg\, m^{-3}$ , Paraffin Oil = $800\ kg\, m^{-3}$

(at pressure $=1.013\times10^{-5}\ N\, m^{-2}$ and Temperature $= 288.15$ K.)

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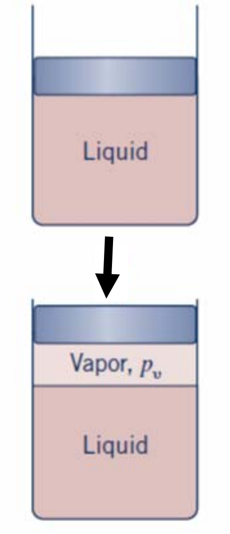
# Vapor Pressure: Evaporation and Boiling

- It is a common observation that liquids such as water and gasoline will evaporate if they are simply placed in a container open to the atmosphere.
- Evaporation takes place because some liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape into the atmosphere.
- If the container is closed with a small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules.
- When an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number entering, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed the ***vapor pressure,*** $P_v$

# Vapor Pressure: Evaporation and Boiling

- Similarly, if the end of a completely liquid-filled container is moved as shown in the figure without letting any air into the container, the space between the liquid and the end becomes filled with vapor at a pressure equal to the vapor pressure.

- Since the development of a vapor pressure is closely associated with molecular activity, the value of vapor pressure for a particular liquid depends on temperature



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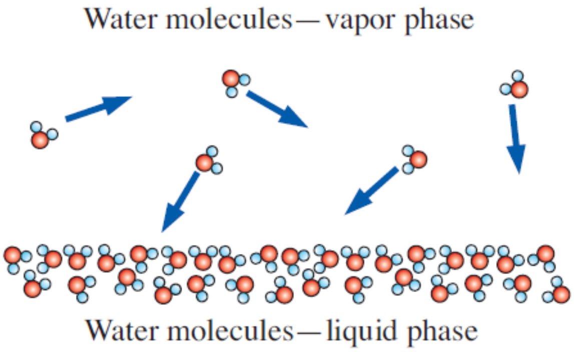
# Vapor Pressure: Evaporation and Boiling

- Boiling, which is the formation of vapor bubbles within a fluid mass, is initiated when the absolute pressure in the fluid reaches the vapor pressure



- Boiling of water occurs at approximately 100 °C, but it is not only a function of temperature, but also of pressure. For example, in Colorado Spring, water boils at temperatures less than 100 °C.

# Vapor Pressure: Evaporation and Boiling

- **Saturation temperature $T_{sat}$**: The temperature at which a pure substance changes phase at a given pressure.
- **Saturation pressure $P_{sat}$**: The pressure at which a pure substance changes phase at a given temperature.
- **Vapor pressure ($P_v$):** The pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature. It is identical to the saturation pressure $P_{sat}$ of the liquid ($P_v = P_{sat}$).
- **Partial pressure:** The pressure of a gas or vapor in a mixture with other gases. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor.

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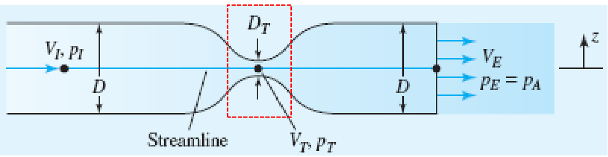
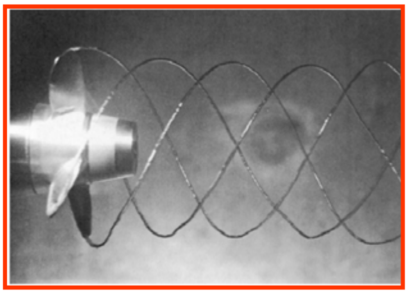
# Cavitation

- In flowing fluids, it is possible to develop very low static pressure due to the fluid motion, routinely occur in regions of high velocity within fluid machinery such as on the inlet (suction) side of a pump and on rapidly rotating propellers and impellers.



Region of high velocity

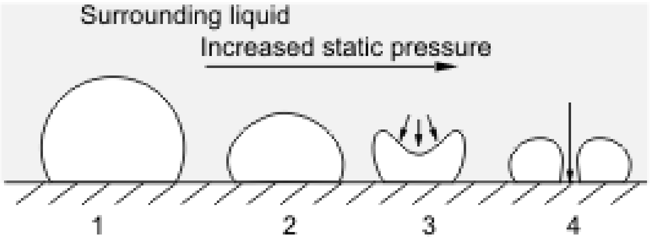


Cavitation can occur at the lower pressure areas of a propeller

- If the pressure is lowered to the vapor pressure, boiling will occur

# Cavitation

- This results in the formation of water vapor bubbles (called cavitation bubbles since they form “cavities of vapor” in the liquid) in certain regions within the flow and on moving surfaces.
- Therefore, cavitation may be thought of as boiling or vapor bubble formation at a point in a moving liquid when the local static pressure is below the vapor pressure at the point.
- If bubbles are then swept to regions of higher pressure, the bubbles suddenly collapse with sufficient intensity to actually cause generating highly destructive, extremely high-pressure waves.



Cavitation bubble imploding close to a fixed surface generating a jet (4) of the surrounding liquid.

# Cavitation

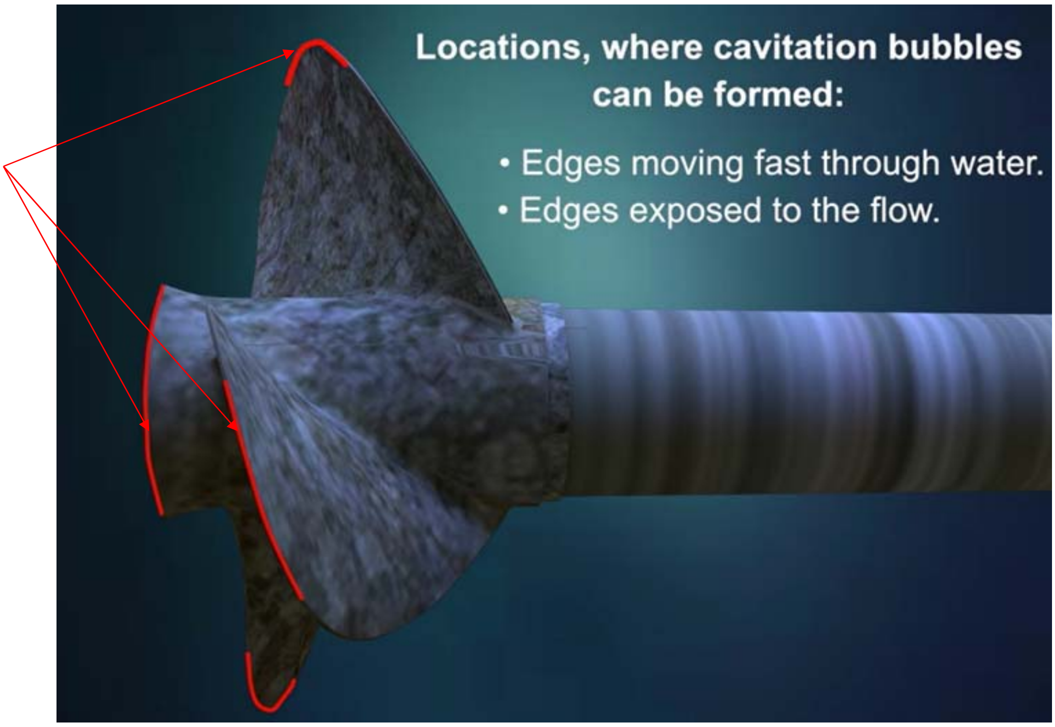
-**Cavitation number** is defined as:

$$\textit{Cavitation number} = \frac{P - P^{v}}{0.5\rho \bar{U}^{2}} \leq 0$$

- When Cavitation number ≤ 0; Cavitation will occur which causes damage of pump.

# Cavitation (Ship Propellers)

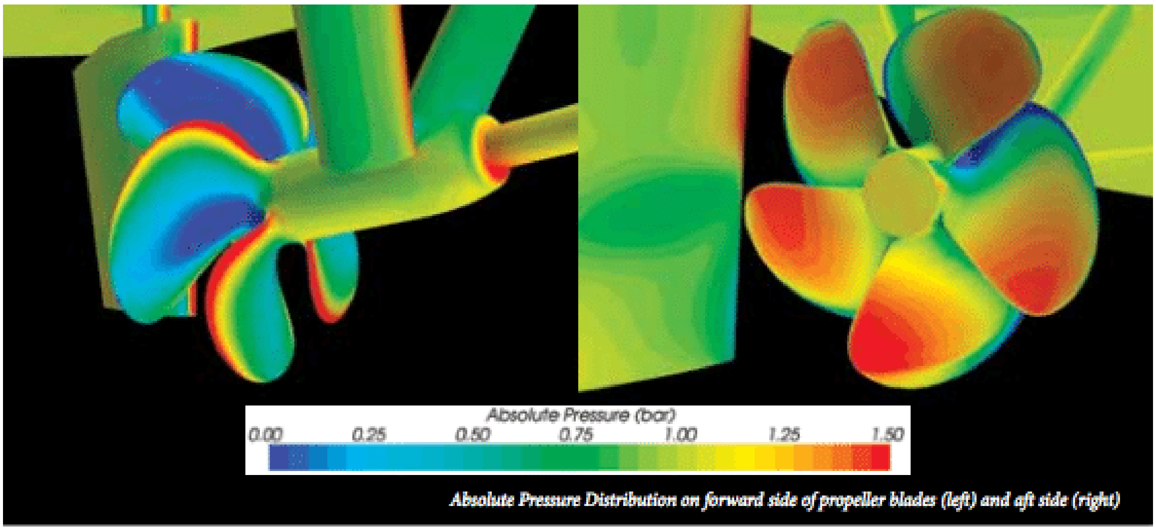
High pressure at the side facing the flow and low pressure at the opposite side



**Locations, where cavitation bubbles can be formed:**

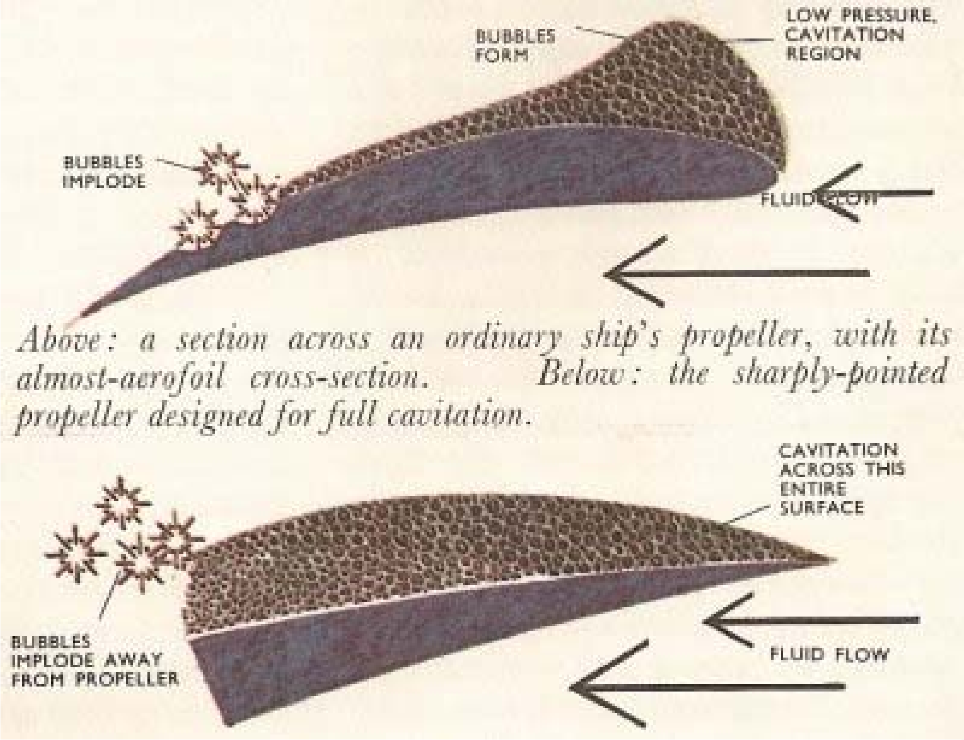
- Edges moving fast through water.
- Edges exposed to the flow.

# Cavitation (Ship Propellers)



*Absolute Pressure Distribution on forward side of propeller blades (left) and aft side (right)*

# Cavitation (Ship Propellers)



*Above: a section across an ordinary ship's propeller, with its almost-aerofoil cross-section. Below: the sharply-pointed propeller designed for full cavitation.*

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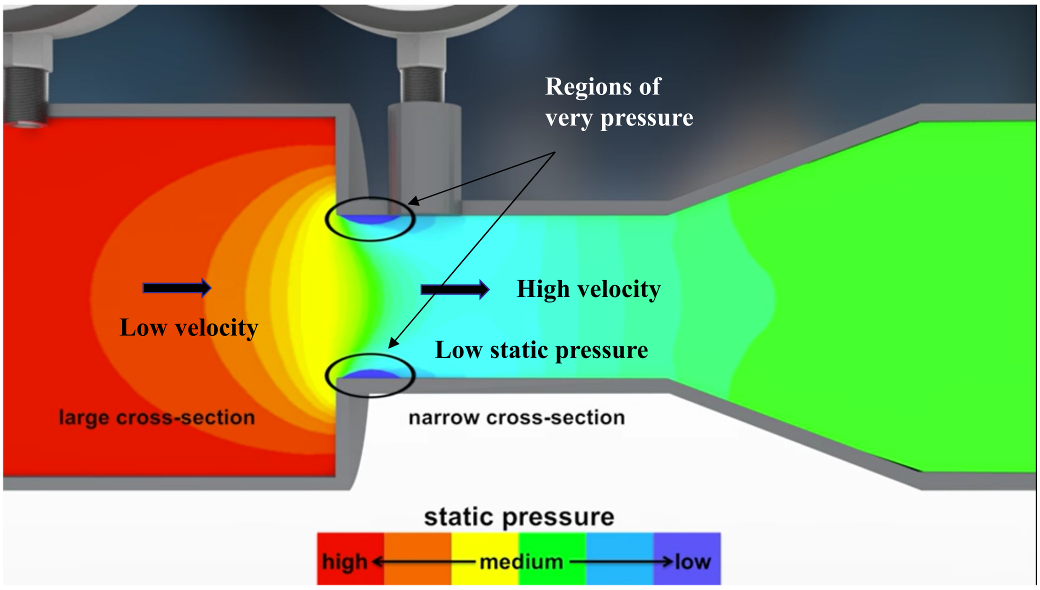
# Cavitation (Ship Propellers)

# Cavitation (Piping System)



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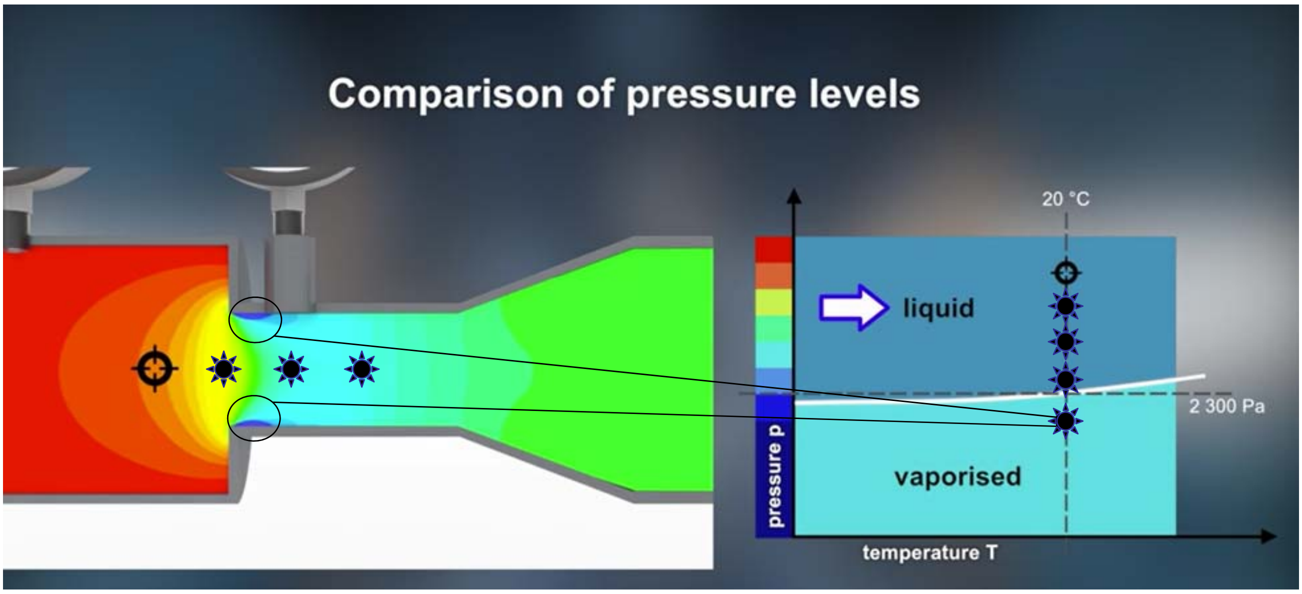
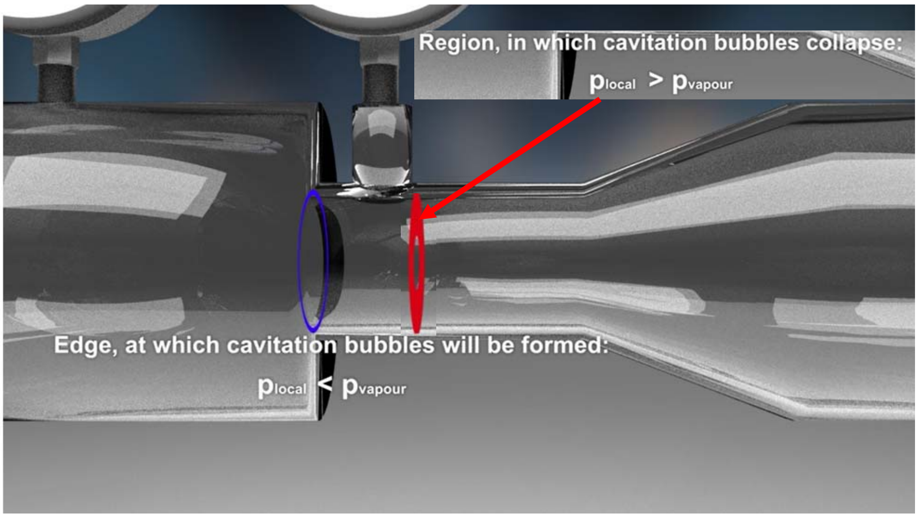
# Cavitation (Piping System)



# Cavitation (Piping System)



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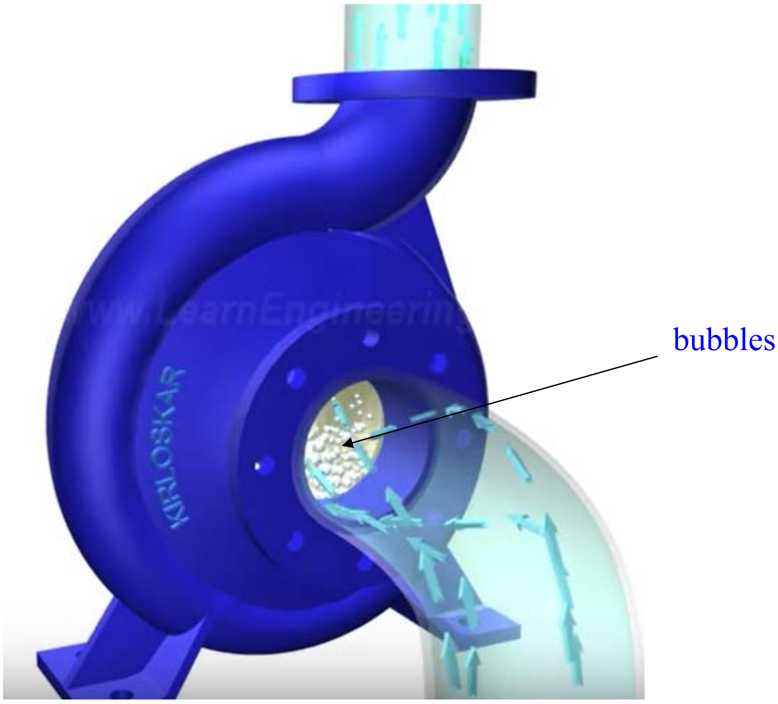
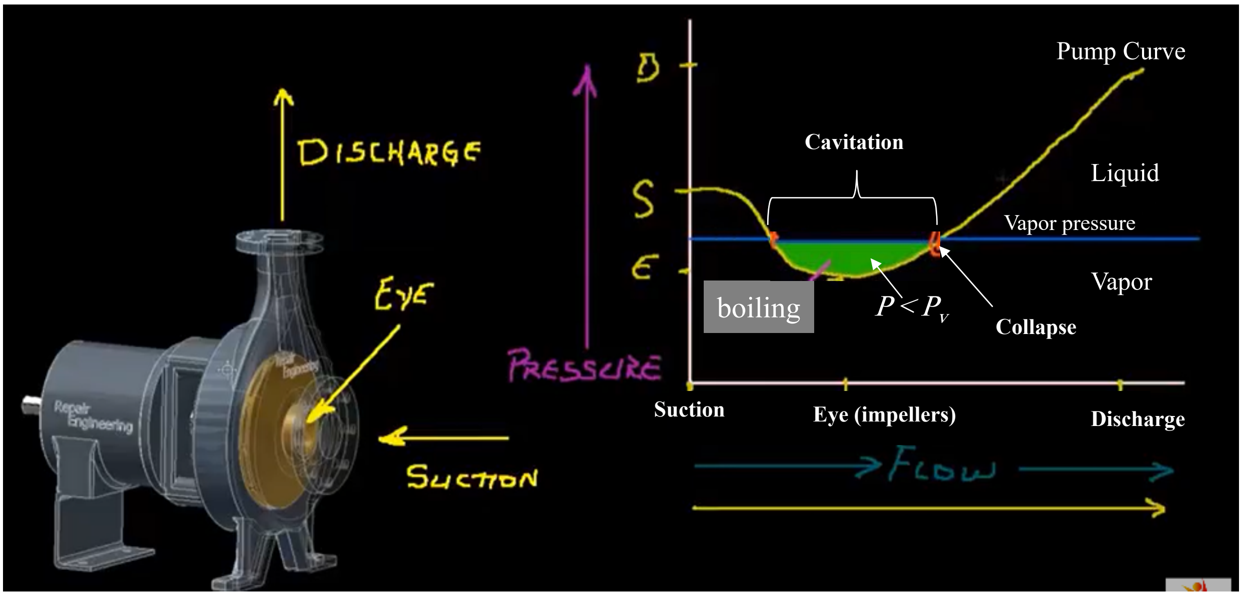
# Cavitation (Pump Suction)



# Cavitation (Pump Suction)



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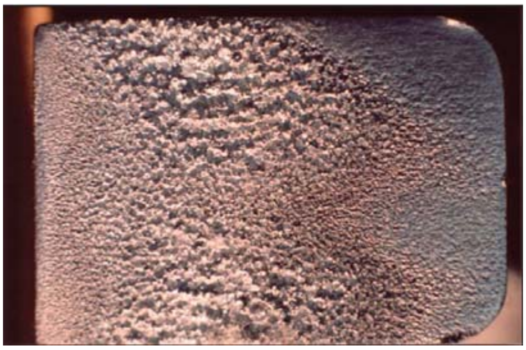
# Cavitation

- Cavitation must be avoided (or at least minimized) in most flow systems since it reduces performance, generates annoying vibrations and noise, and causes damage to equipment.



Damage may result from the cavitation as shown on this turbine blade.

# Cavitation

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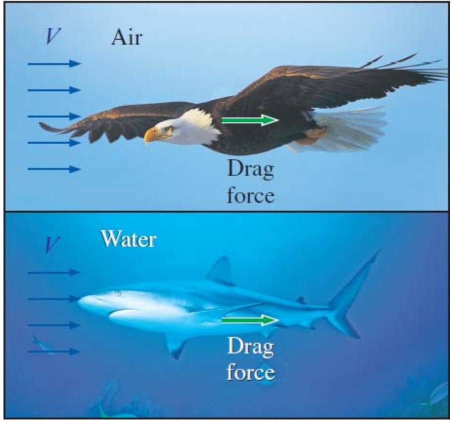
# Example

- In a water distribution system, the temperature of water is observed to be as high as 30°C. Determine the minimum pressure allowed in the system to avoid cavitation.

# Viscosity

- When a fluid moves relative to a solid or when two fluids move relative to each other, a friction force develops at the contact surface in the direction opposite to motion.
- The magnitude of the force needed to move one object relative to another depends on the friction coefficient between them.

**But, is it really a Friction Force?**



- **Viscosity** is a property that represents the internal resistance of a fluid to motion.
- The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity.

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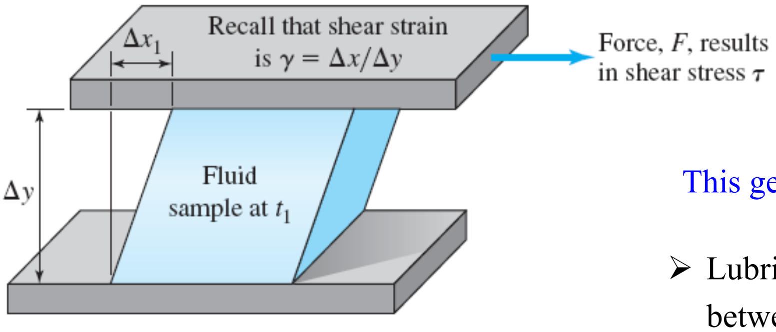
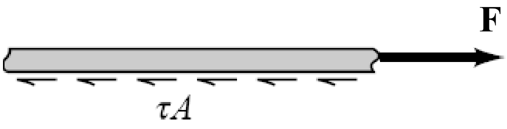
# Viscosity

- The viscosity is measure of the "fluidity" of the fluid which is not captured simply by density or specific weight. A fluid can not resist a shear and under shear begins to flow.

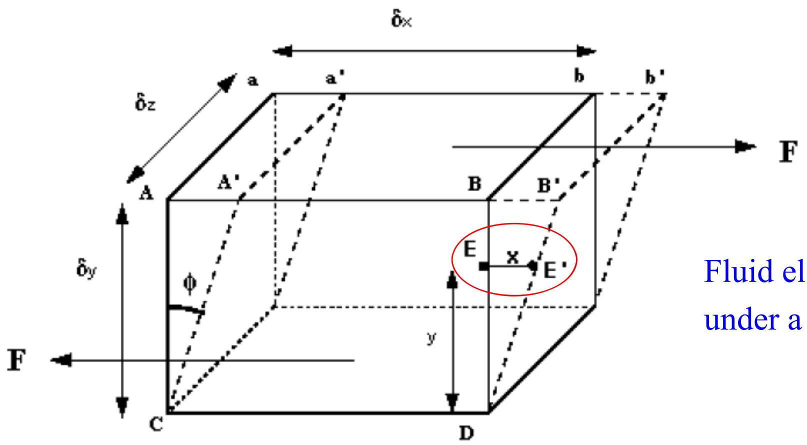




This geometry models

- Lubricant-filled space between a piston and cylinder wall in an engine, in a journal bearing
- Between foot and the wet floor in bathroom

# Viscosity and Shearing Stress: Classical View



Fluid element
under a shear force

- The shearing force F acts on the area on the top of the element $A = \delta s \times \delta x$

$$\text{shear stress, } \tau = \frac{F}{A}$$

- If the particle at point E moves under the shear stress to point E’ and it takes time *t* to get there, it has moved the distance *x.*

For small deformations $\quad \text{shear strain } \gamma = \frac{x}{y}$

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# Viscosity and Shearing Stress

$$\text{rate of shear strain } = \frac{\gamma}{t} = \frac{x}{ty} = \frac{x}{t}\frac{1}{y} = \frac{u}{y}$$

where $\frac{x}{t} = u$ is the velocity of the particle at E

**Experimentally,** $\quad \tau = \text{Constant} \times \frac{u}{y}$

$\frac{u}{y}$ → change in velocity with y, or the velocity gradient

In the differential form
As *t* approach 0.0 ➡ $\frac{du}{dy}$

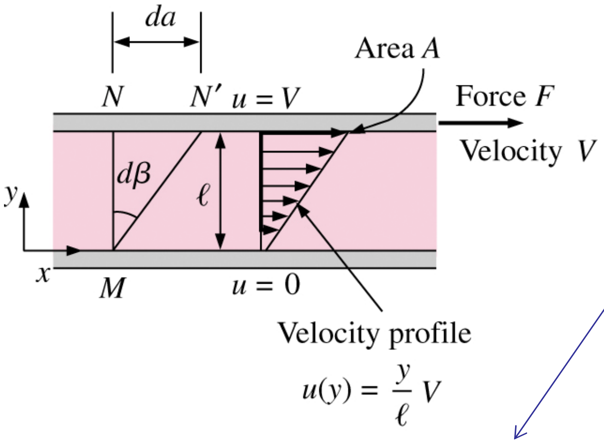
- The constant of proportionality → dynamic viscosity $\mu$

$$\tau = \mu \frac{d\gamma}{dt} \longrightarrow \tau = \mu \frac{du}{dy}$$ **Newton’s law of viscosity**

Fluids for which the rate of deformation is proportional to the shear stress

# Summary: Viscosity and Shear Stress

- Fluids obeying this relation are called Newtonian fluids.



- The no-slip condition: both fluid–solid interfaces the fluid velocity matches the velocity of the solid surface.

- To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates separated by a distance $\ell$
- Definition of shear stress is $\tau = \frac{F}{A}$
- Using the no-slip condition: $u(0) = 0$ and $u(\ell) = V$, the velocity profile and gradient are

$$u(y) = \frac{y}{\ell}V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell}$$

- Shear stress for Newtonian fluid:

$$\boxed{\tau = \mu \frac{du}{dy} \qquad (\text{N/m}^2)}$$

- $\mu$ is the **dynamic viscosity** and has units of $kg/m{\cdot}s$, $Pa{\cdot}s$, or **poise**.

*Shear force*: $\boxed{F = \tau A = \mu A \frac{du}{dy} \qquad (\text{N})}$ The shear force acting on a Newtonian fluid layer

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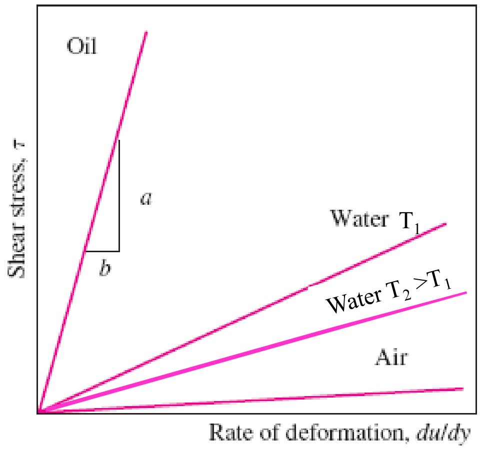
# Example

- A sample of motor oil is tested in a parallel plate shearing device with the following results: $\tau = 1.54\ lb_f/ft^2$, *the plate separation distance is 0.5* in., and the top plate velocity is 10 ft /s. Determine the viscosity of the fluid and shear rate.

# Viscosity and Shear Stress

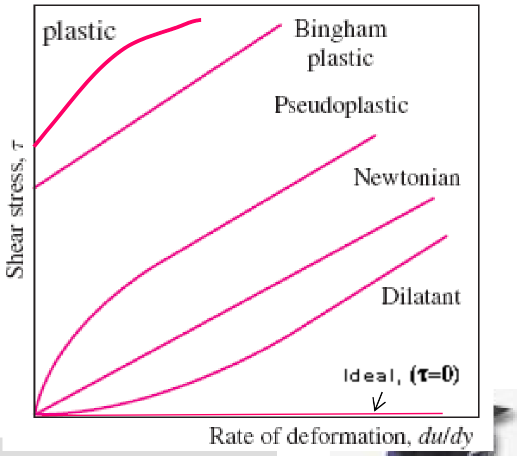


Newtonian fluids (slope is constant)

Newtonian and non-Newtonian fluids

- The slope of the curve is referred to as the apparent viscosity of the fluid.
- Shear-thinning (Pseudoplastic) the apparent viscosity decreases with increasing shear rate (paints and polymer solutions)

$$\tau = A + B\left(\frac{\delta u}{\delta y}\right)^n$$



- Shear-thickening (Dilatant) the apparent viscosity increases with increasing shear rate (solution with suspended starch or sand)
- Bingham plastics can withstand a small shear stress without motion. Once critical shear stress is exceeded, it can flow (Toothpaste and mayonnaise)

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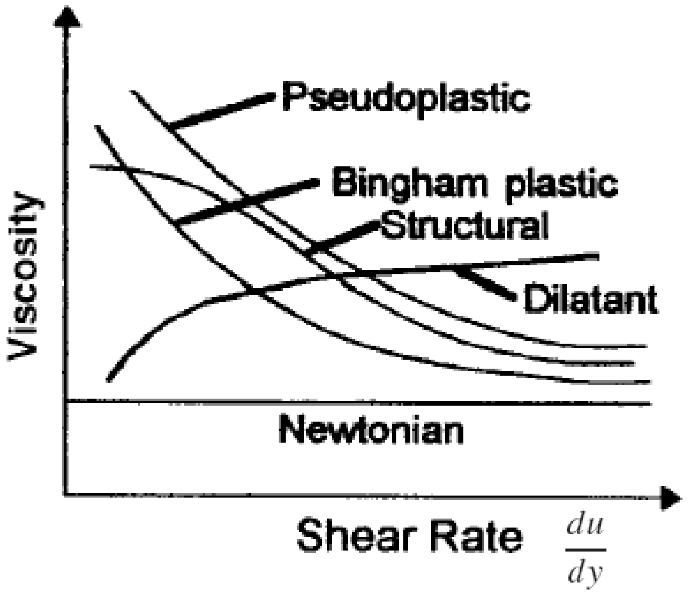
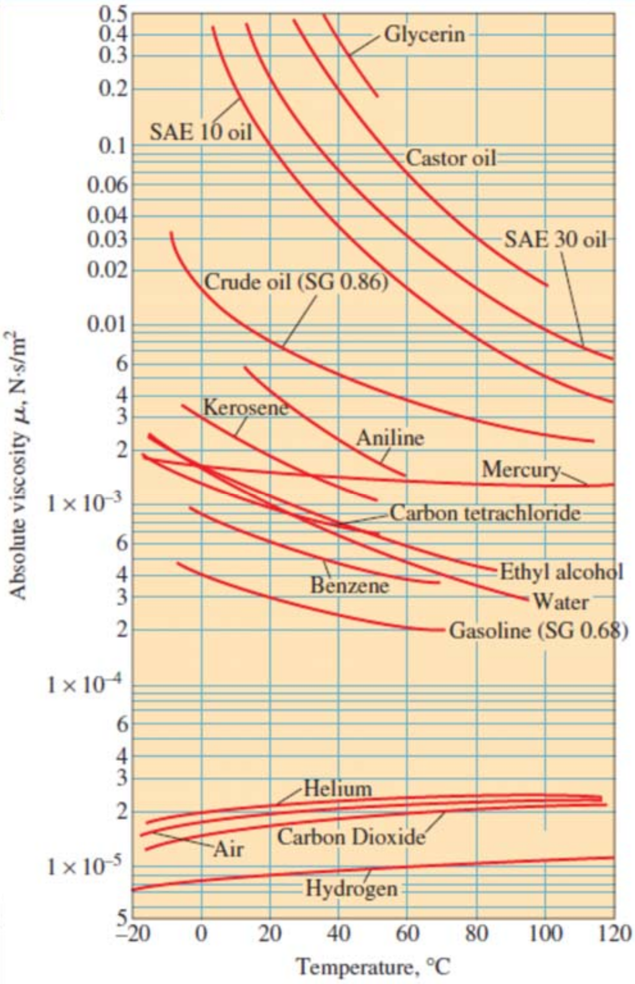
# Viscosity and Shear Stress



Proposed report: Viscosity and viscosity models

# Variation with Temperature

| Fluid | Dynamic Viscosity $\mu$, kg/m · s |
|---|---|
| Glycerin: | |
| −20°C | 134.0 |
| 0°C | 10.5 |
| 20°C | 1.52 |
| 40°C | 0.31 |
| Engine oil: | |
| SAE 10W | 0.10 |
| SAE 10W30 | 0.17 |
| SAE 30 | 0.29 |
| SAE 50 | 0.86 |
| Mercury | 0.0015 |
| Ethyl alcohol | 0.0012 |
| Water: | |
| 0°C | 0.0018 |
| 20°C | 0.0010 |
| 100°C (liquid) | 0.00028 |
| 100°C (vapor) | 0.000012 |
| Blood, 37°C | 0.00040 |
| Gasoline | 0.00029 |
| Ammonia | 0.00015 |
| Air | 0.000018 |
| Hydrogen, 0°C | 0.0000088 |



# Variation with Temperature and Pressure

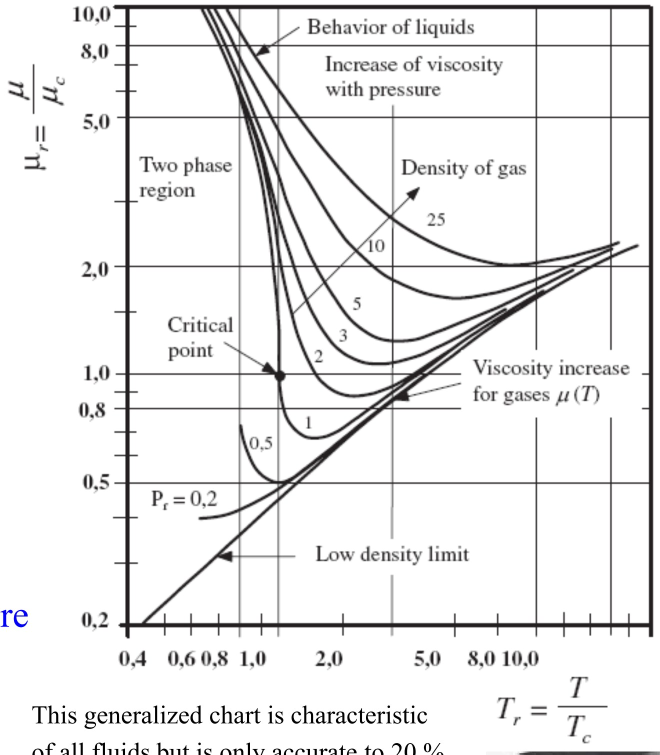
- Pressure does not affect the viscosity of liquids that much.
- The effects of changing temperature are quite large.

Gases: $\mu = \dfrac{aT^{1/2}}{1 + b/T}$

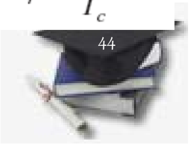
Liquids: $\mu = a10^{b/(T-c)}$

$$\mu_T = \mu_0\left(1 + AT + BT\right)$$

- Shear viscosity increases with temperature for gases but decreases with temperature for liquids.



This generalized chart is characteristic of all fluids but is only accurate to 20 %



# Viscosity Measurement Devices

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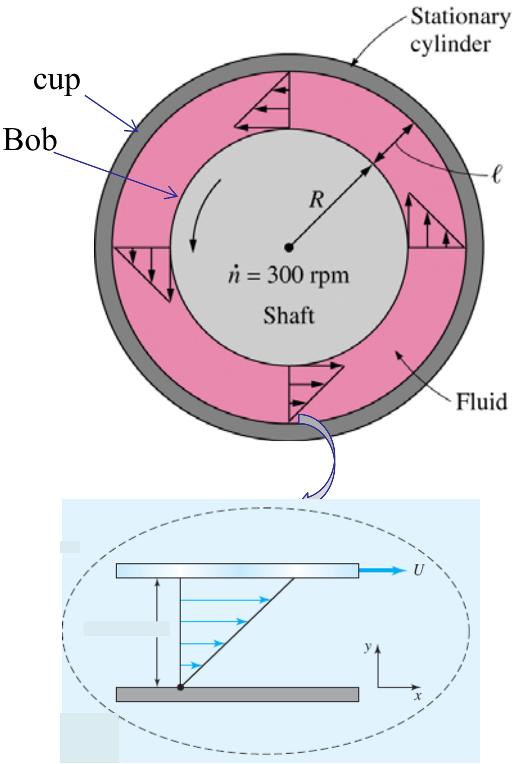
## Rotating-drum viscometer: Cup-and-Bob (Couette) Viscometer



- How is viscosity measured? A rotating viscometer.
  - Two concentric cylinders with a fluid in the small gap $\ell = (R_{out} - R_{in})$
  - Inner cylinder is rotating, outer one is fixed.
- Use definition of shear force:

$$F = \tau A = \mu A \frac{du}{dy}$$

- If $\ell / R << 1$, then cylinders can be modeled as flat plates.
- Torque $T = FR$, and tangential velocity $V = \omega R$,

$$\omega = 2\pi\dot{n}. \quad \frac{du}{dy} = \frac{2\pi\dot{n}R}{l}$$

- Wetted surface area $A = 2\pi RL$

$$T = \tau(2\pi RL)(R)$$

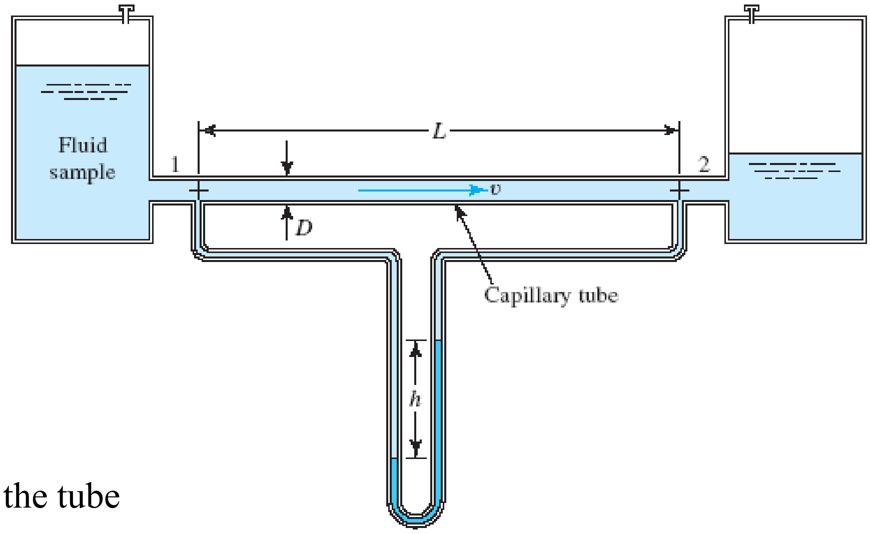
- Measure $T$ and $\omega$ to compute $\mu$

$$T = FR = \mu \frac{2\pi R^3 \omega L}{\ell} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell}$$

# Example

- The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders. The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N.m. Determine the viscosity of the fluid.

# Capillary-tube viscometer



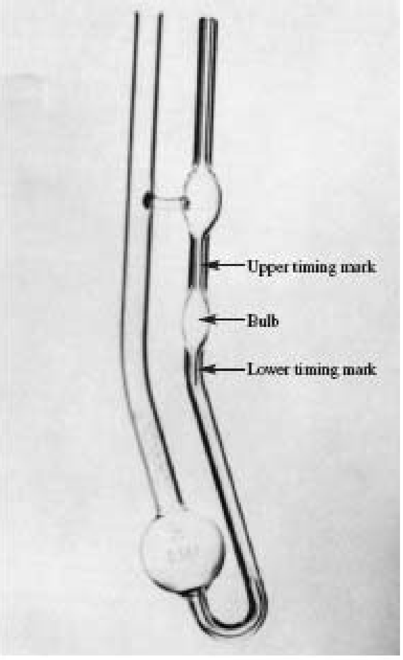
$$\mu = \frac{(p_1 - p_2)D^2}{32vL}$$

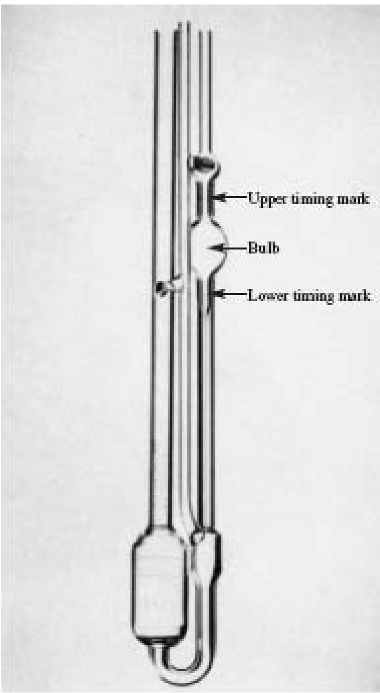
*D:* the inside diameter of the tube

*v* : the fluid velocity

*L* :the length of the tube between points 1 and 2 where the pressure is measured.

# Capillary Viscometers





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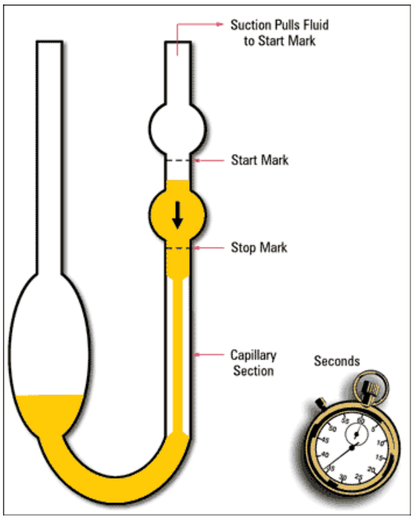
# Capillary Viscometers

- It gives the ‘kinematic viscosity’ of the fluid. It is based on Poiseuille’s law for steady viscous flow in a pipe.
- The kinematic viscosity is computed by multiplying the flow time by the calibration constant of the viscometer supplied by the vendor.

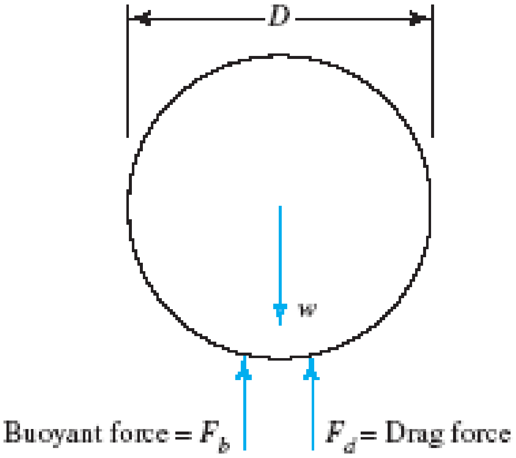


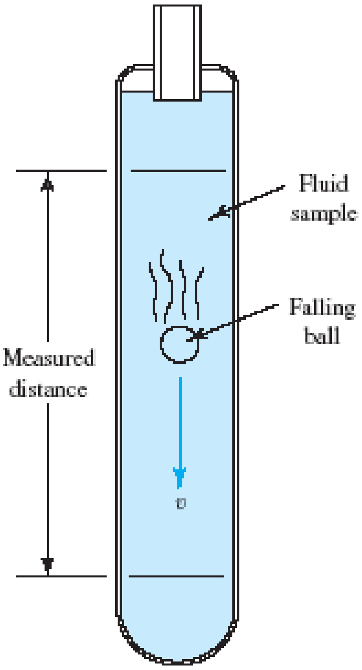
$$\upsilon = \pi r^4 g l t \,/\, 8LV = k(t_2 - t_1)$$

where:

| | |
|---|---|
| $\upsilon$ | is the kinematic viscosity [$m^2/s$]; |
| **r** | is the capillary radius [m]; |
| **l** | is the mean hydrostatic head [m]; |
| **g** | is the earth acceleration [$m/s^2$]; |
| **L** | is the capillary length [m]; |
| **V** | is the flow volume of the fluid [$m^3$]; |
| **t** | is the flow time through the capillary, $\mathbf{t = (t_2 - t_1)}$, [s]; |
| **k** | is the capillary constant which has to be determined experimentally by applying a reference fluid with known viscosity, e.g. by applying freshly distilled water. The capillary constant is usually given by the manufacturer of the viscometer. |

# Falling-Ball Viscometer





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# Falling-Ball Viscometer

“Forces on falling-ball”

Applying the vertical direction force balance on the ball:

$$w - F_b - F_d = 0.$$

where $w$ is the weight of the ball, $F_b$, is the buoyant force, and $F_d$ is the viscous drag force on the ball.

If $\gamma_s$ is the specific weight of the sphere, $\gamma_f$ is the specific weight of the fluid, $V$ is the volume of the sphere, and $D$ is the diameter of the sphere, we have:

$$w = \gamma_s V = \gamma_s \pi D^3/6$$

$$F_b = \gamma_f V = \gamma_f \pi D^3/6$$

# Falling-Ball Viscometer

- For very viscous fluids and small velocity ($Re < 1$), the drag force on the sphere is:

$$F_d = 3\pi\mu v D$$

Where $v$ is the *terminal velocity* ($v=s/t$). Where $s$ is distance in which the ball will travel during time $t$.

Then the equation for calculating viscosity becomes:

$$\mu = \frac{(\gamma_s - \gamma_f)D^2}{18v}$$

Or

$$\mu = gD_p^2(\rho_p - \rho)t_p/18L$$

where $t_p$ is the time required for a sphere to fall a distance, L.

---

# Example

In a falling ball viscometer, a steel ball with a diameter of 2.5 mm is allowed to fall freely in a heavily fuel oil having a specific gravity of 0.9. Steel weighs 77 kN/m$^3$. If the ball is observed to fall 30 mm in 10 seconds, calculate the dynamic viscosity of the oil in Pa.s.

$\text{sg} = 0.90$
$D = 2.5\ \text{mm} = 0.0025\ \text{m}$
$\gamma_s = 77\ \text{kN/m}^3$
$s = 30\ \text{mm} = 0.03\ \text{m}$
$t = 10\ \text{s}$

# Viscosity and Momentum Transport

- $\tau_{ij}$ *terms* physically represent molecular-caused momentum transport terms occurring per unit area and time; the index ***i*** represents the considered molecular transport direction and ***j*** the direction of the considered momentum.

**Viscosity is the ability to transfer momentum**

- In liquids the viscosity is a result of the intermolecular attractive forces between adjacent molecules.
- As the temperature increases, the strength of this cohesive force decreases, and the average separation distance between liquid molecules increases.
- The liquid transfers momentum less effectively, so viscosity decreases.

---

# Viscosity and Momentum Transport

- In gases, molecules are not close enough together for intermolecular forces to be important.
- Gas molecules are far more mobile than those in a liquid.
- Some of these energetic gas molecules move in a direction with a component perpendicular to the motion of the top plate.
- The molecules moving from the “fast” fluid layer to the “slower” moving layer below act to pull the “slower” layer forward.
- In contrast, the molecules moving from the “slower” layer to the “faster” layer exert a drag on the “faster” layer.
- Since a rise in temperature increases the random molecular motion in all directions, the shear viscosity of a gas increases with rising temperature.

# Transport Phenomena and Viscosity

- When treating fluid motions including the transport of heat and momentum as well as mass transport, molecular transport processes occur that cannot be neglected.
- The heat transport occurring, as a consequence of the molecular motion, is given by the Fourier law of heat conduction

$$\dot{q}_i = -\lambda \frac{\partial T}{\partial x_i},$$

- The mass transport occurring analogously given by the Fick's law of diffusion

$$\dot{m}_i = -D \frac{\partial c}{\partial x_i}$$

- In an analogous way, the molecule-dependent momentum transport also has to be described by the Newtonian law

$$\tau_{ij} = -\mu \frac{\partial U_j}{\partial x_i},$$

- The direction *i* indicates the "molecular transport direction", and *j* indicates the components of the velocity vector

---

# Viscous Dissipation

- Viscous dissipation is the frictional energy loss which occur when a fluid is sheared.
- The general action of viscosity in a fluid flow is a tendency to convert the useful energy content of the fluid into heat.
- The useful energy lost appears as an increase in the internal energy of the fluid, corresponding to a rise in temperature.
- The rate of dissipation of energy per unit mass of fluid by the shear viscosity is given by the viscous dissipation

$$\Phi = 2\frac{\mu}{\rho}\left(\frac{du}{dy}\right)^2 \quad \text{J/(s-kg) or W/kg}$$

# Kinematic Viscosity

- In fluid dynamics, many problems involving viscosity are concerned with the magnitude of the viscous forces ($\mu u/L$) compared with the magnitude of the inertia forces ($\rho u^2$), that is, those forces causing acceleration of particles of the fluid.
- Since the viscous forces are proportional to the dynamic viscosity $\mu$ *and the inertia* forces are proportional to the density, the ratio $\mu/\rho$ *is frequently involved*

$$R_e = \frac{\rho U_c L_c}{\mu} \Rightarrow Re = \frac{U_c L_c}{\nu_c} = \frac{\rho_c U_c^2}{\mu_c \frac{U_c}{L_c}} = \frac{\text{acceleration forces}}{\text{viscosity forces}},$$

- Kinematic Viscosity is defined as the ratio of dynamic viscosity to mass density

$$\nu = \frac{\mu}{\rho}$$

- Kinematic viscosity is another way of representing viscosity used in the flow equations
- The units are of $m^2/s$ and $ft^2/s$

Water $=1.14 \times 10^{-6}\ m^2 s^{-1}$, Air $=1.46 \times 10^{-5}\ m^2 s^{-1}$, Mercury $=1.145 \times 10^{-4}\ m^2 s^{-1}$, Paraffin Oil $=2.375 \times 10^{-3}\ m^2 s^{-1}$.

---

# Compressibility & Volume Expansion Coefficients

- How does fluid volume(or density) change with $P$ and $T$?
  - ✓ Fluids expand as $T\uparrow$ or $P\downarrow$
  - ✓ Fluids contract as $T\downarrow$ or $P\uparrow$
- But the amount of volume change is different for different fluids

$\Rightarrow$ Need fluid properties that relate volume changes to changes in $P$ and $T$.

$$v = v(T,P) \quad \rightarrow \quad dv = \underbrace{\left(\frac{\partial v}{\partial T}\right)_P}_{v\beta} dT + \underbrace{\left(\frac{\partial v}{\partial P}\right)_T}_{-\frac{v}{\kappa}} dP$$

- Coefficient of volume expansion

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

- Coefficient of compressibility (**Bulk Modulus)**

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T$$

- In terms of finite changes

$$\beta \approx \frac{\Delta v/v}{\Delta T} = -\frac{\Delta\rho/\rho}{\Delta T} \qquad (\text{at constant } P)$$

$$\kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta\rho/\rho} \qquad (T = \text{constant})$$

# Compressibility Coefficients

- The coefficient of compressibility represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant.
- A large value of $\kappa$ indicates that a large change in pressure is needed to cause a small fractional change in volume,
- Thus a fluid with a large $\kappa$ is essentially incompressible.
- This is typical for liquids, and explains why liquids are usually considered to be *incompressible*.

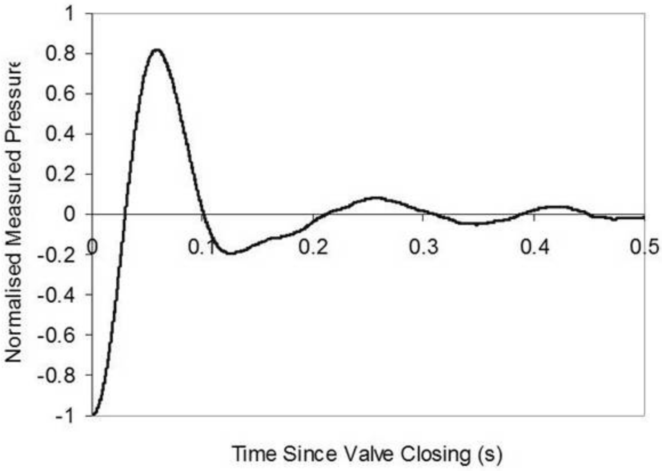
Coefficient of compressibility Values $\kappa$ for Several Common Liquids at $20^{o}C$

| Liquid | $\kappa$ (GPa) |
|---|---|
| Gasoline | 0.958 |
| Mercury | 25.5 |
| Methanol | 0.83 |
| SAE 30W oil | 1.38 |
| Water | 2.19 |
| Seawater (30% salinity) | 2.33 |

- What is the coefficient of compressibility of a truly incompressible substance ($v$ = constant)?

# Compressibility Coefficients

- **Water hammer:** is a pressure surge or wave caused when a fluid (usually a liquid but sometimes also a gas) in motion is forced to stop or change direction suddenly (momentum change) .
- The momentum change causes acoustic waves that strike the pipe surfaces, bends, and valves as they propagate and reflect along the pipe, causing the pipe to vibrate and produce the familiar sound.
- **Water hammering can be quite destructive, leading to leaks or even structural damage. The effect can be suppressed with a *water hammer arrestor*.**



# Compressibility Coefficients

Water hammer arrestors: (a) A large surge tower built to protect the pipeline against water hammer damage. (b) Much smaller arrestors used for supplying water to a household washing machine.

---

# Compressibility Coefficients

For an ideal gas, $P = \rho RT$ and $(\partial P/\partial \rho)_T = RT = P/\rho$, and thus

$$\kappa_{\text{ideal gas}} = P \qquad \text{(Pa)}$$

- ➢ The coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure.

*Ideal gas:*

$$\frac{\Delta\rho}{\rho} = \frac{\Delta P}{P} \qquad (T = \text{constant})$$

- ➢ The percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.

# Compressibility Coefficients

**Isothermal compressibility:** The inverse of the coefficient of compressibility.

The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

$$\alpha = \frac{1}{\kappa} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T \qquad (1/\text{Pa})$$

---

# Coefficient of Volume Expansion

- The density of a fluid depends more strongly on temperature than it does on pressure.
- The variation of density with temperature is responsible for numerous natural phenomena such as winds, currents in oceans, rise of plumes in chimneys, the operation of hot-air balloons, heat transfer by natural convection, and even the rise of hot air and thus the phrase “heat rises”.

Natural convection over a hand.

- To quantify these effects, we need a property that represents the variation of the density of a fluid with temperature at constant pressure.
- Coefficient of volume expansion

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

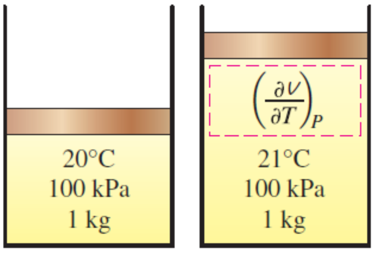
- In terms of finite changes

$$\beta \approx \frac{\Delta v/v}{\Delta T} = -\frac{\Delta \rho/\rho}{\Delta T} \qquad (\text{at constant } P)$$

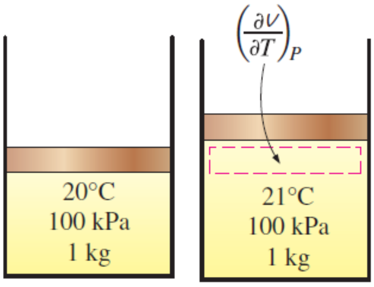
# Coefficient of Volume Expansion

- The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.
- A large value of $\beta$ for a fluid means a large change in density with temperature,
- The product $\beta \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change of $T$ at constant pressure.
- The volume expansion coefficient of an *ideal gas* ($P = \rho RT$) at a temperature $T$ is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K})$$



($a$) A substance with a large $\beta$



($b$) A substance with a small $\beta$

---

# Coefficient of Volume Expansion

- In the study of natural convection currents (*buoyancy force)*, the condition of the main fluid body that surrounds the finite hot or cold regions is indicated by the subscript “infinity” to serve as a reminder that this is the value at a distance where the presence of the hot or cold region is not felt.
- In such cases, the volume expansion coefficient can be expressed approximately as

$$\beta \approx -\frac{(\rho_\infty - \rho)/\rho}{T_\infty - T} \qquad \text{or} \qquad \rho_\infty - \rho = \rho\beta(T - T_\infty)$$

- The larger the temperature difference between the hot or cold fluid pocket and the surrounding main fluid body, the *larger* the buoyancy force and thus the *stronger* the natural convection currents.

The sudden drop in temperature produces condensation of water vapor on a visible vapor cloud when an aircraft flies near the speed of sound.

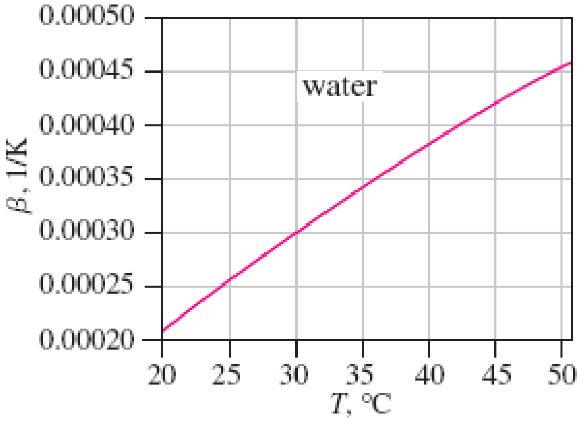
# Coefficient of Volume Expansion

- The combined effects of pressure and temperature changes on the volume change of a fluid can be determined by taking the specific volume to be a function of $T$ and $P$.

$$dv = \left(\frac{\partial v}{\partial T}\right)_P dT + \left(\frac{\partial v}{\partial P}\right)_T dP = (\beta\, dT - \alpha\, dP)v$$

- The fractional change in volume (or density) due to changes in pressure and temperature can be expressed approximately as

$$\frac{\Delta v}{v} = -\frac{\Delta \rho}{\rho} \cong \beta\,\Delta T - \alpha\,\Delta P$$



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# Example

- Consider water initially at 20°C and 1 atm. Determine the final density of water *(a) if it is heated to 50°C at a constant pressure of 1 atm, and (b) if it* is compressed to 100-atm pressure at a constant temperature of 20°C. The isothermal compressibility of water is

$\alpha = 4.80 \times 10^{-5}\ \text{atm}^{-1}$.

$$\alpha = \frac{1}{\kappa}$$

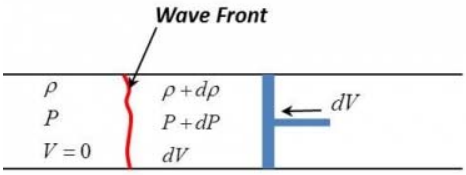
# Example

---

# Speed of Sound and Mach Number

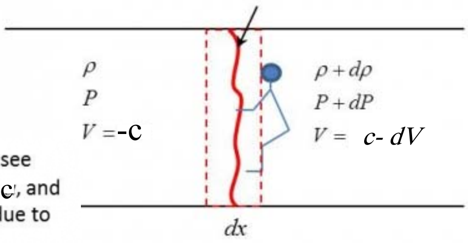
- Another important consequence of the compressibility of fluids is that disturbances introduced at some point in the fluid propagate at a finite velocity.

- Consider a long, constant-area tube filled with fluid and having a piston at one end.
  - The fluid is initially at rest.
  - At a certain instant the piston is given an incremental velocity *dV* to the left.
  - The fluid particles immediately next to the piston are compressed a very small amount as they acquire the velocity of the piston.
  - As the piston and compressed particles continue to move, the next group of fluid particles is compressed and the "wave front" is observed to propagate through the fluid at the characteristic "sonic" velocity of magnitude c.



The wave front propagates at the sonic velocity, c. The fluid to the left of the wave front doesn't know that the piston has moved. The fluid to the right of the wave front is moving at the velocity of the piston. There is an incremental change in pressure and density created by the movement of the piston.

Control volume moving with the wave front



When traveling with the wave front we see the fluid coming at us with a velocity of c, and the velocity behind us is slightly lower due to the forward piston velocity, *dV*.

# Speed of Sound and Mach Number

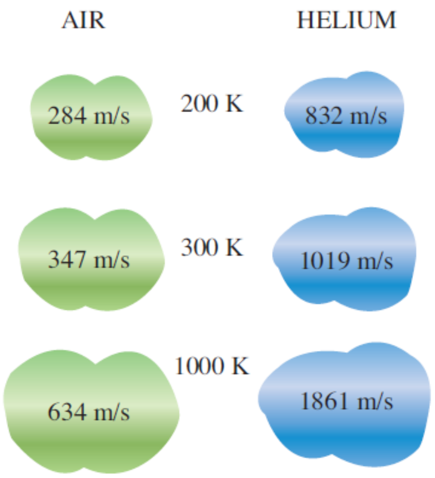
- The velocity at which these small disturbances propagate is called the *acoustic velocity* or the ***speed of sound,*** *c.*

For any fluid

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

For an ideal gas

$$c = \sqrt{kRT}$$



The speed of sound changes with temperature and varies with the fluid.

---

# Speed of Sound and Mach Number

- **Mach number Ma:** The ratio of the actual speed of the fluid (or an object in still fluid) to the speed of sound in the same fluid at the same state.

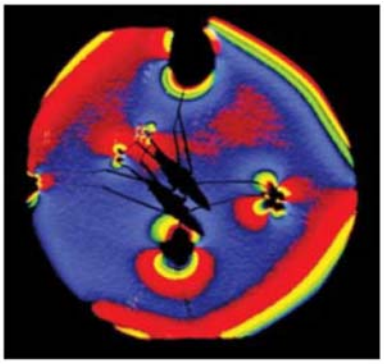
$$\mathrm{Ma} = \frac{V}{c}$$

- The Mach number depends on the speed of sound, which depends on the state of the fluid.

The Mach number can be different at different temperatures even if the flight speed is the same.

# Surface Tension

$$\sigma [=] \frac{J}{m^2} [=] \frac{N.m}{m^2} [=] \frac{N}{m}$$

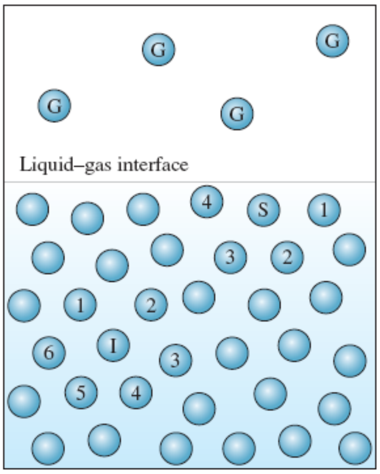


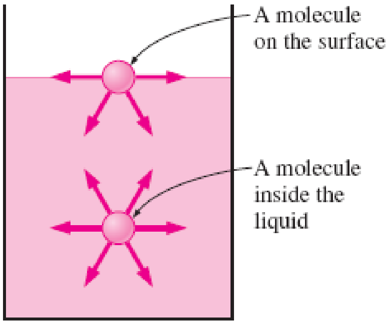
A color schlieren image of the water strider revealing how the water surface dips down where its feet contact the water (it looks like two insects but the second one is just a shadow).

---

# Surface Tension

- Consider the attractive forces between an molecule and its nearest neighbors.
- The interior of the fluid is in balance as molecules of the like fluid are attracting each other, no net force.
- On surface, there is a net inward pulling force.
- Surface molecules of liquid do not have identical molecules above them, they will be more strongly attracted to their neighbors below and in the plane of the interface
- The surface of the liquid will re-arrange until the least number of molecules are present on the surface (i.e. the surface area will be minimized) and attain a spherical shape.
- The surface molecules will pack somewhat closer together than the rest of the molecules in the liquid. The surface molecules will be more ordered.
- Drop of water is like a balloon. The tensile force along the surface gives a drop of water its shape.





# Surface Tension

- It is the net force on a molecule in the interface acts in the plane of the surface in all directions.

$$\therefore \sigma = \frac{F_\sigma}{L} \Rightarrow F_\sigma = \sigma L$$

L: length of cut along the contact line between solid and liquid.

$F_\sigma$: surface tension force

- Surface tension is a property of the liquid type, the temperature, and the other fluid at the interface.

- The surface of the liquid acts like a stretched elastic membrane under tension.
- This membrane can be "broken" with a surfactant which reduces the surface tension.

# Surface Tension

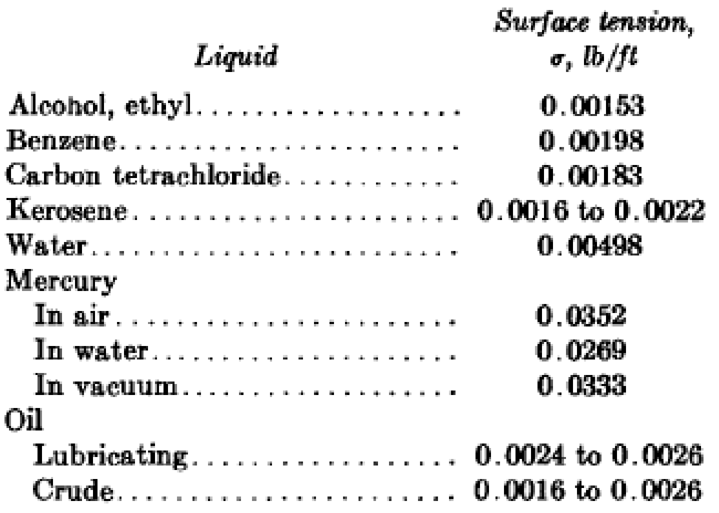
Some consequences of surface tension:

Drops of water beading up on a leaf

A water strider sitting on top of the surface of water,

# Surface Tension

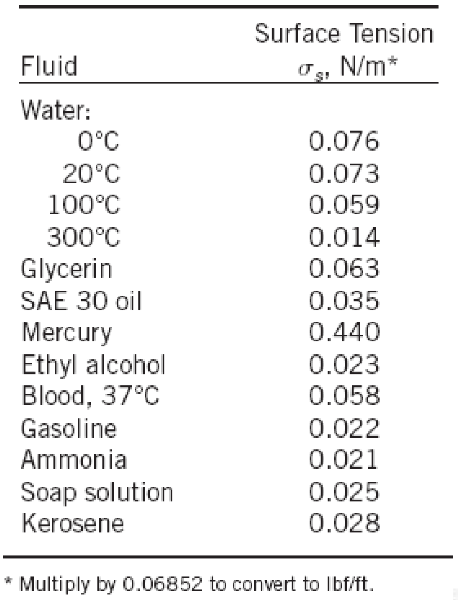
Surface tension of common liquids
in contact with air at 68 °F



| *Liquid* | *Surface tension, σ, lb/ft* |
|---|---|
| Alcohol, ethyl | 0.00153 |
| Benzene | 0.00198 |
| Carbon tetrachloride | 0.00183 |
| Kerosene | 0.0016 to 0.0022 |
| Water | 0.00498 |
| Mercury | |
| In air | 0.0352 |
| In water | 0.0269 |
| In vacuum | 0.0333 |
| Oil | |
| Lubricating | 0.0024 to 0.0026 |
| Crude | 0.0016 to 0.0026 |

- ➢ It is a material property whereby a liquid at a material interface, usually liquid-gas, exerts a force per unit length along the surface.

Surface tension of some fluids
in air at 1 atm and 20°C



| Fluid | Surface Tension $\sigma_s$, N/m* |
|---|---|
| Water: | |
| 0°C | 0.076 |
| 20°C | 0.073 |
| 100°C | 0.059 |
| 300°C | 0.014 |
| Glycerin | 0.063 |
| SAE 30 oil | 0.035 |
| Mercury | 0.440 |
| Ethyl alcohol | 0.023 |
| Blood, 37°C | 0.058 |
| Gasoline | 0.022 |
| Ammonia | 0.021 |
| Soap solution | 0.025 |
| Kerosene | 0.028 |

* Multiply by 0.06852 to convert to lbf/ft.

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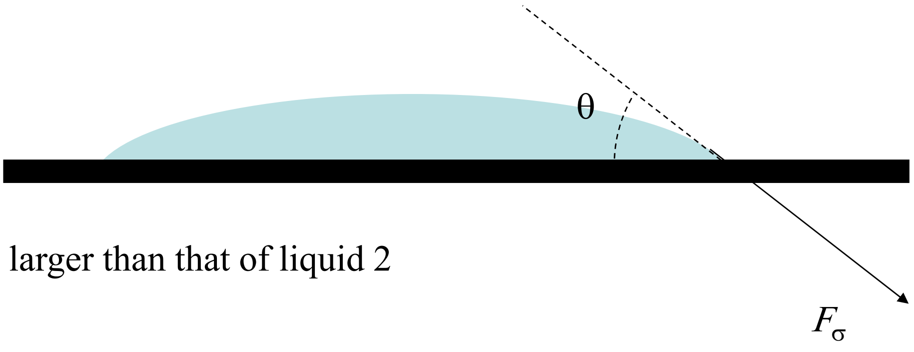
# Surface Tension

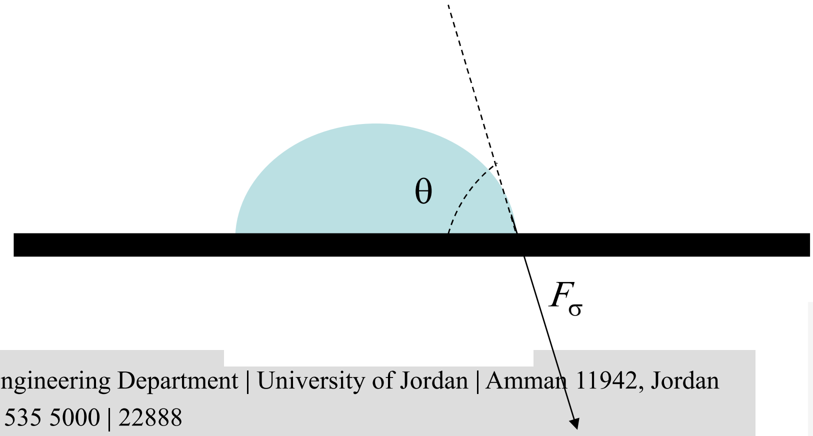
Effect of surface tension on wetting phenomena:

**a) Liquid 1**



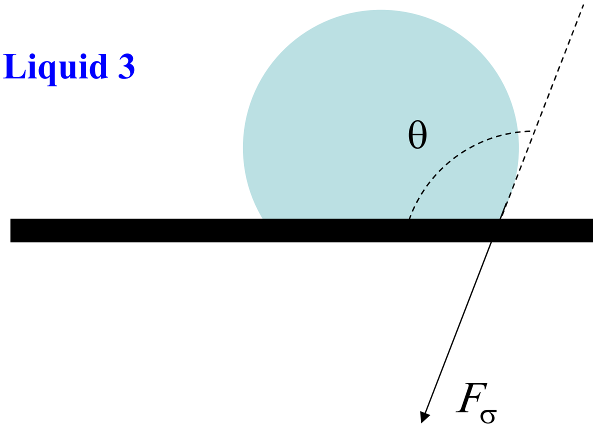
Wetting for liquid 1 is larger than that of liquid 2

**b) Liquid 2**



# Surface Tension

**b) Liquid 3**



Wetting for liquid 3 is smaller that that of liquid 2

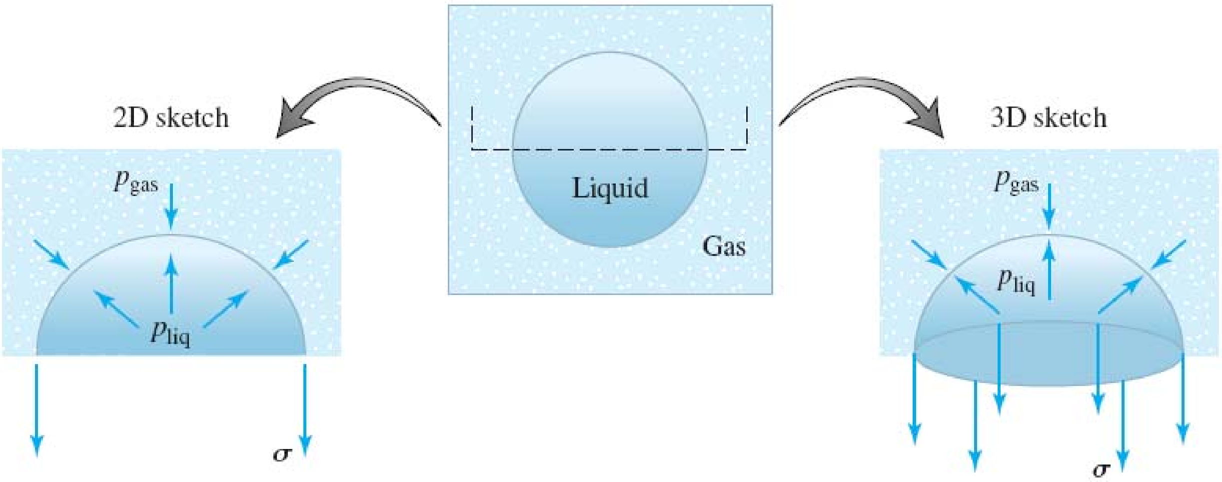
θ: Contact angle between the solid surface and the line tangent to the liquid edge as shown in the above figures; measured from the inside of the liquid.
-It is clear from the figures above that as wetting increases the corresponding contact angle decreases.

- As surface tension increases wetting decreases (θ increases). Ex: $\theta_{water} \approx 0^o$ ; $\theta_{Hg} \approx 140^o$

---

# Surface Tension

- Consider a small spherical drop of one fluid at rest in another
- Since the drop is small, we can neglect the slight hydrostatic pressure variation with height and assume a uniform but different pressure inside and outside the drop
- Cutting the drop in half and performing a force balance in the vertical direction on the hemispherical interface

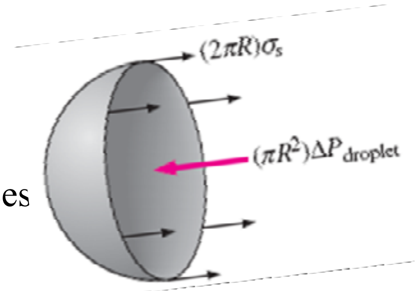


# Surface Tension

- If the drop is at rest,

$$\sum F_{interface} = 0$$

- Neglecting the mass of the droplet (gravity force), the forces acting on the surface are



Half a droplet

i. Net force up due to pressure inside,
ii. Net force down due to pressure outside,
iii. The surface tension force pulling down on the edge of the hemispherical interface.

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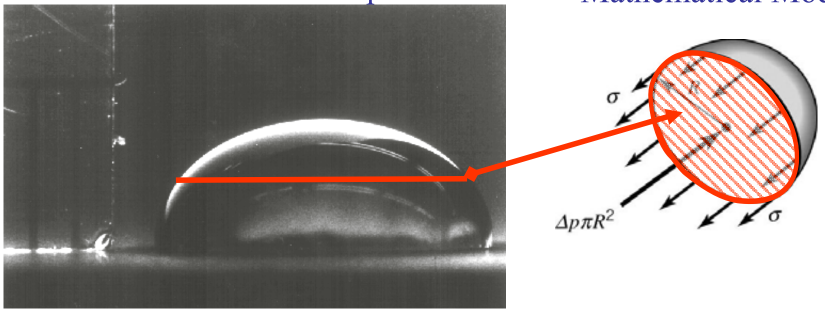
# Surface Tension

- The pressure inside a drop of fluid can be calculated using the free-body diagram



Real Fluid Drops

Mathematical Model

R is the radius of the droplet, σ is the surface tension, Δp is the pressure difference between the inside and outside pressure.

The force developed around the edge due to surface tension along the line:

$$F_{surface} = \boxed{2\pi R}\sigma$$

Applied to Circumference

This force is balanced by the pressure difference Δp:

$$F_{pressure} = \Delta p \boxed{\pi R^2}$$

Applied to Area

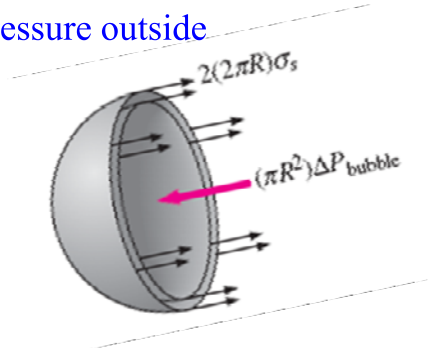
# Surface Tension

$$2\pi R\sigma = \Delta p\ \pi R^2$$

The pressure difference (jump) across the interface yields

$$\Delta p = p_i - p_e = \frac{2\sigma}{R}$$

The pressure inside the drop is larger than the pressure outside



- Is the pressure inside a bubble of water greater or less than that of a droplet of water?
- Show that $\Delta p = \dfrac{4\sigma}{R}$
- The bubble consisting of a film with *two surfaces (inner and outer surfaces)* and thus two circumferences in the cross section

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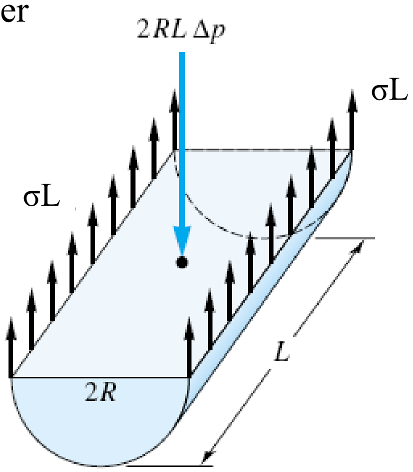
# Surface Tension

- The pressure increase in the interior of a liquid cylinder is balanced by two surface-tension forces

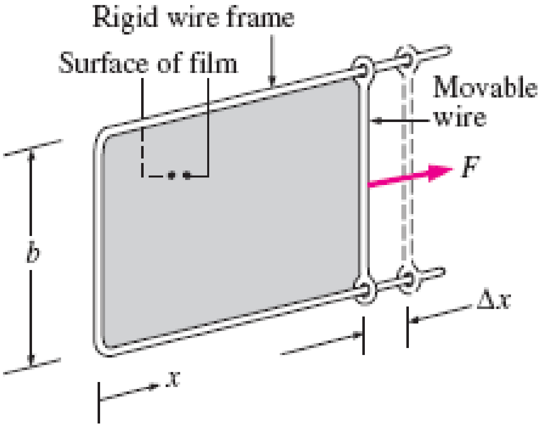


Neglecting the weight of the liquid

$$2RL\ \Delta p = 2\sigma L$$

$$\Delta p = \frac{\sigma}{R}$$

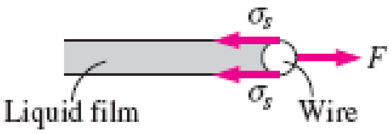
# Surface Tension



- A force *F needs to* be applied on the movable wire in the opposite direction to balance this pulling effect.
- The thin film in the device has two surfaces (the top and bottom surfaces) exposed to air. The length along which the tension acts in this case is 2*b.*

$$F = 2b\sigma_s,$$

- When the movable wire is pulled a distance $\Delta x$



$$W = \text{Force} \times \text{Distance} = F\,\Delta x = 2b\sigma_s\,\Delta x = \sigma_s\,\Delta A$$

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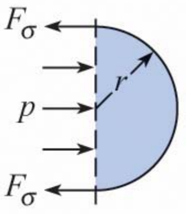
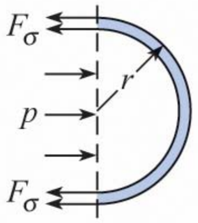
# Surface Tension



$$\Delta p = \frac{2\sigma}{R}$$

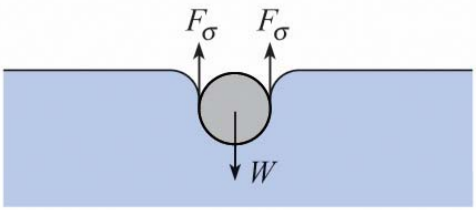
(*a*) Spherical droplet



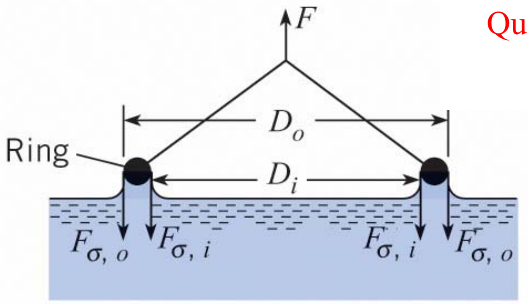
$$\Delta p = \frac{4\sigma}{R}$$

(*b*) Spherical bubble

$$W = 2b\sigma$$



(*c*) Cylinder supported by surface tension (liquid does not wet cylinder)

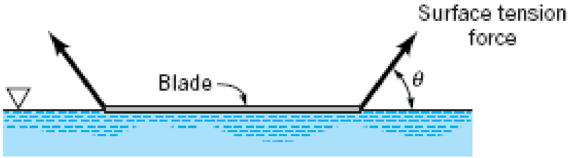


Quiz: Find F

(*d*) Ring pulled out of liquid (liquid wets the ring)

# Example

In the Figure, assume that the surface tension forces act at an angle *relative to the water* surface as shown. **The mass of the double edge** blade is 0.64x$10^{-3}$ kg , and the total length of its sides is 206 mm. Determine the value of *the angle required to maintain* equilibrium between the blade weight and the resultant surface tension force.



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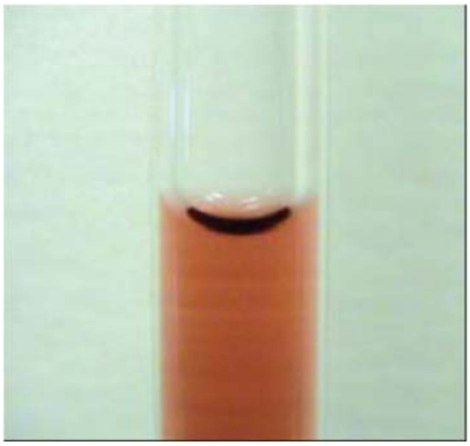
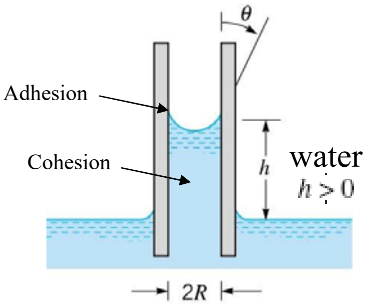


# Capillary Effect

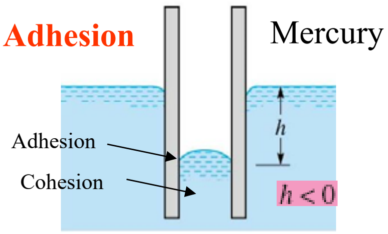
# Capillary Effect

- **Capillary effect** is the rise or fall of a liquid in a small-diameter tube due to surface tension.
- Depends on interaction between solid molecules and fluid molecules
- When the adhesion (force between fluid molecules and wall) is greater than cohesion (force between fluid molecules themselves) the fluid molecules will be pulled up the wall, wetting the surface.

**Adhesion > Cohesion**



**Cohesion > Adhesion**



h is the height, R is the radius of the tube, θ is the angle of contact.

The weight of the fluid is balanced with the vertical force caused by surface tension.

# Capillary Effect

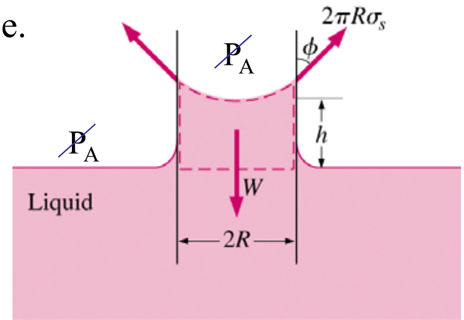
- The curved free surface in the tube is call the **meniscus**.
- Water meniscus curves up because water is a *wetting fluid*.
- Mercury meniscus curves down because mercury is a *nonwetting fluid*.
- Force balance can describe magnitude of capillary rise.

$$\sum F_z = 0 \quad \Longrightarrow \quad F_\sigma \cos\theta - W = 0$$

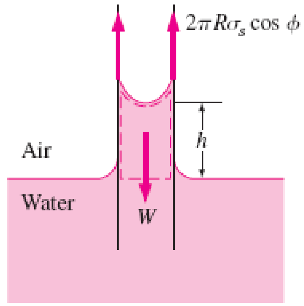
$$W = mg = \rho V g = \rho g(\pi R^2 h)$$

Surface tension upwards is

$$F_{surface} = 2\pi R\sigma\cos\theta$$



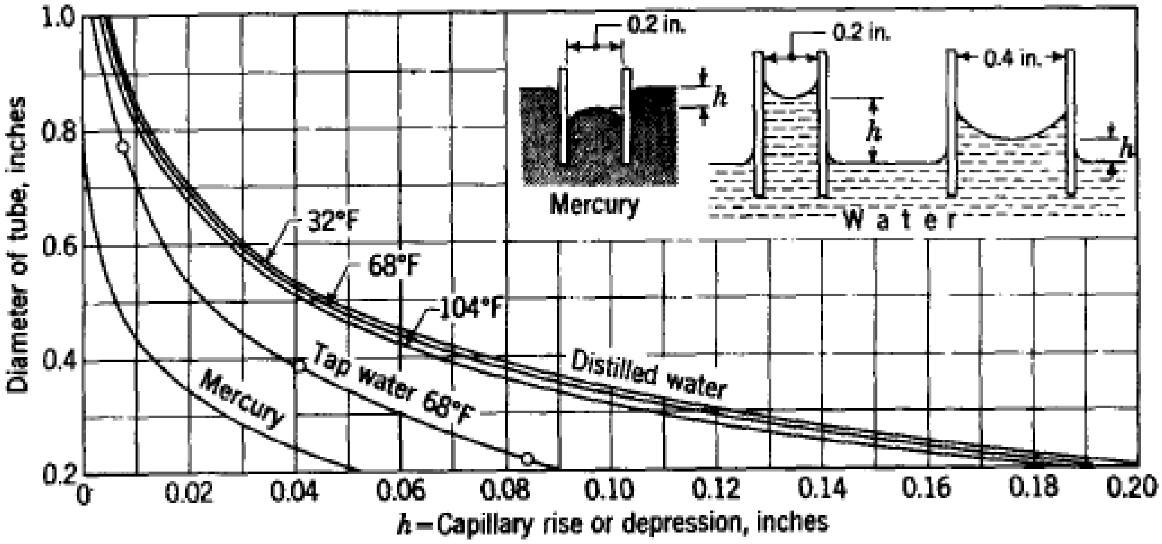
*Φ is the angle that the tangent to the liquid surface makes with the solid surface at the point of contact*



*Capillary rise:* $h = \dfrac{2\sigma_s}{\rho g R}\cos\phi \qquad (R = \text{constant})$

# Capillary Effect

## Capillarity in circular glass tube



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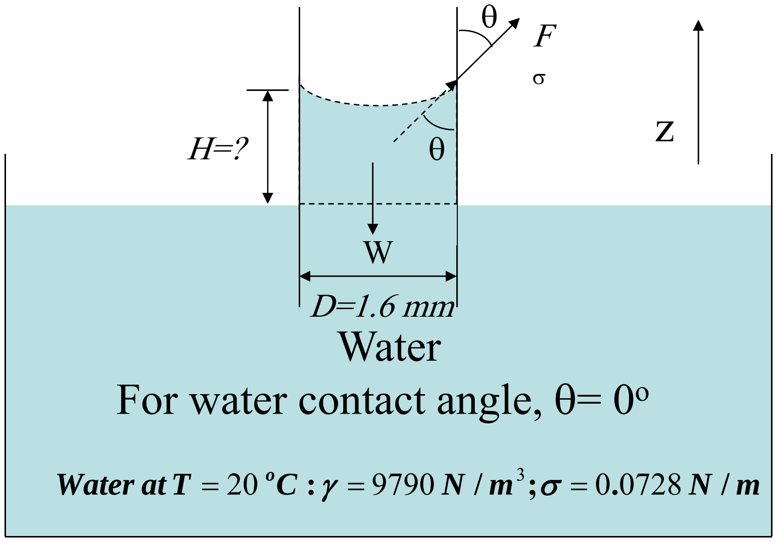
# Example

System: water rises in capillary above the reservoir level

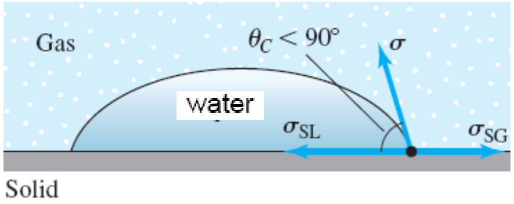


For water contact angle, θ= 0º

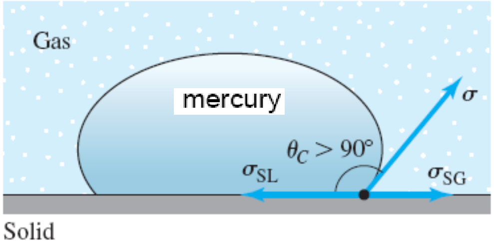
$\boldsymbol{Water\ at\ T} = 20\,^{o}\boldsymbol{C} : \gamma = 9790\ \boldsymbol{N / m^{3}} ; \sigma = 0.0728\ \boldsymbol{N / m}$

-Repeat this example for Mercury?
-How H is affected by surface tension?

# Capillary Effect



Example of a liquid wetting a solid as defined by a contact angle $\theta_c < 90°$



the liquid does not wet the solid, since $\theta_c > 90°$.

- For clean glass in contact with water, $\theta < 90°$, and thus as R decreases, h increases, giving a higher rise.
- For a clean glass in contact with Mercury, $\theta \approx 130°$, and thus h is negative or there is a push down of the fluid.

Surface tension is apparent in many practical problems such as movement of liquid through soil and other porous media, flow of thin films, formation of drops and bubbles, and the breakup of liquid jets.

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# Example

- A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube

# Review: Energy and Specific Heats

- Total energy *E* is comprised of numerous forms: thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, nuclear.
- Units of energy are *joule* (*J*) or *British thermal unit* (BTU).
- Microscopic energy
  - Internal energy *u* is for a non-flowing fluid and is due to molecular activity.
  - Enthalpy $h=u+Pv$ is for a flowing fluid and includes flow energy ($Pv$).
- Macroscopic energy
  - Kinetic energy $k_e=V^2/2$
  - Potential energy $p_e=gz$
- In the absence of electrical, magnetic, chemical, and nuclear energy, the total energy is $e_{flowing}=h+V^2/2+gz$.

$$du = c_v\,dT \qquad \text{and} \qquad dh = c_p\,dT$$

$$c_p - c_V = R,$$